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Adaptive synchronization in networks with heterogeneous uncertain Kuramoto-like units

Ilario A. Azzollini, Simone Baldi, and Elias B. Kosmatopoulos

Abstract— **We analyze adaptive synchronization capabilities in networks with Kuramoto-like units whose dynamical features are unknown and thus synchronization protocols must exhibit co-evolution capabilities. In the presence of heterogeneous and uncertain units, synchronization should be enabled by appropriate adaptive protocols that counteract the effect of heterogeneity. An interaction protocol is presented that is used by the units to communicate with each other: the protocol is based on a distributed disagreement measure. The aim of the protocol is to adapt feedback and coupling gains, so as to guarantee the emergence of a synchronous solution. The adaptive strategy is distributed, i.e. each unit self-determines the strength of its gains by using only neighboring measurements. Convergence of the synchronization error to zero is shown via Lyapunov analysis, and numerical examples demonstrate the effectiveness of the proposed protocol.**

Index Terms— **Adaptive synchronization, Kuramoto-like model, uncertain systems.**

I. Introduction

Synchronization is a collective phenomenon occurring in systems of interacting units, and is ubiquitous in nature, society and technology [1]. Literature has distinguished among two types of synchronization: in the first one, synchronization towards the same evolution which is unknown a priori emerges from the negotiation process taking place on the network (this is sometimes referred to as leaderless synchronization [2]); in the second one, the network is steered in some desired and a priori known solution using a limited set of leader nodes (this is sometimes referred to as leader-follower synchronization or pinning control [3], [4]). In the 80's Kuramoto proposed an exactly solvable model of collective synchronization, which became known as the Kuramoto model [5]. This model has been shown to capture various synchronization phenomena in biological and man-made dynamical systems of coupled oscillators, spanning from flocks of birds and schools of fishes [6], blinks in groups of fireflies [1], the utility power grid [7], to countless other synchronization phenomena [8].

Synchronization research has been first focusing on non-evolving (or non-adaptive) networks of phase oscillators (see [9] and references therein): it was found that synchronization can emerge in the presence of simple static coupling where neighboring nodes adjust their dynamics proportionally to the mismatch between some output function of their states [10], [11], [12], [13]. Most synchronization models have shown that synchronization is favored if the coupling strength is large enough and the spectrum of variety of the oscillators is narrow [1] (almost homogeneous oscillators). In this spirit, the authors in [14] provided a threshold of the couplings that brings from incoherence to synchrony: synchronization occurs when the coupling strength dominates the worst-case dissimilarity over the network. Summarizing, these studies enlighten the crucial role played by the connectivity (interaction topology) and structural properties (parameters and coupling/feedback gains of each unit) in the emergence of synchronized states [15].

However, real-world networks have uncertain and heterogeneous parameters which might even change with time. If uncertainties are large, adaptive-gain approaches are needed to achieve synchronization [16], [17]. In particular, researchers have later been focusing on networks characterized by evolving, adapting couplings which vary in time according to different environmental conditions, leading to the study of evolving (or adaptive) networks [18]. In [19] a simple model of adaptive Kuramoto network is given in which adaptation is taken into account by mechanisms of homophily (reinforcing interactions with correlated units) and homeostasis (preserving the overall connection strength). In [20] a set of adaptive strategies for synchronization and consensus of complex networks of dynamical systems is presented: the main limitation of these approaches is that they address networks composed of identical oscillators. The authors in [21] devise an adaptive scheme to achieve phase synchronization by suppressing the negative effect of the heterogeneity in the network, while in [22] protocols are designed to adaptively interact with system dynamics and preserve the sum of all incoming pairwise coupling strengths. In [23] a co-evolutionary rewiring strategy that depends only on the phase differences of neighboring oscillators is studied for Kuramoto agents. However, synchronization of evolving Kuramoto networks is usually shown numerically but not analytically proven.

In this work we analyze synchronization capabilities in Kuramoto networks whose dynamical features are unknown and thus synchronization protocols must exhibit co-evolution capabilities. With respect to the three ingredients indicated in [24], in this work we consider *(i)* units described by a Kuramoto-like model (like the standard Kuramoto model,

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with an additional inertia term); *(ii)* adaptive interaction protocol based on a disagreement measure with all neighbors and *(iii)* an undirected graph. Convergence of the synchronization error to zero is shown via Lyapunov analysis. Finally, as opposed to state-of-the-art approaches based on a distributed observer [25], [26], the proposed approach does not have to construct observer states, which simplifies the adaptation procedure.

The rest of the paper is organized as follows. In Sect. II we give the problem formulation. The disagreement-based protocol is given in Sect. III. Numerical examples are in Sect. IV, and Sect. V concludes the work.

Notation: The notation in this paper is standard. Matrices are denoted by capital letters, e.g. X, while vectors and scalars by small letters, e.g. x. The transpose of a matrix or of a vector is indicated with X^T and x^T respectively. A vector signal $x \in \mathbb{R}^n$ is said to belong to $\mathscr{L}_2$ class ($x \in \mathscr{L}_2$), if $\int_0^t \left\|x(\tau)\right\|^2 \, d\tau < \infty$, $\forall t \geq 0$. A vector signal $x \in \mathbb{R}^n$ is said to belong to $\mathscr{L}_\infty$ class ($x \in \mathscr{L}_\infty$), if $\max_{t\geq 0} \left\|x(t)\right\| < \infty$, $\forall t \geq 0$. A time-invariant undirected communication graph of order N is completely defined by the pair $\mathscr{G} = (\mathscr{V}, \mathscr{E})$, where $\mathscr{V} = \{1, \ldots, N\}$ is a finite nonempty set of nodes, and $\mathscr{E} \subseteq \mathscr{V} \times \mathscr{V}$ is a set of corresponding non-ordered pair of nodes, called edges. The adjacency matrix of a weighted undirected graph $\mathscr{K} = [k_{ij}]$ is defined as $k_{ii} = 0$ and $k_{ij} = k_{ji} > 0$ if $(i, j) \in \mathscr{E}$, where $i \neq j$. The adjacency matrix $\mathscr{A} = [a_{ij}]$ of an unweighted undirected graph is defined as $a_{ii} = 0$ and $a_{ij} = a_{ji} = 1$ if $(i, j) \in \mathscr{E}$, where $i \neq j$. The Laplacian matrix of the unweighted graph is defined as $\mathscr{L} = [l_{ij}]$, where $l_{ii} = \sum_j a_{ij}$ and $l_{ij} = -a_{ij}$, if $i \neq j$. An undirected graph $\mathscr{G}$ is said to be connected if, taken any arbitrary pair of nodes (i, j) where $i, j \in \mathscr{V}$, there is a path that leads from i to j. In this work we indicate with N the number of nodes (or agents) in the network. The all-ones N-vector is defined as $\mathbf{1}_N = \mathrm{col}(1, 1, \ldots, 1)$. In the same way we define the all-zeros $(N \times n)$-vector $\mathbf{0}_{(N\times n)\times 1} = \mathrm{col}(0, 0, \ldots, 0)$.

II. Problem formulation

The network (1) of heterogeneous coupled oscillators with unknown dynamics is considered in this work. Time index t may be omitted when obvious.

$$m_i\ddot{\theta}_i + d_i\dot{\theta}_i = \tau_i - \sum_{j=1}^{N} k_{ij}\sin(\theta_i - \theta_j), \qquad i \in \mathscr{V} \tag{1}$$

The meaning of the parameters in (1) can be examined via the mechanical analogy of mass points in Figure 1. After

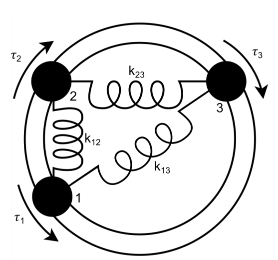

Fig. 1. Mechanical analogy of a network of three coupled oscillators.

neglecting any collision, each point, or agent, will move on the circle describing an angle (or phase, by analogy) θ_i and an angular velocity (or frequency, by analogy) $\dot{\theta}_i$, under the effect of an external driving torque τ_i, an elastic restoring torque $k_{ij}\sin(\theta_i - \theta_j)$ (with $k_{ij} = k_{ji}$), and a viscous damping torque $d_i\dot{\theta}_i$ that is opposite to the direction of motion. All inertial coefficients m_i, damping coefficients d_i and stiffness coefficients k_{ij} have positive but *unknown* value. The external driving torque has two components

$$\tau_i = \omega_i + u_i, \qquad i \in \mathscr{V} \tag{2}$$

where u_i is the actual control torque and ω_i is a term proportional to the natural angular velocity (or frequency) of the agent i, that is the angular velocity it would have if there were no couplings. After defining the state $x_i = [\theta_i, \dot{\theta}_i]^T$, (1) can be rewritten as

$$\dot{x}_i = \underbrace{\begin{bmatrix} 0 & 1 \\ 0 & -\frac{d_i}{m_i} \end{bmatrix}}_{A_i} x_i + \underbrace{\begin{bmatrix} 0 \\ \frac{1}{m_i} \end{bmatrix}}_{b_i} \left(u_i + \omega_i - \sum_{j=1}^{N} k_{ij}\sin(\theta_i - \theta_j) \right). \tag{3}$$

The following connectivity assumption is made.

Assumption 1: The graph $\mathscr{G}$ of the network is undirected and connected.

Problem 1: [Adaptive state synchronization] Consider a network of unknown oscillators (1) satisfying Assumption 1. Find a distributed strategy (i.e. exploiting only state measurements from neighbors) for the control input u_i such that, without any knowledge of the parameters m_i, d_i and k_{ij}, the network state synchronizes to the same behavior, i.e. $x_i - x_j \to 0$, $\forall i, j$.

Two results are now given which are instrumental to solving the problem above.

Proposition 1: [Homogeneization via reference model] For the following reference model

$$\dot{x}_m = \underbrace{\begin{bmatrix} 0 & 1 \\ a^*_{21} & a^*_{22} \end{bmatrix}}_{A_0} x_m + \underbrace{\begin{bmatrix} 0 \\ \frac{1}{m^*} \end{bmatrix}}_{b_0} u \tag{4}$$

with $x_m \in \mathbb{R}^{2\times 1}$, there exist a family of vectors $k^*_i \in \mathbb{R}^{2\times 1}$ and a family of scalars $l^*_i > 0$ such that

$$\begin{cases} A_i + b_i k^{*T}_i = A_0 \\ l^*_i b_i = b_0 \end{cases} \tag{5}$$

Furthermore, there exists an ideal controller

$$u^*_i = k^{*T}_i x_i + l^*_i f^T \sum_{j=1}^{N} a_{ij}(x_i - x_j) + c^*_i + \sum_{j=1}^{N} g^*_{ij} a_{ij} \sin(\theta_i - \theta_j) \tag{6}$$

with $c^*_i = -\omega_i$, $g^*_{ij} = k_{ij}$ and $f \in \mathbb{R}^{2\times 1}$ to be designed, which leads to the following dynamics

$$\dot{x}_i = A_0 x_i + b_0 f^T \sum_{j=1}^{N} a_{ij}(x_i - x_j) \tag{7}$$

Proof: The proof directly follows from applying the control input (6) to agent (3), and using (5).

The following result, allows us to design f to achieve synchronization for the homogeneous dynamics in (7).

Proposition 2: [Homogeneous network synchronization] The homogeneous network (7) synchronizes if

$$A_0+\lambda_i b_0 f^T \text{ is Hurwitz}, \quad \forall i\in\mathscr{V}/\{1\} \tag{8}$$

where λ_i's, $i\in\mathscr{V}/\{1\}$, are the non-zero eigenvalues of the Laplacian, or equivalently if

$$P(A_0+\lambda_i b_0 f^T)+(A_0+\lambda_i b_0 f^T)^T P<\mathbf{0},\ \forall i\in\mathscr{V}/\{1\} \tag{9}$$

where $P\in\mathbb{R}^{2\times 2}$ is a symmetric positive definite matrix.

Proof: Proposition 2 is well known result in synchronization literature [27], and the proof is not given for lack of space. The interested reader is also referred to the companion paper [28] for more details.

Remark 1: Since A_i, b_i, ω_i, k_{ij} are unknown, the ideal control (6) *cannot be implemented to solve Problem 1. Therefore, some adaptation mechanisms must be devised to estimate the unknown ideal gains in Proposition 1.*

The aim of the adaptation mechanism in the following section is to make the heterogeneous network converge to the behavior of the homogeneous network in Proposition 2, estimating the unknown gains by exploiting only measurements from neighbors.

III. Distributed disagreement-based adaptive synchronization

The following synchronizing protocol is proposed

$$\begin{aligned}u_i(t)=&k_i^T(t)x_i+l_i(t)f^T\sum_{j=1}^N a_{ij}(x_i(t)-x_j(t))+c_i(t)+\\&+\sum_{j=1}^N g_{ij}(t)a_{ij}\sin(\theta_i(t)-\theta_j(t))\end{aligned} \tag{10}$$

where k_i, l_i, c_i, g_{ij}, are the (time-dependent) estimates of k_i^*, l_i^*, c_i^*, g_{ij}^*, respectively. The following synchronization result holds.

Theorem 1: Under Assumption 1, the heterogeneous Kuramoto network (3), controlled using the synchronizing protocol (10) and the following adaptive laws

$$\begin{aligned}\dot{k}_i^T&=-\gamma\left(\sum_{j=1}^N a_{ij}(e_i-e_j)\right)^T Pb_0x_i^T\\ \dot{l}_i&=-\gamma\left(\sum_{j=1}^N a_{ij}(e_i-e_j)\right)^T Pb_0f^Te_i\\ \dot{c}_i&=-\gamma\left(\sum_{j=1}^N a_{ij}(e_i-e_j)\right)^T Pb_0\\ \dot{g}_{ij}&=-\gamma\left(\sum_{j=1}^N a_{ij}(e_i-e_j)\right)^T Pb_0\sin(\theta_i-\theta_j)\end{aligned} \tag{11}$$

with adaptive gain $\gamma>0$, and e_i being the local synchronization error

$$e_i=\sum_{j=1}^N a_{ij}(x_i-x_j), \tag{12}$$

reaches synchronization provided that the matrix P and the vector f are chosen such that condition (9) holds.

Proof: The closed-loop network formed by (3) and (10) is given by

$$\begin{aligned}\dot{x}_i=&(A_i+b_ik_i^T)x_i+l_ib_if^T\sum_{j=1}^N a_{ij}(x_i-x_j)+b_ic_i+\\&+b_i\sum_{j=1}^N g_{ij}a_{ij}\sin(\theta_i-\theta_j)\end{aligned} \tag{13}$$

which can be rewritten as a function of the estimation errors,

$$\begin{aligned}\dot{x}_i=&(A_0+b_i\tilde{k}_i^T(t))x_i+(b_0+\tilde{l}_i(t)b_i)f^T\sum_{j=1}^N a_{ij}(x_i-x_j)+\\&+b_i\tilde{c}_i(t)+b_i\sum_{j=1}^N\tilde{g}_{ij}(t)a_{ij}\sin(\theta_i-\theta_j)\end{aligned} \tag{14}$$

where $\tilde{k}_i(t)=k_i(t)-k_i^*$, $\tilde{l}_i(t)=l_i(t)-l_i^*$, $\tilde{c}_i(t)=c_i(t)-c_i^*$, $\tilde{g}_{ij}(t)=g_{ij}(t)-g_{ij}^*$. By defining for compactness

$$\begin{aligned}B_k(t)&=\mathrm{diag}(b_1\tilde{k}_1^T(t),\ldots,b_N\tilde{k}_N^T(t))\\ B_l(t)&=\mathrm{diag}(\tilde{l}_1(t)b_1f^T,\ldots,\tilde{l}_N(t)b_Nf^T)\\ B_c(t)&=\mathrm{diag}(b_1\tilde{c}_1(t),\ldots,b_N\tilde{c}_N(t))\\ B_g(t)&=\mathrm{diag}(b_1\sum_{j=1}^N\tilde{g}_{1j}(t)a_{1j}\sin(\theta_1-\theta_j),\ldots\\&\qquad\ldots,b_N\sum_{j=1}^N\tilde{g}_{Nj}(t)a_{Nj}\sin(\theta_N-\theta_j))\end{aligned} \tag{15}$$

the closed-loop for the overall network can be written as

$$\begin{aligned}\dot{x}=&(I_N\otimes A_0+B_k(t))x+(I_N\otimes b_0f^T+B_l(t))e+\\&+B_c(t)+B_g(t)\end{aligned} \tag{16}$$

where $x=[x_1^T,x_2^T,\ldots,x_N^T]^T$ and $e=[e_1^T,e_2^T,\ldots,e_N^T]^T$. Note that $e_i\in\mathbb{R}^{2\times 1}$ and the error for the overall network can be written as $e=(\mathscr{L}\otimes I_2)x$. Since the graph is undirected and connected, there exists a unitary matrix $\mathscr{U}=[\frac{1}{\sqrt{N}}\mathbf{1}_N\ \mathscr{U}_2]$ with $\mathscr{U}_2\in\mathbb{R}^{N\times(N-1)}$ such that $\mathscr{U}^T\mathscr{L}\mathscr{U}=\mathrm{diag}(0,\lambda_2,\ldots,\lambda_N)\triangleq\Lambda$. This can be used to define $\bar{e}=(\mathscr{U}\otimes I_2)e$. Moreover let $\bar{e}=[\bar{e}_1^T,\bar{e}_2^T,\ldots,\bar{e}_N^T]^T$, it is easily checked that

$$\begin{aligned}\bar{e}_1&=\left(\frac{1}{\sqrt{N}}\mathbf{1}_N\otimes I_2\right)e\\&=\left(\frac{1}{\sqrt{N}}\mathbf{1}_N\otimes I_2\right)(\mathscr{L}\otimes I_2)x=\mathbf{0}_{(N\times 2)\times 1}.\end{aligned}$$

We can now write the overall error dynamics as

$$\begin{aligned}\dot{e}=&[(I_N\otimes A_0)+(\mathscr{L}\otimes b_0f^T)]e+\\&+(\mathscr{L}\otimes I_2)(B_k(t)x+B_l(t)e+B_c(t)+B_g(t)).\end{aligned} \tag{17}$$

The adaptive laws (11) arise from considering the Lyapunov function candidate $V=V_1+V_2+V_3+V_4+V_5$, where

$$
\begin{aligned}
&V_1=e^T(I_N\otimes P)e,\\
&V_2=\sum_{i=1}^{N}\frac{\tilde{k}_i^T(t)\gamma^{-1}\tilde{k}_i(t)}{|l_i^*|},\qquad V_3=\sum_{i=1}^{N}\frac{\tilde{l}_i(t)\gamma^{-1}\tilde{l}_i^T(t)}{|l_i^*|},\\
&V_4=\sum_{i=1}^{N}\frac{\tilde{c}_i(t)\gamma^{-1}\tilde{c}_i^T(t)}{|l_i^*|},\qquad V_5=\sum_{i=1}^{N}\frac{\tilde{g}_{ij}(t)\gamma^{-1}\tilde{g}_{ij}^T(t)}{|l_i^*|}.
\end{aligned}
\tag{18}
$$

Then we have

$$
\begin{aligned}
\dot{V}_1=&[2e^T(I_N\otimes P)]\dot{e}\\
=&2e^T(I_N\otimes P)[(I_N\otimes A_0)+(\mathscr{L}\otimes b_0f^T)]e+\\
&+2e^T(I_N\otimes P)[(\mathscr{L}\otimes I_2)(B_kx+B_le+B_c+B_g)]\\
=&2\bar{e}^T(I_N\otimes PA_0+\Lambda\otimes Pb_0f^T)\bar{e}+\\
&+2e^T(\mathscr{L}\otimes P)(B_kx+B_le+B_c+B_g)\\
=&\sum_{i=2}^{N}\bar{e}_i^T[P(A_0+\lambda_ib_0f^T)+(A_0+\lambda_ib_0f^T)^TP]\bar{e}_i+\\
&+2\sum_{i=1}^{N}\tilde{k}_i^T(t)x_ib_i^TP\left(\sum_{j=1}^{N}a_{ij}(e_i-e_j)\right)+\\
&+2\sum_{i=1}^{N}\tilde{l}_i(t)e_i^Tfb_i^TP\left(\sum_{j=1}^{N}a_{ij}(e_i-e_j)\right)+\\
&+2\sum_{i=1}^{N}\tilde{c}_i(t)b_i^TP\left(\sum_{j=1}^{N}a_{ij}(e_i-e_j)\right)+\\
&+2\sum_{i=1}^{N}\left(\sum_{j=1}^{N}\tilde{g}_{ij}(t)\sin(\theta_i-\theta_j)\right)b_i^TP\left(\sum_{j=1}^{N}a_{ij}(e_i-e_j)\right).
\end{aligned}
\tag{19}
$$

Moreover, by using (11) we have

$$
\dot{V}_2=-2\sum_{i=1}^{N}\frac{\gamma^{-1}}{|l_i^*|}\tilde{k}_i^T(t)x_ib_0^TP\left(\sum_{j=1}^{N}a_{ij}(e_i-e_j)\right)\tag{20}
$$

and similarly for $\dot{V}_3$, $\dot{V}_4$ and $\dot{V}_5$. This leads to

$$
\dot{V}=\sum_{i=2}^{N}\bar{e}_i^T[P(A_0+\lambda_ib_0f^T)+(A_0+\lambda_ib_0f^T)^TP]\bar{e}_i\tag{21}
$$

which is negative semi-definite provided that condition (9) holds. Using standard Lyapunov arguments we can prove boundedness of all closed-loop signals and convergence of e to 0. In fact, since $V>0$ and $\dot{V}\le 0$, it follows that $V(t)$ has a limit, i.e.,

$$
\lim_{t\to\infty}V(e(t),\tilde{\Omega}(t))=V_\infty<\infty\tag{22}
$$

where we have collected all parametric errors in $\tilde{\Omega}$. The finite limit implies V, e, $\tilde{\Omega}\in\mathscr{L}_\infty$. In addition, by integrating $\dot{V}$ it follows that for some $Q>0$

$$
\int_0^\infty e^T(\tau)Qe(\tau)\,\mathrm{d}\tau\le V(e(0),\tilde{\Omega}(0))-V_\infty\tag{23}
$$

from which we establish that $e\in\mathscr{L}_2$. Finally, since $\dot{V}$ is uniformly continuous in time (this is satisfied because $\ddot{V}$ is finite), the Barbalat's lemma implies $\dot{V}\to 0$ as $t\to\infty$ and hence $e\to 0$, from which we derive $x_i\to x_j$, $\forall i,j$. This concludes the proof.

Remark 2: Theorem 1 provides a leaderless synchronization protocol driving the synchronization error (12) *to zero. The final synchronization state to which the network will converge depends on the network initial conditions and cannot be in general imposed a priori. To steer the network in some desired and a priori known state, it is needed to include in the network* (3) *a leader node with dynamics as in* (4) *and without any adaptive law. This can be done in two ways: the most direct approach is to keep using the proposed adaptive laws* (11)*, as shown in Sect. IV. An alternative approach, not elaborated here for lack of space, is to exploit a Lyapunov function depending on the pinning matrix [29].*

Remark 3: In order to implement (11)*, and in particular the term* $\sum_{j=1}^{N}a_{ij}(e_i-e_j)$*, it is required to communicate among neighbors the extra variable* e_i*, which is also local information: note that this is also equivalent to communicating* x_i *to the neighbors of the neighbors (2-hop communication). Communication of extra local variables is often at the core of many synchronization protocols: for example, synchronization based on distributed observer [30], [31] requires communication of extra local variables representing the observer states. Inspired by this idea, let us consider the same synchronizing protocol* (10)*, but this time with the following adaptive version of the distributed observer*

$$
\begin{aligned}
\dot{\chi}_i&=A_0\chi_i+\mu\left(b_0f^T\sum_{j=1}^{N}a_{ij}(\chi_i-\chi_j)\right)\\
\dot{k}_i^T&=-\gamma(x_i-\chi_i)^TPb_0x_i^T\\
\dot{l}_i&=-\gamma(x_i-\chi_i)^TPb_0f^T\sum_{j=1}^{N}a_{ij}(x_i-x_j)\\
\dot{c}_i&=-\gamma(x_i-\chi_i)^TPb_0\\
\dot{g}_{ij}&=-\gamma(x_i-\chi_i)^TPb_0\sin(\theta_i-\theta_j)
\end{aligned}
\tag{24}
$$

with adaptive gain $\gamma>0$ *and distributed observer gain* $\mu>0$*. The following intuition lies behind* (24)*: a virtual homogeneous network in the form* (7) *can be constructed in a distributed way, having the same graph as the heterogeneous network. This is the first equation in* (24)*. Since Proposition 2 guarantees synchronization of the virtual homogenous network, the adaptation laws in* (24) *can now force each agent in the heterogeneous Kuramoto network to behave as its corresponding agent in the homogeneous network* ($x_i-\chi_i\to 0$)*, therefore also achieving synchronization (the proof is not given for lack of space). Actually,* (24) *resembles, with minor modifications, the synchronization protocol adopted in literature for the so-called Euler-Lagrange agents [25], [26]. Now, comparing* (24) *with* (11)*, we see that the proposed disagreement-based protocol is essentially simpler, because it does not require to construct in a distributed manner the observer variables* χ_i*.*

Remark 4: In some applications it is of interest to synchronize the only frequency, while the phase may not synchronize. One possibility to achieve this via (11) *is to* *introduce a phase error in the form $\theta_i - \theta_j = h(\omega_i - \omega_j)$, with $h > 0$ a design parameter: this resembles the idea of velocity-dependent time headway in platooning [32].*

IV. Numerical examples

Simulations using protocol (11) are carried out in the following, considering the weighted graph shown in Figure 2. The parameters and initial conditions for each heterogeneous Kuramoto agent (3) are reported in Table I. Please recall that the agent parameters are unknown to the designer, i.e. the values of Table I are used for simulations but not for control design.

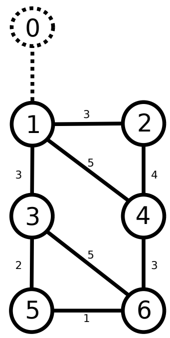

Fig. 2. The undirected weighted communication graph.

TABLE I

Parameters and initial conditions for the Kuramoto agents

	m_i	d_i	ω_i	$\theta_i(0)$	$\dot\theta_i(0)$
agent #1	1.1	0.1	5	0	0.6
agent #2	1.3	0.15	10	π	0.5
agent #3	1.2	0.2	15	$\pi/2$	0.4
agent #4	1.8	0.21	20	$(5/4)\pi$	0.3
agent #5	1.5	0.25	25	$\pi/4$	0.2
agent #6	1	0.3	30	$(3/2)\pi$	0.1

The reference model is chosen as

$$\dot x_m = \underbrace{\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}}_{A_0} x_m + \underbrace{\begin{bmatrix} 0 \\ 0.8 \end{bmatrix}}_{b_0} u\,, \qquad x_m = \begin{bmatrix} \theta_m \\ \dot\theta_m \end{bmatrix} \tag{25}$$

which also represents agent 0 in Figure 2, with chosen initial conditions $x_m(0) = [0, 1]$. The vector f and the matrix P are taken as

$$P = \begin{bmatrix} 1.5824 & 0.5824 \\ 0.5824 & 1.2607 \end{bmatrix}, \qquad f^T = \begin{bmatrix} -1 & -1 \end{bmatrix}, \tag{26}$$

which satisfy condition (9). Finally, the adaptive gain is taken $\gamma = 1$, and all estimated control gains k_i, l_i, c_i, g_{ij}, are initialized to 0.

The adaptive synchronization resulting from (11) is shown in Figure 3. Synchronization is achieved and, due to heterogeneity, note that each agent has different control inputs u_i that reach different steady-state values.

Finally, Figure 4 shows the adaptive control gains of (11) for all the systems. Overall, the protocol (11) shows synchronization capabilities in the presence of both uncertainty and heterogeneity, and without the need to construct a distributed observer.

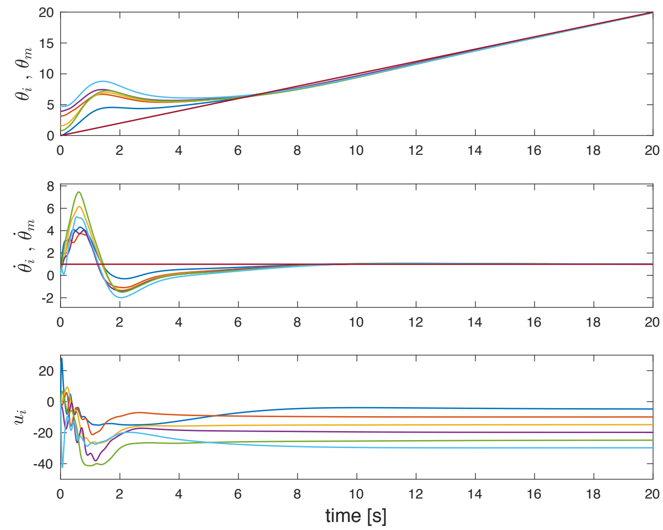

Fig. 3. Protocol (11): synchronization of the states of each agent i to the leader reference state $[\theta_m, \dot\theta_m]$. The control inputs u_i are shown at the bottom.

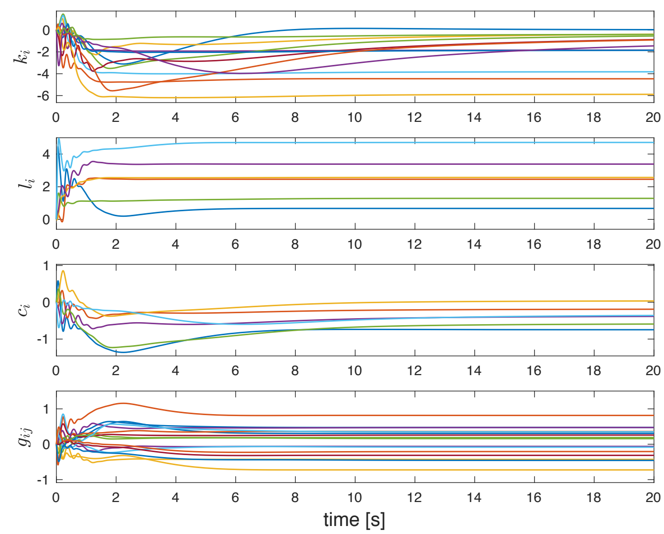

Fig. 4. Protocol (11): adaptive gains.

V. Conclusions

We analyzed the synchronization capabilities in heterogeneous networks with Kuramoto-like agents whose dynamics are unknown and thus synchronization protocols must exhibit co-evolution capabilities. An adaptive synchronization protocol was presented, based on a distributed disagreement measure. Convergence of the synchronization error to zero was shown via Lyapunov analysis, and numerical examples demonstrated the effectiveness of the proposed protocols. We have also shown that the proposed protocol simplifies some observer-based adaptive protocol (e.g. like the one used in Euler-Lagrange agents), since it does not reuire to construct any observer state.

Future work could include saturation constraints so as to model different features of the units: for example, in Kuramoto models of microgrids [33], [34], sources and consumers can only deliver or drain power, respectively. Beyond the microgrid example, future work should be devoted to the following points. The proposed adaptive protocol achieve synchronization by 'cancelling-out' nonlinearities in a sort of adaptive feedback linearization scheme. However, it has been shown that feedback linearization does not lead in general to optimal control inputs [35]: it would be of interest to develop a new adaptive protocol that, while still achieving synchronization, exploits the nonlinearities instead of cancelling them. Finally, a relevant extension could be to consider switching topology: this can be achieved using adaptive switched tools, as in [36].

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