Forward Kinematic Analysis of Tip-Tilt-Piston Parallel Manipulator using Secant-Bootstrap Method

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***Abstract*— This paper, deals with application of the Secant-Bootstrap Method (SBM) to solve the Closed-form forward kinematics of a new three degree-of-freedom (DOF) parallel manipulator with inextensible limbs and base-mounted actuators. The manipulator has higher resolution and precision than the existing three DOF mechanisms with extensible limbs. This methodology has been utilized to achieve approximate solutions for nonlinear equations of kinematic of Tip-Tilt-Piston (T.T.P) Parallel Manipulator. The SBM is a novel methodology which moderate disadvantage of the traditional numerical techniques. The excellent agreement of the Secant-Homotopy Continuation solutions with the traditional numerical methods such as the Newton–Raphson method could be established without aberration. SBM for the forward kinematic of T.T.P Parallel Manipulator leads to 16 solutions that are eight pairs of reflected configurations with respect to the base plane.**

***Index Term*— Forward Kinematic, Tip-Tilt-Piston, Parallel Manipulator, Secant-Bootstrap Method.**

I. INTRODUCTION

Forward kinematics of a parallel manipulator is much more difficult than its inverse kinematics; whereas, for a serial manipulator, the opposite is true. Parallel mechanisms are most suitable for applications in which the requirements for precision, rigidity, load-to-weight ratio, and load distribution are more important than the need for a large workspace [1].The Stewart-Gough platform [2] is probably the first six degree-of-freedom (DOF) parallel mechanism which has been studied in the literature. Several researchers have analyzed the forward kinematic of parallel mechanism [3-8] and it also has growing applications to robotics.

The forward kinematics of serial manipulators is straightforward while their inverse kinematics is quite complicated requiring the solution of a system of nonlinear equations. In contrast, the inverse kinematics of parallel manipulators is relatively straightforward and the forward kinematics is challenging [9].

In the process of solving the kinematics problem of a robot, some troublesome simultaneous equations will be generated, especially simultaneous non-linear equations [10]. To date, we already have many different methods that can deal with simultaneous non-linear equations, such as the Newton–Raphson method [11-12] homotopy continuation method [13-15], secant-bootstrap method [16-17] as advanced model of homotopy continuation method.

Homotopy continuation method was used by kinematicians in the 1960s for solving mechanism synthesis problems. The latest development had been made by Morgan [18-19], Garcia [20] and Allgower [21]. Wu [11-14] presented some techniques by combining Newton's and homotopy methods to avoid divergence in solving nonlinear equations. The homotopy continuation method was known as early as in the 1930s. This method was used by kinematicians in the 1960s was used by kinematicians in the 1960s for solving mechanism synthesis problems. Also, Wu [14] applied the homotopy continuation method to search all the roots of the inverse kinematics problem of a robot and obtained more, but not all, convergence answers than the Newton–Raphson method. Recently this method applied for kinematics problem of robot manipulators [21]. In Ref.[16] by Wu, the traditional Newton–Raphson method has been modified by the secant theory, and the homotopy continuation technique has been applied to a new secant-bootstrap formula.

In this paper secant-bootstrap method has been considered to solve the nonlinear equations in forward kinematic analysis of Tip-Tilt-Piston (T.T.P) Parallel Manipulator.

For this sake, the paper has been organized as follows:

In Section 2, we describe secant-homotopy continuation method; we will study the properties of the Kinematics model of Tip-Tilt-Piston Parallel Manipulator in section 3.Also we applied a numerical example in Section 4. Eventually, the last section contains one of the most significant findings of the paper.

II. DESCRIPTION OF SECANT-BOOTSTRAP METHOD

WhenWhen dealing with any numerical problem, e.g., the Newton–Raphson method, there are two troublesome questions. One is that good initial guesses are not easy to detect and another is related to whether the method we use will converge to useful solutions [16-17]. The secant-bootstrap method can eliminate these shortcomings. As we know, there are two kinds of nonlinear equations: they are nonlinear equations and simultaneous nonlinear equations, shown as

Eqs. (1) and (2), respectively:

$$f(x)=0 \tag{1}$$

And

$$\mathbf{F}(\mathbf{X})=0 \quad i.e., \begin{cases} f(x,y,...,z)=0 \\ g((x,y,...,z)=0 \\ \vdots \\ h((x,y,...,z)=0 \end{cases} \tag{2}$$

The Newton–Raphson iteration forms of these equations are

$$x_{n+1}=x_n-\frac{f(x_n)}{f'(x_n)} \tag{3}$$

And

$$\begin{bmatrix} \frac{\partial f(x_n,y_n,...)}{\partial x} & \frac{\partial f(x_n,y_n,...)}{\partial y} & \cdots \\ \frac{\partial g(x_n,y_n,...)}{\partial x} & \frac{\partial g(x_n,y_n,...)}{\partial y} & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} x_{n+1}-x_n \\ y_{n+1}-y_n \\ \vdots \end{bmatrix} = \begin{bmatrix} -f(x_n,y_n,...) \\ -g(x_n,y_n,...) \\ \vdots \end{bmatrix} \tag{4}$$

If the derivative $f'(x)$ or $\frac{\partial f}{\partial x}$ is difficult or not available, these iteration forms will be not used. Then we can apply the secant theory

$$f'(x_n)=\lim_{x\to x_n}\frac{f(x)-f(x_n)}{x-x_n}\approx\frac{f(x_{n-1})-f(x_n)}{x_{n-1}-x_n} \tag{5}$$

And

$$\frac{\partial f(x_n,y_n,...)}{\partial x}=\lim_{x\to x_n}\frac{f(x,y_n,...)-f(x_n,y_n,...)}{x-x_n} \approx\frac{f(x_{n-1},y_n,...)-f(x_n,y_n,...)}{x_{n-1}-x_n} \tag{6}$$

Substituting Eqs.(5) and (6) into Eqs.(3) and (4) to yield

$$x_{n+1}=x_n-\frac{f(x_n)(x_{n-1}-x_n)}{f(x_{n-1})-f(x_n)} \tag{7}$$

And

$$\begin{bmatrix} \frac{f(x_{n-1},y_n,...)-f(x_n,y_n,...)}{x_{n-1}-x_n} & \frac{f(x_n,y_{n-1},...)-f(x_n,y_n,...)}{y_{n-1}-y_n} & \cdots \\ \frac{g(x_{n-1},y_n,...)-g(x_n,y_n,...)}{x_{n-1}-x_n} & \frac{g(x_n,y_{n-1},...)-g(x_n,y_n,...)}{y_{n-1}-y_n} & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} x_{n+1}-x_n \\ y_{n+1}-y_n \\ \vdots \end{bmatrix} = \begin{bmatrix} -f(x_n,y_n,...) \\ -g(x_n,y_n,...) \\ \vdots \end{bmatrix} \tag{8}$$

The above two equations will become infinite when $f(x_{n-1})=f(x_n)$ or is an even function or the determinate of coefficient in Eq. (8) becomes or approaches zero. So, we have to apply the homotopy continuation technique to improve this defect.

Given a set of equations in variables $x_1,x_2,...,x_n$. We modify the equations by omitting some of the terms and adding new ones until we have a new system of equations, the solutions to which may be easily known. We then deform the coefficients of the new system into the coefficients of the original system by a series of small increments to obtain the solutions. This is called the homotopy continuation technique.

If we wish to find the solutions of Eqs. (1) and (2), we can choose a new simple start system called an auxiliary homotopy function

$$\mathbf{G}(\mathbf{X})=0 \tag{9}$$

The auxiliary homotopy function **G(X)** must be known or controllable and easy to solve. Then, we define the homotopy continuation function as

$$\mathbf{H}(\mathbf{X},t)=t\mathbf{F}(\mathbf{X})+(1-t)\mathbf{G}(\mathbf{X})=0 \quad (4) \tag{10}$$

Where t is an arbitrary parameter and varies from 0 to 1, i.e., $t\in[0,1]$. Therefore, we have the following two boundary conditions:

$$\mathbf{H}(\mathbf{X},0)=\mathbf{G}(\mathbf{X}) \qquad \mathbf{H}(\mathbf{X},1)=\mathbf{F}(\mathbf{X}) \quad (5) \tag{11}$$

This is the famous Bootstrap method.

Our goal in this study is to solve the **H**(**X**, t) = 0 instead of **F**(**X**) = 0 by varying the parameter t from 0 to 1 and thus avoid divergence. Hence, we rewrite Eqs. (7) and (8) as

$$x_{n+1}=x_n-\frac{H(x_n,t)(x_{n-1}-x_n)}{H(x_{n-1},t)-H(x_n,t)} \tag{12}$$

And

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$$\begin{bmatrix} \frac{H_1(x_{n-1},y_n,...)-H_1(x_n,y_n,...)}{x_{n-1}-x_n} & \frac{H_1(x_n,y_{n-1},...)-H_1(x_n,y_n,...)}{y_{n-1}-y_n} & \cdots \\ \frac{H_2(x_{n-1},y_n,...)-H_2(x_n,y_n,...)}{x_{n-1}-x_n} & \frac{H_2(x_n,y_{n-1},...)-H_2(x_n,y_n,...)}{y_{n-1}-y_n} & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} x_{n+1}-x_n \\ y_{n+1}-y_n \\ \vdots \end{bmatrix} = \begin{bmatrix} -H_1(x_n,y_n,...) \\ -H_2(x_n,y_n,...) \\ \vdots \end{bmatrix} \quad (13)$$

Where the divergence occurs at

$$H(x_{n-1},t)-H(x_n,t)=0 \quad or \to 0 \quad (14)$$

And

$$\mathrm{Det}\begin{bmatrix} \frac{H_1(x_{n-1},y_n,...)-H_1(x_n,y_n,...)}{x_{n-1}-x_n} & \frac{H_1(x_n,y_{n-1},...)-H_1(x_n,y_n,...)}{y_{n-1}-y_n} & \cdots \\ \frac{H_2(x_{n-1},y_n,...)-H_2(x_n,y_n,...)}{x_{n-1}-x_n} & \frac{H_2(x_n,y_{n-1},...)-H_2(x_n,y_n,...)}{y_{n-1}-y_n} & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix} = 0 \quad or \to 0 \quad (15)$$

III. Kinematics Model of Tip-Tilt-Piston Parallel Manipulator

Solving the forward kinematics of the manipulator involves finding the location (position and orientation) of the moving platform, given the l_1, l_2 and l_3 lengths. The mechanism described here is a three DOF parallel alignment manipulator with three inextensible limbs and base-mounted actuators. Fig. 1 shows the details of the manipulator. The picture of a manipulator prototype is shown in Fig. 2.

The three inextensible limbs R_1P_1, R_2P_2 ,and R_3P_3 are connected to the output moving platform through spherical joints P_1, P_2 and P_3 .he lower ends of the limbs are connected to links R_1T_1, R_2T_2 , and R_3T_3 through revolute joints at R_1, R_2 , and R_3 .Slider Links R_1T_1, R_2T_2 , and R_3T_3 are connected to the fixed base through base-mounted prismatic actuators N_1T_1 , N_2T_2 , and N_3T_3 , respectively.

The manipulator has three degrees of freedom. Tip, tilt, and piston motions of the moving platform (output link) can be obtained by using the prismatic actuators to vary the O_1R_1 , O_2R_2 and O_3R_3 lengths. Note that the prismatic actuators can be inside or outside of the $R_1R_2R_3$ triangle formed by the lower ends of the limbs.

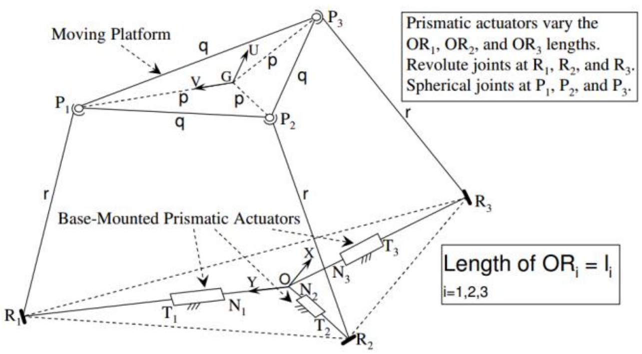

Fig. 1. The new manipulator with base-mounted actuators and inextensible limbs [1].

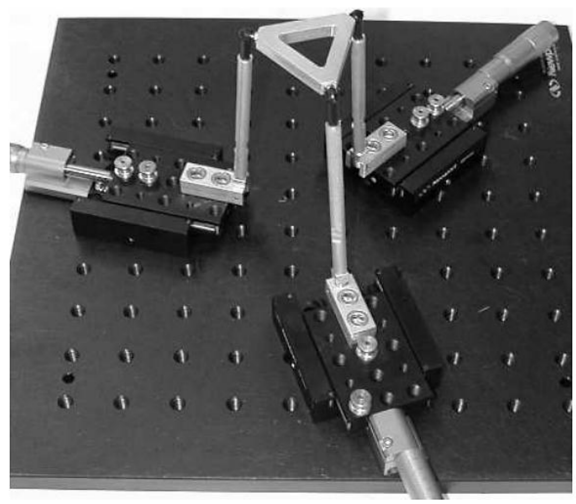

Fig. 2. The new manipulator prototype [1].

The manipulator has symmetrical structure as the angle between the lines ON_i and $ON_{i+1} (i=1,2,3)$ is equal to 120 degrees. As shown in Fig. 1, η_i be the angle from vector $\overline{OR_i}$ to vector $\overline{R_iP_i}$. Also α_i be the angle from the positive X-axis to vector $\overline{OR_i}$. Angle α_i can be found (in radians) from

$$\alpha_i = \pi/2+(i-1)2\pi/3 \quad (16)$$

The X and Y coordinates of point R_i in the fixed reference frame XYZ can be found from the following relationships

$$X_{R,i} = l_i \cos(\alpha_i) \quad (17)$$

$$Y_{R,i} = l_i \sin(\alpha_i) \quad (18)$$

The coordinates of point P_i in the fixed reference frame XYZ are

$$X_{P,i} = r\cos(\alpha_i)\cos(\eta_i)+X_{R,i} \quad (19)$$

$$Y_{P,i} = r\sin(\alpha_i)\cos(\eta_i)+Y_{R,i} \quad (20)$$

$$Z_{P,i} = r\sin(\eta_i) \quad (21)$$

Referring to Fig. 1, we can write

$$(X_{P,i} - X_{P,i+1})^2 + (Y_{P,i} - Y_{P,i+1})^2 + (Z_{P,i} - Z_{P,i+1})^2 = q^2 \quad (22)$$

Substituting from Eqs. (16)-(18) into Eqs. (19), (20); and substituting the resulting expressions for $X_{P,i}$ and $X_{P,i}$ as well as Eq. (21) into Eq. (22) and simplifying, we obtain

$$A_i \sin(\eta_i)\sin(\eta_{i+1}) + B_i \cos(\eta_i)\cos(\eta_{i+1}) + D_i \cos(\eta_i) + E_i \cos(\eta_{i+1}) + F_i = 0 \quad (23)$$

Where

$$\begin{aligned} A_i &= -2r^2 \\ B_i &= r^2 \\ D_i &= 2rl_i + rl_{i+1} \\ E_i &= rl_i + 2rl_{i+1} \\ F_i &= 2r^2 + l_i^2 + l_{i+1}^2 + l_i l_{i+1} - q^2 \end{aligned} \quad (24)$$

The Eq. (22) has been obtained as follows

$$-2r^2 \sin(\eta_1)\sin(\eta_2) + r^2 \cos(\eta_1)\cos(\eta_2) + (2rl_1 + rl_2)\cos(\eta_1) + (rl_1 + 2rl_2)\cos(\eta_2) + 2r^2 + l_1^2 + l_2^2 + l_1 l_2 - q^2 = 0 \quad (25)$$

$$-2r^2 \sin(\eta_2)\sin(\eta_3) + r^2 \cos(\eta_2)\cos(\eta_3) + (2rl_2 + rl_3)\cos(\eta_2) + (rl_2 + 2rl_3)\cos(\eta_3) + 2r^2 + l_2^2 + l_3^2 + l_2 l_3 - q^2 = 0 \quad (26)$$

$$-2r^2 \sin(\eta_3)\sin(\eta_1) + r^2 \cos(\eta_3)\cos(\eta_1) + (2rl_3 + rl_1)\cos(\eta_3) + (rl_3 + 2rl_1)\cos(\eta_1) + 2r^2 + l_3^2 + l_1^2 + l_3 l_1 - q^2 = 0 \quad (27)$$

Now each of Eqs. (25)-(27) is a nonlinear equation in three variables $[\eta_1, \eta_2, \eta_3]$.

IV. NUMERICAL SAMPLE

We can solve this system of nonlinear equations by the secant-bootstrap method. To obtain the result of these equations, the geometric parameters of the manipulator are [1]:

$$l_1 = 1.49 \quad l_2 = 0.18 \quad l_3 = 1.66 \quad r = 1 \quad q = 1.5 \quad p = 0.866 \quad (28)$$

Substituting the values of the geometric parameters from Eq. (28) into Eqs.(25)–(27), upon some simplification, yields:

$$-2\sin(\eta_1)\times\sin(\eta_2) + \cos(\eta_1)\times\cos(\eta_2) + 3.16\times\cos(\eta_1) + 1.85\times\cos(\eta_2) + 2.2707 = 0 \quad (29)$$

$$-2\sin(\eta_2)\times\sin(\eta_3) + \cos(\eta_2)\times\cos(\eta_3) + 2.02\times\cos(\eta_2) + 3.50\times\cos(\eta_3) + 2.8368 = 0 \quad (30)$$

$$-2\sin(\eta_3)\times\sin(\eta_1) + \cos(\eta_3)\times\cos(\eta_1) + 4.18\times\cos(\eta_3) + 4.64\times\cos(\eta_1) + 7.1991 = 0 \quad (31)$$

Thus, we can write the bootstrap function as follows

$$[-2\sin(\eta_1)\times\sin(\eta_2) + \cos(\eta_1)\times\cos(\eta_2) + 3.16\times\cos(\eta_1) + 1.85\times\cos(\eta_2) + 2.2707]\times t + (1-t)\times G_1 = 0 \quad (32)$$

$$[-2\sin(\eta_2)\times\sin(\eta_3) + \cos(\eta_2)\times\cos(\eta_3) + 2.02\times\cos(\eta_2) + 3.50\times\cos(\eta_3) + 2.8368]\times t + (1-t)\times G_2 = 0 \quad (33)$$

$$[-2\sin(\eta_3)\times\sin(\eta_1) + \cos(\eta_3)\times\cos(\eta_1) + 4.18\times\cos(\eta_3) + 4.64\times\cos(\eta_1) + 7.1991]\times t + (1-t)\times G_3 = 0 \quad (34)$$

We solve Eqs. (30)-(34) by the Newton–Raphson method and change the secant-homotopy parameter t from 0 to 1 (dt=0.0001) and choose the initial guesses of unknown parameters as $(\eta_{1,0}, \eta_{2,0}, \eta_{3,0}) = (1,1,1)$.

The result of these equations with change the auxiliary homotopy functions $(G_i, i = 1,2,3)$ are in 16 solutions in the complex domain that out of the 16 solution only 8 ones are real are given in Table I.

TABLE I
THE AUXILIARY HOMOTOPY FUNCTIONS AND 8 REAL SOLUTIONS

Solution	(G_1, G_2, G_3)	***Results*** $(\eta_1, \eta_2, \eta_3)^\circ$
1	(η_1, η_2, η_3)	***(-128.8783,-46.4725,-143.4239)***
2	$(-\eta_1, -\eta_2, -\eta_3)$	***(128.8783,-46.4725,-143.4239)***
3	$(-\eta_1, \eta_2, \eta_3)$	***(169.4349,-50.2868,138.7920)***
4	$(\eta_1, -\eta_2, -\eta_3)$	***(-169.4349, 50.2868,138.7920)***
5	$(\eta_1, -\eta_2, \eta_3)$	***(-133.2253,144.6233,-137.3381)***
6	$(-\eta_1, \eta_2, -\eta_3)$	***(133.2253,-144.6233, 137.3381)***
7	$(\eta_1, \eta_2, -\eta_3)$	***(-129.8439,-45.4474,182.2319)***
8	$(-\eta_1, -\eta_2, \eta_3)$	***(129.8439,45.4475,-182.2319)***

The eight real solutions yield the values shown in Table I for angles η_1, η_2 and η_3 (in degrees) and the coordinates of points P_1, P_2, P_3 and G. As mentioned earlier, triangle $P_1P_2P_3$ is equilateral. Therefore, the *XYZ* coordinates of point G in Tables II, III are calculated using the following relationships:

$$\begin{aligned} X_G &= (X_{P,1} + X_{P,2} + X_{P,3})/3 \\ Y_G &= (Y_{P,1} + Y_{P,2} + Y_{P,3})/3 \\ Z_G &= (Z_{P,1} + Z_{P,2} + Z_{P,3})/3 \end{aligned} \quad (35)$$

The comparison of these results and the results reported by Tahmasebi [1] are in excellent agreement.

TABLE II
FIRST FOUR REAL SOLUTIONS OF THE FORWARD KINEMATICS SAMPLE PROBLEM

Solution	***1***	1[1]	***2***	2[1]	***3***	3[1]	***4***	4[1]
η_1	***129.844***	129.177	***-129.844***	-129.177	***133.237***	133.433	***-133.237***	-133.433
η_2	***45.447***	46.699	***-45.447***	-46.699	***-144.630***	-144.448	***144.630***	144.448
η_3	***142.232***	143.342	***-142.232***	-143.342	***137.34***	137.425	***-137.3484***	-137.425
$x_{P,1}$	***0.000***	0.000	***0.000***	0.000	***0.000***	0.000	***0.000***	0.000
$y_{P,1}$	***0.8493***	0.8628	***0.8493***	0.8628	***0.8051***	0.8070	***0.8051***	0.8070
$z_{P,1}$	***0.7678***	0.7752	***-0.7678***	-0.7752	***0.7286***	0.7262	***-0.7286***	-0.7262
$x_{P,2}$	***-0.7635***	-0.7521	***-0.7635***	-0.7521	***0.5503***	0.5465	***0.5503***	0.5465
$y_{P,2}$	***-0.4408***	-0.4342	***-0.4408***	-0.4342	***0.3177***	0.3155	***0.3177***	0.3155
$z_{P,2}$	***0.7126***	0.7278	***-0.7126***	-0.7278	***-0.5789***	-0.5814	***0.5789***	0.5814
$x_{P,3}$	***0.5722***	0.7422	***0.5722***	0.7422	***0.8007***	0.7992	***0.8007***	0.7992
$y_{P,3}$	***-0.3304***	-0.4285	***-0.3304***	-0.4285	***-0.4623***	-0.4614	***-0.4623***	-0.4614
$z_{P,3}$	***-0.0389***	0.5970	***0.0389***	0.5970	***0.6776***	0.6765	***-0.6776***	-0.6765
x_G	***-0.0638***	-0.0033	***-0.0638***	-0.0033	***0.4503***	0.4486	***0.4503***	0.4486
y_G	***0.0261***	0.0000	***0.0261***	0.0000	***0.2202***	0.2203	***0.2202***	0.2203
z_G	***0.4805***	0.7000	***-0.4805***	-0.7000	***0.2758***	0.2738	***-0.2758***	-0.2738

TABLE III
LAST FOUR REAL SOLUTIONS OF THE FORWARD KINEMATICS SAMPLE PROBLEM

Solution	***5***	5[1]	***6***	6[1]	***7***	7[1]	***8***	8[1]
η_1	***128.885***	130.038	***-128.885***	-130.038	***169.435***	169.757	***-169.435***	-169.757
η_2	***46.473***	45.7923	***-46.473***	-45.7923	***-50.286***	-50.508	***50.286***	-50.508
η_3	***176.827***	177.769	***-176.827***	-177.769	***-138.792***	-138.706	***138.792***	138.706
$x_{P,1}$	***0.000***	0.000	***0.000***	0.000	***0.000***	0.000	***0.000***	0.000
$y_{P,1}$	***0.8623***	0.8512	***0.8623***	0.8512	***0.5072***	0.5104	***0.5072***	0.5104
$z_{P,1}$	***-0.7784***	0.7656	***-0.7784***	-0.7656	***0.1833***	0.1778	***-0.1833***	-0.1778
$x_{P,2}$	***-0.7523***	-0.7620	***-0.7523***	-0.7620	***-0.7092***	-0.7089	***-0.7092***	-0.7089
$y_{P,2}$	***-0.4343***	-0.4399	***-0.4343***	-0.4399	***-0.4094***	-0.4093	***-0.4094***	-0.4093
$z_{P,2}$	***-0.7250***	0.7168	***0.7250***	-0.7168	***-0.7693***	-0.7717	***0.7693***	0.7717
$x_{P,3}$	***0.7422***	0.5716	***0.7422***	0.5716	***0.7861***	0.7863	***0.7861***	0.7863
$y_{P,3}$	***-0.4285***	-0.3300	***-0.4285***	-0.3300	***-0.4538***	-0.4539	***-0.4538***	-0.4539
$z_{P,3}$	***0.5959***	0.0389	***-0.5959***	-0.0389	***-0.6588***	-0.6599	***0.6588***	0.6599
x_G	***-0.0034***	-0.0635	***-0.0034***	-0.0635	***0.0256***	0.0258	***0.0256***	0.0258
y_G	***-0.0002***	0.0271	***-0.0002***	0.0271	***-0.1187***	-0.1176	***-0.1187***	-0.1176
z_G	***0.6998***	0.5071	***-0.6998***	-0.5071	***-0.4150***	-0.4179	***0.4150***	0.4179

V. CONCLUSION

The Secant-Bootstrap Method has been utilized for the forward kinematic problem of a novel Tip-Tilt-Piston (T.T.P) Parallel Manipulator in this paper. The forward kinematic

analysis shown that the 16 solutions are eight pairs of reflected configurations with respect to the plane passing through the lower ends of the manipulator's three limbs .It has been proved that convergence speed this methodology is fast and also the SHCM algorithm is very simple. It has been demonstrated that the Secant-Bootstrap Method could lead to all roots of the system of nonlinear equations and obtained the result more efficiently by this new method.

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