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Service Systems in Ports and Inland Waterways

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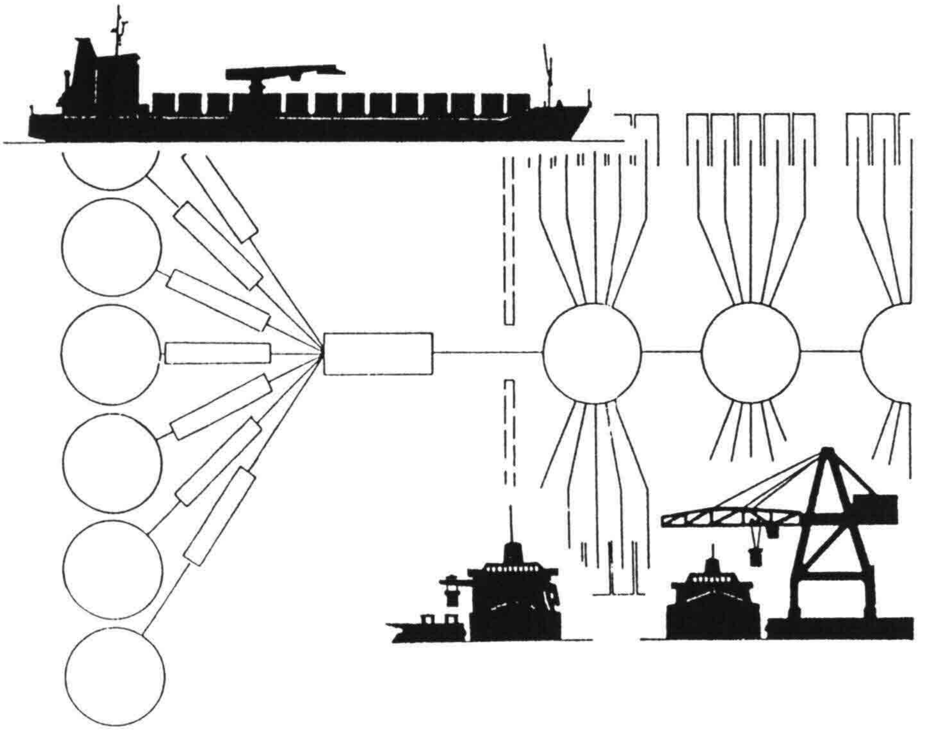
Ir. R. Groenveld



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Queuing notations

P(j)	Steady state probability of having j clients in the system
λ	Arrival rate
μ	Service rate
ρ	ρ/λ
u	Utilization
W	Average waiting time of customers spent in queue
W(t)	p.d.f. waiting times
Nw	Average number of customers in the queue
Na	Average number of customers in the system
ν_a	Degree of variability of arrival intervals - (c.v.)2
ν_s	Degree of variability of service times - (c.v.)2
c.v.	Coefficient of variation of a distribution - standard deviation/mean
s.d.	Standard deviation
T	Average turnaround time (waiting time + service time)

SERVICE SYSTEMS IN PORTS AND INLAND WATERWAYS.

1. INTRODUCTION

Every planning of a port development or design of a new harbour is confrontated with in itself unique physical properties, boundary conditions and problems to be solved. What ports have in common is that they all constitute a link in the transport chain and an interface between transport modes. As a result every port comprises a number of systems:

a. The wet infra structure
   - approach channel(s)
   - manoevring areas
   - mooring basins
b. A system of aids to navigate, to enable the ship to make a safe landfall
   - system of towage
   - pilot system
c. The dry infra structure
   - terminals with cargo handling and storage facilities
   - through transport systems

In port studies, generally two main subjects can be identified, viz. improvement of the existing situation, and design of a required future situation.
For the purpose of optimizing port facilities in relation to capacities demands port operations have to be analyzed, a process which is often facilitated by applying complex port simulation-models. However sometimes a relatively simple-empirical approach or queuing theory can be used. Concerning the choice of the method, always due attention should be paid to the local situation

2. PORT STUDIES

In order to analyze and evaluate the complex system of transport modes which come together at a port, a clear specification of the objectives and criteria is a required starting point for any port study which involves planning and design of future developments.

A port study is normally carried out according the following procedure:
Generation of alternatives:
- analysis of the present and or anticipated cargo flows
- determinations of the transport modes leading to a traffic forecast (as shipping).
- principle dimensions resulting in a number of alternatives

Refinement:
- screening of alternatives
- preliminary engineering and conceptual plans of the selected alternatives
- selection of the most promising alternative.

Finalization
-detailed design and recommendations including the time dependent upgrading steps.

An important criterion for evaluation of alternatives is "minimum total costs". The minimum of total costs is mainly determined by two main components:

1. capital and maintenance costs of port infrastructure and facilities and
2. vessel time in the port and associated with costs,

As these components are interdependent, they have to be duly and jointly investigated for varying cargo and traffic volumes.

The study hinges on the comparison of the various alternatives and the optimization of the most promising alternatives.

3. ASPECTS IN PORT DESIGN

3.1. Introduction

The ultimate symptom of port operational problems is congestion. The cause of congestion is not always easy to discern, but the symptoms can hardly be missed. The symptom referred to, is one of the following signs of congestion, or a combination thereof:

1. every regular storage space is full and a considerable amount of goods is on the quays
2. a long queue of ships is waiting at the anchorage for a berth
3. there are queues of trucks or other means of inland transport.
4. surcharges are levied on cargo carried to or from the port.

These symptoms may aggravate each other. If for example, the quay is for the most part occupied with cargo, the remaining working spaces are inadequate and cargo throughput capacity will be more limited. This may result in fewer ships being served, causing longer queues. The causes may be either physical limitations in ship handling and cargo handling or organisational limitations.

The most important causes are summarized below.

3.2 Organisation

3.2.1 Port management

A thorough study of the local organisation and related procedures is essential when attempting to optimize the throughput of an existing port or when a master plan is being made for future extensions or a new port. By simply improving the procedures a considerable gain in the operation of the port as a whole, can often be obtained. This improved situation may defer the implementation of future physical extensions.
A very common problem is divided management. In some ports the port management isn't under a single authority and divided between central and local governmental institutions.
A special problem for very small ports is that many tasks (port administration, port operations, harbour master and port engineering) may be in the hands of one man, resulting in an overloaded day - to - day program. Consequently little time is available for

planning of future developments or even for short term improvements of operation efficiency.

3.2.2. Cargo handling, stevedoring

Delays in cargo delivery, cumbersome customs procedures and payment methods for duties and port charges greatly contribute to the inefficiency in cargo handling. One reason, which is often overlooked, is the overflow of storage facilities because the storage tariff is too low and the consignee is tempted to use the port facilities as long-term rather than short-term storage. In addition, care should be taken to ensure that stevedoring and the related administration is efficient. Sometimes, the location of cargo is not recorded and a search for goods is necessary when they are due for collection.

3.2.3. Planning, information, communications

With the limited infrastructure and equipment, typical of many ports in developing countries, smooth operation is only possible with a careful planning and execution. This requires quick acquisition of information and efficient communications with foremen, charge hands, etc.

Berth allocation by the harbour master should obviously be coordinated with cargo handling plans. When releasing entrance permit, the harbour master should consult the cargo handling officer to ensure that equipment and manpower are available for the cargo handling and, in case of imports, that enough space is available in transit sheds or other temporary storage.

The full capacity of the infrastructure can only be achieved if entrance criteria are established with due regard to environmental conditions related to safe navigation in the port and its approaches. Consequently, the harbour master should be kept informed about these conditions. The poorer the quality of this information, the greater the safety margins which have to be applied, the longer times are necessary because of the safety margin needed!

3.2.4. Training

The skill of personnel involved in port operations strongly affects the functioning of a port. The extent of training to the level of skill required, and the throughput and/or efficiency of a port (or dimensions of a new port) are very much interrelated. It is stressed that appropriate permanent training facilities should be available locally: a single training course in a developed country may be fruitful for a short period but the experience obtained will fade away quickly. Training courses, adapted to local customs and procedures have to be given to all relevant personnel and at regular intervals. For instance courses for pilots, tugboat crews, harbour masters and terminal operators should preferably and predominantly be given in the home port.

3.3. Ship handling

3.3.1. Berths

A very important item in port operations is the ready availability of adequate berth capacity, when it is required. Too few berths will give rise to queues for ships and delay in cargo delivery. Berths which are too small, limit the maximum ship size, which in turn limits the throughput capacity.

Berths in unprotected or relatively exposed locations give special problems. Under unfavourable wind or swell conditions ships have to leave their berths to prevent damage from impacts with harbour structures and other ships or breaking of mooring lines. In countries which have a storm season, these, what are referred to as 'survival conditions', may be a factor hampering port operations in that season (i.e. conditions for which the port infra structure has to be designed to survive, but during which no cargo handling operations are possible). Breakwaters can help considerably against swell and currents. Berth orientation also plays an important role. For instance, a berth perpendicular to the prevailing storm directions will have a larger downtime because of an earlier attainment of the survival conditions, than a parallel berth.

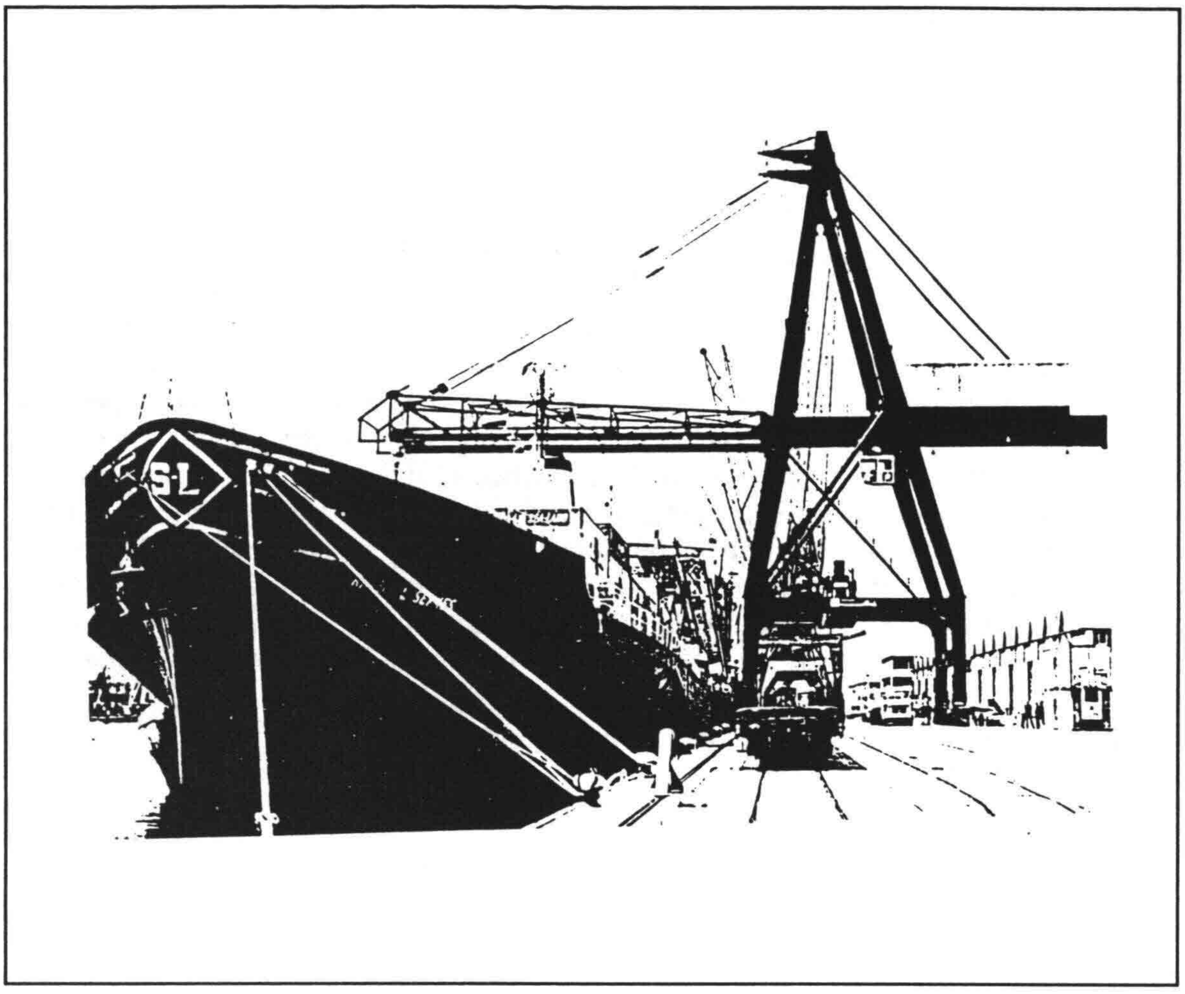
3.3.2. Entrance

The requirement that a ship can enter and leave the port safely is as equally important to port efficiency as the availability of berths. Although this is rather obvious, the consequences are not always fully recognized. The nautical operational limits can effect the port efficiency drastically. These limits are dependent on ship type and class, environmental conditions and the port layout and dimensions.
The conditions for which entry is considered safe or unsafe are referred to as the port entrance regime. If conditions are unsafe for a certain ship when she wants to enter, this will mean that she has to wait for more favourable conditions, which results in a loss of valuable working time at the quays. Entrance safety is mostly based on channel depth (chance of touching the bottom) and width (chance of losing control and touching other ships, obstacles or channel banks which is of course related to the visibility).
These two subjects are discussed below.

a. Depth
The chance that a ship touches the channel bottom is dependent on bottom level, tidal water level, wind set-up or draw-down, ship draught, squat and ship motions due to waves and swell. Entrance criteria are not always given explicitly but when they are, they are basically in the form of a required water depth dependent on ship type and draught and prevailing wave conditions. In situations with a considerable vertical tide, this will result in a 'tidal window'. In other cases, for example in monsoon areas, the consequences might be that, during periods of several months, the design ship can only enter partly loaded, which decreases the throughput capacity.

b. Width.
As safety of manoeuvring is a question of ship manoeuvring characteristics,current and wave conditions but also of human control (including human skills and human failures),

criteria are not easy to establish. In most cases, the pilot will judge according to his own experience. Bottlenecks with respect to manoeuvrability can be narrow bends with possible cross currents or bank effects, but in most cases entering and slowing down are the decisive parts of the ship journey. These are discussed in more detail below.



Picture 1: Besides organizational problems due attention should be paid to ship handling including nautical procedures and berth capacity

c. Steerage, stopping and turning

Wind, cross-currents and waves can reduce the controllability of entering ships, especially since the ships are decelerating, a factor which alone already reduces the steering effectivity. The ship needs to maintain a minimum entrance speed, dependent on the conditions, in order to maintain steerage way. On the other hand, the ship's entrance speed has to be limited, because there is only a limited length available for slowing down and stopping. Whenever stopping length conflicts with minimum entrance speed, the pilot or harbourmaster must decide not to enter.
Other entry limitations are related to tug operations, which is required for assistance in stopping (for big ships) and turning the ship. As a general rule, tugs can only start making fast when the ship speed has decreased below 5 to 6 knots and effective control

by tugs can only start below 4 knots.
This factor may also limit the ship's entrance speed. Moreover, the possibility of tugs to ty up is limited to wave heights below 1.5 m. For ports with a protected inner channel of sufficient length, this condition will not be exceeded but if tugs have to make fast outside the protected area entry can be limited by the wave conditions.
For an outlined overview is referred to the Chapter "Nautical Aspects of Port Planning" of the Lecture notes F12.

3.3.3. Aids to Navigation

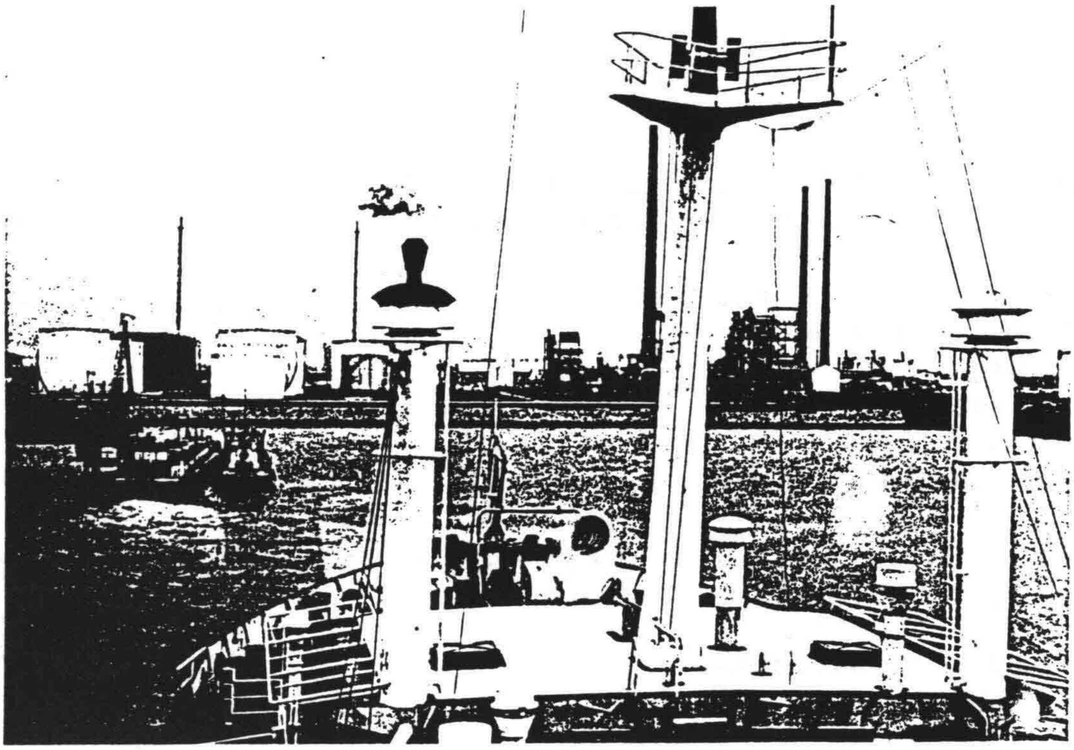
A principle limitation to nautical operations occurs when sailing is restricted to the hours of day-light. This factor is also related to the working conditions in the port itself and if night shifts are operated in the port then entrance and departure manoeuvres should also be carried out around the clock. Even when this is not the case entering and departing during the night result in more efficient use of the infrastructure. In this respect the navigational aids in the approaches and manoeuvring areas are of the greatest importance and should be kept in optimal working condition. When navigating in the (most restricted) access channels or harbour basins regular contact with a shore-based traffic control is generally indispensable but this is only possible if a V.T.S. is available. Unfortunately only a few port may offer this facility. Visibility can also form an important nautical restriction. The extent to which restricted visibility affects safety and thus the decision to enter or leave depends largely on the available navigation aids and possibilities for shore-based traffic control.
The availability of proper equipment and skilled pilotage are important items in restricted visibility navigation.

3.4. Cargo Handling

3.4.1 Loading and unloading

A rather general problem for ports in developing countries is the inadequacy of loading and unloading equipment. This problem is characteristic because the large-scale and fast transport systems originate in developed countries. In these countries, ship types and loading and unloading equipment have grown together. Developing countries, however, find themselves confronted with newer and larger sophisticated ship types, while having only limited conventional facilities which have not had the chance to grow with the ships. As a consequence, in many ports, ships have to unload with their own equipment or with unsuitable port equipment.

The time used for effective loading and unloading is not only dependent on the availability of proper equipment, number of shifts and such but also on the workability conditions. The workability may be determined by, for instance, swell in the harbour basin, wind and rain. It should be noted, that the location and orientation of the moored ships directly affects the workability.



Picture 2: The third main item deals with cargo handling where often problems arise due to the introduction of new ship types

3.4.2 Land transport

In a well designed transport chain, the land transport system is adapted to the sea transport. Different kinds of freight require different transport modes as well as different transport solutions per transport mode.

Not only loading and unloading equipment should be adapted to the commodities and transport modes involved, in order to guarantee smooth and efficient cargo handling. For instance the handling, but also and inland transportation, via road and rail networks and inland waterways, should be adequate for the seaport throughput.

4. SOLUTION OF PROBLEMS

4.1. Introduction

As indicated in the previous chapter congestion may have different causes. It will be clear that in many cases it is very difficult to identify the factors behind the congestion because functioning of the port is affected by so many interdependent parameters.

The complexity of a port system can easily be appreciated by reviewing the factors which determine the length of a ship's stay in a port, for example:

- environmental conditions (tide, wind, wave , currents)
- number of suitable berths
- transhipment system
- storage capacity
- arrival pattern of ships
- service time (efficiency of loading and unloading).

In general, there are three ways of determining some of the answers to the questions related to optimizing port capacity. These are:

- **empirical "rules of thumb"**
- **queuing theory**
- **linear programming techniques**
- **simulation models.**

Generally the choice of the method of solution is based on the following procedure:

- A description of the functioning of the port (real life system) has to be made. To describe this real life system the boundaries of the port system have to be chosen such that a change in the port does not affect the boundary conditions.
- The description of the real life system is then schematized into a verbal model in order to obtain quantitative estimates of the port system.
- Then depending on the complexity of the verbal model the choice can be made between empirical 'rules of thumb' the queuing theory, and simulation models.

This procedure is presented schematically in fig.1.

4.2. Empirical - 'Rules of Thumb'

For more or less isolated problems in small ports with a low traffic intensity it is possible to obtain a good insight into the prevailing conditions without the use of any mathematical techniques whatsoever. Most small ports have, in fact, been designed this way. However, when in case of increasing traffic intensity interactions begin to play a more important role, even with a simple port system it is necessary to use the queuing theory to estimate the basic throughputs involved.

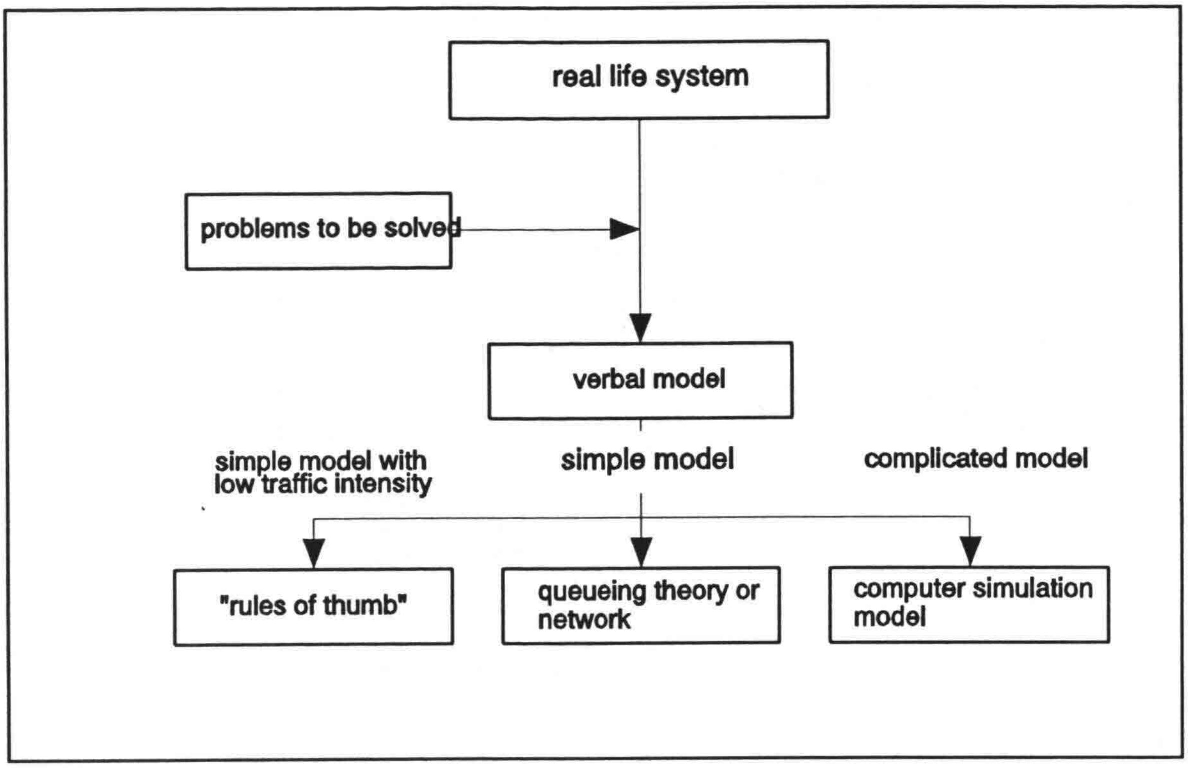


Figure 1: Schematical presentation of choice of method of solution.

4.3. Queuing theory

In the past considerable use has been made of queuing theory. With this theory the port system has to be schematized such that it consists only of a queue (anchorage) and a discrete number of berths. In addition the inter arrival time distribution and service time distribution are expressed mathematically. Assuming that no tidal or meteo windows apply the arrivals, per unit time, are usually found to fit into a Poisson distribution while the servicing operation generally fits a K-Erlang distribution. Such
a queue-delay system can be represented as:

Based upon this delay system queuing theory gives as output:

- The average number of vessels in the arrival queue (anchorage)
- The average number of vessels present in the system
- The chance of delay (all berthing points are occupied)
- The mean waiting time in the arrival queue before being served
- The mean quay utilization
- The mean turn around time (waiting time and servicing time).

The queuing theory will be described in detail in section 5.

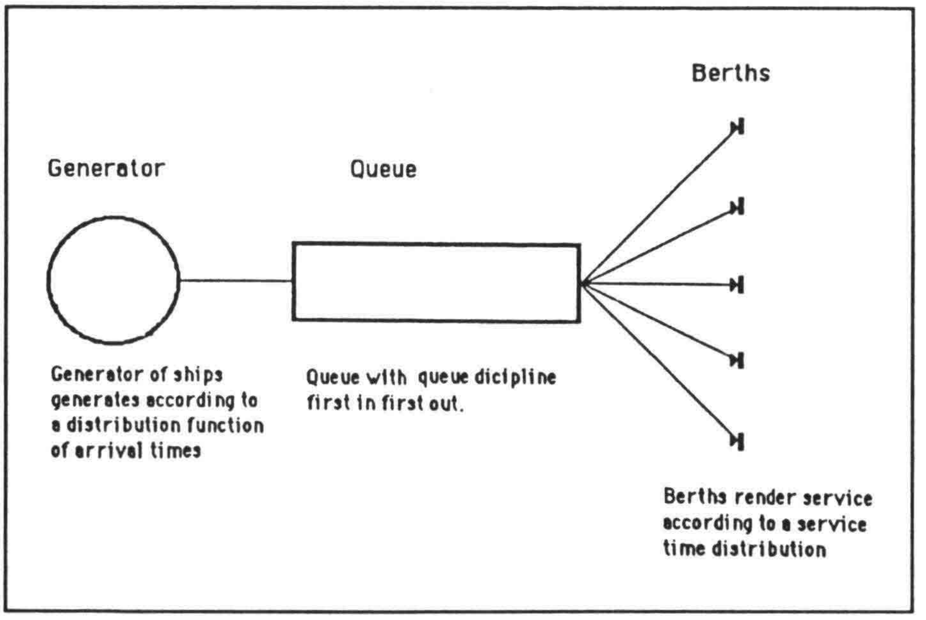


Figure 2: Representation of a queue-delay system

4.4 Simulation Techniques

Simulation techniques have to be used when it is no longer possible to create a simple system such as described above. This can occur, for example when:

- The sailing time from the anchorage to the quay cannot be neglected in relation to the servicing time,
- The number of berths is dependent on the length of the ships and
- The tidal conditions affect the functioning of the system, etc.

Simulation techniques will be described in section 6 and 7

4.5 Systems and System Notations

As indicated in section 1 every port is a service **system** and comprises a number of subsystems. The word system has been derived from the Greek verb which means compose. Boulding defines a system as no chaos. To apply the queuing theory or simulation techniques **models** of reality have to be created.

Models are a simplification of the reality. A model is a description of the "real life system" by leaving out all non relevant aspects. A system consists of several **processes**. A harbour system consists for example out of the process of the ship, pilot, harbour master etc. When a process contains one or more stochastic variables, the process is called a stochastic process (see fig. 3).

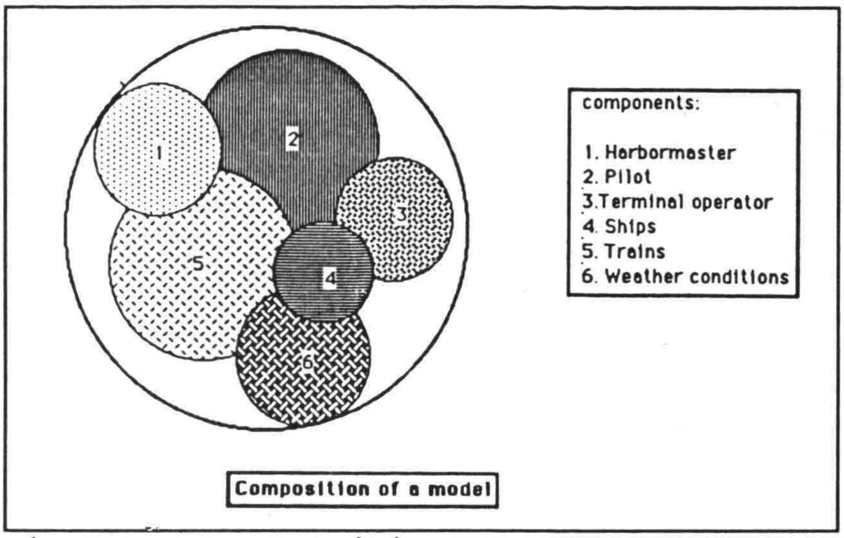


Figure 3: Composition of a model

5. QUEUEING THEORY

5.1 Introduction to the study of queues

D.G. Kendall proposed a notation which covers a wide range of queuing situations. This caters for a queuing system at which customers **require** a **single** service before departure from the system. It doesn't cater for customers requiring service from several service points in sequence. The factors determining the behaviour of such a system are:

1. The customers arrivals
2. The service times of customers
3. The service system (queue - discipline, number of berths).

The customer arrivals and service times are expressed as statistical distributions. The service system can be described by the number of berths in the system and the queue discipline. The queue discipline can be taken as first come first served or first in first out (FIFO) in many cases. This assumption will be made throughout these lectures.

The queuing system can now be described by the inter arrival distribution of customers, the distribution of service times, and the number of servers in the system. Kendall assigned a letter to each of several distributions and was able to describe a queuing theory by a three part code consisting of a letter/letter/number. The first letter specifies the arrival distribution. The second letter specifies the service time distribution and the number specifies the number of servers. The letters Kendall assigned to distribution are:

M - The negative exponential distribution.
The probability density function f(t) of a variate t having a negative exponential distribution is:

$$f(t) = \lambda e^{-\lambda t}$$

The letter M was derived from Markov. This distribution requires the parameter Λ (mean).

E_k - The Erlang distribution

This is a more general distribution than the negative exponential and requires two parameters, μ and k. The Erlang distribution was first used in the study of queues by A.K. Erlang.

$$f(t) = \frac{(k.\mu)^k . t^{k-1} \, e^{-k.\mu.t}}{(k-1)!}$$

D - The Deterministic distribution

The deterministic cumulative distribution function can w expressed in the following formulation.
The variate in this distribution doesn't vary and takes the value a on all occasions.

$$f(t) = \int_0^t f(\mu)\lambda\mu = \begin{matrix} 0 \; if \; t<a \\ 1 \; if \; t>a \end{matrix}$$

G - The general distribution.

This letter is used to cover cases where no assumption is made about the form of the distribution function. The results of studies with this assumption are universally applicable.

The first three mentioned distribution functions are the most tractable for theoretical treatment.

Examples:

M/M/3
- Negative exponential inter arrival time distribution.
- Negative exponential service time distribution.
- 3 servicing points.

M/G/1
- Negative exponential inter arrival time distribution.
- General service time distribution.
- 1 servicing point.

M/E4/4
- Negative exponential inter arrival time distribution .
- Erlang 2 service time distribution.
- 4 servicing points

The arrival process

In general, the arrival process of the ships is stochastic in character. The most convenient way of measuring this uncertainly is to look at the intervals between the successive customers arriving at the service system (port system). The distribution of these intervals is called the inter arrival time distribution. To explain the notations inter arrival time and inter arrival time distribution it is appropriate to give an example. After setting a class width for the inter arrival time, the following inter arrival times are tabulated:

Table 1.

Inter arrival time	number	perc.	cumulative perc.
2h	3	5	5
3h	10	17	22
4h	12	20	42
5h	15	25	67
6h	14	23	90
7h	5	8	98
8h	1	2	100

It is now possible to make two graphs.

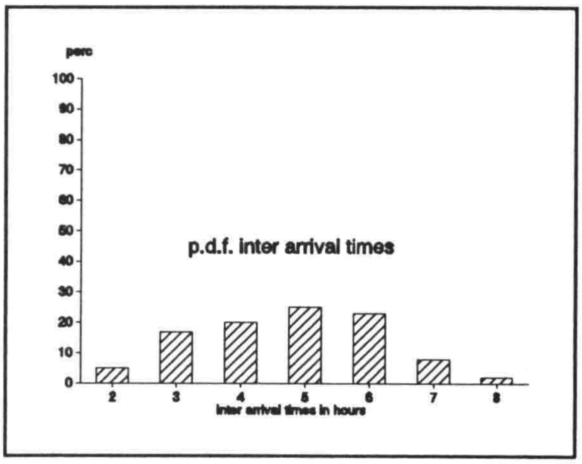


Figure 4: probability density function

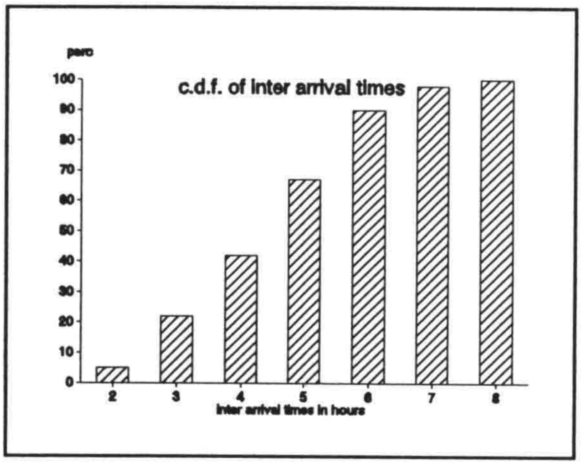


Figure 5: cumulative distribution function

The negative exponential distribution (N.E.D.) has been used to model inter arrival times when arrivals are completely random. The mathematical formulation of the p.d.f. is:

$f(t) = \lambda e^{-\lambda t}$ if $t > 0$
$f(t) = 0$ t else where

and the c.d.f.:

$F(t) = 0$ if $t < 0$
$F(t) = 1 - e^{-\lambda t}$ if $t > 0$

λ = average arrival rate or average number of arrivals per time unit
$1/\lambda = A$ = average inter arrival time.

The service time

The time taken to serve ships along the quay obviously has an effect on the length of the queue, that may form. A system with sufficient berths to meet the average rate of arrivals of ships will still have queue forming.
The distribution of service times must be known before a study can be made. In port engineering systems the total service time often consists of several different-stages and this is also the nature of the Erlang-k distribution. The Erlang-k distribution may thought to be built up out of k negative exponential distributions (N.E.D.). Each stage has an exponential distribution of service time with parameter $k\lambda$.

The mathematic formulation

$$f(t) = \frac{(k \cdot \mu)^k \, t^{k-1}}{(k-1)!} \cdot e^{-k\mu t} \qquad if \ t > 0$$

$$f(t) = 0 \qquad \text{else where}$$

$1/\mu$ = average service time (expected value of the sum of different stages)

μ = average service rate

To describe the measured distribution by means of an Erlang-k distribution is possible by changing k to fit the distribution with the measured distribution (see fig. 6). When the k-value is equal to 1 the Erlang distribution is identically with the negative exponential distribution:

$$f(t) = \lambda e^{-\lambda t}$$

Queue discipline

In situations where a queue of several customers has been formed there must be some way of deciding which (ship) customer is to be served next. The determining rules are called the queue discipline.

The queue discipline can be described:

a. dependent on the arrival time in the queue:
   1. FIFO (First in First out) or
      FCFS (First Come First Served)
   2. LIFO (Last In First Out)
   3. Random

b. dependent on the service time:
   S.P.T (Shortest Processing Time First)

c. dependent on the priority

The chance that the waiting time exceeds a certain value t in a queuing system with different queue disciplines has been indicated in figure 7. It is clear that the variance of a LIFO organisation is much bigger than the variance of a FIFO organisation.

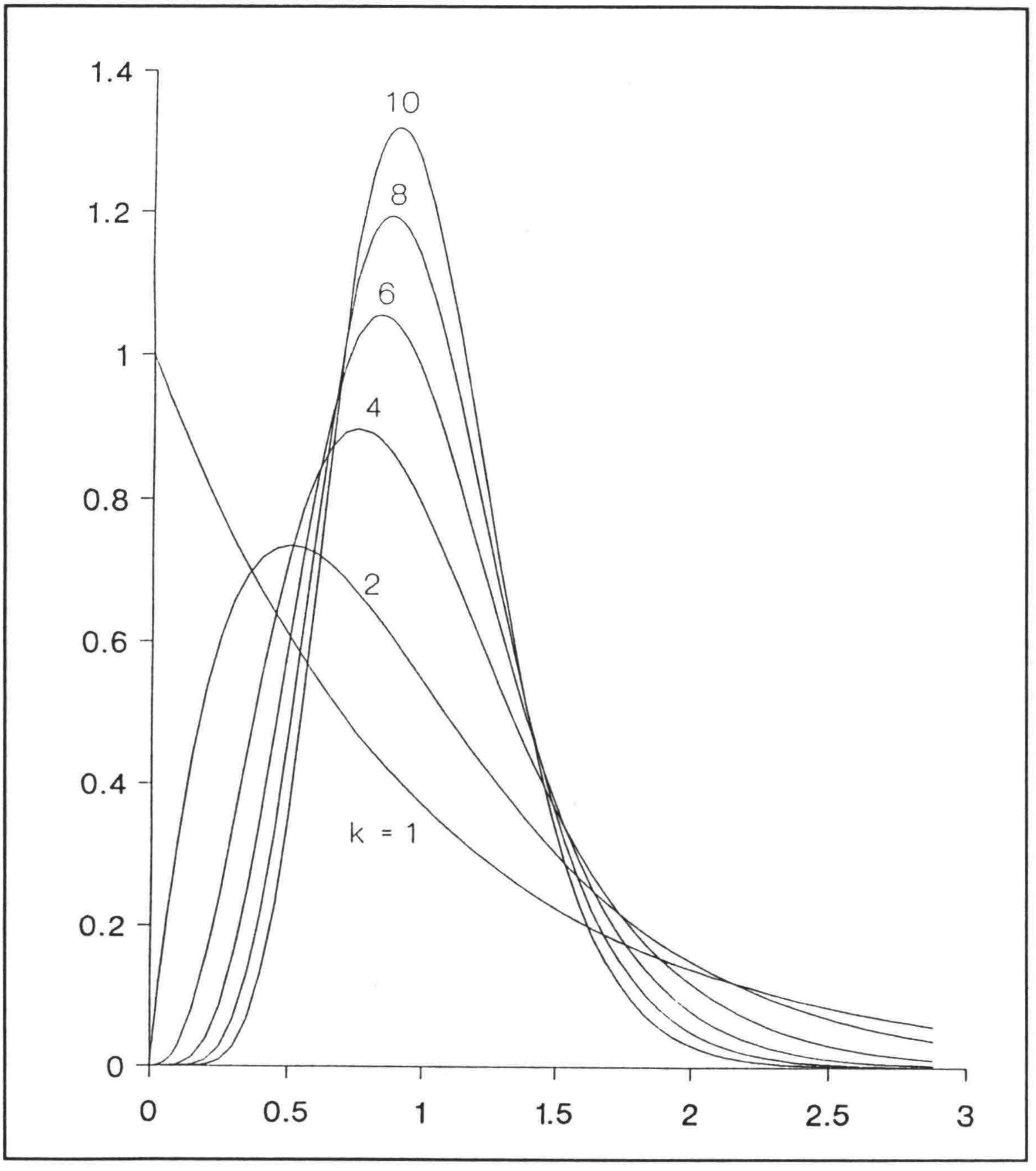


Figure 6: Erlang-k distribution

5.2. A simple queue (M/M/l)

A queue with a negative exponential inter arrival time distribution and negative exponential service time distribution with one berth will be considered in detail. In Kendall's notation this is described by the code M/M/1. Let f(t) be the probability density function of inter arrival times and 1/λ be the average inter arrival time, then:

$$f(t) = \lambda e^{-\lambda t} \qquad (p.d.f.)$$

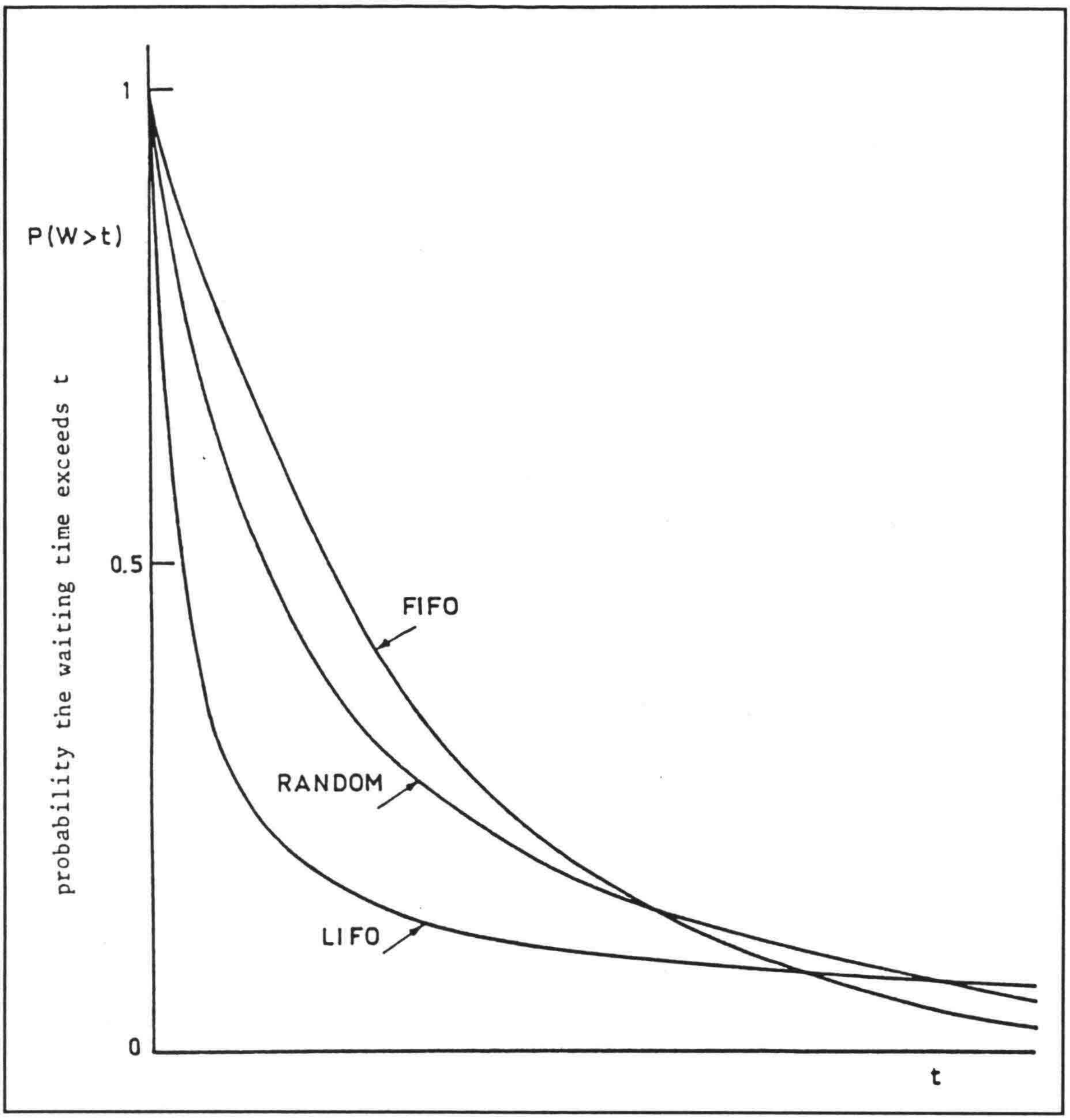


Figure 7: Chance that the waiting time exceeds a value t.

and g(t) the probability density function of the service times:

$$g(t) = \mu e^{-\mu t}$$

where λ = the arrival rate and
μ = the service rate of customers

A small interval of time of length Δt will be considered from t to t + Δt. The probabilities P at the end of the interval ($t+\Delta t$) will be obtained starting from the probabilities P at the beginning of that interval (t). Moreover it is assumed that only one event will occur in the time interval Δt.

So the probability of 0 customers in the system at the end of the interval of time is:

$P_{t+\Delta t}(0)$ = $P_t(0)$ x chance nothing happens in the interval
+ $P_t(l)$ x chance 1 service is completed in the interval

or:

Table 2.

Number of ships in the system	chance	transitional probability	Probability 0 ships in the system
0	P(0)	$(1-\lambda.\Delta t)$	$P(0)=P(0).(1-\lambda.\Delta t)$
1	P(1)	$\mu.\Delta t.(1-\lambda.\Delta t)$	$P(0)=P(1).\mu.\Delta t.(1-\lambda.\Delta t)$

So the total probability of 0 ships in the system is:

$P(0) = P(0).(1-\lambda\Delta t) + P(1).\mu.\Delta t.(1-\lambda\Delta t)$ or
$P(0) = P(0) - P(0).\lambda.\Delta t + P(1).\mu.\Delta t$ or
$\lambda.P(0) - \mu.P(1) = 0 \ldots (1)$

Note: The terms involving Δt at the power 2 are dropped out.

Considering now the probability of j ships in the system.

Table 3.

Number of ships in the system	chance	transitional probability	Probability j ships in the system
j	P(j)	$(1-\Delta t).(1-\mu.\Delta t)$	$P(j)=P(j).(1-\Delta t).(1-\mu.\Delta t)$
j-1	P(j-1)	$\lambda.\Delta t.(1-\mu.\Delta t)$	$P(j)=P(j-1).\lambda.\Delta t.(1-\mu.\Delta t)$
j+1	P(j+1)	$\mu.\Delta t.(1-\lambda.\Delta t)$	$P(j)=P(j+1).\mu.\Delta t.(1-\lambda.\Delta t)$

The total probability there are j ships in the system is:
$P(j) = P(j) - \lambda\Delta t\, P(j) - \mu\Delta t\, P(j) + \lambda\Delta t\, P(j-1) + \mu\Delta t\, P(j+1)$

or

$\lambda\, P(j-l) + \mu\, P(j+l) - (\lambda+\mu)\, P(j) = 0$ .. $j=1,2 \ldots \infty$

$j=1 \rightarrow \lambda\, P(0)+ \mu\, P(2) - \lambda\, P(1) - \mu\, P(0) = 0$ with equation (1):
$\mu\, P(2) - \lambda\, P(l) = 0$ . . . . (2)
$j=2 \rightarrow \lambda\, P(l)+ \mu\, P(3) - \lambda\, P(2) - \lambda\, P(2) = 0$ with equation (2):
$\mu\, P(3) - \lambda\, P(2) = 0$. . . . . (3)

or in general:

$$\mu P(j) = \lambda P(j-1) \qquad j = 1,2,\ldots\infty \qquad \text{or}$$

$$P(j) = \frac{\lambda}{\mu} P(j-1) \qquad j = 1,2,\ldots\infty$$

or

$$P(1) = \frac{\lambda}{\mu} P(0)$$

$$P(2) = \frac{\lambda}{\mu} P(1) = \left[\frac{\lambda}{\mu}\right]^2 P(0)$$

$$P(3) = \frac{\lambda}{\mu} P(2) = \left[\frac{\lambda}{\mu}\right]^3 P(0)$$

$$P(j) = \left[\frac{\lambda}{\mu}\right]^j P(0)$$

The summons of all probabilities must be equal to 1, so :

$$\sum_{j=0}^{\infty} P(j) = \sum_{j=0}^{\infty} \left[\frac{\lambda}{\mu}\right]^j \cdot P(0) = \frac{1}{\left[1-\frac{\lambda}{\mu}\right]} P(0) = 1$$

i.e. $P(0) = 1 - \frac{\lambda}{\mu} = 1 - \rho$ where $\rho = \frac{\lambda}{\mu}$

$$P(j) = (1-\rho)\rho^j \tag{3}$$

Utilisation of the berth

The berth is idle when there are no ships in the system with a probability of P(0). The portion of the time the berth is occupied:

$$1 - P(0) = 1 - 1 + \rho = \frac{\lambda}{\mu} \tag{4}$$

Distribution of ship waiting time

The equation of the chance of having a certain waiting time is given here without prove. Let the p.d.f be W(t), then

$$W(t) = \mu\rho\ (1-\rho)e^{-\mu(1-\rho)t} \tag{5}$$

The average waiting time = $\int_0^\infty t\ \ W(t)\ \ dt = \frac{\rho}{(1-\rho)\mu}$

The average numbers of ships in the queue (anchorage) N_w:

$$N_w = \sum_{i=1}^{\infty}(i-1)\ P(i) = \frac{\rho^2}{1-\rho} \tag{6}$$

Let the average waiting time be equal to W, then

$N_w = W \cdot \lambda$ so

$$W = \frac{N_w}{\lambda} = \frac{\rho^2}{(1-\rho)\lambda} = \frac{\rho}{(1-\rho)-\frac{\mu}{\lambda}\cdot\lambda} = \frac{\rho}{(1-\rho)\mu} \tag{7}$$

Summary of the most important formulae of the M/M/l-system

$P(j) = (1-\rho)\rho^j$ (chance of j customers in the system) (3)

$\Psi = \frac{\lambda}{\mu} = \rho$ (utilization) (4)

$W(t) = \mu\rho(1-\rho)\ e^{-\mu(1-\rho)t}$ (p.d.f. waiting times) (5)

$$N_w = \frac{\rho^2}{1-\rho}$$ (average number of ships in the waiting queue) (6)

$$W = \frac{\rho}{(1-\rho)\mu} = \frac{\rho^2}{(1-\rho)\lambda}$$ (average waiting time) (7)

Example:

1. A transhipment company owns a berth at a port.
2. Ships arrive for unloading on an average of 12 hours with a negative exponential distribution of intervals between ship arrivals.
3. Ships are of a wide range of sizes resulting in a negative exponential service time distribution.
4. Therefore suppose the running costs of the berth is equal to 10000/m per day where m is the average unloading or service time at the berth. The running costs of a berth are directly proportional to the available transhipment capacity. So a short service time means a high transhipment capacity and high running costs of the berth. Therefore suppose the running cost of the berth is equal to 10000/m. Cost of delays to ships are 1000 per ship per day.

Problem:

1. What average unloading time should the equipment maintain for most economic running of the berth.
2. What is the average utilization.
3. What is the average delay per ship.

5.3 The multi-server queue (M/M/n)

One of the simplest methods of controlling queues is to increase the number of berths. The alternatives are either to alter the service times or the arrivals of the ships. The service time of ships is usually difficult to change since this will mean a fundamental change in the method of service. And also the arrivals of ships at most ports are not controllable. The only alternative left in many cases is to increase the number of berths.

5.3.1 Mathematical approach of the-M/M/n - system

Let

- P (f) = the chance there are f ships in the system at time t
- λ = the average arrival rate of ships to the system
- μ = the average service rate of ships
- n = number of berths in the system

The derivation of the probability equations is similar as shown previously for the M/M/l system. The chance of 0 ships in the system:

Table 4.

Number of ships in the system	chance	Transitional probability	Probability 0 ships in the system
0	P(0)	$(1-\lambda.\Delta t)$	$P(0)=P(0).(1-\lambda.\Delta t)$
1	P(1)	$\mu.\Delta t.(1-\lambda.\Delta t)$	$P(0)=P(1).(\mu.\Delta t)$

Hence:

$$P(0) = P(0)\,.\,(1-\lambda\Delta t) + P(1)\,.\,\mu\Delta t \qquad (8)$$
$$\lambda P(0) = \mu P(1)$$

The chance of 1 ship in the system.

Table 5.

Number of ships in the system	chance	Transitional probability	Probability 1 ship in the system
0	P(0)	$\lambda.\Delta t$	$P(1)=P(0).\lambda.\Delta t$
1	P(1)	$(1-\mu.\Delta t).(1-\lambda.\Delta t)$	$P(1)=P(1).(1-\mu.\Delta t-\lambda.\Delta t)$
2	P(2)	$(1-\lambda.\Delta t).2\mu.\Delta t$	$P(1)=P(2).2\mu.\Delta t$

Hence:

$$P(1) = P(0)\,.\,\lambda\Delta t + (1-\mu\Delta t-\lambda\Delta t)\,.\,P(1) + 2\mu\Delta t\,P(2) \qquad (9)$$
$$or\ \ (\mu+\lambda)P(1) = \lambda P(0) + 2\mu P(2)$$

The chance of f ships in the system where $\underline{f \leq n}$:

Table 6.

Number of ships in the system	chance	Transitional probability	Probability f ships in the system (f < n)
f-1	P(f-1)	$\lambda.\Delta t.(1-\mu.\Delta t)^{f-1}$	$P(f)=P(f-1).\lambda\Delta t$
f	P(f)	$(1-\mu.\Delta t)^{f}.(1-\lambda.\Delta t)$	$P(f)=P(f).(1-f.\mu.\Delta t-\lambda.\Delta t)$
f+1	P(f+1)	$(1-\lambda\Delta t).(f+1).\mu\Delta t$	$P(f)=P(f+1).(f+1).\mu.\Delta t$

Hence:

$$P(f) = P(f-1)\,.\,\lambda\Delta t + P(f)\,.\,(1-f\mu\Delta t-\lambda\Delta t) + P(f+1)\,.\,(f+1)\mu\Delta t$$
$$or \quad P(f)\;(\lambda+f\mu) = P(f-1)\lambda + P(f+1)(f+1)\mu \qquad (10)$$

The chance of f ships in the system where <u>f > n</u>:

Table 7.

Number of ships in the system	chance	Transitional probability	Probability f ships in the system (f > n)
f-1	P(f-1)	$\lambda.\Delta t.(1-\mu.\Delta t)^{n}$	$P(f)=P(f-1).\lambda\Delta t$
f	P(f)	$(1-\mu.\Delta t)^{n}.(1-\lambda.\Delta t)$	$P(f)=P(f).(1-n.\mu.\Delta t-\lambda.\Delta t)$
f+1	P(f+1)	$(1-\lambda.\Delta t).n.\mu\Delta t$	$P(f)=P(f+1).n.\mu.\Delta t$

Hence:

$$P(f)=P(f-1)\,.\,\lambda\,.\,\Delta t + P(f)(1-\lambda\Delta t-n\mu\Delta t) + P(f+1)\,.\,n\mu\Delta t$$
$$\text{or} \quad P(f)\,.\,(\lambda+n\mu) = \rho(f-1)\,.\,\lambda + P(f+1)\,n\mu \qquad (11)$$

Taking equations (8) (10) and (11)

$$\lambda P(0) = \mu P(1) \tag{8}$$

$$(\lambda + f\mu) \,.\, P(f) = \lambda \,.\, P(f-1) + (f+1) \,.\, \mu \,.\, P(f+1) \qquad \textit{if } f < n \tag{10}$$

$$(\lambda + n\mu) \,.\, P(f) = \lambda \,.\, P(f-1) + n\mu \,.\, P(f+1) \qquad \textit{if } f > n \tag{11}$$

in turn and adding to the previous equations gives:

$$P(1) = \frac{\lambda}{\mu} \,.\, P(0) = \rho \,.\, P(0) \tag{12}$$

f=1 in (10) gives:

$$(\lambda + \mu) P(1) = \lambda P(0) + 2\mu P(2) \tag{9}$$

with equation (12)

$$(\lambda + \mu)\rho P(0) = \lambda P(0) + 2\mu P(2)$$

$$P(2) = \frac{(\lambda + \mu)\rho \,.\, P(0) - \lambda \,.\, P(0)}{2\mu} = \frac{\{(\rho+1)\rho - \rho\} P(0)}{2} = \frac{\rho^2}{2} \,.\, P(0)$$

f=2 in (10) gives:

$$(\lambda + 2\mu) \,.\, P(2) = \lambda \,.\, P(1) + 3\mu \,.\, P(3)$$

with equation $P(2) = \frac{\rho^2}{2!} P(0)$ and equation 12:

$$(\lambda + 2\mu) \,.\, \frac{\rho^2}{2} \,.\, P(0) = \lambda \,.\, P(0) + 3\mu \,.\, P(3)$$

$$P(3) = \frac{\left[(\lambda+2\mu)\frac{\rho^2}{2} - \lambda\rho\right]}{3\mu} . P(0)$$

$$P(3) = \frac{\rho^3}{3!} P(0)$$

or in general if j < n:

$$P(f) = \frac{\rho^f}{f!} P(0)$$

if f > n then equation (11) is used:

$f = n$ gives $(\lambda+n\mu)\ P(n) = \lambda P(n-1) + n\mu P(n+1)$

and with $P(n-1) = \frac{\rho^{n-1}}{(n-1)!} P(0)$ and $P(n) = \frac{\rho^n}{n!} P(0)$

results in: $P(n+1) = \frac{\rho^n}{n!} \frac{\rho}{n} P(0)$

$f = n+1$ gives $(\lambda+n\mu)\ P(n+1) = \lambda P(n) + n\mu P(n+2)$

and with $P(n) = \frac{\rho^n}{n!} P(0)$ and $P(n+1) = \frac{\rho^n}{n!} . \frac{\rho}{n} P(0)$

results in: $P(n+2) = \left(\frac{\rho}{n}\right)^2 . \frac{\rho^n}{n!} P(0)$

or in general *if* $j > n$: $P(n+j) = \frac{(\rho)^j}{n} \cdot \frac{\rho^n}{n!} P(0)$

since $\sum_{j=0}^{\infty} P(f) = 1$

is $1 = P(0)\left\{1+\rho+\frac{\rho^2}{2!}+\ \dots\ \frac{\rho^{n-1}}{(n-1)!}+\frac{\rho^n}{n!}\cdot(1+\frac{\rho}{n}+\frac{(\rho)^2}{n}+\dots)\right\}$

or provided $\rho/n < 1$

$$1 = P(0)(1+\rho+\frac{\rho^2}{2!}+\ \dots\ +\frac{\rho^{n-1}}{(n-1)!}+\frac{\rho^n}{n!}\left[\frac{1}{1-\rho/n}\right]$$

$$\text{or} \quad P(0) = \left[1+\rho+\frac{\rho^2}{2!}+\ \dots\ +\frac{\rho^{n-1}}{(n-1)!}+\frac{\rho^n}{n!(1-\rho/n)}\right]^{-1} \tag{13}$$

Now the probability of delay meaning all berthing points are occupied is:

$St = P(n+1)+P(n+2)+P(n+3)\ \dots$

$$St = P(0)\cdot\frac{\rho^n}{n!}\cdot\frac{(1)}{1-\rho/n} = P(0)\cdot\frac{\rho^n}{n!}\cdot\frac{n}{n-\rho} \tag{14}$$

The average number of ships in the system:

$$N_a = \sum_0^\infty f\cdot P(f) =$$

$$N_a=P(0)\left[0.1+1.\rho+\frac{2\rho^2}{2!}+\frac{3\rho^3}{3!}\dots+(n-1)\frac{\rho^{n-1}}{(n-1)!}+\frac{\rho^n}{n!}\left[n+(n+1)\frac{\rho}{n}+(n+2)\left[\frac{\rho}{n}\right]^2+\dots\right]\right]$$

or

$$N_a = P(0)\,.\,\rho\,[1+\rho+\frac{\rho^2}{2!}+\frac{\rho^3}{3!}+\dots.\frac{\rho^{n-2}}{(n-2)!}+$$

$$\frac{\rho^{n-1}}{n!}\{n+n\frac{\rho}{n}+n\left[\frac{\rho}{n}\right]^2+n\left[\frac{\rho}{n}\right]^3\dots.+$$

$$\frac{\rho}{n}+2\left[\frac{\rho}{n}\right]^2+3\left[\frac{\rho}{n}\right]^2+4\left[\frac{\rho}{n}\right]^4\}]$$

or

$$N_a = P(0)\cdot\rho\ [\ 1+\rho+\frac{\rho^2}{2!}+\frac{\rho^3}{3!}\cdots\cdots\ \frac{\rho^{n-2}}{(n-2)!}\ +\ \frac{\rho^{n-1}}{n!}\{\frac{n}{1-\rho/n}\ +$$

$$\left[\frac{\rho}{n}\right]+\left[\frac{\rho}{n}\right]^2+\left[\frac{\rho}{n}\right]^3+\left[\frac{\rho}{n}\right]^4+\left[\frac{\rho}{n}\right]^5\cdots\cdots\cdots+$$

$$\left[\frac{\rho}{n}\right]^2+\left[\frac{\rho}{n}\right]^3+\left[\frac{\rho}{n}\right]^4+\left[\frac{\rho}{n}\right]^5\cdots\cdots\cdots+$$

$$\left[\frac{\rho}{n}\right]^3+\left[\frac{\rho}{n}\right]^4+\left[\frac{\rho}{n}\right]^5\cdots\cdots\cdots+$$

$$\left[\frac{\rho}{n}\right]^4+\left[\frac{\rho}{n}\right]^5\cdots\cdots+\text{ enz. }\}\ ]$$

$$N_a = P(0)\cdot\rho\ [\ 1+\rho^2+\frac{\rho^2}{2!}+\frac{\rho^3}{3!}\cdots\cdots\ \frac{\rho^{n-2}}{(n-2)!}\ +$$

$$\frac{\rho^{n-1}}{n!}\ \{\ \frac{n}{1-\rho/n}+\frac{\rho/n}{1-\rho/n}+\frac{\rho^2/n}{1-\rho/n}+\frac{\rho^3/n}{1-\rho/n}+\cdots\ \}\ ]$$

$$N_a = P(0)\cdot\rho\ [\ 1+\rho^2+\frac{\rho^2}{2!}+\frac{\rho^3}{3!}\cdots\cdots\ \frac{\rho^{n-2}}{(n-2)!}\ +$$

$$\frac{\rho^{n-1}}{n!}\ \{\ \frac{n}{1-\rho/n}\ +\frac{\rho/n}{(1-\rho/n)(1-\rho/n)}\ \}\ ]$$

$$N_a = P(0)\cdot\rho\ [\ 1+\rho^2+\frac{\rho^2}{2!}+\frac{\rho^3}{3!}\cdots\cdots\ \frac{\rho^{n-2}}{(n-2)!}\ +$$

$$\frac{\rho^{n-1}}{n!}\ \{\frac{n}{1-\rho/n}+\frac{\rho/n}{(1-\rho/n)^2}\ \}\ ]$$

$$N_a\ =\ P(0)\rho\ [\ 1+\rho+\frac{\rho^2}{2!}+\frac{\rho^3}{3!}\cdots\frac{\rho^{n-2}}{(n-2)!}+\frac{\rho^{(n-1)}}{n!}\ \{\ \frac{n}{1-\rho/n}+ \quad\quad (15)$$

$$+\frac{\rho/\mathrm{n}}{(1-\rho/\mathrm{n})^2}\ \}\ ]$$

In the same way it can be proven that the average number of ships in the queue:

$$N_w = P(0)\frac{\rho^n}{n!}\frac{\rho/n}{(1-\rho/n)^2} \tag{16}$$

or $$N_w = \frac{\rho}{n-\rho}\cdot St$$

And since the average waiting time $W\,x\,\lambda = N_w$

$$W = \frac{1}{n\mu}\cdot\frac{\rho^n}{n!}\cdot\frac{P(0)}{(1-\rho/n)^2} \tag{17}$$

Without proof the waiting time distribution W(t) is given:

$$W(t) = \frac{\rho^n}{n!}\cdot P(0)\,n\mu e^{-(n\mu-\lambda)t} \qquad \text{(p.d.f.)} \tag{18}$$

It is clear that the utilization

$$\Psi = \frac{\lambda}{\mu x n} = \frac{\rho}{n} \tag{19}$$

Summary of the most important formulae of the M/M/n-system

$$P(0) = \left[1+\rho+\frac{\rho^2}{2!}+\frac{\rho^3}{3!}\dots+\frac{\rho^{n-1}}{(n-1)!}+\frac{\rho^n}{n!(1-\rho/n)}\right]^{-1} \tag{13}$$

(chance system is empty)

$$St = P(0)\cdot\frac{\rho^n}{n!}\cdot\frac{n}{n-\rho} \qquad (See\,fig.\,8) \tag{14}$$

(chance an arriving ship has to wait before being served)

$$N_a = P(0)\rho\ [\ 1+\rho+\frac{\rho^2}{2!}+\dots\frac{\rho^{n-2}}{(n-2)!}+\frac{\rho^{n-1}}{n!}\ \{\ \frac{n}{1-\rho/n}+\frac{\rho/n}{(1-\rho/n)^2}\ \}\ ] \tag{15}$$

(average number of ships in the system)

$$N_w = P(0)\frac{\rho^n}{n!}\frac{\rho/n}{(1-\rho/n)^2} \qquad (16)$$

(average number of ships in the queue)

$$W = \frac{1}{n\mu}\cdot\frac{\rho^n}{n!}\frac{P(0)}{(1-\rho/n)^2} \qquad (17)$$

(average waiting time)

$$W(t) = \frac{\rho^n}{n!}\cdot P(0)\, n\mu\, e^{-(n\mu-\lambda)t} \qquad (18)$$

(p.d.f.; waiting time distribution)

$$\Psi = \frac{\rho}{n} \qquad (19)$$

(utilization)

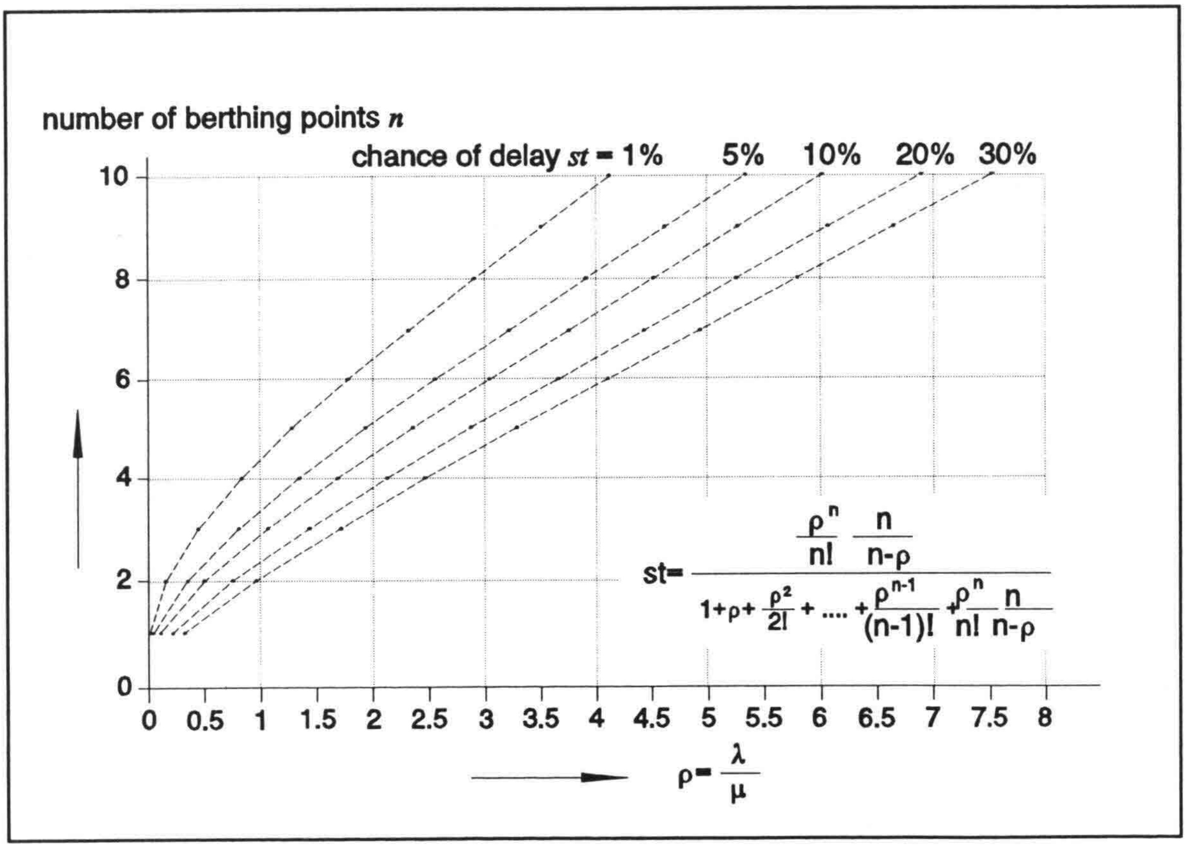


Figure 8: Number of berths versus chance of delay and λ/μ = M/M/N-system

Example 1

Determine the number of berths according to economic criteria when:

a. The costs of a ship per unit time are equal to k_1.

b. The costs of a berth per unit time are equal to k_2.

Suppose we are dealing with n berths.

Obviously the number of berths has to be extended if $\frac{\rho}{n} > 1$

because in this case the capacity is smaller than the arrival rate which means that ultimately the queue will have an infinite length. If $\frac{\rho}{n} > 1$ the extension of the number of berths is justified when:

$$\left(\frac{\rho}{n-\rho}\right) \cdot St \cdot k_1 \quad + \quad n \cdot k_2 \quad < \quad \frac{\rho}{n-1-\rho} \cdot St \cdot k_1 \quad + \quad (n-1)k_2$$

costs of delay to ships in the case of n berths	costs of n berths per time unit	costs of delay to ships in the case of n-1 berths	costs of n-1 berth per time unit

This inequality is presented in figure 9.
Suppose $k_1/k_2 = 0.1$ (this is a realistic value), and with $\rho = 1.5$ it can be found that 4 berths are necessary.

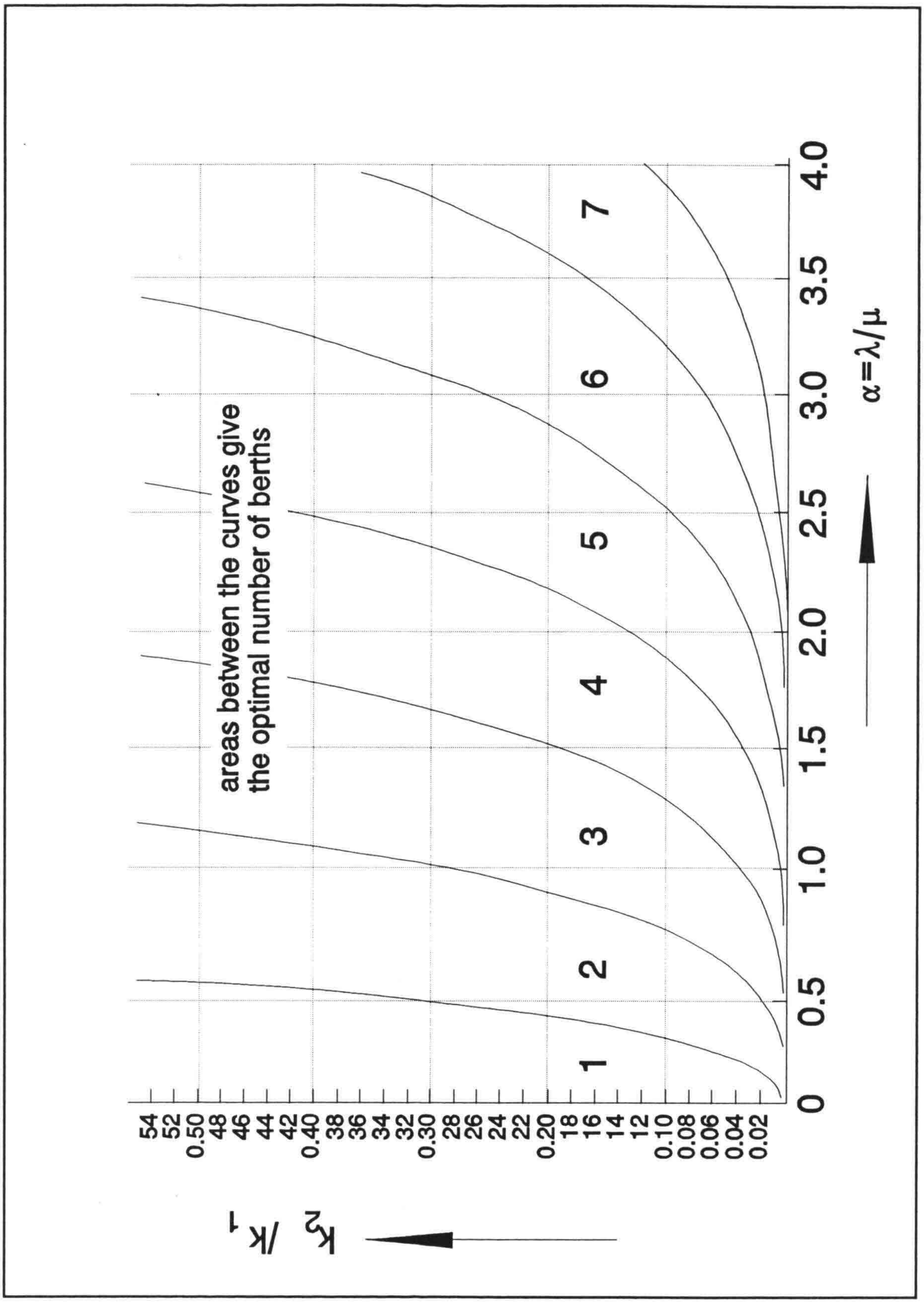


Figure 9: Optimal number of berths in a M/M/n-system

Example 2

When the acceptable chance of delay is known and provided the average arrival rate (λ) and the average service rate (μ) are available, it is possible to determine the number of berths according formula (14):

$$St = P(0)\frac{\rho^n}{n!} \cdot \frac{n}{n-\rho}$$

The relation between $\rho = \frac{\lambda}{\mu}$ and the acceptable chance of delay is shown in figure 8.

Example 3

A transhipment company has three separate berths for general cargo ships, multipurpose ships and reefers. The service times of all three ship types have the same distribution, negative exponential, with a mean of 15 h. The arrivals of the ship types are also negative exponential distributed with averages of 20 h, 18 h, and 30 h respectively.

Problem:

By how much would the average waiting time over all ships drop if each berth was able to deal with any ship type.

Solution:

The present method of operation is as three separate M/M/l systems. The average waiting time of each of the ship types can be found:

$$W = \frac{\rho}{(1-\rho)\mu} \qquad \text{where } \mu = \frac{1}{15}$$

The average waiting time for the general cargo ships

$$\text{with} \quad \rho = \frac{15}{20} = 0{,}75 \quad \text{is} \quad W = \frac{0{,}75}{\frac{1}{15}(1-0{,}75)} = 45h$$

Similarly,
Multi purpose ships:

$$\rho = \frac{15}{18} = \frac{5}{6} \quad \text{and} \quad W = \frac{\frac{5}{6}}{\frac{1}{15}(1-\frac{5}{6})} = 75h$$

Refers:

$$\rho = \frac{15}{30} = 0{,}5 \quad \text{and} \quad W = \frac{0{,}5}{\frac{1}{15}(1-0{,}5)} = 15h$$

Average waiting time over all ships is:

$$\frac{\frac{1}{20}}{\frac{1}{20}+\frac{1}{18}+\frac{1}{30}}\cdot 45 + \frac{\frac{1}{18}}{\frac{1}{20}+\frac{1}{18}+\frac{1}{30}}\cdot 75 + \frac{\frac{1}{30}}{\frac{1}{20}+\frac{1}{18}+\frac{1}{30}}\cdot 15 = 49.8h$$

When all berths can deal with any ship type, the service times are the same and the arrival distribution is still negative exponential with the combined rate of 1/20+1/18+1/30 ships per hour, the system is now a M/M/3 system, with $\rho = 2{,}08$ so the average waiting time for the combined system is:

$$W = \frac{1}{n\mu}\cdot\frac{\rho^n}{n!}\cdot\frac{P(0)}{(1-\rho/n)^2} = 7.9h$$

where $n=3$; $\mu=\frac{1}{15}$ and $P(0) = \left[1+\rho+\frac{\rho^2}{2!}+\frac{\rho^3}{3!(1-\rho/3)}\right]^{-1}$

By extending the three berths so each berth can deal with all ship types the average waiting time is strongly reduced viz from 49.8 to 7.9 h.

5.4 The M/G/l, M/D/l, M/E_k/l-queue system

5.4.1 The M/G/l-queue system

Suppose that service times have mean μ^{-1} and variance $\acute{o}^2$ and there is one server. Without prove the characteristics of a M/G/l-system are given. In general there is no simple expression for the probabilities P(l), P(2), ... of 1, 2, etc.

$$\rho = \frac{\lambda}{\mu}$$

$$N_a = \rho + \frac{\lambda^2(\mu^{-2} + \sigma^2)}{2(1-\rho)} = \rho + \frac{\rho^2(1 + \sigma^2\mu^2)}{2(1-\rho)}$$

$$N_w = \frac{\lambda^2(\mu^{-2} + \sigma^2)}{2(1-\rho)} = \frac{\rho^2(1 + \sigma^2\mu^2)}{2(1-\rho)}$$

$$P(0) = 1-\rho$$

$$W = \frac{\lambda(\mu^{-2} + \sigma^2)}{2(1-\rho)}$$

$$T = \mu^{-1} + \frac{\lambda\mu^{-2}}{2(1-\rho)}$$

5.4.2 The M/D/l-queue system

Assume now the service times have no variability, that is $\sigma^2 = 0$, which means that all service times assume the constant value $1/\mu$. The parameters of the M/D/l-system are given below. They may be derived from the formula of the M/G/1-system.

$$\rho = \frac{\lambda}{\mu}$$

$$N_a = \rho + \frac{\rho^2}{2(1-\rho)}$$

$$N_w = \frac{\rho^2}{2(1-\rho)}$$

$$P(0) = 1-\rho$$

$$W = \frac{\lambda\mu^{-2}}{2(1-\rho)}$$

$$T = \mu^{-1} + \frac{\lambda\mu^{-2}}{2(1-\rho)}$$

5.4.3 Approach to a M/Ek/l-queue system

The parameters for the M/Ek/l-system are given in table 10. They may be computed by substituting $\sigma^2 = 1/k\mu^2$ into the formulas for the M/G/l-system in table 8.

$$N_a = \rho + \frac{1+k}{2k} \cdot \frac{\rho^2}{1-\rho}$$

$$T = \mu^{-1} + \frac{1+k}{2k} \cdot \frac{\rho\mu^{-1}}{1-\rho}$$

$$W = \frac{1+k}{2k} \cdot \frac{\rho\mu^{-1}}{1-\rho}$$

$$N_w = \frac{1+k}{2k} \cdot \frac{\rho^2}{1-\rho}$$

5.5 The M/D/n-and D/M/n-queue system

Results for the M/D/n-system have been obtained from simulation experiments. The results are given in II^a of the table section. This table gives the average waiting time in units of the average service time for 1 to 10 service points with utilisation from 0.1 to 0.9 in steps of 0.1.

The steady state probabilities of a D/M/n-system as the probabilities of a M/D/n-system can only be calculated for specific points in time, for instance at the point of time when ships arrive. The numerical results for this system and the average waiting time are presented in table II^b.

5.6 Systems with more general distributions of arrival and service time

So far two distributions have been discussed, namely being constant and negative exponential. These distributions can be considered as extremes of variability. The constant distribution has no variability and the variation of the negative exponential distribution is unity, while its standard deviation is equal to its mean. In making the models more general, the distribution functions have to be more flexible. A distribution function which can vary from negative exponential to constant has been developed by A.K. Erlang of the Copenhagen Telephone Company. This distribution function is often used to describe the service process.

This distribution is considered to be divided into a fixed number of phases (k) and each phase has a negative exponential distribution, with an average length of each phase of $\frac{1}{k.\mu}$

The mean of the distribution is $\frac{1}{\mu}$

The variance $\sigma^2 = 1/_{k\mu^2}$; standard deviation

$$\sigma = \frac{1}{\sqrt{k} \cdot \mu} \tag{14}$$

k^{-1} = (coefficient of variation)^2
= variance / (average)^2

The value of k must be integer. The Kendall notation for this distribution is E_k, where k is the number of "phases". The shape of the Erlang-k distribution is given in figure 6. For $k \geq 10$, an Erlang random variable is approximately normally distributed and as $k \rightarrow \infty$ an Erlang random variable approaches a constant value of $1/\mu$.

5.6.1 Approach to a E_k/E_l/l-queue system

Because the probability equations are not algebraically soluble other methods have been used. Tables of waiting times have been obtained by simulation (tables V-XII of the table section). The tables can be interpolated to obtain values of average waiting time for intermediate values of:
- utilization (u)
- variability of the Ek distribution of inter arrival times (υ_a = 1/k)
- variability of the El distribution of service times (υ_s = 1/λ).

Linear interpolation on υ_a and υ_s should in most practical cases give sufficiently accurate results (see fig. 10).

Example

The distribution of inter arrival times of ships at a terminal (1 berth) has an average of 6.7 h and a standard deviation of 2.2 h. The service times of the ships have an average of 5.3 h and a standard deviation of 4.3 h. The distributions can be assumed to have the Erlang distribution.

Problem: How much will the average waiting time be with one berth.

Solution:
The utilization of the berth is 5.3/6.7 = 0,791.

The coefficient of variation of arrival time

$$\frac{s.d}{mean} = \frac{2.2}{6.7} = 0.328$$

so v_s = $(c.v)^2$ of arrival time = $(0,328))^2$ = 0,108.
The coefficient of variation of service time

$$\frac{4.3}{5.3} = 0.811$$

and $v_s = (0.811)^2 = 0.658$

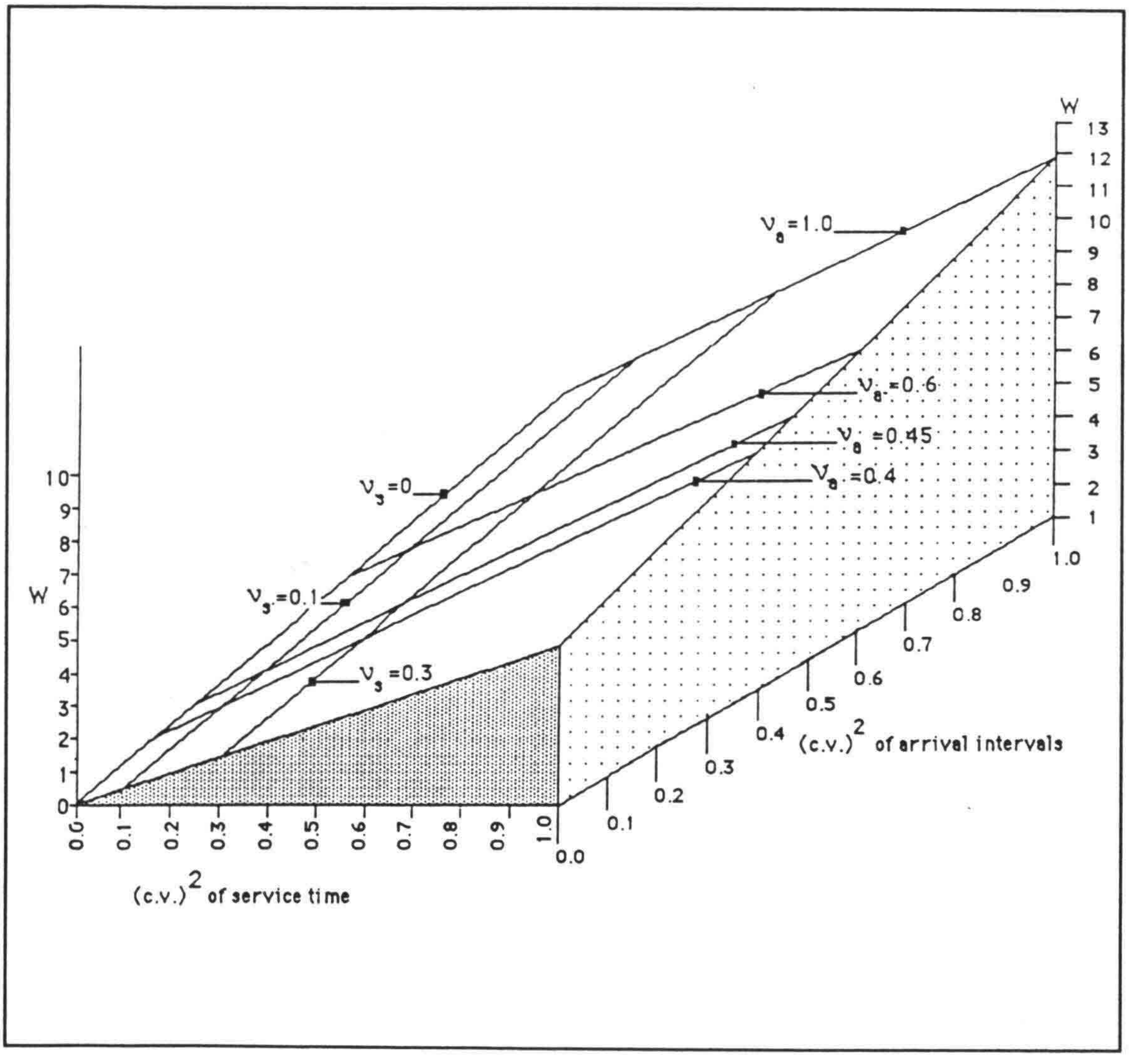


Figure 10: Variability versus waiting times

From the tables of waiting time (tables X, XI) the average waiting times can be read:

υ_a/υ_s	1,0	0,5
0,1	1,922	0,9744
0,2	2,1523	1,1947

with utilization μ = 0,8

υ_a/υ_s	1,0	0,5
0,1	1,0198	0,4908
0,2	1,1642	0,6248

with utilization μ = 0,7

Linear interpolating on the waiting times gives:

υ_a/υ_s	1,0	0,5
0,1	1,8410	0,9309
0,2	2,0634	1,1431

with utilization μ = 0,791

Linear interpolating on υ_s gives:

υ_a/υ_s	0,658
0,1	1,2185
0,2	1,4339

with utilization μ = 0,791

Linear interpolating on υ_s gives the value of the waiting time required:
for μ = 0,791
υ_s = 0,658
υ_a = 0,108
W = 1,235

5.6.2 Approximations to the value of average waiting time in $E_k/E_l/n$-systems

The approximation is based on the linear interpolation on υ_a and υ_s using the queuing systems: M/M/n, D/M/n, M/D/n and D/D/n. Let W_n (υ_a, υ_s, u) be the average waiting time in $E_k/E_l/n$ with utilisation μ, $k^{-1} = \upsilon_a$ and $l^{-1} = \upsilon_s$, then

$W_n(1,1,u)$ = the average waiting time in M/M/n with utilisation u;
$W_n(0,1,u)$ = the average waiting time in D/M/n with utilisation u;
$W_n(1,0,u)$ = the average waiting time in M/D/n with utilisation u.

(waiting times are expressed in units of the average service time)

Assuming linear interpolation on υ_a and υ_s is valid, the average waiting time in general case will be given by:

$$W_n\ (\nu_a,\nu_s,u) = (1-\nu_a)\cdot\nu_s\cdot W_n(0,1,\mu)+\nu_a\cdot(1-\nu_s)\cdot W_n(1,0,\mu)+\nu_a\cdot\nu_s\cdot w_n(1,1,u)$$

The approximation can be used with values of υ_a and υ_s other than 0 or 1. The approximation is always an overestimate of the actual queuing, low ($\pm$ 3%) at high utilisation and 20% at a utilisation of 0.6. If exact results are necessary specially in the ranges of low utilisation the simulation technique should be applied. Exact results obtained by simulation of the $M/E_2/n$-system and the $E_2/E_2/n$-system are shown in table III and IV of the table section. Figure 11 shows the relationship between average ship waiting time and berth utilization.

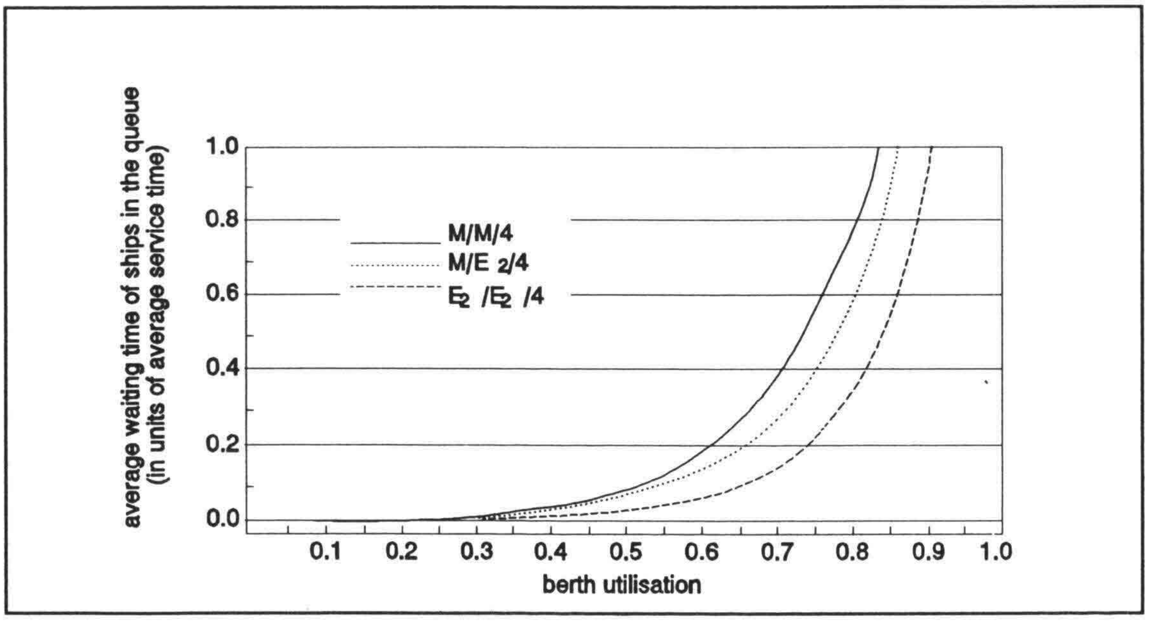


Figure 11: Graph showing relationship between average ship waiting time and berth utilization

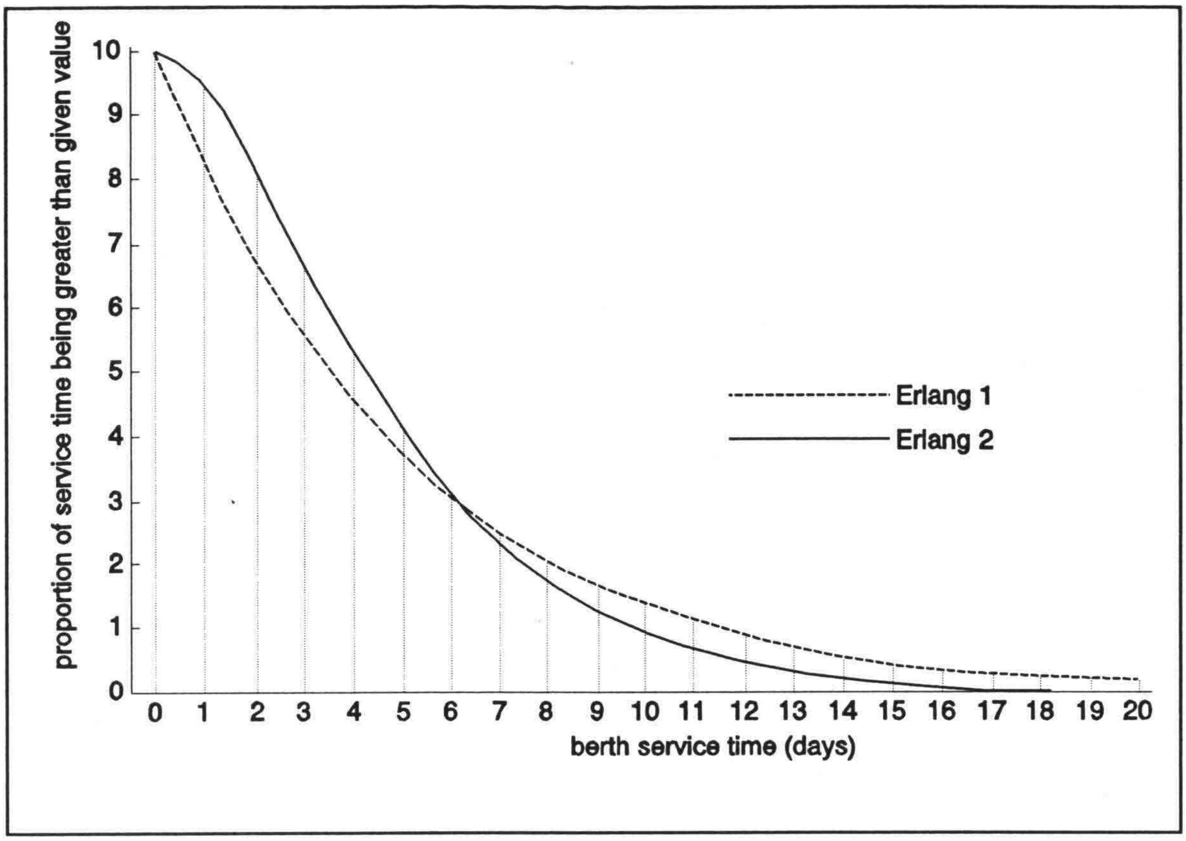


Figure 12 Comparison of Erlang 1 and Erlang 2 distributions for an average vessel service time of five days

<u>Example</u>

Three locomotives are continuously available to transport coal from the mines to a coal terminal. Every 3,7 h a coal train has to be transported from the mines to the terminal with a standard duration of 2.4 h. The turnaround time (service time) of the locomotives is 9,7 h on average with a standard deviation of 7 h. How many minutes does a train have to wait for a locomotive on average if 3 locomotives are used.

$$\nu_a(\textit{the vaiability}) = \left[\frac{s.d.\ of\ arrival}{mean\ arrival\ interval}\right]^2 = \left[\frac{2,4}{3,7}\right]^2 = 0,4207$$

and $\nu_s = \left[\frac{7}{9,7}\right]^2 = 0,5208$

$n = 3 \qquad utilisation\ \frac{9,7}{3X\ 3,7} = 0,8739$

using equation 5.5-1

$$W_3\,(0.4207, 0.5208, 0.9) = 0.5793 \times 0.5208 \times W_3\,(0, 1, 0.9) + $$
$$+ \; 0.4207 \times 0.4792 \times W_3\,(1, 0, 0.9) +$$
$$+ \; 0.4207 \times 0.5208 \times W_3\,(1, 1, 0.9) = 1.2219$$

Similarly W_3 (0.4027, 0.5208, 0.8) = 0.4721

Interpolation for u

W_3 (0.4027, 0.5208, 0.8739) = 0.9560 (in units of the average service time)

Average waiting time = 0.9560 x 9.7 = 9.3 h.

5.7 Some applications

Determination of the number of berths in a M/M/n-queuing system according traffic technical criteria. When the acceptable chance of delay is known the number of berths can be calculated. Of course the average arrival rate and the average service rate should be available.

$$St = P(0)\frac{\rho^n}{n!}\frac{n}{n-\rho} \qquad \text{(formula 13)}$$

From figure 8 the necessary number of berths can be read to satisfy a chance of delay.

Example

Suppose the number of berths of a break bulk terminal has to be determined.
The following data are available:
1. N.E.D. arrival distribution
2. N.E.D. service time distribution
3. one day consists of 2 shifts of 8 hours
4. one week consists out of 6 days
5. the average number of cranes employed per ship is equal to 2,5
6. tonnage forecast is 600.000
7. one year consists out of 50 weeks
8. the number of tons per gang per hour is 12,5
9. the acceptable chance of delay is 0,1.

Determine the number of berths, the average waiting time and the average number of ships waiting in the queue.

Solution: Production per year per berth with 100% utilisation:
16 x 2.5 x 12.5 x 6 x 50 = 150.000 ton
Suppose the average transhipment per ship is A ton, then the average capacity of a berth of this terminal will be 150.000/A ships per year. The average number of ships calling this terminal amounts 600.000/A; so

$$\rho = \frac{600.000/A}{150.000/A} = 4$$

Satisfying the chance of delay of 10% 8 berths have to be built (see fig. 8). The average number of ships in the queue:

$$N_w = \frac{\rho}{n-\rho} \cdot St = \frac{4}{8-4} \cdot 0,1 = 0,1$$

The average waiting time in units of the average service time

$$W = \frac{N_w}{\rho} = 0.025$$

To know the influence of the service time distribution and the inter arrival time distribution we will determine the number of berths in a $E_2/E_2/n$-system and in a $M/E_2/n$-system; again with $\rho = \frac{\lambda}{\mu} = 4$

and an acceptable waiting time of 0,025 in units of the average service time.

Solution: First the $E_2/E_2/n$-system:
Use table IV of the table section.
1. Try 6 berths, then utilisation = ρ/n = 4/6 = 0,67.
2. With table 13 we read an average waiting time of 0.06. This is too long.
3. Try 8 berths; ρ/n = 4/8 = 0,5; W = 0,0031. Maybe too low.
4. Try 7 berths; ρ/n = 4/7; W = 0,015. This satisfies the acceptable waiting time of 0,025.

Next the $M/E_2/n$-system (table III of the table section):
1. Try 8 berths because we are dealing with a somewhat more irregular system.
   Then utilisation 4/8 = 0,5 and W = 0,01.
   This is acceptable but maybe it is to use only 7 berths.
2. ρ/n = 4/7 = 0,574 and W = 0,06 which is not acceptable.
3. We need 8 berths when dealing with a $M/E_2/n$-system.

Figure 12 shows the relationship between the average ship waiting time and berth utilisation dealing with $M/M/4$, $M/E_2/4$ and $E_2/E_2/4$ system.

6. SIMULATION MODELS.

6.1. Introduction.

As stated before simulation techniques have to be used when it is no longer possible to create a simple model such that the queueing theory can be applied.
Simulation models are becoming more and more a common tool for the port planner to use to establish either the most favorable port layout or the most efficient utilization of existing facilities, by simulating an actual or a forecasted situation.

If queueing theory does not seem appropriate, the port planner should consider the use of a simulation model. In this, the first logical step is to formulate precisely the aims of the simulation exercise and to decide whether the complex process or a part of it is to be included in the model. A logical second step is to determine whether an existing model can be used, if necessary with some adaptation, or whether a complete new model has to be developed.
Models used to determine the optimal use of existing facilities are usually restricted to a part of the total port system, e.g. a specific terminal. Other parts of the port system are included in the model as a set of boundary conditions. Results, which are optimal for the part of the system concerned, might be sub-optimal for the total port system. Nevertheless, these models can be quite useful, for example, for terminal operators to determine how to use the facilities under their control most efficiently. In addition, sensitivity analyses of boundary conditions can provide insight into the effects of changes in boundary conditions. This type of analysis can be valuable in discussions with shipping lines, port authority and other port users.

Models used to establish the most favorable layout of the port should take into consideration all relevant components of the port system, and should thus provide an integrated approach to the planning problem.

Integrated port simulation models.
Integrated port simulation models can be useful tools when preparing master plans for the long term development of new and existing ports.
To simulate an integrated port system, models with different aggregation levels have to be used.
An overall model will be used to simulate the entire port process at a rather high aggregation level. The model results will be used to formulate the boundary conditions of the detail models with a lower aggregation levels (for instance models of terminals).

Outlined - versus detailed models.
'Irrespective of the planning objective (terminal, new port, existing port) and irrespective of the availability of a model, a very vital decision is that which concerns the level of detail to be applied in the various components of the model.

Table 12 gives some basic differences between an "outlined" and a detailed model:

Table 12.

The advantages and disadvantages of outlined	and detailed models
Outlined model	**Detailed model**
Advantages	
1. Simple model development 2. Easy data preparation 3. Generally applicable results	1. Basic assumptions are simple 2. Additional details increase the opportunities for studying system response
Disadvantages	
1. Overall assumptions may not be correct under all conditions. 2. Implications of asHxmptions are not clear and are therefore difficult to evaluate 3. Results are not detailed.	1. Complicated model preparation 2. Results are specific for the particular system. Many simulation runs are necessary to check the various possibilities.
Possible seasons for rejection	
1. Results could be invalid under certain conditions	1. Expensive 2. Not sufficient data available.

Initial planning stage.

For new ports, the first planning stage consists of determining the most favorable location along the coastline, the alignments of main basins and channel and the evaluation of the effects of protective measures such as breakwaters (it is evident that for existing ports this first planning stage can be by-passed). Total investments in basins, channel and breakwaters are huge and differences between various alternatives can be considerable. Compared to these amounts, investment in a few extra floating units (tugs, pilot launches), an extra crane and even an additional berth are relatively small.

Overall models are used therefore, in this planning stage and contain rather simplified modules of quay handling and terminal process. The main objective remains, which is to find the most promising alternative based on least economic cost of port investments and ship delays.

Second planning stage.

When the basic decisions on location and orientation have been taken, the second planning stage can start, aiming at detailing the required facilities for the selected alternative. As the location and orientation of the port is more or less fixed. Environment conditions wave climate, tidal movements will be more or less fixed for the selected alternative, resulting in less variation of input variables. Modelling of quay handling, storage and inland transport is required to determine more precisely the optimal berth length, terminal size and equipment.

To explain port simulation, a "hand simulation" of a deterministic model (no uncertainties) and a "hand simulation" of a stochastic model (with random fluctuations) will be discussed. To simulate a stochastic model it is necessary to generate random numbers. Chapter 6.3. therefore is dealing with random numbers while chapter 6.4. will show how random numbers are used to generate a random variable with any desired probability distribution. Applications of stochastic simulation models will be carried out in chapter 6.5. and output analysis is discussed in chapter 6.6.

6.2. Deterministic simulation model.

The example concerns the establishment of a transshipment company. The transshipment company has the disposal of own berths. At first instance it is decided to built a quay with a length of 200 m. Because extension of the quay is very expensive, the length will be checked.
The tables below show the traffic forecast and the characteristics of the vessels.

Table 13.

Company	First arrival	Max. acceptable waiting time [days]	Unloading [days]	Loading time [days]	Ship length [m]
Yellow star	2 Jan.	1	2	1	80
ABC	12 Jan.	2	4	3	100
Oil trade	5 Jan.	1	2	0	100
Gen. Cargo Trade	24 Jan.	2	3	1	160
Fruit Trade	9 Jan.	0	3	3	90
United	2 Jan.	2	1	4	80

Table 14.

Company	Arrival frequency
Yellow Star	one per week
ABC	one per month
Oil Trade	one per 18 days
Gen. Cargo Trade	one per 2 months
Fruit Trade	one per 10 days
United	one per 16 days

The ship will call another port when the maximum waiting time is exceeded and of course in such cases additional costs are involved.

Cost:

A. Calling another port:

- fixed costs 10000
- additional port charges: 50 x length of the ship per day in the port
- additional transport charge:
  - 3000 per unloading day
  - 4000 per unloading day

B. Waiting costs:

300 x length of the ship per day.

The simulation has been executed with a quay length of 200 m during the months January, February and a part of March. The results are shown in fig. 12. The same simulation can be performed with a length of 200, 220, 230 etc. Extension of the length of the quay causes an increase of investment and operational costs but a decrease of ships waiting costs. Extension may even cause an increase of we traffic volume. Considering only the costs the optimum length can easily be determined by showing the costs as a function of the length (see fig. 12).

In general an optimum terminal system is defined as a system which works at minimum costs. However other definitions of the optimum are possible.

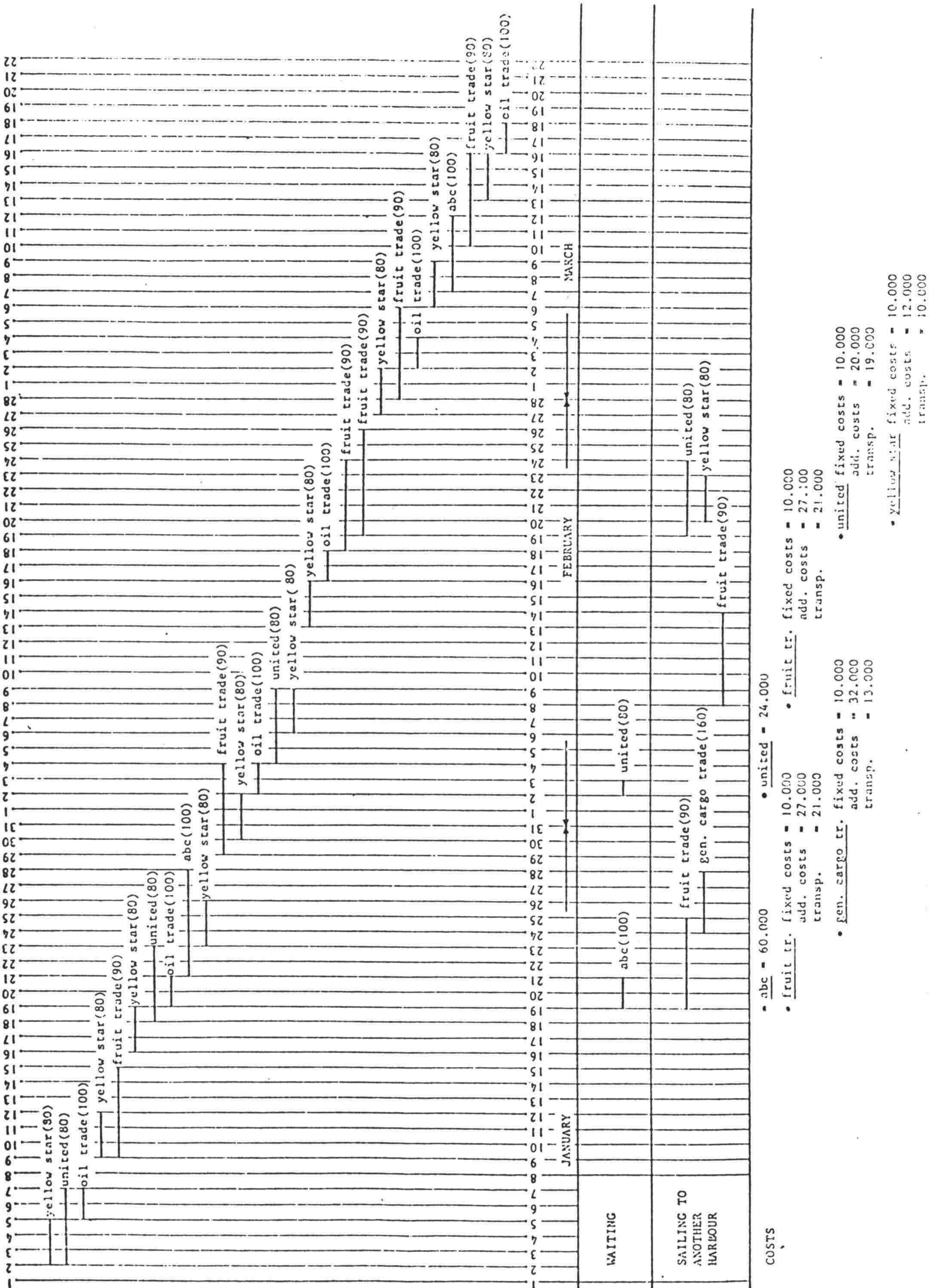

JANUARY
FEBRUARY
MARCH
WAITING
SAILING TO ANOTHER HARBOUR
COSTS
• abc = 60.000
• united = 24.000
• fruit tr. fixed costs = 10.000
add. costs = 27.000
transp. = 21.000
• gen. cargo tr. fixed costs = 10.000
add. costs = 32.000
transp. = 13.000
• fruit tr. fixed costs = 10.000
add. costs = 27.000
transp. = 21.000
• united fixed costs = 10.000
add. costs = 20.000
transp. = 19.000
• yellow star fixed costs = 10.000
add. costs = 12.000
transp. = 10.000

fig. 12.

6.3 Random number generation

Random numbers are a necessary basic ingredient in the simulation of almost all systems. Most computer languages have a subroutine that will generate a random number for asking. Similarly, simulation languages generate random numbers that are to draw samples from distribution functions.

A sequence of random numbers R_1 R_2- - R_i- - - R_n must have two important statistical properties uniformity and independence. Each random number Ri is an independent sample drawn from a continuous uniform distribution between 0 and 1.
The p.d.f. is given by:

f(x)=1 if $0<x<1$ and
f(x)=0 otherwise.

This function is shown in fig. 13.

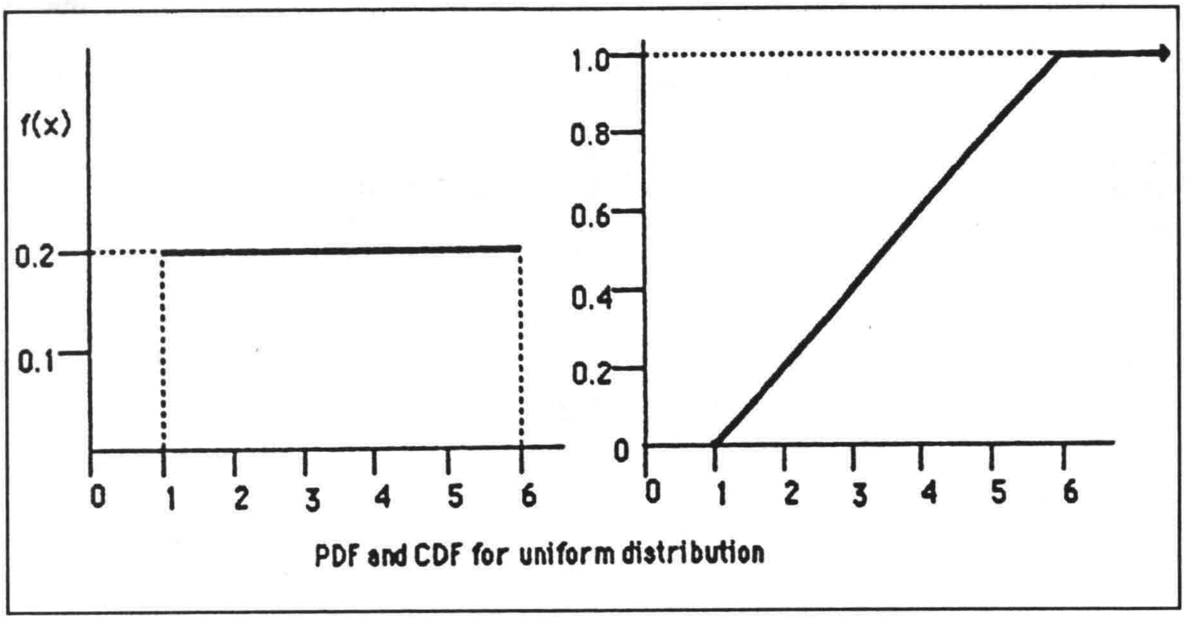


Fig. 13: PDF and CDF of a uniform distribution function

The expected value:

$$E(x)=\int_0^1 x.1dx=\frac{1}{2}$$

The variance:

$$E(x^2)-E(x)^2=\int_0^1 x^2dx-\frac{1}{2}^2=\frac{1}{3}-\frac{1}{4}=\frac{1}{12}$$

Generation of pseudo random numbers.
The goal of any generation scheme is to produce a sequence of numbers between 0 and 1 which simulates or initiates the properties of uniform distribution, independence as closely as possible. In this context independence means that the probability of observing a value in a particular interval is independent of the previous one.

1. The midsquare method.
The midsquare method was proposed by von Neumann and Metropolis in the mid 1940's.
This technique starts with an initial number, or seed.
This number is squared and the middle digits of this square become a random number after replacement of the decimal. Those middle digits are then squared to generate the second random number.
The next example will show how this operates.

Example:
Suppose that a sequence of a two digit random number is needed.
Start with the seed 42.

Table 15.

i	x_i	x_i2	R_i
1	42	1764	0.76
2	76	5776	0.77
3	77	5929	0.92
4	92	8464	0.46
5	46	2116	0.11
6	11	0121	0.12
7	12	0144	0.14
8	14	0196	0.19
9	19	0361	0.36
10	36	1296	0.29

This example is more of historical then of practical interest. The reason is shown in the next table.

Suppose the seed is changed to 5197 and 4 digits will be used.
Then:

Table 16.

i	x_i	x_i^2	R_i
1	5197	27008809	0.0088
2	0088	00007744	0.0077
3	0077	00005929	0.0059
4	0059	00003481	0.0034
5	0034	00001154	0.0011
6	0011	00000121	0.0001
7	0001	00000001	0.0000

2. The additive congruential method.
The additive congruential method is quite fast and what is required is a sequence of n numbers:
$x_1\ x_2\ x_3\ x_4$ - - - - - - x_n . The generator then produces an extension of the sequence, or $x_{n+1}\ x_{n+2}$.

The values are generated as follows:

$$xi=(x_{i-1}+x_{i-n}) \bmod m.$$

This expression tells us to calculate $(x_{i-1}+x_{i-n})$ and take modulo m (i.e., divide $(x_{i-1}+x_{i-n})$ by m and treat the remainder as x_i.

Example:
Let the sequence of integers be $x_1\ x_2\ x_3\ x_4\ x_5$ be 57 34 89 92 and 16 (so n=5) and let m=100. This sequence can be extended by using the additive congruential method as follows:

$x_6 = (x_5 + x_1) \bmod 100 = 73 \bmod 100 = 73$ $R_1 = 0.73$
$x_7 = (x_6 + x_2) \bmod 100 = 107 \bmod 100 = 7$ $R_2 = 0.07$
$x_8 = (x_7 + x_3) \bmod 100 = 96 \bmod 100 = 96$ $R_3 = 0.96$
$x_9 = (x_8 + x_4) \bmod 100 = 188 \bmod 100 = 88$ $R_4 = 0.88$
$x_{10} = (x_9 + x_5) \bmod 100 = 104 \bmod 100 = 4$ $R_5 = 0.04$
$x_{11} = (x_{10} + x_6) \bmod 100 = 77 \bmod 100 = 77$ $R_6 = 0.77$

Remark:
Two integers A and B are said to be congruent modulo m (m being an integer) only if there is an integer k such that A-B=k.m or A-B is divisible by m. This relation is expressed as A=B(mod m).

3. Linear congruential method.

The linear congruential method, initially proposed by Lehmer produces a sequence of integers X_1 X_2 between 0 and m-1 according the following recursive relationship:

$x_{i+1} = (a.x_i+c) \bmod m$ and $R_{i+1} = x_{i+1}/m$

The initial value x_0 is called the seed, a is called the constant multiplier, c the increment and m the modulus.

The selection of the values for a, c, m and x_0 drastically affects the statistical properties and cycle length.

Example:

Let $x_0=27$; $a=17$ $c=43$ and $m=100$

Here the integer values be generated will be all between 0 and 99 because of the value of the modulus.

The sequence of x_i and subsequent R_i values is computed as follows:

$x_0=27$

$x_1=(17*27+43) \bmod 100$	$= 502 \bmod 100$	$=2$	$R_1=x_1/m=0.02$
$x_2=(17*2+43) \bmod 100$	$= 77 \bmod 100$	$=77$	$R_2=x_2/m=0.77$
$x_3=(17*77+43) \bmod 100$	$= 1352 \bmod 100$	$=52$	$R_3=x_3/m=0.52$

The next example in this section is in actual use. It has been extensively tested by Learmonth and Lewis. The values of a c and m have been preselected to ensure that the characteristics of the random stream produced by a generator are most likely to be achieved. By changing x_0, the user can control the repeatability of the stream.

Let:

$a= 7^5 =16807$; $m = 2^{31}-1 = 2147483647$ (a prime number) and $c = 0$.

Since m is a prime number the period p equals m-1

Further let $x_0=123457$ be a seed.

The first numbers generated are as follows:

$x_1=7^5*123457 \bmod (2^{31}-1) = 2074941799$		$R_1=x_1/m=0.9662$
$x_2=7^5*2074941799 \bmod (2^{31}-1) = 559872160$		$R_2=x_2/m=0.2607$
$x_3=7^5*559872160 \bmod (2^{31}-1) = 1645535613$		$R_3=x_3/m=0.7662$

6.4 Samples from distribution functions.

As stated in chapter 6.3 random numbers are a necessary ingredient in simulation. In this chapter it will be shown how to draw samples from a distribution function.

This chapter discusses the inverse transform, the acceptable rejection technique and the convolution method.

6.4.1 The inverse transform technique.

The inverse transform technique can be used for sampling from a wide variety of discrete distributions and can be utilized when the c.d.f., F(x) is of such a simple form that its inverse,

F^{-1}, can be computed analytically. This technique will be explained in detail for the N.E.D.-distribution.

a. The N.E.D-distribution.

The exponential distribution has a probability density function (p.d.f.) given by:

$f(x)=\lambda.e^{-\lambda x}$ if $x > 0$ and

$f(x)=0$ if $x < 0$

And a cumulative distribution function given by:

$$F(x) = \int_{-\infty}^{x} f(t)\,d(t) = 1-e^{-\lambda . x} \qquad \text{if } x \geq 0$$

$$F(x) = 0 \qquad \text{if } x < 0$$

The step by step procedure for the inverse transform technique is as follows:

Step 1: Compute the c.d.f (for the N.E.D. distribution function)

Step 2: Generate random number R, now F(x)=R, so

$R=1-e^{-\lambda . x}$

Step 3:

$$R=1-e^{-\lambda . x}$$

$$-\lambda . x=\ln(1-R)$$

$$x=-\frac{1}{\lambda}.\ln(1-R)$$

in general this last equation is written as:

$x=F^{-1}(R)$

Step 4: Generate random numbers (samples from a uniform distribution function between 0 and 1) R_1, R_2, R_3,---- and draw samples from the exponential distribution function:

$$x_i = -\frac{1}{\lambda}\ln(R_i)$$

Fig. 14 gives a graphical interpretation of the inverse transform technique.

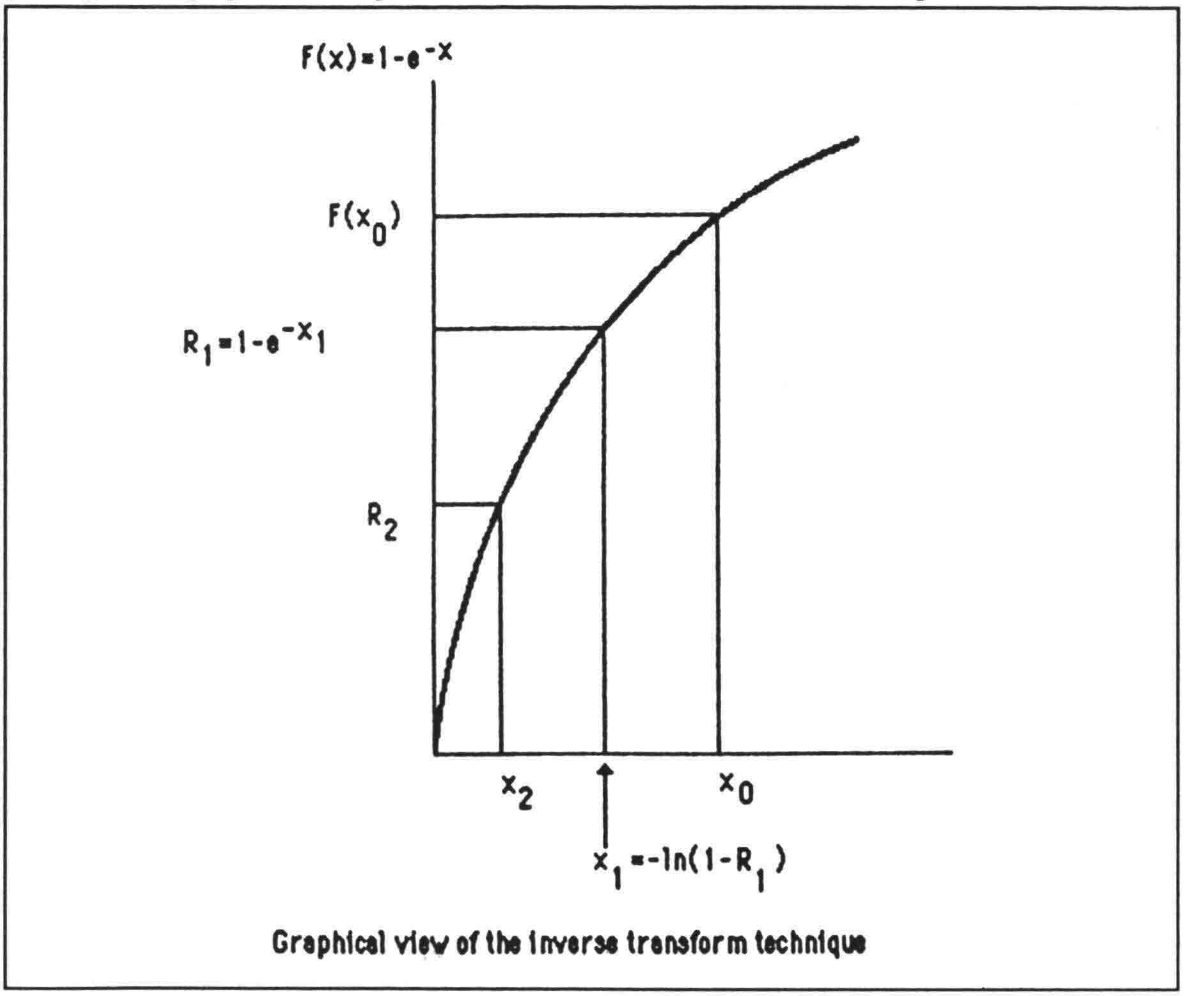


Fig. 14. Graphical view of the inverse transformation technique.

b. Uniform distribution

Step 1. The c.d.f. is given by

$F(x)=0$ if $x<a$

$F(x)=\dfrac{x-a}{b-a}$ if $a \leq x \leq b$

$F(x)=1$ if $x>b$

Step 2: Generate a random number and set:

$$R=\frac{x-a}{b-a}$$

Step 3:

$$x=a+R.(b-a)$$

Step 4:
In general:

$$x_i=a+R_i(b-a)$$

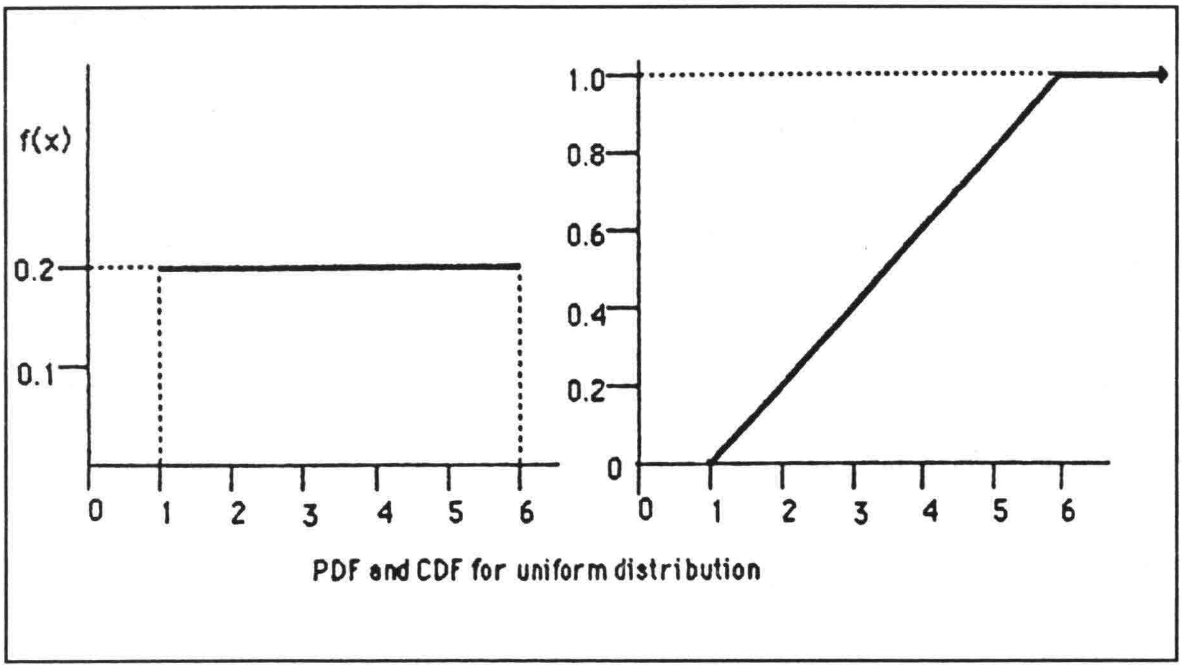


Fig. 15: PDF and CDF for a uniform distribution.

c. Weibull distribution.

$$f(x)=\frac{\beta}{\alpha}\left(\frac{x-\nu}{\alpha}\right)^{\beta-1}.\exp\left[-\left(\frac{x-\nu}{\alpha}\right)^{\beta}\right] \qquad \text{if} \quad x>\nu$$

The three parameters of the Weibull distribution are :

ν···*defined*$(-\infty<\nu<+\infty)$, which is the location parameter

α···$(\alpha>0)$, which is the scale parameter and

β···$(\beta>0)$, which is the shape parameter.

This distribution function is a model for "time to failure" for machines or equipment. When the location parameter is set to 0 its p.d.f. is given by the following equation:

$$f(x)=\frac{\beta}{\alpha^{\beta}}x^{\beta-1}.\exp\left[-\left(\frac{x}{\alpha}\right)^{\beta}\right] \quad \text{if} \quad x\geq 0$$

$f(x)=0$ otherwise

To take a sample from a Weibull distribution the following steps have to be executed:

Step 1: The c.d.f. is given by:

$$F(x)=1-e^{-\left(\frac{x}{\alpha}\right)^{\beta}}=R \qquad \text{if} \quad x\geq 0$$

Step 2:
Let:

$$F(x)=1-e^{-\left(\frac{x}{\alpha}\right)^{\beta}}=R$$

Step 3:

$$x=\alpha.[-\ln(1-R)]^{\frac{1}{\beta}}$$

d. Empirical continuous distributions.
If a modeler has been unable to find a theoretical distribution that provides a good model for the input data, it may be necessary to use the empirical distribution of data.
Suppose 100 repair times of transshipment equipment have been collected. The data are summarized in the table below:

Table 17.

Interval (hours)	Frequency	Relative Frequency	Cumulative Frequency
$0 \leq x \leq 0.5$	31	0.31	0.31
$0.5 \leq x \leq 1.0$	10	0.10	0.41
$1.0 \leq x \leq 1.5$	25	0.25	0.66
$1.5 \leq x \leq 2.0$	34	0.34	1.00

Suppose it is known that all repairs take at least 15 minutes so that $x \geq 0.25$
Then the graph should start in the point (0.25 ;0)
The graph is shown in the fig. 16.
To draw samples from this empirical function it is easy to work with the slopes of the line segments.
Table below can be used to generate samples:

Step 1: Generate a random number R

Step 2: Find the interval i in which R lies, that is find i so that $R_i < R < R_{i+1}$

Step 3: Compute $x = x_i + a(R - R_i)$

Table 18.

	Input	Output	Slope
i	R_i	x_i	a_i
1	0	0.25	0.81
2	0.31	0.5	0.5
3	0.41	1.0	2.0
4	0.66	1.5	1.47
5	1.00	2.00	-

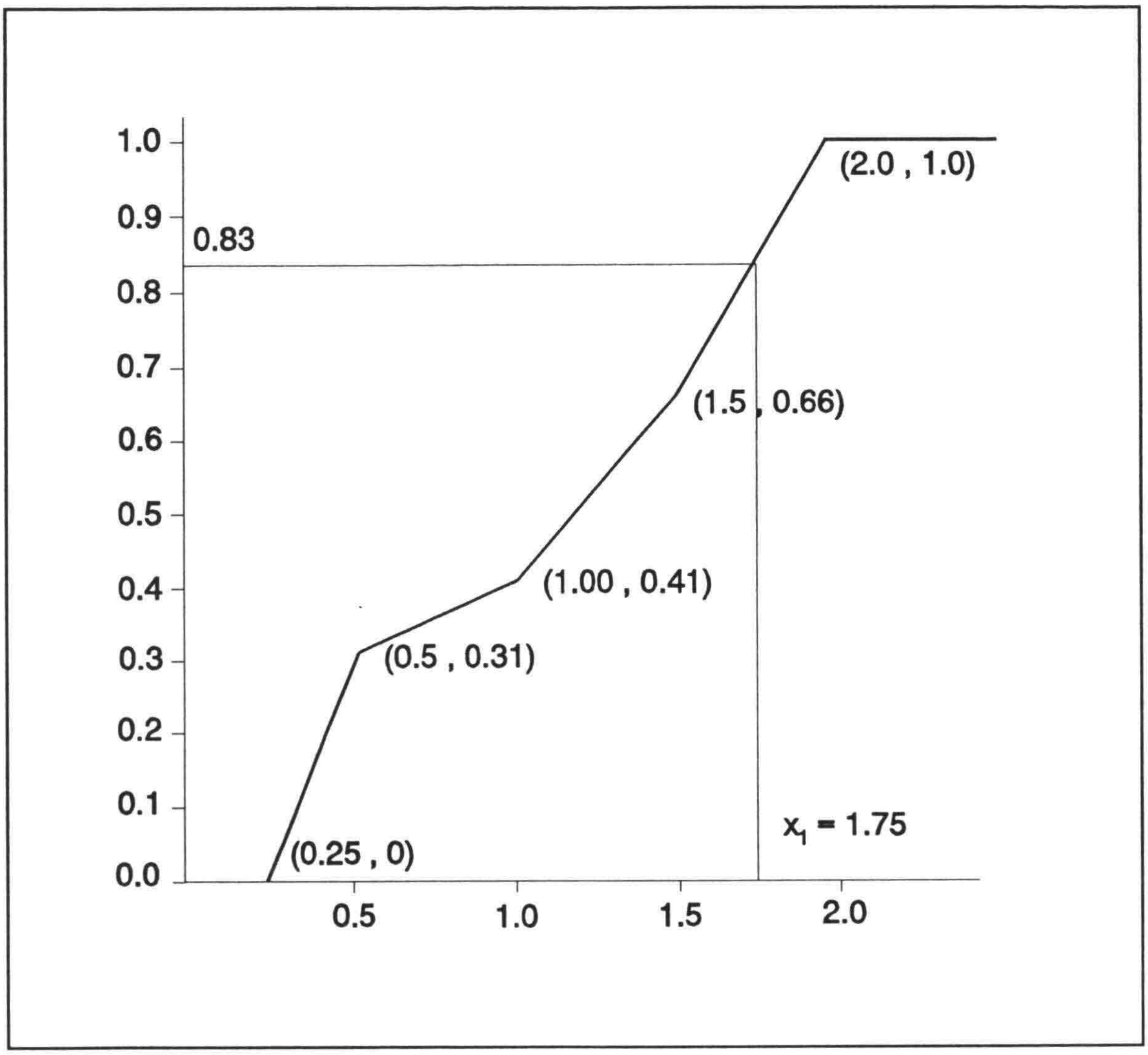


Fig. 16. Samples of an empirical distribution function.

c. Discrete Distributions.

At the end of the day the number of shipments on a quay of a transshipment company is either 0,1 or 2, with an observed relative frequency occurrence of 0.5, 0.3 and 0.20. The distribution function is given in table 19.

Table 19.

x	p(x)	F(x)
0	0.50	0.50
1	0.30	0.80
2	0.20	1.00

The samples can be generated by using the inverse transformation technique through a table-look up procedure.
The generation scheme is summarized as follows.

$x = 0 \ldots R \leq 0.5$

$x = 1 \ldots 0.5 < R \leq 0.8$

$x = 2 \ldots 0.8 < R \leq 1.0$

6.4.2 Direct transformation for the normal distribution.

The inverse transformation technique cannot be applied for the normal distribution function because the inverse c.d.f. cannot be computed analytically.
The standard normal distribution function is given by:

$$F(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2 \cdot \pi}} \cdot e^{-\frac{t}{2}} dt.. \quad , -\infty < X < \infty$$

Here the Box and Muller [1958] method is described.
Consider two standard normal random variable z_1 and z_2, plotted as a point in a plane as shown in fig. 17.
Expressed in polar coordinates:

$z_1 = \beta \cos \theta$
$z_2 = \beta \sin \theta$

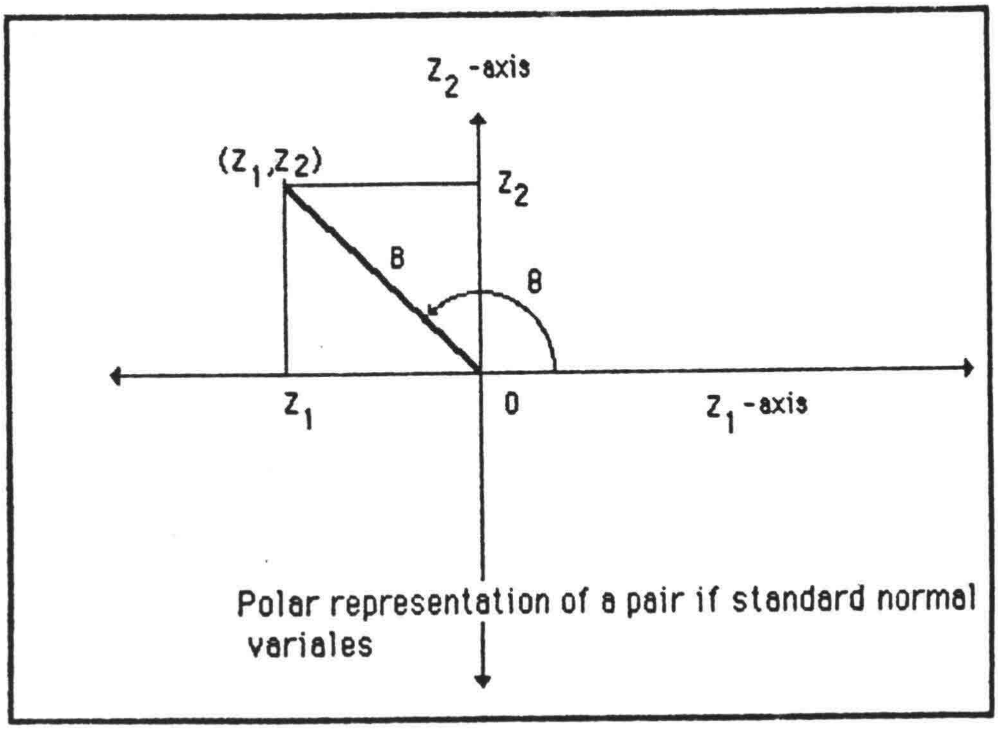


Fig.17. Polar representation of a pair of standard normal variables

It is known that:

$$\beta^2 = z_1^{\,2} + z_2^{\,2}$$

has a chi-square distribution with 2 degrees of freedom, which is equivalent to an exponential distribution with mean 2.

Thus the radius, β can be generated by use of equation:

$$\beta = (-2 \cdot 1nR)^{\frac{1}{2}}$$

And because the angle θ is uniform distributed between 0 and 2π two independent standard normal variates, z_1 and z_2 from two independent random numbers R_1 and R_2 can be generated:

$$z_1 = (-2 \cdot 1nR_1)^{\frac{1}{2}} \cdot \cos(2\pi \cdot R_2)$$

$$z_2 = (-2 \cdot 1nR_1)^{\frac{1}{2}} \cdot \sin(2\pi \cdot R_2)$$

example:
$R_1 = 0.1758$
$R_2 = 0.1489$

$$z_1 = (-2 \cdot \ln(0.1758))^{\frac{1}{2}} \cdot \cos(2\pi \cdot 0.1489) = 1.11$$

$$z_2 = (-2 \cdot \ln(0.1758))^{\frac{1}{2}} \cdot \sin(2\pi \cdot 0.1489) = 1.58$$

6.4.3. Convolution method.

a. Erlang distribution.

As described before an Erlang random variable x with parameters k and μ can be represented as the sum of k independent exponential random variables each having mean $1/k.\mu$ so:

$$x = \sum_{i=1}^{k} \left[-\frac{1}{k \cdot \mu} \ln(R_i) \right] = -\frac{1}{k \cdot \mu} \ln \left(\prod_{l=1}^{k} R_i \right)$$

(In this last equation Π stands for product)

Trucks arrive at a large warehouse in a completely random way which was modeled as a N.E.D. distribution with an arrival rate $\lambda = 10$ trucks per hous. The guard at the entrance sends trucks alternately to the north and south docks. An analyst has developed a model to study the loading and unloading process at the south docks, and needs a model of the arrival process at the south docks alone. An inter arrival time X between successive arrivals at the south docks is equal to the sum of two inter arrival times at the entrance and thus the sum of two exponential random variables each with a mean 0.1 hour or 6 minutes. Thus, X has the Erlang distribution with k = 2 and mean.
2 . 0.1 hour = 0.2 hour. To generate an inter arrival time (a sample from the Eralng 2 distribution) first k = 2 random number have two determines say $R_1 = 0.937$ and $R_2 = 0.217$.
Then using equation:

$$x = \sum_{i=1}^{k} \left[-\frac{1}{k \cdot \mu} \ln(R_i) \right] = -\frac{1}{k \cdot \mu} \left(\prod_{i=1}^{k} R_i \right)$$

$X = - 0.1 \ln [0.937 * 0.217] \sim = \sim 0.159$ hour = 9.56 min.

b. Normal distribution

The central limit theorem asserts that the sum of n independent and identically distributed random variables $x_1, x_2, x_3, \cdots x_n$ each with a mean μ_x and variance σ_x^2 is approximately normal distributed with mean $n.\mu_x$ and variance $n.\sigma_x^2$.
Applying uniform random variables on (0,1) which have a mean E(x)=0.5 and variance $V(x) = \sigma^2 = 1/12$, it follows that

$$\xi = \frac{\sum_{i=1}^{n} R_i - 0.5 \cdot n}{\left[\frac{n}{12}\right]^{\frac{1}{2}}}$$

is approximately normal distributed with mean zero and variance 1.
Using n=12, which is sufficiently according many authors:

$$\xi = \sum_{i=1}^{12} (R_i - 6)$$

If it is desired to generate a normal variate Y with mean μ_y and variance $(\sigma_y)^2$, first generate and then use the transformation:

$$Y = \mu_y + \tau_y \cdot \xi$$

Example:
Service times to strip a container are normally distributed with a mean 21.9 min and a variance $\sigma^2 = 35.1$ minutes. To generate a typical service time, first obtain 12 random numbers:

0.1758	0.1489	0.2774	0.6033	0.9813	0.1052
0.1816	0.7484	0.1699	0.7350	0.6430	0.8830

Then using the previous equations it is possible to obtain:

$$Y = 7.3 + \sqrt{11,7} \left[\sum_{i=1}^{12} R_i - 6\right] = 6.10$$

Many authers recommend this technique for generating approximate normal random variates, but an exact technique, such as the one described in section 6.4.2. is always preferable to an approximate technique. To illustrate one exact generation scheme, consider equations in chapter 6.4.2 with $R_1 = 0.1758$ and $R_2 = 0.1489$. The normal variates are generated as follows:

$$Z_1 = [-2 ln(0.1758)]^{\frac{1}{2}} \cdot \cos(2\pi\ 0.1489) = 1.11$$

$$Z_2 = [-2 ln(0.1758)]^{\frac{1}{2}} \cdot \sin(2\pi\ 0.1489) = 1.50$$

This equation requires one-twelfth the random numbers required in the approximate technique; however, sine, cosine and logarithm calculations are relatively inefficient on a computer.

6.5. Computer simulation models

6.5.1. Procedure to build a model.

The procedure to be followed creating a simulation model is schematically presented in fig. 18.
First the boundaries of the system have to be determined which is of course dependent on the problem to be solved.
In the next step a description of the model has to be given. This means that within the boundaries of the system the reality has to be schematized. The measure of schematization again is dependent on the problem to be solved.
If boundaties have been set and reality schematized, the computer simulation model can be built.

One of the most important and difficult task is the verification and validation of the simulation model.
Verification refers to the questions:
a. Are the imput parameters and logical structure of the model correctly represented and
b. Is the model implemented correctly in computer code.

The goal of the validation process is to produce a model that represents true system behaviour closely enough as a model to be used as a substitute for the actual system for the purpose of experimenting with the system. Validation (tuning of the model) refers to the act of determining a model that is accurate enough to represent a real system. Validation is achieved through calibration of the model, an iterative process of comparing the model to actual system behaviour and using the discrepancies to improve the model.

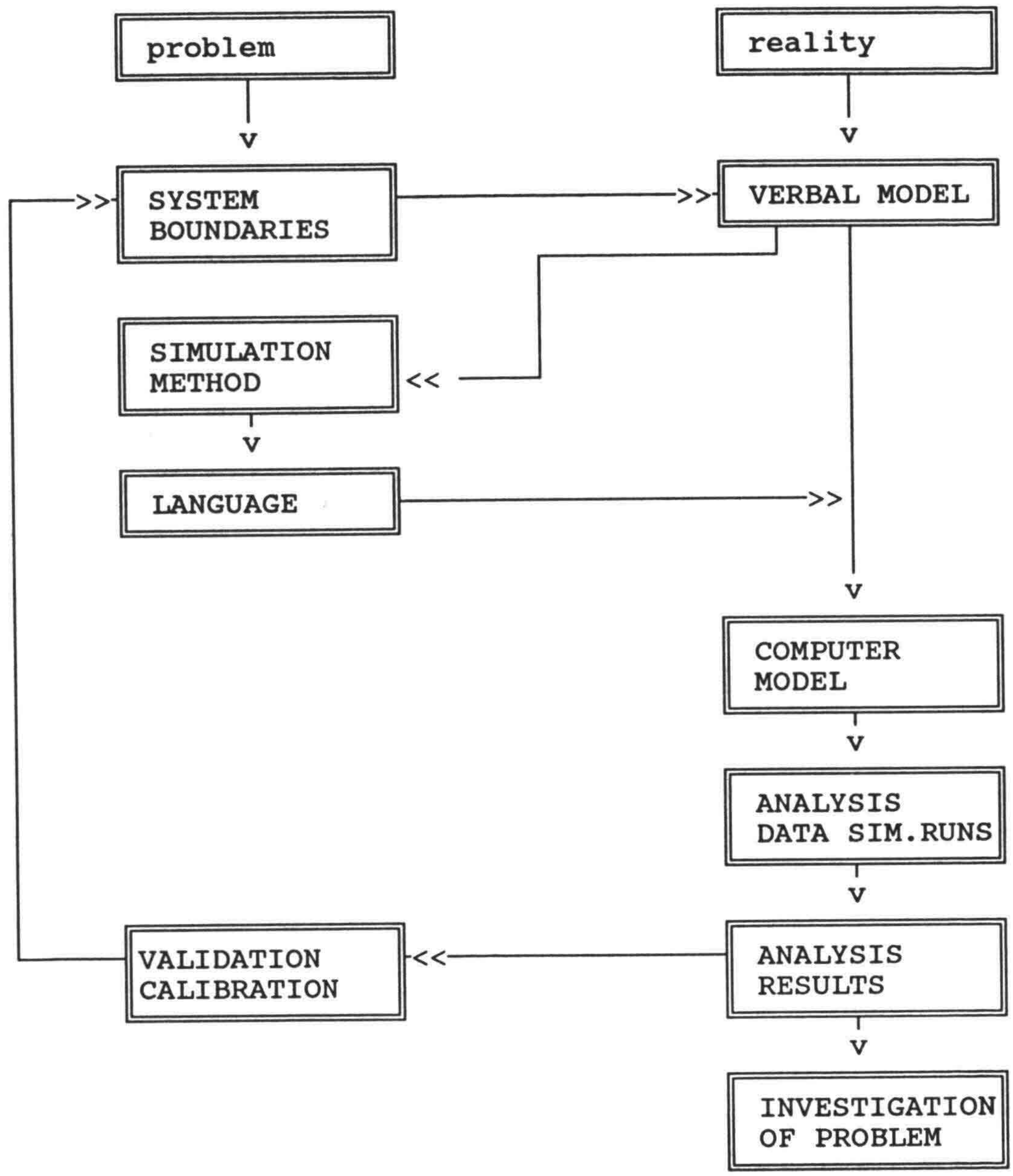


Fig. 18: Steps in the simulation process

6.5.2. Components and Attributes.

Using the process description method the system is considered to be modelled as a set of components that are inter related (see fig. 3 chapter 4.5.)

Prosim (developed at the Delft University of Technology) is using the process description method. The process description method describes the processes of all live components in the model. Live means that these components are executing activities.
Attributes are variables attatched to a component to specify and describe the state of that component.
Components with the same attributes belong to the same class of components; for example the ships in a port system.
So a component will never be an variable or a parameter that will acquire a value.

A ship in a model of a port is a component and belongs to the class of components SHIP and can be specified by the following attributes:

1. LENGTH
2. DRAUGHT
3. AMOUNT OF CARGO
4. ARRIVAL TIME
5. TYPE
6. POSITION
7. BERTH NUMBER

A HARBOURMASTER is, for instance, a single component and does not belong to a class.

6.5.3. The Structure of the Model.

To explain the process description method the simulation language Prosim is used.
Each Prosim model consists of a definition section and a dynamic section.
The definition section consists of 1 module (called DEFINE) and the dynamic section exists of at least 1 module (called MAIN):

1.	DEFINE module	(the definition section)
2.	MAIN module	(part of the dynamic section, mostly used to initialize the system)
3.	Other modules	(part of the dynamic section, necessary to describe the system)

6.5.4. Example of a Simulation Model.

The composition of the model TERMINAL is given in table 20.

The model is using 4 queues:

1. ANCHORAGE; a place where the ship will stay in case entering of the port is not possible.

2. SAILQ; the SAILQ stands for the approach channel. When permission has been granted the ship will leave the ANCHORAGE and will enter the SAILQ
3. BERTHALLC; when a ship is permitted to enter the port a berth will be allocated to that ship. At the same moment a berth was allocated and the ship will enter the queue BERTHALLC. This is necessary to know the number of berths already occupied.
4. BERTH; the queue BERTH will be entered by a ship when a ship has been berthed.

Queues serve also to verify the position of a ship.

Table 20.

COMPONENT	DESCRIPTION	SECTION
DEFINE	Defines: components with attributes, queues, tables, randomstreams time unit, input streams, output streams	DEFINITION
GENERATOR OF SHIPS	generates according distribution functions ships and determines the necessary attributes of that ship	DYNAMIC
TERMINAL MASTER	checks the availability of a berth	DYNAMIC
SHIP (CLASS OF COMPONENTS)	the behaviour of the ship is described in the process of the ship	DYNAMIC

Verbal model.
The terminal master is giving permission to enter the SAILQ (the approach channel) when a berth is available. In this model it is always possible to enter the channel (in reality entering depends on the traffic situation, the tidal conditions, the weather conditions). Allocating berths, the terminal operator will used the so called hunting order (priority of berths). The priority order in this model is berth 1; berth 2; berth 3 and berth 4.
When a ship has been unloaded and loaded (equal to the service time of that ship) the ship will leave the system.

The module DEFINE.
Reference is made to the source text of the module DEFINE of the model TERMINAL in appendix II.

Lines 1 and 2:

Definition of the single components GENERATOR, TERMMAST and the class of components SHIP.

Lines 4 to 6:

Definition of the attributes of the component MAIN:

N, I, J and K are counting variables;

NBBERTH gives the total number of berths belonging to the terminal; the logical BRTH[I] indicates whether a particular berth is free (TRUE) or occupied (FALSE).

Lines 7 to 10:

Definition of the attributes of the class of component SHIP:

SERVICETIME is of course the service time of the ship and will be assigned by the generator of the SHIP in the process of the GENERATOR.

SAILTIME gives the time necessary for the ship to sail from the anchorage to the berth.

SHTURNART, turn around time, gives the time the SHIP spends in the system including the waiting time.

NUMBERSH is a counting variable.

BERTHDEST gives the number of the allocated berth.

BERTHFOUND is a logical which indicates whether a berth has been allocated.

Lines 12 to 14:

Definition of the attributes of the generator.

INTARRTIME gives the inter arrival time between two successive arrivals of ships.

NEXTSHIP is referring to a ship that will be created by the GENERATOR.

Lines 16 to 18:

Definition of the attributes of the component TERMMAST (terminal master).

TERMSHIP is referring to SHIP. The TERMMAST checks the possibility to render the SHIP permission to enter the port.

AVAILBER is an integer variable indicates the number of available berths at a certain point of time.

Line 20:

Line 20 defines the queues as mentioned before.

Line 21:

Line 21 defines the time unit as an hour.

Line 22:

Line 22 defines 2 streams of random numbers (between 0 and 1); INTARRT and SERVICET.

Lines 23 and 24:

Line 23 and 24 are used to define variables for animation purposes which will not be discussed.

The module MAIN

Reference is made to the source text of the module MAIN of the model TERMINAL in appendix II.

The module MAIN is used to initialize the system.

Lines 1 and 2:

The reshape command converts the random stream INTARRT and SERVICET into a distribution function with a mean for the INTARRT and a lower bound of 5, a mean of 24 and a deviation of 3 for the SERVICET.

Line 3:

NBBERTH, the number of berths, is initialized at 4.

Lines 4 to 7

Lines 4 to 7 are used for animation purposes and will not be discussed.

Line 8:

This line activates the GENERATOR from the label GENERATE in the process of the generator called GENPROCESS.

Lines 9 ,10 and 11:

The variable BERTH[I] becomes TRUE for all berths.

Line 12:

The PASSIVATE statement means that the component MAIN will become passive or suspended (a not working state).

The module PROCESS OF THE SHIP

Reference is made to the source text of the module SHIP of the model TERMINAL in appendix II.

This module describes the process of the ship.

Lines 1 to 11:

The SHIP starts her process from the label STARTSHIP, the ship enters the queue ANCHORAGE and activates the TERMMASTER.

The PASSIVATE statement is necessary to enable the TERMMASTER to check the criteria for entrance.

Next the SHWAITT, the waiting time of the SHIP is determined; this waiting time is stored in the set "P" the ship leaves the ANCHORAGE and enters the SAILQ (the approach channel).

The lines 3,7 and 10 are used for animation purposes.

Lines 12 to 26:

After a stay of the SAILTIME in the channel (SAILQ) the ship leaves the SAILQ and enters the BERTH and after the SERVICET (line 21) the ship leaves the BERTH and the BERTHALLC queues.

BRTH[BERTHDEST] becomes TRUE as the BERTH is free again.

The lines 13,17,19,20,24 and 26 are used for animation purposes.

Lines 28 to 32:

The turn around time (SHTURNARRT) is registered and stored in the set "Q". At last the terminal master (TERMMAST) is activated to check whether a next ship can be moored along the quay and the process of the SHIP is TERMINATED in line 32.

The module TERMMPROCESS.

Reference is made to the source text of the module TERMMPROCESS of the model TERMINAL in appendix II.

In the module TERMMPROCESS the process of the TERMMAST (terminal master) is described. The TERMMAST checks the possibility, depending on berth availability, to moor a ship along the quay.

Lines 1 and 2:

The process of the TERMAST starts from the label STTERMM (line 1). In line 2 the TERMMAST takes the first ship from the queue ANCHORAGE and assigns this ship to TERMSHIP.

Lines 3 to 19:

The availability check of a berth starts from the label CHECK.

In line 4 the number of available berths is determined (AVAILBER).

Provided that a ship at present at the ANCHORAGE and one or more berths are available the TERMMAST determines BERTHDEST (number of the berth), changes BERTHFOUND (attribute of ship) to TRUE, changes BRTH[I] to FALSE (means berth I is occupied) and joins the ship under consideration to BERTHALLC (to indicate that from this moment the berth has been assigned to that ship).

Next the TERMMAST takes the next ship from the queue ANCHORAGE (line 16) and repeats from the label CHECK.

Line 20:

When all ships in the queue ANCHORAGE have been checked the TERMMAST stops working (PASSIVATE).

The module GENPROCESS.

Reference is made to the source text of the module GENPROCESS of the model TERMINAL in appendix II.

The generator generates in the process GENPROCESS ships according to a N.E.D inter arrival time pattern.

Lines 1 to 9:

The generator starts from the label GENERATE.

The number of the ship is determined N (for animation purposes) and a number of attributes of the ship will take a value:

BERTHFOUND is FALSE,

NUMBERSH is N

SAILTIME (from ANCHORAGE to the berth) is 1 hour.

Next the ship is activated from the label STARTSHIP in the process of the ship (PROCESSOFTHESHIP).

Lines 10 and 11:

In statement 10 a sample is taken from the distribution INTARRT which represents a N.E.D function (see MAIN module, statement 1) and the GENERATOR INTARRT time units.

After this inter arrival time the generator repeats this process from the label GENERATE and creates the next ship.

7. ANALYSIS OF INPUT AND OUTPUT DATA.

7.1. Introduction.

Output analysis refers to analysis of data generated by a simulation model. Its purpose is to predict the performance of the system or to compare the performance of two or more alternative system designs.
The need for statistical output analysis is based on the observation that the output data from a simulation shows a random variability when random number generators are used to determine the input variables.
First a number of parameters are discussed to evaluate the input (boundary conditions of the model) and output data.

7.2. Characteristics of Distribution Functions

Arithmetic mean.
This is the most common type of average, often referred to simply the "average " or "mean":

$$Mean = \frac{1}{n}\sum_{i=1}^{n} x_i$$

where x_i = a set simulation output,

and n = total number of figures in the set

If the frequency of some observations is greater than 1 then the mean can be written as:

$$mean = \frac{\Sigma f(x_i) \cdot x_i}{\Sigma f(x_i)}$$

Population means.
Population means are important parameters to judge feasibilities of sytems. For instance mean waiting times of vessels for a period of 1 year are important parameters to judge the performance of a port. To estimate the accuracy of this parameter a number of simulation runs have to be carried out.

By placing confidence bands it is possible to calculate the accuracy. Suppose the average waiting time W_{ij} has been determined from one simulation run with a length of 1 year.

In table 21 m sets of n runs are considered where: W_{ij} is the average waiting time of run j in set i;

$$M_i = \frac{\sum_{j=1}^{n} W_{ij}}{n}$$

Because each sample from the Wij population is itself a mean, the central limit theorem holds and normality can be assumed.
This means that the population Mi according to the central limit theorem has a variance

$s = \frac{\sigma^2}{n}$ and an average μ where σ and μ are resp. the standard deviation and average of the population W_{ij}.

Table 21.

sets with n runs	average waiting time during 1 run (n runs per set)	average waiting time in a set
1	W_{11} W_{12} W_{13} ---------- W_{1n}	$M_1 = (W_{11}+W_{12}+W_{13}+$-------$W_{1n})/n$
2	W_{21} W_{22} W_{23} ---------- W_{2n}	$M_2 = (W_{21}+W_{22}+W_{23}+$-------$W_{2n})/n$
3	W_{31} W_{32} W_{33} ---------- W_{3n}	$M_3 = (W_{31}+W_{32}+W_{33}+$-------$W_{3n})/n$
4	W_{41} W_{42} W_{43} ---------- W_{4n}	$M_4 = (W_{41}+W_{42}+W_{43}+$-------$W_{4n})/n$
-		
m	W_{m1} W_{m2} W_{m3} ---------- W_{mn}	$M_m = (W_{m1}+W_{m2}+W_{m3}+$-------$W_{mn})/n$

Suppose we wish to determine an estimate X of the true population mean μ such that:

$$P[\mu-d \leq X \leq \mu+d] = 1-\alpha$$

where X is the sample mean, μ the true population mean and 1-α is the probability that that the interval $\mu \pm d$ contains X

The problem is to determine the number of runs n such that this equation holds.
If the assumption of normality is valid then:

$$n = \frac{(\sigma \cdot Z_{\frac{\alpha}{2}})^2}{d^2}$$

Where $Z_{\alpha/2}$ is the two-tailed normal statistic for the probability we seek.

In many cases the value of must be guessed or run a short pilot experiment. If the lowest and the highest response of the system is known then σ can be estimated because the range is approximately equal to 4σ.

Example:
Suppose we want to estimate the average waiting time with an accuracy of 2 min. and with a probability of 95%.
It is estimated that the range of average waiting times registered during a run covers about 40 min.

So $4\sigma = 40$ min or $\sigma = 10$ d = 2 min and $Z\alpha/2 = 1.96$ (Table II.I).

$$n = \frac{(\sigma \cdot Z_{\frac{\alpha}{2}})^2}{d^2}$$

or $$n = \frac{[10 \cdot 1.96]^2}{2^2}$$

Of course it is also possible rather then estimating the range to use the interval $\mu \pm \frac{\sigma}{4}$

Median.
The median of a set of observations is the middle observation when the observations are ranked or arranged in order of magnitude. Thus a median of 10, 12, 16, 16, 18 is 16. If the number of observations is even the median is taken as the average of the to middle values. For example the median of 17, 21, 22, 22, 26, 31, 31, 34 is (22 + 26)/2 = 24.

Midrange
The mean of the minimum and the maximum of the observations.

Range.
The difference between minimum and maximum of the observations.

Mode.
Mode is the value of observation which occurs most frequently if the variable is discrete or the class interval which has the highest frequency if the distribution is continuous. Thus the mode represents a peak value in a frequency distribution.
Like the median the mode is less affected by extreme values than the mean.

Skewness
The third moment relative to the mean divided by the third power of the standard deviation

$$Skewness = \frac{\sum_{i=1}^{n} (x_i-\mu)^3 \cdot p(x_i)}{\left[\sum_{i=1}^{n} (x_i-\mu)^2 \cdot p(x_i)\right]^{\frac{3}{2}}}$$

The function is skewed to the right when the median is to the right of the mode (tail to the right). Such a curve is said to be positively skewed.

Alternative Skewness
An alternative measure for the skewness is found by subtracting the median from the mean and dividing the result by the standard deviation

$$Skewness = \frac{Mean-Median}{\sqrt{\sum_{i=1}^{n} (x_i-\mu)^2 \cdot p(x_i)}}$$

Kurtosis
The fourth moment relative to the mean divided by the fourth power of the standard deviation.

$$Kurtosis = \frac{\sum_{i=1}^{n} (x_i-\mu)^4 \cdot p(x_i)}{\left[\sum_{i=1}^{n} (x_i-\mu)^2 \cdot p(x_i)\right]^{2}}$$

The kurtosis gives the measure of degree of peakness of a distribution function.
For example the values of a uniform distribution and a normal distribution function are resp. 1.8 and 3.

Covariance

In a manner analogous to the variance a single random variable the covariance can be defined:

$$Cov(x,y) = \sum_{i=1}^{n} [(x_i - u_x) \cdot p(x_i) \cdot (y_i - \mu_y) \cdot p(y_i)]$$

The covariance of x_i and y_i is zero if x_i and y_i are independent.

Correlation

Sometimes an assumption or hypothesis is made about cause and effect relationship between variables. The assumption may or may not be true.

The correlation coefficient informs about this basic assumption.

Correlation coefficients will range from -1 to +1. A correlation coefficient of -1 means a perfect negative correlation. A coefficient of 0 means absolutely no correlation and a coefficient of +1 means a perfect positive correlation.

For a simple linear regression problem the calculation of the coefficient of correlation r is:

$$r = \frac{Cov(x,y)}{\sqrt{Var(x) \cdot Var(y)}} = \frac{Cov(x,y)}{\sigma(x) \cdot \sigma(y)}$$

Auto Correlation.

A typical output variable for instance the total waiting costs of the vessels in a port system during 1 week would be considered as a random variable with an unknown distribution. Thus one week provides just one sample. By taking a run length of n weeks n samples X_1, X_2, X_3, X_4, ------- X_n are collected. In this case the situation of the port system at the end of the week is the same as the situation at the beginning of the next week. Thus the value of X_i has some influence on the value of X_{i+1}.

The sequence of random variables is said to be autocorrelated (i.e. correlated with itself). The required sample sizes are very sensitive to the amount of autocorrelation present within each run. Estimates of the variance in particular are too low if correlation is positive and not taken into account.

basically, there are two methods of dealing with autocorrelated data:

1. Dividing the simulation run into equal subgroups and treat each subgroup as a single independent observation.
2. Estimate the autocorrelation function and include its effects in the estimation of parameters

The second method is considered. During a simulation run n correlated observations, related to time have been carried out.

The estimate of parameters is given by:

$$\mu = \sum_{i=1}^{n} \frac{X_i}{n}$$

$$\rho_{p,x} = \frac{\sum_{i=1}^{n-p} (X_i - \mu) \cdot (X_{i+p})^{-\mu}}{\sigma_x^2} = \frac{Cov(x_i,\ x_{i+p})}{Var\ (x_i)}$$

where:

σ_x^2 = the variance of the population of n observations in the run.

$\rho_{p,x}$ = the lag p coefficient of autocorrelation

As a good rule of thumb the maximum lag for which autocorrelation are computed should be approximately 10 of the number of n observations.

Lag coefficients are used to determine minimum sample sizes.
Geisler [12] shows that the minimum sample size to assure that the estimate of the mean lies within 10% of the true mean with $\alpha = 0.05$

$$n = \frac{(t_{\alpha/2})^2 \cdot var(X_i) \cdot \left[1 + 2 \cdot \sum_{p=1}^{m} \left[(1 - \frac{p}{m+1}) \cdot \rho_{p,x}\right]\right]}{(d\mu)^2}$$

Example:
During a simulation run 500 observations have been carried out with a mean = 205.74 and a variance = 101921.54. Estimates of the lag coefficients were $\rho_{1,x} = 0.3301$, $\rho_{2,x} = 0.2993$ and $\rho_{3,x} = 0.1987$.

The minimum sample size to assure the estimate lies within 10 of the true mean with $\alpha = 0.05$ using for:

$t_{\alpha/2} = 1.96$

$var(x_i) = 101921.54$

$p = 1,2,3$

$m = 3$ the maximum lag for which autocorrelations are computed

$(d\mu)^2 = (0.1)^2 (205.74)^2$

amounts: $n = 1757$

(See appendix 8)

Regression analysis

In a simulation very often output variables are functionally related to one or more input variables.
In trying to establish a mathematical equation to describe the functional relationship between two or more variables the techniques of regression and correlation are very useful. Regression analysis takes a set of data and fits it to an equation whose form is preselected by the analyst. Correlation analysis gives us some indication of how well the data points fit or cluster around the equation derived.
Suppose the functional relationship of the length of a quay X_i and the waiting times of the vessels Y_i using this quay has to be determined. The first step would be to plot these points $(x_1, y_1), (x_2, y_2) \ldots\ldots (x_n, y_n)$.
The analyst must select the curve to be fitted. For reference several common types of approximating curves and the related equations are listed:

Table 22: Approximating curves

1	$y = a_0 + a_1x$	straight line
2	$y = a_1x + a_2x^2$	parabola or quadratic curve
3	y = a0 + a1x + a2x2 + a3x3	cubic curve
4	y = a0 + a1x + a2x2 + a3x3 + a4x4	quartic curve
5	y = a0 + a1x + a2x2 + a3x3 + a4x4 + ... + anxn	nth degree curve
6	y = abx **or** logy = a) + a1x	exponential curve
7	y = 1/(a0 + a1x) **or** 1/y = a0 + a1log x	logarithmic

The method most often used is called the method of the least squares. The measure of the goodness of fit is the value of the sum of the deviations squared. The approximating curve) with a minimum of deviations squared is the best fitting curve.

If a linear relationship is the basic model the equations below are giving the optimum values for a0 and a1.

$$a_0 = \frac{(\Sigma y) \cdot (\Sigma x^2) - (\Sigma x) \cdot (\Sigma x \cdot y)}{n \cdot (\Sigma x^2) - (\Sigma x)^2}$$

$$a_1 = \frac{n \cdot (\Sigma x \cdot y) - (\Sigma x) \cdot (\Sigma y)}{n \cdot (\Sigma x^2) - (\Sigma x)^2}$$

7.3. Identifying distribution functions.

The Kolmogorov Smirnov and the χ^2 (chi square) tests can be used to test the agree of agreement between the distribution function of a set of observations of an input or output variable and some specific theoretical distribution.

Chi square test.
The chi square statistic is give by:

$$\chi^2=\sum_{1}^{k}\frac{(f_o-f_e)^2}{f_e}$$

where
f_0= observed frequency for each class or interval
f_e=expected frequency for each class or interval predicted by a theoretical distribution
k= total classes or intervals

If χ^2=0, then the observed and theoretical frequencies agree exactly, whereas if $\chi^2>0$ they do not. The larger the value of χ^2 the greater is the discrepancy between the observed expected distribution function. If $\chi^2>0$ the calculated value has to be compared against the tabulated value of χ^2 (see table 23) to determine if that much difference could be expected. The χ^2 statistic is tabulated by degrees of freedom versus (1-α) or significance level. If the computed value of χ^2 is greater then the critical or tabulated value at a given significance level and appropriate degrees of freedom the H_0 hypothesis of no significant difference should be rejected.

When using the χ^2 test a number of conditions should be met:

1. Actual frequencies should be used.
2. The expected frequencies for each class or interval should be more then 5.
3. The degrees of freedom are given by ν=k-1-m
   where
   θ=degrees of freedom
   k=number of classes or intervals
   m= number parameters to calculate the expected frequencies.

Example:
Suppose the registered inter arrival times have to be compared against a N.E.D. distribution function: $f(t)=\lambda.e^{-\lambda.t}$ with $\lambda=0.5577$.

Table 23: Chi square test

Inter arrival time	observed frequency f_o	expected frequency f_e	$(f_0-f_e)^2$	$(f_0-f_e)^2/f_e$
0-1	249	252	9	0.036
1-2	162	144	324	2.25
2-3	79	83	16	0.193
3-4	44	47	9	0.192
4-5	22	27	25	0.926
5-6	15	16	1	0.063
6-7	7	9	4	0.444
7-19	12	12	0	0
total				4.104

Degree of freedom:
ν=k-1-m=8-1-1=6

With $\alpha=0.5$ the χ^2 table (see table section) gives the critical value 12.6 so we do not reject H_0 of no significant difference as 12.6 > 4.104

Kolmogorov Smirnov test.

Like the Chi-square test the Kolmogorov-Smirnov test can be used to test the degree of agreement between the distribution of a set of sample data or field observations, or output data and some specified theoretical distribution.
The process is based upon the class in which the theoretical and observed cumulative distributions have the largest absolute deviation. This deviation is then compared to the critical values in the Smirnov's table.

Example:
For demonstration again table 23 of inter arrival times is used.
Again Ho is, "There is no significant difference between the observed data and those which would be given by the N.E.D. with mean=0.5577.
From table 23 the largest absolute deviation is 0.026 in the class 1-2 h.
This value will be tested against the critical value given by the Smirnov's table (see table section).

This gives for n=590 and $\alpha=0.05$ the critical value $\frac{1.36}{\sqrt{n}}=0.056$

Since the largest value is 0.026 the null hypothesis is not rejected.

Table 24: Kolmogorov Smirnov test

Inter arrival time	observed cumulative probability	expected cumulative probability	difference
0-1	0.422	0.427	0.005
1-2	0.697	0.671	0.026
2-3	0.831	0.812	0.019
3-4	0.905	0.892	0.013
4-5	0.942	0.937	0.005
5-6	0.970	0.964	0.006
6-7	0.983	0.979	0.004
total			

See appendix 9

Appendix I[A]

Table Section 1

AVERAGE WAITING OF CUSTOMERS IN THE QUEUE M/M/n, IN UNITS OF AVERAGE SERVICE TIME

utilization (u)	Number of Servers (n)								
	2	3	4	5	6	7	8	9	10
0.1	0.0101	0.0014	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.2	0.0417	0.0103	0.0030	0.0010	0.0003	0.0001	0.0000	0.0000	0.0000
0.3	0.0989	0.0333	0.0132	0.0058	0.0027	0.0013	0.0006	0.0003	0.0002
0.4	0.1905	0.0784	0.0378	0.0199	0.0111	0.0064	0.0039	0.0024	0.0015
0.5	0.3333	0.1579	0.0870	0.0521	0.0330	0.0218	0.0148	0.0102	0.0072
0.6	0.5625	0.2956	0.1794	0.1181	0.0819	0.0589	0.0436	0.0330	0.0253
0.7	0.9608	0.5470	0.3572	0.2519	0.1867	0.1432	0.1128	0.0906	0.0739
0.8	1.7778	1.0787	0.7455	0.5541	0.4315	0.3471	0.2860	0.2401	0.2046
0.9	4.2632	2.7235	1.9693	1.5250	1.2335	1.0285	0.8769	0.7606	0.6687

$$\text{utilization} = \frac{\text{Av.Service Time}}{(\text{n X Av.Arrival Interval})}$$

n = number of servers

Table I

AVERAGE WAITING TIME OF CUSTOMERS IN THE QUEUE M/D/n (IN UNITS OF AVERAGE WAITING TIME)

utilization	Number of Servers (n)								
(u)	2	3	4	5	6	7	8	9	10
0.1	0.0062	0.0009	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.2	0.0242	0.0066	0.0021	0.0007	0.0002	0.0001	0.0000	0.0000	0.0000
0.3	0.0553	0.0201	0.0085	0.0039	0.0019	0.0009	0.0005	0.0002	0.0001
0.4	0.1033	0.0450	0.0227	0.0124	0.0072	0.0043	0.0026	0.0017	0.0011
0.5	0.1767	0.0872	0.0497	0.0307	0.0199	0.0135	0.0093	0.0066	0.0047
0.6	0.2930	0.1584	0.0984	0.0661	0.0467	0.0342	0.0257	0.0197	0.0154
0.7	0.4936	0.2862	0.1897	0.1355	0.1016	0.0788	0.0627	0.0508	0.0419
0.8	0.9030	0.5537	0.3860	0.2890	0.2265	0.1833	0.1519	0.1282	0.1098
0.9	2.0138	1.2887	0.9340	0.7237	0.5848	0.4894	0.4164	0.3606	0.3175

Table II[a]

AVERAGE WAITING TIME OF CUSTOMERS IN THE QUEUE D/M/n (IN UNITS OF AVERAGE SERVICE TIME)

utilization	Number of Servers (n)								
(u)	2	3	4	5	6	7	8	9	10
0.1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.2	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.3	0.0048	0.0008	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.4	0.0223	0.0060	0.0019	0.0007	0.0002	0.0001	0.0000	0.0000	0.0000
0.5	0.0649	0.0239	0.0103	0.0049	0.0024	0.0013	0.0007	0.0004	0.0002
0.6	0.1520	0.0685	0.0360	0.0206	0.0125	0.0079	0.0051	0.0034	0.0023
0.7	0.3257	0.1696	0.1020	0.0665	0.0458	0.0327	0.0240	0.0180	0.0137
0.8	0.7111	0.4114	0.2725	0.1947	0.1461	0.1134	0.0903	0.0734	0.0605
0.9	1.9330	1.2112	0.8612	0.6567	0.5238	0.4310	0.3629	0.3110	0.2703

Table II[b]

Average waiting time of ships in the queue $M/E_1/n$

(In units of average service time)

A: FOR 1 TO 15 BERTHING POINTS

	Number of berthing points														
Utilization	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0.10	.08	.01	0	0	0	0	0	0	0	0	0	0	0	0	0
0.15	.13	.02	0	0	0	0	0	0	0	0	0	0	0	0	0
0.20	.19	.03	.01	0	0	0	0	0	0	0	0	0	0	0	0
0.25	.25	.05	.02	0	0	0	0	0	0	0	0	0	0	0	0
0.30	.32	.08	.03	.01	0	0	0	0	0	0	0	0	0	0	0
0.35	.40	.11	.04	.02	.01	0	0	0	0	0	0	0	0	0	0
0.40	.50	.15	.06	.03	.02	.01	.01	0	0	0	0	0	0	0	0
0.45	.60	.20	.08	.05	.03	.02	.01	0	0	0	0	0	0	0	0
0.50	.75	.26	.12	.07	.04	.03	.02	.01	.01	.01	0	0	0	0	0
0.55	.91	.33	.16	.10	.06	.04	.03	.02	.02	.01	.01	.01	0	0	0

B: FOR 16 TO 30 BERTHING POINTS

	Number of berthing points														
Utilization	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
0.50	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.55	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.60	.01	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.65	.01	.01	.01	.01	.01	.01	0	0	0	0	0	0	0	0	0
0.70	.02	.02	.02	.01	.01	.01	.01	.01	.01	.01	.01	.01	0	0	0
0.75	.04	.04	.03	.03	.03	.02	.02	.02	.02	.02	.02	.01	.01	.01	
0.80	.07	.07	.06	.05	.05	.04	.04	.04	.03	.03	.03	.03	.03	.03	
0.85	.14	.13	.12	.11	.10	.09	.09	.08	.07	.07	.06	.06	.06	.05	
0.90	.28	.26	.24	.22	.21	.19	.18	.17	.16	.15	.14	.14	.13	.12	
0.95	.74	.69	.65	.61	.58	.55	.51	.49	.46	.43	.41	.40	.38	.37	

Table III

AVERAGE WAITING TIME OF CUSTOMERS IN THE QUEUE $E_2/E/n$ (IN UNITS OF AVERAGE SERVICE TIME)

Utilization (u)	Number of Servers (n)								
	2	3	4	5	6	7	8	9	10
0.1	0.0006	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.2	0.0065	0.0011	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.3	0.0235	0.0062	0.0019	0.0007	0.0002	0.0001	0.0000	0.0000	0.0000
0.4	0.0576	0.0205	0.0085	0.0039	0.0019	0.0009	0.0005	0.0003	0.0001
0.5	0.1181	0.0512	0.0532	0.0142	0.0082	0.0050	0.0031	0.0020	0.0013
0.6	0.2222	0.1103	0.0639	0.0400	0.0265	0.0182	0.0128	0.0093	0.0069
0.7	0.4125	0.2275	0.1441	0.0988	0.0712	0.0532	0.0407	0.0319	0.0258
0.8*	0.83	0.46	0.33	0.23	0.19	0.14	0.12	0.09	0.09
0.9*	2.0	1.20	0.92	0.65	0.57	0.44	0.40	0.32	0.30

Table IV

AVERAGE WAITING OF CUSTOMERS IN UNITS OF AVERAGE SERVICE TIME, FOR THE SINGLE SERVER QUEUE $E_k/E_l/1$ WITH UTILISATION $u = 0.1$

$v_a = 1/k$	Value of l for the Erlang Service Distribution = $1/v_s$									
	1	2	3	4	5	6	7	8	9	10
0.1	0.0010	0.0001	0.0001	0.0004	0.0007	0.0012	0.0018	0.0025	0.0033	0.0040
0.2	0.0042	0.0010	0.0004	0.0003	0.0033	0.0002	0.0006	0.0015	0.0076	0.0093
0.3	0.0102	0.0038	0.0023	0.0017	0.0014	0.0012	0.0011	0.0010	0.0010	0.0009
0.4	0.0189	0.0090	0.0064	0.0052	0.0046	0.0042	0.003?	0.0037	0.0035	0.0034
0.5	0.0301	0.0166	0.0128	0.0110	0.0100	0.0093	0.00??	0.0085	0.0082	0.0080
0.6	0.0433	0.0265	0.0214	0.0190	0.0176	0.0167	0.01???	0.0155	0.0151	0.0148
0.7	0.0582	0.0384	0.0321	0.0291	0.0273	0.0261	0.02??	0.0247	0.0242	0.0238
0.8	0.0747	0.0520	0.0446	0.0410	0.0389	0.0374	0.03??	0.0357	0.0351	0.0346
0.9	0.0924	0.0670	0.0587	0.0545	0.0521	0.0504	0.0492	0.0484	0.0477	0.0471
1.0	0.1111	0.0833	0.0741	0.0694	0.0667	0.0648	0.0639	0.0625	0.0617	0.0611

Table V

AVERAGE WAITING TIME, IN UNITS OF AVERAGE SERVICE TIME, FOR THE SINGLES SERVER QUEUE $E_k/E_l/1$ WITH UTILISATION
u = 0.2

	Value of l for the Erlang Service Distribution = $1/v_s$									
$v_a = 1/k$	1	2	3	4	5	6	7	8	9	10
0.1	0.0188	0.0035	0.0013	0.0006	0.0003	0.0002	0.0040	0.0098	0.0109	0.0051
0.2	0.0353	0.0116	0.0064	0.0044	0.0033	0.0027	0.0024	0.0021	0.0019	0.0017
0.3	0.0554	0.0243	0.0162	0.0127	0.0107	0.0095	0.0087	0.0081	0.0076	0.0073
0.4	0.0783	0.0408	0.0301	0.0252	0.0223	0.0205	0.0193	0.0183	0.0176	0.0171
0.5	0.1035	0.0604	0.0474	0.0412	0.0376	0.0353	0.0336	0.0324	0.0315	0.0307
0.6	0.1304	0.0825	0.0675	0.0602	0.0559	0.0531	0.0511	0.0497	0.0485	0.0476
0.7	0.1588	0.1065	0.0898	0.0816	0.0767	0.0735	0.0712	0.0695	0.0682	0.0671
0.8	0.1883	0.1322	0.1140	0.1049	0.0996	0.0960	0.0934	0.0915	0.0901	0.0889
0.9	0.2187	0.1593	0.1397	0.1299	0.1241	0.1202	0.1174	0.1153	0.1137	0.1124
1.0	0.2500	0.1875	0.1667	0.1563	0.1500	0.1458	0.1429	0.1406	0.1389	0.1375

Table VI

AVERAGE WAITING TIME, IN UNITS OF AVERAGE SERVICE TIME, FOR THE SINGLE SERVER QUEUE $E_k/E_l/1$ WITH UTILISATION
u = 0.3

	Value of 1 for the Erlang Service Distribution = $1/v_s$									
$v_a = 1/k$	1	2	3	4	5	6	7	8	9	10
0.1	0.0714	0.0202	0.0093	0.0054	0.0035	0.0025	0.0019	0. ?	0.0053	0.0045
0.2	0.1040	0.0415	0.0253	0.0184	0.0148	0.0125	0.0110	0. ?	0.0091	0.0085
0.3	0.1396	0.0679	0.0475	0.0382	0.0329	0.0296	0.0273	0. ?	0.0243	0.0233
0.4	0.1773	0.0980	0.0742	0.0629	0.0564	0.0522	0.0492	0. ?	0.0453	0.0440
0.5	0.2166	0.1310	0.1043	0.0914	0.0839	0.0789	0.0755	0.0729	0.0708	0.0693
0.6	0.2573	0.1661	0.1371	0.1229	0.1145	0.1090	0.1050	0.1021	0.0998	0.0980
0.7	0.2990	0.2031	0.1720	0.1567	0.1476	0.1415	0.1372	0.1340	0.1315	0.1296
0.8	0.3415	0.2414	0.2086	0.1923	0.1826	0.1761	0.1715	0.1681	0.1654	0.1633
0.9	0.3848	0.2809	0.24?6	0.2295	0.2192	0.2124	0.2075	0.2039	0.2010	0.1988
1.0	0.4286	0.3214	0.2 ?7	0.2679	0.2571	0.2500	0.2449	0.2411	0.2381	0.2357

Table VII

AVERAGE WAITING TIME, IN UNITS OF AVERAGE SERVICE TIME, FOR THE SINGLE SERVER QUEUE $E_k/E_l/1$ WITH UTILISATION
u = 0.4

	Value of l for the Erlang Service Distribution = $1/v_s$									
$v_a = 1/k$	1	2	3	4	5	6	7	8	9	10
0.1	0.1679	0.0596	0.0320	0.0207	0.0149	0.0115	0.0093	0.0078	0.0067	0.0059
0.2	0.2183	0.0979	0.0639	0.0485	0.0400	0.0346	0.0310	0.0283	0.0263	0.0148
0.3	0.2707	0.1408	0.1018	0.0836	0.0731	0.0663	0.0615	0.0580	0.0554	0.0533
0.4	0.3247	0.1869	0.1441	0.1236	0.1116	0.1037	0.0982	0.0941	0.0909	0.0884
0.5	0.3799	0.2355	0.1896	0.1672	0.1541	0.1454	0.1392	0.1346	0.1311	0.1283
0.6	0.4360	0.2859	0.2374	0.2136	0.1995	0.1901	0.1835	0.1785	0.1747	0.1716
0.7	0.4929	0.3378	0.2872	0.2622	0.2472	0.2373	0.2303	0.2250	0.2209	0.2176
0.8	0.5503	0.3909	0.3385	0.3124	0.2968	0.2864	0.2790	0.2735	0.2692	0.2657
0.9	0.6083	0.4451	0.3909	0.3640	0.3478	0.3370	0.3293	0.3236	0.3191	0.3155
1.0	0.6667	0.5000	0.4444	0.4167	0.4000	0.3889	0.3810	0.3750	0.3704	0.3667

Table VIII

AVERAGE WAITING TIME, IN UNITS OF AVERAGE SERVICE TIME, FOR THE SINGLE SERVER QUEUE $E_k/E_l/1$ WITH UTILISATION
u = 0.5

	Value of l for the Erlang Service Distribution = $1/v_s$									
$v_a = 1/k$	1	2	3	4	5	6	7	8	9	10
0.1	0.3246	0.1326	0.0785	0.0545	0.0415	0.0336	0.0282	0.0245	0.0217	0.0195
0.2	0.3962	0.1924	0.1313	0.1029	0.0867	0.0763	0.0691	0.0638	0.0598	0.0566
0.3	0.4692	0.2559	0.1897	0.1581	0.1396	0.1276	0.1192	0.1129	0.1081	0.1043
0.4	0.5432	0.3221	0.2520	0.2150	0.1978	0.1846	0.1753	0.1683	0.1630	0.1587
0.5	0.6180	0.3904	0.3170	0.2810	0.2597	0.2456	0.2356	0.2281	0.2223	0.2177
0.6	0.6935	0.4602	0.3842	0.3467	0.3243	0.3095	0.2990	0.2911	0.2850	0.2801
0.7	0.7696	0.5313	0.4531	0.4142	0.3911	0.3756	0.3647	0.3564	0.3501	0.3450
0.8	0.8460	0.6034	0.5232	0.4833	0.4594	0.4435	0.4322	0.4237	0.4171	0.4118
0.9	0.9229	0.6764	0.5945	0.5537	0.5292	0.5129	0.5012	0.4925	0.4857	0.4803
1.0	1.0000	0.7500	0.6667	0.6250	0.6000	0.5833	0.5714	0.5625	0.5556	0.5500

Table IX

AVERAGE WAITING TIME, IN UNITS OF AVERAGE SERVICE TIME, FOR THE SINGLE SERVER QUEUE $E_k/E_l/1$ WITH UTILISATION

u = 0.6

$v_a = 1/k$	Value of 1 for the Erlang Service Distribution = $1/v_s$									
	1	2	3	4	5	6	7	8	9	10
0.1	0.5786	0.2600	0.1643	0.1200	0.0951	0.0793	0.0684	0.0606	0.0547	0.0501
0.2	0.6786	0.3488	0.2463	0.1973	0.1689	0.1504	0.1375	0.1280	0.1207	0.1149
0.3	0.7996	0.4407	0.3330	0.2808	0.2500	0.2299	0.2156	0.2050	0.1968	0.1903
0.4	0.8813	0.5348	0.4232	0.3684	0.3360	0.3146	0.2994	0.2880	0.2793	0.2723
0.5	0.9835	0.6306	0.5157	0.4590	0.4253	0.4029	0.3870	0.3751	0.3659	0.3586
0.6	1.0862	0.7278	0.6102	0.5519	0.5171	0.4940	0.4775	0.4652	0.4556	0.4480
0.7	1.1892	0.8259	0.7061	0.6465	0.6108	0.5871	0.5702	0.5576	0.5477	0.5399
0.8	1.2926	0.9249	0.8031	0.7424	0.7061	0.6819	0.6646	0.6516	0.6416	0.6335
0.9	1.3962	1.0247	0.9012	0.8395	0.8025	0.7778	0.7603	0.7471	0.7369	0.7287
1.0	1.5000	1.1250	1.0000	0.9375	0.9000	0.8750	0.8571	0.8438	0.8333	0.8250

Table X

AVERAGE WAITING TIME, IN UNITS OF AVERAGE SERVICE TIME, FOR THE SINGLE SERVER QUEUE $E_k/E_l/1$ WITH UTILISATION

u = 0.7

$v_a = 1/k$	Value of 1 for the Erlang Service Distribution = $1/v_s$									
	1	2	3	4	5	6	7	8	9	10
0.1	1.0198	0.4908	0.3252	0.2462	0.2006	0.1711	0.1506	0.1355	0.1240	0.1150
0.2	1.1642	0.6248	0.4526	0.3689	0.3198	0.2876	0.2649	0.2480	0.2350	0.2247
0.3	1.3093	0.7612	0.5840	0.4970	0.4456	0.4116	0.3875	0.3695	0.3556	0.3445
0.4	1.4548	0.8994	0.7183	0.6288	0.5756	0.5404	0.5153	0.4966	0.4821	0.4705
0.5	1.6006	1.0391	0.8547	0.7633	0.7088	0.6726	0.6468	0.6275	0.6125	0.6005
0.6	1.7468	1.1798	0.9927	0.8997	0.8442	0.8072	0.7808	0.7611	0.7458	0.7335
0.7	1.8932	1.3214	1.1321	1.0378	0.9813	0.9437	0.9169	0.8968	0.8812	0.8687
0.8	2.0397	1.4637	1.2724	1.1770	1.1198	1.0817	1.0545	1.0342	1.0183	1.0056
0.9	2.1865	1.6066	1.4136	1.3172	1.2595	1.2209	1.1934	1.1728	1.1568	1.1439
1.0	2.3333	1.7500	1.? 6	1.4583	1.4000	1.3611	1.3333	1.3125	1.2963	1.2833

Table XI

AVERAGE WAITING TIME, IN UNITS OF AVERAGE SERVICE TIME, FOR THE SINGLE SERVER QUEUE $E_k/E_l/1$ WITH UTILISATION
u = 0.8

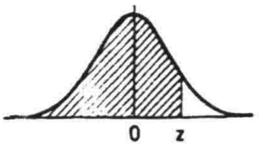
υ_a = 1/k	Value of 1 for the Erlang Service Distribution = $1/\upsilon_s$									
	1	2	3	4	5	6	7	8	9	10
0.1	1.9222	0.9744	0.6692	0.5205	0.3916	0.3758	0.3355	0.3056	0.2826	0.2644
0.2	2.1523	1.1947	0.8831	0.7299	0.6391	0.5791	0.5366	0.5049	0.4803	0.4608
0.3	2.3826	1.4168	1.1005	0.9440	0.8509	0.7891	0.7452	0.7124	0.6869	0.6667
0.4	2.6132	1.6405	1.3203	1.1614	0.0665	1.0035	0.9586	0.9250	0.8989	0.8781
0.5	2.8440	1.8653	1.5419	1.3811	1.2849	1.2209	1.1753	1.1411	1.1146	1.0934
0.6	3.0750	2.0910	1. ?	1.6026	1.5053	1.4406	1.3944	1.3598	1.3329	1.3114
0.7	3.3061	2.3174	1. ?	1.8254	1.7273	1.6620	1.6153	1.5804	1.5532	1.5351
0.8	3.3573	2.5444	2. ?	2.0494	1.9506	1.8847	1.8377	1.8025	1.7750	1.7531
0.9	3.7686	2.7720	2.4402	2.2743	2.1749	2.1086	2.0612	2.0257	1.9981	1.9760
1.0	4.0000	3.0000	2.6667	2.5000	2.4000	2.3333	2.2857	2.2500	2.2222	2.2000

Table XII

AVERAGE WAITING TIME, IN UNITS OF AVERAGE SERVICE TIME, FOR THE SINGLE SERVER QUEUE $E_k/E_l/1$ WITH UTILISATION
u = 0.9

υ_a = 1/k	Value of 1 for the Erlang Service Distribution = $1/\upsilon_s$									
	1	2	3	4	5	6	7	8	9	10
0.1	4.6603	2.4602	1.7374	1.3800	1.1674	1.0266	0.9267	0.8522	0.7944	0.7484
0.2	5.1421	2.9330	2.2042	1.8424	1.6265	1.4831	1.3811	1.3047	1.2454	1.1981
0.3	5.6241	3.4072	2.6738	2.3089	2.0907	1.9456	1.8422	1.7647	1.7045	1.6564
0.4	6.1061	3.8827	3.1456	2.7783	2.5584	2.4120	2.3076	2.2294	2.1685	2.1199
0.5	6.5883	4.3590	3.6189	3.2497	3.0285	2.8811	2.7760	2.6972	2.6359	2.5869
0.6	7.0705	4.8362	4.0935	3.7227	3.5004	3.3523	3.2466	3.1674	3.1058	3.0565
0.7	7.5528	5.3139	4.5690	4.1969	3.9738	3.8251	3.7189	3.6393	3.5774	3.5279
0.8	8.0352	5.7922	5.0454	4.6722	4.4483	4.2991	4.1926	4.1127	4.0505	4.0008
0.9	8.5176	6.2709	5.5224	5.1482	4.9238	4.7741	4.6673	4.5871	4.5248	4.4750
1.0	9.0000	6.7500	6.0000	5.6250	5.4000	5.2500	5.1429	5.0625	5.0000	4.9500

Table XIII



Area under the standard normal curve from $-\infty$ to z.

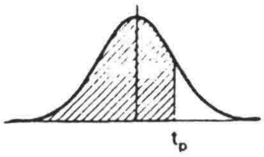
z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767

Area under the standard normal curve from $-\infty$ to z (*cont.*).

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

Table II-I

Values of students t distribution with ν degrees of freedom (shaded area = $p = 1 - \alpha$).



ν	$t_{0.995}$	$t_{0.99}$	$t_{0.975}$	$t_{0.95}$	$t_{0.90}$
1	63.66	31.82	12.71	6.31	3.08
2	9.92	6.96	4.30	2.92	1.89
3	5.84	4.54	3.18	2.35	1.64
4	4.60	3.75	2.78	2.13	1.53
5	4.03	3.36	2.57	2.02	1.48
6	3.71	3.14	2.45	1.94	1.44
7	3.50	3.00	2.36	1.90	1.42
8	3.36	2.90	2.31	1.86	1.40
9	3.25	2.82	2.26	1.83	1.38
10	3.17	2.76	2.23	1.81	1.37
11	3.11	2.72	2.20	1.80	1.36
12	3.06	2.68	2.18	1.78	1.36
13	3.01	2.65	2.16	1.77	1.35
14	2.98	2.62	2.14	1.76	1.34
15	2.95	2.60	2.13	1.75	1.34
16	2.92	2.58	2.12	1.75	1.34
17	2.90	2.57	2.11	1.74	1.33
18	2.88	2.55	2.10	1.73	1.33
19	2.86	2.54	2.09	1.73	1.33
20	2.84	2.53	2.09	1.72	1.32
21	2.83	2.52	2.08	1.72	1.32
22	2.82	2.51	2.07	1.72	1.32
23	2.81	2.50	2.07	1.71	1.32
24	2.80	2.49	2.06	1.71	1.32
25	2.79	2.48	2.06	1.71	1.32
26	2.78	2.48	2.06	1.71	1.32
27	2.77	2.47	2.05	1.70	1.31
28	2.76	2.47	2.05	1.70	1.31
29	2.76	2.46	2.04	1.70	1.31
30	2.75	2.46	2.04	1.70	1.31
40	2.70	2.42	2.02	1.68	1.30
60	2.66	2.39	2.00	1.67	1.30
120	2.62	2.36	1.98	1.66	1.29
∞	2.58	2.33	1.96	1.645	1.28

Table II-II

Kolmogorov-Smirnov critical values.

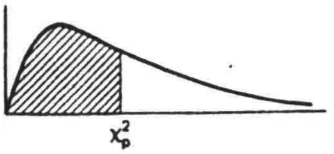
Degrees of Freedom (N)	One Sample Test*			Two Sample Test†	
	$D_{0.10}$	$D_{0.05}$	$D_{0.01}$	$D_{0.05}$	$D_{0.01}$
1	0.950	0.975	0.995		
2	0.776	0.842	0.929		
3	0.642	0.708	0.828		
4	0.564	0.624	0.733	1.000	1.000
5	0.510	0.565	0.669	1.000	1.000
6	0.470	0.521	0.618	0.833	1.000
7	0.438	0.486	0.577	0.857	0.857
8	0.411	0.457	0.543	0.750	0.875
9	0.388	0.432	0.514	0.667	0.778
10	0.368	0.410	0.490	0.700	0.800
11	0.352	0.391	0.468	0.636	0.727
12	0.338	0.375	0.450	0.583	0.667
13	0.325	0.361	0.433	0.538	0.692
14	0.314	0.349	0.418	0.571	0.643
15	0.304	0.338	0.404	0.533	0.600
16	0.295	0.328	0.392	0.500	0.625
17	0.286	0.318	0.381	0.471	0.588
18	0.278	0.309	0.371	0.500	0.556
19	0.272	0.301	0.363	0.474	0.526
20	0.264	0.294	0.356	0.450	0.550
25	0.24	0.27	0.32	0.40	0.48
30	0.22	0.24	0.29	0.37	0.43
35	0.21	0.23	0.27	0.34	0.39
Over 35	$\frac{1.22}{\sqrt{N}}$	$\frac{1.36}{\sqrt{N}}$	$\frac{1.63}{\sqrt{N}}$	$1.36\sqrt{\frac{n_1+n_2}{n_1 n_2}}$	$1.63\sqrt{\frac{n_1+n_2}{n_1 n_2}}$

*Used for testing goodness of fit of a sample to a theoretical distribution where N = sample size.

†Used to determine if two samples are from the same distribution. For small samples (up to 35), $N = n_1 = n_2$.

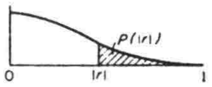
Table II-III

Percentile values (χ^2_p) for the chi-square distribution, with ν degrees of freedom (shaded area = p).



ν	$\chi^2_{0.995}$	$\chi^2_{0.99}$	$\chi^2_{0.975}$	$\chi^2_{0.95}$	$\chi^2_{0.90}$
1	7.88	6.63	5.02	3.84	2.71
2	10.60	9.21	7.38	5.99	4.61
3	12.84	11.34	9.35	7.81	6.25
4	14.96	13.28	11.14	9.49	7.78
5	16.7	15.1	12.8	11.1	9.2
6	18.5	16.8	14.4	12.6	10.6
7	20.3	18.5	16.0	14.1	12.0
8	22.0	20.1	17.5	15.5	13.4
9	23.6	21.7	19.0	16.9	14.7
10	25.2	23.2	20.5	18.3	16.0
11	26.8	24.7	21.9	19.7	17.3
12	28.3	26.2	23.3	21.0	18.5
13	29.8	27.7	24.7	22.4	19.8
14	31.3	29.1	26.1	23.7	21.1
15	32.8	30.6	27.5	25.0	22.3
16	34.3	32.0	28.8	26.3	23.5
17	35.7	33.4	30.2	27.6	24.8
18	37.2	34.8	31.5	28.9	26.0
19	38.6	36.2	32.9	30.1	27.2
20	40.0	37.6	34.2	31.4	28.4
21	41.4	38.9	35.5	32.7	29.6
22	42.8	40.3	36.8	33.9	30.8
23	44.2	41.6	38.1	35.2	32.0
24	45.6	43.0	39.4	36.4	33.2
25	49.6	44.3	40.6	37.7	34.4
26	48.3	45.6	41.9	38.9	35.6
27	49.6	47.0	43.2	40.1	36.7
28	51.0	48.3	44.5	41.3	37.9
29	52.3	49.6	45.7	42.6	39.1
30	53.7	50.9	47.0	43.8	40.3
40	66.8	63.7	59.3	55.8	51.8
50	79.5	76.2	71.4	67.5	63.2
60	92.0	88.4	83.3	79.1	74.4
70	104.2	100.4	95.0	90.5	85.5
80	116.3	112.3	106.6	101.9	96.6
90	128.3	124.1	118.1	113.1	107.6
100	140.2	135.8	129.6	124.3	118.5

Table II-IV



Values of Correlation Coefficient r

ν	5 Percent Level of Significance				1 Percent level of Significance				ν
	Total number of variables				Total number of variables				
	2	3	4	5	2	3	4	5	
1	.997	.999	.999	.999	1.000	1.000	1.000	1.000	1
2	.950	.975	.983	.987	.990	.995	.997	.998	2
3	.878	.930	.950	.961	.959	.976	.983	.987	3
4	.811	.881	.912	.930	.917	.949	.962	.970	4
5	.754	.836	.874	.898	.874	.917	.937	.949	5
6	.707	.795	.839	.867	.834	.886	.911	.927	6
7	.666	.758	.807	.838	.798	.855	.885	.904	7
8	.632	.726	.777	.811	.765	.827	.860	.882	8
9	.602	.697	.750	.786	.735	.800	.836	.861	9
10	.576	.671	.726	.763	.708	.776	.814	.840	10
11	.553	.648	.703	.741	.684	.753	.793	.821	11
12	.532	.627	.683	.722	.661	.732	.773	.802	12
13	.514	.608	.664	.703	.641	.712	.755	.785	13
14	.497	.590	.646	.686	.623	.694	.737	.768	14
15	.482	.574	.630	.670	.606	.677	.721	.752	15
16	.468	.559	.615	.655	.590	.662	.706	.738	16
17	.456	.545	.601	.641	.575	.647	.691	.724	17
18	.444	.532	.587	.628	.561	.633	.678	.710	18
19	.433	.520	.575	.615	.549	.620	.665	.698	19
20	.423	.509	.563	.604	.537	.608	.652	.685	20
21	.413	.498	.552	.592	.526	.596	.641	.674	21
22	.404	.488	.542	.582	.515	.585	.630	.663	22
23	.396	.479	.532	.572	.505	.574	.619	.652	23
24	.388	.470	.523	.562	.496	.565	.609	.642	24
25	.381	.462	.514	.553	.487	.555	.600	.633	25
26	.374	.454	.506	.545	.478	.546	.590	.624	26
27	.367	.446	.498	.536	.470	.538	.582	.615	27
28	.361	.439	.490	.529	.463	.530	.573	.606	28
29	.355	.432	.482	.521	.456	.522	.565	.598	29
30	.349	.426	.476	.514	.449	.514	.558	.591	30
35	.325	.397	.445	.482	.418	.481	.523	.556	35
40	.304	.373	.419	.455	.393	.454	.494	.526	40
45	.288	.353	.397	.432	.372	.430	.470	.501	45
50	.273	.336	.379	.412	.354	.410	.449	.479	50
60	.250	.308	.348	.380	.325	.377	.414	.442	60
70	.232	.286	.324	.354	.302	.351	.386	.413	70
80	.217	.269	.304	.332	.283	.330	.362	.389	80
90	.205	.254	.288	.315	.267	.312	.343	.368	90
100	.195	.241	.274	.300	.254	.297	.327	.351	100
125	.174	.216	.246	.269	.228	.266	.294	.316	125
150	.159	.198	.225	.247	.208	.244	.270	.290	150
200	.138	.172	.196	.215	.181	.212	.234	.253	200
300	.113	.141	.160	.176	.148	.174	.192	.208	300
400	.098	.122	.139	.153	.128	.151	.167	.180	400
500	.088	.109	.124	.137	.115	.135	.150	.162	500
1,000	.062	.077	.088	.097	.081	.096	.106	.116	1,000

The critical value of r at a given level of significance, total number of variables, and degrees of freedom ν, is read from the table. If the computed $|r|$ exceeds the critical value, then the null hypothesis that there is no association between the variables is rejected at the given level. The test is an equal-tails test, since we are usually interested in either positive or negative correlation. The shaded portion of the figure is the stipulated probability as a level of significance.

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Table II-V

Appendix II
Simulation model TERMINAL

```
COMPONENT   : GENERATOR TERMMAST
CLASS       : SHIP

ATTRIBUTE OF MAIN:
                INTEGER: N I J K NBBERTH
                LOGICAL: BRTH[4]
ATTRIBUTE OF SHIP :
                  REAL : SERVICETIME SAILTIME  SHTURNARRT SHWAITT
                INTEGER: NUMBERSH BERTHDEST
                LOGICAL:BERTHFOUND

ATTRIBUTE OF GENERATOR :
               REAL     : INTARRTIME
    REFERENCE TO SHIP   : NEXTSHIP

ATTRIBUTE OF TERMMAST :
     REFERENCE TO SHIP  : TERMSHIP
               INTEGER  : AVAILBER

QUEUE   : ANCHORAGE SAILQ BERTH BERTHALLC
TIMEUNIT : HOUR
RANDOMSTREAM : INTARRT  SERVICET
FIGURE        : TURNARRT WAITT QUAYOCC  POS[100] BD[100]
FIGURE        : ANCHOR
```

MODEL TERMINAL
MOD MAINMOD

Date: 91/10,
Time: 13:00

```
RESHAPE INTARRT AS SAMPLED FROM DISTRIBUTION EXPONENTIAL WITH
PARAMETERS  MEAN(7)
RESHAPE SERVICET AS SAMPLED FROM DISTRIBUTION GAMMASHAPE WITH
PARAMETERS  LB(15) MEAN(24) DEVIATION(3)
NBBERTH←4
MOVE TURNARRT TO SINK
MOVE WAITT TO SINK
MOVE QUAYOCC TO SINK
MOVE ANCHOR TO SINK
ACTIVATE GENERATOR FROM GENERATE IN GENPROCESS
FOR I←1 TO NBBERTH
 BRTH[I]←TRUE
END
PASSIVATE
```

```
STARTSHIP:
ENTER ANCHORAGE
MOVE ANCHOR TO LENGTH OF ANCHORAGE
ACTIVATE TERMMAST FROM STTERMM IN TERMMPROCESS IF TERMMAST IS NOT ACTIV]
PASSIVATE
SHWAITT←NOW-ARRIVALTIME
MOVE WAITT TO SHWAITT
STORE SHWAITT AS "P"
LEAVE ANCHORAGE
MOVE ANCHOR TO LENGTH OF ANCHORAGE
ENTER SAILQ
FOR J←1 TO 4
 MOVE POS[NUMBERSH] TO J
 WORK SAILTIME÷4
END
LEAVE SAILQ
MOVE POS[NUMBERSH] TO SINK
ENTER BERTH
MOVE BD[NUMBERSH] TO BERTHDEST
MOVE QUAYOCC TO LENGTH OF BERTH
WORK SERVICETIME
BRTH[BERTHDEST]←TRUE
LEAVE BERTH
MOVE BD[NUMBERSH] TO SINK
LEAVE BERTHALLC
MOVE QUAYOCC TO LENGTH OF BERTH

SHTURNARRT←NOW-ARRIVALTIME
STORE SHTURNARRT AS "Q"
MOVE TURNARRT TO SHTURNARRT
ACTIVATE TERMMAST FROM STTERMM IN TERMMPROCESS IF TERMMAST IS NOT ACTIV]
TERMINATE
```

```
STTERMM :
TERMSHIP < FIRST SHIP IN ANCHORAGE
CHECK :
AVAILBER<NBBERTH-LENGTH OF BERTHALLC
IF TERMSHIP IS NOT NONE ^ (AVAILBER>0)
 REACTIVATE TERMSHIP
 FOR I<1 TO NBBERTH
  IF BRTH[I]=TRUE
   IF BERTHFOUND OF TERMSHIP=FALSE
    BERTHDEST OF TERMSHIP<I
    BRTH[I]<FALSE
    BERTHFOUND OF TERMSHIP<TRUE
    JOIN TERMSHIP TO BERTHALLC
   END
  END
 END
 TERMSHIP < SUCC OF TERMSHIP IN ANCHORAGE
 REPEAT FROM CHECK
END
PASSIVATE
```

```
GENERATE:
NEXTSHIP < NEW SHIP
N<N+1
N<1 IF N>100
BERTHFOUND OF NEXTSHIP<FALSE
NUMBERSH OF NEXTSHIP<N
SERVICETIME OF NEXTSHIP < SERVICET
SAILTIME OF NEXTSHIP < 1
ACTIVATE NEXTSHIP FROM STARTSHIP IN PROCESSOFTHESHIP
WAIT INTARRT
REPEAT FROM GENERATE
```

PERSONAL PROSIM STATISTICS SUPPLEMENT ═ FILE MMNKADE = STREAM INTARRT
Licensed to TUD Faculteit der Civiele Techniek, vakgroep Waterbouwkunde

== BASIC STATISTICS ==

Mean :	1.775808	Number of entries:	590
St. deviation:	1.863449		
Mn. deviation:	1.278229	Minimum :	0.000972
Skewness :	2.861250	Maximum :	18.147995
Alt. Skewness:	0.278524	Range :	18.147023
Kurtosis :	17.370150	Midrange:	9.074484

Confidence intervals for the mean: 90% (1.649 - 1.902)
95% (1.625 - 1.927)

Correlation according to the independant

Linear (Pierson): 0.02
Rank (Spearman) : 6.94E˜3
Lags (max= 164) : 20

Percentiles

10.000 0.232455 Median: 1.256792 90.000 3.898927

═ F3:Correlation ═ F4:Kolmogorov-Smirnov ═ F5:Percentiles ═ F10:Quit
<&10H

PERSONAL PROSIM HISTOGRAM FACILITY = MODEL KADE == SELECTION INTARRT
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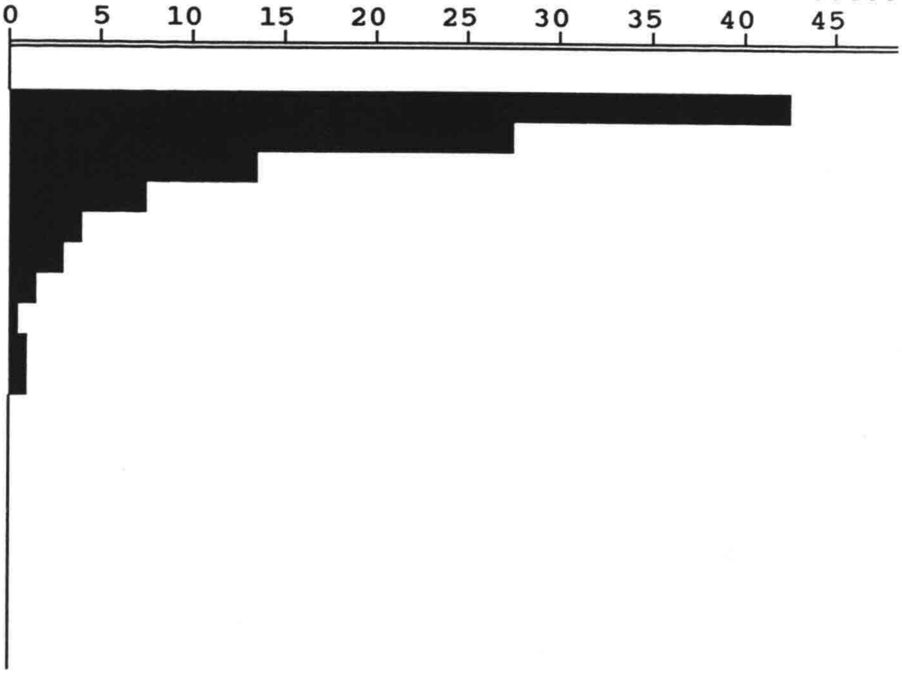
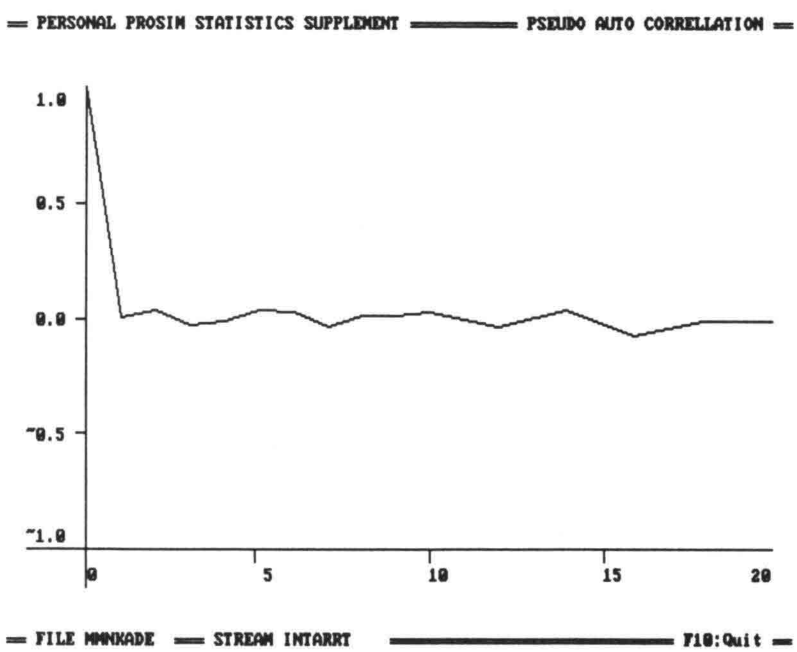
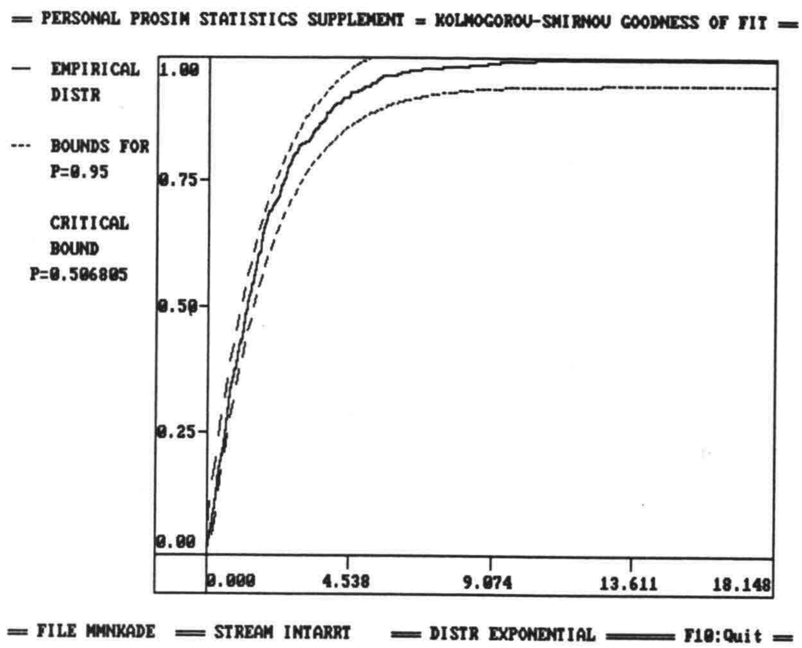
Mean :	1.775807	Minimum:	0.000
Deviation:	1.863449	Maximum:	18.147
90% :	3.931818	95% :	5.300

Upper Bound	Cum Perc	590 Entries
0.0000	0.00	0
1.0000	42.20	249
2.0000	69.66	162
3.0000	83.05	79
4.0000	90.51	44
5.0000	94.24	22
6.0000	96.78	15
7.0000	97.97	7
8.0000	98.31	2
9.0000	98.98	4
10.0000	99.49	3
11.0000	99.66	1
12.0000	99.66	0
13.0000	99.66	0
14.0000	99.83	1
15.0000	99.83	0
16.0000	99.83	0
17.0000	99.83	0
18.0000	99.83	0
19.0000	100.00	1





auto corr. versus lag value (p)


= PERSONAL PROSIM STATISTICS SUPPLEMENT = KOLMOGOROV-SMIRNOV GOODNESS OF FIT =
— EMPIRICAL DISTR
--- BOUNDS FOR P=0.95
CRITICAL BOUND P=0.506805
1.00
0.75
0.50
0.25
0.00
0.000
4.538
9.074
13.611
18.148
= FILE MMNKADE = STREAM INTARRT = DISTR EXPONENTIAL = F10:Quit =

934185