Train Speed Profile tracking using Linear Quadratic Regulator

CIE 5050-09 Additional graduation work, research project

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Contents

Abstract

In order to track the speed profile, a case study using a linear quadratic regulator (LQR) is addressed in this paper. A time-invariant nonlinear train motion model is constructed. To apply LQR, the quadratic term of the aerodynamic drag and the rolling mechanical resistance is linearized. Gradient resistance, curve resistance, and other environmental disturbance are not considered in this paper. An LQR algorithm with constraints is proposed to track a given reference speed profile. Maximum error and cumulative error are used in the evaluation of the tracking performance. The tracking results in plots, parameters tuning, and performance analysis are shown. The results show that the proposed LQR algorithm is able to track the given 1200 seconds speed profile, the cumulative error can be controlled within $\mathbf{4.7478}\boldsymbol{m/s}$, and the maximum absolute error is $\mathbf{0.90839}\boldsymbol{m/s}$. The output of the control variable $\boldsymbol{u}$ is also given.

1 Introduction

The railway industry has been a crucial mode of transportation for many years, serving millions of passengers and transporting large quantities of goods. With the increasing demands for higher efficiency, safety, and reliability, Automated Train Operation (ATO) may be a potential solution for satisfying the growing demand through increasing frequency and without building new lines and tracks.

ATO aims to improve the efficiency and reduce the energy consumption of railway traffic operations by automatically generating and following the reference speed profile that consists of a combination of driving regimes accelerating, coasting, cruising, and braking commands. This not only benefits the public transport service level including punctuality, reliability, and parking precisely but also will essentially be the backbone of a climate-compatible European transport system catering to the policy of a major reduction in pollution and carbon emissions from the European Union.

It is believed that, initially, the driver has to cautiously operate the control handle of the train. The next level of train operation involves manually following a speed profile, which is generated by matching with the schedule satisfying a certain punctuality level. The reasonable speed limits are determined by the European Train Control System (ETCS). At this level, reliability and punctuality can be improved by following an optimal speed profile. While ATO allows the machine to automatically follow the optimal speed profile, which can result in a smaller buffer time and bandwidth, and predictability, therefore leading to a higher frequency and capacity for public service.

According to TNO (Poulus et al. (2018)), from GoA2 onwards, ATO is able to optimize capacity utilization by taking over braking and acceleration. As a result, trains with ATO can follow the curve of one another more closely for shorter headways which allows for more trains on the same amount of rail infrastructure. In the UK, Thameslink with ATO proposes to drive 24 trains per hour which are 70% more seats through the city center of London. Thameslink also shows that ATO can minimize energy consumption by coasting before braking to save energy. Alstom's simulation tests show that energy can be reduced by 15% on intercity lines, and 45% on regional trains. There are other benefits such as reducing operational costs, operational flexibility, potential safety improvement, punctuality improvement, and service level improvement.

This paper mainly focuses on reference speed profile tracking in ATO using the Linear Quadratic Regulator algorithm. The paper structure is as follows. Section 2 presents the literature review of algorithms for speed profile tracking and the justification for the research gap. Section 3 shows the mathematical formulation of the train motion. Section 4 describes the general LQR and shows the constraints and calculations in LQR for the train speed profile tracking. Section 5 carries out the implementation using Matlab, gives a demonstration and case study, and discusses the reference speed profile, parameters, and evaluation of performance. Future research directions and the conclusion is presented in section 6.

2 Literature review

As discussed in the introduction, ATO has so many benefits, in terms of how to achieve speed profile tracking, several algorithms have been developed in recent years. These are model predictive control Moaveni et al. (2020), iterative learning control Sun et al. (2011), adaptive iterative learning control Ji et al. (2015) Li et al. (2021) Sun et al. (2012), finite-iteration adaptive ILC(Iterative Learning Control) Yu et al. (2022), neural network Claviere et al. (2019), deep neural network Li et al. (2017), adaptive controller based on neural network and PID control Pu et al. (2020), RMP(Robust Model Predictive) anti-slip controller based on the LMI (Linear Matrix Inequality) Molavi and Rashidi Fathabadi (2022), fuzzy logic control Zhu et al. (2020) Moaveni et al. (2022), Multi-Modal Fuzzy PID (MM-FPID) control Yang et al. (2017), PID control Xu et al. (2019), adaptive terminal sliding mode controller (ATSMC) Wang et al. (2020) and etc.

This paper is interested in how to successfully track the speed profile smoothly and control the error at a certain level to improve performance. Linear Quadratic Regulator (LQR) is a suitable algorithm to linearize the quadratic terms in a nonlinear train motion model. Therefore, this paper aims to use the LQR algorithm to track the given reference speed profile. One of the key points is linearizing the nonlinear term of aerodynamic drag and rolling mechanical resistances, another key point is fine-tuning the cost function weights. The research is executed using Matlab. The literature review of LQR algorithm is discussed in the following.

Gruber and Bayoumi (1982) designed an LQR controller to optimize the in-train forces and/or speed deviation from the reference speed. The paper gave weights matrices Q and R for throttling and braking representatively.

Ahmed and Bayoumi (1983) designed a controller that achieves asymptotic tracking of a given velocity profile and minimizes a quadratic performance index subject to physical and practical constraints for freight trains. The multivariable proportional + integral controller was built using the standard linear quadratic regulator theory.

Chou and Xia (2007) employed an LQR controller to minimize the in-train force, fuel consumption, and traveling time. The cost function weight R determined the fuel consumption as well as brake usage while Q was to penalize the in-train forces experienced by the couplers as well as the traveling speed tracking of the whole train. This paper showed the tracking results of the open-loop controller, the closed-loop controller with generic tuning parameters, the closed-loop controller optimized for velocity tracking, the closed-loop controller optimized for in-train force, and the closed-loop controller optimized for energy usage.

Ide et al. (2013) showed an observer-based LQR in order to stabilize the system and solve the reference tracking problem. In the paper, the controller gain Kc is evaluated using LQR while the weighting matrices were chosen with the purpose of ensuring a fast response of the first state.

Tian et al. (2021) developed a linear quadratic Gaussian (LQG) optimal control which is used for trajectory tracking in order to achieve the optimal output of the system while anti-disturbance. The LQG control combined linear quadratic regulator (LQR) and Kalman filter to improve the robustness of the formation system.

As was mentioned above, LQR is popular when dealing with nonlinear train motion models. However, most of the existing research focuses on the overall tracking, there is a research gap in studying how the different parameters affect the detail performance for different regimes of the speed profile. It is almost certain that no existing research gives specific cost function weights in LQR for the coasting regime in the train speed profile tracking research. The LQR controller in Chou and Xia (2007) is referred to in the paper. One of the differences is this paper only considers penalizing the in-train force and the speed. Another difference is this paper is using a time scale for the tracking problem while in Chou and Xia (2007) was using a distance scale. Another difference is, in this paper, the speed profile is divided into different regimes of acceleration, cruising, coasting, and braking. The reason why dividing the speed profile is the coasting regime has different characteristics from other regimes, and dividing is easier to study the difference.

3 Train motion model

The section below describes the train motion model, in which the train is represented as a single mass point and its longitudinal motion can be described using Newton's equation. We consider the train can output any continuous value within limits, the formulation of train motion is:

$$m\dot{v}(t) = U(t) - R(v) - g(s) \tag{1}$$

Let v be the speed of the train, m the total mass, and s is the location. $R(v)$ is the aerodynamic drag and the rolling mechanical resistances which are represented by the Davis formula (2), $g(s)$ the gradient and curve resistance which is considered as zero. The traction force minus braking force is considered as the single input $U(t)$.

$$R(v) = m(r_0 + r_1 v + r_2 v^2) \tag{2}$$

To bring the $U(t)$ and $R(v)$ to the same magnitude, let both of them be divided by m:

$$r(v) = R(v)/m = r_0 + r_1 v + r_2 v^2 \tag{3}$$

$$u(t) = U(t)/m \tag{4}$$

Thus, the equation (1) can be rewrite into:

$$\dot{v}(t) = u(t) - (r_0 + r_1 v + r_2 v^2) \tag{5}$$

3.1 Constraints

In the reality, the control variable $u(t)$ is constrained by physical limits, due to the fact that we could not have infinite traction power or infinite braking power. $bMax$ is the maximum braking force, $uMax$ is the maximum tractive force and $pMax$ is the maximum power.

$$bMax \leq u(t) \leq min\{\frac{pMax}{v}, uMax\} \tag{6}$$

Variable	Notation	unit
Mass	m	kg
Velocity	v	m/s
Time	t	s
Traction force - braking force	$U(t)$	N
Control variable	$u(t)$	m/s^2
Aerodynamic drag and rolling mechanical resistances	$R(v)$	N
Aerodynamic drag and rolling mechanical resistances divided by mass	$r(v)$	m/s^2
Resistance parameters	$r0, r1, r2$	N
location	s	m
Gradient and curve resistance	$g(s)$	N
Maximum braking force	$bMax$	m/s^2
Maximum power	$pMax$	W

Table 1: Table of notation for train motion model

The units of variables can be found in the notation table (1). The next section describes the LQR algorithm.

4 Linear Quadratic Regulator (LQR)

In the following pages, I will present the general LQR algorithm for a closed-loop single-input system, and the applied LQR for speed profile tracking. The latter consists of the description of a single-input, single-output nonlinear system, cost function, the choosing of the equilibrium point, and the general calculation of A, B C, and D matrices.

4.1 General Linear Quadratic Regulator

A general linear quadratic regulator selects the closed-loop eigenvalue by optimizing a cost function. Given a single-input linear system (Åström and Murray (2021)):

$$\frac{dx}{dt} = Ax + Bu \tag{7}$$

$$y = Cx + Du \tag{8}$$

Attempting to minimize the quadratic cost function:

$$J = \int_0^\infty (x^T Q_x x + u^T Q_u u)\, dt \tag{9}$$

where Q_x and Q_u are the weight matrices. We assume $Q_x > 0$, $Q_u > 0$ to guarantee that a solution exists. Q_x and Q_u are chosen using the knowledge of the system. The cost function indicates the trade-off between the rate of convergence of the solution and the cost of control. The solution to the LQR is given by a linear control law of the form:

$$u = -Q_u^{-1} B^T P x \tag{10}$$

where P satisfies the algebraic Riccati equation (Åström and Murray (2021)):

$$PA + A^T P - PBQ_u^{-1} B^T P + Q_x = 0 \tag{11}$$

In the next section, the application of LQR for train speed profile tracking is shown.

4.2 Linear Quadratic Regulator for speed profile tracking

The block diagram (figure 1) shows a control system of the train. Let v be the speed of the train and v_r be the reference speed which in our case is given. The difference between $v(m/s)$ and $v_r(m/s)$ is shown as error w. The controller receives the signal v and v_r and generates a control signal that is sent to an actuator that controls the throttle position. The aerodynamic drag and rolling mechanical resistances are represented by the Davis formula $R(v)$. The gradient and curve resistance $g(s)$ is simplified as zero in this paper.

The train motion model (5) is a quadratic dynamical system of the first order. The state variable is velocity v, while $u(t)$ is the control variable. The quadratic term is $r(v)$. We assume that all parameters are static.

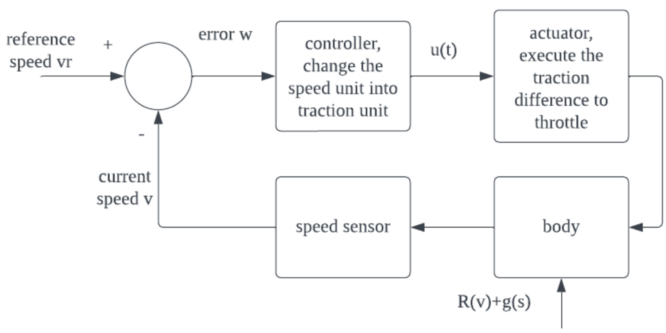


Figure 1: Block diagram of a cruise control system for a train

In this paper, consider a single-input, single-output nonlinear system:

$$\frac{dv}{dt} = f(v,u) = u - (r_0 + r_1 v + r_2 v^2) \tag{12}$$

$$y = h(v,u) \tag{13}$$

The cost function:

$$J = \int_0^{\infty} (v^T Q v + u^T R u) \, dt \tag{14}$$

Q and R are the cost function weights, Q indicates the convergence rate of v while R indicates the cost of control u. With an equilibrium point at $v = v_e$, $u = u_e$. For the local behavior around the equilibrium point (v_e, u_e), we suppose that $v - v_e$ and $u - u_e$ are small so that nonlinear perturbations around this equilibrium point can be ignored compared with the lower order linear terms. We define a new set of state variables $w = v - v_e$, $z = u - u_e$, $x = y - v_e$. These variables are all close to zero when near the equilibrium point. Formally, the Jacobian linearization (Åström and Murray (2021)) of the nonlinear system (12, 13) is:

$$\frac{dw}{dt} = Aw + Bz \tag{15}$$

$$x = Cw + Dz \tag{16}$$

where

$$A = \left.\frac{\partial f}{\partial v}\right|_{(v_e, u_e)}, B = \left.\frac{\partial f}{\partial u}\right|_{(v_e, u_e)}, C = \left.\frac{\partial h}{\partial v}\right|_{(v_e, u_e)}, D = \left.\frac{\partial h}{\partial u}\right|_{(v_e, u_e)} \tag{17}$$

The system (15,16) approximates the original system (12, 13) when near the equilibrium point about which the system was linearized.

The tracking problem can be categorized into acceleration, cruising and braking, and coasting regimes. For the acceleration regime, the system attempts to speed up to a speed limit using a comfortable acceleration rate. For the cruising regime, the system attempts to maintain a constant velocity in the presence of the aerodynamic drag and rolling mechanical resistances in the track. The controller compensates for these unknowns by measuring the speed of the train and adjusting the throttle appropriately. For the coasting regime, the control variable $u(t)$ is set to zero, which means the aerodynamic drag and rolling mechanical resistances contribute as the braking force.

When the train is cruising, the acceleration rate is zero, where $\dot{v} = 0$, and the equilibrium velocity is the cruise speed. While the acceleration regime and the braking regime are similar to the cruising regime. When choosing the equilibrium speed v_e, it is the value of the end of each regime in the speed profile. The equilibrium u_e is calculated with given m, r_0, r_1, r_2, v_e:

$$u_e - r_0 - r_1 v_e - r_2 {v_e}^2 = 0 \tag{18}$$

$$u_e = r_0 + r_1 v_e + r_2 {v_e}^2 \tag{19}$$

With the known equilibrium point (v_e, u_e) and equation (5), the A, B, C, and D matrices can be calculated:

$$A = \left.\frac{\partial f}{\partial v}\right|_{(v_e, u_e)} = -r_1 - 2r_2 v_e \tag{20}$$

$$B = \left.\frac{\partial f}{\partial u}\right|_{(v_e, u_e)} = 1 \tag{21}$$

$$C = \left.\frac{\partial y}{\partial v}\right|_{(v_e, u_e)} = 1 \tag{22}$$

$$D = \left.\frac{\partial y}{\partial u}\right|_{(v_e, u_e)} = 0 \tag{23}$$

Therefore,

$$\frac{dw}{dt} = Aw + Bz = [-r_1 - 2r_2 v_e]w + [1]z \tag{24}$$

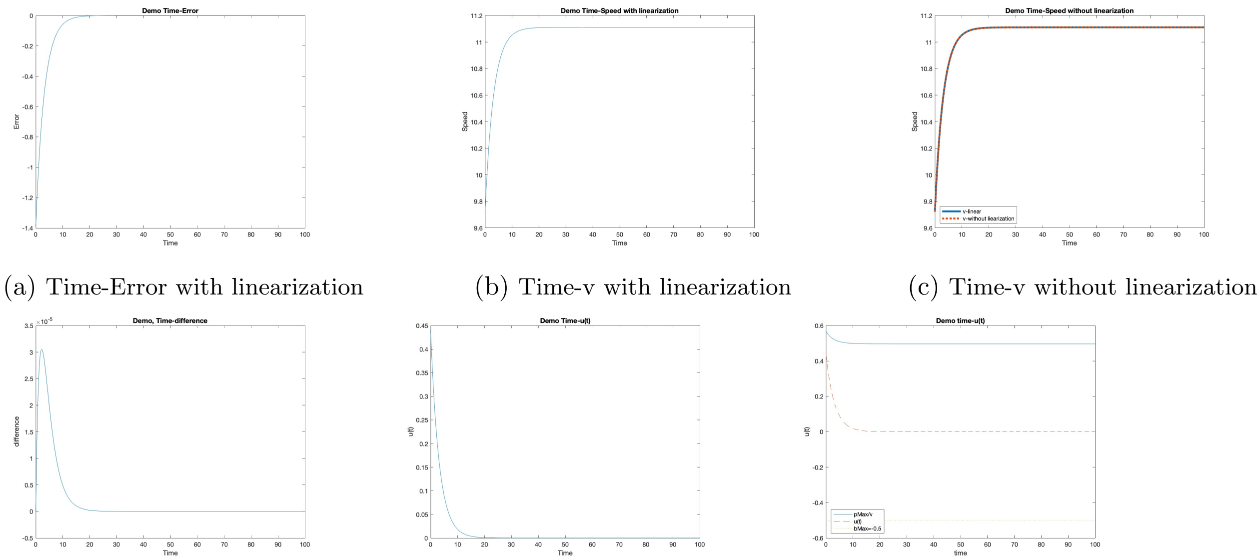
$$x = Cw + Dz = [1]w + [0]z \tag{25}$$

The procedure is suitable for the acceleration, cruising, and braking regimes by setting the different original velocities and different v_e. Except for the coasting regime, the u_e need to be zero.

5 Case study

The following part of this paper moves on describe a tracking demonstration, the given reference speed profile, and parameters tuning in greater detail.

5.1 Demonstration



(a) Time-Error with linearization (b) Time-v with linearization (c) Time-v without linearization

(d) Time-difference (e) Time-u(t) (f) Time-u(t), pMax/v, bMax

Figure 2: Demonstration, time scale

As a demonstration, LQR is applied to a cruising problem. With tuning the cost function weights Q and R, the acceleration rate should be limited within the range from $-0.5m/s^2$ to $0.6m/s^2$ considering the comfortable level of the public service.

The origin speed is defined as v_0 and the reference speed is defined as v_e. We take an example of $v_0 = 9.72m/s(35km/h)$ which needs to converge to $v_e = 11.11m/s(40km/h)$. In figure (2), the results of error, speed, acceleration rate, and $T - u$ are shown. Figure (2a) indicates the speed difference between the reference speed and the current speed, which is defined as an error. It takes about 20 seconds to reduce the error from $-1.38888m/s$ to zero. Figure (2b) shows the speed with linearization converged from $9.72m/s$ to $11.11m/s$ within 20 seconds. While in figure (2c), the smash red line indicates the speed without linearization, and the blue line indicates the speed with linearization. As shown, the smash red line and the blue line almost coincide. In figure (2d), the speed difference between linearization and without linearization is below 0.000031, therefore, we conclude that linearizing the quadratic part has a good performance in this demonstration. Figure (2e) shows the control variable $u(t)$ along the time scale, which decreases from $0.45m/s^2$ to a value near zero within 20 seconds. The value near zero is $r(v)$, which indicates the aerodynamic drag and rolling mechanical resistance. When the train is cruising, let $u(t) = r(v)$ to keep the $\dot{v} = 0$. In figure (2f), the $bMax$ and $min\{\frac{pMax}{v}, uMax\}$ are the lowbound and upbound of the control variable $u(t)$, in the demonstration, the output control variable is within the bound, otherwise we need to constraint the $u(t)$ within the physical bound.

5.2 Reference Speed profile

Section 5.1 shows a demonstration of a single cruising segment, while in the following section, a 1200-second reference time speed profile (figure (3)) is tracked using the algorithm. The track characteristics, train characteristics, and speed limits are shown in the following table (2).

The reference speed profile starts at $v_0 = 0m/s$ and accelerates to $11.11m/s$ ($40km/h$), then cruising till $36.9668s$. And then performs the second acceleration to $22.22m/s$ ($80km/h$), cruising till $76.2248s$, and begins the third acceleration to $35.83m/s$ ($129km/h$). After cruising to $272.202s$,

Train characteristics	Notation	Value	unit
Mass	m	391000	kg
Rotating Mass Factor		1.06	
Aerodynamic drag and rolling mechanical resistances parameter	r_0	2711.3	N
Aerodynamic drag and rolling mechanical resistances parameter	r_1	43.43	Ns/m
Aerodynamic drag and rolling mechanical resistances parameter	r_2	7.82	Ns^2/m^2
Train length	L	162	m
Maximum braking force	$bMax$	0.5	m/s^2
Maximum power	$pMax$	2157000	W
Maximum u	$uMax$	219000	N
Track characteristics			
Time period	t	1200	s
Location	s	35000	m
Speed			
Original speed	v_0	0	m/s
The first speed limit	v_{e40}	11.11	m/s
The second speed limit	v_{e80}	22.22	m/s
The third speed limit	v_{e129}	35.83	m/s
The fourth speed limit	v_{e120}	33.32	m/s
The speed after the second coasting	v_{e77}	21.57	m/s

Table 2: Track characteristics, train characteristics, speed limits

it starts coasting to $33.32m/s$ ($120km/h$) and then cruising at $33.32m/s$ ($120km/h$) until $374.305s$. Then it accelerates to $35.83m/s$ ($129km/h$) again, cruising till $603.676s$, starts coasting to $21.57m/s$ ($77.652km/h$) and finally braking to $v = 0m/s$. The whole procedure is divided into 8 segments, processed, and combined together later. The vertical lines in figure (3) indicate the 8 segments.

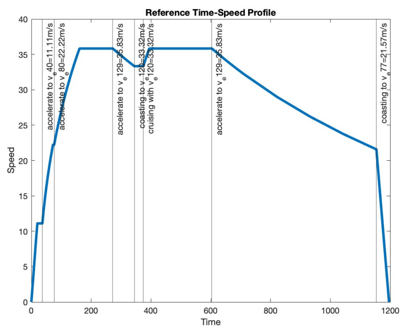


Figure 3: Reference Speed Profile

When executing the algorithm with Matlab, the speed at the end of each segment will be set as the equilibrium point for each segment, for example, in the first acceleration regime, $v_e = 11.11m/s$ and $u_e = r0 + r1v_e + r2v_e^2$ will be set as the equilibrium for the first regime tracking process. For the coasting regimes, $u_e = 0$ is applied. With the equilibrium point, the A, B, C, and D matrices can be calculated, and the controller gain is obtained. Note that, the control variable needs to follow the constraints in equation (6).

5.3 Performance and parameters tuning

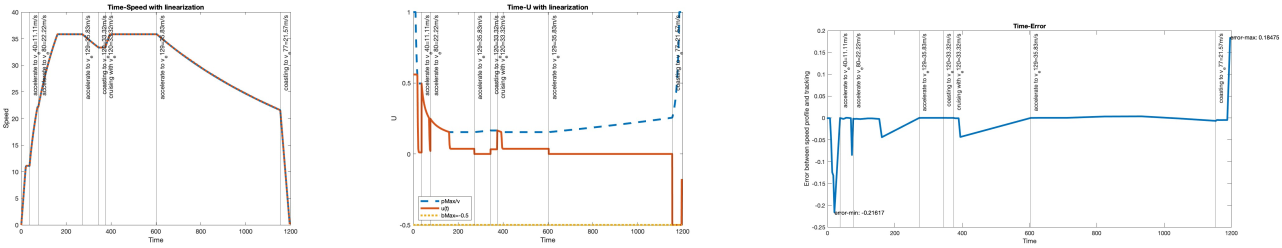
Before proceeding to examine the parameters, it is important to state that there are 4 tunable parameters in this paper which are the cost function weights $Q1, Q2, R1, R2$. Considering the difference between the coasting regime and the others, $Q1, R1$ are set for acceleration, cruising, and braking regimes, while $Q2, R2$ are set for coasting regimes. The $Q1, Q2$ indicate the cost function weights for speed v, while the $R1, R2$ indicate the cost function weights for control variable u. In the rough tuning stage, generally speaking, the smaller value of Q the smoother the tracking, while in terms of R, the larger value of R, the smoother the tracking.

Moving on now to consider the evaluation of the tracking performance, the maximum error, and the cumulative error are used. The error indicates the difference between the tracking blue line and the reference speed profile smash red line. It is very intuitive that the smaller the error, the better the tracking performance. In this paper, one of the goals is to have reasonably small errors.

5.3.1 Base case tracking

In the base case (see figure (4a)), the smash red line indicates the given reference speed profile, note that the speed limits are shown in the table (2). The blue line shows the tracking results with all cost function weights $Q1, Q2, R1, R2$ equal to 1. The linearized speed fits the reference speed profile successfully. Zoom in the figure (4a), the result is close to the reference speed profile, which can be proved that the maximum absolute error of the base case is $0.21617 m/s$, and the cumulative error is $0.9784 m/s$ (figure (4c)). In figure (4b), the $u(t)$ is limited in the upbound of $min\{pMax/v, uMax\}$ (the smash blue line indicates $pMax$) and the lowbound of $bMax$ (the smash yellow line). The control output shows that the u is equal to 0 from $272.202s$ to $344.305s$ and $603.676s$ to $1153.98s$, which are the two coasting regimes in the speed profile, which conforms to the previous setting.

Having discussed the parameters, evaluation, and base case, the following section will focus on tuning $Q1, Q2, R1, R2$ separately and give an optimal combination of Q, R.



(a) Base case tracking (b) Base case: time-u(t) (c) Base case: time-error

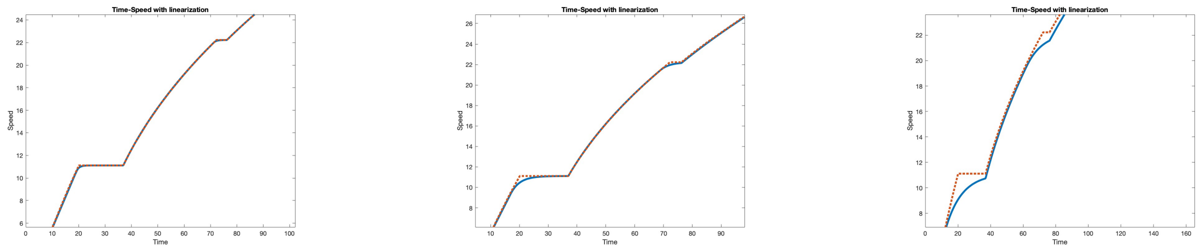
Figure 4: Base case tracking

5.3.2 Q1

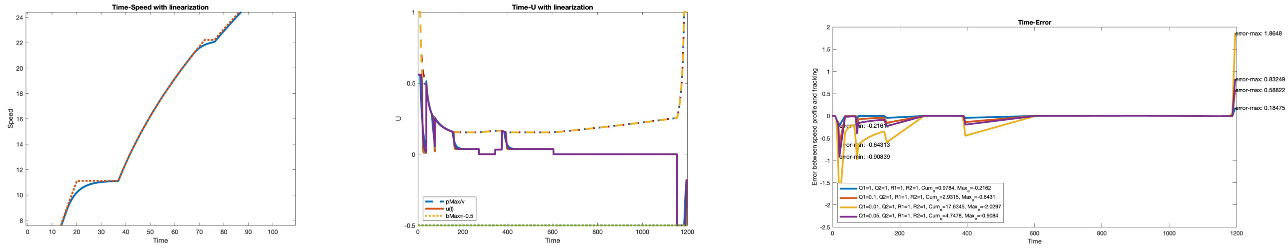
$Q1$ indicates the weight for v in the acceleration, cruising, and braking regime. According to the definition, we can speculate that the larger the $Q1$, the more aggressive the tracking. As one of the goals of this paper is to have smooth tracking considering the human sensation of acceleration, the tuning starts from $Q1 = 1$ and gradually decreases by powers of 10. Figure (5) shows that the smaller value of Q1, the smoother tracking, and the larger the error, which verifies the conjecture in the previous discussion. For $Q1 = 0.1$, $R1, Q2, R2$ equal to 1, the maximum error is $-0.6431 m/s$, and the cumulative error is $2.9315 m/s$. For $Q1 = 0.01$, $R1, Q2, R2$ equal to 1, the maximum error is $-2.0297 m/s$, and the cumulative error is $17.6345 m/s$. For $Q1 = 0.05$, $R1, Q2, R2$ equal to 1, the maximum error is $-0.9084 m/s$, and the cumulative error is $4.7478 m/s$. The time-u plot is shown in figure (5e). Although the control variable u does not have a severe change with tuning Q1, there is still a small difference, which is the larger value of $Q1$, the sharper the change of u. In reality, the reaction time of a machine to give traction force is limited, this can be studied later to find out the optimal value of parameter $Q1$. In figure (5c), the tracking is too slow that could not reach the speed limit of $11.11 m/s$ and $22.22 m/s$ within the time limits, though the tracking is very smooth. Considering the trade-off between smooth tracking and errors, Q1=0.05 is chosen as the relatively optimal value without considering the reaction time in reality.

5.3.3 R1

Regarding $R1$, which indicates the penalty for u in the acceleration, cruising, and braking regime. Tuning starts from $R1 = 1$ and gradually increases by powers of 10. Figure (6) shows that the larger value of $R1$, the smoother tracking, and the larger the error, which verifies the conjecture in the definition of $R1$. With the $Q1, Q2, R2$ equal to 1, for $R1 = 10$, the maximum error is $-0.6431 m/s$, and the cumulative error is $2.9315 m/s$; for $R1 = 100$, the maximum error is $-2.0297 m/s$, and the cumulative error is $17.6345 m/s$; for $R1 = 20$, the maximum error is $-0.9084 m/s$, and the cumulative error is $4.7478 m/s$. This is almost the same result as tuning $Q1$. Thus, there exists matching pairs of $Q1, R1$ that have the same effect on the tracking results. Next, the superimposed effect needs to



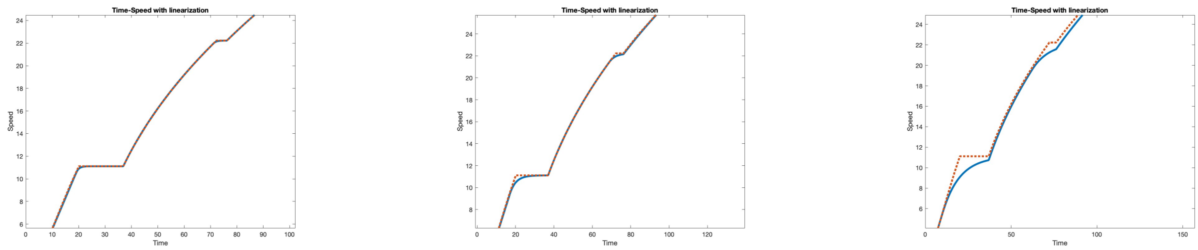
(a) Detail-1, Q1=1 (b) Detail-2, Q1=0.1 (c) Detail-3, Q1=0.01



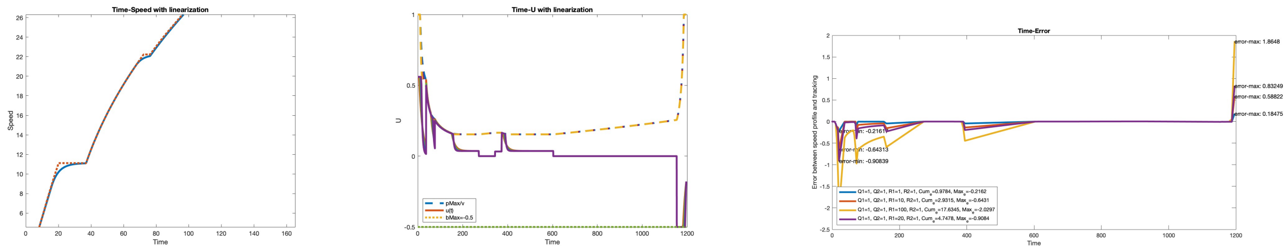
(d) Detail-4, Q1=0.05 (e) Time-u(t) (f) Time-error tuning Q1

Figure 5: Performance with tuning Q1

be studied. Figure (7e) shows the smaller the $R1$, the sharper the change in u. Although we want the machine reacts as fast as possible in the ideal case, we still need to consider the real reaction time limit. Considering the trade-off between smooth tracking and errors, $R1 = 20$ is chosen as the optimal value regarding $Q1, Q2, R2$ equal to 1 without considering the machine reaction time in reality.



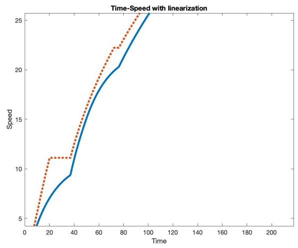
(a) Detail-1, R1=1 (b) Detail-2, R1=10 (c) Detail-3, R1=100



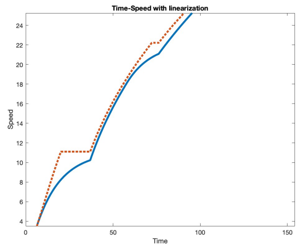
(d) Detail-4, R1=20 (e) Time-u(t) (f) Time-error tuning R1

Figure 6: Performance with tuning R1

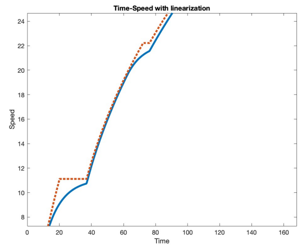
Next, assuming that $Q1$ is equal to 0.05 and $Q2, R2$ equal to 1 to find out the superimposed effect of $Q1, R1$ pair, figure (7) shows the maximum error and cumulative error for $R1$ tuning from 20 to 1. With $R1$ equal to 20, the cumulative error is $58.5404 m/s$; with $R1$ equal to 10, the cumulative error is $32.0845 m/s$; while with $R1$ equal to 5, the cumulative error is $17.6345 m/s$. The performance of $Q1 = 0.05, R1 = 1, Q2 = 1, R2 = 1$ and $Q1 = 1, R1 = 20, Q2 = 1, R2 = 1$ are the same, however, on the contrary, there is no better additive effect of $Q1 = 0.05, R1 = 20, Q2 = 1, R2 = 1$. A likely explanation is over-smoothing. Figure (7a) shows the $R1$ equal to 20 has relatively smoother tracking, however, it may be not true that the smoother the better. There is a trade-off between smooth tracking and errors. It could be an interesting research gap in studying the optimal smooth degree or giving



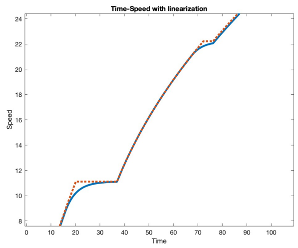
(a) Detail-1, R1=20 with Q1=0.05



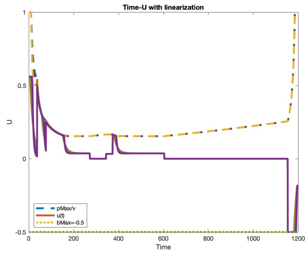
(b) Detail-2, R1=10 with Q1=0.05



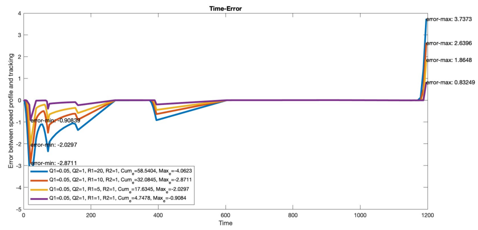
(c) Detail-3, R1=5 with Q1=0.05



(d) Detail-4, R1=1 with Q1=0.05



(e) Time-u,tuning R1 with Q1=0.05



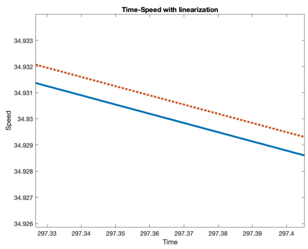
(f) Time-error tuning R1 with Q1=0.05

Figure 7: Performance with tuning R1 with Q1=0.05

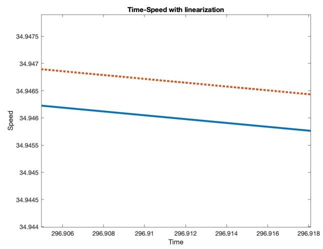
weights for the smooth degree of tracking and weights for the error. Regarding the goal of minimizing the error as possible in this paper, $R1$ equal to 1 is chosen with Q1=0.05 as the relatively optimal value.

5.3.4 Q2

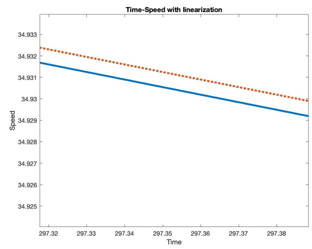
Regarding $Q2$, which indicates the weight for v in the coasting regime. The conjecture is the larger the $Q2$, the more aggressive the tracking in the coasting regime. Tuning starts from $Q2 = 1$, and tuning gradually decreases by powers of 100. In figures (8a - 8d), the differences between the speed profile and the coasting tracking are smaller than 0.001. Figure (8f and 8h) shows the errors of $Q2 = 1, Q2 = 0.01, Q2 = 0.0001$ with $Q1, R1, R2$ equal to 1 and with $Q1 = 0.05, R1 = 1$ representatively. Note that the maximum error does not happen in the coasting regime, thus it can not evaluate the performance of coasting. It may be an interesting potential research to develop different variables for the evaluation of coasting regimes. The results show that the error curves almost completely coincide, which indicates that the change in $Q2$ almost does not affect the performance. This is different from the conjecture. A possible explanation could be after setting $u_e = 0$ for the coasting regime (figure (8g) shows the result), the tracking is very close to the speed profile, thus there is not much room for adjustment. Therefore, Q2=1 is chosen.



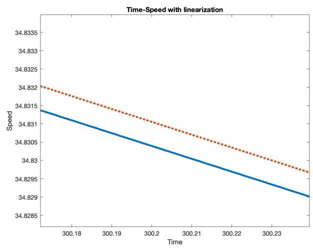
(a) Detail-1, Q2=0.01 with Q1=1, R1=1, R2=1



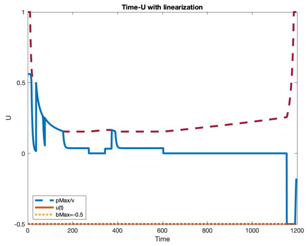
(b) Detail-2, Q2=0.0001 with Q1=1, R1=1, R2=1



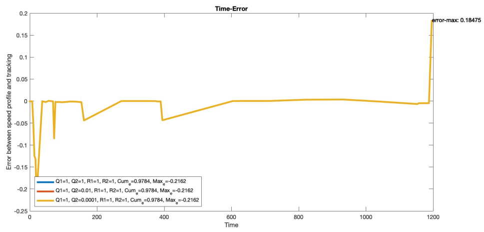
(c) Detail-3, Q2=0.01 with Q1=0.05, R1=1



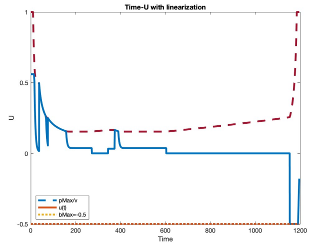
(d) Detail-4, Q2=0.0001 with Q1=0.05, R1=1



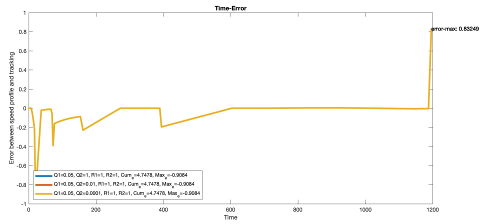
(e) Time-u(t), tuning Q2 with Q1=1, R1=1



(f) Time-error, tuning Q2 with Q1=1, R1=1, R2=1



(g) Time-u(t), tuning Q2 with Q1=0.05, R1=1

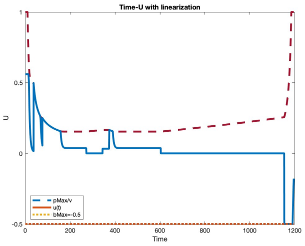


(h) Time-error, tuning Q2 with with Q1=0.05, R1=1

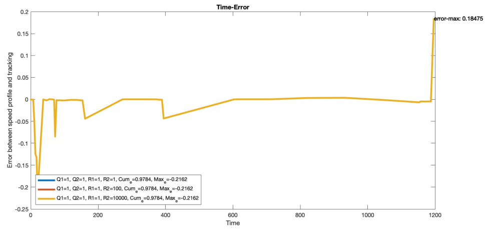
Figure 8: Performance with tuning Q2: Maximum error and Cumulative error

5.3.5 R2

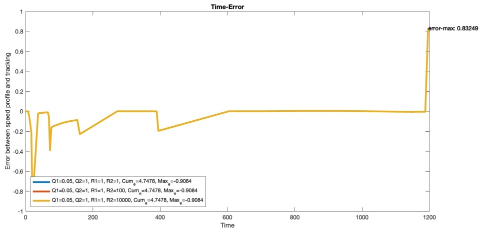
In the coasting regime, the u_e is set to zero. $R2$ indicates the penalty for u when coasting, which could lead to a result that the larger the better. Figure (9) shows the results of tuning $R2$ from $R2 = 1, R2 = 100, R2 = 10000$ with $Q1, R1, Q2$ equal to 1 and with $Q1 = 0.05, R1 = 1, Q2 = 1$. The results indicate that the change in R2 almost does not affect the performance which is the same conclusion as tuning $Q2$. Thus, R2=1 is chosen.



(a) Time-u(t), tuning R2 with Q1=0.05, R1=1, Q2=1



(b) Time-error, tuning R2 with Q1=1, R1=1, Q2=1

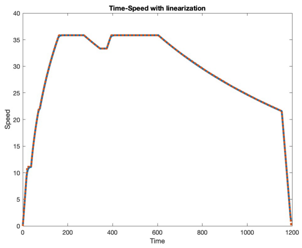


(c) Time-error, tuning R2 with with Q1=0.05, R1=1, Q2=1

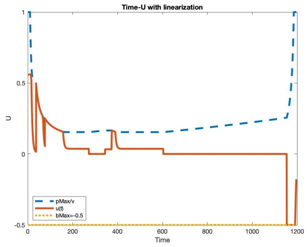
Figure 9: Performance with tuning R2: Maximum error and Cumulative error

5.3.6 Optimal Result

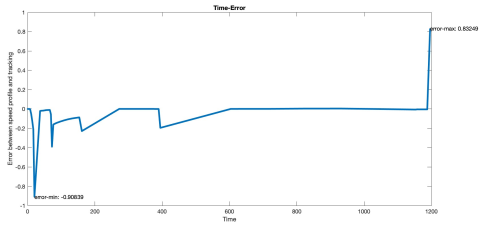
With $Q1 = 0.05, Q2 = 1, R1 = 1, R2 = 1$, the relatively optimal result without considering machine reaction time and smooth degree shown in figure (10). The maximum error is $-0.90839m/s$, and the cumulative error is $4.7478m/s$. Figure (10b) shows the control variable u along with time.



(a) Optimal tracking



(b) Optimal tracking: Time-u



(c) Optimal tracking: Time-error

Figure 10: Optimal Result with Q1=0.05, Q2=1, R1=1, R2=1

6 Conclusion

Most of the results in this paper are reasonable and speculative. Using the proposed LQR algorithm, it is intuitive and relatively easy to get a relatively nice result in train speed profile tracking for regimes of acceleration, cruising, coasting, and barking without considering gradient resistance, curve resistance, and other environmental disturbance. The relatively optimal result has a cumulative error of $4.7478m/s$ and a maximum error of $-0.90839m/s$ which is acceptable.

As discussed in the case study, the definition of a smooth degree of tracking and the optimal parameters considering the limits of machine reaction time could be studied further. The obvious conflict is the trade-off between smooth tracking and tracking error. It is also interesting to develop other evaluation variables for the coasting regime.

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