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# Fast universal quantum gate above the fault-tolerance threshold in silicon

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Fault-tolerant quantum computers that can solve hard problems rely on quantum error correction<sup>1</sup>. One of the most promising error correction codes is the surface code<sup>2</sup>, which requires universal gate fidelities exceeding an error correction threshold of 99 per cent<sup>3</sup>. Among the many qubit platforms, only superconducting circuits<sup>4</sup>, trapped ions<sup>5</sup> and nitrogen-vacancy centres in diamond<sup>6</sup> have delivered this requirement. Electron spin qubits in silicon<sup>7–15</sup> are particularly promising for a large-scale quantum computer owing to their nanofabrication capability, but the two-qubit gate fidelity has been limited to 98 per cent owing to the slow operation<sup>16</sup>. Here we demonstrate a two-qubit gate fidelity of 99.5 per cent, along with single-qubit gate fidelities of 99.8 per cent, in silicon spin qubits by fast electrical control using a micromagnet-induced gradient field and a tunable two-qubit coupling. We identify the qubit rotation speed and coupling strength where we robustly achieve high-fidelity gates. We realize Deutsch–Jozsa and Grover search algorithms with high success rates using our universal gate set. Our results demonstrate universal gate fidelity beyond the fault-tolerance threshold and may enable scalable silicon quantum computers.

Electron spins in silicon quantum dots are an attractive platform for a quantum computer with a long coherence time<sup>7–9</sup>, the capability of high-temperature operation<sup>10,11</sup> and potential scalability<sup>12–15</sup>. Single-qubit gate fidelity higher than the fault-tolerance threshold is now routinely achieved<sup>7,8,17</sup>. Two-qubit gate fidelity, however, still remains limited to 98% (ref. <sup>16</sup>), which is below the threshold, because of the complexity of operation and/or slow operation compared with the coherence time<sup>16,18–20</sup>. Native two-qubit gates for spin qubits include  $\sqrt{\text{SWAP}}$ <sup>21–25</sup>, controlled phase<sup>9,19,20</sup> and controlled rotation (CROT)<sup>18,19</sup>, all of which rely on the exchange coupling. Rapid control of exchange coupling by gate voltage pulses enables  $\sqrt{\text{SWAP}}$ <sup>21–25</sup> and controlled-phase gates<sup>9,19,20</sup> at the cost of requiring high-bandwidth and precise pulse engineering, which obstructs a high-fidelity gate. In contrast, a CROT<sup>18,19</sup> can be implemented with less demanding pulse engineering in a fixed coupling<sup>16</sup>. With additional adjustments of single-qubit phases, a controlled-NOT (CNOT) gate with a fidelity of 98% is demonstrated. As the fidelity is mostly limited by dephasing<sup>16</sup>, it is crucial to mitigate the dephasing effect by a faster operation to go beyond the fault-tolerance threshold. Furthermore, a reliable and efficient tuning strategy for the high-fidelity gate is desired for scaling up the silicon spin qubits.

Here we realize a CNOT gate fidelity of 99.5% with a gate time of 103 ns in an isotopically enriched silicon quantum dot array. Our device satisfies three key elements to achieve this. First, the exchange coupling  $h/J$  is widely controllable to make it large enough at the charge-symmetry point where the effect of charge noise during a fast operation is

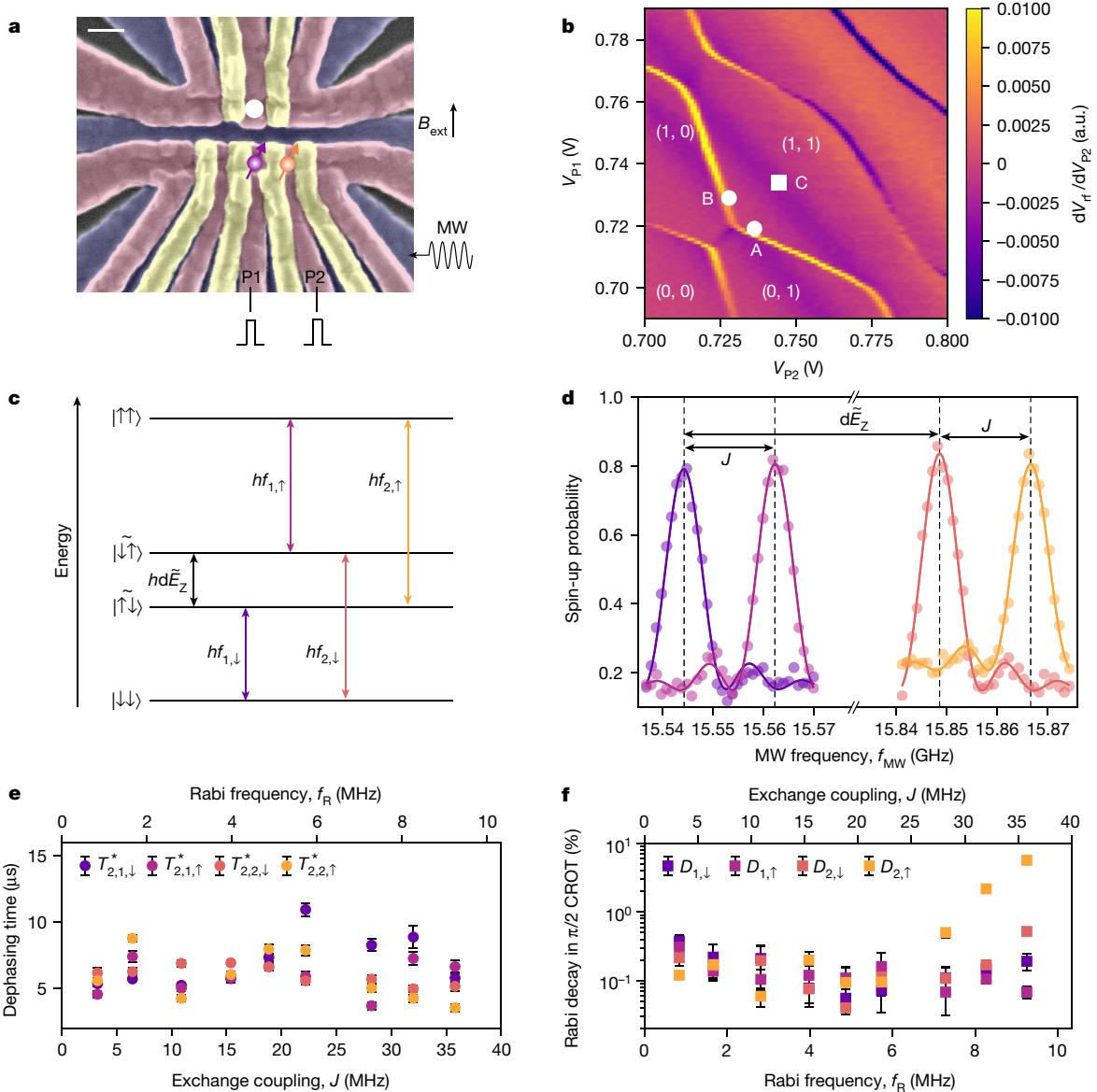
suppressed<sup>18,23</sup>. Here  $h$  is the Planck constant. Second, the Zeeman energy difference between the qubits  $h\Delta E_Z$  induced by a micromagnet is also large enough to allow a large  $h/J$ . Finally, the CROT gates are implemented by fast electric-dipole spin resonance (EDSR) controls of single spins driven in the slanting magnetic field induced by the micro-magnet. These device features enable us to assess the single-qubit and two-qubit gate performances over a wide range of parameters that were not accessible in previous work<sup>16</sup>. A comprehensive study of the gate performances reveals that they mainly depend on the gate speed, from which we identify the gate condition where a CNOT gate fidelity higher than 99% is robustly achieved. In the same gate condition, single-qubit gate fidelities reach 99.8% for both qubits. Using the high-fidelity universal quantum control, we implement the two-qubit Deutsch–Jozsa algorithm<sup>26</sup> and the Grover search algorithm<sup>27</sup> with success rates of 96–97%. These results demonstrate universal quantum control fidelity that exceeds the surface code error correction threshold, showing that high-fidelity quantum processing is feasible in silicon spin qubits.

The device is a linearly coupled triple quantum dot fabricated on an isotopically enriched silicon/silicon-germanium heterostructure (Methods). Three layers of aluminium gates create confinement potentials to define the quantum dots<sup>15</sup> (Fig. 1a). The centre (right) quantum dot has an electron that is operated as a qubit  $Q_1$  ( $Q_2$ ) whereas the left dot is not formed but used as an extension of the left reservoir. On top of the aluminium gates, a cobalt micromagnet is fabricated to generate the magnetic field gradient required for the fast EDSR control of both qubits<sup>28</sup> and also induce  $dE_Z$  for the CROT gates.

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**Fig. 1 | Two-qubit system.** **a**, False-colour scanning microscope image of a device identical to the one measured. The qubits are located underneath the P1 and P2 gate electrodes. The white circle shows a charge sensor quantum dot embedded in a radio-frequency tank circuit<sup>39,40</sup>. The white scale bar indicates 100 nm. **b**, Charge stability diagram around the operation condition. Initialization and measurement for  $Q_1$  ( $Q_2$ ) is performed at the white circle labelled A (B). Qubit manipulation is performed at the charge-symmetry point shown in the white square labelled C. **c**, Energy diagram of the two-qubit system. Each coloured arrow shows the state transition driven by EDSR with the

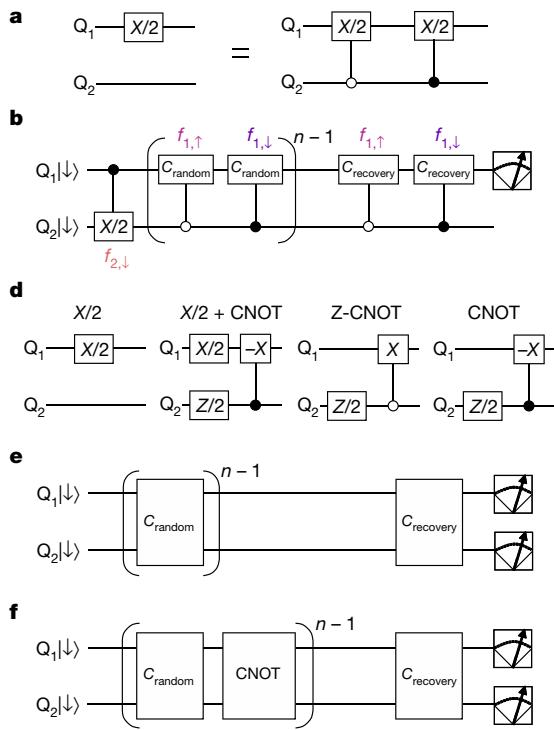
microwave frequency  $f_{1,\uparrow}$ ,  $f_{1,\downarrow}$ ,  $f_{2,\uparrow}$  and  $f_{2,\downarrow}$ . **d**, EDSR spectra for  $Q_1$  when  $Q_2$  is spin-down (purple) and spin-up (magenta) and for  $Q_2$  when  $Q_1$  is spin-down (orange) and spin-up (yellow). **e**, The  $J$  (and  $f_R = J/\sqrt{15}$ ) dependence of the dephasing times (Extended Data Fig. 2d–f). We choose the charge-symmetry point as the operation condition and control  $J$  by modifying the tunnel coupling between the quantum dots. The errors represent the estimated standard errors for the best-fit values. **f**, The  $f_R$  (and  $J = \sqrt{15}f_R$ ) dependence of the Rabi decay during a  $\pi/2$  CROT obtained from the Rabi decay curves (Extended Data Fig. 2j–l). The errors represent the estimated standard errors for the best-fit values.

Figure 1b shows a typical charge stability diagram around the charge configurations to define the qubits. Each qubit is sequentially initialized and measured in a single-shot manner by energy-selective electron tunnelling between the quantum dots and neighbouring reservoirs<sup>29,30</sup> at the gate voltage conditions shown by the white circles in Fig. 1b. The qubits are manipulated around the charge-symmetry point in the (1, 1) charge state (Extended Data Fig. 1) shown by the white square to suppress charge-noise sensitivity during operations<sup>18,23</sup>. Here,  $(c, r)$  denotes the number of electrons in the centre ( $c$ ) and right ( $r$ ) dots.

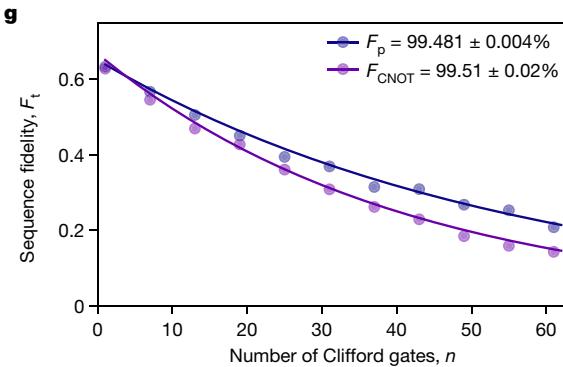
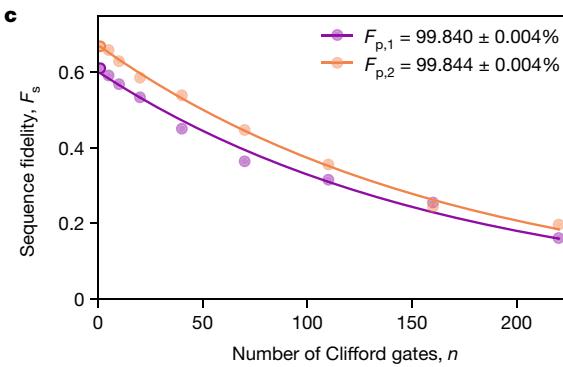
This two-qubit system is capable of implementing the universal gate set. Under an EDSR control with a Rabi frequency of  $f_{R,1}$  and  $f_{R,2}$  for  $Q_1$  and  $Q_2$ , respectively, a microwave (MW) frequency of  $f_{MW}$  and its phase  $\phi$ , a Hamiltonian for the two-qubit system in the basis of  $|↑↑⟩$ ,  $|↓↑⟩$ ,  $|↑↓⟩$  and  $|↓↓⟩$  can be approximated as<sup>16</sup>

$$H = \frac{\hbar}{2} \begin{pmatrix} 2E_Z & \Omega_2 & \Omega_1 & 0 \\ \Omega_2^* & -d\tilde{E}_Z - J & 0 & \Omega_1 \\ \Omega_1^* & 0 & d\tilde{E}_Z - J & \Omega_2 \\ 0 & \Omega_1^* & \Omega_2^* & -2E_Z \end{pmatrix},$$

where  $\hbar E_Z$  is the average Zeeman energy,  $hd\tilde{E}_Z = h\sqrt{dE_Z^2 + J^2}$  is the effective Zeeman energy difference between the qubits and  $\Omega_m = f_{R,m} e^{i2\pi f_{MW} t + i\phi}$  ( $m=1$  or  $m=2$ ) is the EDSR driving with driving time  $t$ . The tilde indicates the hybridization of the spineigenstates  $|↓↑⟩$  and  $|↑↓⟩$  due to the exchange coupling. Then each EDSR frequency is given by  $f_{m,\sigma} = E_Z \pm (d\tilde{E}_Z \pm J)/2$  (Fig. 1c) where  $m$  is the index of the target qubit  $Q_m$  ( $m=1$  or  $m=2$ ) and  $\sigma$  is the control qubit state  $|↑\rangle$  or  $|↓\rangle$ . When the separations of  $f_{m,\sigma}$  are larger than



**Fig. 2 | Characterization of universal quantum control performances by randomized benchmarking.** **a**, Example of a single-qubit gate constructed from two CROT gates to make it unconditional spin rotation. **b**, Quantum circuit to measure the single-qubit gate fidelity of  $Q_1$  by the Clifford-based randomized benchmarking. As the spin rotation under finite exchange coupling is a CROT, we synthesize our unconditional single-qubit rotations to assess the averaged performance of single-qubit gate of  $Q_1$  with various  $Q_2$  states. The roles of  $Q_1$  and  $Q_2$  are swapped to measure the single-qubit gate fidelity of  $Q_2$ . **c**, Single-qubit Clifford-based randomized benchmarking for  $Q_1$  (purple) and  $Q_2$  (orange). The visibilities are 0.602 and 0.672 for  $Q_1$  and  $Q_2$ , respectively. The uncertainty in the gate fidelities is obtained by a Monte Carlo method<sup>4</sup>. **d**, Quantum circuits for two-qubit primitive gates that rotate  $Q_1$ .



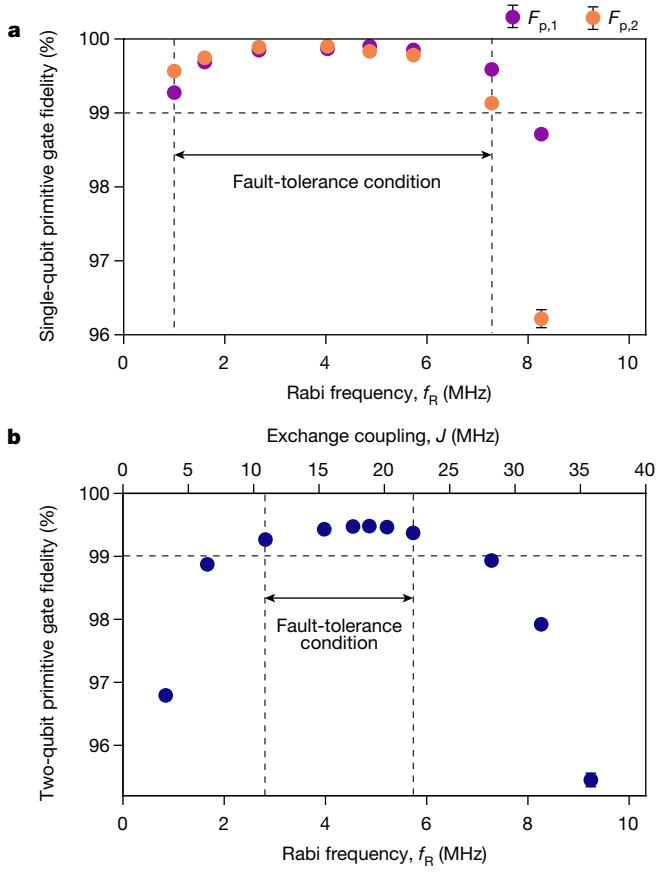
CNOT and zero-CNOT (Z-CNOT) gates flip the target qubit  $Q_1$  when the control qubit  $Q_2$  is spin-down and spin-up<sup>16</sup>. We construct our Clifford gates so that the number of primitive gates per one Clifford gate is minimum, which results in 2.57 primitive gates per one Clifford gate on average<sup>16</sup>. As the single-qubit phase gates are implemented by changing the reference frame of all subsequent microwave pulses in the software, we do not include them in the primitive gate count. **e**, Quantum circuit for two-qubit randomized benchmarking to measure the Clifford gate fidelity and the primitive gate fidelity  $F_p$ . **f**, Quantum circuit for interleaved randomized benchmarking to measure the CNOT gate fidelity  $F_{CNOT}$ . **g**, Results of the two-qubit Clifford-based randomized benchmarking. The uncertainty in the gate fidelities is obtained by a Monte Carlo method<sup>4</sup>.

the width of each EDSR spectrum, a CROT gate can be implemented by driving one of the EDSR transitions<sup>16,18</sup> (Fig. 1d). An in-plane external magnetic field of  $B_{ext} = 0.408$  T results in  $E_z \approx 15.70$  GHz. The micromagnet induces  $dE_z \approx 300$  MHz and we can control it from a few megahertz to tens of megahertz by varying the barrier gate voltage. As  $f_{R,1}$  and  $f_{R,2}$  for the target qubit rotations are tuned independently by adjusting the applied microwave amplitude (Methods), we always calibrate both Rabi frequencies to the same target Rabi frequency  $f_R$ . Although a larger  $f_R$  is desired for a high-fidelity CROT gate, it also results in unwanted rotation of the off-resonant states. To cancel this unwanted rotation in both  $\pi$  and  $\pi/2$  CROT gates (Methods)<sup>16,31</sup>, hereafter we use  $f_R = J/\sqrt{15}$  unless specifically noted.

Two important characteristics that can influence both single-qubit and two-qubit gate performances are the dephasing and the decay of Rabi oscillation during the gate time. Figure 1e shows the  $J$  (and  $f_R$ ) dependence of the dephasing times  $T_{2,m,o}^*$  measured for each transition (see also Extended Data Fig. 2d–f for details). We find that  $T_{2,m,o}^*$  is almost constant in the measured range of  $J$  as they are mostly limited by single-qubit frequency noise rather than the fluctuation in  $J$  as corroborated by noise measurement (Extended Data Fig. 5a). This implies that increasing  $f_R$  with keeping  $J = f_R \sqrt{15}$  is favourable to suppress the dephasing effect, as a larger  $J$  does not introduce extra dephasing. In contrast, we find that the Rabi decay depends on  $f_R$ . Figure 1f shows the  $f_R$  dependence of Rabi decay  $D_{m,o}$  during a  $\pi/2$  CROT (see Extended Data Fig. 2j–l for details). One can expect that the coherence-limited single-qubit and two-qubit gate performances<sup>16,17,32</sup> are improved with decreasing  $D_{m,o}$  (refs. <sup>8,33</sup>). At small ( $f_R \leq 2$  MHz) Rabi frequencies,  $D_{m,o}$

decreases with increasing  $f_R$  as the effect of dephasing is suppressed. In contrast,  $D_{m,o}$  increases with  $f_R$  above approximately 5 MHz as the Rabi decay becomes faster, possibly due to heating and/or population leakage<sup>8,33</sup>. In between the two regimes,  $D_{m,o}$  is minimized. This simple observation indicates that the best performance of single-qubit and two-qubit gates should be obtained at  $f_R \approx 5$  MHz.

Then, we measure basic qubit properties and assess single-qubit and two-qubit gate fidelities at  $f_R = 4.867$  MHz and  $J = 18.85$  MHz. The spin relaxation times for both qubits are much longer than the maximum operation time of 100  $\mu$ s used to characterize the gate fidelities (Extended Data Fig. 2c) and therefore the spin relaxation effect is negligible in the gate performances. The dephasing times  $T_{2,m,o}^*$  are several microseconds (Fig. 1e) and they are enhanced by the Hahn echo sequence up to about 30  $\mu$ s (Extended Data Fig. 2i). Single-qubit gate fidelities are characterized by the Clifford-based randomized benchmarking (Fig. 2b, Methods)<sup>34</sup>. In this system, a single-qubit gate is constructed from two CROT gates (Fig. 2a). We obtain primitive gate fidelities of  $F_{p,1} = 99.840 \pm 0.004\%$  for  $Q_1$  and  $F_{p,2} = 99.844 \pm 0.004\%$  for  $Q_2$  (Fig. 2c). The two-qubit gate fidelity is also characterized by the Clifford-based two-qubit randomized benchmarking. All of the Clifford gates are constructed from the primitive gates shown in Fig. 2d, another set of gates where the roles of  $Q_1$  and  $Q_2$  are swapped and single-qubit phase gates acting on each qubit<sup>16</sup>. Using the quantum circuit shown in Fig. 2e, we obtain a Clifford gate fidelity  $F_C = 98.67 \pm 0.01\%$ , which corresponds to a primitive gate fidelity  $F_p = 99.481 \pm 0.004\%$  as shown in Fig. 2g (Methods). As all of the primitive gates similarly comprise two  $\pi/2$  CROT gates<sup>16</sup>,



**Fig. 3 | Rabi frequency dependence of single-qubit and two-qubit primitive gate fidelities.** **a**, The  $f_R$  dependence of the single-qubit primitive gate fidelity extracted from single-tone single-qubit primitive gate fidelities (Extended Data Fig. 3d) as  $F_{p,m} = F_{p,m,\downarrow}F_{p,m,\uparrow}$ . Here we measure single-tone single-qubit primitive gate fidelities to assess the impact of  $f_R$  on the single-qubit gate performance without involving the effect of  $J$  (Extended Data Fig. 3d). **b**, The  $f_R$  and  $J$ /dependence of the two-qubit primitive gate fidelity.  $f_R = J/\sqrt{15}$  is always satisfied. When the Rabi frequency is between 2.8 MHz and 5.7 MHz, both dephasing and Rabi decay effects are suppressed and the two-qubit primitive gate fidelity exceeds the fault-tolerance threshold. Similar Rabi frequency dependence on the two-qubit primitive gate fidelity is observed by modifying detuning to control  $J$  (Extended Data Fig. 6). The uncertainty in the fidelities is obtained by a Monte Carlo method<sup>4</sup>.

each primitive gate including the CNOT gate should have similar gate fidelity. To confirm this, we directly assess the CNOT gate fidelity  $F_{\text{CNOT}}$  by the interleaved randomized benchmarking<sup>4,35</sup>. By comparing the sequence fidelity decay using the quantum circuit shown in Fig. 2e, f, we obtain  $F_{\text{CNOT}} = 99.51 \pm 0.02\%$  (Fig. 2g, Methods) which agrees with  $F_p$ .

Next, we measure the impact of  $f_R$  on the single-qubit and two-qubit gate performances to study robustness of the high-fidelity gates. Figure 3a shows the  $f_R$  dependence of single-qubit primitive gate fidelities  $F_{p,m}$ . The best performance of  $F_{p,m} \approx 99.8\%$  is obtained at  $f_R = 2-5$  MHz, in agreement with the  $f_R$  dependence of the Rabi decay (Fig. 1f). The two-qubit primitive gate fidelity  $F_p$  also depends on  $f_R$  as shown in Fig. 3b. For small  $f_R$  ( $\leq 2.8$  MHz),  $F_p$  is below 99% and mostly limited by dephasing<sup>16</sup>. In this regime,  $F_p$  is much lower than  $F_{p,m}$  as the dephasing effect mainly affects the control qubit that is left idle while the target qubit is driven in CROT gates. Therefore, suppressing the dephasing effect is more important in the two-qubit gates. By increasing  $f_R$ , the dephasing effect can be suppressed and we obtain  $F_p$  above 99%. By further increasing  $f_R$ ,  $F_p$  sharply drops due to the fast Rabi decay. As expected from  $f_R$  dependence of the dephasing time and the Rabi decay (Fig. 1e, f), we obtain the best values of  $F_p \approx 99.5\%$  at  $f_R = 4-5$  MHz.

To consider the limiting factor of  $F_p$  in this condition, we simulate the effect of dephasing (Methods, Extended Data Fig. 5a, b). We find that the infidelity owing to dephasing is only 0.1%. The effect of Rabi decay is also small as indicated by the high-fidelity (99.84%) single-qubit gates (Fig. 2c). The remaining infidelity could originate from pulse calibration errors and long-term fluctuations in the device condition. These results indicate that the optimal gate condition is efficiently searched only from simple measurements of dephasing time and Rabi decay, which will be useful to tune up a large qubit array.

Finally, we implement the two-qubit Deutsch–Jozsa algorithm<sup>26</sup> and Grover search algorithm<sup>27</sup> to demonstrate the feasibility of high-fidelity quantum processing. The Deutsch–Jozsa algorithm (Fig. 4a) determines whether an unknown function  $f_i(x)$  ( $i \in \{0,1,2,3\}$ ) mapping a single-bit input  $x \in \{0,1\} = \{\uparrow, \downarrow\}$  to a single-bit output is constant ( $f_0(x) = 1$ ,  $f_1(x) = 0$ ) or balanced ( $f_2(x) = x$ ,  $f_3(x) = 1 - x$ ) by a single call of the function. The Grover search algorithm (Fig. 4b) finds the unique input two-bit string  $x_0 = ij$  ( $i, j \in \{0, 1\}$ ) of a function  $f_{ij}(x)$ , which outputs 0 for  $x \neq x_0$  but 1 for  $x = x_0$  by a single call of the function. Figure 4c–f (Fig. 4g–j) shows the real part of the density matrix (Methods) measured at each stage for  $f_2(f_{11})$ . All through the processing, the state fidelity is kept high (more than 96%), compared with the ideal state, which surpasses the state fidelities previously obtained in silicon spin qubits<sup>19,36</sup>. We also obtain similar output-state fidelities for the other functions (Extended Data Fig. 8). These results demonstrate that high-fidelity quantum processing is feasible in silicon spin qubits.

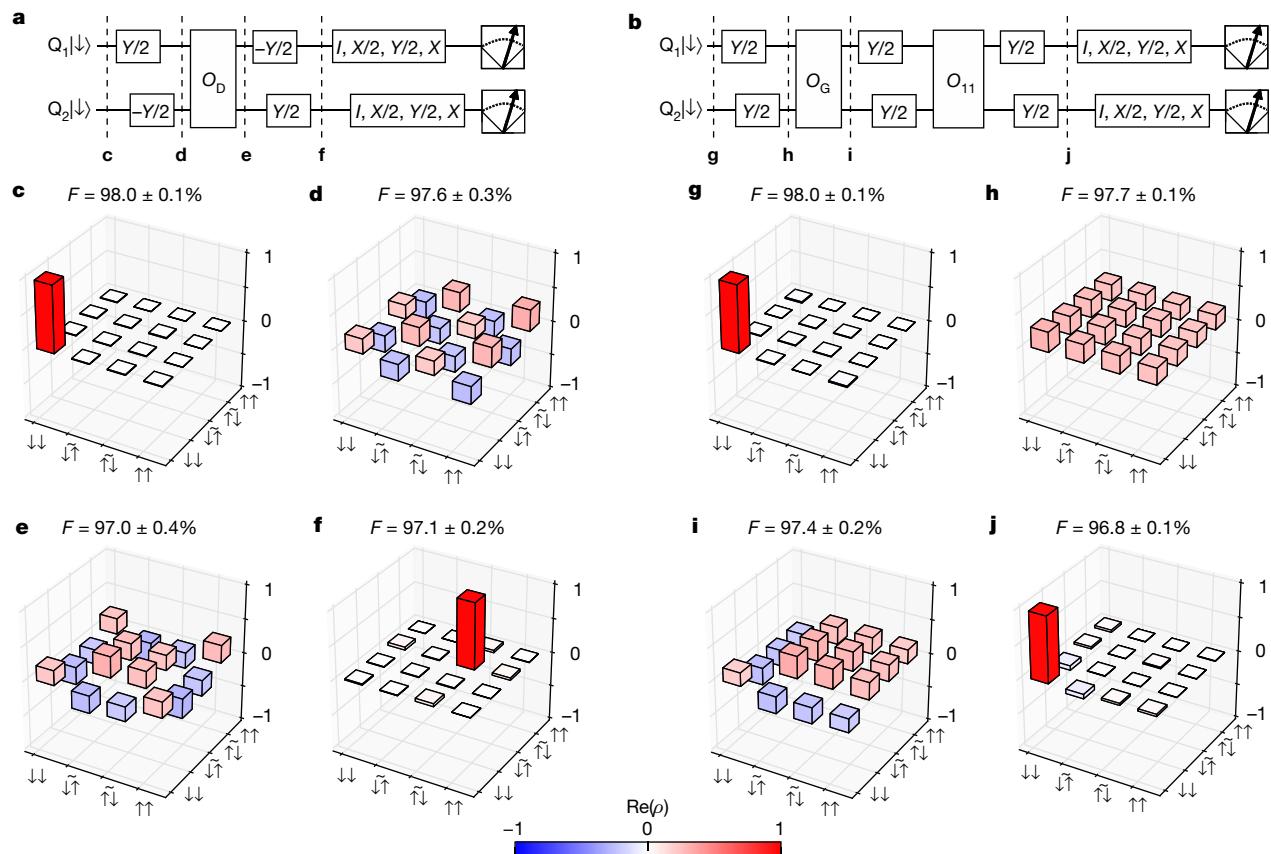
In conclusion, we demonstrate single-qubit and two-qubit primitive gate fidelities of 99.8% and 99.5%, respectively, which are beyond the surface code error correction threshold<sup>3</sup>. Micromagnet-induced gradient field and tunable exchange coupling allow us to assess the single-qubit and two-qubit gate fidelities with a variety of gate conditions and reveal a relationship between them. We identify that the Rabi frequency for single-qubit rotation influences both single-qubit and two-qubit gate fidelities. We find a range of Rabi frequencies where we robustly achieve the two-qubit primitive and CNOT gate fidelities higher than 99%. The demonstrated universal quantum gate set allows us to implement two-qubit quantum algorithms with high fidelities. Our results are an important step towards realizing fault-tolerant quantum computation in silicon spin qubits.

*Note added in proof:* During the completion of this article, we became aware of related experiments that demonstrate universal quantum control fidelity exceeding the fault-tolerance threshold in two electron spin qubits<sup>37</sup> and two nuclear spin qubits<sup>38</sup> in silicon.

## Online content

Any methods, additional references, Nature Research reporting summaries, source data, extended data, supplementary information, acknowledgements, peer review information; details of author contributions and competing interests; and statements of data and code availability are available at <https://doi.org/10.1038/s41586-021-04182-y>.

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**Fig. 4 | Two-qubit quantum processing.** **a**, Quantum circuit for the two-qubit Deutsch-Jozsa algorithm<sup>19</sup>.  $f_i(x)$  is implemented by an oracle  $O_D = I_2$  for  $f_{0i}$ ,  $X_2$  for  $f_{1i}$ ,  $Z\text{-CNOT}_2$  for  $f_{2i}$  and  $\text{CNOT}_2$  for  $f_{3i}$ , where the subscript 2 after the gates indicates that the target qubit is  $Q_2$ ,  $I$ ,  $X/2$ ,  $Y/2$  and  $X$  acting on both qubits at the end change the measurement axis to implement the state tomography (Methods). **b**, Quantum circuit for the two-qubit Grover search algorithm<sup>19</sup>.  $f_{ij}(x)$  is implemented by an oracle  $O_G = O_{11} = (Y_2/2)(\text{CNOT}_2)(-Y_2/2)$  for  $f_{11}$ ,  $O_{10} = (Y_1/2)(Z\text{-CNOT}_1)(-Y_1/2)$  for  $f_{10}$ ,  $O_{01} = (-Y_1/2)(\text{CNOT}_1)(Y_1/2)$  for  $f_{01}$  and

$O_{00} = (-Y_2/2)(Z\text{-CNOT}_2)(Y_2/2)$  for  $f_{00}$ . **c-f**, Real part of the measured density matrix (Methods) for  $f_2$  after initialization (**c**), preparation of the input state (**d**), application of the oracle (**e**) and the completion of the processing (**f**). **g-j**, Real part of the measured density matrix for  $f_{11}$  at each stage shown in **b**, respectively. The absolute values of the matrix elements for the imaginary parts are less than 0.028 (**c**), 0.046 (**d**), 0.051 (**e**), 0.050 (**f**), 0.013 (**g**), 0.072 (**h**), 0.081 (**i**) and 0.079 (**j**). The uncertainties in the state fidelities  $F$  are obtained by a Monte Carlo method<sup>16,19,41</sup>.

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## Methods

### Measurement setup

The sample was cooled in a dry dilution refrigerator (Oxford Instruments Triton) with a base temperature of about 20 mK. The electron temperature was about 60 mK. The d.c. gate voltages were supplied by a 24-channel digital-to-analogue converter (QDevil ApS QDAC), which was low-pass filtered at a cut-off frequency of 800 Hz. The voltage pulses applied to the P1 and P2 gate electrodes were generated by an arbitrary waveform generator (Tektronix AWG5014C). The EDSR microwave pulses were generated using an I/Q modulated signal generator (Anapico APMS20G with a Marki microwave MLIQ-0218 I/Q mixer) and applied to the bottom screening gate. The I/Q modulation signals were generated by another arbitrary waveform generator (Tektronix AWG70002A) triggered by the arbitrary waveform generator used for generating the gate voltage pulses. The microwave signals were side-band-modulated by frequencies ranging from -240 MHz to 180 MHz from the baseband frequency to avoid the unintentional spin rotation due to leakage (the typical isolation was about 50 dBc after calibration of the I/Q imbalances and the d.c. offsets) as well as switch the microwave frequencies rapidly. During the initialization and measurement stages, additional pulse modulations were used to provide further isolation of the microwave signals. The Rabi frequencies of single-qubit rotations were controlled by the amplitudes of I/Q modulation signals.

### Sample fabrication

The quantum dots were defined at the isotopically enriched silicon quantum well (residual <sup>29</sup>Si concentration of 800 ppm) 50 nm below the wafer surface. Three layers of overlapping aluminium gates were fabricated by electron-beam lithography and lift-off processes<sup>15</sup>. Each layer was insulated by thin native aluminium oxide. The micromagnet made of a stack of titanium and cobalt films with thicknesses of 5 nm and 250 nm, respectively, was placed on top of the overlapping gates with a 30-nm-thick insulating layer (aluminium oxide grown by atomic layer deposition) in between. The micromagnet design was similar to those in previous reports<sup>8,18,28,41</sup>.

### Sequence-fidelity and gate-fidelity extraction in randomized benchmarking

The sequence fidelity of single-qubit randomized benchmarking was obtained by the following procedure. According to the standard randomized benchmarking protocol, we measured the spin-up probability as a function of the number of Clifford gates  $n$ . Then the spin-up probability  $P_{\uparrow}$  follows  $P_{\uparrow}(n) = A_s p_s^n + C_s$ , where  $p_s$  is the depolarizing parameter, and  $A_s$  and  $C_s$  are the constants to absorb the state preparation and measurement errors. Here the recovery Clifford gate is chosen so that the final ideal state is spin-up. We also obtained another dataset where the final ideal state was spin-down by choosing different recovery gates. In this case, the spin-up probability  $P'_{\uparrow}$  follows  $P'_{\uparrow}(n) = B_s p_s^n + C_s$ , where  $B_s$  is the constant determined by state preparation and measurement errors. Then the sequence fidelity  $F_s(n)$  was obtained from  $F_s(n) = P_{\uparrow}(n) - P'_{\uparrow}(n) = (A_s - B_s)p_s^n$ , eliminating the uncertainty of determining  $C_s$  (refs. <sup>7,19,41,42</sup>). Here the fitting parameter  $A_s - B_s$  represents the readout visibility and absorbs the state preparation and measurement errors. We averaged 16 random sequences, each of which was repeated 400 times to measure  $F_s(n)$ . The Clifford gate fidelity  $F_{C,m}$  was obtained by  $F_{C,m} = (1 + p_s)/2$  where  $m$  is the qubit number 1 or 2. As a Clifford gate contains 1.875 primitive gates on average, we extracted the primitive

gate fidelity  $F_{p,m}$  as  $F_{p,m} = 1 - (1 - F_{C,m})/1.875$ . We obtained  $A_s - B_s = 0.602$  for  $Q_1$  and  $A_s - B_s = 0.672$  for  $Q_2$  (Fig. 2c). As  $F_{p,m}$  was obtained independently of the readout visibility, we did not carefully optimize the readout condition. The visibilities are most likely limited by unoptimized ratios of the tunnelling in/out rates between the quantum dots and the adjacent reservoirs, the readout time and the measurement bandwidth.

Similarly, the sequence fidelity of two-qubit randomized benchmarking was extracted by the following procedure. We measured  $P_{\uparrow\uparrow}(n) = A_t p_t^n + C_t$  as a function of  $n$  where  $P_{\uparrow\uparrow}$  is the joint probability of spin-up in both qubits,  $p_t$  is the depolarizing parameter, and  $A_t$  and  $C_t$  are the constants to absorb the state preparation and measurement errors. Here the recovery Clifford gate was chosen so that the final ideal state is spin-up for both qubits. We also measured another dataset where the final ideal state is spin-down for both qubits and obtain  $P'_{\uparrow\uparrow}(n) = B_t p_t^n + C_t$ ,  $B_t$  is the constant determined by state preparation and measurement errors. Then the sequence fidelity  $F_t(n)$  was extracted from  $F_t(n) = P_{\uparrow\uparrow}(n) - P'_{\uparrow\uparrow}(n) = (A_t - B_t)p_t^n$ . We averaged 60 random sequences, each of which was repeated 400 times to measure  $F_t(n)$ . The two-qubit Clifford gate fidelity was obtained by  $F_C = (1 + 3p_t)/4$ . As a Clifford gate contains 2.57 primitive gates on average, we extracted the primitive gate fidelity  $F_p$  as  $F_p = 1 - (1 - F_C)/2.57$ . The obtained gate fidelity using this protocol agrees with that obtained using the standard protocol<sup>4,16</sup> (only measures  $P_{\uparrow\uparrow} = A_t p_t^n + C_t$ ) as shown in Extended Data Fig. 4.

The fidelity of the CNOT gate was obtained as follows<sup>4</sup>. We first measured  $F_t(n)$  by applying random Clifford gates (Fig. 2e) and obtain the depolarizing parameter  $p_{ref}$  as a reference. We also measured  $F_t(n)$  by applying the CNOT gate between each random Clifford gates (Fig. 2f) and obtained the depolarizing parameter  $p_{CNOT}$ . Then we extracted the CNOT gate fidelity as  $F_{CNOT} = (1 + 3p_{CNOT}/p_{ref})/4$ .

The errors of the gate fidelities were obtained by a Monte Carlo method<sup>4</sup> assuming that the measured single-shot probabilities follow multinomial distributions. Single-shot probabilities from these distributions were sampled randomly and we obtained the gate fidelity by the above procedure. We repeated this procedure 1,000 times to obtain the fidelity distribution. Then, the obtained fidelity distribution was fitted by the Gaussian distribution and its standard deviation was extracted.

### Estimation of resonance frequencies fluctuations

The time dependence of the resonance frequencies  $f_{1,\downarrow}$ ,  $f_{1,\uparrow}$ ,  $f_{2,\downarrow}$ , and  $f_{2,\uparrow}$  were extracted from repeated Ramsey fringe measurements. We sequentially measured Ramsey fringes for  $Q_1$  ( $Q_2$ ) when  $Q_2$  ( $Q_1$ ) was spin-down and spin-up by changing the evolution time from 0.04  $\mu$ s to 4.0  $\mu$ s with a 0.04- $\mu$ s step. Then we estimated each resonance frequency from a single record of the Ramsey fringe by Bayesian estimation<sup>32,43</sup>. This single cycle takes 1.706 s. We repeated the measurement for 10,000 cycles and extracted the time dependence of  $f_{1,\downarrow}$ ,  $f_{1,\uparrow}$ ,  $f_{2,\downarrow}$ , and  $f_{2,\uparrow}$ . The fluctuation of  $J/2$  ( $\Delta J/2 = (\Delta f_{1,\uparrow} - \Delta f_{1,\downarrow})/2$ ) and the single-qubit frequencies of  $Q_1$  ( $\Delta f_1 = (\Delta f_{1,\uparrow} + \Delta f_{1,\downarrow})/2$ ) and  $Q_2$  ( $\Delta f_2 = (\Delta f_{2,\uparrow} + \Delta f_{2,\downarrow})/2$ ) are shown in Extended Data Fig. 5a. As the fluctuation of each resonance frequency is  $\Delta f_{m,a} = \Delta f_m \pm \Delta J/2$  and  $\Delta f_m$  is larger than  $\Delta J/2$ , the dephasing times are mostly limited by the noise in single-qubit frequencies rather than that of  $J$  and therefore  $J$  does not have a significant impact on the dephasing times as shown in Fig. 1e.

### Theoretical description of controlled-rotation

To understand the time evolution of the system under EDSR control, it is simpler to consider a time-dependent rotating frame  $R = \text{diag}(e^{-i2\pi E_Z t}, e^{-i\pi(-dE_Z - J)t}, e^{-i\pi(dE_Z - J)t}, e^{i2\pi E_Z t})$ . Then the Hamiltonian is described by

$$H_R(t) = RHR^\dagger - \frac{i\hbar}{2\pi} \frac{\partial R}{\partial t} R^\dagger = \frac{\hbar}{2} \begin{pmatrix} 0 & f_{R,2} e^{-i2\pi(f_{2,\uparrow} - f_{MW})t + i\phi} & f_{R,1} e^{-i2\pi(f_{1,\uparrow} - f_{MW})t + i\phi} & 0 \\ f_{R,2} e^{i2\pi(f_{2,\uparrow} - f_{MW})t - i\phi} & 0 & 0 & f_{R,1} e^{-i2\pi(f_{1,\downarrow} - f_{MW})t + i\phi} \\ f_{R,1} e^{i2\pi(f_{1,\uparrow} - f_{MW})t + i\phi} & 0 & 0 & f_{R,2} e^{-i2\pi(f_{2,\downarrow} - f_{MW})t + i\phi} \\ 0 & f_{R,1} e^{i2\pi(f_{1,\downarrow} - f_{MW})t - i\phi} & f_{R,2} e^{i2\pi(f_{2,\downarrow} - f_{MW})t - i\phi} & 0 \end{pmatrix}.$$

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The off-diagonal terms result in a CROT gate by choosing one of the resonance frequencies  $f_{1,\downarrow}$ ,  $f_{1,\uparrow}$ ,  $f_{2,\downarrow}$  and  $f_{2,\uparrow}$ . Here the effect of fast-oscillating terms (with  $f_{R,1}$  ( $f_{R,2}$ ) when  $f_{MW} = f_{2,\downarrow}$  or  $f_{2,\uparrow}$  ( $f_{1,\downarrow}$  or  $f_{1,\uparrow}$ )) is averaged out during the  $\pi/2$  CROT time  $t_{hp}$ . In general,  $f_{R,1}/f_{R,2} \neq 1$  as the gyromagnetic ratios of  $Q_1$  and  $Q_2$  are different and also spin-electric coupling is different between the qubits. Nevertheless, we can tune the Rabi frequencies for both qubits to a single target value of  $f_R$  when they are rotated as the target qubit as the terms for the control qubit vanish. When  $J$  is comparable to  $f_R$ , the terms oscillating with a frequency of  $J$  results in unwanted off-resonant rotation of the target qubit. The off-resonant rotation follows the Hamiltonian  $\frac{\hbar}{2} \begin{pmatrix} \pm J & f_R \\ f_R & \mp J \end{pmatrix}$  and therefore

the target qubit rotates along a tilted axis with an effective Rabi frequency  $\tilde{f}_R = \sqrt{f_R^2 + J^2}$ . To suppress the population transfer by the off-resonant rotation,  $\tilde{f}_R t_{hp}$  must be an integer, and therefore we use the Rabi frequency such that  $f_R = J/\sqrt{16k^2 - 1}$  where  $k$  is an integer<sup>16,31</sup>.

## Simulation of two-qubit gate infidelity by quasi-static noise in resonance frequencies

We simulate the effect of the resonance frequency noise<sup>16</sup> on the two-qubit primitive gate fidelity. We assume that the noise is quasi-static in the single measurement of 100  $\mu$ s but changes in between the measurements. We use the measured time dependence of  $\Delta J$ ,  $\Delta d\tilde{E}_Z = \Delta f_2 - \Delta f_1$  and  $\Delta E_Z = (\Delta f_1 + \Delta f_2)/2$  (Extended Data Fig. 5a). We calculated  $\pi/2$  CROT operators by  $U_{\text{CROT}} = \prod_{k=0}^{k=N} e^{-i2\pi(H_R(k\Delta t) + \Delta H_R)\Delta t/\hbar}$  where  $\Delta H_R = \text{diag}(2h\Delta E_Z, -h\Delta d\tilde{E}_Z - h\Delta J, h\Delta d\tilde{E}_Z - h\Delta J, -2h\Delta E_Z)/2$ ,  $\Delta t = t_{hp}/N$ , and  $N$  is a large integer ( $N = 1,000$  in our calculation) and obtained operators of the primitive gates. Then we calculated the probability of the ideal final state as a function of the number of randomly chosen Clifford gates. We averaged 60 random sequences, each of which was repeated 100 times with different  $\Delta J$ ,  $\Delta d\tilde{E}_Z$  and  $\Delta E_Z$ . Then we extracted the two-qubit primitive gate infidelity. In the simulation,  $J$  is fixed at  $\sqrt{15}f_R$ . The  $f_R$  dependence of the infidelity is shown in Extended Data Fig. 5b. Around the optimal gate condition  $f_R = 4\text{--}5$  MHz, the infidelity is only about 0.1%.

Future studies will include a CNOT gate with pulsed exchange control<sup>18</sup> to make the system suitable for scaling up. In addition to switching the exchange coupling, this requires an additional idle time of  $(2 - \sqrt{15}/2)\Delta t$  in all of the primitive gates to make the total gate time  $2\Delta t$  to remove unwanted controlled-phase accumulation during the exchange pulse<sup>18,31</sup>. We simulated this case as shown in Extended Data Fig. 5c. Around  $f_R = 4\text{--}5$  MHz, the infidelity caused by the additional idle time is less than 0.1%. Combined with a high-fidelity pulsed exchange control<sup>37</sup>, we anticipate a CNOT gate fidelity higher than 99% with exchange pulses within reach.

## State tomography

First, we removed the measurement error from the measured joint probabilities  $P_M = (P_{\downarrow\downarrow}, P_{\downarrow\uparrow}, P_{\uparrow\downarrow}, P_{\uparrow\uparrow})$  and obtained the joint probabilities  $P = (P_{\downarrow\downarrow}, P_{\downarrow\uparrow}, P_{\uparrow\downarrow}, P_{\uparrow\uparrow})$ , which was used to extract its density matrix. To do this, we measured a readout correction matrix  $C$  as shown in Extended Data Fig. 7. Here four computational basis states ( $|\uparrow\uparrow\rangle$ ,  $|\uparrow\downarrow\rangle$ ,  $|\downarrow\uparrow\rangle$  and  $|\downarrow\downarrow\rangle$ )

were prepared and joint probabilities were measured<sup>44</sup>. Then,  $P$  was obtained such that  $P = C^{-1}P_M$ .

Next, we performed a maximum likelihood estimation to make the density matrix physical<sup>16,18,19,41</sup>. A physical density matrix  $\rho$  can be described using a complex lower triangular matrix having real diagonal elements  $T$  as  $\rho = TT^\dagger/\text{Tr}(TT^\dagger)$ . Then we minimized the cost function

$$C(\mathbf{t}) = \sum_{v=1}^{16} \frac{(\langle \psi_v | \rho(\mathbf{t}) | \psi_v \rangle - P_v)^2}{2\langle \psi_v | \rho(\mathbf{t}) | \psi_v \rangle},$$

where  $\mathbf{t} = (t_1, t_2, \dots, t_{16})$  is the real parameters of  $T$ ,  $v$  is the state number (1, 2, ..., 16),  $P_v$  is the probability projected at a state  $|\psi_v\rangle$  obtained by averaging measurement results of 10,000 shots with measurement error correction. To extract  $\mathbf{t}$ , 16 combinations of  $(I, X/2, Y/2, Z)$  pre-rotations acting on  $Q_1$  and  $Q_2$  were used<sup>4,19</sup>. The uncertainty of the state fidelity was obtained by a Monte Carlo method assuming that the measured single-shot probabilities follow multinomial distributions<sup>16,19,41</sup>. Then, the obtained fidelity distribution is fitted by the Gaussian distribution and its standard deviation extracted. We find that just after preparing  $|\downarrow\downarrow\rangle$ , the state fidelity is only 98% (Fig. 4c, g) and subsequent qubit controls do not decrease the state fidelity much (less than 2%) (Fig. 4d–f, h–j). We also obtain the Bell state fidelities of 96–97% (Extended Data Fig. 9b–e). This indicates that the imperfection of state preparation and measurement error removal contributes about 1–2% infidelity to the obtained state infidelities.

## Data availability

All data in this study are available from the Zenodo repository at <https://doi.org/10.5281/zenodo.5508362>.

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**Author contributions** A.N. and K.T. fabricated the device and performed the measurements. T.N. and T.K. contributed the data acquisition and discussed the results. A.S. and G.S. developed and supplied the  $^{28}\text{Si}$ /silicon-silicon-germanium heterostructure. A.N. wrote the manuscript with inputs from all co-authors. S.T. supervised the project.

**Competing interests** The authors declare no competing interests.

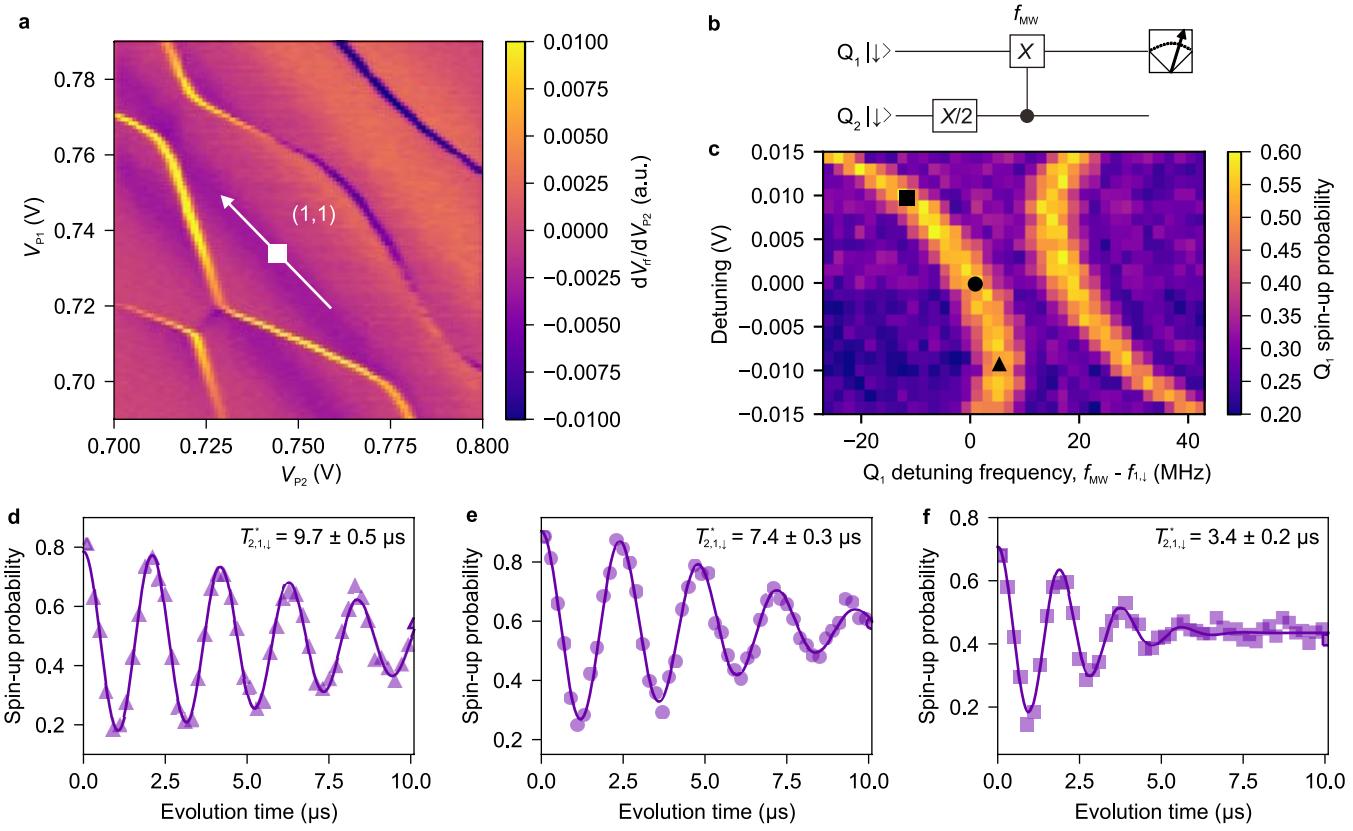
## Additional information

**Supplementary information** The online version contains supplementary material available at <https://doi.org/10.1038/s41586-021-04182-y>.

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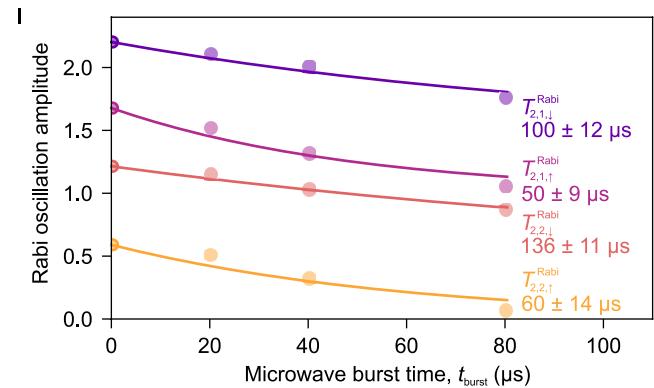
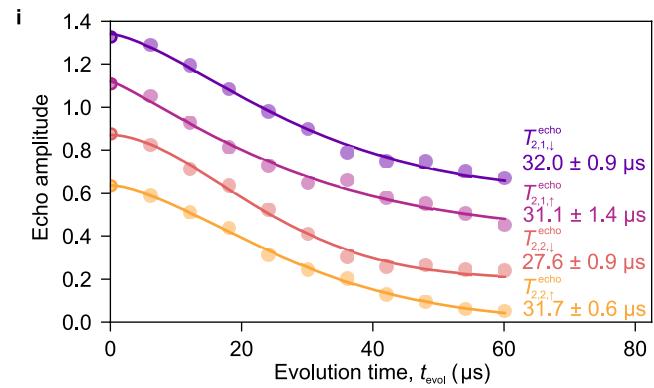
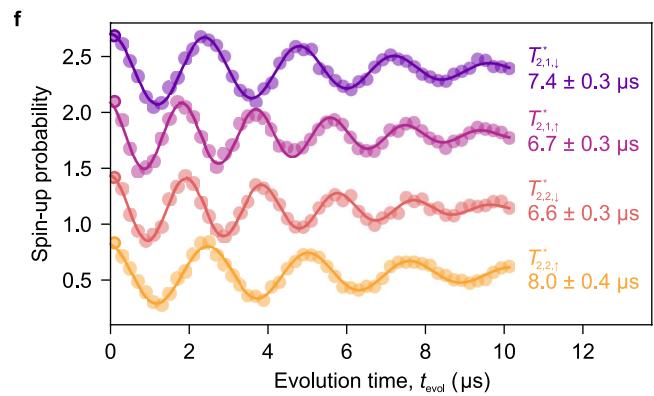
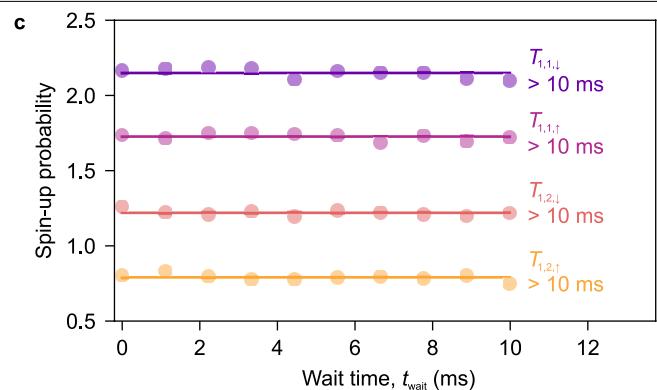
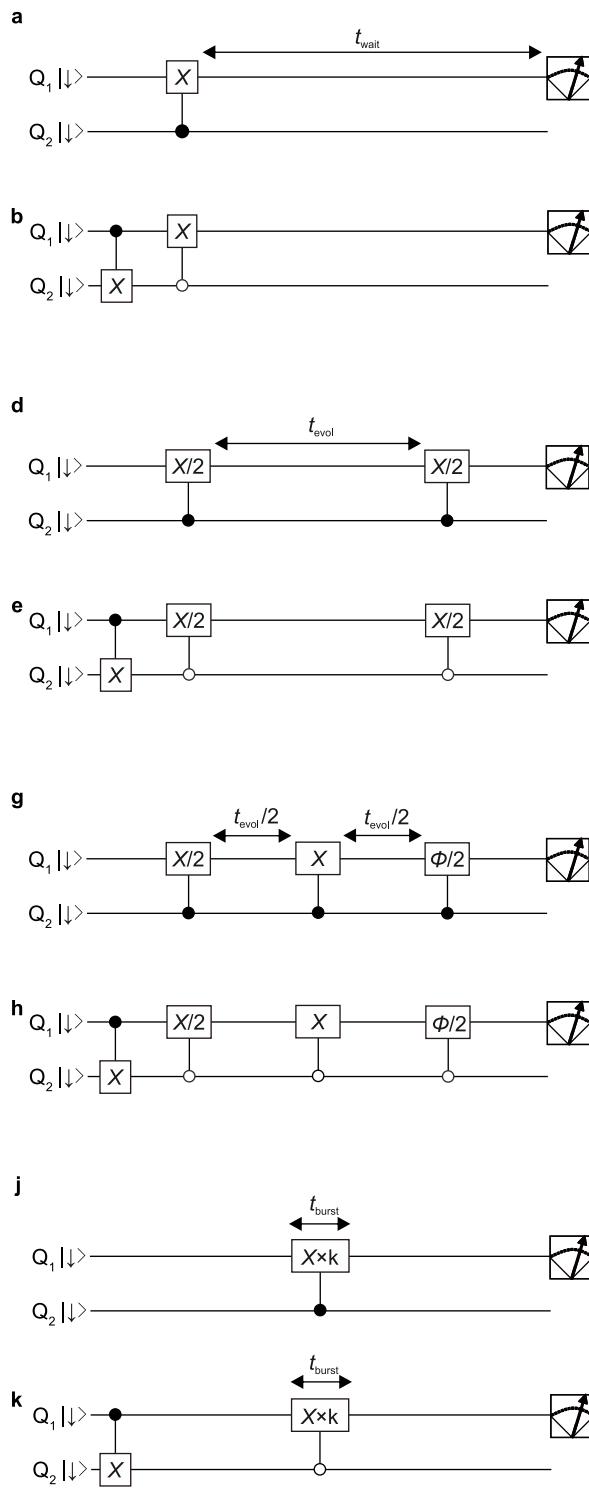
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**Extended Data Fig. 1 | Detuning dependence of EDSR spectra.** **a**, Stability diagram around the (1,1) charge state. **b**, Quantum circuit for producing **c**. The microwave frequency of the  $\pi$  CROT on  $Q_1$  is varied to measure EDSR spectra. **c**, Detuning dependence of EDSR spectra of  $Q_1$ . The detuning axis and its origin are shown as the white arrow and square in **a**. Three black symbols show the conditions where the dephasing times  $T_{2,1,\downarrow}^*$  shown in **d–f** are measured. **d–f**, Ramsey fringes of  $Q_1$  when  $Q_2$  is spin-down measured at the

detuning =  $-0.009\text{ V}$  (**d**),  $0\text{ V}$  (**e**), and  $0.009\text{ V}$  (**f**). The integration time is  $87\text{ s}$  for all of the traces. The errors in  $T_{2,1,\downarrow}^*$  represent the estimated standard errors for the best-fit values. We observe longer (shorter)  $T_{2,1,\downarrow}^*$  when the slope of the EDSR frequency against the detuning is smaller (larger), indicating the detuning charge noise limits  $T_{2,1,\downarrow}^*$  at the charge-symmetry point where a finite slope exists due to the micromagnet-induced gradient field. A similar tendency is also observed in all the  $T_{2,m,o}^*$ .

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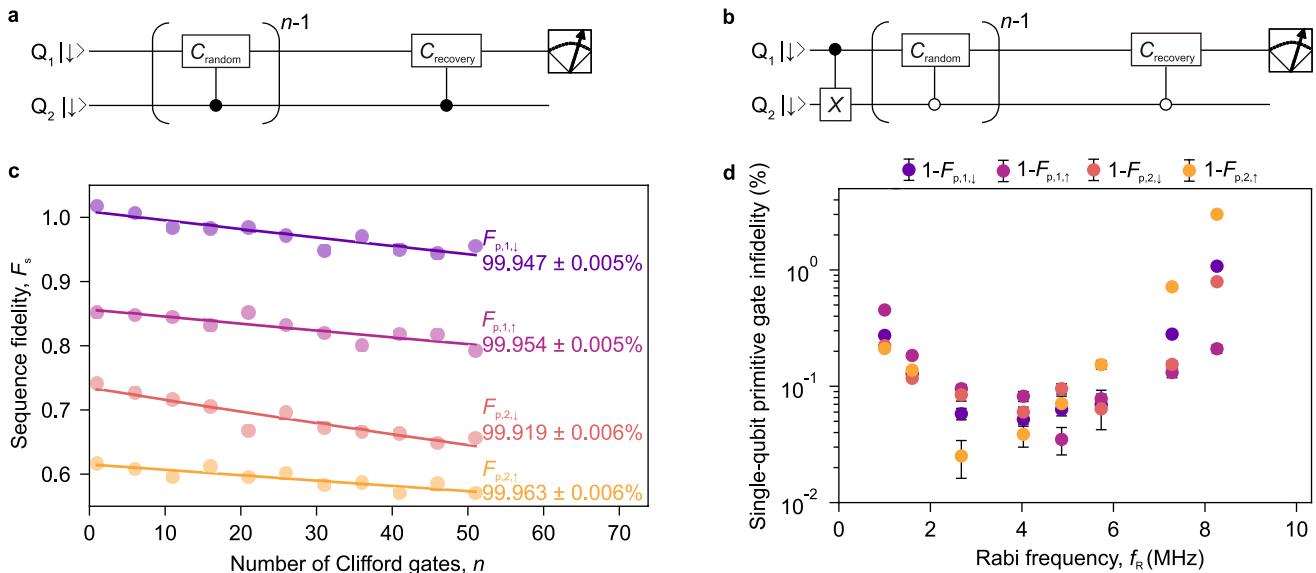


**Extended Data Fig. 2** | See next page for caption.

**Extended Data Fig. 2 | Qubits characterizations.** **a, b**, Sequences to measure spin relaxation times for  $Q_1$  when  $Q_2$  is spin-down,  $T_{1,1,\downarrow}$  (**a**) and -up,  $T_{1,1,\uparrow}$  (**b**). **c**, Spin-up probability as a function of the wait time. All of the traces do not show a decaying property indicating that spin relaxation is negligible for both qubits. The purple (magenta) curve is obtained using the sequence shown in **a** (**b**). The roles of  $Q_1$  and  $Q_2$  are swapped to measure the data for  $Q_2$ . Each trace is offset by 0.45 for clarity. All of the measurements are performed with  $J = 18.85\text{MHz}$  and  $f_R = 4.867\text{MHz}$ . **d, e**, Ramsey sequences to measure dephasing times for  $Q_1$ ,  $T_{2,1,\downarrow}$  and  $T_{2,1,\uparrow}$ . **f**, Ramsey fringes of  $Q_1$  and  $Q_2$  fitted with Gaussian decaying oscillation functions. The integration time is 87 s for all of the traces. The errors represent the estimated standard errors for the best-fit values. Each trace is offset by 0.6 for clarity. **g, h**, Echo sequences to measure echo times for  $Q_1$ ,  $T_{2,1,\downarrow}^{\text{echo}}$  and  $T_{2,1,\uparrow}^{\text{echo}}$ . The phase of the final  $\pi/2$  rotation is varied and the amplitude of the measured oscillation as a function of the phase is plotted in **i**. **i**, Echo

amplitudes as a function of the evolution time. The exponent of the decay is 1.5, 1.2, 1.8, and 1.6 for  $T_{2,1,\downarrow}^{\text{echo}}$ ,  $T_{2,1,\uparrow}^{\text{echo}}$ ,  $T_{2,2,\downarrow}^{\text{echo}}$ , and  $T_{2,2,\uparrow}^{\text{echo}}$ . The errors represent the estimated standard errors for the best-fit values. Each trace is offset by 0.2 for clarity. **j, k**, Measurement of Rabi decay time for  $Q_1$ ,  $T_{2,1,\downarrow}^{\text{Rabi}}$ , and  $T_{2,1,\uparrow}^{\text{Rabi}}$ . We measure Rabi oscillations by varying microwave burst time  $t_{\text{burst}}$  from 0.01  $\mu\text{s}$  to 0.41  $\mu\text{s}$  with a separation of 0.01  $\mu\text{s}$ . Rabi oscillations for longer  $t_{\text{burst}}$  (offset by 20, 40, and 80  $\mu\text{s}$ ) are also measured and the amplitudes of the oscillations are plotted in **l**. **l**, Rabi oscillation amplitude as a function of the microwave burst time with decaying fits. The decay follows  $R_{m,o}(t) = \exp(-t/T_{2,m,o}^{\text{Rabi}})W(t)$  where  $W(t) = \left(1 + t^2/(f_R(T_{2,m,o}^{\text{Rabi}})^2)\right)^{-1/4}$  represents the effect of dephasing<sup>32</sup>. From the fit, we extract the Rabi decay during a  $\pi/2$  CROT as  $D_{m,o} = R_{m,o}(t=1/(4f_R))$ . The errors represent the estimated standard errors for the best-fit values. Each trace is offset by 0.5 for clarity.

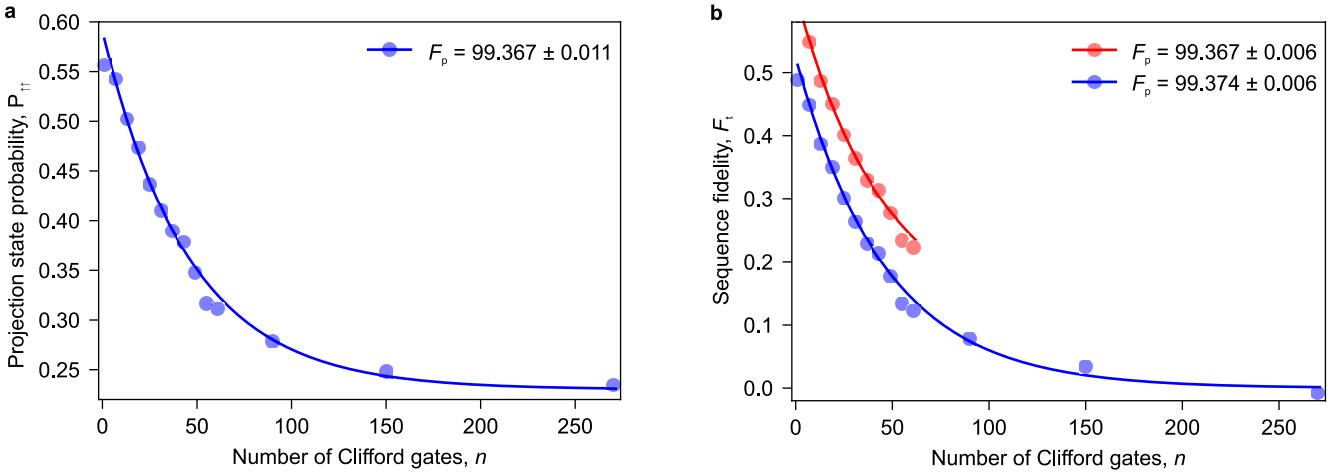
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## Extended Data Fig. 3 | Single-tone single-qubit gate performance.

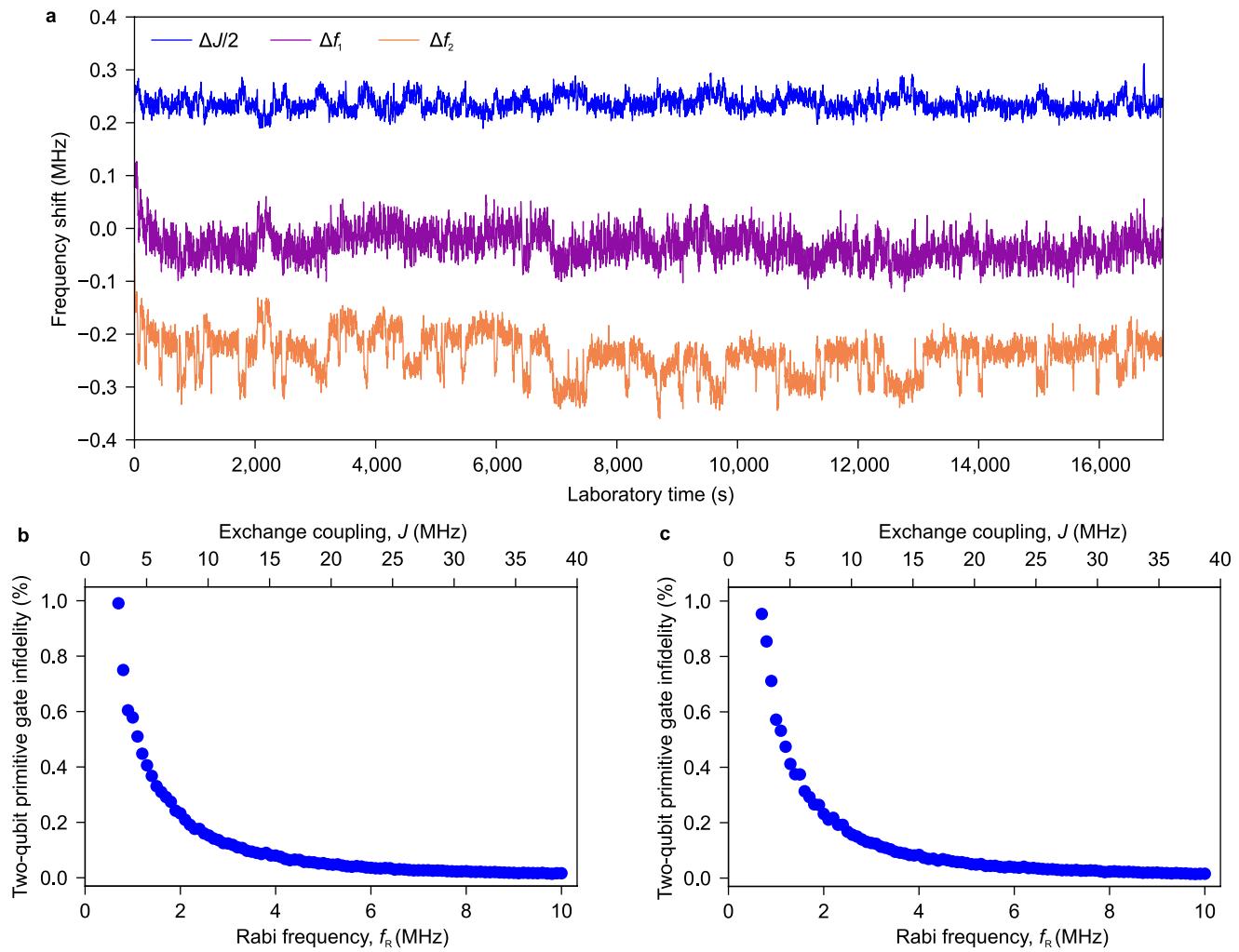
**a, b**, Quantum circuits of single-tone single-qubit Clifford-based randomized benchmarking for  $Q_1$  when  $Q_2$  is spin-down (a) and -up (b). **c**, Single-tone single-qubit primitive gate fidelities  $F_{p,m,\sigma}$  assessed by the Clifford-based randomized benchmarking. The purple (magenta) curve is obtained using the sequence shown in **a** (**b**). The roles of  $Q_1$  and  $Q_2$  are swapped to measure the data for  $Q_2$ .  $f_R = 4.867$  MHz and  $J = 18.85$  MHz =  $\sqrt{15}f_R$  are used. Each trace is offset by 0.15 for clarity. The uncertainty in the gate fidelities are obtained by a Monte Carlo method<sup>4</sup>. The obtained fidelities are consistent with those obtained in Fig. 2c as  $F_{p,m} \approx F_{p,m,\downarrow}F_{p,m,\uparrow}$ . **d**, Rabi frequency dependence of single-tone

single-qubit primitive gate infidelities. Since the control qubit state is fixed in this measurement, the off-resonant rotation does not matter so that  $f_R$  can be varied under a fixed  $J$  of 32.0 MHz. Therefore, the impact of  $f_R$  on the single-qubit gate performance is assessed without involving the effect of  $J$ . We find that the fidelities depend on  $f_R$  and the best values are obtained at  $f_R = 2\text{--}5$  MHz. Around the best condition, the fidelities are uniformly high suggesting that the fidelity is mostly limited by pulse imperfections and calibration errors rather than dephasing and Rabi decay effects. The uncertainty in the gate fidelities are obtained by a Monte Carlo method<sup>4</sup>.



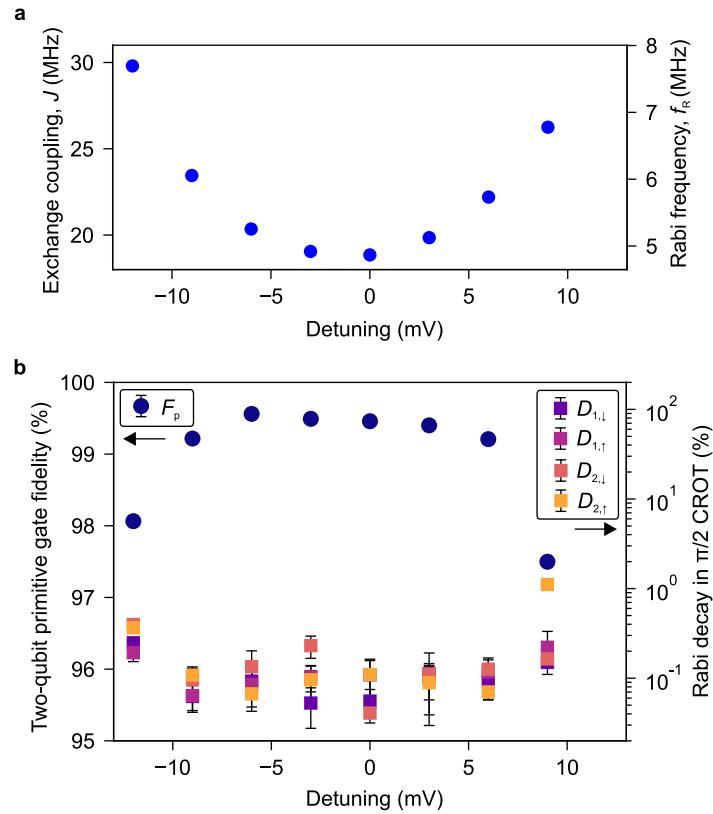
**Extended Data Fig. 4 | Two-qubit gate fidelity extraction.** **a**, Number of Clifford gates  $n$  dependence of the projection state probability  $P_{\uparrow\uparrow}$ <sup>4,16</sup>. The ideal final state is spin-up for both qubits. To extract gate fidelity, we need to measure the saturation value of  $P_{\uparrow\uparrow}$  with a large  $n$  (Methods). The uncertainty in the gate fidelity is obtained by a Monte Carlo method<sup>4</sup>. **b**, Gate fidelity extraction from the sequence fidelity  $F_t$ . In addition to the data in **a**, we measure another data set where the final ideal state is spin-down for both qubits and

then obtain  $F_t$  as shown in blue (Methods). The saturation value of  $F_t$  is almost zero ( $F_t(271) = -0.007$ ) as expected. Gate fidelity extraction using only the data up to  $n = 62$  is shown in red. The uncertainty in the gate fidelities are obtained by a Monte Carlo method<sup>4</sup>. The trace is offset by 0.1 for clarity. The obtained gate fidelities agree well with that obtained in the standard protocol in **a**. The uncertainty in the fidelity is larger in **a** due to the uncertainty of the saturation value of  $P_{\uparrow\uparrow}$ .  $f_R = 5.732$  MHz and  $J = 22.2$  MHz are used.



**Extended Data Fig. 5 | Estimation of two-qubit primitive gate infidelity by resonance frequency noise.** **a**, Time dependence of  $\Delta J/2 = (\Delta f_{1,\uparrow} - \Delta f_{1,\downarrow})/2$  (blue),  $\Delta f_1 = (\Delta f_{1,\uparrow} + \Delta f_{1,\downarrow})/2$  (purple), and  $\Delta f_2 = (\Delta f_{2,\uparrow} + \Delta f_{2,\downarrow})/2$  (orange) extracted from repeated Ramsey fringe measurements (Methods).  $J$  is fixed at 18.85 MHz. Each trace is offset by 0.25 MHz for clarity. Single-qubit frequency

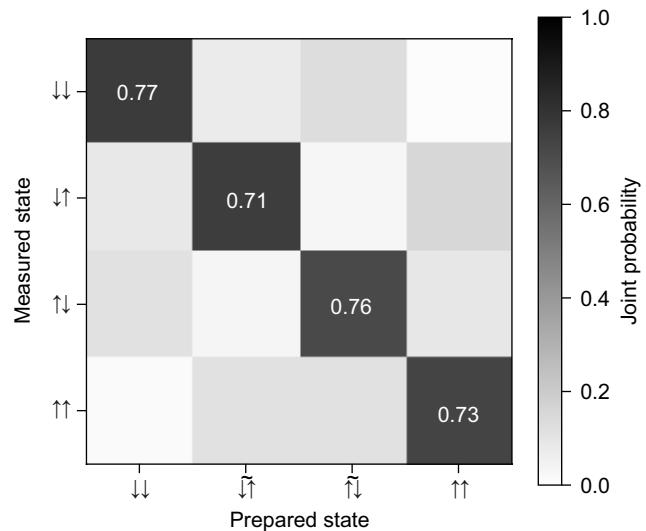
noises ( $\Delta f_1$  and  $\Delta f_2$ ) are larger than that of the exchange noise  $\Delta J/2$ . **b**, Simulation of a two-qubit primitive gate infidelity by the frequency noises obtained in **a** (Methods). **c**, Similar to **b** but the case with inserting an idle time for both qubits to remove the controlled-phase accumulation during the CROT when switching  $J$  on and off<sup>18,31</sup>.



**Extended Data Fig. 6 | Detuning dependence of the two-qubit gate performance.** **a**, Detuning dependence of  $J$ .  $J$  at the charge-symmetry point (detuning = 0 mV) is 18.85 MHz. **b**, Detuning dependence of the two-qubit primitive gate fidelity  $F_p$  (indigo circles) and the Rabi decay during the  $\pi/2$  CROT (colored squares) obtained similarly to Fig. 1f. Around the

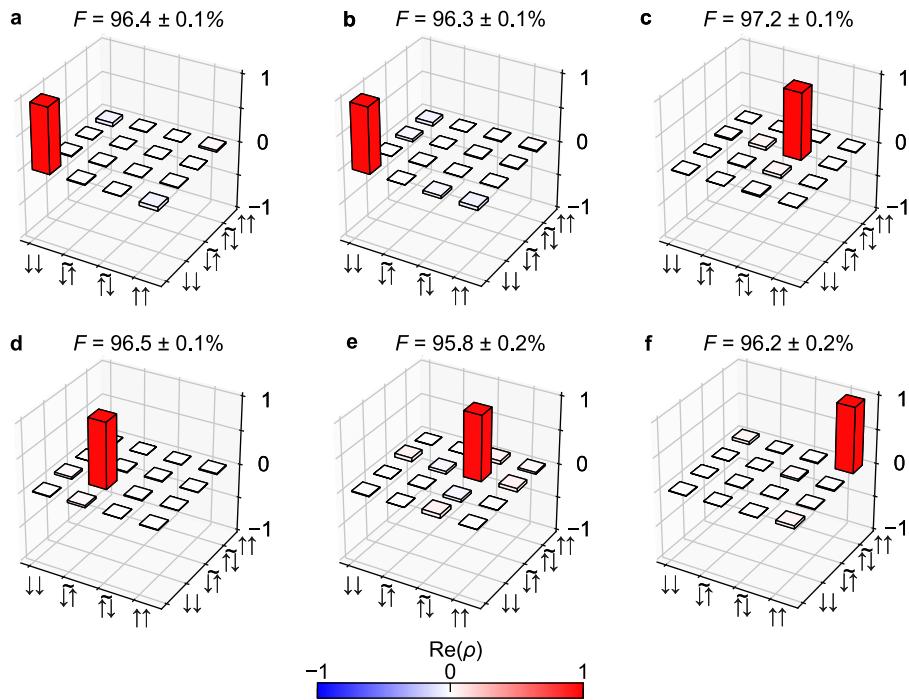
charge-symmetry point, we reproducibly obtain  $F_p$  higher than 99%. In large positive and negative detuning,  $F_p$  sharply drops mainly due to the fast Rabi decay. The uncertainty in the gate fidelity is obtained by a Monte Carlo method<sup>4</sup>. The errors in the Rabi decay represent the estimated standard errors for the best-fit values.

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**Extended Data Fig. 7 | Measurement error calibration in state tomography.**

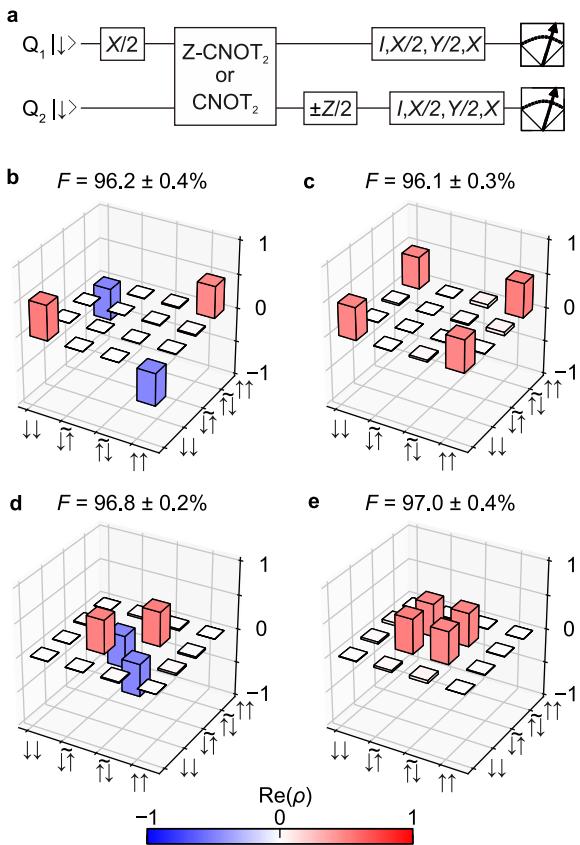
Typical joint probabilities measured with preparing  $|\uparrow\uparrow\rangle$ ,  $|\uparrow\downarrow\rangle$ ,  $|\downarrow\uparrow\rangle$ , and  $|\downarrow\downarrow\rangle$ .  
At  $J = 18.85$  MHz,  $|\downarrow\uparrow\rangle = 0.9995|\downarrow\uparrow\rangle + 0.0310|\uparrow\downarrow\rangle$ .



**Extended Data Fig. 8 | Output state of Deutsch–Jozsa algorithm and Grover search algorithm.** **a–c**, Real part of the measured density matrix for the final output states for  $f_0$  (**a**),  $f_1$  (**b**), and  $f_3$  (**c**) in the Deutsch–Jozsa algorithm (Fig. 4a).

(**d–f**),  $f_{01}$  (**e**), and  $f_{00}$  (**f**) in the Grover search algorithm (Fig. 4b). The absolute values of the matrix elements for the imaginary parts are less than 0.055 (**a**), 0.056 (**b**), 0.040 (**c**), 0.111 (**d**), 0.072 (**e**), and 0.081 (**f**). The uncertainty in the state fidelities  $F$  are obtained by a Monte Carlo method<sup>16,19,41</sup>.

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**Extended Data Fig. 9 | Bell state tomography.** **a**, Quantum circuit for the Bell state tomography. After the first  $\pi/2$  rotation,  $Z\text{-CNOT}_2$  and  $-Z_2/2$  ( $Z_2/2$ ) are applied for **b** (**c**),  $\text{CNOT}_2$  and  $Z_2/2$  ( $-Z_2/2$ ) are applied for **d** (**e**).  $I$ ,  $X/2$ ,  $Y/2$ , and  $X$  acting on both qubits at the end change the measurement axis to implement the state tomography (Methods). **b–e**, Real part of the measured density matrix for the prepared Bell states for  $\Phi^-$  (**b**),  $\Phi^+$  (**c**),  $\Psi^-$  (**d**), and  $\Psi^+$  (**e**), respectively. The absolute values of the matrix elements for the imaginary parts are less than 0.038 (**b**), 0.093 (**c**), 0.100 (**d**), 0.113 (**e**). The uncertainty in the state fidelities  $F$  are obtained by a Monte Carlo method<sup>16,19,41</sup>.