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A topological characterisation of looped drainage networks

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ABSTRACT

Hydrodynamic models are used to analyse water networks (water distribution, drainage, surface water, district heating, etc.). The non-linear nature of water flows necessitates the use of iterative solution methods in hydraulic modelling. This requires a relatively large computational effort. To reduce this effort, networks, network forcing and/or the flow in networks are often simplified and analysed using the Graph Theory. The simplification options depend on the network characteristics. There are many topological features to describe Graph-based networks. In this paper, these characteristics are summarised, applied on 7 urban drainage networks and discussed. As the topological features do not describe the networks in a uniform manner, a new type of topological characterisation of looped drainage networks (Network Linearisation Parameter, NLP) is proposed based on linearized hydraulics and bottlenecks identified in paths to outfalls.

1. Introduction

Networks (e.g. internet, communication, transport, power grids, water distribution networks [WDN], urban drainage networks [UDN]) are crucial for the functioning of today's society and especially for urban areas. The performance level of these networks is under pressure due to: (i) an increasing load as result of urbanisation and population growth, (ii) deterioration as result of ageing (and a lack of maintenance), (iii) terrorist attacks and (iv) for some networks, climate change (Reyes-Silva, Helm, & Krebs, 2020).

To reclaim, or at least maintain, the desired level of service, proper maintenance and rehabilitation of infrastructure are essential (see e.g. Le Gauffre et al., 2007; Wirahadikusumah, Abraham, & Iseley, 2001). Achieving the desired service level is extra important for networks that, because of the interdependence with other networks or co-location, could result in the failure of other networks (D'Agostino & Scala, 2014; Klinkhamer et al., 2019). For developing adequate maintenance strategies and to improve the reliability of networks, understanding network failures is necessary (Klinkhamer et al., 2019; Reyes-Silva et al., 2020).

The influence of the network structure on the performance of the network in terms of efficiency, reliability and robustness is studied for many kinds of networks (ants, cells, internet, power supply, social, transport, water distribution, sewer, etc.) (see e.g. Albert & Barabási, 2002; Barthélemy, 2011; Buhl et al., 2004; Buhl et al., 2006, Giudicianni et al., 2018; Klinkhamer, 2019; Yazdani, Otoo, & Jeffrey, 2011). As the structure of each

network is unique, an unambiguous description of the network is required to make statements on network performance. Therefore, the structures (topology) in combination with the performance of networks have been investigated in a broad range of disciplines (see e.g. Albert & Barabási, 2002; Barthélemy, 2011; Klinkhamer et al., 2019). This has resulted in numerous characteristics, all of which describe one or more aspects of a network (see section 2.3).

The Graph Theory is widely used as starting point for characterising networks and to identify critical elements in networks. This theory, in combination with linearisation of hydrodynamic processes, is also used in the Graph Based Weakest Link Method (GBWLM) (Meijer, Korving, Langeveld, & Clemens-Meyer, 2022). The GBWLM is a method to analyse urban water systems (combination of gully pots, drainage systems and surface water) with long time rainfall series. In order to predict the applicability of the GBWLM to a network in advance, many methods have been reviewed. The topology of 7 urban drainage networks has been analysed with various methods to evaluate if the existing methods offer indicators that could be used to determine the potential for applying a linearized approach. However, none of them meets the following three criteria:

1. Is applicable on spatial networks (these are networks with spatial constraints).
2. Takes into account the functions of the elements of the network.
3. Gives insight in the linearisation possibilities.

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Given these results, a new concept (i.e. Network Linearisation Parameter, NLP) is proposed to classify looped drainage networks based on the network structure, linearized hydraulics and bottlenecks in drainage paths to outfalls. This paper is structured as follows: section 2 presents an overview of existing network concepts and topologies, including the proposed Network Linearisation Parameter (NLP) and a description of the case studies. The application of this concept to 7 UDN is described in section 3. The topological features are discussed in section 4. In section 5, the key findings are summarised along with an outlook and recommendations for future research.

2. Materials and methods

The GBWLM (Meijer et al., 2022) is used to analyse the robustness of urban water systems against capacity reduction and load increase. The GBWLM combines the structure of networks with linearized hydrodynamics. The quadratic hydraulic gradient (Equation (1)) has been simplified to a linear relationship (Equation (2)). Based on the maximum available hydraulic gradient, the maximum capacity of each pipe is computed (Equation (1)). α_{linear} is determined for each pipe by linearising the discharge (between zero and the maximum pipe capacity) and the maximum available hydraulic gradient with Equation (2):

$$I = \alpha_{quadratic}Q^2 \qquad (1)$$
$$I = \alpha_{linear}|Q| \qquad (2)$$

where I is the hydraulic gradient (–), Q is the discharge (m³/s), $\alpha_{quadratic}$ is a quadratic hydraulic parameter (s²/m¹/₄), and α_{linear} is a linear hydraulic parameter (s/m³).

To determine the types of networks for which hydraulics can be linearized, network models and topological characteristics of networks have been analysed. Much literature exists about network models, network topology and failure. Networks have been studied from numerous perspectives (e.g. mathematics, transport, civil engineering (water, energy), computer science, biology, (tele)communication, sociology). The following sections describe successively: (i) important concepts in complex networks analyses, (ii) impact of spatial constraints on networks, (iii) topological features to characterise networks, (iv) the Dual Graph Approach to convert spatial networks to scale free networks, (v) the NLP, a new topology parameter for urban drainage networks and (vi) the case studies.

2.1. Important concepts in complex networks analyses

Albert and Barabási (2002) have presented an overview of the advances in the field of complex networks, focussing on the statistical mechanics of network topology and dynamics. They discuss the main models and analytical tools, covering random graphs and small-world and scale-free networks, as well as the interactions between topology and the network's robustness against failures and terrorist attacks.

Until the 1950s, complex networks were mainly studied as regular graphs using graph theory. Since the 1950s, complex networks have been described as random graphs. A widely used model for studying random graphs is the Erdós-Rényi model. This model starts with N nodes and every edge is formed with probability p independently of every other edge.

This results in a graph with approximately $p\frac{N(n-1)}{2}$ edges distributed randomly (Albert & Barabási, 2002).

Albert and Barabási (2002) also observe that since the 1990s, three concepts play an important role in network analyses:

1. The small-world concept: there is a relatively short path between any two nodes, even in large networks. The path length is defined as the number of edges along the shortest path (Watts, 1999; Watts & Strogatz, 1998).
2. Clustering: cliques are common in social networks. In these circles of acquaintances, every member knows every other member. The degree of clustering can be expressed with the clustering coefficient:

$$C_i = \frac{2E_i}{k_i(k_i - 1)} \qquad (3)$$

where k_i is the number of edges connected to node i and E_i is the total number of edges.

3. Scale-free networks and degree distribution (degree is the number of edges that are connected to the node): the distribution of the node degree $P(k)$ for large networks has a power-law tail as follows:

$$P(k) \sim k^{-\gamma} \qquad (4)$$

where $P(k)$ is the distribution of the node degree and k is the node degree.

The three concepts (small worlds, clustering and scale-free networks) result in three main classes of modelling paradigms (Albert & Barabási, 2002):

1. Random graphs based on the Erdös–Rénvi model serve as benchmarks for modelling and empirical studies.
2. Small-world models are situated between random graphs and highly clustered regular lattices.
3. Scale-free models are used to explain the origin of non-Poisson degree distributions (such as power-law tails) as seen in real systems by focussing on the network dynamics.

Scale-free models are widely used to test the robustness of networks. Albert and Barabási (2002) have shown that networks that meet the criteria of scale-free models display a high degree of robustness against random failure of edges. However, nodes with a high degree (hubs are nodes connected to many other nodes) are crucial for the functioning of scale-free networks. Therefore, the functioning of these networks is susceptible to (terrorist) attacks on hubs.

2.2. Impact of spatial constraints on networks

Barthélemy (2011) has presented an overview of the influence of spatial constraints on the structure and properties of networks (transportation and mobility networks, internet, mobile phone networks, power grids and social and contact networks). The topological structure is strongly influenced by space, as an important consequence of space is that costs are associated

with the length and location of the edges. Two main categories of spatial networks are planar graphs and non-planar graphs.

"A plane graph is a graph drawn in the plane in such a way that any pair of edges meet only at their end vertices (if they meet at all). A planar graph is a graph which is isomorphic to a plane graph, i.e. it can be (re)drawn as a plane graph" (Clark & Holton, 1991, p. 157). In contrast to planar graphs, non-planar graphs can have intersection links (e.g. rail networks or flyovers, airline networks, cargo ship networks or the internet) (Barthélemy, 2011).

Spatial constraints affect the network characteristics of planar graphs in the following ways (Barthélemy, 2011):

- In planar graphs, $P(k)$ is peaked because space restricts the existence of high degrees.
- In planar graphs, the length of links is limited, and the distribution is peaked.
- The tendency to connect to hubs is limited, but there is a tendency of cliques to form between spatially close nodes leading to higher clustering coefficients.
- In 2D planar networks, the average shortest paths scale as $\sqrt{\text{No. of nodes}}$.

According to Barthélemy (2011), the five most important models of spatial networks are the following:

1. Random geometric graph: the nodes in a plane are connected according to a given geometric rule (e.g. distance).
2. Spatial Erdös-Rénvi graph: the probability to connect two nodes depends on the distance between these nodes.
3. Spatial small-world graph: based on a given probability distribution for their length, random links are added to a d-dimensional lattice.
4. Spatial growth model: a spatial extension of the scale-free models in which new nodes have a preference to be connected to already well-connected nodes.
5. Optimal networks: networks obtained by the minimisation of a certain cost function.

The optimal network model is important in many practical engineering issues related to both the problem of optimal networks and optimal flow through the networks. This has resulted in a large variety of hub-and-spoke networks. In the case of optimal networks, fluctuations and resilience to sabotage attempts naturally lead to the formation of loops. However, loops can also have adverse effects. For example, in WDNs, loops might lead to prolonged stagnation of water, which can have a negative effect on water quality.

2.3. Topological features to characterise networks

Barthélemy (2011) has presented an overview of the main empirical features that can be used to characterise all types of networks with spatial constrains:

- Number of nodes N;

- Average (node) degree $\langle k \rangle$ (degree = the number of edges connected to a node);
- Average clustering coefficient C Equation (5);
- Average shortest path l;
- The degree distributions $P(k)$ (scale-free models: broad with a power-law tail; spatial constraints model: peaked);
- The weight distributions of the edges $P(w)$ (peaked or broad);
- The scaling of the strength with the degree $s(k) \sim k^\beta$;
- The relationship between the centrality (number of shortest paths through a link or node) and the degree;
- Meshedness (Buhl et al., 2004):

$$M = \frac{E - N + 1}{2N - 5} \qquad (5)$$

where M is meshedness, E is the number of edges and N is the number of nodes.

The trend of describing networks based on topological characteristics has continued over the past 10 years (Giudicianni et al., 2018; Johnson, Flage, & Guikema, 2019; Johnson, Reilly, Flage, & Guikema, 2021; Meng, Fu, Farmani, Sweetapple, & Butler, 2018; Metcalfe, 2020; Reyes-Silva et al., 2020; Yazdani et al., 2011). Table 1 summarises the parameters used to describe networks.

To clarify the relationship between the structure of a network and the topological features, these values have been calculated for two networks: an unstructured grid of 10×10 and the minimum spanning tree of this grid (see Figures 1 and 2 and Table 2).

2.4. Converting spatial networks to scale free networks with dual graph approach

Over the past 10–15 years, the dual graph approach has been used to analyse urban road, drainage and water distribution networks. A dual graph or line graph (Harary & Norman, 1960) is a graph in which nodes are replaced by edges and edges by nodes. In a dual graph, each vertex represents an edge of the graph. The dual graph has an edge for each pair of vertices in G that are separated from each other by an edge (Harary & Norman, 1960).

To cluster edges, the dual graph approach is combined with the Intersection Continuity Negotiation model (ICN model) (Porta, Crucitti, & Latora, 2006) or Hierarchical Intersection Continuity Negotiation (HICN) (Masucci, Stanilov, & Batty, 2013). The ICN model is a generalisation model based on the principle of continuity for urban street networks. At each graph node, the two edges forming the largest convex angle are assigned the highest continuity and are coupled together. This is repeated for other edges. In nodes with an odd degree, the remaining edge receives the lowest continuity value (Porta et al., 2006). In the HICN, the roads are first categorised to four hierarchical levels, which broadly reflect capacity. The ICN is then applied at each road category (Masucci et al., 2013).

Table 1. Empirical features for characterising networks.

Category	Topological parameters	Description/explanation
Size	Number of nodes	
	Number of links	
Structure	Maximum degree	
	Average degree	
	Degree assortativity	The degree variance in a network.
	Node closeness	The inverse of the sum of shortest paths from a node to every other node.
	Density	Ratio between the number of edges and to the maximal number of edges.
Redundancy	Meshedness coefficient 1	Equation (5)
	Meshedness coefficient 2	Meshedness based on the inner nodes degree (Reyes-Silva et al., 2020).
	Clustering coefficient (or transitivity)	A redundancy measure by quantifying the density of triangular loops and the degree to which junctions in a Graph tend to be linked (Wasserman & Faust, 1994). In grid-like structures and networks with structural loops different from a simple triangle the clustering coefficient is normally small (Yazdani et al., 2011).
Robustness	Algebraic connectivity	The second smallest eigenvalue of the Laplacian matrix. The Laplacian matrix is a matrix with the node degrees minus the adjacency matrix. The adjacency matrix describes the connections between nodes. An eigenvector is a vector which direction does not change in a transformation. The eigen value is the factor by which the eigenvector is scaled. A large algebraic connectivity implies (Wang & Van Mieghem, 2010): (i) a relatively large number of links should be deleted to generate a bipartition. (ii) a robust synchronised state. (iii) a more optimal performance of dynamic processes, e.g. synchronisation of dynamic processes at the nodes of a network and random walks on Graphs. (iv) efficient movement and dissemination of random walks.
	Spectral gap	Difference between first and second eigen values of Graph's adjacency matrix. A large spectral gap is usually associated with high network performance (Watanabe & Masuda, 2010) and a relatively short distance between nodes (Donetti, Neri, & Muñoz, 2006).
	Density of bridges	Number of bridges (edge that does not belong to any cycle) and the total number of links. The higher the density the smaller the meshedness.
	Density of articulation points	Ratio of the number of articulation points (node variant of bridge) and the total number of nodes. The higher the density the smaller the meshedness.
	Inverse spectral radius	Inverse of the largest absolute eigenvalue of the adjacency matrix. The lower the inverse spectral radius, the better is the communication within a network. A low number implies many hubs and a high number many loops (Giudicianni et al., 2018).
	Central point dominance	The average difference in betweenness centrality of the node with the maximum betweenness centrality and all other nodes. A high value means that one node is much more often in a path than the other nodes of the network.
Distance measures	Average path length l	
	Network diameter	The maximum eccentricity. Eccentricity of node n is the maximum distance from n to all other nodes in G.
	Network radius	The minimum eccentricity.
	Average hop count	The average shortest-path between all node pairs.
Centrality/ importance	Node betweenness (max)	Number of shortest paths through a node.
	Link betweenness (max)	Number of shortest paths through a link.

A way of schematising a UDN or WDN as a graph is to assign a node for each manhole or connection point and an edge for each pipe or connection. In the dual graph approach, groups of pipes with, for example, the same diameter are schematised as a node, and the manholes are schematised as connections between the groups of pipes. The diameters are applied as an HICN criterion (Zischg et al., 2017, 2019). According to Klinkhamer et al. (2019), high node degrees represent collector pipes in UDNs; however, Zischg et al. (2019) have shown that high node degrees do not necessarily correlate with large pipe diameters.

According to Krueger et al. (2017), the benefit of dual mapping (based on pipe diameters) of urban water networks is that separate pipes are mapped as "functional pipe units". The node-degree distribution is affected by representing a network as a dual graph. Kalapala, Sanwalani, Clauset, and Moore (2006) and Klinkhamer et al. (2019) analysed urban road networks, drainage networks and water supply networks with the dual graph approach and the HICN. They show that the node-degree distribution of these networks changes from a peaked distribution to a highly variable degree distribution with a heavily tailed $P(k)$. They conclude that these properties suggest a robustness against random failures and a vulnerability to the loss of high node-

degree hubs. The overlap in locations between the networks introduces the possibility for cascading failures affecting multiple infrastructure networks. Klinkhamer et al. (2017) conclude that co-located large degree elements can be applied as a criterion to allocate resources for maintenance and investment.

2.5. A New topology parameter for urban drainage networks

The GBWLM is developed to analyse large comprehensive networks with multi-annual precipitation series (Meijer et al., 2022). To determine in advance whether the hydrodynamics can be linearised and the GBWLM is suitable for analysing a network, the NLP has been developed as an indicator. The NLP is based on the network geometry, pipe characteristics and the runoff area.

In optimal designed networks, the pipe diameters are matched to the design flow rates. In flat areas, this results in uniform hydraulic gradients, especially in pipes toward outflow locations. In pipes furthest from the outfalls, other design criteria influence the minimum pipe diameter, resulting in lower gradients. At bottlenecks (pipes with larger

Figure 1. Unstructured 10×10 grid and its minimum spanning tree.

hydraulic gradients at high flows than adjacent upstream pipes), the gradient increases to allow a larger discharge.

If full hydraulic equations are used, each pipe gets a specific hydraulic gradient in order to use the maximum capacity of the network. In the GBWLM, each pipe has a fixed maximum capacity based on a fixed gradient. The linearisation in the GBWLM is valid when the hydraulic gradients in the network are approximately equal. The degree of homogeneity of the gradients in a catchment draining into an overflow determines whether linearisation can be applied successfully. If there are many bottlenecks, especially in the main pipes to the outfall locations, the outcomes of the GBWLM are less valid.

The hydraulic gradient in the GBWLM depends on the α_{linear} and the discharge (Equation (2)). The required capacity of each pipe is unknown at the start. To estimate the discharge, the shortest paths between all manholes and the outflow structure are determined. For each pipe, the runoff area that discharges via a pipe (RA_{pipe}) is calculated based on the runoff area of each manhole and the shortest path from the manhole to an outfall. Based on the α_{linear} and RA_{pipe}, the NLP is determined by:

$$\text{NLP} = \frac{1}{\alpha_{linear}\ RA_{pipe}} \tag{6}$$

where NLP is the network linearisation parameter (m/s), RA_{pipe} is the runoff area that discharges via a pipe (m^2) and α_{linear} is a hydraulic parameter (s/m^3).

For each pipe, the NLP is quantified along a path from the manhole to the outflow location by calculating the NLP-factor (Equation (7)). If the NLP-factor of a pipe is larger than 1, the head loss is larger than the head loss of the directly upstream pipe. If the NLP-factor exceeds a threshold, the pipe is labelled

as a "bottleneck". The part of a UDN that drains to an outflow location is defined as an "outflow catchment". For each bottleneck, two parameters are determined:

1. Bottleneck location, Equation (8).
2. Percentage of affected manholes, Equation (9):

$$\text{NLP} - \text{factor}\ = \frac{\alpha_2\ RA_{\text{down stream pipe}}}{\alpha_2\ RA_{\ \text{up stream pipe}}} \tag{7}$$

$$\text{Bottleneck location}\ = \frac{\text{Pipe No.}}{\text{Max. path length}} \tag{8}$$

$$\text{\% affected manholes}\ = \frac{\text{No. manholes}}{\text{Tot. No. manholes}} \tag{9}$$

where the bottleneck location is the position of the bottleneck in a path relative to the maximum path length of all manholes in an outflow catchment, Pipe No. is the location number of a pipe counted from outflow location (–), Max. path length is the maximum path length to the outfall location of all manholes in an outflow catchment (–), % affected manholes is the percentage of manholes affected by the bottleneck, No. Manholes is the number of manholes with a path to an outlet crossing the bottleneck and Tot. No. Manholes is the total number of manholes in a UDN (see Figure 3).

2.6. Case studies

The network parameters are applied on 7 urban drainage systems. Figure 4 and Table 3 show the structure (manholes and pipes).
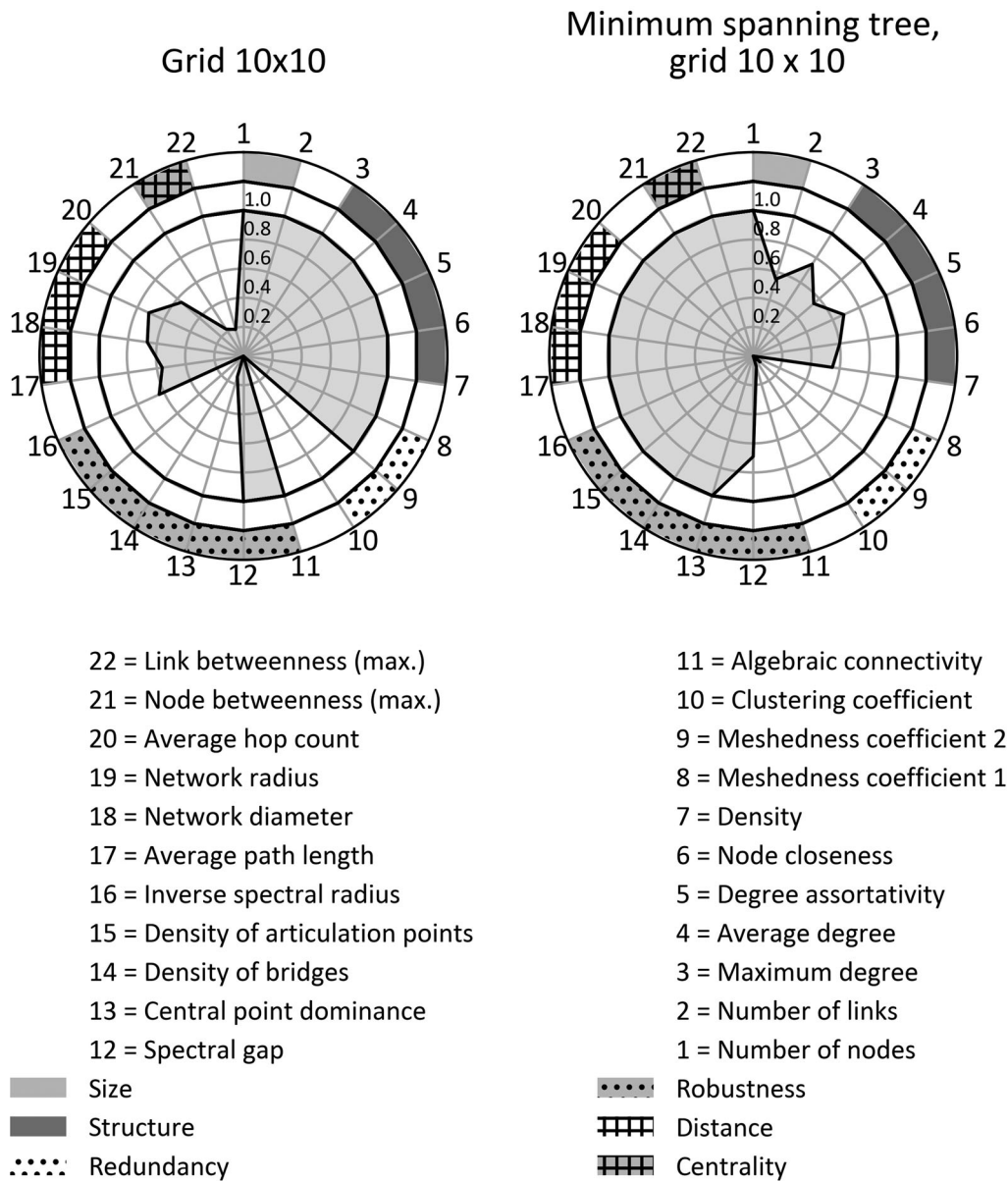
Grid 10x10

Minimum spanning tree, grid 10 x 10

22 = Link betweenness (max.)

21 = Node betweenness (max.)

20 = Average hop count

19 = Network radius

18 = Network diameter

17 = Average path length

16 = Inverse spectral radius

15 = Density of articulation points

14 = Density of bridges

13 = Central point dominance

12 = Spectral gap

11 = Algebraic connectivity

10 = Clustering coefficient

9 = Meshedness coefficient 2

8 = Meshedness coefficient 1

7 = Density

6 = Node closeness

5 = Degree assortativity

4 = Average degree

3 = Maximum degree

2 = Number of links

1 = Number of nodes

	Size		Robustness
	Structure		Distance
	Redundancy		Centrality

Figure 2. Normalised topological features of an unstructured 10 × 10 grid and its minimum spanning tree.

The systems used are based on the Dutch context. The catchment area of Loenen is mildly sloping. The other catchments are flat.

3. Results and interpretation

3.1. Reflection on the applicability of the topological parameters for urban drainage systems

Underground piped systems, as UDNs, are examples of planar graphs. They have spatial constraints, and connections between distant nodes normally do not occur (Buhl et al., 2004). UDNs have some typical characteristics that distinguish them from other spatial networks: (i) UDN are supply driven, (ii) UDN have multi origins and few destinations, and (iii) UDN are mainly gravity driven. Free surface and pressurised flow may both occur during a storm event.

Because of spatial constraints, the characteristics of UDNs do not correspond to the characteristics of small-world and scale-free networks (Barthélemy, 2011). The topological features mentioned

in Subsection 2.3 to describe networks can be applied to describe UDNs, but these turn out to be very specific for each network (see Figure 5 and Table 4). In Figure 5, values have been normalised based on the minimum and maximum values of the parameters for the seven networks.

Parameters 1 and 2 are measures for the size of the network. The larger networks Vlijmen and Drunen, holding many nodes and pipes, have a high score. Parameters 3–6 describe the structures of the UDNs. The differences in the parameter values for the tested networks are limited. The average degree of the less-looped system of Loenen is slightly lower than that of the other looped networks. The parameter values "node closeness" and "density" of the Almere UDNs are slightly higher than those of the other networks. This may be related to the fact that the Almere networks are smaller (less nodes and pipes) and more looped than the other networks.

Parameters 8–10 are indicators for redundancy. As expected, the meshedness parameters for the partly branched and partly looped UDN of Loenen are slightly less than those of the other

Table 2. Topological features of an unstructured grid (10 × 10), its minimum spanning tree and the differences between these characteristics for both networks.

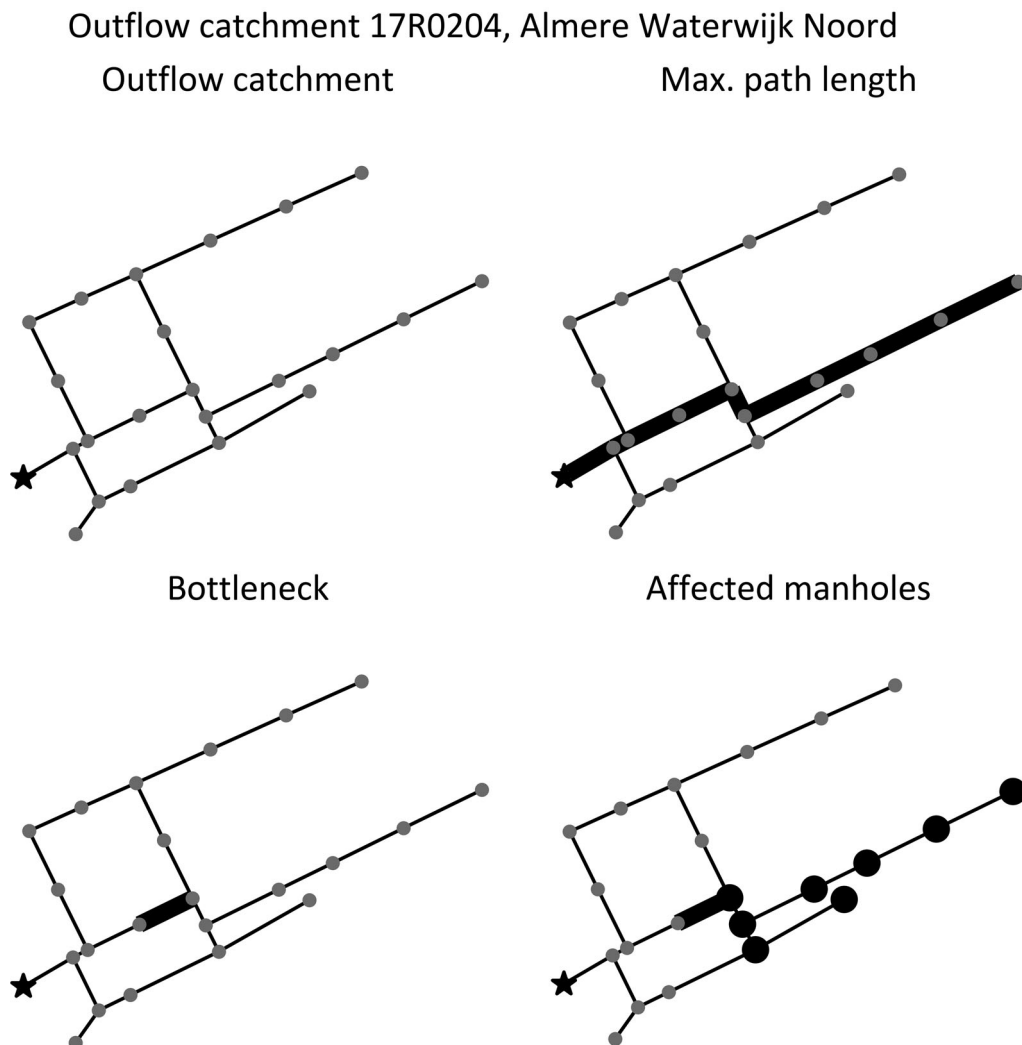| Parameter	Category	Grid 10 × 10	Minimum spanning tree, grid 10 × 10	Difference
Number of nodes	Size	100	100	0
Number of links	Size	180	99	81
Maximum degree	Structure	4	3	1
Average degree	Structure	3.6	1.98	1.62
Degree assortativity	Structure	0.57	0.39	0.18
Node closeness	Structure	0.15	0.09	0.06
Density	Structure	0.0364	0.02	0.0164
Meshedness coefficient 1	Redundancy	0.42	0	0.42
Meshedness coefficient 2	Redundancy	0.8	0.05	0.75
Clustering coefficient	Redundancy	0	0	0
Algebraic connectivity	Robustness	0.0979	0.0076	0.0903
Spectral gap	Robustness	0.2365	0.1633	0.0732
Central point dominance	Robustness	0.07	0.47	0.4
Density of bridges	Robustness	0	1	1
Density of articulation points	Robustness	0	0.9	0.9
Inverse spectral radius	Robustness	0.2606	0.4098	0.1492
Average path length	Distance measures	6.67	11.85	5.18
Network diameter	Distance measures	18	27	9
Network radius	Distance measures	10	14	4
Average hop count	Distance measures	6.67	11.85	5.18
Node betweenness (max)	Centrality	616.21	2810	2193.79
Link betweenness (max)	Centrality	475	2500	2025

Figure 3. Example of the network linearisation parameter for outflow catchment 17R0204 of the urban drainage network of Almere Waterwijk Noord. The upper-left subplot depicts the outflow catchment 17R0204 and the outfall is indicated by a star. The wide black line in the upper-right subplot is the maximum path length. The wide black line in the lower-left figure is the location of the bottleneck. The dots in the lower-right figure are the affected manholes.

networks. The clustering coefficient indicates the absence of tri-angle loops in six of the seven tested networks. The robustness parameters (11–16) contain three parameters (density of bridges and articulation points and inverse spectral radius) that have an inverse relationship with the meshedness parameters. The UDN of Loenen scores highly for these parameters. The smaller and more meshed networks of Almere have high values for algebraic connectivity and spectral gap.

This is consistent with the characteristic that a relatively large number of links should be deleted to generate a biparti-tion (characteristic of algebraic connectivity) and the relatively short distance between nodes (characteristic of spectral gap). The central point dominance of the UDNs of Loenen, Vlijmen and Heusden is relatively high. These networks are either partly branched (Loenen) or made of different parts connected by a few pipes only (see Figure 4). The distance parameters (17–20) and the centrality parameters (21–22) are high for the larger tested networks and low for the smaller ones.

3.2. Reflection on the applicability of the dual graph approach

The dual graph approach, in combination with the HICN (based on pipe diameters), is used to convert UDNs into networks with the characteristics of small-world and scale-free networks (Zischg et al., 2019). Figure 6 shows an example of the UDN of Loenen and its dual graph represen-tation. Loenen's UDN is used as an example because, of the seven UDNs, it is the most branched one. The structure of the network is still partly recognisable in this example. Applying the dual graph approach on the Loenen network has two important consequences. First, the pipes in the looped part of the network have the same diameter and are replaced by one node (rectangles with a solid line in Figure 6). This node has a central place in the network and has the highest degrees because it is interconnected with many other parts of the network. A part of the network around which
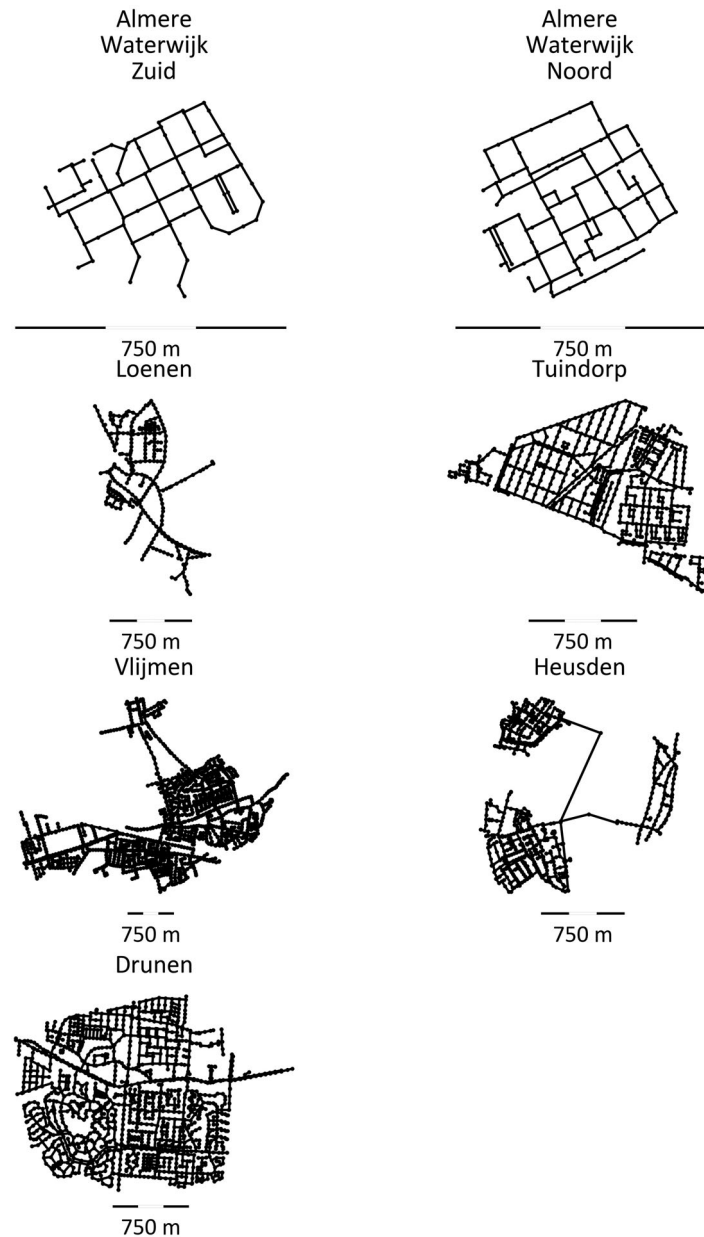

Figure 4. Structure (pipes and manholes) of 7 urban drainage systems.

Table 3. Characteristics of 7 urban drainage systems.

Characteristics	Almere Waterwijk Zuid	Almere Watewijk Noord	Loenen	Tuindorp	Vlijmen	Heusden	Drunen
Catchment area	Flat	Flat	Mildly sloping	Flat	Flat	Flat	Flat
System type	Storm water	Storm water	Combined	Combined	Combined	Combined	Combined
System structure	Looped	Looped	Partly branched	Looped	Looped	Looped	Looped
Contributing area (ha)	9.2	14.6	20.5	56.2	208.74	41.34	103.01
Number of inhabitants (–)	0	0	2.1	10.656	15740	4786	11600
Number of combined sewer overflow (CSO) structures (–)	4	3	2	5	10	6	4
Number of pumping stations (–)	0	0	1	1	1	1	2
Number of edges (–)	92	118	352	778	1,776	787	2,053
Number of nodes (–)	80	102	337	684	1,556	722	1,843

1 = Number of nodes
2 = Number of links
3 = Maximum degree
4 = Average degree
5 = Degree assortativity
6 = Node closeness
7 = Density
8 = Meshedness coefficient 1
9 = Meshedness coefficient 2
10 = Clustering coefficient
11 = Algebraic connectivity
12 = Spectral gap
13 = Central point dominance
14 = Density of bridges
15 = Density of articulation points
16 = Inverse spectral radius
17 = Average path length
18 = Network diameter
19 = Network radius
20 = Average hop count
21 = Node betweenness (max.)
22 = Link betweenness (max.)

Centrality Redundancy
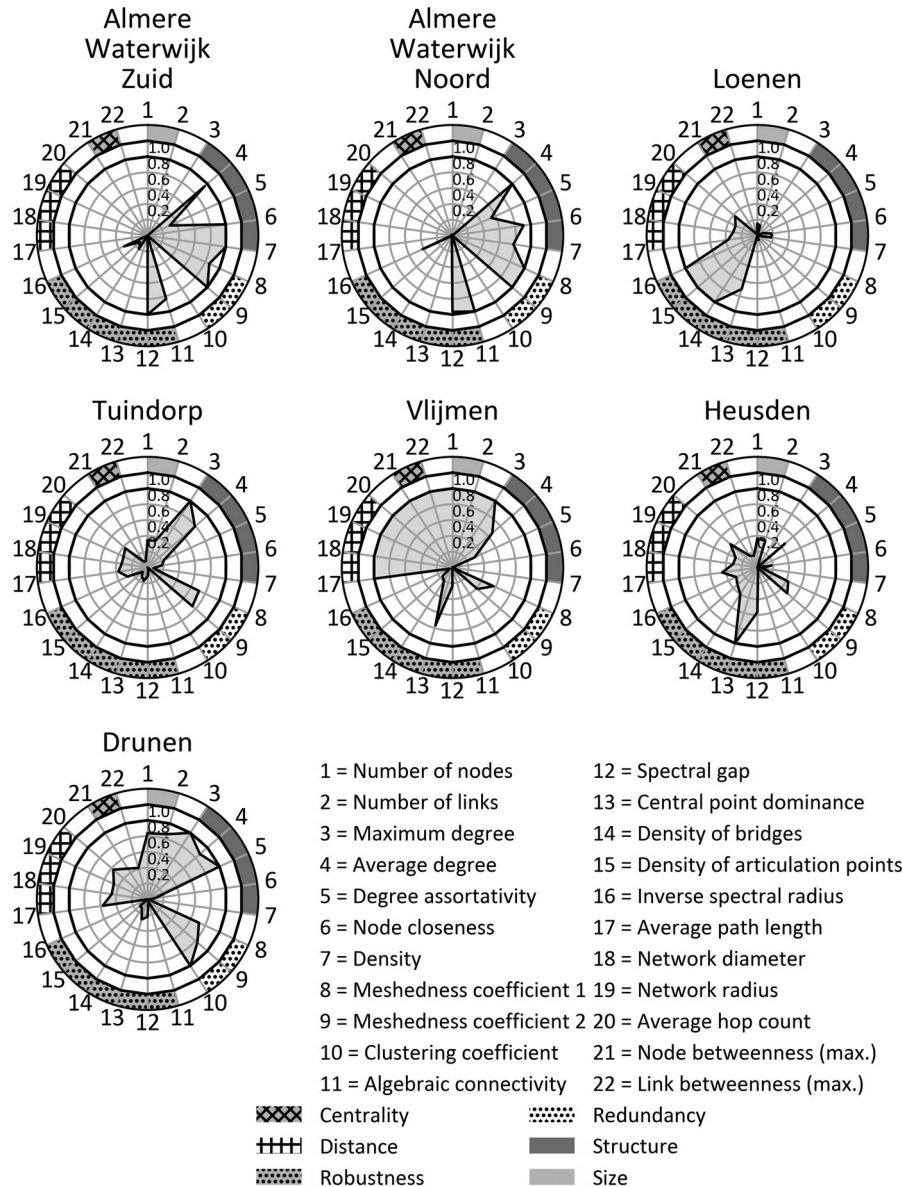Distance Structure
Robustness Size

Figure 5. Normalised values of 22 different network parameters for seven urban drainage networks. For each parameter, the normalisation is based on the minimum and maximum values of the seven networks.

water can flow in case of a blockage is therefore represented as one element in the dual graph approach. Second, the part in the dotted rectangle are two branches that come together. Because the diameter of these branches is equal, the branches are also displayed as one node in the dual graph. This means that pipes that drain different parts of the network are represented as one node in the dual graph if the diameter of the pipes is the same.

The degree distribution *P(k)* of the UDN of Loenen is presented is Figure 7. The graph representation shows a peak at a degree of two. The degree distribution of the dual graph representation has a power-law tail for larger degrees. The two nodes

Table 4. Overview of 22 parameters of seven sewer networks.

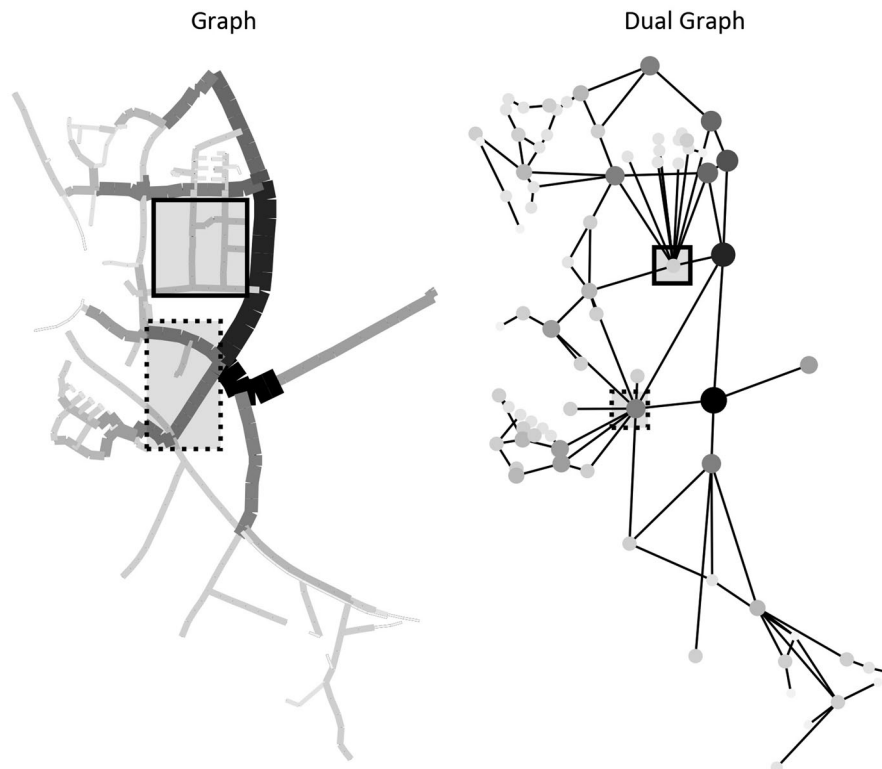|	Almere Waterwijk Zuid	Almere Watewijk Noord	Loenen	Tuindorp	Vlijmen	Heusden	Drunen
Number of Nodes	80	102	337	684	1,843	722	1,556
Number of links	92	118	352	778	2,053	787	1,776
Maximum degree	4	4	4	5	5	4	5
Average degree	2.3	2.31	2.07	2.26	2.23	2.18	2.28
Degree assortativity	−0.11	−0.08	−0.14	−0.12	−0.11	−0.15	−0.02
Node closeness	0.13	0.12	0.04	0.04	0.02	0.04	0.03
Density	0.0291	0.0229	0.0061	0.0033	0.0012	0.003	0.0015
Meshedness coefficient 1	0.08	0.09	0.02	0.07	0.06	0.05	0.07
Meshedness coefficient 2	0.23	0.23	0.11	0.2	0.16	0.17	0.21
Clustering coefficient	0	0	0	0	0	0	0.01
Algebraic connectivity	0.0161	0.0193	0.0012	0.0008	0.0001	0.0007	0.0007
Spectral gap	0.1724	0.1673	0.0289	0.0456	0.0624	0.1102	0.0607
Central point dominance	0.24	0.25	0.48	0.3	0.5	0.58	0.33
Density of bridges	0.26	0.2	0.5	0.24	0.26	0.32	0.24
Density of articulation points	0.25	0.22	0.46	0.24	0.26	0.3	0.25
Inverse spectral radius	0.3616	0.3661	0.3961	0.3591	0.3468	0.3599	0.3449
Average path length	7.96	8.71	23.99	23.95	50.84	27.24	32.53
Network diameter	21	22	62	65	154	62	82
Network radius	11	11	31	33	77	31	42
Average hop count	7.96	8.71	23.99	23.95	50.84	27.24	32.53
Node betweenness (max)	1,013	1,641	31,052	80,025	901,142	159,114	417,366
Link betweenness (max)	1,069	1,323	16,933	70,599	837,172	130,312	346,459

Graph Dual Graph

Figure 6. Urban drainage network in Loenen, shown as a graph and a dual graph. Larger and darker dots and lines indicate larger pipe diameters. The pipes with the same diameter (colour) in the rectangles are replaced by one node in the dual graph.

in the rectangles in dual graph representation in Figure 6 have the highest degree and should, according to the scale-free theory, and can be crucial for the functioning of the network.

3.3. Results of the network linearisation parameter

The NLP was determined for the seven UDNs. Table 5 and Figure 8 present the results. In Figure 8, each dot represents a bottleneck. The size and greyscale colouring visualise the number of nodes of the UDN that discharge via the bottleneck see % affected manholes in Figure 8). The larger and darker

the dot, the higher is the percentage of affected manholes. On the x-axis is the applied threshold for the bottleneck. The bottlenecks per combined sewer outfall or storm sewer outflow of the UDN have been plotted side by side for each threshold. The position of the bottleneck is on the y-axis. The position has been normalised per outfall based on the maximum path length to the outfall. The numbers on top of each plot indicate the number of bottlenecks per threshold for the outfall with the highest number of bottlenecks.

For Tuindorp's UDN, Figure 8 shows that there is one major bottleneck that affects approximately 40% of the manholes that

drain to one of the outflow locations if the "bottleneck threshold" is smaller than three. The bottleneck is located at approximately 0.25 of the maximum path length. The other bottlenecks only influence a few percent of the manholes. For Heusden's UDN, there is a relatively large bottleneck (impacting 60% of the manholes) at a threshold value of 100 relatively close to an outfall. At a threshold value of 50, a second bottleneck can be identified with similar characteristics.

Table 5 presents an overview of the maximum thresholds at which more than 1% of the manholes of an outflow catchment would be affected by a bottleneck. It also includes the corresponding position of the bottleneck, the percentage of manholes influenced by the bottleneck and the number of catchments. The table shows the following:

- For the UDNs of Vlijmen, Heusden and Drunen, the maximum threshold is higher than that for the networks of Almere, Loenen and Tuindorp.
- The percentage of affected manholes of the UDNs of Heusden and Drunen is larger than that of the other networks.
- The bottlenecks of the UDNs of Almere Zuid, Almere Noord and Tuindorp are situated at a greater distance from the outflow than those of the other networks.
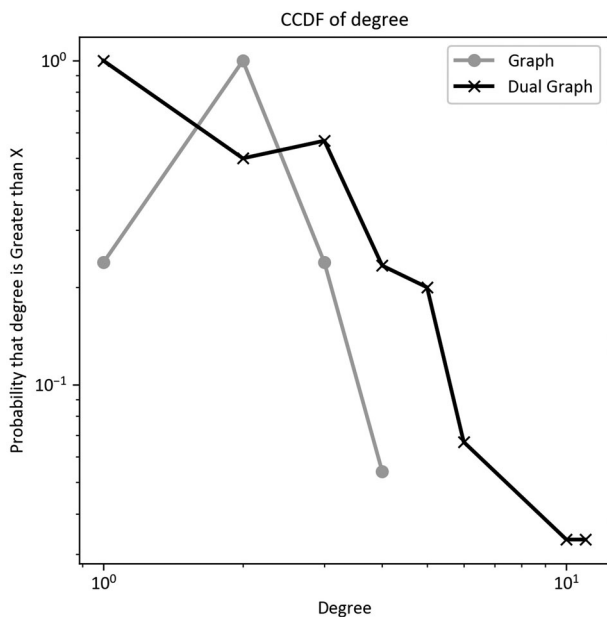
For precipitation intensities between 10 and 140 l/(s.ha), in steps of 10 l/(s.ha), the HBWLM and the GBWLM were used to identify flooding. The results were mutually compared using the Matthews Correlation Coefficient (MCC). For each catchment, the calculated MCC values of the different precipitation intensities for which the MCC was not equal to zero were analysed. The mean 95% confidence interval and median of the MCC values were calculated. Table 6 summarises the results of the MCC analysis. Please note that the confidence interval is based on a small sample and should therefore be regarded as an indication, at best.

Table 6 shows that the mean, the median and the lower and upper limit of the 95% confidence interval of the MCC values of the UDNs of Vlijmen, Heusden and Drunen were lower than those of the other four UDNs. Based on this observation, it can be concluded that for the UDNs of Almere Waterwijk Zuid, Almere Waterwijk Noord, Tuindorp and Loenen, the results of GBWLM were more in agreement with the HBWLM results than those for the UDNs of Vlijmen, Heusden and Drunen.

When the results of the NLP and the comparison of the GBWLM with the HBWLM were compared, the following observations could be made:

- The mean MCC of Heusden's UDN was the lowest, and the NLP showed several major bottlenecks close to an outfall.
- The mean MCC values for the UDNs of Vlijmen, Heusden and Drunen were relatively small, and the NLP showed multiple bottlenecks for high thresholds (i.e. 100, 1000).
- The UDNs of Almere, Loenen and Tuindorp had relatively high MCC values and a relatively small number of bottlenecks, occurring at smaller threshold values and affecting fewer manholes.

4. Discussion

4.1. Value of the topological network parameters

Many parameters have been proposed in the literature to describe the topological features of networks. The seven networks tested exhibit three distinctive features, each of which can be described by one or more parameters.

1. Size of the network (parameters are, e.g. number of nodes, number of pipes, network diameter or network radius).
2. Meshedness of the network (parameters are, e.g. meshedness coefficient, density of bridges, density of articulation points and inverse spectral radius).
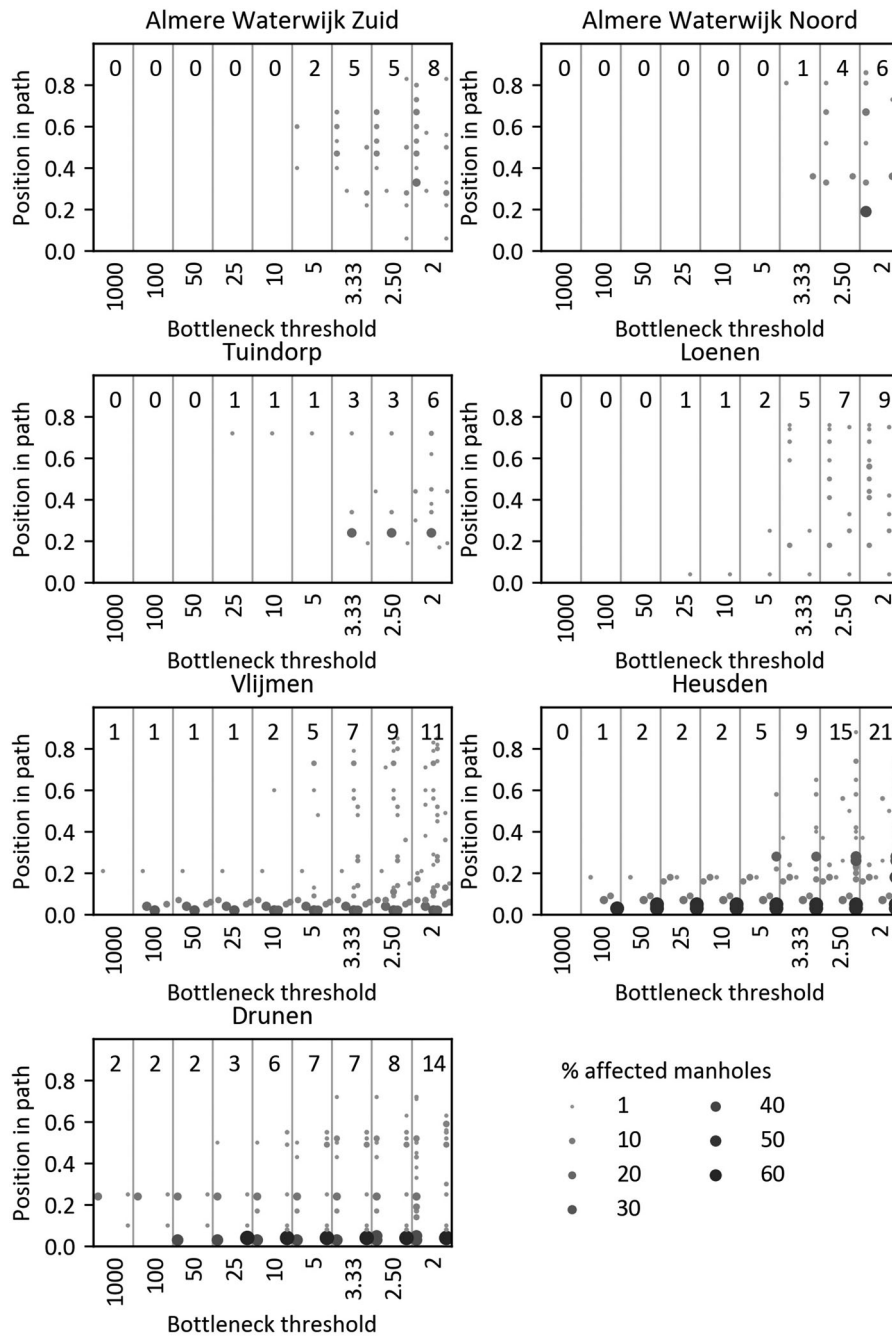

Figure 7. The probability of the degree distribution of Loenen's urban drainage network, represented as a graph and a dual graph.

Table 5. Overview of Urban Drainage Networks (UDNs) with a threshold at which more than 1% of a catchment of the UDN is affected by a bottleneck including the location, the percentage affected manholes and the number of bottlenecks.

	Threshold at which >1% of no. of manholes is affected by a bottleneck	Bottleneck location in path (0 = downstream, 1 = upstream)	% of affected manholes	Total number of bottlenecks
Almere Waterwijk Zuid	5	0.4	0.01	2
Almere Waterwijk Noord	3.33	0.36	0.07	2
Tuindorp	25	0.44	0.01	1
Loenen	25	0.04	0.04	1
Vlijmen	1000	0.07	0.07	1
Heusden	100	0.03	0.57	4
Drunen	1000	0.1	0.14	3

Figure 8. Bottlenecks in seven Urban Drainage Networks (UDNs) that affect more than 1% of the manholes of the UDN. The applied threshold is on the *x*-axis, and the normalised position is on the *y*-axis. The numbers in top of each plot indicate per threshold the number of bottlenecks for the outfall with the highest number of bottlenecks.

3. Structure of the network (parameters are, e.g. central point dominance).

Even using a combination of the above parameters, it is not possible to describe the characteristics of a UDN in an unambiguous manner. The parameters describe the structure and not the flow (hydrodynamics) in the network. Special characteristics of UDNs that are not taken into account by the parameters mentioned are:

1. The flow directions (multiple origins, few destinations).
2. The capacity (geometry) of the pipes.

3. The transportation "costs" (required energy, head loss).

UDNs that produce more or less similar results in the GBWLM may have very different network parameter values. Therefore, other indicators are needed to characterise UDNs in order to predict whether or not the GBWLM can be applied successfully.

4.2. The dual graph approach

With the Dual Graph Approach, adjacent pipes with the same diameter are merged into one node. If meshed parts

of the network are merged to one node, the Dual Graph Approach neglect the fact that these pipes function partially as each other's backup. It is not plausible that they will fail all at the same moment. If a failure occurs in this part of the UDN, it will usually only be a partial failure.

If at the confluence, pipes of equal diameters, are merged to one node, the Dual Graph approach does not distinguish between the upstream areas. The consequences of failure depend on the exact location of the failure. Failure of one branch will lead to other consequences than failure of another branch. This information is lost in the Dual Graph Approach. These examples show that the assumption of the scale free network theory, that nodes with a high degree are crucial for the functioning of scale-free networks cannot be applied automatically on Dual Graph representation of UDN. Information is lost when the network structure is changed. This could lead to invalid conclusions if the analysis is not carried out carefully, especially for looped networks.

4.3. Network linearisation parameter

The NLP combines linearised hydraulics and network structure to classify networks. There was no general value for the NLP for all tested networks. There were differences in the following:

- The threshold at which bottlenecks might occur.
- The position of the bottlenecks in the paths.
- The percentage of manholes affected by the bottlenecks.

The importance of these three sub-parameters is not yet entirely clear. The threshold value is an indication of the severity of the bottleneck. The higher the value, the larger is the difference in capacity between two adjacent pipes. However, the consequences also depend on the position. The consequences are smaller if the bottleneck occurs in the upstream parts of the network than in the downstream parts. The consequences are larger if the block is in a "main path" to an outflow location rather than in a path used only to discharge water from a few manholes. More networks should be analysed to determine the importance of the three sub-parameters more precisely and to determine whether or not these parameters set specific boundaries for the successful implementation of the GBWLM.

Table 6. The mean, 95% confidence interval and median of the available Matthews correlation coefficient values for 14 rainfall intensities in seven urban drainage networks.

	Mean	95% confidence interval	Median
Almere Waterwijk Zuid	0.714	[0.517, 0.911]	0.839
Almere Waterwijk Noord	0.784	[0.641, 0.926]	0.836
Tuindorp	0.687	[0.591, 0.783]	0.728
Loenen	0.610	[0.481, 0.739]	0.656
Vlijmen	0.478	[0.388, 0.568]	0.523
Heusden	0.343	[0.195, 0.49]	0.364
Drunen	0.453	[0.392, 0.514]	0.471

5. Conclusions

This paper presents the results of an analysis of the topology of 7 UDN. As described in the results and discussion sections, the existing topological features to describe networks can be applied to describe UDN but prove to be very specific for each individual network. A rather large overlap between the different features makes the use of all topological parameters at the same time meaningless. On the other hand, special characteristics of UDNs (multiple origins but few destinations, required transport energy, etc.) are not (fully) considered.

The Dual Graph method in combination with the HICN could be useful for the analysis of branched networks. However, this method is less applicable to looped networks where many pipes have the same diameter. By merging multiple pipes into one node, the network structure is strongly affected, and the bypass function of the loops is ignored.

Seven urban drainage networks (UDN) have been analysed using the GBWLM and hydrodynamic models. The MCC value of 4 of the 7 UDNs (Almere Zuid and Noord, Tuindorp and Loenen) is larger than 0.6 indicating a relatively good match between the outcomes of the GWBLM and hydrodynamic models (see Table 6). These UDN have different characteristics (see Figure 5, Table 4). The Network Linearisation Parameter is used to describe the UDNs. The NLP combines the network structure and the flow characteristics. The UDNs that according to the MCC match well with the outcomes of hydrodynamic models meet the following criteria:

- A low (<5) bottleneck threshold for larger (impact on >5% manholes).
- No large bottlenecks (impact on >5% manholes) close to outfalls (<20% path length).

More networks have to be analysed to determine more precisely the importance of the sub-parameters (bottleneck threshold, the bottleneck position, the percentage affected manholes and the number of bottlenecks (see Figure 8) and the applicability of the NLP on other networks.

Disclosure statement

No potential conflict of interest was reported by the authors.

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