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Initialization Strategies for Energy Management System Optimization

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Preface

This thesis is written from a strong personal motivation and a dedication to applying science to solve real-world problems. Driven by difficulties and passionate about explaining complex matters in inclusive terms, I chose to study Applied Mathematics with a specialization in Discrete Mathematics and Optimization. During and after my Bachelor, I contributed to several innovative projects and organizations focused on finding creative solutions to initiate and realize the urgent transitions within today's complex transport, energy, waste and supply systems.

Among these experiences were the TU Delft Hyperloop Dream Team, an impact investment internship, and a Management of Innovation exchange program in Paris. These projects strengthened my belief that the energy transition, affecting both public and private sectors, industry, households, and individuals, can only be achieved collectively. My earlier contributions in the energy domain include the development of a model to calculate the greenhouse gas emission savings of clean molecules, based on the methodology of the European Commission. Furthermore, I participated in a conference in Paris that brought together policy makers and key stakeholders from the French and Dutch energy sectors to discuss innovation, demand reduction, and education. The preparatory workshop on the integration of values in climate and energy citizen assemblies greatly inspired me to continue collaborating and seeking synergies to address our shared challenge.

During my time in Paris, I actively searched for a graduation project in the energy transition, as I consider it to be one of the most impactful applications in my field of expertise. Through the Dutch Embassy in Paris, I came in contact with Energy Pool, a front-runner in the valorization of flexibility, controlling and optimizing industrial sites, (islanded) microgrids, prosumers and hybrid powerplants. I was instantly impressed by their portfolio, experience, and expertise and it soon became clear that we share common values and a mission to contribute to mitigating climate change. The product of this fruitful collaboration is now in front of you.

I would like to thank Joshua Leduc, Nicolas Bergevin, and all my colleagues in the Software Business Line at Energy Pool for their guidance, French lessons, and the welcoming environment in Lyon. Their openness and support helped me greatly in my professional and personal development. I am equally grateful for the supervision from the TU Delft, I feel fortunate to have been supported by Leo van Iersel, my enthusiastic Professor in Optimization, who trusted me and this project from the very beginning.

Abstract

The increasing integration of Renewable Energy Sources (RES), rising global electricity demand, and ongoing developments in power market structures collectively increase the complexity of Energy Management Systems (EMS). The tight scheduling of interdependent decisions in a Rolling Horizon (RH) Mixed Integer Linear Programming (MILP) environment requires efficient formulations to remain scalable and flexible to future innovations. This thesis investigates initialization strategies (warm starts) that leverage previous optimal system configurations to reduce computational complexity and solution time. Iterative cycles of variable selection, warm start execution, and problem reformulation are evaluated across multiple scenarios. These scenarios vary in modeling horizon, day-ahead price profiles, market engagement strategies, and environmental and system conditions. Problem reformulations include adjustments in the treatment of violation decision variables, linear reformulations, and the use of Benders decomposition. The results demonstrate that successful warm start implementations can substantially reduce solution times and provide valuable insights for further tightening problem formulations. Overall, the study provides guidance on efficient formulations that support effective initialization and enhance solver performance across a wide range of users and system configurations, thereby contributing to more scalable and widely applicable energy optimization practices.

Keywords: Rolling Horizon (RH), Warm Start, Unit Commitment (UC), Mixed Integer Linear Programming (MILP), Benders Decomposition

Contents

Abbreviations

- B&B: Branch and Bound
- B&C: Branch and Cut
- BCM: Balancing Capacity Market
- BEM: Balancing Energy Market
- BESS: Battery Energy Storage System
- BRP: Balance Responsible Party
- BRP: Balance Responsible Party
- BSO: Brainstorm Optimization
- CCHP: Combined Cooling Heating and Power
- CHP: Combined Heat Power
- CPU: Central Processing Unit
- DA: Day Ahead
- DER: Distributed Energy Resources
- DHPP: District heating power plant
- DSO: Distribution System Operator
- DSO: Distribution System Operators
- EEX: Energy Exchange Exchange
- EFC: Equivalent Full Cycle
- EMS: Energy Management System
- ESS: Energy Storage System
- EV: Electric Vehicles
- FCR: Frequency Containment Reserve
- GA: Genetic Algortihm
- GBD: Generalized Benders Decomposition
- L-MAPF: Lifelong Multi-Agent Path Finding
- LP: Linear Programming

- MCP: Market Clearing Price
- MCP: Market Clearing Price
- MG: Microgrid
- MILP: Mixed-Integer Linear Programming
- MIR: Mixed Integer Rounding
- MOCGO: Multi Objective Chaos Game Optimization
- MOPSO: Multi-Objective Particle Swarm Optimisation
- MPC: Model Predictive Control
- P2G: Power To Grid
- PSO: Partical Swarm Optimization
- RES: Renewable Energy Sources
- RH: Rolling Horizon
- RHC: Receding Horizon Control
- RINS: Relaxation-Induced Neighborhood Search
- RL : Reinforcement Learning
- RR: Replacement Reserve
- SDG: Sustainable Development Goals
- SOC: State of Charge
- TSO: Transmission System Operators
- UC: Unit Commitment
- WS: Warm Start
- aFRR: automated Frequency Restoration Reserve
- mFRR: manual Frequency Restoration Reserve

1 Introduction

1.1 Relevance

1.1.1 Energy Management Systems and Initialization Strategies

The decarbonization of energy systems is one of the most pressing challenges of the twenty-first century. To meet the internationally established Sustainable Development Goals and the Paris Agreement, global warming needs to be mitigated. Energy Management Systems (EMS) are software- and hardware based systems that monitor, control, and optimize the generation, storage, and consumption of energy. Figure 1 shows a schematic representation of an EMS.

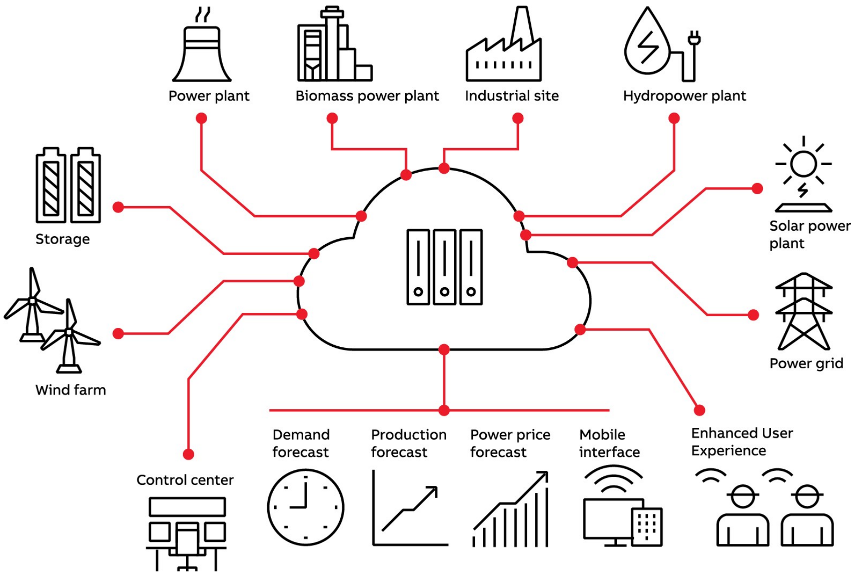


Figure 1: Schematic representation of an Energy Management System (EMS) [1]

More efficient and advanced EMSs are needed as the transformation from conventional energy sources such as coal and gas toward Renewable Energy Sources (RES) such as solar, wind, and hydrogen takes place. This is due to the fact that the variable and weather-dependent availability of RES makes it difficult to ensure the stability and reliability of the power grid [84] [78] [28] [24].
This thesis provides a road map of strategies that works around the compounded complexity of EMSs, by investigating initialization strategies in a Rolling Horizon (RH) energy optimization context. The energy optimization problem is formulated as a Mixed Integer Linear Program (MILP). In the MILP, the EMS is represented by decision variables and constraints, and the optimization problem is formulated as an objective function that minimizes the cost of the system. In the RH framework, the MILP has to be solved repeatedly over short intervals, which is problematic in practice because the running time of solvers grows exponentially with instance size. The main goal of this thesis is to present opportunities to benefit from initialization strategies in various scenarios across problem reformulations and algorithm designs to improve the scalability of the RH MILP approach.

1.1.2 Stakeholders and Electricity Markets

The increasing global demand for sustainable energy is driving innovation in smart grids, energy storage and scheduling, and demand-response flexibility solutions. The intermittent behavior of RES, combined with increasing electricity demand, has led to the implementation of ancillary services to handle variability and prevent grid congestion. These services involve many actors operating across various time frames, depending on system needs. Key stakeholders include Transmission and Distribution System Operators (TSO, DSO), prosumers, consumers, power exchanges, (islanded) microgrids, (hybrid) power plants, and households. Together, they aim to guarantee transparency and security of supply across all levels of operation, from resource to consumption [35] [56].
To predict and anticipate electricity demand and supply, developments in forecasting, storage and decision-making methods unfold rapidly. Policy makers, grid operators and major producers and consumers are committed to designing robust frameworks integrating economic, environmental, and social considerations [75]. On an international scale, countries benefit from raw materials and interconnections, complementing one another to reduce the risk of grid failures and congestion. Participation in wholesale markets is encouraged, as it generates profits while simultaneously providing system stability. An increasing number of participants enhances market liquidity, which improves transparency and provides a clearer picture of available volumes for distribution, thereby facilitating more accurate predictions of supply and demand [67][55][15].

1.1.3 Importance of Efficient Modeling

On a local industrial and generational level, the co-optimization of energy and reserves is referred to as the Unit Commitment (UC) or dispatch problem. With the increasing complexity and diversity of EMSs and flexibility market mechanisms, it is crucial to integrate the assets both internally on-site and with the surrounding environment, while responding in a timely manner to system events. The ensemble must therefore adhere to timeseries, system constraints, and ambient conditions at all times.
A reactive and adaptive EMS can incorporate the latest developments and innovations in the field, while also allowing for creative solutions fitted for their specific application. In this way, financial and technical gains and losses can be controlled in response to evolving conditions. EMSs typically involve batteries, generators, loads, converters, and the site as a whole, and continuously update the target settings for each asset. It is crucial that trade-offs are safe, realistic, smooth, and efficient. To limit losses and maximize gains while protecting the system, decision-making needs to occur as close to real-time as possible. An EMS is considered that optimizes at regular intervals over a Rolling Horizon (RH) into the near future.
The cost of calculations is influenced by the quantity of both functionalities and data points depending on the chosen modeling horizon, but also by the inherent complexity and variety of UC and Dispatch optimization. The relevance of fast and efficient optimization is therefore twofold: it enables EMSs to remain adaptive to ongoing developments and environmental changes, while also integrating innovative solutions and contributing to system stability [49][91].

1.2 Motivation: Computational Limits

The more computationally expensive calculations in RH optimization become, the less time remains to anticipate the next step. This issue is amplified by the growing number and complexity of market mechanisms, which operate on overlapping time frames. In such a setting, accurate knowledge system details, like storage levels and forecast quality, is essential before placing bids and committing volumes. As the frequency of decision updates increases in the RH environment, the benefits of repetition and tight control are challenged by rising system complexity and computational cost. This complexity does not only stem from the sheer scale of assets, services, and exceptions, but also from the need to preserve a tractable MILP formulation. Avoiding nonlinearities often requires the introduction of auxiliary variables, constraints, and reformulations, which expand the problem size. Thus, as more physical enhancements and adaptive mechanisms are incorporated, the MILP grows denser and harder to solve within the available computational budget and strict cycle deadlines [24].

A practical failure mode arises when the solver is still computing the previous optimization at the moment the next RH cycle begins. Forecasts and time series then become desynchronized from the computed setpoints, degrading the realized asset targets and control quality. Operators are forced into a trade-off: either runtime is prioritized—allowing the solver to continue until a desired solution quality is achieved—or solution quality is relaxed by accepting a larger optimality gap to meet the real-time cycle deadline [30].

Contemporary research explores ways to manage this tension, for example, selecting forecast horizons that are long enough to produce effective setpoints without inflating data volume, employing multiobjective formulations, or adopting multistage RH schemes to partition problem scale [91][101][31]. Yet, maintaining timely and high-quality dispatch without fragmenting the optimization, and thus without sacrificing adaptability, remains a central challenge in today's energy optimization landscape. In this setting, initialization strategies hold particular promise: By providing good starting points, they can offset the compounded complexity introduced by both the growing scale of services and the linearization of non-linear physical behaviors. This opens opportunities to reduce solve times and latency without compromising solution quality.

1.3 Thesis Contribution

1.3.1 Initialization Strategies

This thesis investigates initialization strategies for reducing the running times of energy optimization problems while preserving solution quality. In this context, initialization strategies use parts of previously obtained solutions as starting points for subsequent Rolling Horizon (RH) runs, providing high-quality offsets for decision variables. This phenomenon is referred to as a warm start. The RH environment is particularly suitable for such approaches, as not all variables are expected to change significantly between consecutive high-resolution runs. The effectiveness of the proposed strategies is systematically tested, analyzed, and repeated across representative environments and scenarios, covering a wide range of system needs and configurations.

1.3.2 Reformulations and Algorithm Design

To extensively analyze the performance of initialization strategies, this work examines reformulations of constraints and the removal of nonlinearities in MILP models, as well as the influence of algorithmic choices in solver usage. The reformulations contain the deletion of violation decision variables, and additions as well as reformulations of constraints. The alorithmic design choices contain the investigation of a Benders Decomposition of the optimization problem. These aspects are included because they contribute to the compounded complexity of RH Unit Commitment and dispatch problems, and therefore directly impact both runtime and solution quality.

1.3.3 Implementation

The analysis provides extensive information on the potential of initialization techniques and related methods in practice. The findings are intended to inspire the adoption of successful outcomes in compatible settings and to motivate further refinements of solution approaches. Moreover, the increasing heterogeneity and complexity of EMSs highlights the need for conditional decision-making, where time frames, initialization strategies, constraint formulations, and algorithmic choices can be creatively combined to enhance optimization performance.
Close-to-real-time solving not only provides financial benefits, but also improves robustness and the quality of dispatch optimization. In addition, it enables the integration of new functionalities, supporting ongoing innovation. Enhancements in system efficiency and problem formulation also stimulate competition and market participation, thereby contributing to the development of flexibility services and the stabilization of an increasingly renewable energy system.

1.4 Thesis Structure

The remainder of this thesis is organized according to the following Sections:

- *Literature Review* reviews relevant research on initialization strategies, Rolling Horizon optimization, stochastic programming, algorithms, and decomposition approaches for EMS purposes and related fields.
- *Energy Markets* introduces the market design, system operators, and ancillary services that provide context for the modeling work.
- *Energy Management System Description* describes the EMS setup, including assets, variables, and constraints, and formulates the objective function minimizing operational and market participation costs.
- *warm start strategies* applies warm start initialization first to violation variables in a baseline scenario, and then extends the analysis across multiple scenarios to study solver dynamics, variable sets, effort levels, and modeling horizons.
- *Linearity and Reformulations* analyzes the removal of nonlinearities and the impact of constraint reformulations on solver behavior and evaluates the performance of warm start strategies under these changes.
- *Model Conversion and Benders Decomposition* implements the model in Python's docplex, compares formulations with and without violations, and investigates Benders decomposition as an alternative approach.
- *Conclusion and Discussion* discusses contributions and challenges, and outlines directions for future research.

2 Literature Review

2.1 Drivers and Challengers of Renewable Energy Integration

The global transition towards a clean renewable energy system is led by issues of environmental degradation, finite quantity and uneven distribution of fuels, as well as growing energy demand [22]. Revolution is needed to secure affordable energy globally, tackle climate-change related risks and elevate long-term economic growth. Specific goals and guidelines, established internationally in the Sustainable Development Goals (SDG) and the European Green Deal, among others, require the implementation of measures to eliminate or reduce the negative impact of human activity on the environment [83].
A key aspect of the energy transition is the replacement of non-renewable energy sources such as coal and natural gas with Renewable Energy Sources (RES). The integration of RES has a significant impact on the reliability and stability of the power grid, since their output is inherently variable and weather-dependent. This variability presents challenges in balancing supply and demand, maintaining frequency stability, and ensuring adequate reserve margins. On our way to climate neutrality, the increase in the share of RES in the electricity grid requires new means of securing and stocking energy due to the intermittent production of RES [24]. Moreover, rapid urbanization, population growth, and technological advancements cause a rise in demand for electricity, in cities as well as in expanding suburban areas and more isolated areas. The existing power grid networks are not equipped to meet the increasing demands of the 21st century and struggle to keep up with the increasing load [35] [56].

2.2 Grid Balancing and Flexibility Services

The increasing penetration of RES and the growing electricity demand have led to important developments in the operation of the electricity grid. Flexibility services are implemented to enable the electricity system to respond to variability and uncertainty in supply and demand, and to prevent grid congestion. These services rely on flexible resources such as generation units, energy storage systems, and controllable demand. The operation and coordination of these resources involve multiple actors, including electricity generators, Transmission System Operators (TSOs), Distribution System Operators (DSOs), and consumers. Flexibility is activated over various time frames depending on system needs [45]. Industrial loads can provide demand-side flexibility by adjusting their power consumption in response to market signals or grid needs, such as temporarily reducing or shifting production during peak demand periods or high prices. Storage systems offer a solution to mitigate output power fluctuations, maintain frequency, and provide voltage stability [19]. The usage of Battery Energy Storage Systems (BESS) for energy arbitrage is evolving in the electric sector, driven by increasing participation in wholesale markets. More competitive market structures contribute to grid stability, as a large number of price signals reflect real-time system conditions and resource availability, ensuring an efficient balance between demand and supply [41] [15] [69]. Energy markets and their participants are discussed in further detail in Section 3.
To locally provide reliable and efficient power, increasing self-consumption of RES is emerging, turning consumers into prosumers using energy production systems. Surplus

electricity is traded with grid companies, a concept that is known as Power To Grid (P2G). This decentralized approach can lead to grid instability, increasing the number of interactions and unpredictable fluctuations. The exchange of electricity among users and the reduction of main grid dependency of Energy Communities consisting of Distributed Energy Resources (DERs) rise as solutions to meet collective benefits. [39] Discovers the role of prosumer energy communities in providing flexibility and grid balancing services in alignment with EU goals. Microgrids (MG) are localized energy systems that can operate independently or in conjunction with the main electrical grid [104]. Microgrids can be seen as energy communities and consist of DERs, storage systems, and control systems.
The increasing complexity of energy distribution networks has implicated the development of efficient and intelligent Energy Management Systems (EMS). The implementation of an EMS that integrates RES and operates flexibility services requires profound decision making [94]. Optimization algorithms, modeling horizons, objective functions, constraints, data pre-processing, and variable definition are to be considered. With the increasing complexity of EMSs, scalability challenges arise in terms of the number of variables, multi-objectivity, differences in order magnitude of variables, computational time, solution quality, robustness in case of erratic input parameters, and many other modeling segments. Multiple best practices and recent trends are discussed, in energy systems as well as other domains that face similar optimization challenges.

2.3 Rolling Horizon MILP optimization

Optimization methods aim to find the best available solution to a mathematical problem, in which an objective function is minimized or maximized. The feasible solution set is defined by constraints on the variables of the problem. In Linear Programming (LP), both the objective function and the constraints must be linear. Mixed-Integer Linear Programming (MILP) is a variant of LP where one or more variables are restricted to be integer [33].
In order to implement an optimization method in an EMS, some parameters of the model have to be estimated. For example power generation from intermittent RES and consumer activity have uncertain natures, whereas system limits and day-ahead electricity prices are known. In deterministic optimization methods, the uncertainty of the variables is not taken into account, that is, the forecasts of the estimated values are assumed to be perfect. In stochastic optimization methods, on the other hand, multiple scenarios are generated in which the to be estimated parameters take different values [90].
An optimization problem is often formulated to find the best solution for a specific time series. The length of this so-called modeling horizon is determinative for the compatibility of computing resources and the running time of the model that represents your problem [32] [74]. Rolling Horizon (RH) algorithms split the modeling horizon and operation problem into multiple time slots and solve the corresponding subproblems in sequence. This approach is typically adopted in complex operation problems that use forecasts for uncertain input data and for large-scale optimization problems. In the energy domain, rolling horizons have shown to be suitable for power system operation schedules for Unit Commitment (UC) [79] [95], Combined heat and power (CHP) [24] and storage systems [31][32].

To satisfy the European qualification framework for high-efficiency cogeneration (CHP), yearly-basis energy savings indexes need to be reported, that is, the modeling horizon needs to be extended to a year. Normally, extending the modeling horizon implies increasing the computational requirements of the model. A rolling horizon model either overcomes the latter or decreases the computational requirements in case of an unchanged modeling horizon. [23] presents a MILP for a Combined Cooling Heat Power (CCHP) energy system, minimizing operation and maintenance costs subject to system constraints. An extension of this MILP applies a RH algorithm for a cogeneration (CHP) energy system with time-variable loads, system limits, financial incentives, and ambient conditions. Each subproblem takes into account information from past and future periods with aggregated estimates from both typical weeks in the first phase and previous solutions of the solver in the second phase of model testing. The use of previous solutions by the solver is called a warm start strategy, which helps guide the search. The concept is described in more detail in Section 2.4. The solution of the first phase of model testing is found 60 times faster than to optimize the whole year problem without division into weekly time slots. The second phase even speeds up the optimization by a factor of nearly 2,500, which shows great potential for the implementation of warm start strategies. Notably, the inclusion of heat storage in the energy system leads to an increase in running time; nevertheless this can be mitigated by allowing a higher MILP gap [24]. To guarantee computational tractability in multi-staged optimization, the problem can be solved for a limited set of typical and extreme periods, which can be selected by a k-MILP clustering model [102].

[31] increases self-consumption of RES and provides a more robust microgrid EMS by executing a RH strategy with two different time periods. In the first stage of the strategy, an optimization for the BESS settings is performed each 15 minutes over a modeling horizon of 24 hours. The second stage RH of 48 hours follows the reference values from the first optimization with a sampling time of 1 minute, allowing an accurate response to load changes and reducing errors associated with load predictions. By integrating the solution, the costs of drawing energy from the main grid are reduced by 45% per day, increasing the independence and efficiency of the system.

The choice of the modeling horizon and forecast horizons of intermittent sources and electricity prices affects the quality and computational time of the solution. After comparison with 24 and 48 hour periods, forecasting up to 36 hours in each subproblem is found to guarantee effective dispatch scheduling of electricity generators and BESS for multiple-day periods. Compared with Pareto search and Genetic Algorithm (GA), MILP is the only algorithm that guarantees optimum identification in the case of increased model complexity following increased horizons. MILP also reports the best performance in terms of solution computation time [91].

An extension of the Rolling-Horizon Collision Resolution algortihm (exRHCR) solves instances 39% faster than RHCR in a Lifelong Multi-Agent Path Finding (L-MAPF) application where a team of agents visits multiple locations on a shared graph avoiding collisions with each other. The approach employs an extension of Priority-Based Search (exPBS), and allows to warm start the search with the priorities used in previous MAPF instances.[62]. Search algorithms are commonly implemented in priority-based multi-objective optimization problems [34] [85] [101].

2.4 Warm Starts and Initialization Strategies

Warm starts use the data from a prior solution to provide initial values and therefore avoid costly initialization between runs. Warm start initialization is a well-known concept in Mixed Integer Linear Programming as often consecutive problem instances are not strongly divergent and the B&B algorithm allows efficient reuse of its search tree and the dual bounds of its leaf nodes. This way, convergence can be strongly accelerated. Warm start algorithms are given an instance of the problem and a prediction or guess of the solution based on the solution space of the problem. The method is commonly used in applications where related instances of the same optimization problem are solved in sequence. The runtime of the algorithm is bound by the distance between the predicted solution and the true solution, so high quality predictions can strongly improve algorithm run time [81].

Unit commitment (UC) is one of the biggest optimization challenges in energy management, since switching between generators, dispatched power depending on the (predicted) demand and availability of renewable energy, and electricity-market participation, all have to be optimized simultaneously. A Dantzig-Wolfe decomposition strategy can break the UC problem down by generators. The reformulated decomposed problem can be solved with a column generation procedure that can be seen as the dual of a cutting plane approach. A warm started column generation procedure uses a pre-trained model to generate initial dual variables. Numerical experiments demonstrate that solving a UC problem with decomposition is always faster than solving it without decomposition using IBM ILOG CPLEX. Warm starting the column generation procedure reduces the number of iterations and computational time of the solution [82]. [27] presents a MILP formulation for the UC problem of thermal assets requiring a single set of binary variables, each associated with one unit per period. This is a significantly lower number of binary variables compared with other MILP formulations and therefore proposes an efficient framework reducing modeling complexity.

2.5 Model predictive control (MPC)

Model Predictive Control (MPC) is a model-based optimization method that contains three classical steps: prediction of evolution of the system, optimization at each sampling time and control by means of a rolling time horizon policy. MPC can be seen as a form of control in which the current control action is obtained by solving at each sampling instant a finite horizon open-loop optimal control problem. The technique is capable of considering state- and input constraints in the control of linear, nonlinear and uncertain systems [59][52][51]. The approach overcomes the shortcomings of static optimization problems by using disturbance models and a receding horizon [92]. In MPC we expect consecutive instances to be nearly identical and embrace the idea that if we are able to solve trajectory optimization problems quickly enough we can replan the future of the system at each sampling time and achieve a reactive behavior [58]. Advances in MPC allow the inclusion of discrete decisions in many MPC optimization problems [60].

To include MPC in a real-time system, it is crucial to know the worst-case iterations and size of the Branch and Bound (B&B) tree of the MILP solver, since the MPC requires a solution at each sampling time [80]. The B&B method relies on solving convex relaxations of the problem in a binary search tree to approach the solution to the MILP. Each node corresponds to a convex relation that either contains relaxations of the binary constraints or not. The idea of the method is to use the result from the relaxations to prune parts of the search tree before explicit exploration. Nodes are cut in one or multiple of the following cases:

- infeasibility of the relaxation
- the optimal solution of the relaxation is *worse* than the best known integer solution so far and/or
- the solution to the relaxation is *integer infeasible*

Many optimization solvers like ILOG's CPLEX use the B&B algorithm to solve MILP optimization problems [65]. The algorithm terminates and delivers a solution that is globally optimal. Most of the B&B schemes make use of a warm start within a single B&B solve, where the general approach is to start each B&B search from the leaves of the previous optimization. [58] introduces a warm start procedure for MPC by partly shifting the leaves of the B&B of the previous tree in time and using duality to obtain cost limits for the new subproblems. Properly shifted and combined with feasibility arguments this approach greatly outperforms approaches that solve optimization problems from scratch. MPC appears suitable in microgrid operation planning in order to cost-efficiently manage its energy resources. [65] solves a microgrid optimization problem by defining a MPC problem that uses the cost function associated to the MILP that describes the system containing dispatchable- and storage units, (non)controllable loads and RESs. The approach assumes perfect knowledge of the microgrid state and RES production and embeds inevitable forecast errors in an MPC framework. The method shows great cost- and demanded power reductions while maintaining an effective trade-off between computational time and solution quality. Furthermore, approaches that optimize MILPs using MPC are used for District Heating Power Plants (DHPP), thermal energy storage (TES) and CHP plants [92].
Solving Hybrid MPC with both continuous and discrete variables can take a long time due to the offline computation of discontinuous variables for MPC as a consequence of the combinatorial complexity. Generalized Benders Decomposition (GBD) strongly accelerates Hybrid MPC by separating the problem into a master problem which solves the so-called complicating variables and a subproblem which solves the rest [53]. The complicating are identified such that when those would be fixed, the problem is easy to solve; In MILP applications these are often the discrete variables (integers and booleans). The strategy can be seen as Danzig-Wolfe decomposition applied to the dual.

2.6 Particle Swarm Optimization (PSO)

Particle Swarm Optimization (PSO) emerges in the energy landscape and field of micro-grid systems due to its applicability, quick convergence time, and high performance [56]. PSO is a heuristic programming approach inspired by the choreography of a bird flock where particles decide their optimal position continuously during flight. The approach can be seen as a distributed behavior algorithm that manages and coordinates agents spread across a system. The fast convergence compared to traditional evolutionary algorithms is due to the introduction of the use of flying potential solutions through hyperspace. PSO also allows individuals to learn from their past experiences, whereas other evolutionary algorithms tend to focus only on the current population. The short running time for single-objective optimization shows potential for multi-objectivity. Multi Objective Particle Swarm Optimization (MOPSO) is considered for sizing hybrid renewable energy systems with multiple storage technologies [21]. The method discusses multiple genetic algorithms for Hybrid Energy Storage Systems.

Genetic algorithms (GA) are stochastic global search and are based on heuristic methods. GA are bio-inspired and known to be robust in multi-objective applications [54]. PSO and GA are also considered in the design optimization of Combined Cooling Heating and Power (CCHP) systems. [73] introduces a Multi-Objective Chaos Game Optimization (MOCGO) two-layer MILP algorithm that also couples RES, unlike previous PSO techniques. Convergence behavior of the MOCGO again shows efficacy in optimizing multi-objective problems when convergence to the Pareto front is observed when increasing the number of algortihm iterations. The method distinguishes economic, energy and environmental performance, which underlines the conflicts of interest in multi-objective EMS optimization. Pareto solutions are often used in combination with GA and PSO in problems with a multi-objective nature [54]. Pareto ranking schemes can be used to extend a PSO approach to multi-objectivity where the updated memory of each individual corresponds to an objective function value. The production of hypercubes to divide the explored space and the identification of leaders that guide the search appear to be successful and faster compared to other genetic and evolutionary algorithms [29]. Another double layer optimization of GA and MILP provides robustness for erratic input data considering uncertainties of electricity demand and production of intermittent sources. The model optimizes profit after the participation of a MG on the wholesale market [72]. PSO is also examined in the optimization of microgrid operations in island mode. Together with a dynamic adjustment algorithm, PSO sets charge limits of the batteries by the diesel generator and determines the optimal size of the PV fields and battery capacity. The dependence of the diesel generators is minimized, reducing over all production costs [104].

2.7 Reinforcement Learning

A Reinforcement Learning (RL) strategy to minimize energy optimizes the performance of a Central Processing Unit (CPU) through Voltage Frequency Scaling (VFS) in three phases:

- A chosen predictor is used to identify the Q-value of the future system state, using previous observed CPU workloads, providing a predicted CPU workload
- Based on the current system state and informed by Q- values, appropriate VFS control actions are explored to meet performance requirements
- The state-action relationships are exploited. In case of performance offset caused by mispredictions, the RL algorithms learns. The offset is minimized through the action rewarding mechanism that considers both energy efficiency and performance.

The case study shows that RL is a powerful tool in systems with erratic future states and that it is capable of learning from its environment in order to guarantee performance efficiently [77].
Reinforcement Learning (RL) is integrated into an existing MPC framework (see Section 2.5) to enhance adaptability and reduce the engineering effort required for controller implementation [18]. The goal of the proposed RL-MPC algorithm is to learn from the environment while satisfying constraints and this is done by effectively combining the machine learning and control communities. The solution method of MPC takes the union of estimated- states and disturbances as input and the objective function is inferred every control step. The value function of RL is used to shorten the non-linear program from MPC and to enable learning. In the value based approach of RL, the optimal action is derived from a specific state and from there the policy based approach of RL parametrizes a policy to map states into action. [103] shows an application of a RL strategy for a multi-objective distributed manufacturing optimization problem where energy consumption and assembly completion time are minimized. In particular, the feedback-based Q-learning method effectively identifies rewards for potential future actions, and the next action with optimal return is chosen accordingly. RL is combined with a Brainstorm Optimization (BSO) algorithm: a swarm intelligence algorithm that applies clustering to speed up convergence. An alternative clustering method than the standard k-means clustering mechanism in BSO is chosen to reduce computational complexity and save evaluation time.
[79] shows the successful result of the implementation of Reinforcement Learning (RL) in a Rolling Horizon Unit Commitment (RHUC) optimization. RL methods are chosen to solve optimization problems because of their ability to reduce computational time and improve performance. Given the capacity of RL methods to find optima in unknown environments, the methods are specifically contemporary relevant for power systems integrating an increasing amount of uncertainties.

2.8 Conclusion and Future Research

The reviewed literature outlines a diverse and promising landscape of methods for managing the increasing complexity of modern Energy Management Systems (EMS). Key technical challenges arise from the variability and uncertainty of RES, the rise of decentralized and remote generation, and the coordination required among a growing number of stakeholders in the energy sector. These include grid operators, policy makers, aggregators, and prosumers, each with distinct responsibilities, constraints, and optimization objectives. This underlines the need for EMS solutions that are not only robust and scalable, but also capable of balancing multiple objectives and responding flexibly to diverse system needs.

MILP-based optimization, applied within a rolling horizon framework, remains a central technique for solving operational planning problems such as Unit Commitment (UC). Model Predictive Control enhances this approach by embedding short-term reactivity, while metaheuristics such as PSO and learning-based methods like Reinforcement Learning contribute to adaptability and resilience in non-convex or data-rich settings.

A particularly relevant focus across studies is the use of warm start strategies: methods that leverage information from previous optimization runs to accelerate convergence in future iterations. These strategies show strong potential in improving solver performance, especially in EMS contexts with high temporal resolution or frequent re-optimization. However, the current body of literature lacks a structured understanding of when and how warm starts are most effective, particularly in the case of the Unit Commitment problem, where solution times remain a bottleneck for large-scale and high-resolution EMS deployment.

Three key factors affecting warm start performance remain underexplored:

- The length and resolution of the (rolling) optimization horizon appear to have a significant, but poorly quantified, impact on warm start effectiveness. The trade-off between warm start benefit and forecast accuracy degradation over longer horizons remains insufficiently studied.

- Although combinations of warm starts with decomposition, MPC, or metaheuristics are occasionally proposed, there is no systematic evaluation of hybrid strategies or guidance on how to tune their integration for specific EMS applications.

- Most studies rely on a single commercial solver (e.g., CPLEX, Gurobi), without comparative analysis of how warm start strategies interact with different solver heuristics, presolve routines, and node selection policies.

A unified framework that maps the interaction between warm start techniques, solver architecture, horizon configuration, and auxiliary heuristics in EMS problems, particularly for UC, will strongly improve both academic insight and practical deployment. This framework applied to an integrated energy system will be essential to meet the operational demands of the energy transition within the set 2030 and 2050 timelines.

3 Energy Markets

The energy market was liberated in the 1990s with the intention of securing supply by efficiently organizing the provision of electricity and gas by introducing regulated competitive forces [67]. To maintain the balance of demand and supply, electricity is traded on Power Exchanges, like European Energy Exchange (EEX) in Europe [55][15]. Members submit orders for buying and/ or selling power that reflect supply and demand for a certain market area at that given moment in time. Power Exchanges are major contributors to transparent electricity flow across borders. Multiple responsibility schemes considering key actors, market participants, and flexibility solutions are presented.

3.1 Shared Responsibilities between TSOs and DSOs

The rise of Distributed Energy Resources (DERs) including RES generation, demand-side response, electric vehicles (EVs) and batteries (BES) have motivated the development of agents procuring flexibility services for a reliable and cost-efficient power system. It is a TSO's responsibility to respond to unexpected demand, meet transmission demand, reduce frequency fluctuations, and prevent power cuts and network congestion on a national level. Distributed System Operators (DSOs) seek alternatives that can follow the high rate of DER penetration on a regional level as local grid operator [88]. The problem of optimal TSO-DSO coordination through market-based mechanisms remains unsolved. [93] considers the following responsibility mechanism: DSOs for local congestion management, TSO for system-wide reserve deployment and retailers for hedging against network usage tariffs based upon peak-load pricing. [43] presents multiple different models representing potential responsibility relations between TSOs, DSOs and DERs.

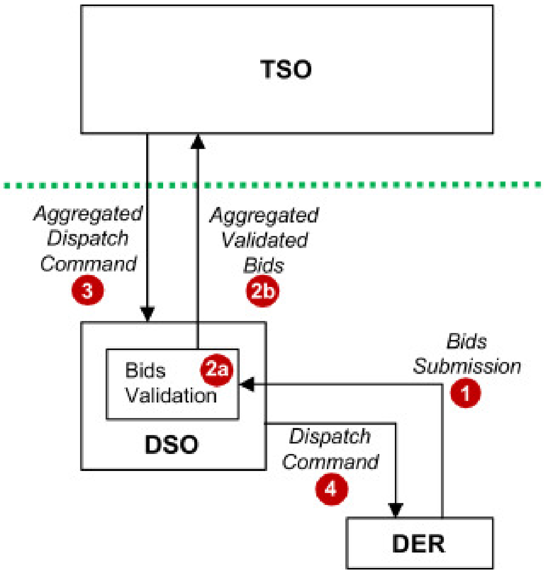


Figure 2: DSO managed model presented in [43]

In a distributed energy system, the flexibility of the market is essential for the increasing number of producers, consumers and prosumers of greatly varying scales [87]. In the European context, energy supply and system balancing is guaranteed through wholesale energy markets at various timeframes. Every market participant is associated with a Balance Responsible Party (BRP) that makes up for under-/overproduction of the participant. TSOs rely on BRPs to pre-balance their system. The authors of [88] work out the analogy between DSOs and BRP as middle players between TSOs and DERs or market participants. They pose a model that makes DSOs financially responsible for imbalance, like BRPs, by submitting price-taking offers that represent the net outcome of DSO scheduled actions at the distribution level. This is done with the goal to increase the market coordination and promote wider participation of DERs in both the balancing market and the proposed DSO market.

3.2 Day-Ahead and Intraday

As mentioned before, flexibility services operate over various time frames, each corresponding to different markets. The Day-Ahead (DA) market, like EPEX SPOT in the case of the EEX group, operates daily through a blind auction that trades all hours of the following day. Orders are logged in by the market participants before the order book closes at 12:00 CET (see Figure 6). There are two types of orders that represent the volumes participants are willing to buy or sell for the lowest to highest price ticks coming out of the auction. Aggregated demand- and supply curves are established based on buy- and sell- orders respectively for each hour of the following day. The DA market is a wholesale market where the marginal production cost of the last accepted unit determines the electricity price, according to a merit order (see Figure 3). The matching algorithm of the Power Exchange determines the legally binding agreements to purchase or sell for the Market Clearing Price (MCP) of the given hour (see Figure 4). The market is cleared and settled every day by a clearing house for energy and commodity products, ECC in the case of the EPEX SPOT market. ECC conducts all financial settlement and connects multiple European markets, by maintaining relations with banks. The clearing house manages transactional risks by collateral payment mechanisms for participants.

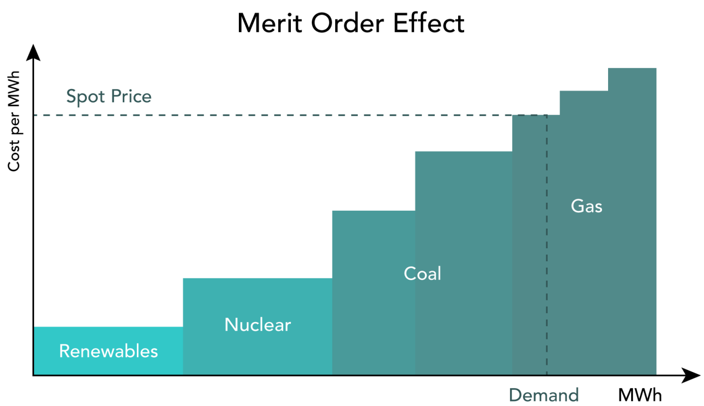


Figure 3: The merit order pricing of the Day-Ahead market [2]

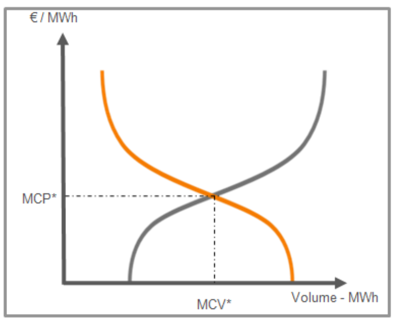


Figure 4: MCP and MCV as intersection of demand- and supply curves for a specific hour [15]

With the incorporation of RES, power markets need to be able to accomodate short-term forecasts and quick turn transactions. In order to make changes to trading contracts in the case of shifting weather conditions or other unforeseen events, the day ahead markets are complemented by intraday and balancing markets. Where Day-Ahead Markets trades $\sim$ 24 hours before delivery, Intraday Markets allow purchasing and selling of electricity throughout the whole day, up to minutes before the physical delivery. Members use the Intraday Market to make last minute adjustments and change their market position close to real-time. Trading on the intraday market allows for energy arbitrage for assets, purchasing electricity and charge when prices are low and discharge/produce during peak demand hours [47] [90].

3.3 Balancing services

The final balancing of the demand and supply is achieved through balancing markets, that are controlled by TSOs [55] [26]. There exist multiple ancillary services to maintain a stable frequency of 50 Hz of the European electricity grid. Frequency ancillary services in Europe are divided into four main reserve categories. Short-term deviations are balanced through the primary reserve, also known as the Flexibility Containment Reserve (FCR) that intervenes within seconds. FCR providers must be able to ramp up/down their generation/consumption within 30 seconds after a disturbance in supply and demand. FCR is automatically activated and TSOs provide financial compensations for offered amounts of flexible power by FCR market participants [68]. Frequency Restoration Reserve (FRR) is activated either automatically (aFRR) or manually (mFRR) and restores the frequency to its nominal value after successful stabilization by FCR. Furthermore, a TSO can use Replacement Rerserve (RR) to free activated capacities in mFRR. The automatically activated FCR and aFRR have a fast response and short but frequent activation events, whereas the manually activated mFRR and RR have a slow response and longer but less frequent activation events. In accordance with the response time of the reserve services between the overall system imbalance and the engagement in frequency control, aFRR and mFRR are secondary and tertiary reserve respectively [66] [47] (see Figure 5).

3.4 Capacity- and Energy Market

Reserve markets are organized in two stages, referred to as the market for reservation of the balancing *capacity* (unit currency in EUR/MW) firstly, and the market for activation of the balancing *energy* (unit currency EUR/MWh) secondly (see Figure 6). FCR does not have a balancing energy market (BEM), but only a balancing capacity market (BCM). FCR is activated proportionally among all accepted capacity bids. For aFRR, mFRR and RR holds that participants who are cleared in the balancing capacity stage must submit their offers in the balancing energy state at a desired price.

3.5 Participation in Flexibility Services

To participate in flexibility services, assets on energy sites need to work together in so-called pools to offer the required capacities at the required moment to the market. If we examine aFRR participation restrictions in the Netherlands, an engagement of at least 24 hours is required to place bids. The ramp rates of CHP assets are not fast enough for aFRR response time of 30 seconds (see Section 3.3), and the capacity of batteries is

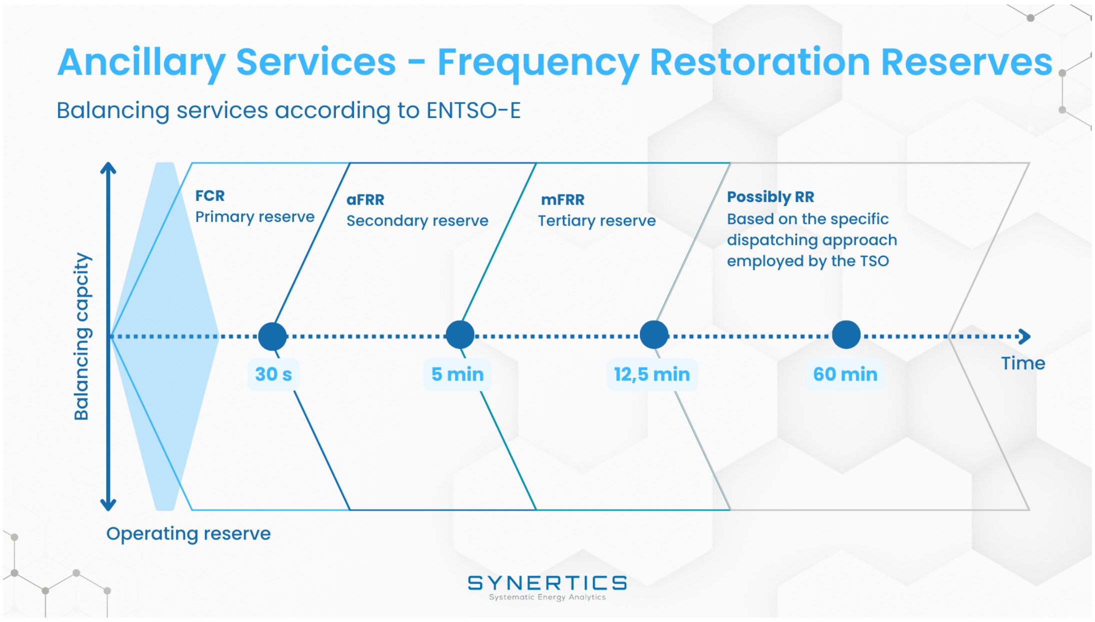


Figure 5: Ancillary services ordered by activation time [3]

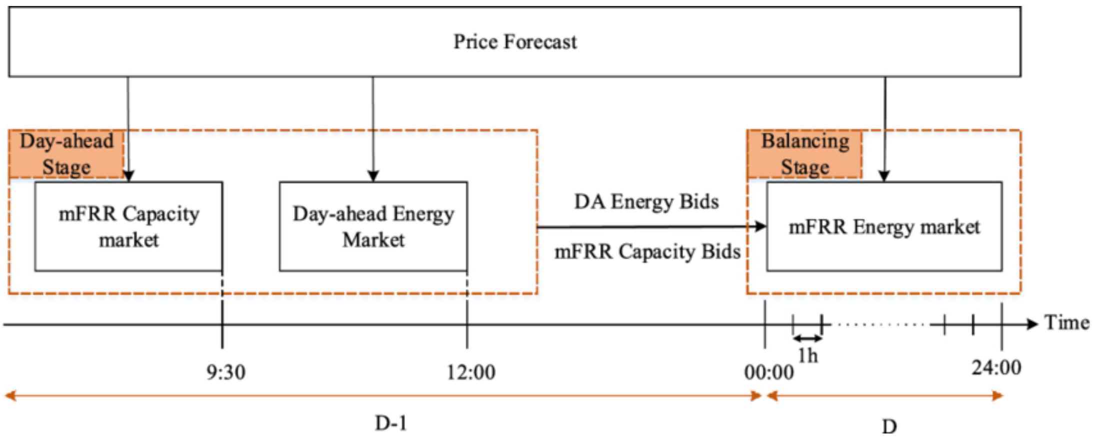
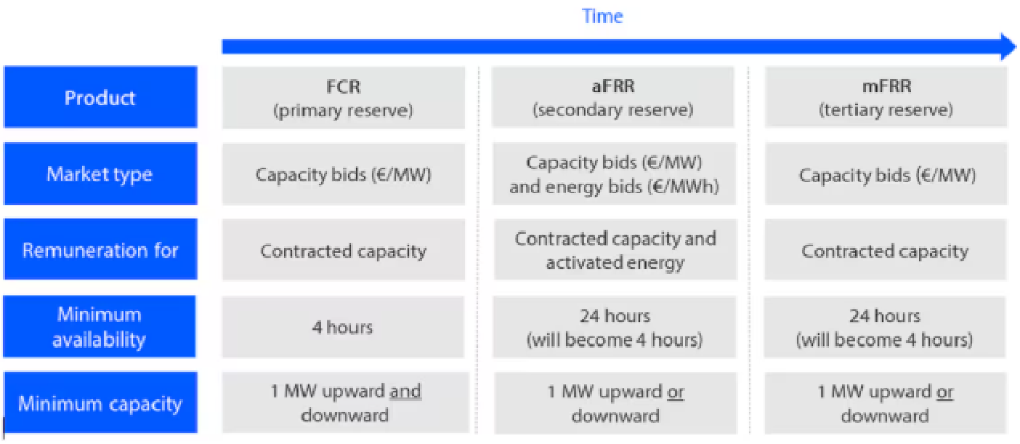


Figure 6: Electricity market timeline presented in [50]

not sufficient to deliver 24 hours, as shown in Figure 7. The pool benefits from the fast response time of batteries to deliver FCR and first phase aFRR, and the sustained output capacity of CHP assets for the long term engagement.



Time			
Product	FCR (primary reserve)	aFRR (secondary reserve)	mFRR (tertiary reserve)
Market type	Capacity bids (€/MW)	Capacity bids (€/MW) and energy bids (€/MWh)	Capacity bids (€/MW)
Remuneration for	Contracted capacity	Contracted capacity and activated energy	Contracted capacity
Minimum availability	4 hours	24 hours (will become 4 hours)	24 hours (will become 4 hours)
Minimum capacity	1 MW upward and downward	1 MW upward or downward	1 MW upward or downward

Figure 7: Engagement requirements of Dutch Balancing markets [4]

4 Energy Management System Description

To control a set of energy assets efficiently, a smart integrated system is required. An Energy Management System (EMS) optimizes the ensemble, adhering to timeseries, system constraints and ambiant conditions. This section describes the optimization problem, formulated as a MILP with a Rolling Modeling Horizon (Section 2.3). System details and a mathematical formulation are provided.

4.1 Input Data

To describe energy sites, assets $a \in A$ are classified by the following types, where A represents the set of all assets on the controlled site.

$$\forall a \in A : a = \begin{cases} \text{SITE} & \text{if } a \text{ represents the whole site,} \\ \text{LOAD} & \text{if } a \text{ consumes electricity,} \\ \text{GENERATOR} & \text{if } a \text{ produces electricity,} \\ \text{STORAGE} & \text{if } a \text{ stores electricity,} \\ \text{CONVERTER} & \text{if } a \text{ converts (other inputs to) electricity,} \\ \text{INTERMITTENT} & \text{if } a \text{ produces electricity from a RES.} \end{cases} \tag{1}$$

To contribute to maintaining grid stability and generate extra profits, assets can be connected to ancillary services (Section 2.2). The EMS simulates asset operation and revenue potential based on system parameters. Parameters represent system characteristics and remain constant during simulation. They are hard coded in the model describing the optimization problem. Examples of parameters are modeling horizons, efficiency rates, state-of-charge (SOC) limits, operational costs, consumption- and production forecasts, electricity market engagements, wheather- and electricity forecasts, and heat congestion details.
Considering electricity market engagement, positive engagement means purchase and negative engagement means sale. Imbalances occur in the case that an asset was committed, or engaged, in energy markets (Section 3), and can somehow not follow the agreed transaction. Positive imbalance (or being long) means that there is less energy consumed than purchased, and/or there is more energy produced than sold. In the case of a negative imbalance (or being short), there is more energy consumed than purchased and/or less energy produced than sold. The classification of electricity markets is presented in Equation 2.

$$\forall f \in F : f = \begin{cases} \text{Long Term} & \text{if } f \text{ is engaged in the long term aggregated electricity market,} \\ \text{Day Ahead} & \text{if } f \text{ is engaged in the Day Ahead electricity market,} \\ \text{Intraday} & \text{if } f \text{ is engaged in the Intraday electricity market,} \\ \text{FCR} & \text{if } f \text{ is engaged in the FCR balancing electricity market,} \\ \text{aFRR} & \text{if } f \text{ is engaged in the aFRR balancing electricity market,} \\ \text{mFRR} & \text{if } f \text{ is engaged in the mFRR balancing electricity market.} \end{cases} \tag{2}$$

4.2 Problem Statement

A Mixed Integer Linear Program (MILP) to solve an optimization problem consists of an objective function and constraints. In order to formulate the objective function, decision variables are introduced.

4.2.1 Decision Variables

The decision variables are the variables the solver seeks to determine in order to optimize the objective function while satisfying the given constraints. Decision variables can either be boolean, integer, or continuous, depending on the possible values they can take at the given decision step $t \in T$. T represents the set of decision steps of equal length, covering the modeling horizon, as discussed in Sections 1 and 2.3. Boolean variables x_t are also referred to as flags, indicating whether a certain condition is met over decision step t.

$$\forall t \in T : x_t = \begin{cases} 1 & \text{if the condition is satisfied,} \\ 0 & \text{otherwise.} \end{cases} \tag{3}$$

Integer variables y_t at decision step $t \in T$ are defined as follows

$$\forall t \in T, y_t \in \mathbb{Z} \tag{4}$$

Continuous variables or floats z_t at decision step $t \in T$ are defined as follows

$$\forall t \in T, z_t \in \begin{cases} \mathbb{R}^+ & \text{if the variable only takes positive values,} \\ \mathbb{R} & \text{otherwise.} \end{cases} \tag{5}$$

When developing an objective function that translates the intensions of the optimization, the problem is formulated as a minimization problem. The decision variables represent costs that describe system needs. These costs stand for operational costs, trading costs and violation costs. Operational costs stand for start-up costs, (dis)charge costs, curtailment costs (in case an intermittent generation asset is forced to reduce output), and electricity costs. Trading costs represent DA- FCR- and aFRR- trading network costs, see Section 3 for more details on energy trading- and market concepts. Violation costs are the category of imbalance costs, physical violations, reserve violation costs, and artificial penalty costs. Imbalance costs arise in the case that there is a difference between real grid exchanges and electricity market engagements. Physical violation costs represent the exceeding of physical limits of the system. Reserve violation costs are penalties for failing to deliver contracted reserve capacities. As discussed in Section 1, constraints are formulated with decision variables to translate the characteristics and limits of the system. To include customized preferences, artificial penalty costs are introduced to hierarchy constraints based on importance or to encourage certain solver behavior. Section 5.3 treats the usefulness and application of violation decision variables in more detail.

4.2.2 Objective Function and Constraints

The mathematical formulation of the optimization problem is presented in equation 6 and Table 1 provides descriptions of the used variables. The physical balance of the system is preserved with power- and heat balance constraints (6d, 6d). The market engagement is controlled by imbalance- and certification constraints (6e, 6g), and it is ensured that if an asset participates in an electricity market, it will not inject power into the system (6f). The latter will be explained in further detail in Section 5.2.

As explained in Section 4.2.1, decision variables describe the system and are combined such that they represent costs that are minimized in the objective. In the objective function (6a), a generalized notation for the costs is used, e.g. $x_{(a,t)_i}$ represents one of the m costs that are related to an asset $a \in A$ at decision step $t \in T$, for some $i \in \{1, ..., m\}$. Likewise, $w_{(a,f,t)_k}$ represents one of the l costs related to an asset $a \in A$ that can be used to participate in electricity market $f \in F$ at decision step $t \in T$, for some $k \in \{1, ..., l\}$.

Cost variables are expressed in currency units and Engagement-, Imbalance, Heat- and Power variables are expressed in kW. Model 6 presents a selection of system constraints in order to maintain a global overview of the most relevant functionalities considered in this research. The omitted constraints include import and generation limits for power, consumption and injection limits for heat, network and heat congestion constraints, bounds on the minimum and maximum number of consecutive decision steps for asset operation, grid operation mode constraints, non-linear variable cost segments for generators, charge and discharge rate constraints for storage assets, and SOC bounds that account for the minimum storage level required to initiate engagement periods.

$$
\begin{aligned}
\text{minimize} \quad & \sum_{t \in T} \sum_{a \in A} x_{(a,t)_1} + ... + x_{(a,t)_m} + y_a + z_{t_1} + ... + z_{t_n} + \sum_{f \in F} w_{(a,f,t)_1} + ... + w_{(a,f,t)_l} && \text{(6a)} \\
\text{subject to} \quad & \forall t \in T, \forall f \in F, \forall a \in A && \text{(6b)} \\
& \sum_{a \in A_p} PowerOut_{a,t} + E_t = \sum_{a \in A_c} PowerIn_{a,t} - PowerDeficit_t + PowerExcess_t && \text{(6c)} \\
& \sum_{a \in B_p} HeatOut_{a,t} = \sum_{a \in B_c} Heatin_{a,t} - HeatDeficit_t + HeatExcess_t && \text{(6d)} \\
& E_t = \sum_{a \in A} Engagement_{a,f,t} - Imbalance_t && \text{(6e)} \\
& PowerOut_{a,t} \leq (1 - EngagementFlag_{a,f,t}) * MaxPowerOut_{a,t} && \text{(6f)} \\
& \sum_{t \in T} Engagement_{a,f,t} \leq CertifiedPower_{a,f} && \text{(6g)} \\
& PowerOut_{a,t} \in \mathbb{R} \quad \forall a \in A_c && \text{(6h)} \\
& \{E_t, Imbalance_t\} \subseteq \mathbb{R} && \text{(6i)} \\
& PowerIn_{a,t} \in \mathbb{R} \quad \forall a \in A_p && \text{(6j)} \\
& HeatOut_{a,t} \in \mathbb{R} \quad \forall a \in B_c && \text{(6k)} \\
& HeatIn_{a,t} \in \mathbb{R} \quad \forall a \in B_p && \text{(6l)} \\
& \{PowerDeficit_t, PowerExcess_t, HeatDeficit_t, HeatExcess_t\} \subset \mathbb{R}_{\geq 0} && \text{(6m)} \\
& Engagement_{a,f,t} \in \mathbb{R} && \text{(6n)} \\
& EngagementFlag_{a,f,t} \in \{0, 1\} && \text{(6o)} \\
& MaxPowerOut_{a,t} \in \mathbb{R} && \text{(6p)} \\
& CertifiedPower_{a,f} \in \mathbb{R} && \text{(6q)}
\end{aligned}
$$

Notation	Description
Set	
A	set of assets on site
T	set of decision steps in the optimization window
F	set of electricity markets
A_c	set of flexible and non-flexible load units and converter assets consuming power, $A_c \subset A$
A_p	set of (intermittent) dispatchable generators and storage assets producing power, $A_p \subset A$
B_c	set of non-flexible load units consuming heat, $B_c \subset A$
B_p	set of dispatchable generators, storage assets and converter assets producing heat, $B_p \subset A$
Decision variables	
$x_{(a,t)_i}$	costs x for asset $a \in A$ at decision step $t \in T$, $i = 1...m$
y_a	costs y for asset $a \in A$
z_{t_j}	costs z at decision step $t \in T$, $j = 1...n$
$w_{(a,f,t)_k}$	costs w for asset $a \in A$ and electricity market $f \in F$ at decision step $t \in T$, $k = 1...l$
$PowerOut_{a,t}$	Power produced by asset $a \in A_p$ at decision step $t \in T$
$PowerIn_{a,t}$	Power consumed by asset $a \in A_c$ at decision step $t \in T$
E_t	Power imported from the main grid at decision step $t \in T$
$PowerDeficit_t$	Average power deficit over decision step $t \in T$
$PowerExcess_t$	Average power excess over decision step $t \in T$
$HeatIn_{a,t}$	Heat consumed by asset $a \in B_c$ at decision step $t \in T$
$HeatOut_{a,t}$	Heat produced by asset $a \in B_p$ at decision step $t \in T$
$HeatDeficit_t$	Average heat deficit over decision step $t \in T$
$HeatExcess_t$	Average heat excess over decision step $t \in T$
$Imbalance_t$	Difference between real grid exchanges and market engagements at decision step $t \in T$
$EngagementFlag_{a,f,t}$	Flag indicating if asset $a \in A$ is engaged (1) on electricity market $f \in F$ at decision step $t \in T$ or not (0)
Input parameters	
m	number of cost variables that are indexed over $a \in A, t \in T$
n	number of cost variables that are indexed over $t \in T$
l	number of cost variables that are indexed over $a \in A, f \in F, t \in T$
$Engagement_{a,f,t}$	Total engagement of asset $a \in A$ on electricity market $f \in F$ at decision step $t \in T$
$MaxPowerOut_{a,t}$	Maximum power produced by asset $a \in A_p$ at decision step $t \in T$
$CertifiedPower_{a,f}$	Maximum engagement certified of asset $a \in A$ on electricity market $f \in F$ over the optimization window

Table 1: Notation and Description of Elements in Model 6

5 Warm Start Strategies

As discussed in Section 2.4, using warm starts are an appropriate way to efficiently model energy problems. For the EMS described in Section 4, we discuss multiple configurations of warm starts for numerous problem instances. The goal of a warm start is to efficiently use previous solutions as an input for a next optimization, resulting in a reduced running time.

5.1 MIP Start Configurations in CPLEX

IBM ILOG CPLEX Optimization Studio offers diverse built-in warm start strategies that help CPLEX find an initial solution [5]. We discover multiple configurations of *AddMIPStart* and *ReadMIPStart*. *AddMIPStart* helps CPLEX using an initial solution to find the optimal solution faster and is suitable for multi-staged energy management applications. The built-in function allows you to assign values to decision variables. The values can be derived from earlier feasible solutions, but do not have to be [63][6]. A combination of *readMIPStart* and *writeMIPStart* offers another environment to apply warm start strategies in CPLEX using an mst file format [7] and shows great potential for MILP scheduling optimization problems [76][8][9]. Applying the *writeMIPStart* function, if not specified otherwise, all solutions of the discrete decision variables are stored in an mst file. For $writeLevel = 1$, all decision variables of the current solution are stored, including the continuous ones.
As discussed in section 2.5, CPLEX applies a branch and bound algorithm to arrive at optimal solutions. Facet defining inequalities in the problem formulation, together with other combinatorial and mixed-integer inequalities, are used as cutting planes [44]. The incorporation of cutting planes enhances the B&B algorithm, and the hybrid result is referred to as the Branch and Cut (B&C) method. CPLEX processes the data of provided MIP starts before launching B&C during an optimization. The best of the potential solutions defined by the MIP Starts is fed to the algorithm as an incumbent solution, strongly reducing the size of B&C trees.
You can provide CPLEX with multiple MIPStarts and specify the way they are treated. CPLEX decides how to construct starting points from the supplied combinations of discrete and continuous variables and their assigned values, combined with the specified Effort Level. The Effort level of the supplied MIPStarts for the current problem can be specified individually and their implications can be found in Table 2. In case multiple MIPStarts are applied, CPLEX treats the first MIPStarts with effort level 4, and the others with effort level 1. In the mst file format, for each provided solution, the Effort level can be specified individually.
In order to perform operations on decision variables before solving the model, model needs to be defined and created in a separate file. To this end, we consider an orchestration configuration that calls the model from this file, loads MIPstarts and consequently solves it. Although warm start strategies can significantly accelerate convergence, repeated applications can cause computational overhead. It is therefore crucial to balance informativeness and conciseness, providing the algorithm with effective initial guidance without overwhelming it with excessive or low-quality starting points.

Level	Effect
0	AUTO: Lets CPLEX choose level 1-5 automatically.
1	Checks feasibility of the corresponding MIP start. Values for all variables in the problem, both discrete and continuous need to be specified. If any values are missing, CPLEX does not process this MIP start.
2	Solves the fixed LP problem specified by the MIP start. Values for all discrete variables need to be provided for this effort level. If values for any discrete variables are missing, CPLEX does not process the MIP start
3	Solves a subMIP. The value of at least one discrete variable at this effort level needs to be specified.
4	Attempts to repair the MIP start if it is infeasible, according to the parameter that sets the number of attempts to repair infeasible MIP start. The value of at least one discrete variable at this effort level needs to be specified, too.
5	Does not delay processing to perform the usual checks. CPLEX checks only whether the MIP start is a complete solution; if the MIP start is not a complete solution, CPLEX rejects it. If the MIP start is a complete solution, CPLEX performs no further checks. At this level, CPLEX does not delay processing to check whether any constraints in the MIP start were designated as lazy constraints in the model, for example. If the solution defined by the MIP start is infeasible, behavior is undefined, as a consequence of this lack of checking

Table 2: MIPStart Effort Levels and their effect [5]

5.2 Nominal Scenario Description

An appropriate nominal case that could serve as baseline for run time experiments is chosen to be an optimization of the Energy Management System described in Section 4 that takes approximately 7 minutes to converge. This is considered to be a manageable duration, on which the impact of improvement will be clearly visible. The chosen baseline scenario is the following: An increased modeling horizon (See section 2.3) of 48 hours is chosen, enlarging the input data set (See section 4.1). This results in a greater number of forecast values to be handled, a larger feasible solution set, and an increased computational complexity. The standard configuration of the input data of our baseline problem instance contains aFRR engagements. The market engagement blocks energy generation from the assets that are capable of providing aFRR service. This constraint prevents these assets from producing as discussed in Section 4, precisely in constraint 6f. aFRR engagements are thus removed from the scenario to bring in degrees of freedom to the optimization, giving the model the option to discover power production for these assets. This enlarges the feasible region by reducing constraint tightness, making convergence to the optimal solution more difficult. The difference is shown in Table 3, where the engagement of aFRR speeds up optimization by 47.73%.

Baseline Running time: 413.50 s	
Configuration	**Difference in Running Time (%)**
Scenario with aFRR engagements	-47.73

Table 3: Running Time Reduction for Scenario with aFRR engagements

5.3 Model without Violation Decision Variables

5.3.1 Handling of Violations

As discussed in Section 4, the EMS handles violations and violation costs. A violation is introduced to temporarily allow the relaxation of constraints, against an artificial penalty described by a violation cost that is minimized in the objective function.
A generalized framework for handling violation variables is presented as follows.

$$\forall t \in T, \quad LHS_t = RHS_t \tag{7}$$

becomes

$$\forall t \in T, \quad LHS_t = RHS_t + Deficit_t - Excess_t \tag{8}$$

In equation 8, LHS_t and RHS_t stand for the left-hand side and right-hand side of the constraint, respectively, and are linear combinations of the problem decision variables. Furthermore, $\{Deficit_t, Excess_t\} \subset \mathbb{R}_{\geq 0}$. A customized penalty cost is assigned to each violation and total penalty costs are minimized in the objective function.
Two implementations are presented in Equations 6d and 6c. Other examples of violations contain deficits in the pool compared to what is engaged (see Section 3.5), the exceeding of battery limits (see Section 5.5.1), and violations of the constraints described at the end of Section 4.2.2.

In a cost minimazation problem, each violation is assigned an artificial penalty cost. Violations improve robustness as the model remains feasible under slight bending of a constraint, in a controlled and penalized way. The inclusion of violation decision variables gives the model more decisions to make, and therefore elevates complexity. Removing violations hence takes away robustness, but reduces complexity. For optimization of instances where no violations were addressed, feasibility is maintained when removing all violation variables from the objective function and constraints. After observing that the violations were not activated—that is, all violation variables were zero in the optimal solution—they were removed from the model, and the baseline scenario was re-evaluated.

5.3.2 Removal of Violations

Removing the violation decision variables from the model shows significant running time reduction for the baseline problem instance as presented Test case 1 in Table 4, and feasibility is maintained. It is also examined whether warm starting power- and storage targets in the violation-free model can further reduce computation time. The results are presented in Table 4, test case 2 and 3, and we observe no further computational gains with respect to the baseline. The same holds for applying warm starts to discrete variables in the model without violations, as observed in case 5 to 9 for different configurations discussed in Section 5.1.
Based on the result of Test case 1, further testing is conducted using warm starts in which the violation variables are initialized at zero in the model with violations. The motivation behind this approach is to create a model that is both robust and efficient, maintaining feasibility at all times while minimizing the computational effort associated with exploring the solution space introduced by the violation decision variables.

Baseline Running Time: 413.50 s			
Test case	**Warm Start Strategy**	**Decision Variables**	**Difference in Running Time (%)**
1	N.A.	N.A.	−33.63
2	*AddMIPStart*	$PowerTarget_p$, $StorageTarget_s$	−7.12
3	*AddMIPStart*	$PowerTarget_p$	−10.02
4	*AddMIPStart*	$IsGenOn$	−11.00
5	*AddMIPStart*	$IsCharging$	−30.8
6	*readMIPStart*	$IsGenOn(0)$	−8.52
7	*readMIPStart*	$IsCharging(4), isDisCharging(4)$	−34.01
8	*readMIPStart*	*All discrete decision variables*(4)	−31.60
9	*readMIPStart*	*All discrete decision variables*(0)	+13.68

Table 4: Warm Start Strategy Tests for the Model without Violations for the Baseline described in Section 5.2

5.4 Warm Starts of Violation Decision Variables

Based on the observations in Section 5.3, the hypothesis is tested whether performing warm starts on violation decision variables improves the running time of the EMS optimization.

5.4.1 Deployment Strategy and Results

With the *AddMIPStart* configuration described in Section 5.1, the warm start of Power Targets of Dispatchable Generators, Intermittent Generators, and Converter Assets is tested and the results are displayed in Table 5. The Storage Targets are considered for BESS and a heat buffer, that stores heat. See Section 4.1 for more details on the classification of the assets and input data. The order in which the *AddMIPStart* functions are called in the orchestration matters, as discussed in Section 5.1. In case multiple orders of decision variable configurations are tested, for example for Test case 6, the average Difference in Running time is mentioned in Table 5.
In Test case 9, warm starting PowerDeficit before PowerTarget slightly speeds up the code, where the reversed order slowed down the code by 11.01%, in Test case 3. This underlines the importance of regulating effort levels of MIPStarts combinations, using mst files (See section 5.1). In the mst files, there exist multiple configurations to store the solutions of the decision variables selected for warm start initialization. They can eather be provided as multiple separate solutions each corresponding to small sets of the selected decision variables, or as one solution containing the values of all decision variables. Each solution provided in the mst file is accompanied with its own EffortLevel, denoted in brackets in Table 5, starting from test case 15.
Case 8 supports the hypothesis of Section 5.3, showing a 16.15% running time reduction compared to the Baseline by applying a warm start of the violation variable $PowerDeficit$. As observed in cases 9 up to 19, not all warm starts of (combinations of) violation decision variables are beneficial for reducing running time. Test case 13 presents another running time reduction, this time of 19.27%, by performing a warm start of $SOCmaxExcess$. Multiple (combinations of) warm start configurations for $SOCmaxExcess$ and $PowerDeficit$ are tested in case 14 to 19.
The results support the conclusion that a synergy is observed in case 16 for the warm start of $PowerDeficit$ in combination with $SOCmaxExcess$, compared to individual warm

starts in cases 8 and 13. It can be concluded that a duo warm start with *SOCMaxExcess* is more beneficial for *PowerDeficit*, whereas for *SOCMaxExcess* it is more beneficial to warm start individually. For *SOCmaxExcess*, the most successful configuration is at effort level 0, reducing the running time by 26.17%.

Baseline Running time: 413.50 s			
Test case	**Warm Start Strategies**	**Decision Variables**	**Difference in Running Time (%)**
1	*AddMIPStart*	$PowerTarget_p$	+21.4
2	*AddMIPStart*	$PowerTarget_p$	+169.28
3	*AddMIPStart*	$PowerTarget_p, PowerDeficit$	+11.01
4	*AddMIPStart*	$StorageTarget_s$	+7.90
5	*AddMIPStart*	$StorageTarget_s$	+14.12
6	*AddMIPStart*	$PowerTarget_p$, $StorageTarget_s$	+10.42
7	*AddMIPStart*	$PowerTarget_p$, $StorageTarget_s$	+17.70
8	*AddMIPStart*	$PowerDeficit$	−16.15
9	*AddMIPStart*	$PowerDeficit, PowerTarget_p$	−1.85
10	*AddMIPStart*	$PowerExcess$	+9.43
11	*AddMIPStart*	$HeatExcess$	−1.43
12	*AddMIPStart*	$SOCstrictMinDeficit$	+213.78
13	*AddMIPStart*	$SOCMaxExcess$	−19.27
14	*AddMIPStart*	$PowerDeficit, SOCMaxExcess$	−5.41
15	*readMIPStart*	$\{PowerDeficit, SOCmaxExcess\}(4)$	−7.24
16	*readMIPStart*	$SOCmaxExcess(4), PowerDeficit(4)$	−22.62
17	*readMIPStart*	$SOCmaxExcess(4)$	−26.20
18	*readMIPStart*	$SOCmaxExcess(0)$	−26.71
19	*readMIPStart*	$PowerDeficit(0)$	−12.55

Table 5: Warm Start Strategy Tests for the Model with Violations for the Baseline described in Section 5.2

Test case	**Index**
2	$p = \{\text{GENERATOR}, \text{CONVERTER}, \text{INTERMITTENT}\}$
	$s = \{\text{STORAGE (E)}, \text{STORAGE (H)}\}$
3	$p = \text{GENERATOR}$

Table 6: Index Specification representing Assets for the Decision Variables in Table 4

Test case	**Index**
1	$p = \text{GENERATOR}$
2	$p = \text{CONVERTER}$
3	$p = \text{INTERMITTENT} \cap \text{GENERATOR}$
4	$s = \text{STORAGE (E)}$
5	$s = \text{STORAGE (H)}$
6	$p = \{\text{GENERATOR}, \text{CONVERTER}, \text{INTERMITTENT}\}$
	$s = \{\text{STORAGE (E)}, \text{STORAGE (H)}\}$
7	$p = \text{GENERATOR}$
	$s = \text{STORAGE (E)}$
9	$p = \text{GENERATOR}$

Table 7: Index Specification representing Assets for the Decision Variables in Table 5

5.5 Reformulations

Building on the findings presented in previous Sections, this Section revisits two key components of the baseline scenario: the SOC violation limits and the Day-Ahead electricity prices. In both cases, warm start experiments are repeated to empirically assess the effects of the adjustments.

5.5.1 Bounding SOC Limit Violations

In Section 5.4, the strong impact of a warm start of $SOCMaxExcess$ can be attributed to the occcurrence of negative Day-Ahead prices in the baseline scenario. In case of negative Day Ahead prices, there is no incentive to generate electricity locally and inject it into the main grid, as this would result in financial losses. Instead, the system takes advantage of the low Day-Ahead prices and charging of storage assets is pushed to the maximum. As a consequence, possible violations of SOC limits are explored. Precise and tight bounding of the violation of storage capacity is therefore crucial to prevent excessive search of the solver. Motivated by this insight, the SOC bounds presented in Equations 9a and 9b were revised to explicitly include limits on the violations, presented in Equations 9c and 9d.

$\forall s \in S \quad \forall t \in T \quad \forall f \in F$

$$SOCTarget_{s,t} \leq MaxSOC_s - EngPower_{f,s,t} * \frac{EngUpEnergy_f}{MaxEnergy_{s,t}} * 100 + SOCMaxExcess_{s,t} \tag{9a}$$

$$SOCTarget_{s,t} \geq MinSOC_s + EngPower_{f,s,t} * \frac{EngDwnEnergy_f}{MaxEnergy_{s,t}} * 100 - SOCMinDeficit_{s,t} \tag{9b}$$

$$SOCMaxExcess_{s,t} \leq 100 - MaxSOC_s + EngPower_{f,s,t} * \frac{EngUpEnergy_f}{MaxEnergy_{s,t}} * 100 \tag{9c}$$

$$SOCMinDeficit_{s,t} \leq MinSOC_s + EngPower_{f,s,t} * \frac{EngDwnEnergy_f}{MaxEnergy_{s,t}} * 100 \tag{9d}$$

$$\{SOCMaxExcess_{s,t}, SOCMinDeficit_{s,t}\} \subset \mathbb{R}_+ \tag{9e}$$

$$\{MaxSOC_s, MinSOC_s, EngUpEnergy_f, EngDwnEnergy_f, MaxEnergy_{s,t}, SOCTarget_{s,t}\} \subset \mathbb{R} \tag{9f}$$

$$EngPower_{f,s,t} \in \mathbb{Z} \tag{9g}$$

Notation	Description
Set	
S	set of electrical storage assets
T	set of decision steps in the optimization window
F	set of electricity markets that allow engaged volumes coming from BESSs
Decision variables	
$SOCTarget_{s,t}$	SOC Target expressed as a % of asset's $s \in S$ maximum electrical energy storage capacity at decision step $t \in T$
$SOCMaxExcess_{s,t}$	SOC max excess expressed as a % for asset $s \in S$ at decision step $t \in T$
$SOCMinDeficit_{s,t}$	SOC min deficit expressed as a % for asset $s \in S$ at decision step $t \in T$
$EngPower_{f,s,t}$	asset's $s \in S$ engaged power expressed in MW on market $f \in F$ at decision step $t \in T$
$EngUpEnergy_f$	battery's minimum energy upward needed to enter energy market $f \in F$ expressed in kWh/MW engaged
$EngDwnEnergy_f$	battery's minimum energy downward needed to enter energy market $f \in F$ expressed in kWh/MW engaged
Input parameters	
$MaxSOC_s$	maximum state of charge of $s \in S$ expressed as a % of asset's maximum electrical energy storage capacity
$MinSOC_s$	minimum state of charge of $s \in S$ expressed as a % of asset's maximum electrical energy storage capacity
$MaxEnergy_{s,t}$	maximum energy capacity expressed in kWh for $s \in S$ over $t \in T$

Table 8: Notation and Description of Elements in Equation 9

Constraints 9c and 9d allow temporary crossing of SOC limits and regard for the committed upward and downward power band for participation in electricity markets like FCR (see Section 3.3). Upward energy is energy that will be injected from the battery to the grid, and downward energy will be absorbed from the main grid in the battery. As discussed in Seciton 5.2, aFRR engagement blocks battery usage, hence $aFRR \notin F$ for Equation 9 for this EMS.

In Table 9, we observe that the addition of bounds for $SOCMaxExcess$ and $SOCMinDeficit$ reduces the running time by 36.60% and therefore has a greater impact on the running time than performing a MIPStart on $SOCMaxExcess$. Beneficial warm starts can hence point out potential improvements in your problem definition, after revision of limit constraints that concern the warm started decision variable.

Baseline Running time: 413.50 s	
Configuration	**Difference in Running Time (%)**
Upperbound $SOCmaxExcess$ and $SOCMinDeficit$	-36.60

Table 9: Running Time Reduction from introducing Upperbounds for Violation Decision Variables

When taking the model with upper-bounded SOC violation limits, as described above, as the baseline, a significant slowdown is observed when a warm start is applied to $SOCMaxExcess$ (See Table 10). This suggests that warm starting violation decision variables that are already tightly constrained by the model offers no added value in this case. It is important to note that bounding violation decision variables is not always straightforward. As such, exploring warm start strategies for violation decision variables remains relevant and potentially beneficial.

Baseline Running time: 262.16 s			
Test case	**Warm Start Strategies**	**Decision Variables**	**Difference in Running Time (%)**
1	*readMIPStart*	$SOCMaxExcess(0)$	+199.42

Table 10: Warm Start Strategy Tests for the Model with Violations for the Baseline described in Section 5.5.1

5.5.2 Model Comparison for Scenario with Positive Day-Ahead Prices

As discussed in Section 5.5.1, scenarios with negative Day-Ahead (DA) prices significantly influence the utilization of storage assets. To mitigate this effect and provide a more neutral test environment, a scenario with strictly positive DA prices is introduced. This adjustment also allows for a clearer investigation of warm start strategies applied to decision variables beyond the ones previously tested.

A baseline for the same EMS from Section 4 is considered without aFRR engagements. The modeling horizon is 50 hours and the DA prices are strictly positive. The versions of the model with and without the bounds presented in Section 5.5.1 are considered for this scenario. The hypothesis that the bounds on the SOC limits violations have a smaller effect is supported in Table 11, which shows only a slight decrease in running time for the model including the bounds.

Variation of the Model	**Running time**
Model without bounds on SOC limit violation variables	220.58s
Model with bounds on SOC limit violation variables	209.71s

Table 11: Model comparison for the Model with Violations for the Baseline described in Section 5.5.2

From Table 11 we observe that for the positive Day-Ahead prices scenario, the model with bounds on the SOC limit violation variables is favorable compared to the one without bounds. As a baseline for the next experiment is thus chosen the model with bounds and from now on warm start strategy *readMIPStart* is consistently use to control Effort Levels described in Section 2.

Baseline Running time: 209.71 s		
Test case	**Decision Variables**	**Difference in Running Time (%)**
1	*All decision variables*(0)	+4.92
2	*All decision variables* (1)	−3.74
3	$IsGenOn$, $PowerTarget_p$	−
4	*All discrete decision variables* (0)	−2.43
5	*All discrete decision variables* (5)	−12.32

Table 12: Warm Start Tests for the Model with Violations and for the Baseline described in Section 5.5.2

We observe for case 3 where p = GENERATOR that CPLEX can reject the warm start of certain (combinations of) variables at all Effort Levels. The results in Table 12 show a running rime reduction for test cases 2, 4 and 5, where large sets of decision variables are warm started. This observation supports the further exploration of warm start strategies for all variables, as well as discrete variables exclusively, at different Effort Levels.

5.6 Solver Behavior under Warm Start Strategies

This section examines convergence to the optimal solution of three different warm start strategies in various scenarios and analyzes the results using solver logs and the theoretical principles underlying solver dynamics. The algorithmic structures and CPLEX solver progression follow the descriptions in [61] and [10].

5.6.1 Solver Dynamics

To understand how providing warm starts can potentially improve solver performance, it is important to initially elaborate on the solution path of the CPLEX Solver. Traditionally, Mixed Integer programming for a minimization problem can be viewed as having two main parts, one in which the upper bound for the solution is decreased using heuristics, and the other in which the lower bound is increased using cutting planes [25].
The Branch and Bound method exploits both approaches and proceeds as follows: All integer and binary variables of the MILP are relaxed and this LP relaxation is referred to as the root node. If its solution is integral, the optimum is found. If not, the objective value of the fractional solution is a dual bound for the root node. Primal search is conducted at the root node to find a first, hence best, feasible integer solution, or incumbent. The incumbent can be obtained through various methods, including heuristics, as well as from user-provided MIP Starts. The objective value of the incumbent becomes the primal bound. Interleaved with the primal search, CPLEX generates cutting planes at the root, that represent valid inequalities satisfied by all feasible integer solutions, but violated by the current fractional LP solution. The tightened LP provides a new dual bound closer to the best found integer solution and heuristics are applied to find better feasible integer solutions. If the obtained integral solution results in a better objective value than the primal bound, the Best Integer is updated accordingly. This process continues recursively until either the LP solution is integer-feasible and matches the Best Integer, or the LP solution is not integer-feasible and no further cuts can be found. In the first case, the optimal solution is found and the algorithm terminates.
If the LP solution is not integer-feasible and no further cuts can be found, the algorithm proceeds by branching: it creates child nodes by tightening the bounds of discrete variables. At each node, the LP relaxation is solved to obtain a new dual bound for that subproblem. If the solution is integer-feasible, Best Integer is updated. If the bound is worse than the Best Integer, the node is pruned. In addition to solving the LP, CPLEX may generate local cutting planes that are valid for only the current node's feasible region. These local cuts further tighten the node's LP relaxation, potentially improving the bound or enabling earlier pruning. The Best Bound is the best of all dual bounds among active nodes. The algorithm continues until all nodes are either solved or pruned and the optimal solution is obtained from.

5.6.2 Warm Start Setup and Logging

This section discusses the warm start strategies for discrete variables for multiple scenarios and Effort Levels. A Baseline scenario similar to the ones described in Sections 5.2 and 5.5.2 is created. The nominal scenario has positive Day Ahead prices, a modeling horizon of 36 hours and is configured without aFRR participation. Afterwards, a warm start on Effort Level 0 is applied to all decision variables from the mst file that was created in the prior Baseline run. Subsequently, the optimal solutions of the discrete variables are extracted, and stored in a separate mst file. The problem is likewise warm started from this renewed mst file, this time on Effort Level 2. While converging to the optimal solution, the solver keeps track of the Solution Gap between the primal and dual bound, defined as:

$$\text{Solution Gap} = \frac{|\text{Best Integer} - \text{Best Bound}|}{|\text{Best Integer}|} \tag{10}$$

The CPLEX execution log also displays the presolve summary, the number of generated cuts, and intermediate values of the objective function, the best integer solution, and the best bound. The results are presented Tables 13 and 14. To calculate the running times, the average is taken from multiple runs of the same instance. The CPLEX execution logs supporting the data can be found in Appendix B.1.

Strategy	**Baseline**	**WS All Variables** (0)	**WS Discrete Variables (2)**
First presolve Elimination Count			
Rows	14114	14114	14114
Columns	15287	15287	15287
Initial Value			
Best Integer	$8.01272e^{9}$	-2004.5594	-2004.5594
Best Bound	-2114.6839	-2114.6839	-2114.6839
Gap	100%	5.49%	5.49%
Second presolve Elimination Count			
Rows	837	1408	1349
Columns	1293	1535	1513

Table 13: Comparison of Log Data before the Optimal Solution is obtained

Strategy	**Baseline**	**WS All Variables (0)**	**WS Discrete Variables (2)**
Result			
Running Time	142.36 s	166.79 s	142.35 s
Objective	-2004.5594	-2004.5594	-2004.5594
Cut Count			
Clique cuts	1	15	15
Implied bound cuts	1407	1964	2972
MIR cuts	733	325	325
Total	4640	4453	6393

Table 14: Comparison of Log Data After the Optimal Solution is obtained

5.6.3 Solver Path under Warm Start Strategies

As observed in Table 13, the initial presolve phase is unaffected by warm start strategies. However, their application substantially enhances the incumbent solution. In Table 14, a total number of generated cuts and a time of convergence of equivalent order is observed across all scenarios. This raises the question how the small initial gap influences solver dynamics.

The equal problem size of the Reduced MIP After Presolve in all three scenarios indicates that providing a warm start does not change the presolve outcome. After the first restart, however, we observe in Table 13 that more rows and columns are eliminated during presolve in the warm started scenarios. This is consistent with the notion that a strong incumbent enables reduced-cost fixing and additional presolve tightening [10] [20].

The most striking difference between the Baseline log and warm startede scenarios is the progression of the MIP Gap, calculated according to Equation 10. As explained in Section 5.6.1, the first integer solution for the baseline is obtained via primal search Starting heuristics, such as Fix and Propagate and Simple Local Search, which often yield solutions far from optimal [37], like $8.01272e^9$ in Table 13. Afterwards, the algorithm relies on primal search Improvement heuristics, such as Rounding and Diving Heuristics and Local Branching. These heuristics exploit LP-relaxations, and rapidly improve the primal bound in early iterations [11]. As described in Section 5.6.1, the branch-and-bound algorithm reduces the primal–dual gap by interleaving these primal heuristics with cut generation. Once a reasonably good incumbent is found, the solver alternates between improving the incumbent and tightening the dual bound from LP relaxations, which eventually closes the gap and proves optimality.

For the warm start scenarios, the optimal integer solution is provided. Consequently, the initial Gap is very small at the root node and uniquely defined by the dual bound. Primal search to find a better incumbent is no longer relevant, so primal search heuristics are not invoked in the same way. The algorithm immediately focuses on lifting the lower bound by generating cutting planes and branching to close the gap and prove optimality of the incumbent [57][42].

This explains the relatively high number of cuts observed in warm start runs, given the small initial gap observed in Table 14. Progress in gap closure must come exclusively from strengthening the dual bound. While a strong incumbent prunes large suboptimal regions of the search space, it can also reduce the role of primal heuristics and thus shift the computational burden toward bound proving. This aligns with prior findings that proving optimality with a known incumbent may still require extensive cut generation and branching [17][42].

5.7 Deployment of Warm Start Strategies across Scenarios

To see whether the events described in Section 5.6.2 occur consistently when applying warm start strategies, the same tests were conducted for other scenarios. A selection of the log output data and a description of the scenarios are presented in Tables 15 and 16, respectively.

Data	**Scenario**	BL	WSA (0)	WSD (2)
Initial Gap (%)	1	100	5.49	5.49
	2	100	N.A.	36.52
	3	100	4.98	4.98
	4	100	4.10	4.10
	5	100	4.66	4.66
Running Time* (%)	1	142.36 s	+17.16	$-7 * 10^{-3}$
	2	262.17 s	N.A.	+42.79
	3	425.39 s	−11.74	−2.13
	4	39.41 s	−53.92	−8.17
	5	209.71 s	+6.76	−12.11
Clique Cuts	1	1	15	15
	2	97	N.A.	84
	3	283	249	226
	4	73	147	58
	5	200	304	49
Cover Cuts	1	1917	1646	2265
	2	3265	N.A.	4544
	3	6932	6240	6198
	4	2290	1960	2704
	5	5876	4125	4547
Implied Bound Cuts	1	1407	1964	2972
	2	3282	N.A.	3785
	3	5353	5119	5669
	4	4089	3941	4243
	5	8475	8025	8634
Mixed Integer Rounding Cuts	1	733	325	325
	2	558	N.A.	666
	3	1725	2196	2695
	4	1165	1604	1486
	5	3849	4093	4962
Total Cut Count	1	4640	4453	6393
	2	7722	N.A.	9489
	3	16191	15844	16583
	4	9852	9694	10429
	5	20321	19429	20914

Table 15: Comparison of Logs of Warm Start Strategies for different Scenarios
* Relative difference compared with baseline of scenario

Scenario	Description
1	EMS instance with a horizon of 36 hours from Section 5.6.2, only positive DA prices
2	EMS instance with a horizon of 48 hours from Section 5.5.1, includes negative DA prices
3	EMS instance with a horizon of 48 hours, only positive DA prices
4	EMS instance with a horizon of 36 hours, only positive DA prices
5	EMS instance with a horizon of 50 hours, only positive DA prices
Abbreviation	
BL	Baseline
WSA (0)	Warm Start Strategy for **A**ll Decision Variables at Effort Level 0
WSD (2)	Warm Start Strategy for all **D**iscrete Decision Variables at Effort Level 2

Table 16: Specification of Scenarios and Abbreviations in Table 15. None of the tested scenarios is aFRR engaged.

5.7.1 Discussion of Results

Table 15 unfolds important insights on the application of Warm Start in different environments. A striking result is the poor solver performance in scenario 2 for a warm start on Effort Level 2. The input data of scenario 2 contains several negative Day Ahead prices. The fast convergence of the baseline scenario is related to the fact that the model is heavily constrained in scenarios with negative Day Ahead prices, as discussed in Section 5.5. The system relies on storage mechanisms and other assets, like generators, are blocked. This reduces computational complexity, and the solution is found rather fast, following the steps from Section 5.6.1. The optimal solution values of the discrete variables provided in WSD (2), in contrast, clearly provide less guidance for the LP-relaxation of the root node than in the other scenarios, resulting in a high initial gap compared to the other scenarios. Proving optimality following the steps from Section 5.6.3 while closing a gap of 36.52% initiated extensive generation of cuts and a costly convergence, as observed in Table 15.
Across the five tested scenarios, the WSD (2) strategy proves to be beneficial for all tested horizons, provided that Day-Ahead prices remain strictly positive. In contrast, the WSA (0) strategy is less favorable, as it is rejected in scenario 2 and shows adverse effects in scenarios 1 and 5. Nevertheless, the acceleration of convergence by 53.92% observed in scenario 4 represents a promising outcome, which merits closer examination in future analyses.

5.8 Rolling Horizon Environment

As discussed in Sections 2.3 and 2.4, warm starting instances in Rolling Horizon (RH) applications can provide significant benefits in solver performance. In the practical application, the baseline scenario adopts an RH framework with a 15-minute resolution and a planning horizon of 36 hours. In the empirical evaluation, by contrast, the problem is tested in a static setting: individual instances are provided with their own optimal feasible solution for a selected subset of variables.
A central challenge in this context arises from the fact that consecutive RH instances are never identical. While warm starts can improve efficiency, they also introduce risks if the incumbent from the previous run is no longer feasible or representative of current system conditions. This makes the design of effective warm start strategies crucial for ensuring both solver efficiency and robustness. This section therefore discusses solver dynamics in RH optimization, and how both convergence and consistency are maintained.

5.8.1 Incumbents and Primal Heuristics

Considering the optimal solution of $t-1$ as a warm start for the problem instance at decision step t can greatly improve solver performance, if applied correctly. Due to environmental changes, the incumbent coming from the optimal solution at $t-1$ will deviate from the objective function value for decision step t. As discussed in Section 5.6.1, the initial gap will change compared to the solver log for decision step t accompanied with its own incumbent. Recall that the numerical value of the root LP is untouched by warm start strategies, as it is obtained purely from the model's constraints and objective. As discussed in Section 5.6.3, a perfect warm start incumbent shifted the solver's workload primarily to lower bound improvement, or optimality verification, rather than incumbent improvement. In the static empirical evaluations, the upper bound is fixed by an optimal incumbent, and further effort is devoted to improving the lower bound. In RH practical applications, there will be room for upper bound improvement, and the small $\sim 5\%$ initial gaps observed in Table 15 will increase, since the objective function value of t, referred to as O, will land between this the LB from its root LP relaxation, and the suboptimal accepted incumbent from $t-1$, referred to as U. This is captured in

$$LB \leq O \leq U \tag{11}$$

A high quality, but imperfect incumbent can, once accepted and feasible, trigger primal heuristics in the search for better feasible solutions to close the gap observed from the latter inequality in Equation 11. The primal heuristics relaxation-induced Neighborhood search (RINS) requires an incumbent and becomes available [5].

5.8.2 Feasibility Requirements

As discussed in Section 5.8.1, O needs to be found between LB and U. If an incumbent with costs U is provided, MILP solvers apply a global cutoff: implicit constraints prevent the solver from exploring nodes where $LB \geq U$. As a consequence, the solver immediately performs strong branching or cutting with knowledge of the upper bound U for the objective function. Furthermore, the objective cut can be installed, in the form of the

constraint $O \leq U$ [97].
To benefit from warm start strategies, U needs to be a feasible incumbent. This is why partial warm starts are considered to derive initial feasible incumbents. A fixed-LP test fixes the binary variables of the problem with the submitted binary values and solves the LP to complete the continuous variables [37]. An elastic safety net in the form of slack variables with large penalties can be included so that feasibility of the incumbent is always guaranteed [99].

5.8.3 Effort Levels and Stability Strategies

In Energy Management System (EMS) optimization, multiple solutions often exist with nearly identical costs. However, these solutions can differ substantially in operational profiles, such as charging and discharging patterns, ramping decisions, or investment strategies. Warm start strategies provide a mechanism to favor consistency across runs by preventing the solver from deviating toward vastly different but equally optimal solutions. This consistency not only avoids undesirable oscillations but also reduces transition costs, for example those related to ramping, startups, or shutdowns.
With a partial warm start that proposes incumbents derived from a selection of variables, you can build in feasibility checks before choosing to adopt the incumbent. With Warm Start Level 2, an LP is solved, where the discrete variables are fixed by the provided solution. If it is feasible, this solution will serve as the incumbent for the rest of the run, as discussed in Section 5.8.2 [5].
More advanced stability strategies that avoid oscillations in RH solutions contain a warm fix and trust region. Warm fix for scheduling purposes involves setting the first steps of your optimization to the solution previously obtained such that only beyond that window the solution can diverge. This strategy not only improves continuity, but also leaves fewer free binaries in the MILP and, therefore, reduces computational complexity [57]. Another approach is to constrain the level of deviation of the new solution from the old by specifying a trust region. An example of a trust region is a percentage of binary decisions that needs to remain unchanged [37]. Stability can also by preserved by penalizing deviation in the objective function. A complex but very effective method is to carry over cuts and reuse the branch-and-bound information of previous runs between separate solves [42] [46]. This method re-optimizes branch-and-bound, using the reduced feasible region or final search frontier of the preceding run. For nearly identical consecutive runs, the reuse of the search tree and dual bounds of the search tree can greatly improve solver performance [57].

5.8.4 Resolution and Recency Stability

The higher the resolution, the smaller the difference between consecutive optimal solutions. The stability or rate of change between consecutive runs is often measured in the number of changes in binary decisions between runs. Successive optimal schedules that differ only marginally illustrate the concept of recency stability.. In high-resolution RH applications, [57] states that warm start methods eventually make the solver choose the same integer decisions as before, except for the last time step added at the end.

5.9 Cutting Planes

5.9.1 Effectiveness

The addition of cutting planes is a very useful way to increase the Best Bound and remove suboptimal branches of the search tree [61]. However, each cut is an additional constraint that makes the LP-relaxation larger and potentially harder to solve. Hence, the more cuts added, the larger the inherent matrix, possibly increasing the processing time at the nodes. Generating too many cutting planes that barely tighten the relaxation can lead to slower convergence, hence the solver must balance cut effectiveness and computational complexity [89]. Moreover, different classes of cuts target different structures of the problem, making their usefulness strongly depend on the problem [98].

5.9.2 Impact of Warm Starts, Horizons, and Scenario

Providing an incumbent objective value helps the solver to prune portions of the search tree in an early stage. Any node with a worse LP-relaxation than the incumbent, where $LB > U$ (see Section 5.8.1), can be pruned without branching [5]. Furthermore, both the structure of the problem and the suggested solution greatly influence the underlying solver algorithm.
The manner in which a MILP instance is expressed greatly affects the optimal basis returned by the LP solver, guiding further cutting-plane generation, primal heuristics, and branching. The selection of the optimal basis, even the first one within the optimal face of the very first LP relaxation, appears to be a crucial decision for the evolution of the whole MIP enumeration. [36] proposes an algorithm that samples the optimal face of the initial LP relaxation, and for each of the samples, executes the solver's default cutting plane loop and applies the default primal heuristics. Afterwards, for every different initial optimal basis, cutting planes and feasible solutions are collected and used as input for a final run. The algorithm greatly reduced the variability of the root node, execution time, and the number of branch-and-bound nodes and is adopted by CPLEX.
Larger optimization windows imply larger MILPS with more variables and constraints, leading to weaker initial LP-relaxations. Thus, there are more degrees of freedom to tighten, promoting increased cut generation. Alternating scenario input data, however, can also drastically guide cut generation in other directions. The same model structure with different data can be easy or hard to solve, depending on how the data influence the tightness of the formulation [36]. Machine Learning approaches improve state based node selection within a branch-and-bound tree. [100] applies a method that considers multiple state-action pairs that lead to good solutions, instead of only the top solution. Moreover, the algorithm chooses only to discover only the node's children, instead of the entire sub-tree below the child. These contributions respectively help the solver in a deep learning context, and to find solutions quickly.

5.9.3 Empirical Analysis

Tables 14 and 15 report the total number of cuts generated in each scenario and demonstrate varying amounts for the different classes of cuts across the scenarios. The differences in the types and numbers of cuts generated in the scenarios are discussed.
As discussed in Section 5.9.2, a strong LP-relaxation for a short horizon is confirmed in Scenario 4. Furthermore, scenarios 1 and 4, optimized over a window of 36 hours, generated significantly less cuts in total compared with the 48 and 50 horizon instances, except for scenario 2. This can be explained by the difference in supplied scenario data, in this case negative day-ahead prices, influencing unit commitment and binary decision patterns. As observed, this can lead to fewer fractional values and *less* cutting planes over a *longer* horizon.
The substantial choice for cover cuts, implied bound cuts, and mixed integer rounding cut in EMS optimization, can be explained as follows, where definitions and applications comply with [98], [96], [10].

- Cover cuts strengthen the LP relaxation in knapsack-like constraints by eliminating infeasible subsets of binary variables whose summed contributions exceed capacity. These are common in scheduling and commitment problems, making them relevant to treat capacity-related constraints in EMS models.

- Implied Bound cuts reflect relationships between constraints, linking binary and continuous variables by enforcing implications. The model has over 18000 indicator constraints, that generate many implied bound inequalities. The subject of indicator constraints will be treated in more detail in Section 6.

- Mixed Integer Rounding (MIR) cuts are highly effective for tightening mixed-integer knapsack constraints. CPLEX contains a parameter specifying if and to what extent the user wants the solver to generate MIR cuts. The increased generation of MIR Cuts for warm start strategies across all scenarios accept scenario 1 motivates to investigate alternative efforts, potentially improving efficiency.

6 Linearity and Reformulations

In the context of Mixed Integer optimization models, the linear programming formulation or LP-formulation, arises from relaxing the integral values. The so-called LP-relaxation of the problem has polyhedral properties. That is, the feasible region or set of variables that satisfies the relaxed constraints is a polytope and the optimal solution always lies at a vertex. In some problems, absolute value expressions require workarounds to define the feasible region of a MILP. The statements in Section 6.1 up to 6.3 follow [16], [12] and [13].

6.1 Absolute Value Expressions

Implementing absolute values of decision variables removes the linearity of your model. Requiring an absolute value forces positive values, creating a V shape in previously linear graphs that had a constant rate of change. The handling of absolute values in (MI)LP formulations requires the introduction of auxiliary binary variables and indicator constraints, which is done as follows:
Consider the absolute value of variable x,

$$|x| = \begin{cases} -x, & \text{if } x < 0 \\ x, & \text{if } x \geq 0 \end{cases} \tag{12}$$

When dealing with constraints involving absolute value expressions, such as $|X| \leq C$ with $C \geq 0$, the inequality can be reformulated as a system of linear constraints

$$X \leq C \tag{13}$$
$$-X \leq C \tag{14}$$

creating a convex feasible region $[-C, C]$. However, in order to satisfy the constraint $|X| \geq C$ with $C > 0$, X must satisfy one of the following equations.

$$X \geq C \tag{15}$$
$$-X \geq C \tag{16}$$

which results in a non-convex feasible region $(-\infty, -C] \cup [C, \infty)$, where non-convexity is caused by the gap between $-C$ and C. X cannot satisfy Equations 15 and 16 at the same time if $C > 0$, which makes it impossible to reformulate constraints of the form $|X| \geq C$ to a linear equivalent. This could be reformulated as that only one of the Equations 15 and 16 can be *active* at a given time. The same holds for equality constraints $|X| = C$, where the feasible region $\{C, -C\}$ is also non-convex and the solution needs to be approximated using disjunctions.
The introduction of indicator constraints is used to handle absolute values in MILP. Indicator constraints enable the expression of absolute values by identifying a binary variable b and a large constant M. Equations 15 and 16 are reformulated as

$$X + Mb \geq C \tag{17a}$$

$$-X + M(1 - b) \geq C \tag{17b}$$

$$b \in \{0, 1\} \tag{17c}$$

The introduction of indicator constraints enables the user to reformulate and express particular modeling constructs, such as absolute value expressions, in MILP applications. However, it comes with the introduction of binary variables and indicator constraints, increasing the computational complexity of models.

6.2 Global and Local Optima

As concluded from Table 19, both the linear and non-linear problem formulation provide the same, global optimal objective function value. This touches on a critical aspect in the MILP and MINLP optimization domain, namely the ability to guarantee globally optimal solutions. In convex optimization, where the feasible region is convex, every local optimum is also a global optimum. This guarantee is broken once non-convex constraints , like $|X| \geq C$ as discussed in Section 6.1, are introduced. Other examples of non-convex constraints include constraints that multiply decision variables, called bilinear terms, or piecewise constraints. Introducing constraints that lead to non-convexity—such as those creating multiple disjoint or curved feasible regions—can prevent local-search-based solvers from finding the global optimum. The Branch and Bound method used by CPLEX, however, always guarantees to find a global optimum, which can be explained by the process of how solutions are treated, as presented in Section 5.6.1. The algorithm recursively examines each branch and the final solution is guaranteed to be globally optimal, because no better solution in any part of the feasible region can exist.
As demonstrated in Section 6.3, non-convex formulations can require workarounds and reformulations that place considerable demands on the solver, resulting in large-scale models and extended run times.

6.3 Empirical Validation of Efficient Formulation

We investigate a Unit Commitment Energy problem in which absolute value expressions arise. The Section explores their linear reformulation and evaluates the efficiency of different formulations in terms of performance and structure.

6.3.1 Battery Cycles in Energy Optimization

Battery degradation has a large economic impact in energy applications. The degradation of batteries therefore constraints Unit Commitment or Dispatch Scheduling problems. The degradation is quantified and monitored with factors including state of charge, ramp rates, and maximum daily number of cycles. The lifetime of Battery Energy Storage Systems is limited by the maximum number of cycles it can perform. Short-term quantification and control of degradation can have a large impact on long-term profits and results [40].

A battery cycle is defined as one full charge and one full discharge of the battery. To quantify cycles partially, industries and research refer to the concept of Equivalent Full Cycles (EFC) [86]. An EFC equals the energy throughput of a BESS relative to its total available capacity. For example, charging and discharging half of the available capacity twice uses ~1 EFC. Mathematically, this results in the following variable definitions and constraints in a MILP energy optimization

$\forall t \in T \quad \forall s \in S$

$$\Delta E_{s,t} = (Charge_{s,t} - Discharge_{s,t}) * StepInHours \tag{18a}$$

$$CycleContribution_{s,t} = \frac{|\Delta E_{s,t}|}{2E_{max_s}} \quad \forall E_{max_s} \neq 0 \tag{18b}$$

$$E_{s,t}, \quad CycleContribution_{s,t} \in \mathbb{R} \tag{18c}$$

$$Charge_{s,t}, \quad Discharge_{s,t} \in \mathbb{R}_+ \tag{18d}$$

$$E_{max_s} \in \mathbb{R} \tag{18e}$$

$$StepInHours \in \mathbb{R} \tag{18f}$$

We obtain a factor 2 in the denominator to secure that we need two full transfers of E_{max_s} to obtain a cycle. The transferred charged or discharged energy hence contributes to half cycles.

Battery operational constraints prevent BESSs from both charging and discharging at a given time step t. This is captured in the following system of constraints:

$\forall t \in T \quad \forall s \in S$

$$Charge_{s,t} \leq Charge_{max_{s,t}} * IsCharging_{s,t} \tag{19a}$$

$$Charge_{s,t} \geq Charge_{min_{s,t}} * IsCharging_{s,t} \tag{19b}$$

$$Disharge_{s,t} \leq Discharge_{max_{s,t}} * IsDischarging_{s,t} \tag{19c}$$

$$Disharge_{s,t} \geq Discharge_{min_{s,t}} * IsDischarging_{s,t} \tag{19d}$$

$$IsCharging_{s,t} + IsDischarging_{s,t} \leq 1 \tag{19e}$$

$$\{Charge_{s,t}, \quad Discharge_{s,t}\} \subset \mathbb{R}_+ \tag{19f}$$

$$\{IsCharging_{s,t}, \quad IsDischarging_{s,t}\} \subset \{0, 1\} \tag{19g}$$

From the systems of equations 18 and 19, we observe that

$\forall t \in T \quad \forall s \in S$

$$|\Delta E_{s,t}| = (Charge_{s,t} + Discharge_{s,t}) * StepInHours \tag{20}$$

and conclude

$\forall t \in T \quad \forall s \in S$

$$CycleContribution_{s,t} = \frac{(Charge_{s,t} + Discharge_{s,t}) * StepInHours}{2E_{max_s}} \quad \forall E_{max_s} \neq 0 \quad (21)$$

Notation	Description
Set	
S	set of electrical storage assets
T	set of decision steps in the optimization window
Decision variables	
$E_{s,t}$	Incremental energy charge of storage asset $s \in S$ at decision step $t \in T$ expressed in kWh
$CycleContribution_{s,t}$	The number of cycles performed by storage asset $s \in S$ at decision step $t \in T$
$Charge_{s,t}$	Charged power of storage asset $s \in S$ at decision step $t \in T$ expressed in kW
$Discharge_{s,t}$	Discharged power of storage asset $s \in S$ at decision step $t \in T$ expressed in kW
$IsCharging_{s,t}$	Flag indicating whether storage asset $s \in S$ is charging (1) or not (0)at decision step $t \in T$
$IsDischarging_{s,t}$	Flag indicating whether storage asset $s \in S$ is discharging (1) or not (0) at decision step $t \in T$
Input parameters	
E_{max_s}	Maximal nominal energy capacity of storage asset $s \in S$ expressed in kWh
$StepInHours$	Optimization step duration (15 minutes) expressed in hours (0.25)

Table 17: Notation and Description of Elements in Equations 18 up to 21

6.3.2 Solver Behavior Comparison

IBM ILOG CPLEX Optimization Studio has an *abs* function and, therefore, allows absolute value expressions of decision variables. The solver linearizes them by introducing indicator constraints, as discussed in Section 6.1. As discussed in Section 6.3.1, multiple formulations are possible to define the EFC-constraint of a BESS. To analyze the impact of the constraint formulation, different Branch and Cut proceedings are compared using the absolute value formulation of the Cycle Contribution Constraint in Equation 18b and the linear formulation in Equation 21, respectively.
The Energy Management System described in Section 4 serves as the basis for the comparison of the Cycling Constraint formulation for 3 scenarios with shared properties. The scenarios are stripped from aFRR engagements to authorize the usage of the BESS, as explained in Section 5.2. Furthermore, it is shown that negative prices push storage capacities to their limits. Thus, scenarios with positive Day-Ahead prices are compared, such that potential differences in Branch and Bound proceedings between scenarios cannot be related to negative prices. The pre- and post-solve data is presented in Table 18 and Table 19, respectively. The CPLEX execution logs supporting the reported data can be found in Appendix B.2.
We observe an impressive reduction in the size of the problem in the linear formulation in Table 18. As discussed in Section 6.1, CPLEX no longer introduces auxiliary variables and indicator constraints presented in Equation 17 to approach the cycle contribution. On average in the scenarios, as a consequence, the solver produces ~ 71 times fewer indicator constraints in the linear formulation. The rest of the counts together reflect an inflation by a factor ~ 9.7 of structural elements such as rows, columns and binaries across all scenarios in the case of the non-linear formulation. The notion of activation indicates that the indicator constraints inequalities in the non-linear formulation are not all valid. The high number of integer and binary variables that do not all represent

	Scenario	Horizon 50 hours	Horizon 48 hours	Horizon 36 hours
Count				
Rows	Non-linear (abs) formulation	64664	61274	41664
	Linear formulation	5921	5610	4481
Columns	Non-linear (abs) formulation	65095	61657	41835
	Linear formulation	6817	6458	5087
Binary Variables	Non-linear (abs) formulation	16382	15531	10576
	Linear formulation	1742	1659	1312
Indicator constraints	Non-linear (abs) formulation	29678	28124	18816
	Linear formulation	398	380	288

Table 18: Comparison of Pre-Solve Data

valid inequalities in the non-linear formulation results in a dense constraint matrix. This gives the LP relaxation more chances to produce non-integral solutions, increasing the integrality gap. Therefore, it is harder for the LP relaxation to closely approximate the integer feasible region, which weakens the relaxation. Consequently, the solver needs to add valid inequalities, or cuts, to remove fractional solutions and the need for extensive branching of these nodes.

	Scenario	Horizon 50 hours	Horizon 48 hours	Horizon 36 hours
Count				
Cover Cuts	Non-linear (abs) formulation	5876	6932	2290
	Linear formulation	60	97	72
Implied Bound Cuts	Non-linear (abs) formulation	8475	5353	4089
	Linear formulation	228	89	203
Flow Cuts	Non-linear (abs) formulation	1707	1764	2179
	Linear formulation	352	260	183
Mixed Integer Rounding Cuts	Non-linear (abs) formulation	3849	1725	1165
	Linear formulation	259	307	215
Objective Value	Non-linear (abs) formulation	-2605.93	-1305.63	-2848.62
	Linear formulation	-2605.93	-1305.63	-2848.62
Total Solve time	Non-linear (abs) formulation	207.89 sec	420.75 sec	39.41 sec
	Linear formulation	12.47 sec	18.64 sec	5.27 sec

Table 19: Comparison of After-Solve Data

In Table 19, we observe a consistent and significant reduction in total runtime when using the linear formulation of the cycling constraint compared to the formulation using absolute value expressions. This difference can be attributed to the weaker LP relaxation of the absolute value formulation. In the non-linear case, absolute value constraints are typically modeled via indicator constraints and auxiliary binary variables, resulting in a non-convex feasible region, as explained in Section 6.1. When the problem is relaxed, the integer restrictions are temporarily ignored and the LP solver allows artificial solutions that interpolate between disjunctive branches of the model, such as values that lie between the positive and negative branches of the absolute value. As stated before, these do not correspond to feasible integer solutions and as a result, the LP relaxation is weak, the integrality gap is large, and the solver must perform extensive cutting to converge to an optimal integer solution. Table 19 confirms this reasoning, showing that the nonlinear formulation yields, on average, ~ 19.5 times more cuts across all scenarios.
By contrast, Equation 21 in the linear formulation provides a tight LP relaxation. It defines a hyperplane that lies on a facet of the feasible polytope of the LP relaxation,

forming part its boundary. This means that the constraint directly bounds the solution space in a way that is both valid and tight for the integer model. As a result, there is no need to simulate disjunctive behavior using binary variables and indicator logic as the LP relaxation already captures the structure of the solution space well. The linear formulation converges ~ 18.4 times faster on average over the scenarios, corresponding to a 94.57% decrease in running time.

6.4 Warm Start Strategies for Linear Configuration

A larger modeling horizon of 72 hours is considered to potentially observe a significant impact on running time when applying warm start strategies, now that the running times for smaller modeling horizons have considerably decreased. The results of the deployment of warm start strategies in the reformulated problem are presented in Table 20. The extended horizon leaves more room for the LP-relaxation to explore low costs across numerous data points, causing a large initial gap.
As discussed in Section 5.7.1, closing a large initial solution gap, following the steps in Section 5.6.3, is costly. It is observed in Tables 15 (scenario 2) and 20 that proving optimality of a high quality incumbent, provided from a warm start, starting from a low quality LB, results in an increased running time. In case of a warm start, a shift of focus of the solver to repairing a weak initial LP-relaxation without using primal search heuristics occurs. This shift results in an increased amount of generated Cover cuts. This corresponds to the description of the function of Cover cuts in Section 5.9.3, and can equally be obtained in Tables 15 (scenario 2) and 20

6.4.1 Log data

Strategy	BL	WSA (0)	WSD (2)
Data			
Initial Gap (%)	100	14.36	14.36
Maximal Reported Gap* (%)	150.15	14.36	14.36
Running Time** (%)	235.75 s	+58.18	+65.91
Clique Cuts	1	5	4
Cover Cuts	67	134	153
Implied Bound Cuts	148	77	102
Flow Cuts	670	459	449
Mixed Integer Rounding Cuts	733	564	519
Total Cut Count	2084	1475	1442

Table 20: Log Data of Warm Start Strategies for Scenario with a Modeling Horizon of 72 Hours
* During incumbent finding, the Gap can deviate from 100%
** relative difference compared with baseline of scenario

7 Model Conversion and Benders Decomposition

7.1 Conversion to Python

In order to access parameters defined for the Python API of CPLEX, the model described in Section 4 is converted to python, using the *docplex* package [10]. The usage of the package is presented in Appendix C.1.

7.2 Model Without Violations

In the converted model, the violation decision variables are not concluded, as it is observed in Section 5.3 that their removal caused running time reduction of 33.63% compared to the model with violations. Their removal is safe regarding feasibility, since their optimal solution value was zero for all tested scenarios. Tables 21, 22 and 23 respectively show the comparison of the model sizes, the log data, and the scenario descriptions for the models with- and without violations across different scenarios.

	Configuration		With Violations	Without Violations
S	**Data**			
1	Presolve Count	Row Elimination	19122	20975
		Column Elimination	21449	15128
		Coefficient Modification	4	517
	Reduced MIP Count	Rows	61275	59807
		Columns	61658	59143
		Nonzeros	169076	164232
2	Presolve Count	Row Elimination	13809	17067
		Column Elimination	15150	14101
		Coefficient Modification	18	834
	Reduced MIP Count	Rows	4481	3196
		Columns	5087	2665
		Nonzeros	104070	18927
3	Presolve Count	Row Elimination	36075	42148
		Column Elimination	39614	34451
		Coefficient Modification	18	1645
	Reduced MIP Count	Rows	12486	9548
		Columns	14187	8024
		Nonzeros	104070	60069
4	Presolve Count	Row Elimination	26830	31477
		Column Elimination	29591	25763
		Coefficient Modification	4	1116
	Reduced MIP Count	Rows	9462	7264
		Columns	10748	6100
		Nonzeros	76247	44400

Table 21: Comparison of Log Pre-Solve Data of Different Scenarios of Models with and without Violation Decision Variables

	Configuration	With Violations	Without Violations
Data	**Scenario**		
Maximal Reported Gap * (%)	1	115.04	171.23
	2	109.77	435.00
	3	391.77	264.80
	4	150.15	159.24
Running Time** (%)	1	413.50 s	−33.63
	2	5.27 s	−74.76
	3	2419.20 s	−45.89
	4	235.75 s	+35.70
Clique Cuts	1	120	183
	2	N.A.	15
	3	N.A	40
	4	1	3
Cover Cuts	1	5683	3234
	2	72	N.A.
	3	18	6
	4	67	5
Implied Bound Cuts	1	5004	3325
	2	203	47
	3	86	191
	4	148	49
Mixed Integer Rounding Cuts	1	2060	511
	2	215	153
	3	339	1044
	4	959	425
Total Cut Count	1	14578	7846
	2	732	404
	3	1155	2295
	4	2084	1292

Table 22: Comparison of Logs of Different Scenarios of Models with and without Violation Decision Variables

* During incumbent finding, the Gap can deviate from 100%

** Relative difference compared with model with violations

Scenario	Description
1	EMS instance from Section 5.4, with a horizon of 48 hours that solves the non-linear model configuration, without bounds on SOC limit violation decision variables. The scenario contains negative Day Ahead prices, and no aFRR engagements.
2	EMS instance from Section 6, with a horizon of 36 hours that solves the linear model configuration, with bounds on SOC limit violation decision variables. The scenario contains strictly positive Day Ahead prices, and no aFRR engagements.
3	EMS instance from Section 6, with a horizon of 96 hours that solves the linear model configuration, with bounds on SOC limit violation decision variables. The scenario contains strictly positive Day Ahead prices, and no aFRR engagements.
4	EMS instance from Section 6, with a horizon of 72 hours that solves the linear model configuration, with bounds on SOC limit violation decision variables. The scenario contains strictly positive Day Ahead prices, and no aFRR engagements.

Table 23: Specification of Scenarios in Tables 21 and 15. None of the tested scenarios contain aFRR engagements.

Table 21 indicates that removing violation decision variables from your model, reduces the number of rows, columns, and nonzeros in the reduced MIP, thereby decreasing the overall model size. The absence of violation decision variables also triggers the modification of coefficients during Presolve. The number of eliminated rows and columns during Presolve, however, remains of the same order of magnitude for both models with and without violations.

Table 22 shows that the removal of violation variables improves runtime performance in three out of four scenarios. The increased convergence time observed for the model with violations in scenario 4 is difficult to attribute to model size, presolve activity, reported MIP gaps, or cutting-plane generation, as its behavior is otherwise consistent with the other scenarios. Notably, this specific instance with a horizon of 72 hours converges very quickly in the model with violations. Even with an increase of 35.60%, the model still converges rapidly given the instance size.

7.3 Empirical Study of Benders Decomposition

7.3.1 Algorithmic Structure

Section 2.5 describes how Benders Algorithm can strongly accelerate models with both continuous and discrete variables. Benders Decomposition separates the problem into a master and one or more subproblems. The Master problem solves the complicating variables, supported by the idea that if these will be fixed, the problem would be easier to solve. In a MILP EMS optimization, the discrete decisions contain dispatch decisions like *GeneratorOn/Off, Is(dis)Charging* and market engagements (*DayAheadOffer*), and therefore offer valuable insights for the optimization. The continuous variables represent power- and heat flows and energy levels of the assets. Benders Decomposition isolates the discrete variables in the master problem and the continuous variables in a subproblem and generates Benders Cuts to link them.

7.3.2 Implementation and Results

The Python API allows to define models and use the CPLEX solver in a Python environment using the package *docplex*. The package allows you to annotate your decision variables with 0 for the variables in the Master Problem, and 1, 2, 3,... according to the subproblem division, where the variables in the same subproblem should get equal annotation. Variables that participate in the same constraint should be annotated to the same subproblem. Once the model is annotated, a Benders Strategy could be defined, that will be treated according to Table 24. Table 25 presents the results of the deployment of Benders Algorithm with strategies 1, 2, and 3 for a scenario with a horizon of seven days. Consequently, Table 26 presents the results of the deployment of Benders Algorithm with strategies -1, 1, and 2 for a scenario with a horizon of four days. The Execution Log of one of the runs is presented in Appendix C.2.

Strategy	**Name**	**Effect**
-1	OFF	Ignores Benders annotations and executes conventional Branch and Bound
0	AUTO	CPLEX uses available annotations of the master problem and attempts to partition the subproblems further before applying Benders to solve the model.
1	USER	Benders algorithm is applied according to the annotations specified by the user.
2	WORKERS	CPLEX accepts the master as given and attempts to decompose the remaining elements into disjoint subproblems to assign to workers before solving with Benders.
3	FULL	CPLEX ignores annotations and performs presolve. Subsequently the solver atuomatically generates a Benders partition with integer variables in the master and continuous linear variables into disjoint subproblems. If the problem contains either only or no integers, CPLEX reports an error.

Table 24: Benders Strategies and their Effect [14]

Strategy	1⋆	2⋆⋆	3⋆⋆
Data			
Reduced MIP Model Size			
Rows	20179	20179	20179
Columns	16880	16880	16880
Nonzeros	118051	118051	118051
Binaries	5122	5122	5122
First cycle			
Initial Best Integer	9917.50	9916.82	9917.89
Final Best Integer	9917.50	9916.82	9917.89
Initial Best Bound	-10000.00	-10000.00	-10000.00
Final Best Bound	-5928.89	-5949.62	-4195.38
Iteration Count	25541	29033	30346
Final Gap (%)	159.80	159.98	142.18
Elapsed time	82.00	74.17	76.27
Termination			
Total Tree Size	54612	42278	41091
Best Integer	-561.13	-1528.18	-1400.78
Best Bound	-4206.91	-4087.34	-3974.35
Final gap (%)	649.73	167.47	193.05
Iteration Count	1574465	1421717	902192
Benders Cuts Count	2325	N.A.⋆	N.A.⋆
Other Cuts Count	63	N.A.⋆	N.A.⋆

Table 25: Log Data of Benders Strategies for Scenario with a Modeling Horizon of 168 Hours

⋆ Terminated after $\sim 1230s$

⋆⋆ Terminated with exit code -1 after $\sim 1000s$

Strategy	-1	1*	2**
Data			
Reduced MIP Model Size			
Rows	9548	11209	11209
Columns	8024	9394	9394
Nonzeros	60069	64289	64289
Binaries	2879	2880	2880
First cycle			
Initial Best Integer	6068.07	6637.67	6638.50
Final Best Integer	-2410.66	6637.67	6638.50
Initial Best Bound	-10000.00	-10000.00	-10000.00
Final Best Bound	-2675.68	-3137.76	-3061.74
Iteration Count	25541	26776	23140
Final Gap (%)	10.99	147.27	146.12
Elapsed time	5.55	20.88	17.91
Termination			
Total Tree Size	674900	22115	77072
Best Integer	-2602.12	-1125.67	-1643.02
Best Bound	-2602.55	-3115.91	-3041.18
Final gap (%)	0.02	176.81	85.10
Iteration Count	12269594	382916	1454124
Benders Cuts Count	N.A.	1607	3984
Other Cuts Count	2295	48	32
Solve Time	1308.92	N.A.	N. A.

Table 26: Log Data of Benders Strategies for Scenario with a Modeling Horizon of 96 Hours

* Terminated with exit code 0 after 213s

** Terminated with exit code 0 after time limit 1800s

7.3.3 Discussion of Results

The MILP representing the EMS described in Section 4 includes 18 discrete and 83 continuous decision variables, all stored in dictionaries. For most of these variables, four target values are determined each hour, leading to a substantial overall variable count for the solver, as observed in the model sizes in Tables 25 and 26.

Note that the gap increases while the algorithm proceeds, as the gap represents the difference of the best bound relative to the incumbent. The absolute difference between the Best Integer and the Best Bound, decreases. An identical initial Best Bound is observed across all strategies, since the same initial LP relaxation is solved, regardless of whether Strategy -1 (conventional Branch and Cut) or a Benders Strategy is chosen. Later bound improvements in Benders strongly depend on the strength of the master problem formulation. Weak masters and subsequent Benders cuts that do not constrain the feasible space sufficiently lead to poor progress unless continuous components are integrated more tightly [70]. Continuous variables can be integrated by adding constraints that connect discrete and continuous variables, for example.

The constraint coupling inherent to energy optimization models is weakly represented by Benders cuts. As discussed in Sections 5.6.1, 5.8.1 and 5.9, standard Branch and Cut exploits cross-variable interactions through global cuts and heuristics. Benders algorithm does not succeed in linking the Master and Subproblem in the same profound way, and needs many Benders cuts, without reaching the same pace of convergence as conventional Branch and Cut. Benders decomposition tend to perform poorly in tightly coupled MILPs, since cuts generated from subproblems fail to capture cross-variable dependencies. The art is to integrate enough continuous relaxations to strengthen the master, and to avoid excessive interdependencies among continuous variables that result in over-coupled decompositions [71].

It is observed that the incumbent does not improve in early iterations when Benders strategies are applied. Benders strategies mainly improve the lower bound, but do not contribute to finding good primal feasible solutions. Strategy -1 in Table 26, however, benefits from the primal heuristics mentioned in Sections 5.6.1 and 5.6.3 in generating strong incumbents. Enhancing Benders with advanced techniques, such as Pareto-optimal cuts or stabilization methods, can mitigate this drawback [48] [64]. Hybrid strategies combining primal heuristics and cut generation have also been proposed to accelerate convergence [38]. In both the four- and seven-day scenario, Strategy 2 provides the best feasible integer solution in the given time, out of the tested Benders strategies. This can be related to the strategy's feature described Table 24, that assigns disjoint subproblems to workers, reducing cut generation overhead. This hybrid approach of decomposition and parallel subproblem solving allows Strategy 2 to achieve better dual bounds while efficiently identifying feasible integer solutions.

It is concluded that the success of Benders is highly sensitive to decomposition quality, cut strength and solver integration.

8 Challenges and Sugggestions for Future Research

The results presented throughout this research inspire to explore more problem formulations, methods, strategies, and scenarios.

8.1 Custom Environments for Warm Start Strategies

The extensive testing of warm start strategies in a wide range of environments and scenarios reveals great potential, once applied in the right context. EMSs are known to demonstrate complex behavior as many combinations of input data are possible. The variability can be thought of as its strength that, provided an efficient formulation for a specific EMS, a warm start in the right configuration greatly improves solver performance. Choosing the Effort Level however, is delicate as we observe in Tables 12 and 15, where warm starts are rejected by CPLEX. Other solvers like Gurobi can be considered in such cases to execute fully supported warm start strategies.
The beneficial configurations of initialization strategies and problem formulations within the scope of this research motivate investigating conditional guidance of the solver towards the most beneficial strategy, based on the provided scenario input data. By exploiting past experience across scenarios, horizons, and environments, the solver can be guided to decide whether a warm start is likely to be beneficial or whether an alternative initialization should be preferred. Future research could further develop and test such conditional warm start mechanisms, particularly those leveraging machine learning to recognize patterns of success and dynamically adapt initialization strategies to the given EMS context. Repeated large-scale testing across a broader set of scenarios could provide valuable insights into consistency, guidelines, and general principles in RH MILP energy optimization field.
As discussed in Section 5.9, the type and quantity of cuts are closely related to performance and solvers continuously improve their effective cutting strategies. The variability of input data can be used to guide the solver in a favorable direction at an early stage. Supervised learning strategies for node selection could, for example, prevent extensive branching in suboptimal directions of large branch-and-bound search trees caused by large amounts of fractional variables. Observations in Section 5.9.3, however, inspire to avoid such extensive sampling for scenarios with few fractional variables, as a consequence of the provided input data. Root LP-relaxation sampling strategies and the usage of machine learning can greatly improve node selection guiding the solver into the efficient generation of cutting planes.

8.2 Rolling Horizon

Feasibility conditions play a central role in the rolling horizon application of warm start strategies, as discussed in Section 5.8.2. Robust warm starts in a rolling horizon environment require careful testing of the proposed initialization methods. Real-time evaluation will help identify the challenges and opportunities associated with providing (parts of) the solution from $t-1$. At the solver-specific level, actions could be informed by prior knowledge or learning-based methods, guiding the solver towards the most effective initialization strategy in light of the incumbent from $t-1$ and the scenario input data. Such mechanisms may include conditional rules or loop structures that, for example, assess whether the incumbent from $t-1$, when used as a warm start at t, produces an acceptable initial gap relative to the LP relaxation of t. If the initial gap violates a predefined limit, the solver could discard or adjust the warm start and instead rely on violation variables or alternative initialization strategies providing a faster route to feasibility and optimality. In addition, varying temporal horizons may be considered for industrial processes or assets that are constrained on different temporal bases. The conditional mechanisms suggested in Section 8.1 can be used to decide the most efficient temporal resolutions, given the provided input data.

8.3 Problem Formulation and Algorithm Design

Problem formulations were consistently observed to have a decisive impact on solver performance. Furthermore, the (non-)beneficial applications of warm start strategies pointed towards opportunities for more efficient formulations. For instance, Table 15 shows a high initial gap for scenario 2, once provided with a warm start of the discrete decision variables on Effort Level 2. This observation suggests that the problem should be bounded more tightly using valid inequalities or reformulations that restrict infeasible fractional solutions. This would narrow the search space for unrealistically low costs causing the weak Lower Bound. Reformulation aims to improve the LB in scenarios in which discrete decisions are monotone, such as the total blockage of assets in case of negative DA prices and aFRR engagements. In general, tighter bounding is expected to improve initial bounds across a wide range of EMS scenarios, and more specifically to reduce initial gaps when warm starts provide a high-quality incumbent.
Benders decomposition is most effective when the master problem provides strong guidance to the subproblems. However, when the master is too weak, either due to a high number of complicating variables or excessive interdependencies between constraints, generated cuts carry limited information. In such cases, convergence can be slow and the benefits of decomposition weaken compared to solving the full MILP directly. Alternative annotations could improve the guidance level of the master problem. Similar to warm start strategies for certain Effort Levels, Benders strategies can be rejected in the *docplex* environment, and other solvers like Gurobi can be considered to implement custom Benders using callbacks, an approach that provides great control over the formulation. Hybrid formulations that combine Benders with Dantzig-Wolfe or Branch and Cut can be explored to leverage both primal heuristics and multiple types of cut generation.

9 Discussion and Conclusion

The contribution of this research lies in providing an extensive performance analysis of initialization strategies for EMS optimization across a broad range of scenarios. Numerous cycles of testing, reflection, and reformulation of warm start strategies have been performed throughout this study. Strategies can work accelerating or decelerating, depending on environmental conditions and problem formulations. The baseline comparisons, reformulations, and validation experiments confirmed both the potential benefit of a warm start strategy, once accurately targeted, and the importance of efficient problem formulation. The study systematically explored diverse problem formulations, from which key insights are derived. The conclusion of this work is structured into categories of problem formulations and initialization strategies that demonstrated the greatest impact on solver performance throughout the research. For each category, the key results are presented.

9.1 Market Engagement, DA Prices, and Horizons

EMS optimization problem instances vary greatly in degrees of freedom, as discussed in Sections 5.2, 5.5.2 and 5.9.2. Engagement in the aFRR market narrows the feasible solution region by deactivating a group of assets. A scenario with aFRR participation converged 48% faster than its equivalent without aFRR engagements, as a consequence of fewer degrees of freedom for the solver to search for feasible solutions. Moreover, it is observed throughout this research that DA price profiles have a great impact on solver performance. We observed in Table 15 for scenarios 2 and 3, identical except for their DA price profiles, that scenario 2 containing negative DA prices converged 38% faster than the same scenario with a strictly positive DA price profile. Across the tested cases with strictly positive DA price profiles, we observed an increase in cut generation for further modeling horizons. Note that augmented cut generation did not always imply slow convergence, as the 50 hours horizon scenario outperformed both scenarios with a horizon of 48 hours in terms of running time.

9.2 Violation Decision Variables

Sections 5.3 and 7.2 discussed the impact on solver performance and model robustness as a consequence of the inclusion of violation decision variables. Table 22 showed an acceleration of at least 34% when the violation decision variables were removed of the model in three out of four scenarios and a deceleration of 36% of the last scenario. Sections 5.5.1 and 5.5.2 showed a reduction in running time by 37% and 5%, respectively, once bounds for violation decision variables were provided.

9.3 Linearity

Solver dynamics concerning the treatment of nonlinearities was analyzed in Section 6 and reformulations were presented to avoid them. Reformulation accelerated convergence to the optimal solution, and the linear formulation converged 18 times faster on average over the tested scenarios, corresponding to a 95% decrease in running time. It is therefore advisable to eliminate nonlinearities where possible and apply linear reformulations, provided that solution quality is not compromised.

9.4 Benders Decomposition

Section 7 presented an empirical analysis and reflection of a Benders Decomposition. A Benders Decomposition was applied to the linear formulation of the model without violation decision variables. Across three tested Benders Strategies for the scenario with a seven-day horizon, Strategy 2 provided the best feasible integer solution in the given time frame. The four-day horizon scenario converged for conventional Branch and Cut (B&C) and obtained no benefits in terms of running time from adopting a Benders Decomposition with Benders Strategies 1 and 2. Across the tested Benders Strategies for this scenario, Strategy 2 again provided the best feasible integer solution in the given time frame, that deviated 163% from the optimal feasible solution obtained from B&C.

9.5 Warm Start Strategies and Outlook

Results of warm start strategies for different variable sets and Effort Levels for diverse scenarios, alternating market engagements, price profiles, and tightness of problem definitions were presented in Tables 5, 15, 20. For an optimization with a horizon of 48 hours and a scenario containing negative DA prices, the most effective warm start accelerated convergence by 27%. Across five different scenarios ((Table 15)), varying in horizon length, DA price profile, and environmental and system conditions, the warm start of all variables on Effort Level 0 and the warm start of all discrete variables on Effort Level 2 were tested. The latter strategy appeared to be beneficial in terms of running time for all scenarios with strictly positive DA prices. The most effective warm start of this comparison improved the running time by 54% by adopting a warm start of all variables on Effort Level 0. The success of the warm start strategy depended on the size and formulation of the problem, the specified Effort Level, and is scenario-specific for the optimization of Energy Management Systems. Also, note that for the same scenario, but with a different or more efficient problem formulation, warm starts may lose their effect. This was observed after adding bounds in Table 10, or in the linear formulation in Table 20.

The synthesis of the findings of this research into categorized takeaways, together with the proposed directions for future work in Section 8, provides guidance on efficient formulations that support effective initialization and enhance solver performance. The broad range of strategies, formulations, algorithms, and scenarios considered ensures that the results of this thesis remain relevant across diverse users and system configurations, thereby contributing to scalable and widely applicable optimization practices.

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A CPLEX OPL Codes

A.1 Orchestration to run warm start strategies

```
// Florine 04/06/2025 14:48

    main {

    var filename = "C:/Dev/ems-optimizer/optimizer/Data/windowsData/warmstart/MOPA3D - Copie.mst";
    var source = new IloOplModelSource("C:/Dev/ems-optimizer/optimizer/MicrogridOptimizer5.mod");
    var cplex = new IloCplex();
    var def;
    var opl;
    def = new IloOplModelDefinition(source);
    opl = new IloOplModel(def,cplex);
    var data = new IloOplDataSource("C:/Dev/ems-optimizer/optimizer/Data/windowsDatFiles/MicrogridOptimizer.dat");
    opl.addDataSource(data);
    opl.generate();
    cplex.bendersstrategy
    cplex.tilim = 30*60;
    cplex.epgap = 1.5E-4;
    cplex.WriteLevel = 1;
    cplex.readMIPStarts(filename); //comment if you want to run without warm start
    writeln("3D scenario afrr zero, mip start all variables on level 0");
    cplex.tuningdisplay = 3;
    cplex.solve();
    strObjVal = cplex.getObjValue().toString();
    strSolStatus = cplex.getCplexStatus().toString();
    writeln("OBJ = " + strObjVal);
    writeln("Status = "+ strSolStatus);
//      cplex.writeMIPStarts(filename); //comment if you don't want to write mst file
    opl.postProcess();
    cplex.clearModel();
    opl.end();
    data.end();
    def.end();
    cplex.end();
    source.end();
}
```

A.2 MST file format and Constraint Formulation

```xml
<?xml version = "1.0" encoding="UTF-8" standalone="yes"?>
<CPLEXSolutions version="1.2">
 <CPLEXSolution version="1.2">
  <header
    problemName="MicrogridOptimizer5"
    solutionName="m1"
    solutionIndex="0"
    MIPStartEffortLevel="0"
    writeLevel="1"/>
  <variables>
   <variable name="ImportTarget#0" index="0" value="0"/>
   <variable name="ImportTarget#1" index="1" value="0"/>
```

Is the mst file layout

Reformulations of constraints in CPLEX

```
//dexpr float CyclingStepContribution[s in isE_STORAGES][t in
isDECISION_STEPS] = (nomEnergyMax[s] > 0.0 ? abs(StorStepDCEnergyIn[s][t])
/ (2 * nomEnergyMax[s]) : 0.0);
dexpr float CyclingStepContribution[s in isE_STORAGES][t in
isDECISION_STEPS] = (nomEnergyMax[s] > 0.0 ? (StorDCPowerCharge[s][t] +
StorDCPowerDischarge[s][t]) * assetStepDurationInHours / (2 *
nomEnergyMax[s]) : 0.0);


// DC Charge Power
//    forall (s in isE_STORAGES, t in isDECISION_STEPS)
//      ctStorDCPowerCharge: StorDCPowerCharge[s][t] ==
//          sum(se in 1..chargeStorSegNbr[chargeVarEffModelId[s]])
//            (  (storChargeSegSlope[chargeVarEffModelId[s]][se] > 0.0 ?
StorACSegPowerCharge[s][se][t] /
storChargeSegSlope[chargeVarEffModelId[s]][se] : 0.0)
//             - (storChargeSegSlope[chargeVarEffModelId[s]][se] > 0.0 ?
storChargeSegOrdinate[chargeVarEffModelId[s]][se] /
storChargeSegSlope[chargeVarEffModelId[s]][se] : 0.0)
//             * StorACSegChargeFlag[s][se][t]
//            );
      forall (s in isE_STORAGES, t in isDECISION_STEPS)
        ctStorDCPowerCharge: StorDCPowerCharge[s][t] ==
StorACPowerCharge[s][t] * assetChargeEfficiency[s];

// DC Discharge Power
//    forall (s in isE_STORAGES, t in isDECISION_STEPS)
//      ctStorDCPowerDischarge: StorDCPowerDischarge[s][t] ==
//          sum(se in 1..dischStorSegNbr[dischVarEffModelId[s]])
//            ( (storDischSegSlope[dischVarEffModelId[s]][se] > 0.0 ?
StorACSegPowerDischarge[s][se][t] /
storDischSegSlope[dischVarEffModelId[s]][se]: 0.0)
//             - (storDischSegSlope[dischVarEffModelId[s]][se] > 0.0 ?
storDischSegOrdinate[dischVarEffModelId[s]][se] /
storDischSegSlope[dischVarEffModelId[s]][se] : 0.0)
//             * StorACSegDischFlag[s][se][t]);
      forall (s in isE_STORAGES, t in isDECISION_STEPS)
        ctStorDCPowerDischarge: StorDCPowerDischarge[s][t] ==
      StorACPowerDischarge[s][t] / assetChargeEfficiency[s];

// Upperbound SOCmaxExcess
      forall (s in isE_STORAGES, t in isDECISION_STEPS)
        ctSOCmaxExcessbound: SOCmaxExcess[s][t] <= 100 - storElecMaxSOC[s]
+ FCRPower_MW[s][assetStepFCRStep[t]] * (storEnergyUp1MWFCR /
storMaxDCEnergy[s][t])*100;

// Upperbound SOCstrictMinDeficit
      forall (s in isE_STORAGES, t in isDECISION_STEPS)
        ctSOCstrictMinDeficitbound: SOCstrictMinDeficit[s][t] <=
storStrictElecMinSOC[s] + FCRPower_MW[s][assetStepFCRStep[t]] *
(storEnergyDwn1MWFCR / storMaxDCEnergy[s][t]) * 100 - 0;
```

B CPLEX Execution Logs

B.1 Warm Start Execution Logs from Section 5.6.2

```
Model with violations, 36h scenario no ws
OBJ = -2004.559432251
Status = 102

Checking license ...
License found. [0,06 s]
Version identifier: 22.1.2.0 | 2024-12-09 | 8bd2200c8
CPXPARAM_Tune_Display                            3
CPXPARAM_Output_WriteLevel                       1
CPXPARAM_MIP_Tolerances_MIPGap                   0.00014999999999999999
Legacy callback                                  pi
Tried aggregator 3 times.
MIP Presolve eliminated 14114 rows and 15287 columns.
MIP Presolve modified 18 coefficients.
Aggregator did 1583 substitutions.
Reduced MIP has 41892 rows, 42390 columns, and 116704 nonzeros.
Reduced MIP has 10633 binaries, 102 generals, 0 SOSs, and 18814 indicators.
Presolve time = 0,31 sec. (540,86 ticks)
Probing fixed 0 vars, tightened 286 bounds.
Probing time = 0,70 sec. (27,27 ticks)
Tried aggregator 1 time.
Detecting symmetries...
MIP Presolve eliminated 120 rows and 0 columns.
Reduced MIP has 41772 rows, 42390 columns, and 116464 nonzeros.
Reduced MIP has 10634 binaries, 420 generals, 0 SOSs, and 18814 indicators.
Presolve time = 0,12 sec. (102,99 ticks)
Probing time = 0,11 sec. (10,84 ticks)
Clique table members: 19316.
MIP emphasis: balance optimality and feasibility.
MIP search method: dynamic search.
Parallel mode: deterministic, using up to 12 threads.
Root relaxation solution time = 1,67 sec. (2995,09 ticks)

        Nodes                                         Cuts/
   Node  Left     Objective  IInf  Best Integer    Best Bound    ItCnt
Gap

*     0+    0                      8,01272e+09
---
*     0+    0                      1,59078e+08
---
      0     0    -2114,6839   489  1,59078e+08    -2114,6839       35
100,00%
*     0+    0                      1,46489e+08    -2114,6839
100,00%
*     0+    0                      1,46483e+08    -2114,6839
100,00%
      0     0    -2065,8626   400  1,46483e+08    Cuts: 1729     5364
100,00%
*     0+    0                     7471299,5743    -2065,8626
100,03%
      0     0    -2056,6027  1539  7471299,5743    Cuts: 9285     9429
100,03%
*     0+    0                     6031944,8371    -2056,6027
100,03%
      0     0    -2044,6550   901  6031944,8371    Cuts: 5009    13163
100,03%
*     0+    0                     5428216,6318    -2044,6550
100,04%
      0     0    -2042,9573   948  5428216,6318    Cuts: 1256    14061
100,04%
*     0+    0                     2394385,5485    -2042,9573
100,09%
      0     0    -2042,6685  1071  2394385,5485     Cuts: 503    14370
100,09%
      0     0    -2042,4244   926  2394385,5485     Cuts: 373    14689
100,09%
      0     0    -2041,3315  1165  2394385,5485     Cuts: 452    15269
100,09%
*     0+    0                     1872103,5855    -2041,3315
100,11%
      0     0    -2040,9186  1123  1872103,5855     Cuts: 133    15413
100,11%
*     0+    0                     1321251,0607    -2040,9186
100,15%
      0     0    -2040,6874   948  1321251,0607     Cuts: 234    15660
100,15%
*     0+    0                     1321251,0400    -2040,6874
100,15%
      0     0    -2040,6874   946  1321251,0400       Cuts: 4    15665
100,15%
      0     0    -2040,6874   944  1321251,0400     Covers: 1    15668
100,15%
*     0+    0                        4883,7485    -2040,6874
141,79%
*     0+    0                       -1498,2172    -2040,6874
36,21%
      0     2    -2040,6874   944   -1498,2172    -2040,6874    15668
36,21%
Elapsed time = 18,17 sec. (23122,56 ticks, tree = 0,02 MB, solutions = 13)
     11     9    -1873,0705   772   -1498,2172    -2040,5926    17300
36,20%
     39    22    -1921,0048   883   -1498,2172    -2029,5009    20641
35,46%
    107    79    -1989,1106   239   -1498,2172    -2027,9620    34670
35,36%
    194   142    -1871,5616   357   -1498,2172    -2027,9620    45024
35,36%
    293   262    -1998,3737   298   -1498,2172    -2027,9620    50788
35,36%
    362   313    -1896,3755   460   -1498,2172    -2027,9620    56367
35,36%
    473   394    -1900,9793   160   -1498,2172    -2027,9620    58888
35,36%
    583   510    -1977,3723   354   -1498,2172    -2027,9620    61968
35,36%
    693   580    -1966,5812   225   -1498,2172    -2027,9620    65279
35,36%
*   854+  723                       -1885,0250    -2027,9620
7,58%
*   925+  794                       -1903,4233    -2027,9620
6,54%
*   928+  804                       -1961,4237    -2027,9620
3,39%
    959   796    -1906,1581     6   -1961,4237    -2027,9620    74704
3,39%
Elapsed time = 31,52 sec. (26263,14 ticks, tree = 15,79 MB, solutions = 15)
*  1019+  818                       -1981,4763    -2027,9620
2,35%
   1120   164    -1994,7191   271   -1981,4763    -2027,9620    90810
2,35%
   1334   249    -1983,4549   246   -1981,4763    -2025,5693   102949
2,23%
   1607   430    -1983,5345   229   -1981,4763    -2014,1263   121442
1,65%
*  1688+  449                       -1982,5673    -2014,1263
1,59%
*  1754+  448                       -1982,8136    -2014,1263
1,58%
*  1795+  533                       -1988,2981    -2014,1263
1,30%
*  1855+  533                       -1988,3403    -2014,1263
1,30%
   1918   614    -1991,0112    53   -1988,3403    -2014,1263   141735
1,30%
*  2182+  379                       -1990,3018    -2014,1263
1,20%
*  2204+  565                       -1994,0717    -2014,1263
1,01%
   2232   434    -1992,9326     2   -1994,0717    -2014,1263   153887
1,01%
*  2344+  358                       -1996,4621    -2014,1263
0,88%
*  2449+  444                       -1998,7811    -2013,7368
0,75%
   2547   325    -2001,7534   154   -1998,7811    -2012,1484   162357
0,67%
*  2585+  444                       -1998,8234    -2012,1484
0,67%
*  2697+  321                       -2004,3394    -2011,2590
0,35%
*  2715+  345                       -2004,3816    -2011,2590
0,34%
   2865   262    -2004,4373     2   -2004,3816    -2010,7946   170329
0,32%
   3216   289    -2008,0465   316   -2004,3816    -2010,4075   177815
0,30%
   3540   468    -2004,3836     1   -2004,3816    -2010,4075   181984
0,30%
   3898   653    -2004,3836     1   -2004,3816    -2010,4075   192029
0,30%
Elapsed time = 71,36 sec. (35815,85 ticks, tree = 11,65 MB, solutions = 42)

Performing restart 1

Repeating presolve.
Tried aggregator 2 times.
MIP Presolve eliminated 837 rows and 1293 columns.
MIP Presolve modified 153 coefficients.
Aggregator did 177 substitutions.
Reduced MIP has 40758 rows, 40920 columns, and 113469 nonzeros.
Reduced MIP has 10500 binaries, 172 generals, 0 SOSs, and 18814 indicators.
Presolve time = 0,19 sec. (126,95 ticks)
Tried aggregator 1 time.
MIP Presolve modified 111 coefficients.
Reduced MIP has 40900 rows, 40920 columns, and 113753 nonzeros.
Reduced MIP has 10500 binaries, 172 generals, 0 SOSs, and 18672 indicators.
Presolve time = 0,09 sec. (91,64 ticks)
Represolve time = 0,95 sec. (330,80 ticks)
   3930     0    -2035,6599  1079   -2004,3816    Cuts: 3361   226183
0,30%
   3930     0    -2033,1649  1089   -2004,3816    Cuts: 1234   227067
0,30%
   3930     0    -2031,3167  1411   -2004,3816     Cuts: 735   228188
0,30%
   3930     0    -2029,3291  1455   -2004,3816    Cuts: 1465   229094
0,30%
   3930     0    -2026,7252  1449   -2004,3816    Cuts: 1902   230298
0,30%
   3930     0    -2024,1830  1405   -2004,3816    Cuts: 1179   231126
0,24%
   3930     0    -2022,4757  1489   -2004,3816    Cuts: 1706   232456
0,22%
   3930     0    -2021,8468  1510   -2004,3816    Cuts: 1179   232906
0,18%
   3930     2    -2021,8464  1510   -2004,3816    -2007,7009   233004
0,17%
   3952     4    -2005,3194   825   -2004,3816    -2007,7009   236398
0,17%
   4009    19    -2007,1091   537   -2004,3816    -2007,7009   240792
0,17%
   4413   242    -2005,2863   215   -2004,3816    -2007,7009   255697
0,17%
   4827   427    -2004,5114    50   -2004,3816    -2007,7009   270117
0,17%
   5329   653    -2004,4214    45   -2004,3816    -2007,7009   283435
0,17%
*  5528   777      integral     0   -2004,4507    -2007,7009   292469
0,16%
*  5536   736      integral     0   -2004,5113    -2007,7009   287584
0,16%
   5878   618        cutoff         -2004,5113    -2006,7035   308825
0,11%
```

```
   6272   517        cutoff          -2004,5113     -2006,2103   335992
0,08%
Elapsed time = 139,44 sec. (95983,85 ticks, tree = 8,04 MB, solutions = 44)
*  6319+  552                        -2004,5594     -2006,2103
0,08%

GUB cover cuts applied:  4
Clique cuts applied:  1
Cover cuts applied:  1917
Implied bound cuts applied:  1407
Flow cuts applied:  425
Mixed integer rounding cuts applied:  733
Lift and project cuts applied:  76
Gomory fractional cuts applied:  77

Root node processing (before b&c):
  Real time             =   17,72 sec. (22930,34 ticks)
Parallel b&c, 12 threads:
  Real time             =  123,17 sec. (73629,84 ticks)
  Sync time (average)   =   27,37 sec.
  Wait time (average)   =    0,06 sec.
                          ------------
Total (root+branch&cut) =  140,89 sec. (96560,18 ticks)

XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX

Model with violations, ws m1 effort level 0
OBJ = -2004.559432251
Status = 102


Checking license ...
License found. [0,06 s]
Version identifier: 22.1.2.0 | 2024-12-09 | 8bd2200c8
CPXPARAM_Tune_Display                            3
CPXPARAM_Output_WriteLevel                       1
CPXPARAM_MIP_Tolerances_MIPGap                   0.00014999999999999999
Legacy callback                                  pi
Reduced MIP has 39349 rows, 59548 columns, and 119960 nonzeros.
Reduced MIP has 14458 binaries, 144 generals, 0 SOSs, and 19104 indicators.
Presolve time = 0,08 sec. (455,04 ticks)
1 of 1 MIP starts provided solutions.
MIP start 'm1' defined initial solution with objective -2004,5594.
Tried aggregator 3 times.
MIP Presolve eliminated 14114 rows and 15287 columns.
MIP Presolve modified 18 coefficients.
Aggregator did 1583 substitutions.
Reduced MIP has 41892 rows, 42390 columns, and 116704 nonzeros.
Reduced MIP has 10633 binaries, 102 generals, 0 SOSs, and 18814 indicators.
Presolve time = 0,17 sec. (540,86 ticks)
Probing fixed 0 vars, tightened 286 bounds.
Probing time = 0,44 sec. (27,27 ticks)
Tried aggregator 1 time.
Detecting symmetries...
```

```
MIP Presolve eliminated 120 rows and 0 columns.
Reduced MIP has 41772 rows, 42390 columns, and 116464 nonzeros.
Reduced MIP has 10634 binaries, 420 generals, 0 SOSs, and 18814 indicators.
Presolve time = 0,11 sec. (102,99 ticks)
Probing time = 0,09 sec. (10,84 ticks)
Clique table members: 19316.
MIP emphasis: balance optimality and feasibility.
MIP search method: dynamic search.
Parallel mode: deterministic, using up to 12 threads.
Root relaxation solution time = 1,28 sec. (2995,09 ticks)

        Nodes                                         Cuts/
   Node  Left     Objective  IInf  Best Integer    Best Bound    ItCnt
Gap

*     0+    0                        -2004,5594
---
      0     0    -2114,6839   489    -2004,5594    -2114,6839       35
5,49%
      0     0    -2052,8748   473    -2004,5594    Cuts: 3100     5413
2,41%
      0     0    -2041,1164  1308    -2004,5594    Cuts: 6452     8619
1,82%
      0     0    -2038,8089  2028    -2004,5594    Cuts: 3634    11080
1,71%
      0     0    -2038,2992  1936    -2004,5594     Cuts: 505    11659
1,68%
      0     0    -2038,0591  1933    -2004,5594     Cuts: 301    11813
1,67%
      0     0    -2038,0090  1930    -2004,5594     Cuts: 181    11870
1,67%
      0     0    -2037,9798  1952    -2004,5594      Cuts: 35    11907
1,67%
      0     2    -2037,9798  1952    -2004,5594    -2037,9798    11907
1,67%
Elapsed time = 11,25 sec. (15083,45 ticks, tree = 0,02 MB, solutions = 1)
      5     3    -2027,8877  1867    -2004,5594    -2037,6489    12491
1,65%
     17     5    -2006,2421   732    -2004,5594    -2030,2806    15212
1,28%
     35     6        cutoff          -2004,5594    -2013,7108    21726
0,46%
     50     9    -2006,1928   511    -2004,5594    -2013,7108    22780
0,46%
     74    10        cutoff          -2004,5594    -2013,6904    24471
0,46%
     94    14    -2008,2029   535    -2004,5594    -2013,0194    25845
0,42%
    146    25        cutoff          -2004,5594    -2012,5867    26872
0,40%
    227    59        cutoff          -2004,5594    -2012,5867    28469
0,40%
    305    94        cutoff          -2004,5594    -2012,5867    32055
0,40%
```

```
    684   247    -2006,2296   336    -2004,5594     -2012,5867   100649
0,40%
Elapsed time = 23,98 sec. (18539,82 ticks, tree = 3,44 MB, solutions = 1)
   1087   447    -2005,5067   269    -2004,5594     -2011,6400   117701
0,35%
   1439   641    -2007,6792   402    -2004,5594     -2011,5994   152068
0,35%
   1680   735    -2006,0835   328    -2004,5594     -2011,3309   169634
0,34%
   2031   816    -2005,2997   394    -2004,5594     -2009,5654   182489
0,25%
   2379   923    -2007,2363   323    -2004,5594     -2008,9978   198591
0,22%
   2736   974    -2007,8866   430    -2004,5594     -2008,5982   211535
0,20%
   3104  1057    -2004,9620   269    -2004,5594     -2008,1899   233223
0,18%
   3494  1094    -2005,4022   277    -2004,5594     -2007,6225   239889
0,15%
   3839  1144        cutoff          -2004,5594     -2007,4188   260922
0,14%
   4245  1112    -2005,6782   322    -2004,5594     -2007,1429   272136
0,13%
Elapsed time = 70,25 sec. (28109,29 ticks, tree = 18,79 MB, solutions = 1)
   4645  1065    -2005,4159   247    -2004,5594     -2006,3062   285374
0,09%
   4955  1011        cutoff          -2004,5594     -2006,0594   302062
0,07%
   5295   994    -2004,7074   374    -2004,5594     -2005,8204   321380
0,06%
   5609  1057        cutoff          -2004,5594     -2005,7203   348949
0,06%
   5920  1078    -2005,4269   303    -2004,5594     -2005,6183   399448
0,05%
   6198  1065    -2004,7153   282    -2004,5594     -2005,5922   448021
0,05%
   6494  1041        cutoff          -2004,5594     -2005,5063   494215
0,05%
   6783   932        cutoff          -2004,5594     -2005,3664   568617
0,04%
   7082   850    -2004,8739   280    -2004,5594     -2005,1769   613730
0,03%
   7463   815    -2005,0180   452    -2004,5594     -2005,0730   663598
0,03%
Elapsed time = 114,36 sec. (37672,42 ticks, tree = 16,00 MB, solutions = 1)

Performing restart 1

Repeating presolve.
Tried aggregator 2 times.
MIP Presolve eliminated 1408 rows and 1535 columns.
MIP Presolve modified 2165 coefficients.
Aggregator did 279 substitutions.
Reduced MIP has 40208 rows, 40576 columns, and 111247 nonzeros.
```

```
Reduced MIP has 10390 binaries, 30 generals, 0 SOSs, and 18691 indicators.
Presolve time = 0,19 sec. (152,98 ticks)
Tried aggregator 2 times.
MIP Presolve eliminated 8 rows and 2 columns.
MIP Presolve modified 13987 coefficients.
Aggregator did 123 substitutions.
Reduced MIP has 40096 rows, 40451 columns, and 111013 nonzeros.
Reduced MIP has 10267 binaries, 30 generals, 0 SOSs, and 18672 indicators.
Presolve time = 0,11 sec. (111,72 ticks)
Represolve time = 0,89 sec. (372,15 ticks)
   7695     0    -2033,8690  1464    -2004,5594    Cuts: 5152   744232
0,02%
   7695     0    -2031,3002  1370    -2004,5594     Cuts: 742   745418
0,02%
   7695     0    -2029,3038  1416    -2004,5594    Cuts: 1217   746114
0,02%
   7695     0    -2028,0688  1767    -2004,5594    Cuts: 1215   747216
0,02%
   7695     0    -2025,8716  1958    -2004,5594     Cuts: 956   747975
0,02%
   7695     0    -2025,0377  2133    -2004,5594    Cuts: 1099   748774
0,02%
   7695     2    -2025,0377  2133    -2004,5594    -2005,0180   748774
0,02%
   7702     4    -2011,0689   755    -2004,5594    -2005,0180   751359
0,02%
   7705     3    -2021,5623  1420    -2004,5594    -2005,0180   749251
0,02%
   7711     3    -2017,6034   783    -2004,5594    -2005,0180   756044
0,02%
   7740    10    -2007,5453   504    -2004,5594    -2005,0180   760670
0,02%
   7942    40        cutoff          -2004,5594    -2005,0180   768618
0,02%

GUB cover cuts applied:  3
Clique cuts applied:  15
Cover cuts applied:  1646
Implied bound cuts applied:  1964
Flow cuts applied:  400
Mixed integer rounding cuts applied:  325
Lift and project cuts applied:  42
Gomory fractional cuts applied:  58

Root node processing (before b&c):
  Real time             =   10,80 sec. (14792,59 ticks)
Parallel b&c, 12 threads:
  Real time             =  146,72 sec. (71950,32 ticks)
  Sync time (average)   =   39,07 sec.
  Wait time (average)   =    0,05 sec.
                          ------------
Total (root+branch&cut) =  157,51 sec. (86742,91 ticks)
```

XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX

Model with violations, ws discrete vars level 2

OBJ = -2004.559432251

Status = 102

```
Checking license ...
License found. [0,03 s]
Version identifier: 22.1.2.0 | 2024-12-09 | 8bd2200c8
CPXPARAM_Tune_Display                            3
CPXPARAM_Output_WriteLevel                       1
CPXPARAM_MIP_Tolerances_MIPGap                   0.00014999999999999999
Legacy callback                                  pi
Reduced MIP has 39349 rows, 59548 columns, and 119960 nonzeros.
Reduced MIP has 14458 binaries, 144 generals, 0 SOSs, and 19104 indicators.
Presolve time = 0,06 sec. (455,04 ticks)
1 of 1 MIP starts provided solutions.
MIP start 'All discrete variables from m1' defined initial solution with
objective -2004,5594.
Tried aggregator 3 times.
MIP Presolve eliminated 14114 rows and 15287 columns.
MIP Presolve modified 18 coefficients.
Aggregator did 1583 substitutions.
Reduced MIP has 41892 rows, 42390 columns, and 116704 nonzeros.
Reduced MIP has 10633 binaries, 102 generals, 0 SOSs, and 18814 indicators.
Presolve time = 0,17 sec. (540,86 ticks)
Probing fixed 0 vars, tightened 286 bounds.
Probing time = 0,38 sec. (27,27 ticks)
Tried aggregator 1 time.
Detecting symmetries...
MIP Presolve eliminated 120 rows and 0 columns.
Reduced MIP has 41772 rows, 42390 columns, and 116464 nonzeros.
Reduced MIP has 10634 binaries, 420 generals, 0 SOSs, and 18814 indicators.
Presolve time = 0,09 sec. (102,99 ticks)
Probing time = 0,08 sec. (10,85 ticks)
Clique table members: 19316.
MIP emphasis: balance optimality and feasibility.
MIP search method: dynamic search.
Parallel mode: deterministic, using up to 12 threads.
Root relaxation solution time = 1,28 sec. (3004,12 ticks)

        Nodes                                         Cuts/
   Node  Left     Objective  IInf  Best Integer    Best Bound    ItCnt
Gap

*     0+    0                        -2004,5594
---
      0     0    -2114,6839   489    -2004,5594    -2114,6839       35
5,49%
      0     0    -2057,4959   586    -2004,5594    Cuts: 2953     6194
2,64%
      0     0    -2041,1702  1860    -2004,5594    Cuts: 9366     9769
1,83%
      0     0    -2040,1836  1836    -2004,5594    Cuts: 1186    10871
1,78%
      0     0    -2039,6099  1870    -2004,5594     Cuts: 318    11186
1,75%
      0     0    -2039,4375  1863    -2004,5594     Cuts: 283    11345
1,74%
      0     0    -2038,9032  1889    -2004,5594      Cuts: 81    11481
1,71%
      0     0    -2038,8726  1884    -2004,5594      Cuts: 40    11490
1,71%
      0     0    -2038,8219  1860    -2004,5594     Cuts: 114    11598
1,71%
      0     0    -2038,6836  1882    -2004,5594      Cuts: 31    11629
1,70%
      0     2    -2038,6836  1882    -2004,5594    -2038,6836    11629
1,70%
Elapsed time = 11,25 sec. (14690,35 ticks, tree = 0,02 MB, solutions = 1)
      5     3    -2028,2951  1882    -2004,5594    -2038,4791    11928
1,69%
     12     4    -2023,7855   769    -2004,5594    -2030,6103    12905
1,30%
     23     7        cutoff          -2004,5594    -2023,0400    15737
0,92%
     52    11        cutoff          -2004,5594    -2013,2398    20099
0,43%
     96    29    -2009,1312   486    -2004,5594    -2012,8228    21747
0,41%
    189    51    -2007,2346   515    -2004,5594    -2011,9973    24410
0,37%
    275    80    -2005,2157   409    -2004,5594    -2011,9973    27931
0,37%
    359    91    -2004,6993   479    -2004,5594    -2011,9973    30303
0,37%
    471    95    -2006,1419   491    -2004,5594    -2011,9973    32429
0,37%
    872   232    -2004,6093   181    -2004,5594    -2010,7463    41246
0,31%
Elapsed time = 22,91 sec. (18109,33 ticks, tree = 3,08 MB, solutions = 1)
   1204   327    -2005,6508   405    -2004,5594    -2010,7463    62884
0,31%
   1538   471        cutoff          -2004,5594    -2010,0032    97483
0,27%
   1796   542    -2006,2727   360    -2004,5594    -2009,3926   121879
0,24%
   2167   653    -2004,9215   253    -2004,5594    -2009,1795   142549
0,23%
   2679   832    -2004,6109   171    -2004,5594    -2006,8949   167116
0,12%
   3066   966    -2006,4546   222    -2004,5594    -2006,6053   196512
0,10%
   3403  1169    -2004,9143   269    -2004,5594    -2006,6052   241787
0,10%
   3714  1275        cutoff          -2004,5594    -2006,6052   269594
0,10%
   4133  1417    -2004,9395   313    -2004,5594    -2006,6052   298055
0,10%
   4536  1587    -2005,5844   313    -2004,5594    -2006,6052   345078
0,10%
Elapsed time = 64,58 sec. (27677,84 ticks, tree = 28,83 MB, solutions = 1)
   4970  1714    -2006,3014   378    -2004,5594    -2006,6052   383843
0,10%
   5395  1931        cutoff          -2004,5594    -2006,6052   430260
0,10%
   5800  2008    -2004,6755   258    -2004,5594    -2006,3842   450792
0,09%

Performing restart 1

Repeating presolve.
Tried aggregator 2 times.
MIP Presolve eliminated 1349 rows and 1513 columns.
MIP Presolve modified 1953 coefficients.
Aggregator did 273 substitutions.
Reduced MIP has 40278 rows, 40604 columns, and 111560 nonzeros.
Reduced MIP has 10403 binaries, 40 generals, 0 SOSs, and 18686 indicators.
Presolve time = 0,16 sec. (126,57 ticks)
Tried aggregator 2 times.
MIP Presolve eliminated 2 rows and 2 columns.
MIP Presolve modified 13140 coefficients.
Aggregator did 128 substitutions.
Reduced MIP has 40162 rows, 40474 columns, and 111327 nonzeros.
Reduced MIP has 10275 binaries, 40 generals, 0 SOSs, and 18672 indicators.
Presolve time = 0,11 sec. (108,64 ticks)
Represolve time = 0,86 sec. (337,05 ticks)
   5824     0    -2033,9019   982    -2004,5594    Cuts: 3440   509333
0,09%
   5824     0    -2029,0664  1282    -2004,5594     Cuts: 976   510742
0,09%
   5824     0    -2028,1444  1836    -2004,5594    Cuts: 1451   512201
0,09%
   5824     0    -2026,3530  1595    -2004,5594    Cuts: 3162   513441
0,09%
   5824     0    -2024,7278  1963    -2004,5594    Cuts: 1960   514294
0,09%
   5824     0    -2024,2133  2095    -2004,5594    Cuts: 1951   515120
0,09%
   5824     0    -2023,4747  2338    -2004,5594    Cuts: 1235   515965
0,09%
   5824     2    -2023,4747  2338    -2004,5594    -2006,3842   515965
0,09%
   5833     3    -2011,3395  1415    -2004,5594    -2006,3842   517876
0,09%
   5903    31    -2013,4879  1648    -2004,5594    -2006,3842   521357
0,09%
   6030   105    -2006,6353   545    -2004,5594    -2006,3842   539469
0,09%
   6245   205    -2004,6215   178    -2004,5594    -2006,3842   566634
0,09%
   6556   239    -2012,8821  1350    -2004,5594    -2006,3842   615808
0,09%
   6895   255        cutoff          -2004,5594    -2006,3842   683334
0,09%
Elapsed time = 138,78 sec. (90353,11 ticks, tree = 5,49 MB, solutions = 1)

GUB cover cuts applied:  5
Clique cuts applied:  15
Cover cuts applied:  2265
Implied bound cuts applied:  2972
Flow cuts applied:  663
Mixed integer rounding cuts applied:  325
Lift and project cuts applied:  72
Gomory fractional cuts applied:  73

Root node processing (before b&c):
  Real time             =   10,86 sec. (14393,32 ticks)
Parallel b&c, 12 threads:
  Real time             =  128,89 sec. (76490,81 ticks)
  Sync time (average)   =   26,24 sec.
  Wait time (average)   =    0,04 sec.
                          ------------
Total (root+branch&cut) =  139,75 sec. (90884,13 ticks)
```

B.2 Execution Logs for the (Non-) Linear Formulations from Section 6.3.2

SCENARIO 1 :

LINEAR

```
Checking license ...
License found. [0,05 s]
Version identifier: 22.1.2.0 | 2024-12-09 | 8bd2200c8
Legacy callback                                     pi
Tried aggregator 3 times.
MIP Presolve eliminated 19519 rows and 21330 columns.
MIP Presolve modified 4 coefficients.
Aggregator did 1885 substitutions.
Reduced MIP has 5921 rows, 6817 columns, and 46983 nonzeros.
Reduced MIP has 1742 binaries, 65 generals, 0 SOSs, and 398 indicators.
Presolve time = 0,09 sec. (90,41 ticks)
Probing fixed 0 vars, tightened 398 bounds.
Probing time = 0,01 sec. (1,91 ticks)
Cover probing fixed 0 vars, tightened 1 bounds.
Tried aggregator 1 time.
Detecting symmetries...
Reduced MIP has 5921 rows, 6817 columns, and 46983 nonzeros.
Reduced MIP has 1742 binaries, 272 generals, 0 SOSs, and 398 indicators.
Presolve time = 0,02 sec. (18,56 ticks)
Probing time = 0,02 sec. (1,79 ticks)
Cover probing fixed 0 vars, tightened 1 bounds.
Clique table members: 479.
Tightened 1 constraints.
MIP emphasis: balance optimality and feasibility.
MIP search method: dynamic search.
Parallel mode: deterministic, using up to 12 threads.
Root relaxation solution time = 0,09 sec. (64,08 ticks)

        Nodes                                         Cuts/
   Node  Left     Objective  IInf  Best Integer    Best Bound    ItCnt
Gap

      0     0    -2727,4332   436                 -2727,4332      906
*     0+    0                  1,15041e+10     -2727,4332
100,00%
*     0+    0                  3,77280e+08     -2727,4332
100,00%
      0     0    -2694,2250   526  3,77280e+08    Cuts: 1218     2108
100,00%
*     0+    0                  1,75310e+07     -2694,2250
100,02%
      0     0    -2667,0175   295  1,75310e+07     Cuts: 982     2702
100,02%
      0     0    -2655,1678   242  1,75310e+07     Cuts: 385     2971
100,02%
*     0+    0                  1,53273e+07     -2655,1678
100,02%
      0     0  -1,00000e+75     0  1,53273e+07     -2655,1678     2971
100,02%
      0     0    -2650,1648   281  1,53273e+07     Cuts: 156     3115
100,02%
*     0+    0                  1,23045e+07     -2650,1648
100,02%
      0     0    -2645,1722   262  1,23045e+07      Cuts: 98     3198
100,02%
*     0+    0                  1,13766e+07     -2645,1722
100,02%
Detecting symmetries...
      0     0    -2641,9385   243  1,13766e+07      Cuts: 76     3266
100,02%
*     0+    0                  1,08779e+07     -2641,9385
100,02%
      0     0    -2641,6540   243  1,08779e+07      Cuts: 30     3290
100,02%
      0     0    -2641,6362   243  1,08779e+07       Cuts: 9     3305
100,02%
*     0+    0                 3064894,1918     -2641,6362
100,09%
      0     0    -2641,5839   440 3064894,1918      Cuts: 12     3317
100,09%
*     0+    0                  477383,6585     -2641,5839
100,55%
Detecting symmetries...
      0     2    -2641,5839   440  477383,6585    -2636,4472     3317
100,55%
Elapsed time = 2,49 sec. (1861,79 ticks, tree = 0,02 MB, solutions = 9)
    280   190    -2575,0900   364  477383,6585    -2625,7352     9592
100,55%
    921   810    -2370,8396   220  477383,6585    -2625,7352    14065
100,55%
   1493  1285    -2538,1049   182  477383,6585    -2625,7352    24833
100,55%
*  2099+ 1635                  226151,2997    -2625,7352
101,16%
   2426  1989    -2443,5473   117  226151,2997    -2625,7352    36246
101,16%
*  2970+ 2381                  112033,9018    -2625,7352
102,34%
*  3215+ 2599                   -2514,7356    -2625,7352
4,41%
*  3364  2795      integral     0   -2531,6311    -2625,7352    41026
3,72%
*  3449  2759      integral     0   -2586,0906    -2625,7352    40901
1,53%
   3852   380    -2608,4240   307   -2586,0906    -2625,5708    47192
1,53%

Performing restart 1

Repeating presolve.
Tried aggregator 3 times.
MIP Presolve eliminated 553 rows and 1036 columns.
MIP Presolve modified 246 coefficients.
Aggregator did 506 substitutions.
Reduced MIP has 4862 rows, 5275 columns, and 43710 nonzeros.
Reduced MIP has 1711 binaries, 99 generals, 0 SOSs, and 398 indicators.
Presolve time = 0,08 sec. (110,20 ticks)
Tried aggregator 1 time.
MIP Presolve eliminated 6 rows and 4 columns.
MIP Presolve modified 73 coefficients.
Reduced MIP has 4856 rows, 5271 columns, and 43690 nonzeros.
Reduced MIP has 1707 binaries, 99 generals, 0 SOSs, and 398 indicators.
Presolve time = 0,03 sec. (24,71 ticks)
Represolve time = 0,14 sec. (165,77 ticks)
*  3891+    0                   -2586,3318    -2625,5705
1,52%
   3891     0    -2624,0279   156   -2586,3318     Cuts: 399    54113
1,46%
   3891     0    -2620,0988   124   -2586,3318     Cuts: 211    54304
1,31%
   3891     0    -2618,2842   149   -2586,3318     Cuts: 167    54446
1,24%
   3891     0    -2617,2728   147   -2586,3318     Cuts: 116    54544
1,20%
*  3891+    0                   -2587,7112    -2617,2728
1,14%
*  3891+    0                   -2596,7449    -2617,2728
0,79%
   3891     0    -2615,9257   153   -2596,7449     Cuts: 166    54657
0,74%
*  3891+    0                   -2599,6219    -2615,9257
0,63%
*  3891+    0                   -2601,2439    -2615,9257
0,56%
   3891     0    -2614,3067   148   -2601,2439     Cuts: 111    54781
0,50%
*  3891+    0                   -2601,3564    -2614,3067
0,50%
   3891     0    -2611,9954   148   -2601,3564     Cuts: 131    54901
0,41%
*  3891+    0                   -2601,3857    -2611,9954
0,41%
*  3891+    0                   -2601,5204    -2611,9954
0,40%
   3891     0  -1,00000e+75     0   -2601,5204    -2611,9954    54901
0,40%
   3891     0    -2611,2521   127   -2601,5204      Cuts: 68    54991
0,37%
*  3891+    0                   -2601,6621    -2611,2521
0,37%
   3891     0    -2611,0632   104   -2601,6621      Cuts: 78    55072
0,36%
   3891     0    -2610,9490   131   -2601,6621      Cuts: 60    55126
0,36%
*  3891+    0                   -2602,0524    -2610,9490
0,34%
   3891     0    -2610,2371   130   -2602,0524      Cuts: 54    55189
0,31%
*  3891+    0                   -2603,5739    -2610,2371
0,26%
   3891     0    -2609,7426   123   -2603,5739      Cuts: 33    55248
0,24%
*  3891+    0                   -2603,5747    -2609,7426
0,24%
*  3891+    0                   -2605,0911    -2609,7426
0,18%
*  3891+    0                   -2605,3536    -2609,7426
0,17%
   3891     0  -1,00000e+75     0   -2605,3536    -2609,7426    55248
0,17%
   3891     0    -2609,4923   128   -2605,3536      Cuts: 47    55322
0,16%
   3891     0    -2609,4299   121   -2605,3536      Cuts: 35    55347
0,16%
   3891     0    -2609,3659   121   -2605,3536      Cuts: 59    55401
0,15%
   3891     0    -2609,3511   120   -2605,3536      Cuts: 27    55423
0,15%
   3891     0    -2608,9171    90   -2605,3536      Cuts: 40    55507
0,14%
*  3891+    0                   -2605,9241    -2608,9171
0,11%
*  3891+    0                   -2605,9264    -2608,9171
0,11%
   3891     0  -1,00000e+75     0   -2605,9264    -2608,9171    55507
0,11%
   3891     0    -2608,5952    81   -2605,9264      Cuts: 37    55549
0,10%
   3891     0    -2608,2266    94   -2605,9264      Cuts: 59    55600
0,09%
   3891     0    -2608,1817   101   -2605,9264      Cuts: 20    55645
0,09%
   3891     1    -2608,1468   101   -2605,9264    -2608,1817    55645
0,09%
   3900     2    -2605,9997    49   -2605,9264    -2606,1937    55769
0,01%

GUB cover cuts applied:  2
Clique cuts applied:  4
Cover cuts applied:  60
Implied bound cuts applied:  228
Flow cuts applied:  352
Mixed integer rounding cuts applied:  259
Zero-half cuts applied:  3
Lift and project cuts applied:  76
Gomory fractional cuts applied:  130

Root node processing (before b&c):
  Real time             =    2,27 sec. (1847,70 ticks)
Parallel b&c, 12 threads:
```

```
  Real time             =   10,20 sec. (6807,97 ticks)
  Sync time (average)   =    0,92 sec.
  Wait time (average)
=    0,03 sec.
                          ------------
Total (root+branch&cut) =   12,47 sec. (8655,67 ticks)
```

XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX

SCENARIO 1 :

NONLINEAR

```
Checking license ...
License found. [0,05 s]
Version identifier: 22.1.2.0 | 2024-12-09 | 8bd2200c8
CPXPARAM_Tune_Display                            3
CPXPARAM_Output_WriteLevel                       1
CPXPARAM_TimeLimit                               1800
CPXPARAM_MIP_Tolerances_MIPGap                   0.00014999999999999999
Legacy callback                                  pi
Tried aggregator 3 times.
MIP Presolve eliminated 20349 rows and 22225 columns.
MIP Presolve modified 4 coefficients.
Aggregator did 2072 substitutions.
Reduced MIP has 64664 rows, 65095 columns, and 178438 nonzeros.
Reduced MIP has 16382 binaries, 65 generals, 0 SOSs, and 29678 indicators.
Presolve time = 0,30 sec. (1160,40 ticks)
Probing fixed 0 vars, tightened 398 bounds.
Probing time = 0,78 sec. (44,84 ticks)
Tried aggregator 1 time.
Detecting symmetries...
MIP Presolve eliminated 465 rows and 0 columns.
Reduced MIP has 64199 rows, 65095 columns, and 177508 nonzeros.
Reduced MIP has 16382 binaries, 272 generals, 0 SOSs, and 29678 indicators.
Presolve time = 0,22 sec. (239,71 ticks)
Probing time = 0,17 sec. (15,35 ticks)
Clique table members: 30143.
MIP emphasis: balance optimality and feasibility.
MIP search method: dynamic search.
Parallel mode: deterministic, using up to 12 threads.
Root relaxation solution time = 3,14 sec. (5938,72 ticks)

        Nodes                                         Cuts/
   Node  Left     Objective  IInf  Best Integer    Best Bound    ItCnt
Gap

*     0+    0                      8,91716e+09
---
*     0+    0                      4,07409e+07
---
      0     0    -2727,4332   496   4,07409e+07    -2727,4332    16348
100,01%
*     0+    0                      3,60006e+07    -2727,4332
100,01%
      0     0    -2694,5696   546   3,60006e+07    Cuts: 1805    18403
100,01%
      0     0    -2683,3806  1329   3,60006e+07    Cuts: 9015    19947
100,01%
*     0+    0                      1,91662e+07    -2683,3806
100,01%
      0     0    -2663,6625  3075   1,91662e+07    Cuts: 9685    25793
100,01%
      0     0    -2652,4738  3720   1,91662e+07    Cuts: 5194    29895
100,01%
*     0+    0                      1,77766e+07    -2652,4738
100,01%
      0     0    -2650,5920  3702   1,77766e+07    Cuts: 2014    30652
100,01%
      0     0    -2647,7131  3971   1,77766e+07    Cuts: 1111    31354
100,01%
      0     0    -2645,9520  3949   1,77766e+07     Cuts: 371    31803
100,01%
*     0+    0                      1,20391e+07    -2645,9520
100,02%
      0     0    -2641,2223  3821   1,20391e+07     Cuts: 555    32245
100,02%
      0     0    -2640,8633  3822   1,20391e+07     Cuts: 497    32499
100,02%
      0     0    -2640,4958  3904   1,20391e+07      Cuts: 88    32537
100,02%
      0     0    -2640,4855  3904   1,20391e+07      Cuts: 46    32559
100,02%
      0     0    -2640,4855  3904   1,20391e+07      Cuts: 14    32565
100,02%
*     0+    0                       -2544,0257    -2640,4855
3,79%
      0     2    -2640,4855  3904    -2544,0257    -2640,4855    32565
3,79%
Elapsed time = 21,81 sec. (27809,54 ticks, tree = 0,02 MB, solutions = 7)
      7     9    -2565,5888  3456    -2544,0257    -2639,3831    32929
3,75%
*    10+    1                       -2584,3853    -2639,3831
2,13%
     12     3    -2633,2555  3832    -2584,3853    -2639,3831    32642
2,13%
     36    11    -2587,0302  3695    -2584,3853    -2627,5409    33569
1,67%
     54    20    -2620,4317  3053    -2584,3853    -2626,8714    34754
1,64%
    111    56    -2604,9560  3089    -2584,3853    -2625,0266    36718
1,57%
    164   114    -2602,4525  2499    -2584,3853    -2625,0266    39179
1,57%
    193   132    -2597,7900  2141    -2584,3853    -2625,0266    40612
1,57%
    249   147    -2601,0892  2802    -2584,3853    -2625,0266    41388
1,57%
    353   239    -2587,8659  2746    -2584,3853    -2625,0266    44038
1,57%
*   599+  422                       -2587,7234    -2625,0266
1,44%
    685   522    -2611,3449  2412    -2587,7234    -2625,0266    54597
1,44%
Elapsed time = 37,06 sec. (30960,84 ticks, tree = 10,55 MB, solutions = 10)
*   690+  501                       -2587,8675    -2625,0266
1,44%
*   707+  547                       -2592,7261    -2625,0266
1,25%
    975   652    -2601,0108   192    -2592,7261    -2625,0266    67606
1,25%
*   981+  602                       -2594,4905    -2625,0266
1,18%
*  1089+  651                       -2596,3355    -2625,0266
1,11%
   1223   731    -2599,2490   106    -2596,3355    -2625,0266    72834
1,11%
*  1254+  707                       -2597,6020    -2625,0266
1,06%
*  1361+  763                       -2601,0652    -2625,0266
0,92%
   1429   863    -2605,9789  1552    -2601,0652    -2625,0266    77416
0,92%
*  1495+  760                       -2601,1816    -2625,0266
0,92%
*  1543+  759                       -2601,2094    -2625,0266
0,92%
   1603   837    -2605,7745  1546    -2601,2094    -2625,0266    87573
0,92%
*  1674+  847                       -2601,4113    -2625,0266
0,91%
*  1688+  880                       -2602,0080    -2625,0266
0,88%
   1843   885    -2611,1716  2525    -2602,0080    -2624,3358    92275
0,86%
*  1987+ 1065                       -2602,1243    -2624,3358
0,85%
*  2018+ 1064                       -2602,1522    -2624,3358
0,85%
   2018  1116    -2602,8158   365    -2602,1522    -2624,3358    96859
0,85%
*  2091+ 1114                       -2602,1790    -2624,3358
0,85%
*  2097+ 1080                       -2602,8724    -2624,3358
0,82%
   2261  1107    -2605,3024   187    -2602,8724    -2624,3358   102839
0,82%
*  2420+ 1228                       -2603,7849    -2620,2037
0,63%
   2491  1276    -2605,9535  1657    -2603,7849    -2620,0625   107476
0,63%
*  2622+ 1204                       -2603,9290    -2620,0625
0,62%
*  2699+ 1203                       -2604,1063    -2620,0625
0,61%
   2752  1236    -2604,2507     2    -2604,1063    -2620,0625   113588
0,61%
*  2768+ 1302                       -2604,2950    -2620,0071
0,60%
   3011  1371    -2606,5749  2548    -2604,2950    -2618,3314   121111
0,54%
Elapsed time = 85,86 sec. (40539,95 ticks, tree = 33,82 MB, solutions = 34)
   3266  1597    -2604,7906  1709    -2604,2950    -2617,5654   129045
0,51%
   3543  1801    -2604,7641   227    -2604,2950    -2617,3005   134360
0,50%
   3831  2022        cutoff          -2604,2950    -2616,9459   141832
0,49%
*  3908+ 1990                       -2604,4920    -2616,9459
0,48%
   3967  2074    -2604,4391     0    -2604,4920    -2616,9459   143325
0,48%

Performing restart 1

Repeating presolve.
Tried aggregator 3 times.
MIP Presolve eliminated 2174 rows and 2579 columns.
MIP Presolve modified 688 coefficients.
Aggregator did 389 substitutions.
Reduced MIP has 61636 rows, 62127 columns, and 170494 nonzeros.
Reduced MIP has 15865 binaries, 52 generals, 0 SOSs, and 29678 indicators.
Presolve time = 0,70 sec. (794,44 ticks)
Tried aggregator 1 time.
MIP Presolve eliminated 8 rows and 0 columns.
MIP Presolve modified 2818 coefficients.
Reduced MIP has 61826 rows, 62127 columns, and 170862 nonzeros.
Reduced MIP has 15865 binaries, 52 generals, 0 SOSs, and 29480 indicators.
Presolve time = 0,17 sec. (143,74 ticks)
Represolve time = 1,95 sec. (1110,58 ticks)
   3969     0    -2629,7246  1639    -2604,4920    Cuts: 8520   193627
0,48%
   3969     0    -2622,7513  1327    -2604,4920    Cuts: 1142   194182
0,48%
   3969     0    -2620,8302  1160    -2604,4920     Cuts: 795   194933
0,48%
   3969     0    -2618,7300  1080    -2604,4920     Cuts: 549   195708
0,48%
   3969     0    -2616,9205  1111    -2604,4920     Cuts: 295   196169
0,48%
   3969     0    -2615,8753  1079    -2604,4920     Cuts: 494   196681
0,44%
```

```
   3969     0    -2615,0798  1137    -2604,4920     Cuts: 585   197261
0,41%
   3969     0    -2614,1962   949    -2604,4920     Cuts: 646   197636
0,37%
   3969     0    -2613,7147   958    -2604,4920     Cuts: 503   198106
0,35%
   3969     0    -2613,5487  1025    -2604,4920     Cuts: 489   198621
0,35%
   3969     0    -2613,2055  1010    -2604,4920     Cuts: 430   199005
0,33%
*  3969+    0                        -2604,9575    -2613,2055
0,32%
   3969     0    -2612,7337  1024    -2604,9575     Cuts: 503   199529
0,30%
   3969     0    -2612,5449   989    -2604,9575     Cuts: 132   199633
0,29%
   3969     0    -2612,4050  1004    -2604,9575     Cuts: 382   199737
0,29%
   3969     0    -2611,8855  1004    -2604,9575     Cuts: 125   199960
0,27%
   3969     0    -2611,7196   946    -2604,9575     Cuts: 178   200230
0,26%
   3969     0    -2611,5325   944    -2604,9575     Cuts: 140   200409
0,25%
   3969     0    -2611,4603   940    -2604,9575     Cuts: 302   200594
0,25%
   3969     0    -2611,3946   984    -2604,9575      Cuts: 45   200681
0,25%
   3969     0    -2611,3664   929    -2604,9575     Cuts: 128   200726
0,25%
*  3969+    0                        -2605,0601    -2611,3664
0,24%
   3969     0    -2611,2578   902    -2605,0601      Cuts: 63   200789
0,24%
   3969     0    -2611,2192   905    -2605,0601      Cuts: 34   200819
0,24%
   3969     0    -2611,1693   905    -2605,0601      Cuts: 74   200898
0,23%
   3969     2    -2611,1642   901    -2605,0601    -2611,1693   200898
0,23%
   3990     4    -2605,5741   587    -2605,0601    -2609,8542   201558
0,18%
   4040    19    -2605,8454   232    -2605,0601    -2607,0521   204008
0,08%
   4214    82        cutoff          -2605,0601    -2606,8871   207433
0,07%
*  4297+  101                        -2605,4085    -2606,8871
0,06%
   4493   162    -2605,5253    58    -2605,4085    -2606,8871   212440
0,06%
*  4549+  194                        -2605,7956    -2606,8871
0,04%
*  4634+  242                        -2605,8157    -2606,6594
0,03%
```

```
*  4637+  229                        -2605,9258    -2606,6594
0,03%

GUB cover cuts applied:  9
Clique cuts applied:  200
Cover cuts applied:  5876
Implied bound cuts applied:  8475
Flow cuts applied:  1707
Mixed integer rounding cuts applied:  3849
Lift and project cuts applied:  57
Gomory fractional cuts applied:  148

Root node processing (before b&c):
  Real time             =    21,36 sec. (27479,95 ticks)
Parallel b&c, 12 threads:
  Real time             =   186,53 sec. (116935,58 ticks)
  Sync time (average)   =    34,10 sec.
  Wait time (average)   =     0,10 sec.
                          ------------
Total (root+branch&cut) =   207,89 sec. (144415,53 ticks)
```

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SCENARIO 2 :

LINEAR

```
Checking license ...
License found. [0,06 s]
Version identifier: 22.1.2.0 | 2024-12-09 | 8bd2200c8
Legacy callback                                   pi
Tried aggregator 3 times.
MIP Presolve eliminated 18707 rows and 20570 columns.
MIP Presolve modified 4 coefficients.
Aggregator did 1825 substitutions.
Reduced MIP has 5610 rows, 6458 columns, and 44550 nonzeros.
Reduced MIP has 1659 binaries, 56 generals, 0 SOSs, and 380 indicators.
Presolve time = 0,11 sec. (85,70 ticks)
Probing fixed 0 vars, tightened 380 bounds.
Probing time = 0,02 sec. (1,82 ticks)
Cover probing fixed 0 vars, tightened 1 bounds.
Tried aggregator 1 time.
Detecting symmetries...
Reduced MIP has 5610 rows, 6458 columns, and 44550 nonzeros.
Reduced MIP has 1659 binaries, 236 generals, 0 SOSs, and 380 indicators.
Presolve time = 0,03 sec. (17,55 ticks)
Probing time = 0,01 sec. (1,70 ticks)
Cover probing fixed 0 vars, tightened 1 bounds.
Clique table members: 457.
Tightened 1 constraints.
MIP emphasis: balance optimality and feasibility.
MIP search method: dynamic search.
```

```
Parallel mode: deterministic, using up to 12 threads.
Root relaxation solution time = 0,09 sec. (59,17 ticks)

        Nodes                                         Cuts/
   Node  Left     Objective  IInf  Best Integer    Best Bound    ItCnt
Gap

      0     0    -1370,6485   347                  -1370,6485      888
*     0+    0                      1,75622e+10     -1370,6485
100,00%
*     0+    0                      3,91874e+08     -1370,6485
100,00%
      0     0    -1350,4109   437  3,91874e+08     Cuts: 1025     1867
100,00%
*     0+    0                      6,03047e+07     -1350,4109
100,00%
*     0+    0                      3,45039e+07     -1350,4109
100,00%
      0     0    -1331,3691   387  3,45039e+07      Cuts: 989     2657
100,00%
*     0+    0                      3,16263e+07     -1331,3691
100,00%
      0     0    -1327,8803   345  3,16263e+07      Cuts: 949     3075
100,00%
*     0+    0                      3,16263e+07     -1327,8803
100,00%
      0     0  -1,00000e+75     0  3,16263e+07     -1327,8803     3075
100,00%
      0     0    -1325,9308   260  3,16263e+07      Cuts: 558     3362
100,00%
*     0+    0                     1293202,1184     -1325,9308
100,10%
      0     0    -1325,4425   249  1293202,1184     Cuts: 145     3472
100,10%
Detecting symmetries...
      0     0    -1325,1513   241  1293202,1184      Cuts: 81     3544
100,10%
      0     0    -1323,4529   236  1293202,1184      Cuts: 84     3665
100,10%
      0     0    -1323,0398   234  1293202,1184      Cuts: 97     3737
100,10%
*     0+    0                      886240,6772     -1323,0398
100,15%
*     0+    0                      886240,5239     -1323,0398
100,15%
*     0+    0                      886220,7250     -1323,0398
100,15%
      0     0  -1,00000e+75     0  886220,7250     -1323,0398     3737
100,15%
      0     0    -1322,8187   228  886220,7250      Cuts: 45     3779
100,15%
      0     0    -1322,6919   223  886220,7250      Cuts: 27     3804
100,15%
```

```
*     0+    0                      709742,9862     -1322,6919
100,19%
      0     0    -1322,6177   309   709742,9862      Cuts: 10     3811
100,19%
*     0+    0                      -1267,8734     -1322,6177
4,32%
      0     0  -1,00000e+75     0   -1267,8734     -1322,6177     3811
4,32%
Detecting symmetries...
      0     2    -1322,6177   337   -1267,8734     -1322,5200     3811
4,31%
Elapsed time = 2,86 sec. (2145,21 ticks, tree = 0,02 MB, solutions = 12)
*    20+    3                       -1274,8855     -1322,4915
3,73%
*    79+   11                       -1277,3367     -1322,2726
3,52%
    347   248    -1279,3203   169   -1277,3367     -1322,2726     9915
3,52%
*   393+  267                       -1284,4637     -1322,2726
2,94%
*   578+  327                       -1285,0890     -1322,2726
2,89%
    765   522    -1285,3810   164   -1285,0890     -1322,2726    16152
2,89%
*  1329+  790                       -1291,8303     -1322,2726
2,36%
   1376   926    -1316,0580   231   -1291,8303     -1322,2726    20109
2,36%
*  1549+  930                       -1295,7698     -1322,2726
2,05%
*  1663+  930                       -1300,3509     -1322,2726
1,69%
*  1862+  923                       -1304,2182     -1322,2726
1,38%
*  2012+  921                       -1304,6476     -1322,2726
1,35%
   2067  1263    -1298,7883   125   -1304,6476     -1322,2726    24216
1,35%
   2714   914    -1309,8407   189   -1304,6476     -1320,7480    29504
1,23%
   3374  1424    -1315,8493   235   -1304,6476     -1320,7480    36381
1,23%
   3880  1768    -1309,9602   138   -1304,6476     -1320,4183    43393
1,21%

Performing restart 1

Repeating presolve.
Tried aggregator 3 times.
MIP Presolve eliminated 875 rows and 1255 columns.
MIP Presolve modified 1326 coefficients.
Aggregator did 385 substitutions.
Reduced MIP has 4350 rows, 4818 columns, and 40373 nonzeros.
Reduced MIP has 1556 binaries, 20 generals, 0 SOSs, and 376 indicators.
```

```
Presolve time = 0,09 sec. (103,22 ticks)
Tried aggregator 1 time.
MIP Presolve modified 84 coefficients.
Reduced MIP has 4538 rows, 4818 columns, and 40749 nonzeros.
Reduced MIP has 1556 binaries, 20 generals, 0 SOSs, and 188 indicators.
Presolve time = 0,02 sec. (16,43 ticks)
Represolve time = 0,16 sec. (155,62 ticks)
   3898     0    -1319,7694   203    -1304,6476      Cuts: 727     50205
1,16%
   3898     0    -1318,0361   196    -1304,6476      Cuts: 244     50394
1,03%
   3898     0    -1316,7294   191    -1304,6476      Cuts: 215     50593
0,93%
   3898     0    -1315,2042   182    -1304,6476      Cuts: 163     50767
0,81%
*  3898+    0                        -1304,6811    -1315,2042
0,81%
   3898     0  -1,00000e+75     0    -1304,6811    -1315,2042     50767
0,81%
   3898     0    -1314,4521   213    -1304,6811      Cuts: 162     50911
0,75%
   3898     0    -1313,7328   223    -1304,6811      Cuts: 204     51084
0,69%
   3898     0    -1313,2596   203    -1304,6811      Cuts: 123     51234
0,66%
   3898     0    -1313,0150   194    -1304,6811      Cuts: 109     51341
0,64%
*  3898+    0                        -1305,0588    -1313,0150
0,61%
   3898     0    -1312,5828   203    -1305,0588      Cuts: 131     51461
0,58%
   3898     0    -1312,0914   208    -1305,0588      Cuts: 100     51569
0,54%
   3898     0    -1311,7814   196    -1305,0588       Cuts: 87     51667
0,52%
   3898     0    -1311,4759   180    -1305,0588      Cuts: 116     51791
0,49%
   3898     0    -1311,3811   193    -1305,0588       Cuts: 53     51862
0,48%
   3898     0    -1311,2520   185    -1305,0588       Cuts: 89     51936
0,47%
   3898     0    -1311,1484   188    -1305,0588       Cuts: 47     51994
0,47%
   3898     0    -1310,9169   182    -1305,0588       Cuts: 28     52038
0,45%
   3898     0    -1310,8706   188    -1305,0588       Cuts: 75     52074
0,45%
   3898     0    -1310,8135   163    -1305,0588       Cuts: 46     52107
0,44%
   3898     0    -1310,7928   161    -1305,0588       Cuts: 29     52138
0,44%
*  3898+    0                        -1305,1112    -1310,7928
0,44%
```

```
   3898     0    -1310,7616   159    -1305,1112      Cuts: 31     52165
0,43%
   3898     0    -1310,7507   158    -1305,1112     MIRcuts: 9    52175
0,43%
   3898     1    -1310,5657   116    -1305,1112    -1310,7507     52184
0,43%
   3921     8    -1306,5934   105    -1305,1112    -1309,8608     52632
0,36%
*  4358+   41                        -1305,3827    -1307,4264
0,16%
*  4443+   21                        -1305,6317    -1307,2275
0,12%

GUB cover cuts applied:  4
Clique cuts applied:  5
Cover cuts applied:  97
Implied bound cuts applied:  89
Flow cuts applied:  260
Mixed integer rounding cuts applied:  307
Zero-half cuts applied:  2
Lift and project cuts applied:  54
Gomory fractional cuts applied:  41

Root node processing (before b&c):
  Real time             =    2,77 sec. (2128,99 ticks)
Parallel b&c, 12 threads:
  Real time             =   15,88 sec. (11413,51 ticks)
  Sync time (average)   =    1,71 sec.
  Wait time (average)   =    0,02 sec.
                          ------------
Total (root+branch&cut) =   18,64 sec. (13542,51 ticks)
```

XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX

SCENARIO 2 :

NONLINEAR

```
Checking license ...
License found. [0,05 s]
Version identifier: 22.1.2.0 | 2024-12-09 | 8bd2200c8
CPXPARAM_Tune_Display                            3
CPXPARAM_Output_WriteLevel                       1
CPXPARAM_MIP_Tolerances_MIPGap                   0.00014999999999999999
Legacy callback                                  pi
Tried aggregator 3 times.
MIP Presolve eliminated 19505 rows and 21449 columns.
MIP Presolve modified 4 coefficients.
Aggregator did 2003 substitutions.
Reduced MIP has 61274 rows, 61657 columns, and 169075 nonzeros.
Reduced MIP has 15531 binaries, 56 generals, 0 SOSs, and 28124 indicators.
Presolve time = 0,25 sec. (1057,48 ticks)
Probing fixed 0 vars, tightened 380 bounds.
```

```
Probing time = 0,69 sec. (42,54 ticks)
Tried aggregator 1 time.
Detecting symmetries...
MIP Presolve eliminated 465 rows and 0 columns.
Reduced MIP has 60809 rows, 61657 columns, and 168145 nonzeros.
Reduced MIP has 15531 binaries, 236 generals, 0 SOSs, and 28124 indicators.
Presolve time = 0,19 sec. (227,05 ticks)
Probing time = 0,12 sec. (14,69 ticks)
Clique table members: 28777.
MIP emphasis: balance optimality and feasibility.
MIP search method: dynamic search.
Parallel mode: deterministic, using up to 12 threads.
Root relaxation solution time = 2,89 sec. (4749,41 ticks)

        Nodes                                         Cuts/
   Node  Left     Objective  IInf  Best Integer    Best Bound    ItCnt
Gap

*     0+    0                      1,37966e+10
---
*     0+    0                      1,93670e+08
---
      0     0    -1370,6485   398   1,93670e+08    -1370,6485     16317
100,00%
*     0+    0                      4,98459e+07    -1370,6485
100,00%
      0     0    -1350,3530   448   4,98459e+07    Cuts: 2449     19540
100,00%
*     0+    0                      3,59090e+07    -1350,3530
100,00%
      0     0    -1334,5070  1755   3,59090e+07    Cuts: 9309     23292
100,00%
*     0+    0                     7219987,3709    -1334,5070
100,02%
      0     0    -1329,9822  3292  7219987,3709    Cuts: 5688     27993
100,02%
      0     0    -1327,8490  3516  7219987,3709    Cuts: 3506     30326
100,02%
*     0+    0                     4042204,1268    -1327,8490
100,03%
      0     0    -1327,1232  3663  4042204,1268     Cuts: 834     31567
100,03%
      0     0    -1326,5047  3510  4042204,1268    Cuts: 1872     32388
100,03%
      0     0    -1326,1327  3746  4042204,1268     Cuts: 424     33160
100,03%
*     0+    0                     3792713,4627    -1326,1327
100,03%
      0     0    -1326,0238  3908  3792713,4627     Cuts: 811     33624
100,03%
      0     0    -1325,9556  4005  3792713,4627     Cuts: 136     33736
100,03%
      0     0    -1325,9556  4004  3792713,4627      Cuts: 14     33743
100,03%
```

```
*     0+    0                     3316347,8081    -1325,9556
100,04%
*     0+    0                     2493592,4896    -1325,9556
100,05%
*     0+    0                     2145887,3102    -1325,9556
100,06%
*     0+    0                      404077,3738    -1325,9556
100,33%
*     0+    0                       -1187,3115    -1325,9556
11,68%
      0     2    -1325,9556  4004    -1187,3115    -1325,9556     33743
11,68%
Elapsed time = 22,12 sec. (26498,63 ticks, tree = 0,02 MB, solutions = 12)
      7     5    -1300,2503  3702    -1187,3115    -1325,6535     34311
11,65%
     15     9    -1306,7491  3108    -1187,3115    -1325,6535     35217
11,65%
     31    25    -1316,4082  3290    -1187,3115    -1325,1811     40545
11,61%
     60    27    -1296,6989  2812    -1187,3115    -1325,1811     42196
11,61%
    115    75    -1286,5972  2330    -1187,3115    -1325,0170     53613
11,60%
    162   132    -1321,0294  2918    -1187,3115    -1325,0170     67533
11,60%
*   171+  122                       -1229,7842    -1325,0170
7,74%
    185    85    -1293,4217  2134    -1229,7842    -1325,0170     56692
7,74%
*   186+  103                       -1260,4299    -1325,0170
5,12%
*   200+   87                       -1270,3015    -1325,0170
4,31%
    219   188    -1289,4564   965    -1270,3015    -1325,0170     82469
4,31%
*   239+  131                       -1274,4362    -1325,0170
3,97%
    264   209    -1289,1816  1921    -1274,4362    -1325,0170     88396
3,97%
    415   318    -1293,7174  1763    -1274,4362    -1325,0170    110277
3,97%
Elapsed time = 35,12 sec. (29772,51 ticks, tree = 6,67 MB, solutions = 17)
*   470+  348                       -1281,3419    -1325,0170
3,41%
*   543+  358                       -1282,1542    -1325,0170
3,34%
    616   396    -1298,0079   889    -1282,1542    -1325,0170    135433
3,34%
*   625+  382                       -1287,0533    -1325,0170
2,95%
*   692+  377                       -1288,9133    -1325,0170
2,80%
    828   418    -1310,8617  1928    -1288,9133    -1325,0170    171245
2,80%
```

```
    978   514    -1289,5304   450    -1288,9133    -1325,0170   185078
2,80%
   1128   616    -1302,2776  1842    -1288,9133    -1323,3964   200256
2,68%
   1302   766    -1306,3406  1959    -1288,9133    -1323,3964   223607
2,68%
   1488   949    -1292,3664   217    -1288,9133    -1323,3964   249495
2,68%
*  1498+  765                        -1299,4077    -1323,3964
1,85%
*  1510+  758                        -1305,2914    -1323,3964
1,39%
*  1528+  878                        -1305,2914    -1323,3964
1,39%
   1642   289        cutoff          -1305,2914    -1323,3964   266930
1,39%
   1802   350    -1308,9735  1407    -1305,2914    -1323,0514   285568
1,36%
   1968   496    -1307,1748  1158    -1305,2914    -1322,1032   322611
1,29%
   2178   607    -1305,6930  1641    -1305,2914    -1321,8582   347223
1,27%
Elapsed time = 79,16 sec. (39341,91 ticks, tree = 10,95 MB, solutions = 24)
   2340   770    -1306,2248   214    -1305,2914    -1321,6055   370474
1,25%
   2543   814    -1309,1821  1657    -1305,2914    -1321,3035   379589
1,23%
*  2609+  890                        -1305,3258    -1321,3035
1,22%
   2717   951    -1314,6938  2219    -1305,3258    -1320,3562   395964
1,15%
   2930  1086    -1310,5304  1726    -1305,3258    -1318,4810   420894
1,01%
   3130  1213    -1313,2943  1937    -1305,3258    -1318,0254   440973
0,97%
   3316  1317    -1315,0735  2022    -1305,3258    -1317,6617   455409
0,95%
   3529  1530    -1307,3298  1902    -1305,3258    -1317,4682   492271
0,93%
   3692  1716    -1308,2987  1669    -1305,3258    -1317,2567   524921
0,91%
   3825  1746    -1305,5589   244    -1305,3258    -1317,2567   529293
0,91%
   3831  1759    -1311,4602  2526    -1305,3258    -1317,2567   533279
0,91%
Elapsed time = 118,61 sec. (48959,36 ticks, tree = 42,94 MB, solutions =
25)

Performing restart 1

Repeating presolve.
Tried aggregator 3 times.
MIP Presolve eliminated 2175 rows and 2551 columns.
MIP Presolve modified 350 coefficients.
Aggregator did 384 substitutions.
Reduced MIP has 58250 rows, 58722 columns, and 161099 nonzeros.
Reduced MIP has 14997 binaries, 20 generals, 0 SOSs, and 28124 indicators.
Presolve time = 0,53 sec. (747,29 ticks)
Tried aggregator 1 time.
MIP Presolve modified 1739 coefficients.
Reduced MIP has 58440 rows, 58722 columns, and 161479 nonzeros.
Reduced MIP has 14997 binaries, 20 generals, 0 SOSs, and 27934 indicators.
Presolve time = 0,16 sec. (139,06 ticks)
Represolve time = 1,56 sec. (1039,28 ticks)
   3832     0    -1323,4037  3526    -1305,3258    Cuts: 9726   603952
0,90%
   3832     0    -1321,0413  3984    -1305,3258    Cuts: 4272   607299
0,90%
   3832     0    -1319,5270  3273    -1305,3258    Cuts: 2606   609365
0,90%
   3832     0    -1318,3592  3596    -1305,3258    Cuts: 1440   610970
0,90%
   3832     0    -1317,2873  3476    -1305,3258    Cuts: 1972   612284
0,90%
   3832     0    -1316,5356  3459    -1305,3258     Cuts: 814   613131
0,86%
   3832     0    -1315,7164  3567    -1305,3258     Cuts: 854   614330
0,80%
   3832     0    -1315,3055  3548    -1305,3258    Cuts: 1125   615354
0,76%
   3832     0    -1314,8519  3570    -1305,3258     Cuts: 338   616245
0,73%
   3832     0    -1314,3847  3563    -1305,3258    Cuts: 1023   616942
0,69%
   3832     0    -1313,9391  3731    -1305,3258     Cuts: 541   617858
0,66%
   3832     0    -1313,2570  3843    -1305,3258     Cuts: 808   619105
0,61%
   3832     0    -1312,7673  3938    -1305,3258     Cuts: 346   619455
0,57%
   3832     0    -1312,4758  3500    -1305,3258     Cuts: 387   619880
0,55%
   3832     0    -1312,2769  3334    -1305,3258     Cuts: 132   620448
0,53%
   3832     0    -1312,1334  3417    -1305,3258     Cuts: 614   620906
0,52%
   3832     0    -1311,9518  3237    -1305,3258     Cuts: 514   621613
0,51%
   3832     0    -1311,9315  3360    -1305,3258     Cuts: 419   622034
0,51%
   3832     0    -1311,7999  3340    -1305,3258      Cuts: 36   622173
0,50%
   3832     0    -1311,7227  3338    -1305,3258      Cuts: 29   622196
0,49%
   3832     0    -1311,6727  3336    -1305,3258      Cuts: 15   622220
0,49%
   3832     0    -1311,4963  3520    -1305,3258      Cuts: 92   622305
0,47%
   3832     0    -1311,4454  3239    -1305,3258      Cuts: 52   622398
0,47%
   3832     0    -1311,4295  3243    -1305,3258     Cuts: 130   622424
0,47%
   3832     2    -1311,4295  3243    -1305,3258    -1311,4295   622424
0,47%
   3844     7    -1306,7781  3120    -1305,3258    -1311,2192   622625
0,45%
   3968    65        cutoff          -1305,3258    -1311,2192   641137
0,45%
   4083   111        cutoff          -1305,3258    -1311,2192   664312
0,45%
   4225   171    -1305,5488  1816    -1305,3258    -1310,8963   680034
0,43%
   4397   278    -1307,4115  1330    -1305,3258    -1310,7915   701257
0,42%
   4589   359    -1306,2706  2973    -1305,3258    -1310,2292   731879
0,38%
   4743   399    -1306,9360  1521    -1305,3258    -1310,2292   754103
0,38%
   4950   452    -1306,1916  1028    -1305,3258    -1309,1211   768753
0,29%
   5210   620    -1305,4801  1039    -1305,3258    -1309,1211   804894
0,29%
Elapsed time = 280,72 sec. (204065,62 ticks, tree = 9,77 MB, solutions =
25)
   5432   770        cutoff          -1305,3258    -1309,1211   832560
0,29%
   5650   903    -1306,4967   762    -1305,3258    -1307,9111   853770
0,20%
   5938  1014    -1305,6319   748    -1305,3258    -1307,9111   866941
0,20%
   6185  1160    -1305,9061   922    -1305,3258    -1307,9111   898890
0,20%
   6473  1413    -1305,6319   653    -1305,3258    -1307,9111   928527
0,20%
   6760  1588    -1305,6319   474    -1305,3258    -1307,9111   948606
0,20%
   7083  1783    -1305,6318   141    -1305,3258    -1307,9111   972172
0,20%
   7433  1983    -1305,3261   785    -1305,3258    -1307,9111   991678
0,20%
   7846  2403    -1305,6318    48    -1305,3258    -1307,9111  1014783
0,20%
*  8183  2605      integral     0    -1305,6317    -1307,9111  1026566
0,17%
   8193  2687        cutoff          -1305,6317    -1307,9111  1029572
0,17%
Elapsed time = 321,39 sec. (213631,77 ticks, tree = 64,45 MB, solutions =
26)
   8394  1600    -1305,6546   885    -1305,6317    -1307,5344  1042658
0,15%
   8435  1657    -1305,6320   772    -1305,6317    -1307,5344  1044069
0,15%
   8512  1723    -1305,6320   728    -1305,6317    -1307,4844  1045591
0,14%
   8621  1710    -1306,8265  1378    -1305,6317    -1307,4653  1052663
0,14%
   8720  1830    -1305,6320   632    -1305,6317    -1307,4653  1056029
0,14%
   8791  1869    -1306,3178  1032    -1305,6317    -1307,4621  1068376
0,14%
   8864  1923    -1306,1818   873    -1305,6317    -1307,4621  1075582
0,14%
   8980  1969    -1306,0199   922    -1305,6317    -1307,4621  1082349
0,14%
   9072  2014    -1305,6319   330    -1305,6317    -1307,4621  1091156
0,14%
   9192  2028    -1306,5891  1049    -1305,6317    -1307,4096  1100715
0,14%
Elapsed time = 338,56 sec. (223311,61 ticks, tree = 46,03 MB, solutions =
26)
   9423  2176    -1305,6319   760    -1305,6317    -1307,4096  1112967
0,14%
   9685  2306    -1305,6319   958    -1305,6317    -1307,4096  1125035
0,14%
   9944  2520    -1305,6320   757    -1305,6317    -1307,4096  1136631
0,14%
  10205  2646    -1305,6319   639    -1305,6317    -1307,4096  1138247
0,14%
* 10256  2712      integral     0    -1305,6318    -1307,4096  1144311
0,14%
  10468  2988    -1305,6320   672    -1305,6318    -1307,4096  1164627
0,14%
  10683  3142    -1305,6319   673    -1305,6318    -1307,2401  1174169
0,12%
  10840  3180    -1305,6319   635    -1305,6318    -1307,2201  1174207
0,12%
  10939  3331    -1305,8761   925    -1305,6318    -1307,2201  1191866
0,12%
  11020  3336    -1306,2171  1049    -1305,6318    -1307,2201  1197291
0,12%
  11111  3374    infeasible          -1305,6318    -1307,1433  1222291
0,12%
Elapsed time = 366,31 sec. (232898,25 ticks, tree = 82,31 MB, solutions =
27)
  11208  3348    -1306,1687   740    -1305,6318    -1307,0800  1249163
0,11%
  11312  3329    -1306,3556  1980    -1305,6318    -1307,0772  1261928
0,11%
  11376  3318    -1305,9056  1139    -1305,6318    -1306,9483  1273313
0,10%
  11462  3314    -1305,6567   818    -1305,6318    -1306,9260  1292099
0,10%
  11564  3291        cutoff          -1305,6318    -1306,9260  1329204
0,10%
  11654  3293    -1306,1571   736    -1305,6318    -1306,8339  1338104
0,09%
```

```
  11769  3290        cutoff          -1305,6318    -1306,8115  1359983
0,09%
  11864  3256        cutoff          -1305,6318    -1306,8101  1384781
0,09%
  11956  3230        cutoff          -1305,6318    -1306,7967  1406172
0,09%
  12049  3206        cutoff          -1305,6318    -1306,7967  1421600
0,09%
Elapsed time = 387,36 sec. (242540,15 ticks, tree = 77,31 MB, solutions =
27)
  12125  3263    -1306,6074  1246    -1305,6318    -1306,7361  1388061
0,08%
  12156  3196    -1305,9774   920    -1305,6318    -1306,6383  1422621
0,08%
  12189  3128        cutoff          -1305,6318    -1306,6383  1463847
0,08%
  12247  3123        cutoff          -1305,6318    -1306,6279  1472539
0,08%
  12315  3110    -1306,1254   752    -1305,6318    -1306,5961  1484907
0,07%
  12386  3059        cutoff          -1305,6318    -1306,5789  1499944
0,07%
  12462  3024        cutoff          -1305,6318    -1306,5743  1520441
0,07%
  12549  2984        cutoff          -1305,6318    -1306,5446  1545208
0,07%
  12612  2992    -1305,6389  1043    -1305,6318    -1306,4401  1543913
0,06%
  12690  2916        cutoff          -1305,6318    -1306,4401  1570473
0,06%
Elapsed time = 405,06 sec. (252362,31 ticks, tree = 71,35 MB, solutions =
27)
  12783  2872        cutoff          -1305,6318    -1306,3404  1583188
0,05%
  12913  2835    -1306,0514   758    -1305,6318    -1306,3404  1591455
0,05%
  13114  2744        cutoff          -1305,6318    -1306,2834  1617719
0,05%
  13397  2519        cutoff          -1305,6318    -1306,1058  1670330
0,04%
  13595  2443    -1305,6658   932    -1305,6318    -1305,9319  1682385
0,02%

GUB cover cuts applied:  8
Clique cuts applied:  283
Cover cuts applied:  6932
Implied bound cuts applied:  5353
Flow cuts applied:  1764
Mixed integer rounding cuts applied:  1725
Zero-half cuts applied:  1
Lift and project cuts applied:  76
Gomory fractional cuts applied:  49

Root node processing (before b&c):
  Real time             =   21,67 sec. (26195,67 ticks)
Parallel b&c, 12 threads:
  Real time             =  399,08 sec. (231458,50 ticks)
  Sync time (average)   =   76,68 sec.
  Wait time (average)   =    0,09 sec.
                          ------------
Total (root+branch&cut) =  420,75 sec. (257654,17 ticks)
```

XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX

SCENARIO 3 :

LINEAR

```
Checking license ...
License found. [0,70 s]
Version identifier: 22.1.2.0 | 2024-12-09 | 8bd2200c8
Legacy callback                                   pi
Tried aggregator 3 times.
MIP Presolve eliminated 13809 rows and 15150 columns.
MIP Presolve modified 18 coefficients.
Aggregator did 1388 substitutions.
Reduced MIP has 4481 rows, 5087 columns, and 32022 nonzeros.
Reduced MIP has 1312 binaries, 72 generals, 0 SOSs, and 288 indicators.
Presolve time = 0,05 sec. (60,41 ticks)
Probing fixed 0 vars, tightened 289 bounds.
Probing time = 0,00 sec. (1,34 ticks)
Cover probing fixed 0 vars, tightened 1 bounds.
Tried aggregator 1 time.
Detecting symmetries...
MIP Presolve eliminated 1 rows and 1 columns.
MIP Presolve modified 1 coefficients.
Reduced MIP has 4480 rows, 5086 columns, and 31991 nonzeros.
Reduced MIP has 1313 binaries, 300 generals, 0 SOSs, and 288 indicators.
Presolve time = 0,02 sec. (13,34 ticks)
Probing time = 0,00 sec. (1,27 ticks)
Clique table members: 371.
MIP emphasis: balance optimality and feasibility.
MIP search method: dynamic search.
Parallel mode: deterministic, using up to 12 threads.
Root relaxation solution time = 0,05 sec. (45,97 ticks)

        Nodes                                         Cuts/
   Node  Left     Objective  IInf  Best Integer    Best Bound    ItCnt
Gap

      0     0    -2965,5239   410                 -2965,5239      762
*     0+    0                       9,20186e+09   -2965,5239
100,00%
*     0+    0                       4,21656e+08   -2965,5239
100,00%
      0     0    -2906,7786   421   4,21656e+08     Cuts: 857     1657
100,00%
      0     0    -2882,3189   135   4,21656e+08     Cuts: 732     2014
100,00%
*     0+    0                       1,76759e+07   -2882,3189
100,02%
      0     0    -2873,3978   121   1,76759e+07     Cuts: 205     2133
100,02%
*     0+    0                      5225708,7541   -2873,3978
100,05%
      0     0    -2872,3603   104  5225708,7541      Cuts: 84     2181
100,05%
*     0+    0                      4914100,1486   -2872,3603
100,06%
      0     0  -1,00000e+75     0  4914100,1486   -2872,3603     2181
100,06%
      0     0    -2872,0269   107  4914100,1486      Cuts: 31     2214
100,06%
Detecting symmetries...
*     0+    0                      2470628,2655   -2872,0269
100,12%
      0     0    -2871,8066   105  2470628,2655      Cuts: 18     2231
100,12%
      0     0    -2871,6423   105  2470628,2655      Cuts: 10     2237
100,12%
      0     0    -2871,5878   186  2470628,2655     MIRcuts: 1    2239
100,12%
*     0+    0                      1452373,5520   -2871,5878
100,20%
*     0+    0                      1452373,4327   -2869,9945
100,20%
*     0+    0                       418037,6190   -2869,9945
100,69%
*     0+    0                       106451,8479   -2869,9945
102,70%
      0     0  -1,00000e+75     0   106451,8479   -2869,9945     2239
102,70%
Detecting symmetries...
      0     2    -2871,5878   186   106451,8479   -2869,9945     2239
102,70%
Elapsed time = 1,09 sec. (946,54 ticks, tree = 0,02 MB, solutions = 10)
*    16+    2                       106446,2040   -2855,7129
102,68%
*    17+   10                        29288,4956   -2855,7129
109,75%
*    19+    6                        29255,3690   -2855,7129
109,76%
*    29+    8                        29250,1962   -2853,9621
109,76%
*    63+    8                        29250,1961   -2853,9621
109,76%
*   162+   95                        29244,4130   -2853,3746
109,76%
*   163+  110                        29240,0234   -2853,3746
109,76%
*   180+  129                        29204,5524   -2853,3746
109,77%
*   276+  167                        29204,3871   -2853,3746
109,77%
*   279+  178                        29194,2378   -2853,3746
109,77%
    380   283    -2835,4193   100    29194,2378   -2853,3746     6395
109,77%
*   637+  394                        -2789,2101   -2853,3746
2,30%
*   671+  434                        -2832,1795   -2853,3746
0,75%
   1150   515    -2841,9076    73    -2832,1795   -2853,3746    10727
0,75%
*  1165+  420                        -2832,5497   -2853,3746
0,74%
*  1316   584      integral     0    -2838,3061   -2853,3746    11464
0,53%
*  1634   643      integral     0    -2846,7366   -2851,8789    13654
0,18%
*  1701   721      integral     0    -2847,7574   -2851,8623    13882
0,14%
*  1734+  389                        -2848,3615   -2851,8623
0,12%
   1774   301    -2850,6483    93    -2848,3615   -2851,8623    14183
0,12%
*  2372   390      integral     0    -2848,6245   -2851,4944    16998
0,10%
   2728   329    -2851,1522    67    -2848,6245   -2851,3341    19438
0,10%
   3365   511    -2848,7729    57    -2848,6245   -2851,2060    30412
0,09%
   3836   517        cutoff          -2848,6245   -2850,2679    40724
0,06%
   4500   355        cutoff          -2848,6245   -2849,6609    53531
0,04%

Cover cuts applied:  72
Implied bound cuts applied:  203
Flow cuts applied:  183
Mixed integer rounding cuts applied:  215
Lift and project cuts applied:  5
Gomory fractional cuts applied:  54

Root node processing (before b&c):
  Real time             =    1,05 sec. (949,33 ticks)
Parallel b&c, 12 threads:
  Real time             =    4,22 sec. (1740,95 ticks)
  Sync time (average)   =    0,85 sec.
  Wait time (average)   =    0,00 sec.
                          ------------
Total (root+branch&cut) =    5,27 sec. (2690,29 ticks)
```

XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX

SCENARIO 3 :

NONLINEAR

```
Checking license ...
License found. [0,03 s]
Version identifier: 22.1.2.0 | 2024-12-09 | 8bd2200c8
CPXPARAM_Tune_Display                            3
CPXPARAM_Output_WriteLevel                       1
CPXPARAM_MIP_Tolerances_MIPGap                   0.00014999999999999999
Legacy callback                                  pi
Tried aggregator 3 times.
MIP Presolve eliminated 14415 rows and 15903 columns.
MIP Presolve modified 18 coefficients.
Aggregator did 1519 substitutions.
Reduced MIP has 41664 rows, 41835 columns, and 115008 nonzeros.
Reduced MIP has 10576 binaries, 72 generals, 0 SOSs, and 18816 indicators.
Presolve time = 0,17 sec. (535,60 ticks)
Probing fixed 0 vars, tightened 289 bounds.
Probing time = 0,42 sec. (27,31 ticks)
Tried aggregator 1 time.
Detecting symmetries...
MIP Presolve eliminated 435 rows and 0 columns.
MIP Presolve modified 1 coefficients.
Reduced MIP has 41229 rows, 41835 columns, and 114138 nonzeros.
Reduced MIP has 10577 binaries, 300 generals, 0 SOSs, and 18816 indicators.
Presolve time = 0,12 sec. (150,34 ticks)
Probing time = 0,11 sec. (10,73 ticks)
Clique table members: 19091.
MIP emphasis: balance optimality and feasibility.
MIP search method: dynamic search.
Parallel mode: deterministic, using up to 12 threads.
Root relaxation solution time = 1,56 sec. (2791,76 ticks)

        Nodes                                         Cuts/
   Node  Left     Objective  IInf  Best Integer    Best Bound    ItCnt
Gap

*     0+    0                       9,02352e+09
---
*     0+    0                       2,97149e+08
---
      0     0    -2965,5239   385   2,97149e+08    -2965,5239       41
100,00%
*     0+    0                      1811606,6221    -2965,5239
100,16%
      0     0    -2898,1959   443  1811606,6221     Cuts: 988     3158
100,16%
      0     0    -2879,5567   228  1811606,6221    Cuts: 9459     3992
100,16%
      0     0    -2873,9978   382  1811606,6221    Cuts: 1042     4837
100,16%
*     0+    0                      1811606,6219    -2873,9978
100,16%
      0     0  -1,00000e+75     0  1811606,6219    -2873,9978     4837
100,16%
      0     0    -2872,8199   500  1811606,6219     Cuts: 447     5007
100,16%
      0     0    -2872,7707   394  1811606,6219     Cuts: 214     5111
100,16%
      0     0    -2872,7588   487  1811606,6219     Cuts: 108     5199
100,16%
      0     0    -2872,7282   477  1811606,6219     Cuts: 122     5308
100,16%
      0     0    -2872,7282   475  1811606,6219       Cuts: 5     5312
100,16%
*     0+    0                        926367,5343    -2872,7282
100,31%
*     0+    0                         29212,3929    -2872,7282
109,83%
      0     2    -2872,7282   475    29212,3929    -2872,7282     5312
109,83%
Elapsed time = 8,92 sec. (10876,07 ticks, tree = 0,02 MB, solutions = 6)
      4     4    -2855,7634   480    29212,3929    -2860,5543     5836
109,79%
     17    18    -2852,1856   271    29212,3929    -2854,2059     9438
109,77%
     41    25    -2828,2763   257    29212,3929    -2853,8297    11320
109,77%
     72    61    -2814,6385   208    29212,3929    -2853,8297    23295
109,77%
    128    96    -2829,4909   104    29212,3929    -2853,8297    34531
109,77%
    215   168    -2750,9324   628    29212,3929    -2853,8297    40826
109,77%
*   236+  122                        -2814,6737    -2853,8297
1,39%
    265   145    -2850,8141   255    -2814,6737    -2853,8297    48476
1,39%
*   271+  127                        -2824,7015    -2853,8297
1,03%
    340   125    -2848,1058    36    -2824,7015    -2853,8297    50566
1,03%
    413   165    -2844,6750    14    -2824,7015    -2853,8297    54044
1,03%
*   648+  264                        -2828,1011    -2853,5442
0,90%
*   717   373      integral     0    -2840,9986    -2853,5442    62111
0,44%
Elapsed time = 20,70 sec. (13753,67 ticks, tree = 4,29 MB, solutions = 9)
*   749   333      integral     0    -2841,9001    -2853,5442    60862
0,41%
*   855+  280                        -2847,0990    -2853,5442
0,23%
*  1070   142      integral     0    -2847,2027    -2853,5442    68331
0,22%
*  1120   146      integral     0    -2848,6245    -2853,5442    71062
0,17%
*  1182   132      integral     0    -2848,6245    -2852,6036    70924
0,14%
   1440   143    -2848,6251     1    -2848,6245    -2851,6696    75391
0,11%
   1958   345    -2848,6263     3    -2848,6245    -2851,6615    84441
0,11%
   2420   425        cutoff          -2848,6245    -2851,4644    89948
0,10%

Clique cuts applied:  73
Cover cuts applied:  2290
Implied bound cuts applied:  4089
Flow cuts applied:  2179
Mixed integer rounding cuts applied:  1165
Lift and project cuts applied:  1
Gomory fractional cuts applied:  55

Root node processing (before b&c):
  Real time             =    8,61 sec. (10689,36 ticks)
Parallel b&c, 12 threads:
  Real time             =   30,80 sec. (7637,98 ticks)
  Sync time (average)   =   11,09 sec.
  Wait time (average)   =    0,00 sec.
                          ------------
Total (root+branch&cut) =   39,41 sec. (18327,34 ticks)
```

C Python Codes

C.1 Benders Decomposition from Section 7.1

```python
import docplex.mp
import pandas as pd
from collections import defaultdict
import math
### MODEL ###
from docplex.mp.model import Model
from numpy.distutils.extension import cxx_ext_re
from docplex.mp.solution import SolveSolution
sol = SolveSolution #(name="MicrogridOtimizerSolution")
m = Model(name = "MicrogridOptimizer")
# m.parameters.benders.strategy = 3 #CPLEX ignores any annotation file supplied with the model; CPLEX applies presolve; CPLEX then automatically generates a Benders partition, putting integer variables in master and continuous linear variables into disjoint subproblems. CPLEX then solves the Benders decomposition of the model
# m.parameters.benders.strategy = 1 #of m.context.cplex_parameters... Benders is guaranteed and own selection of annotations are used
# m.parameters.benders.strategy = 2 #CPLEX accepts the master as given and attempts to decompose the remaining elements into disjoint subproblems to assign to workers. It then solves the Benders decomposition of the model
# m.parameters.benders.strategy = 0 #AUTO If annotations specifying a Benders partition of the current model are available, CPLEX attempts to decompose the model. CPLEX uses the master as given by the annotations, and attempts to partition the subproblems further, if possible, before applying Benders algorithm to solve the model
m.parameters.benders.strategy = -1 #Ignore any .benders_annotation values and solve monolithically
m.time_limit = 30*60
m.parameters.mip.tolerances.mipgap.set(1.5e-4)

#decision variable annotations

InterGenActivePower = m.continuous_var_dict([(i, t) for i in isINTER_E_GENS for t in isDECISION_STEPS], name='InterGenActivePower', lb=0)
for v in InterGenActivePower.values():
    v.benders_annotation = 1  # continuous variables in Subproblem 1


IsCharging = m.binary_var_dict([(s, t) for s in isE_STORAGES.union(isH_STORAGES) for t in isDECISION_STEPS], name='IsCharging')
for v in IsCharging.values():
    v.benders_annotation = 0  # discrete variables in Masterproblem 0
```

C.2 Benders Execution Log from Section 7.3.2

```
Python log 4D scenario Benders Strategy 2:

C:\Users\florine.laseur\Envs\venv36\Scripts\python.exe
C:\Users\florine.laseur\Downloads\MicrogridOptimizer.py

constraint atteint

CPXPARAM_TimeLimit                  1800

CPXPARAM_Read_DataCheck              1

CPXPARAM_Benders_Strategy            2

CPXPARAM_MIP_Tolerances_MIPGap       0.00014999999999999999

Tried aggregator 2 times.

MIP Presolve eliminated 41474 rows and 34065 columns.

MIP Presolve added 3 rows and 0 columns.

MIP Presolve modified 974 coefficients.

Aggregator did 4398 substitutions.

Reduced MIP has 11209 rows, 9394 columns, and 64289 nonzeros.

Reduced MIP has 2880 binaries, 278 generals, 0 SOSs, and 0 indicators.

Presolve time = 0.06 sec. (77.50 ticks)

Found incumbent of value 6638.501746 after 0.14 sec. (176.46 ticks)

Tried aggregator 1 time.

MIP Presolve eliminated 291 rows and 0 columns.

MIP Presolve added 3 rows and 0 columns.

MIP Presolve modified 289 coefficients.

Reduced MIP has 10921 rows, 9394 columns, and 63703 nonzeros.

Reduced MIP has 2881 binaries, 277 generals, 0 SOSs, and 0 indicators.

Presolve time = 0.02 sec. (19.66 ticks)


    Nodes                    Cuts/

  Node  Left   Objective  IInf  Best Integer   Best Bound   ItCnt   Gap

0   0   -10000.0000       Benders: 1   0
0   0   -10000.0000       Benders: 1   2
0   0   -9981.9257        Benders: 1   391
0   0   -8555.7099        Benders: 1   743
0   0   -8555.7099        Benders: 1   744
0   0   -8552.8823        Benders: 1   748
0   0   -8552.8823        Benders: 1   749
0   0   -7787.6659        Benders: 1   981
0   0   -7787.6659        Benders: 1   982
0   0   -7787.6659        Benders: 1   983
0   0   -7787.6659        Benders: 1   984
0   0   -7787.6659        Benders: 1   985
0   0   -7787.6659        Benders: 1   986
0   0   -7586.8181        Benders: 1   1084
0   0   -7586.8181        Benders: 1   1088
0   0   -7586.8181        Benders: 1   1092
0   0   -7586.8181        Benders: 1   1095
0   0   -7586.8181        Benders: 1   1097
0   0   -7586.8181        Benders: 1   1100
0   0   -5753.9000        Benders: 1   1455
0   0   -5745.7640        Benders: 1   1498
0   0   -5745.7640        Benders: 1   1499
0   0   -5638.8802        Benders: 1   1664
0   0   -5638.8802        Benders: 1   1665
0   0   -5638.8802        Benders: 1   1666
0   0   -5638.8802        Benders: 1   1667
0   0   -5638.8802        Benders: 1   1668
0   0   -5429.4721        Benders: 1   1787
0   0   -4767.9968        Benders: 1   2196
0   0   -4767.9968        Benders: 1   2197
0   0   -4767.9968        Benders: 1   2198
0   0   -4767.9968        Benders: 1   2199
0   0   -4767.9968        Benders: 1   2200
0   0   -4767.9968        Benders: 1   2201
0   0   -4036.9365        Benders: 1   2443
0   0   -3998.6244        Benders: 1   2513
0   0   -3998.6244        Benders: 1   2514
0   0   -3998.6244        Benders: 1   2515
0   0   -3998.6244        Benders: 1   2516
0   0   -3998.6244        Benders: 1   2517
0   0   -3998.6244        Benders: 1   2518
0   0   -3913.5818        Benders: 1   2594
0   0   -3913.5818        Benders: 1   2595
0   0   -3913.5818        Benders: 1   2596
0   0   -3913.5818        Benders: 1   2597
0   0   -3913.5818        Benders: 1   2598
0   0   -3913.5818        Benders: 1   2599
0   0   -3823.0472        Benders: 1   2638
0   0   -3785.3986        Benders: 1   2648
0   0   -3785.3986        Benders: 1   2649
0   0   -3785.3986        Benders: 1   2650
0   0   -3447.8010        Benders: 1   2875
0   0   -3447.8010        Benders: 1   2938
0   0   -3446.5949        Benders: 1   2986
0   0   -3441.6310        Benders: 1   3014
0   0   -3440.2138        Benders: 1   3044
0   0   -3433.9332        Benders: 1   3101
0   0   -3432.8156        Benders: 1   3115
0   0   -3428.2316        Benders: 1   3154
0   0   -3423.4392        Benders: 1   3192
0   0   -3404.4283        Benders: 1   3238
0   0   -3401.5072        Benders: 1   3260
0   0   -3397.0912        Benders: 1   3291
0   0   -3360.4188        Benders: 1   3359
0   0   -3357.1261        Benders: 1   3385
0   0   -3350.5524        Benders: 1   3416
0   0   -3349.0237        Benders: 1   3431
0   0   -3339.0067        Benders: 1   3492
0   0   -3337.4171        Benders: 1   3510
0   0   -3329.5977        Benders: 1   3549
0   0   -3325.2051        Benders: 1   3574
0   0   -3315.7861        Benders: 1   3629
0   0   -3315.7861        Benders: 1   3630
0   0   -3312.0503        Benders: 1   3658
0   0   -3309.2843        Benders: 1   3683
0   0   -3306.7112        Benders: 1   3715
0   0   -3302.4107        Benders: 1   3765
0   0   -3300.5003        Benders: 1   3787
0   0   -3297.4654        Benders: 1   3821
0   0   -3295.4696        Benders: 1   3843
0   0   -3293.8666        Benders: 1   3861
0   0   -3286.5462        Benders: 1   3917
0   0   -3280.4774        Benders: 1   3948
0   0   -3277.3876        Benders: 1   3995
```

0	0	-3274.8096	Benders: 1	4032
0	0	-3272.5525	Benders: 1	4082
0	0	-3266.6328	Benders: 1	4167
0	0	-3265.5703	Benders: 1	4193
0	0	-3264.0914	Benders: 1	4224
0	0	-3260.8495	Benders: 1	4265
0	0	-3259.7125	Benders: 1	4283
0	0	-3257.6007	Benders: 1	4320
0	0	-3257.2427	Benders: 1	4336
0	0	-3255.8026	Benders: 1	4367
0	0	-3254.9246	Benders: 1	4398
0	0	-3249.8774	Benders: 1	4432
0	0	-3248.9549	Benders: 1	4450
0	0	-3248.2493	Benders: 1	4470
0	0	-3245.3022	Benders: 1	4529
0	0	-3244.2895	Benders: 1	4557
0	0	-3242.7284	Benders: 1	4587
0	0	-3241.9477	Benders: 1	4610
0	0	-3241.0976	Benders: 1	4648
0	0	-3239.3743	Benders: 1	4683
0	0	-3238.3677	Benders: 1	4709
0	0	-3236.7286	Benders: 1	4746
0	0	-3236.3708	Benders: 1	4758
0	0	-3235.2569	Benders: 1	4786
0	0	-3233.1829	Benders: 1	4826
0	0	-3232.5889	Benders: 1	4851
0	0	-3232.5235	Benders: 1	4863
0	0	-3228.4322	Benders: 1	4910

0	0	-3228.0011	Benders: 1	4936
0	0	-3226.4407	Benders: 1	4986
0	0	-3226.2776	Benders: 1	4998
0	0	-3223.6762	Benders: 1	5049
0	0	-3222.8324	Benders: 1	5086
0	0	-3222.5138	Benders: 1	5107
0	0	-3222.1887	Benders: 1	5123
0	0	-3221.2933	Benders: 1	5159
0	0	-3220.6231	Benders: 1	5200
0	0	-3220.3420	Benders: 1	5212
0	0	-3219.8894	Benders: 1	5231
0	0	-3219.8198	Benders: 1	5232
0	0	-3219.8198	Benders: 1	5236
0	0	-3219.8198	Benders: 1	5241
0	0	-3219.0483	Benders: 1	5270
0	0	-3190.2110	Benders: 1	5450
0	0	-3190.2110	Benders: 1	5451
0	0	-3190.2110	Benders: 1	5452
0	0	-3190.2110	Benders: 1	5453
0	0	-3190.2110	Benders: 1	5454
0	0	-3190.2110	Benders: 1	5455
0	0	-3188.9774	Benders: 1	5502
0	0	-3165.3285	Benders: 1	5632
0	0	-3165.3285	Benders: 1	5633
0	0	-3165.3285	Benders: 1	5634
0	0	-3165.3285	Benders: 1	5635
0	0	-3165.3285	Benders: 1	5665
0	0	-3165.3285	Benders: 1	5666

0	0	-3164.7955	Benders: 1	5697
0	0	-3163.2869	Benders: 1	5708
0	0	-3161.7550	Benders: 1	5746
0	0	-3161.7550	Benders: 1	5755
0	0	-3161.7550	Benders: 1	5756
0	0	-3161.7550	Benders: 1	5757
0	0	-3161.7550	Benders: 1	5758
0	0	-3161.7550	Benders: 1	5759
0	0	-3135.8323	Benders: 1	5833
0	0	-3135.7373	Benders: 1	5839
0	0	-3135.7373	Benders: 1	5840
0	0	-3135.7373	Benders: 1	5841
0	0	-3135.7373	Benders: 1	5842
0	0	-3135.7373	Benders: 1	5843
0	0	-3134.1608	Benders: 1	5848
0	0	-3133.1999	Benders: 1	5851
0	0	-3133.1999	Benders: 1	5852
0	0	-3133.1999	Benders: 1	5894
0	0	-3130.2908	Benders: 1	5914
0	0	-3130.0108	Benders: 1	5927
0	0	-3130.0108	Benders: 1	5928
0	0	-3128.7059	Benders: 1	5951
0	0	-3127.3721	Benders: 1	5975
0	0	-3125.7926	Benders: 1	6005
0	0	-3124.4240	Benders: 1	6069
0	0	-3124.0034	Benders: 1	6087
0	0	-3123.0005	Benders: 1	6118
0	0	-3121.4226	Benders: 1	6172

0	0	-3120.5049	Benders: 1	6205
0	0	-3120.5049	Benders: 1	6206
0	0	-3120.2811	Benders: 1	6222
0	0	-3117.9365	Benders: 1	6246
0	0	-3117.0126	Benders: 1	6282
0	0	-3116.4047	Benders: 1	6319
0	0	-3114.6715	Benders: 1	6367
0	0	-3114.0409	Benders: 1	6390
0	0	-3112.7446	Benders: 1	6454
0	0	-3112.1845	Benders: 1	6489
0	0	-3111.8791	Benders: 1	6524
0	0	-3111.4309	Benders: 1	6562
0	0	-3110.7168	Benders: 1	6600
0	0	-3110.0873	Benders: 1	6636
0	0	-3109.5627	Benders: 1	6674
0	0	-3109.2508	Benders: 1	6697
0	0	-3108.7090	Benders: 1	6748
0	0	-3108.0374	Benders: 1	6790
0	0	-3107.3688	Benders: 1	6844
0	0	-3107.1320	Benders: 1	6869
0	0	-3106.3718	Benders: 1	6927
0	0	-3105.9564	Benders: 1	6991
0	0	-3105.6469	Benders: 1	7018
0	0	-3105.0954	Benders: 1	7069
0	0	-3104.5700	Benders: 1	7102
0	0	-3104.2340	Benders: 1	7140
0	0	-3103.1973	Benders: 1	7173
0	0	-3096.3569	Benders: 1	7262

```
0  0  -3095.4750          Benders: 1   7336
0  0  -3094.9722          Benders: 1   7414
0  0  -3093.7220          Benders: 1   7497
0  0  -3093.2321          Benders: 1   7542
0  0  -3093.0204          Benders: 1   7597
0  0  -3092.8250          Benders: 1   7667
0  0  -3092.5408          Benders: 1   7726
0  0  -3092.2579          Benders: 1   7778
0  0  -3091.9543          Benders: 1   7822
0  0  -3091.9261          Benders: 1   7850
0  0  -3091.9261          Benders: 1   7851
0  0  -3091.6421          Benders: 1   7913
0  0  -3091.5600          Benders: 1   7944
0  0  -3091.4956          Benders: 1   7958
0  0  -3090.1957          Benders: 1   8058
0  0  -3089.5760          Benders: 1   8169
0  0  -3089.5760          Benders: 1   8170
0  0  -3089.2630          Benders: 1   8239
0  0  -3089.2630          Benders: 1   8240
0  0  -3088.3845          Benders: 1   8312
0  0  -3087.8485          Benders: 1   8396
0  0  -3087.5320          Benders: 1   8434
0  0  -3087.5320          Benders: 1   8435
0  0  -3086.5535          Benders: 1   8502
0  0  -3086.1974          Benders: 1   8549
0  0  -3085.9462          Benders: 1   8601
0  0  -3085.5086          Benders: 1   8674
0  0  -3085.0942          Benders: 1   8741
0  0  -3084.3066          Benders: 1   8806
0  0  -3084.1651          Benders: 1   8844
0  0  -3083.8044          Benders: 1   8924
0  0  -3083.4778          Benders: 1   8986
0  0  -3083.1802          Benders: 1   9046
0  0  -3082.2689          Benders: 1   9163
0  0  -3082.2330          Benders: 1   9195
0  0  -3081.5713          Benders: 1   9280
0  0  -3080.9625          Benders: 1   9368
0  0  -3080.6468          Benders: 1   9440
0  0  -3080.5027          Benders: 1   9497
0  0  -3080.3015          Benders: 1   9567
0  0  -3079.9342          Benders: 1   9630
0  0  -3072.9773          Benders: 1   9856
0  0  -3072.7514          Benders: 1   9882
0  0  -3072.7514          Benders: 1   9883
0  0  -3071.7701          Benders: 1   9934
0  0  -3071.6486          Benders: 1   9969
0  0  -3070.8300          Benders: 1   10069
0  0  -3070.8300          Benders: 1   10070
0  0  -3070.2310          Benders: 1   10136
0  0  -3069.8839          Benders: 1   10195
0  0  -3069.0022          Benders: 1   10301
0  0  -3068.5549          Benders: 1   10363
0  0  -3068.2031          Benders: 1   10408
0  0  -3067.2432          Benders: 1   10512
0  0  -3066.6661          Benders: 1   10583
Tried aggregator 1 time.

MIP Presolve eliminated 5 rows and 1 columns.
MIP Presolve modified 206 coefficients.
Reduced MIP has 1753 rows, 3158 columns, and 338002 nonzeros.
Reduced MIP has 2880 binaries, 277 generals, 0 SOSs, and 0 indicators.
Presolve time = 0.13 sec. (170.69 ticks)
Probing fixed 0 vars, tightened 3 bounds.
Probing time = 0.00 sec. (4.08 ticks)
Tried aggregator 1 time.
MIP Presolve eliminated 2 rows and 0 columns.
MIP Presolve modified 2 coefficients.
Reduced MIP has 1751 rows, 3158 columns, and 337982 nonzeros.
Reduced MIP has 2882 binaries, 275 generals, 0 SOSs, and 0 indicators.
Presolve time = 0.09 sec. (83.93 ticks)
Probing time = 0.00 sec. (3.64 ticks)
Clique table members: 474.
MIP emphasis: balance optimality and feasibility.
MIP search method: dynamic search.
Parallel mode: deterministic, using up to 12 threads.
Root relaxation solution time = 0.44 sec. (415.83 ticks)
*   0+  0              6638.5017  -3066.2477        146.19%
    0   0  -3065.9992  133   6638.5017  -3065.9992  11456  146.19%
    0   0  -3061.7457  133   6638.5017     Cuts: 5  11519  146.12%
    0   0  -3061.7396  131   6638.5017     Cuts: 2  11529  146.12%
    0   0  -3061.7396  131   6638.5017  Benders: 1  11530  146.12%
    0   0  -3061.7396  131   6638.5017  Benders: 1  11531  146.12%

Repeating presolve.
Tried aggregator 1 time.
Reduced MIP has 1751 rows, 3158 columns, and 337982 nonzeros.
Reduced MIP has 2882 binaries, 275 generals, 0 SOSs, and 0 indicators.
Presolve time = 0.05 sec. (66.32 ticks)
Probing time = 0.06 sec. (3.64 ticks)
Tried aggregator 1 time.
Reduced MIP has 1751 rows, 3158 columns, and 337982 nonzeros.
Reduced MIP has 2882 binaries, 275 generals, 0 SOSs, and 0 indicators.
Presolve time = 0.06 sec. (66.30 ticks)
Represolve time = 0.22 sec. (163.43 ticks)
Probing time = 0.00 sec. (3.64 ticks)
Clique table members: 468.
MIP emphasis: balance optimality and feasibility.
MIP search method: dynamic search.
Parallel mode: deterministic, using up to 12 threads.
Root relaxation solution time = 0.56 sec. (504.79 ticks)
*   0+  0              6638.5017  -3061.7396        146.12%
    0   0  -3061.7396  131   6638.5017  -3061.7396  23136  146.12%
    0   0  -3061.7396  131   6638.5017     Cuts: 5  23139  146.12%
    0   0  -3061.7396  131   6638.5017  Benders: 1  23140  146.12%
    0   2  -3061.7396  131   6638.5017  -3061.7396  23140  146.12%
Elapsed time = 17.91 sec. (15856.85 ticks, tree = 0.01 MB, solutions = 0)
    2   4  -3020.2047   69   6638.5017  -3061.7300  23380  146.12%
   12  14  -3053.1932   73   6638.5017  -3060.4201  23786  146.10%
   24  25  -3016.8925   45   6638.5017  -3060.4201  24382  146.10%
  104  55  -3041.4095   53   6638.5017  -3053.2381  25783  145.99%
  139  75  -2927.2862   58   6638.5017  -3053.2381  27100  145.99%
  164 104  -2677.0603   62   6638.5017  -3053.2381  28691  145.99%
  191 128  -3017.5263   36   6638.5017  -3053.2381  29421  145.99%
```

```
234 163 -3016.8488 42 6638.5017 -3053.2381 30430 145.99%
307 219 -2996.4301 42 6638.5017 -3053.2381 31544 145.99%
601 450 -3016.6635 26 6638.5017 -3053.2381 36127 145.99%
Elapsed time = 24.25 sec. (19175.83 ticks, tree = 6.45 MB, solutions = 0)
841 738 -3015.6719 35 6638.5017 -3053.2381 41062 145.99%
1152 1011 -3014.7547 21 6638.5017 -3053.2381 47521 145.99%
1493 1227 -2993.3153 36 6638.5017 -3053.2381 52332 145.99%
1747 1574 -2988.9404 74 6638.5017 -3053.2381 63610 145.99%
1938 1772 -2996.4153 18 6638.5017 -3053.2381 73341 145.99%
1988 1845 -3027.6009 37 6638.5017 -3053.2381 78831 145.99%
1990 1992 -3056.0792 153 6638.5017 -3053.2381 112103 145.99%
1991 1993 -3052.1029 147 6638.5017 -3052.0419 112176 145.97%
1996 1284 -2988.9377 66 6638.5017 -3048.2038 112559 145.92%
2009 274 -3030.5373 42 6638.5017 -3042.5974 113946 145.83%
Elapsed time = 46.89 sec. (39779.78 ticks, tree = 5.22 MB, solutions = 0)
2085 58 -2978.8559 45 6638.5017 -3042.5974 115288 145.83%
2219 153 -2980.4389 27 6638.5017 -3042.5620 119057 145.83%
2308 245 -3028.3325 41 6638.5017 -3042.5620 123303 145.83%
2451 376 -2637.1714 4 6638.5017 -3042.5620 127641 145.83%
2619 507 -2979.2952 35 6638.5017 -3042.5620 133275 145.83%
2754 659 -3025.6056 36 6638.5017 -3042.5620 138117 145.83%
2941 867 -2980.6201 37 6638.5017 -3042.5620 144142 145.83%
3130 946 -3019.8885 22 6638.5017 -3042.5620 146071 145.83%
3406 1166 -3019.5543 12 6638.5017 -3042.5620 151244 145.83%
3612 1347 -3027.9928 35 6638.5017 -3042.5620 155295 145.83%
Elapsed time = 68.39 sec. (49427.79 ticks, tree = 2.31 MB, solutions = 0)
3846 1649 -3014.8560 18 6638.5017 -3042.5620 162523 145.83%
4080 1789 -3023.6079 37 6638.5017 -3042.5620 168531 145.83%

4311 2089 -3014.1305 10 6638.5017 -3042.5620 171379 145.83%
4516 2326 -3018.4538 29 6638.5017 -3042.5620 177770 145.83%
4658 2412 -3016.5305 18 6638.5017 -3042.5620 180665 145.83%
4785 2666 -2949.7345 33 6638.5017 -3042.5620 185742 145.83%
5077 2916 -2598.6448 12 6638.5017 -3042.5620 189063 145.83%
5331 3162 -2598.4419 14 6638.5017 -3042.5620 192600 145.83%
5577 3268 -2997.0915 30 6638.5017 -3042.5620 194329 145.83%
5846 3461 -2991.3714 11 6638.5017 -3042.5620 196569 145.83%
Elapsed time = 90.92 sec. (59387.74 ticks, tree = 5.94 MB, solutions = 0)
6087 3681 -2947.1477 19 6638.5017 -3042.5620 200685 145.83%
6456 4171 -2863.2466 8 6638.5017 -3042.5620 208529 145.83%
6837 4276 -3026.5455 20 6638.5017 -3042.5620 210262 145.83%
7126 4884 -2908.9267 12 6638.5017 -3042.5620 214924 145.83%
7468 5187 -2978.3357 7 6638.5017 -3042.5620 219032 145.83%
7796 5413 -2378.6818 7 6638.5017 -3042.5620 220680 145.83%
8218 5768 -2539.1406 10 6638.5017 -3042.5620 222530 145.83%
8567 5736 -2456.9393 11 6638.5017 -3042.5620 222820 145.83%
8852 6568 -3027.8064 11 6638.5017 -3042.5620 228038 145.83%
9103 6797 -2455.0237 8 6638.5017 -3042.5620 230681 145.83%
Elapsed time = 116.09 sec. (68966.77 ticks, tree = 11.77 MB, solutions = 0)
9348 7138 -3022.7292 9 6638.5017 -3042.5620 234194 145.83%
9589 7389 -2078.4385 9 6638.5017 -3042.5620 237967 145.83%
9809 7583 -3039.7535 56 6638.5017 -3042.5620 241626 145.83%
9959 7626 -2420.2291 12 6638.5017 -3042.5620 241959 145.83%
10127 7908 -909.9732 1 6638.5017 -3042.5620 245707 145.83%
10215 8048 -2661.1041 20 6638.5017 -3042.5620 249315 145.83%
10381 8155 -2651.8692 10 6638.5017 -3042.5620 249937 145.83%
10509 8353 -2017.2033 14 6638.5017 -3042.5620 253846 145.83%

10619 8454 -2385.8274 12 6638.5017 -3042.5620 255343 145.83%
10743 8469 -2379.0054 12 6638.5017 -3042.5620 255524 145.83%
Elapsed time = 139.95 sec. (78571.28 ticks, tree = 14.79 MB, solutions = 0)
10823 8651 -3014.2345 31 6638.5017 -3042.5620 259098 145.83%
10969 8708 -2374.2926 15 6638.5017 -3042.5620 259181 145.83%
11056 8841 -3034.1695 22 6638.5017 -3042.5620 262910 145.83%
11168 8952 -2300.9785 10 6638.5017 -3042.5620 264400 145.83%
11274 9053 -2407.3358 8 6638.5017 -3042.5620 264741 145.83%
11420 9258 -2405.1936 5 6638.5017 -3042.5620 267879 145.83%
* 11455+ 9263 -1627.4764 -3042.5620 86.95%
11535 9196 -3027.5112 8 -1627.4764 -3042.5620 270250 86.95%
11613 9224 -3026.5703 12 -1627.4764 -3042.5620 270506 86.95%
11698 9327 -3033.1504 85 -1627.4764 -3042.5620 273489 86.95%
11761 9445 -3029.6234 12 -1627.4764 -3042.5620 277180 86.95%
Elapsed time = 163.98 sec. (88216.08 ticks, tree = 16.86 MB, solutions = 1)
11848 9434 -3023.8696 25 -1627.4764 -3042.5620 277809 86.95%
11914 9510 -3041.2849 74 -1627.4764 -3042.5620 281966 86.95%
12068 9545 -3037.4966 68 -1627.4764 -3042.5620 285417 86.95%
12141 9756 -3022.7155 35 -1627.4764 -3042.5620 293877 86.95%
12245 9790 -3036.0696 63 -1627.4764 -3042.5620 295483 86.95%
12319 9943 -3035.7727 50 -1627.4764 -3042.5620 302734 86.95%
12414 10010 -3008.1774 10 -1627.4764 -3042.5620 304726 86.95%
12511 10041 -3024.3048 26 -1627.4764 -3042.5620 306820 86.95%
12638 10132 -3024.1690 56 -1627.4764 -3042.5620 312098 86.95%
12751 10272 -3036.2534 72 -1627.4764 -3042.5620 316012 86.95%
Elapsed time = 192.84 sec. (97862.08 ticks, tree = 18.54 MB, solutions = 1)
12839 10495 -3038.7220 30 -1627.4764 -3042.5620 321616 86.95%
12973 10462 -3019.2880 38 -1627.4764 -3042.5620 321305 86.95%

13112 10672 -3038.1835 29 -1627.4764 -3042.5620 328030 86.95%
13260 10637 -2895.8139 8 -1627.4764 -3042.5620 327139 86.95%
13436 10834 -3022.2727 28 -1627.4764 -3042.5620 332255 86.95%
13628 10989 -2881.2777 7 -1627.4764 -3042.5620 332842 86.95%
13830 11290 -3036.8090 23 -1627.4764 -3042.5620 340656 86.95%
13986 11456 -2631.9426 8 -1627.4764 -3042.5620 344667 86.95%
14175 11648 -3035.6237 12 -1627.4764 -3042.5620 347095 86.95%
14306 11823 -2315.8835 3 -1627.4764 -3042.5620 351691 86.95%
Elapsed time = 223.13 sec. (107429.54 ticks, tree = 21.33 MB, solutions = 1)
14437 11765 -3032.4358 5 -1627.4764 -3042.5620 350243 86.95%
14579 12098 -3024.5292 44 -1627.4764 -3042.5620 356799 86.95%
14714 12166 -3025.3230 15 -1627.4764 -3042.5620 357145 86.95%
14882 12406 -3029.2104 19 -1627.4764 -3042.5620 361387 86.95%
14977 12432 -3024.9127 15 -1627.4764 -3042.5620 362068 86.95%
15115 12597 -3006.7035 26 -1627.4764 -3042.5620 363166 86.95%
15219 12713 -3035.8133 34 -1627.4764 -3041.7867 366719 86.90%
15321 12808 -1827.1548 6 -1627.4764 -3041.7867 367969 86.90%
15404 12829 -3029.4716 33 -1627.4764 -3041.7867 369702 86.90%
15784 13299 -3032.1466 49 -1627.4764 -3041.7867 382329 86.90%
Elapsed time = 261.59 sec. (119973.59 ticks, tree = 24.07 MB, solutions = 1)
16291 13726 -3034.0793 18 -1627.4764 -3041.7867 391277 86.90%
16530 14129 -2953.2067 18 -1627.4764 -3041.7867 399187 86.90%
16986 14247 -1964.4663 3 -1627.4764 -3041.6694 401193 86.89%
17387 14709 -3038.6037 52 -1627.4764 -3041.6694 411547 86.89%
17658 15182 -2795.4929 1 -1627.4764 -3041.6694 424147 86.89%
17833 15297 -3002.7809 1 -1627.4764 -3041.6694 425636 86.89%
18043 15544 -2377.6311 6 -1627.4764 -3041.6694 433856 86.89%
18255 15698 -2310.2334 4 -1627.4764 -3041.6694 438922 86.89%
```

18519 16107 -1638.0274 1 -1627.4764 -3041.6694 449052 86.89%

18833 16201 -3035.2516 17 -1627.4764 -3041.6694 452947 86.89%

Elapsed time = 379.33 sec. (158222.20 ticks, tree = 29.88 MB, solutions = 1)

19135 16579 -2877.9128 49 -1627.4764 -3041.6694 462586 86.89%

19456 16863 -3015.9011 40 -1627.4764 -3041.6694 472390 86.89%

19754 17117 -2037.3597 7 -1627.4764 -3041.6694 474669 86.89%

20010 17508 -1646.2419 1 -1627.4764 -3041.6694 483325 86.89%

20204 17732 -3007.6057 23 -1627.4764 -3041.4388 488673 86.88%

20450 17783 -3040.1870 44 -1627.4764 -3041.4388 494059 86.88%

20649 18157 -3025.6465 15 -1627.4764 -3041.4388 509378 86.88%

20852 18315 -2587.7090 1 -1627.4764 -3041.4388 518430 86.88%

21057 18637 -2671.2811 1 -1627.4764 -3041.4388 532354 86.88%

21228 18729 -2614.3454 14 -1627.4764 -3041.4388 536421 86.88%

Elapsed time = 507.97 sec. (196559.51 ticks, tree = 35.20 MB, solutions = 1)

21419 18896 -3040.7255 69 -1627.4764 -3041.4388 546851 86.88%

21603 19042 -3036.0740 29 -1627.4764 -3041.4388 552884 86.88%

21793 19151 -3041.2279 77 -1627.4764 -3041.3305 559011 86.87%

21985 19391 -2985.9142 34 -1627.4764 -3041.3305 571852 86.87%

22143 19567 -2984.8577 43 -1627.4764 -3041.3305 582593 86.87%

22308 19802 -3038.5139 38 -1627.4764 -3041.3305 590637 86.87%

22530 19827 -3037.7789 34 -1627.4764 -3041.2653 591732 86.87%

22730 20077 -2984.0851 28 -1627.4764 -3041.2653 603187 86.87%

22893 20295 -3037.3613 48 -1627.4764 -3041.2653 612317 86.87%

23068 20520 -2779.6258 23 -1627.4764 -3041.2653 622403 86.87%

Elapsed time = 639.36 sec. (234951.37 ticks, tree = 38.74 MB, solutions = 1)

23200 20766 -3038.3912 52 -1627.4764 -3041.2653 635945 86.87%

23364 20753 -3034.5951 34 -1627.4764 -3041.2653 635030 86.87%

23567 20978 -2896.6289 47 -1627.4764 -3041.2653 648452 86.87%

23740 20986 -2774.5374 67 -1627.4764 -3041.2591 649371 86.87%

23906 21276 -2698.0103 32 -1627.4764 -3041.2591 662866 86.87%

24134 21462 -3035.1884 19 -1627.4764 -3041.2591 673292 86.87%

24380 21637 -3040.1268 58 -1627.4764 -3041.2591 685279 86.87%

24513 21871 -3040.6512 68 -1627.4764 -3041.2591 693545 86.87%

24655 21931 -2868.6235 48 -1627.4764 -3041.2591 695617 86.87%

24800 22136 -3038.0563 53 -1627.4764 -3041.2591 709383 86.87%

Elapsed time = 762.63 sec. (273344.86 ticks, tree = 42.83 MB, solutions = 1)

25022 22301 -3034.0829 56 -1627.4764 -3041.2591 715109 86.87%

25305 22574 -2868.1206 34 -1627.4764 -3041.2591 725484 86.87%

25520 22693 -2865.2732 22 -1627.4764 -3041.2591 730836 86.87%

25624 23139 -3036.1771 31 -1627.4764 -3041.2591 743664 86.87%

25803 23169 -3035.6788 25 -1627.4764 -3041.2591 744216 86.87%

25980 23254 -2684.3773 5 -1627.4764 -3041.2591 746446 86.87%

26130 23479 -3041.1481 124 -1627.4764 -3041.2591 756564 86.87%

26320 23597 -2918.6500 55 -1627.4764 -3041.2278 766231 86.87%

26540 23819 -3008.9873 26 -1627.4764 -3041.2270 779079 86.87%

* 26676+23982 -1643.0230 -3041.2270 85.10%

26726 24081 -3039.6587 64 -1643.0230 -3041.2270 791395 85.10%

Elapsed time = 890.94 sec. (311673.35 ticks, tree = 46.26 MB, solutions = 2)

26933 24050 -3027.9893 70 -1643.0230 -3041.2270 792547 85.10%

27155 24182 -3028.3826 25 -1643.0230 -3041.2268 796070 85.10%

27319 24510 -3026.9509 31 -1643.0230 -3041.2268 813771 85.10%

27504 24575 -3036.3860 33 -1643.0230 -3041.2268 820211 85.10%

27721 24787 -3020.5402 44 -1643.0230 -3041.2102 832100 85.10%

27961 25265 -2888.3978 28 -1643.0230 -3041.2102 859683 85.10%

28151 25193 -3039.2814 63 -1643.0230 -3041.2102 855685 85.10%

28272 25602 -3031.4794 45 -1643.0230 -3041.2102 878900 85.10%

28376 25742 -3040.5636 81 -1643.0230 -3041.2102 887524 85.10%

28487 25886 -3038.5752 54 -1643.0230 -3041.2102 895019 85.10%

Elapsed time = 1022.77 sec. (350022.20 ticks, tree = 49.86 MB, solutions = 2)

28663 25887 -2972.0587 49 -1643.0230 -3041.2102 894587 85.10%

28920 26191 -3040.1359 64 -1643.0230 -3041.2102 914263 85.10%

29124 26288 -2748.5795 56 -1643.0230 -3041.2102 918297 85.10%

29324 26405 -2955.2053 44 -1643.0230 -3041.2102 927624 85.10%

29525 26448 -3031.6764 41 -1643.0230 -3041.2102 930527 85.10%

29730 27041 -3037.5623 43 -1643.0230 -3041.2102 964818 85.10%

29917 26941 -3033.3667 80 -1643.0230 -3041.2102 960268 85.10%

30114 27368 -3032.8517 77 -1643.0230 -3041.2102 981279 85.10%

30317 27506 -3031.1342 50 -1643.0230 -3041.2102 990248 85.10%

30524 27692 -3024.4796 19 -1643.0230 -3041.2102 994730 85.10%

Elapsed time = 1158.47 sec. (388366.95 ticks, tree = 54.52 MB, solutions = 2)

30731 27713 -3040.6744 84 -1643.0230 -3041.2102 995545 85.10%

30934 27776 -3034.4965 94 -1643.0230 -3041.2067 1001946 85.10%

31142 28279 -3034.0730 75 -1643.0230 -3041.2067 1028072 85.10%

31354 28581 -3000.9784 39 -1643.0230 -3041.2067 1041609 85.10%

31628 28514 -3037.7121 46 -1643.0230 -3041.2067 1035343 85.10%

31843 28705 -3032.0714 58 -1643.0230 -3041.2024 1050573 85.10%

32110 29225 -3031.1558 41 -1643.0230 -3041.2024 1072513 85.10%

32364 29345 -3040.1616 62 -1643.0230 -3041.2024 1079395 85.10%

32631 29809 -3032.8109 70 -1643.0230 -3041.2024 1099966 85.10%

32887 29928 -3039.4534 84 -1643.0230 -3041.2024 1105002 85.10%

Elapsed time = 1292.02 sec. (426624.44 ticks, tree = 58.99 MB, solutions = 2)

33098 30315 -3030.4702 43 -1643.0230 -3041.1954 1122262 85.10%

33316 30507 -3038.8484 52 -1643.0230 -3041.1954 1131067 85.10%

33566 30403 -3028.6751 31 -1643.0230 -3041.1954 1123612 85.10%

33801 30809 -3038.4279 61 -1643.0230 -3041.1924 1146499 85.10%

34016 30823 -3024.6896 70 -1643.0230 -3041.1924 1149496 85.10%

34233 31050 -3035.9753 64 -1643.0230 -3041.1924 1156464 85.10%

34419 31218 -3033.0093 64 -1643.0230 -3041.1924 1168628 85.10%

34586 31540 -3034.9001 47 -1643.0230 -3041.1887 1185927 85.10%

34791 31741 -3039.9764 54 -1643.0230 -3041.1887 1198534 85.10%

34923 32010 -3034.1485 44 -1643.0230 -3041.1887 1209450 85.10%

Elapsed time = 1431.94 sec. (465033.24 ticks, tree = 63.54 MB, solutions = 2)

35062 32200 -3030.3662 40 -1643.0230 -3041.1887 1218800 85.10%

35253 32319 -3037.9065 48 -1643.0230 -3041.1887 1227668 85.10%

35402 32335 -3034.1741 33 -1643.0230 -3041.1887 1224499 85.10%

35516 32729 -3038.2208 50 -1643.0230 -3041.1879 1253654 85.10%

35624 32960 -2976.5849 46 -1643.0230 -3041.1879 1261362 85.10%

35756 33047 -3009.5439 8 -1643.0230 -3041.1879 1265741 85.10%

35948 33168 -3011.3400 25 -1643.0230 -3041.1877 1274374 85.10%

36212 33154 -2952.0175 27 -1643.0230 -3041.1877 1271735 85.10%

36376 33328 -2931.5141 19 -1643.0230 -3041.1877 1279263 85.10%

36550 33551 -3016.8751 62 -1643.0230 -3041.1874 1294004 85.10%

Elapsed time = 1557.95 sec. (503469.95 ticks, tree = 68.10 MB, solutions = 2)

36733 33857 -3040.6732 103 -1643.0230 -3041.1874 1307904 85.10%

36991 33661 -2920.5611 55 -1643.0230 -3041.1874 1302799 85.10%

37269 34322 -3023.3226 33 -1643.0230 -3041.1867 1336348 85.10%

37552 34502 -3013.7918 61 -1643.0230 -3041.1867 1342903 85.10%

37801 34672 -3032.6161 26 -1643.0230 -3041.1867 1347920 85.10%

37996 34783 -3022.0815 25 -1643.0230 -3041.1836 1350813 85.10%

38278 35294 -3012.0703 28 -1643.0230 -3041.1836 1370630 85.10%

38440 35557 -3034.7666 65 -1643.0230 -3041.1836 1383482 85.10%

38563 35715 -3019.3819 37 -1643.0230 -3041.1826 1389481 85.10%

```
 38709 35807   -3040.6002   82   -1643.0230   -3041.1826  1392056   85.10%
Elapsed time = 1694.34 sec. (541716.67 ticks, tree = 72.22 MB, solutions = 2)
 38866 35820   -3040.0085   66   -1643.0230   -3041.1826  1392875   85.10%
 39039 36138   -3034.3812   61   -1643.0230   -3041.1826  1409749   85.10%
 39207 36362   -3039.1743   60   -1643.0230   -3041.1826  1419086   85.10%
 39370 36209   -3030.8644   28   -1643.0230   -3041.1768  1414371   85.10%
 39501 36479   -3013.6799   40   -1643.0230   -3041.1768  1428252   85.10%
 39669 36671   -3029.1664   23   -1643.0230   -3041.1768  1437866   85.10%
 39853 36767   -3035.9410   46   -1643.0230   -3041.1768  1442185   85.10%
 40108 36964   -3038.5100   62   -1643.0230   -3041.1768  1454124   85.10%

Benders cuts applied:  3984
Cover cuts applied:  11
Implied bound cuts applied:  2
Flow cuts applied:  2
Mixed integer rounding cuts applied:  8
Zero-half cuts applied:  8
Gomory fractional cuts applied:  1

Root node processing (before b&c):
  Real time             =   17.78 sec. (15836.47 ticks)
Parallel b&c, 12 threads:
  Real time             = 1783.31 sec. (559252.66 ticks)
  Sync time (average)   =  188.30 sec.
  Wait time (average)   =    0.07 sec.
                          ------------
Total (root+branch&cut) = 1801.09 sec. (575089.14 ticks)
Model: MicrogridOptimizer
 - number of variables: 47857
   - binary=12681, integer=402, continuous=34774
 - number of constraints: 57078
   - linear=57078
 - annotations: 47857
   - variables: 47857
 - parameters:
     parameters.timelimit = 1800.00000000000000
     parameters.benders.strategy = 2
     parameters.mip.tolerances.mipgap = 0.00015000000000
 - objective: minimize
 - problem type is: MILP
Solve status:  JobSolveStatus.FEASIBLE_SOLUTION
Model status: JobSolveStatus.FEASIBLE_SOLUTION
Objective value: -1643.0229762199456

Process finished with exit code 0
```