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Relaxing the Control-gain Assumptions of DSC Design for Nonlinear MIMO Systems

Yong Chen, Maolong Lv, Simone Baldi, Zongcheng Liu, Wenqian Zhang and Yang Zhou

Abstract— **This work focuses on adaptive neural dynamic surface control (DSC) for an extended class of nonlinear MIMO strict-feedback systems whose control gain functions are continuous and possibly unbounded. The method is based on introducing a compact set which is eventually proved to be an invariant set: thanks to this set, the restrictive assumption that the upper and lower bounds of control gain functions must be bounded is removed. This method substantially enlarges the class of systems for which DSC can be applied. By utilizing Lyapunov theorem and invariant set theory, it is rigorously proved that all signals in the closed-loop systems are semi-globally uniformly ultimately bounded (SGUUB) and the output tracking errors converge to an arbitrarily small residual set. A simulation example is provided to demonstrate the effectiveness of the proposed approach.**

I. INTRODUCTION

In recent years, approximation-based adaptive control of uncertain nonlinear systems has attracted much attention [1-3]. When combined with the backstepping technique, approximation-based adaptive approaches have been shown to obtain global stability for many classes of nonlinear systems [1-5]. However, it is well known that, due to repeatedly differentiating the virtual controllers at each step, the complexity of conventional backstepping controller drastically grows as the order of the systems increases. The DSC technique has been proposed to avoid this problem by introducing a first-order low-pass filter in the conventional backstepping design procedure. Approximation-based adaptive controllers stemming from this technique have been successfully constructed for many nonlinear systems and their applications, see [5-18] and references therein. To list a few, for example, a novel adaptive neural control is designed for a class of nonlinear MIMO time-delay systems in [5]. In [6], adaptive fuzzy hierarchical sliding-mode control is conducted for MIMO input-constrained nonlinear systems, etc.

However, it should be pointed out that, for all above schemes [5-9] to work, upper and lower bounds of the control gain functions must be assumed to exist. In order to remove this restrictive assumption, some efforts have been made: most notably, in [3] the upper bound is relaxed to a known positive function, while the lower bound is still assumed to exist. However, the lower and upper bounds of the control gain functions maybe difficult to acquire in practical applications, or even nonexistent [4]. This motivates us to explore new approaches to remove this restrictive assumption from the control gain functions. The main contributions of this work are as follows:

(1) Only the signs of the control gain functions are assumed to be known: in other words, the control gain functions are only required to be positive (and possibly unbounded), rather than a priori bounded by positive constants. The main challenge arising from this setting is that the states cannot be assumed to be bounded a priori before obtaining system stability.

(2) A novel set-invariance neural adaptive design is carried out for MIMO nonlinear dynamic systems. The challenge of this design is to construct appropriate compact sets via Lyapunov stability and invariant set theory, which guarantee that the states of the closed-loop system will stay in those sets all the time, even in the presence of possibly unbounded control gain functions.

The rest of this paper is organized as follows. Section II presents the problem formulation and preliminaries. The control design and stability analysis are given in Section III. In Section IV simulation results are presented to show the effectiveness of the proposed scheme. Finally, Section V concludes the work.

II. PROBLEM FORMULATION AND PRELIMINARIES

A. Problem Formulation

Consider a class of MIMO strict-feedback nonlinear systems given by [7]:

$$\begin{cases} \dot{x}_{j,i_j} = \varphi_{j,i_j}(\bar{x}_{j,\rho_j}) + g_{j,i_j}(\bar{x}_{j,i_j})x_{j,i_j+1} + d_{j,i_j}(x,t) \\ \qquad\qquad\qquad\qquad\qquad\qquad 1 \le i_j \le \rho_j - 1 \\ \dot{x}_{j,\rho_j} = \varphi_{j,\rho_j}(\bar{x}_{j,\rho_j}) + g_{j,\rho_j}(\bar{x}_{j,\rho_j})u_j + d_{j,\rho_j}(x,t) \\ y_j = x_{j,1} \qquad\qquad\qquad\qquad\qquad j = 1, ..., m \end{cases} \tag{1}$$

where $x_{j,i_j} \in \mathbb{R}$ is the state of the jth subsystem, $x = [\bar{x}_{1,\rho_1}^T, ..., \bar{x}_{j,\rho_j}^T, ..., \bar{x}_{m,\rho_m}^T]^T \in \mathbb{R}^N$ is the state vector of the whole system ($N = \rho_1 + \cdots + \rho_m$), where $\bar{x}_{j,\rho_j} = [x_{j,1}, ..., x_{j,\rho_j}]^T \in \mathbb{R}^{\rho_j}$ and ρ_j is the order of the jth subsystem. $\bar{x}_{j,i_j} = [x_{j,1}, ..., x_{j,i_j}]^T \in \mathbb{R}^{i_j}$, u_j and $y_j \in \mathbb{R}$ are the input and output of the jth subsystem respectively. $\varphi_{j,i_j}(\bar{x}_{j,\rho_j})$ are unknown continuous functions with

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$\varphi_{j,i_j}(0) = 0$, $g_{j,i_j}(\bar{x}_{j,i_j})$ are unknown continuous control gain functions, and $d_{j,i_j}(x,t)$, $i_j = 1,...,\rho_j, j = 1,...,m$ are uncertainties consisting of dynamical coupling terms and external disturbances.

Assumption 1: Only the signs of nonlinear functions $g_{j,i_j}(\bar{x}_{j,i_j})$ are known. Without loss of generality, it is further assumed that $g_{j,i_j}(\bar{x}_{j,i_j}) > 0$ for $i_j = 1,2,\ldots,\rho_j$ and $j = 1,...,m$.

Remark 1: It has to be noted that, in all the existing methods, e.g., [5-9], the control gain functions $g_{j,i_j}(\bar{x}_{j,i_j})$ are assumed to satisfy $0 < a \le g_{j,i_j}(\bar{x}_{j,i_j}) \le b$, with a and b being unknown constants. In fact, this assumption is sufficient for controllability of system (1). However, the assumption $a \le g_{j,i_j}(\bar{x}_{j,i_j}) \le b$ is too restrictive since such a priori knowledge of $g_{j,i_j}(\bar{x}_{j,i_j})$ may be difficult or even impossible to be acquired in practice. In addition, the lower bound a and upper bound b of $g_{j,i_j}(\bar{x}_{j,i_j})$ may be nonexistent: for example, the control gain functions $g_{j,i_j}(\bar{x}_{j,i_j}) = x_{j,1}^2 + e^{x_{j,i_j}}$ does not admit any a and b do not exist for all states: however, Assumption 1 holds since $g_{j,i_j}(\bar{x}_{j,i_j}) = x_{j,1}^2 + e^{x_{j,i_j}} > 0$ for all $\bar{x}_{j,i_j}$. Therefore, Assumption 1 allows the functions $g_{j,i_j}(\bar{x}_{j,i_j})$ to be possibly unbounded, which require new stability tools.

Assumption 2 [7]: For $\forall t > 0$, there exist positive constants d_{j,i_j}^* such that $\left|d_{j,i_j}(x,t)\right| \le d_{j,i_j}^*$, for $i_j = 1,\ldots,\rho_j$ and $j = 1,\ldots,m$.

Assumption 3 [8]: The reference signal $y_{j,d}(t)$ is a sufficiently smooth function of t, and there exist positive constants B_{j0} such that $\Omega_{j0} := \left\{ \left[y_{j,d}, \dot{y}_{j,d}, \ddot{y}_{j,d}\right]^T \middle| \left(y_{j,d}\right)^2 + \left(\dot{y}_{j,d}\right)^2 + \left(\ddot{y}_{j,d}\right)^2 \le B_{j0} \right\}$.

Lemma 1 [4]: Consider the dynamic system

$$\dot{\chi}(t) = -\alpha\chi(t) + \beta\upsilon(t) \tag{2}$$

where α and β are positive constants, and $\upsilon(t)$ is a positive function. For any given bounded initial condition $\chi(0) \ge 0$, we have $\chi(t) \ge 0$, for $\forall t \ge 0$.

Lemma 2 [4]: For any $\varrho \in R$ and $\varpi > 0$, the hyperbolic tangent function fulfills $\forall \varpi > 0$

$$\begin{cases} 0 \le |\varrho| - \varrho \tanh(\varrho/\varpi) \le 0.2785\varpi \\ 0 \le \varrho \tanh(\varrho/\varpi) \end{cases} \tag{3}$$

B. Properties of RBF NNs

The radial basis function neural networks (RBF NNs) is used to approximate the unknown continuous functions $\varphi_{j,i_j}(\bar{x}_{j,\rho_j})$ in this study. As is well known, for a given $\varepsilon^* > 0$ and any continuous function $h(Z)$ defined on a compact set $\Omega_z \subset \mathbb{R}^n$, there exists a RBF NN $\Theta^{T*}\bar{\phi}(Z)$ such that

$$h(Z) = \Theta^{*T}\bar{\phi}(Z) + \varepsilon(Z), \qquad |\varepsilon(Z)| \le \varepsilon^* \tag{4}$$

where $Z \in \Omega_z \subset \mathbb{R}^n$ is the input vector, Θ^* is the ideal constant weight vector, $\varepsilon(Z)$ is the approximation error, and $\bar{\phi}(Z) = [\phi_1(Z),...,\phi_l(Z)]^T$ with $l > 1$ being the number of neural network nodes and $\phi_i(Z)$ being commonly taken as Gaussian functions

$$\phi_i(Z) = \exp\left[\frac{-(Z-\omega_i)^T(Z-\omega_i)}{\Theta_i^2}\right], \quad i = 1,2,...,l \tag{5}$$

where $\omega_i = [\omega_{i1}, \omega_{i2}, \cdots, \omega_{in}]^T$ and $\Theta_i \in \mathbb{R}$ are the center and the width of the Gaussian function, respectively.

III. Control Design and stability analysis

A. Adaptive dynamic surface tracking controller design

The DSC technique is employed to design the adaptive neural controller for system (1) under the framework of backstepping. The control design is carried out based on the following changes of coordinates:

$$\begin{cases} z_{j,1} = x_{j,1} - y_{j,d} \\ z_{j,i_j} = x_{j,i_j} - \chi_{j,i_j} \end{cases} \tag{6}$$

where $z_{j,1}$ is the output tracking error and χ_{j,i_j} is the output of the first-order filter with ψ_{j,i_j} as the input, where ψ_{j,i_j} is the virtual controller defined in the step i_j. The recursive design includes ρ_j steps. From step 1 to step $\rho_j - 1$, the virtual control ψ_{j,i_j} will be constructed in step j, i_j and the actual control input u_j will be designed in the step ρ_j.

Since $\varphi_{j,i_j}(\bar{x}_{j,\rho_j})$, $i_j = 1,...,\rho_j$, are unknown continuous functions, they cannot be used in the control design directly. Therefore, throughout this note, we use RBF NNs to approximate the continuous functions $\varphi_{j,i_j}(\bar{x}_{j,\rho_j})$ as follows:

$$\varphi_{j,i_j}(\bar{x}_{j,\rho_j}) = \Theta_{j,i_j}^{*T}\bar{\phi}_{j,i_j}(\bar{x}_{j,\rho_j}) + \varepsilon_{j,i_j}(\bar{x}_{j,\rho_j}), \quad \bar{x}_{j,\rho_j} \in \Omega_{\bar{x}_{j,\rho_j}} \tag{7}$$

where $\bar{\phi}_{j,i_j}(\bar{x}_{j,\rho_j}) = [\phi_{j,i_j,1}(\bar{x}_{j,\rho_j}),...,\phi_{j,i_j,l_{i_j}}(\bar{x}_{j,\rho_j})]^T$ with $\phi_{j,i_j,n}(\bar{x}_{j,\rho_j})$, for $n = 1,...,l_{j,i_j}$, being Gaussian functions defined in (5), and ε_{j,i_j} are the approximation errors, satisfying $\left|\varepsilon_{j,i_j}\right| \le \varepsilon_{j,i_j}^*$ with ε_{j,i_j}^* being unknown positive constants. For compactness, we let ε_{j,i_j} and d_{j,i_j} denote $\varepsilon_{j,i_j}(\bar{x}_{j,\rho_j})$ and $d_{j,i_j}(x,t)$ respectively.

Step $j,1$: To begin with, it follows from (1) and (7) that the dynamics of $z_{j,1}$ is

$$\begin{aligned} \dot{z}_{j,1} =& \Theta_{j,1}^{*T}\bar{\phi}_{j,1}(\bar{x}_{j,\rho_j}) + \varepsilon_{j,1} + g_{j,1}(x_{j,1})x_{j,2} \\ &+ d_{j,1} - \dot{y}_{j,d} \end{aligned} \tag{8}$$

where $\varepsilon_{j,1}$ is the approximation error satisfying $|\varepsilon_{j,1}| \le \varepsilon_{j,1}^*$ with $\varepsilon_{j,1}^* > 0$ being an unknown constant.

To consider the stabilization of (8), we consider the following quadratic function

$$\dot{V}_{z_{j,1}} = \frac{1}{2}z_{j,1}^2. \tag{9}$$

Thus the time derivative of (9) can be given by

$$\begin{aligned} \dot{V}_{z_{j,1}} =& z_{j,1}(\Theta_{j,1}^{*T}\bar{\phi}_{j,1}(\bar{x}_{j,\rho_j}) + \varepsilon_{j,1} + g_{j,1}(x_{j,1})x_{j,2} \\ &+ d_{j,1} - \dot{y}_{j,d}). \end{aligned} \tag{10}$$

Define a compact set $\Omega_{j,1} := \left\{z_{j,1} \middle| V_{z_{j,1}} \le p\right\}$, with $p > 0$ being any positive constant. For $\Omega_{j,1} \times \Omega_{j0}$ and $g_{j,1}(x_{j,1})$, the following lemma holds.

Lemma 3: The continuous control gain function $g_{j,1}(x_{j,1})$ has maximum and minimum in $\Omega_{j,1} \times \Omega_{j0}$, namely, there exist positive constants $\underline{g}_{j,1}$ and $\bar{g}_{j,1}$ satisfying $\underline{g}_{j,1} = \min\limits_{\Omega_{j,1}\times\Omega_{j0}} g_{j,1}(x_{j,1})$ and $\bar{g}_{j,1} = \max\limits_{\Omega_{j,1}\times\Omega_{j0}} g_{j,1}(x_{j,1})$.

Proof: Observing $z_{j,1} = x_{j,1} - y_{j,d}$, we obtain $x_{j,1} = y_{j,d} + z_{j,1}$. Hence continuous function $g_{j,1}(x_{j,1})$ can be expressed by

$$g_{j,1}(x_{j,1}) = \mu_{j,1}(z_{j,1}, y_{j,d}) \tag{11}$$

with $\mu_{j,1}(\cdot)$ being a continuous function. Note that $\Omega_{j,1} \times \Omega_{j0}$ is a compact set since $\Omega_{j,1}$ and Ω_{j0} are compact sets respectively. It is possible to derive from (11) that all the variables of $\mu_{j,1}(\cdot)$ are included in the compact set $\Omega_{j,1} \times \Omega_{j0}$, thus we have

$$0 < \underline{g}_{j,1} \le g_{j,1}(x_{j,1}) \le \bar{g}_{j,1}, \quad x_{j,1} \in \Omega_{j,1} \times \Omega_{j0}. \tag{12}$$

□

Choose the virtual control law $\psi_{j,1}$ and parameters adaptation laws $\hat{\vartheta}_{j,1}$ and $\hat{\delta}_{j,1}$ as follows

$$\begin{aligned}\psi_{j,1} = &-c_{j,1}z_{j,1} - \frac{\hat{\vartheta}_{j,1}z_{j,1}}{2a_{j,1}^2} - \hat{\delta}_{j,1}\tanh\left(\frac{z_{j,1}}{\nu_{j,1}}\right) \\ &- \xi_{j,1}\dot{y}_{j,d}\tanh\left(\frac{z_{j,1}\dot{y}_{j,d}}{\nu_{j,1}}\right)\end{aligned} \tag{13}$$

$$\dot{\hat{\vartheta}}_{j,1} = \frac{\beta_{j,1}z_{j,1}^2}{2a_{j,1}^2} - \sigma_{j,1}\beta_{j,1}\hat{\vartheta}_{j,1} \tag{14}$$

$$\dot{\hat{\delta}}_{j,1} = \gamma_{j,1}z_{j,1}\tanh\left(\frac{z_{j,1}}{\nu_{j,1}}\right) - \sigma_{j,1}\gamma_{j,1}\hat{\delta}_{j,1} \tag{15}$$

where $c_{j,1} > 0, a_{j,1} > 0, \nu_{j,1} > 0, \beta_{j,1} > 0, \sigma_{j,1} > 0, \gamma_{j,1} > 0$ and $\xi_{j,1} \ge \underline{g}_{j,1}^{-1}$ are design parameters. $\hat{\vartheta}_{j,1}$ and $\hat{\delta}_{j,1}$ are estimates of the unknown constants $\vartheta_{j,1} = \underline{g}_{j,1}^{-1}\left\|\Theta_{j,1}^*\right\|^2 l_{j,1}$ and $\delta_{j,1} = \underline{g}_{j,1}^{-1}\left(\varepsilon_{j,1}^* + d_{j,1}^*\right)$ respectively, with $l_{j,1}$ being the dimension of $\bar{\phi}_{j,1}(\bar{x}_{j,\rho_j})$. By recalling Lemma 1, we can obtain $\hat{\vartheta}_{j,1}(t) \ge 0$ and $\hat{\delta}_{j,1}(t) \ge 0$ for $\forall t \ge 0$ by choosing $\hat{\vartheta}_{j,1}(0) = 0$ and $\hat{\delta}_{j,1}(0) = 0$.

To avoid repeatedly differentiating $\psi_{j,1}$, in line with the DSC in [10], we introduce a first-order filter with positive time constant $\tau_{j,2}$, as follows

$$\tau_{j,2}\dot{\chi}_{j,2} + \chi_{j,2} = \psi_{j,1},\ \chi_{j,2}(0) = \psi_{j,1}(0). \tag{16}$$

Now, by defining the output error of filter (16) as $e_{j,2} = \chi_{j,2} - \psi_{j,1}$, which yields $\dot{\chi}_{j,2} = -e_{j,2}/\tau_{j,2}$ and

$$\dot{e}_{j,2} = -\frac{e_{j,2}}{\tau_{j,2}} + \zeta_{j,2}\left(z_{j,1}, z_{j,2}, e_{j,2}, \hat{\vartheta}_{j,1}, \hat{\delta}_{j,1}, y_{j,d}, \dot{y}_{j,d}, \ddot{y}_{j,d}\right) \tag{17}$$

where $\zeta_{j,2}(\cdot)$ is a continuous function, which will be used in the stability analysis.

In view of Young's inequality[1], one has

$$\begin{aligned}z_{j,1}\Theta_{j,1}^{*T}\bar{\phi}_{j,1}(\bar{x}_{j,\rho_j}) \le &\frac{z_{j,1}^2\left\|\Theta_{j,1}^*\right\|^2}{2a_{j,1}^2}\bar{\phi}_{j,1}^T(\bar{x}_{j,\rho_j})\bar{\phi}_{j,1}(\bar{x}_{j,\rho_j}) \\ &+ \frac{a_{j,1}^2}{2}.\end{aligned} \tag{18}$$

Note that $\bar{\phi}_{j,1}^T(\bar{x}_{j,\rho})\bar{\phi}_{j,1}(\bar{x}_{j,\rho}) \le l_{j,1}$ since $\bar{\phi}_{j,1}(\bar{x}_{j,\rho_j}) = [\phi_{j,1,1}(\bar{x}_{j,\rho_j}), ..., \phi_{j,1,l_{j,1}}(\bar{x}_{j,\rho_j})]^T$ and $\left|\phi_{j,1,n}(\bar{x}_{j,\rho_j})\right| \le 1$, for $n = 1, ..., l_{j,1}$, with $l_{j,1}$ being the dimension of $\bar{\phi}_{j,1}(\bar{x}_{j,\rho})$. Thus we have

$$z_{j,1}\Theta_{j,1}^{*T}\bar{\phi}_{j,1}(\bar{x}_{j,\rho_j}) \le \frac{z_{j,1}^2\left\|\Theta_{j,1}^*\right\|^2}{2a_{j,1}^2}l_{j,1} + \frac{a_{j,1}^2}{2}. \tag{19}$$

Using $x_{j,2} = z_{j,2} + e_{j,2} + \psi_{j,1}$ and substituting (19) and (13) into (10), we obtain the time derivative of $V_{z_{j,1}}$ as

$$\begin{aligned}\dot{V}_{z_{j,1}} \le &-c_{j,1}\underline{g}_{j,1}z_{j,1}^2 - \frac{\underline{g}_{j,1}\hat{\vartheta}_{j,1}z_{j,1}^2}{2a_{j,1}^2} + \frac{z_{j,1}^2\left\|\Theta_{j,1}^*\right\|^2}{2a_{j,1}^2}l_{j,1} \\ &- \underline{g}_{j,1}z_{j,1}\hat{\delta}_{j,1}\tanh\left(\frac{z_{j,1}}{\nu_{j,1}}\right) + z_{j,1}z_{j,2}g_{j,1}(x_{j,1}) \\ &- z_{j,1}\dot{y}_{j,d}\tanh\left(\frac{z_{j,1}\dot{y}_{j,d}}{\nu_{j,1}}\right) + z_{j,1}g_{j,1}(x_{j,1})e_{j,2} \\ &+ \frac{a_{j,1}^2}{2} + |z_{j,1}|\left(\varepsilon_{j,1}^* + d_{j,1}^*\right) - z_{j,1}\dot{y}_{j,d}.\end{aligned} \tag{20}$$

Choose the Lyapunov function candidate as

$$V_{j,1} = V_{z_{j,1}} + \frac{\underline{g}_{j,1}\tilde{\delta}_{j,1}^2}{2\gamma_{j,1}} + \frac{\underline{g}_{j,1}\tilde{\vartheta}_{j,1}^2}{2\beta_{j,1}} + \frac{1}{2}e_{j,2}^2 \tag{21}$$

where $\tilde{\delta}_{j,1} = \delta_{j,1} - \hat{\delta}_{j,1}$ and $\tilde{\vartheta}_{j,1} = \vartheta_{j,1} - \hat{\vartheta}_{j,1}$ are the estimation errors of $\delta_{j,1}$ and $\theta_{j,1}$, respectively.

Substituting (14), (15) and (20) into (21), the time derivative of $V_{j,1}$ is

$$\begin{aligned}\dot{V}_{j,1} \le &- c_{j,1}\underline{g}_{j,1}z_{j,1}^2 + z_{j,1}z_{j,2}g_{j,1}(x_{j,1}) \\ &+ \sigma_{j,1}\underline{g}_{j,1}\left(\tilde{\vartheta}_{j,1}\hat{\vartheta}_{j,1} + \tilde{\delta}_{j,1}\hat{\delta}_{j,1}\right) - \frac{e_{j,2}^2}{\tau_{j,2}} \\ &+ 0.2785\nu_{j,1}\left(\varepsilon_{j,1}^* + d_{j,1}^* + 1\right) + \frac{a_{j,1}^2}{2} \\ &+ \left|e_{j,2}\zeta_{j,2}(\cdot)\right| + z_{j,1}g_{j,1}(x_{j,1})e_{j,2}.\end{aligned} \tag{22}$$

Step j, i_j $(2 \le i_j \le \rho_j - 1,\ j = 1, \ldots, m)$: The design process for step i_j is similar to Step 1. From $z_{j,i_j} = x_{j,i_j} - \psi_{j,i_j}$ and (4), the dynamics of z_{j,i_j} is

$$\begin{aligned}\dot{z}_{j,i_j} = &\Theta_{j,i_j}^{*T}\bar{\phi}_{j,i_j}(\bar{x}_{j,\rho_j}) + \varepsilon_{j,i_j} + g_{j,i_j}(\bar{x}_{j,i_j})x_{j,i_j+1} \\ &+ d_{j,i_j} - \dot{\psi}_{j,i_j}\end{aligned} \tag{23}$$

with ε_{j,i_j} being the approximation error satisfying $\left|\varepsilon_{j,i_j}\right| \le \varepsilon_{j,i_j}^*$, where $\varepsilon_{j,i_j}^* > 0$ is an unknown constant.

[1] $xy \le \frac{\epsilon^2}{\alpha}|x|^2 + \frac{1}{\beta\epsilon^2}|y|^2$ $(\alpha > 1, \beta > 1, \epsilon > 0$ and $(\alpha-1)(\beta-1) = 1)$

Choose the following quadratic function

$$V_{z_{j,i_j}} = \frac{1}{2} z_{j,i_j}^2. \tag{24}$$

From (23), the time derivative of $V_{z_{j,i_j}}$ is

$$\begin{aligned}\dot{V}_{z_{j,i_j}} =& z_{j,i_j}(\Theta_{j,i_j}^{*T}\bar{\phi}_{j,i_j}(\bar{x}_{j,\rho_j}) + g_{j,i_j}(\bar{x}_{j,i_j})x_{j,i_j+1} \\ &+ d_{j,i_j} + \varepsilon_{j,i_j} - \dot{\psi}_{j,i_j}).\end{aligned} \tag{25}$$

Design the virtual control law ψ_{j,i_j} and adaptation laws $\hat{\delta}_{j,i_j}$ and $\hat{\vartheta}_{j,i_j}$ as

$$\begin{aligned}\psi_{j,i_j} =& -c_{j,i_j}z_{j,i_j} - \frac{\hat{\vartheta}_{j,i_j}z_{j,i_j}}{2a_{j,i_j}^2} - \hat{\delta}_{j,i_j}\tanh\left(\frac{z_{j,i_j}}{\nu_{j,i_j}}\right) \\ &- \xi_{j,i_j}\frac{e_{j,i_j}}{\tau_{j,i_j}}\tanh\left(\frac{z_{j,i_j}e_{j,i_j}}{\tau_{j,i_j}\nu_{j,i_j}}\right)\end{aligned} \tag{26}$$

$$\dot{\hat{\vartheta}}_{j,i_j} = \frac{\beta_{j,i_j}z_{j,i_j}^2}{2a_{j,i_j}^2} - \sigma_{j,i_j}\beta_{j,i_j}\hat{\vartheta}_{j,i_j} \tag{27}$$

$$\dot{\hat{\delta}}_{j,i_j} = \gamma_{j,i_j}z_{j,i_j}\tanh\left(\frac{z_{j,i_j}}{\nu_{j,i_j}}\right) - \sigma_{j,i_j}\gamma_{j,i_j}\hat{\delta}_{j,i_j} \tag{28}$$

where the parameters are chosen similar to (13)-(15).

Next, let ψ_{j,i_j} pass through a first-order filter with time constant τ_{j,i_j+1} as follows

$$\tau_{j,i_j+1}\dot{\chi}_{j,i_j+1} + \chi_{j,i_j+1} = \psi_{j,i_j}, \chi_{j,i_j+1}(0) = \psi_{j,i_j}(0). \tag{29}$$

Define the filter errors $e_{j,i_j+1} = \chi_{j,i_j+1} - \psi_{j,i_j}$. We have $\dot{\chi}_{j,i_j+1} = -e_{j,i_j+1}/\tau_{j,i_j+1}$ and

$$\begin{aligned}\dot{e}_{j,i_j+1} =& -\frac{e_{j,i_j+1}}{\tau_{j,i_j+1}} \\ &+ \zeta_{j,i_j+1}\left(\bar{z}_{j,i_j+1}, \bar{e}_{j,i_j+1}, \bar{\hat{\vartheta}}_{j,i_j}, \bar{\hat{\delta}}_{j,i_j}, y_{j,d}, \dot{y}_{j,d}, \ddot{y}_{j,d}\right)\end{aligned} \tag{30}$$

with $\zeta_{j,i_j+1}(\cdot)$ being a continuous function.

Following similar lines as Lemma 3, we find that the continuous control gain function $g_{j,i_j}(\bar{x}_{j,i_j})$ can be rewritten as

$$g_{j,i_j}(\bar{x}_{j,i_j}) = \mu_{j,i_j}(\bar{z}_{j,i_j}, \bar{e}_{j,i_j}, \bar{\hat{\vartheta}}_{j,i_j-1}, \bar{\hat{\delta}}_{j,i_j-1}, y_{j,d}) \tag{31}$$

where $\mu_{j,i_j}(\cdot)$ is a continuous function.

Then, define the following compact sets Ω_{j,i_j}

$$\Omega_{j,i_j} := \left\{ \left[\bar{z}_{j,i_j}^T, \bar{e}_{j,i_j}^T, \bar{\hat{\vartheta}}_{j,i_j-1}^T, \bar{\hat{\delta}}_{j,i_j-1}^T\right]^T | z_{j,i_j}^2 + \sum_{k=1}^{i_j-1} \left(z_{j,k}^2 + e_{j,k+1}^2 + \frac{\underline{g}_{j,i_j}\tilde{\delta}_{j,i_j}^2}{\gamma_{j,i_j}} + \frac{\underline{g}_{j,i_j}\tilde{\vartheta}_{j,i_j}^2}{\beta_{j,i_j}} \right) \le 2p \right\}$$

where p is an arbitrary positive constant. For Ω_{j,i_j} and $g_{j,i_j}(\bar{x}_{j,i_j})$, in a similar fashion as Lemma 3 was derived, we have that the continuous function $g_{j,i_j}(\bar{x}_{j,i_j})$ has maximum and minimum in $\Omega_{j,i_j} \times \Omega_{j0}$, namely, there exist positive constants $\underline{g}_{j,i_j}$ and $\bar{g}_{j,i_j}$ satisfying

$$\underline{g}_{j,i_j} \le g_{j,i_j}(\bar{x}_{j,i_j}) \le \bar{g}_{j,i_j},\ \bar{x}_{j,i_j} \in \Omega_{j,i_j} \times \Omega_{j0}. \tag{32}$$

Consider the Lyapunov function candidate

$$V_{j,i_j} = V_{z_{j,i_j}} + \frac{\underline{g}_{j,i_j}\tilde{\delta}_{j,i_j}^2}{2\gamma_{j,i_j}} + \frac{\underline{g}_{j,i_j}\tilde{\vartheta}_{j,i_j}^2}{2\beta_{j,i_j}} + \frac{1}{2}e_{j,i_j+1}^2 \tag{33}$$

where $\tilde{\delta}_{j,i_j} = \delta_{j,i_j} - \hat{\delta}_{j,i_j}$ and $\tilde{\vartheta}_{j,i_j} = \vartheta_{j,i_j} - \hat{\vartheta}_{j,i_j}$.

With the help of Young's inequality, we get

$$z_{j,i_j}\Theta_{j,i_j}^{*T}\bar{\phi}_{j,i_j}(\bar{x}_{j,\rho_j}) \le \frac{z_{j,i_j}^2\left\|\Theta_{j,i_j}^*\right\|^2}{2a_{j,i_j}^2}l_{j,i_j} + \frac{a_{j,i_j}^2}{2} \tag{34}$$

where a_{j,i_j} and l_{j,i_j} are designed constants in line with (19).

Substituting (26) (28) and (34) into (33) and using Lemma 2 and $\xi_{j,i_j}\underline{g}_{j,i_j} \ge 1$, we have

$$\begin{aligned}\dot{V}_{j,i_j} \le& -c_{j,i_j}\underline{g}_{j,i_j}z_{j,i_j}^2 + z_{j,i_j}g_{j,i_j}(\bar{x}_{j,i_j})e_{j,i_j+1} - \frac{e_{j,i_j+1}^2}{\tau_{j,i_j+1}} \\ &+ \sigma_{j,i_j}\underline{g}_{j,i_j}\left(\tilde{\vartheta}_{j,i_j}\hat{\vartheta}_{j,i_j} + \tilde{\delta}_{j,i_j}\hat{\delta}_{j,i_j}\right) + \frac{a_{j,i_j}^2}{2} \\ &+ \left|e_{j,i_j+1}\zeta_{j,i_j+1}(\cdot)\right| + z_{j,i_j}z_{j,i_j+1}g_{j,i_j}(\bar{x}_{j,i_j}) \\ &+ 0.2785\nu_{j,i_j}\left(\varepsilon_{j,i_j}^* + d_{j,i_j}^* + 1\right).\end{aligned} \tag{35}$$

Step j, ρ_j $(j = 1, ..., m)$: From (1), (6) and (7), one has

$$\begin{aligned}\dot{z}_{j,\rho_j} =& \Theta_{j,\rho_j}^{*T}\bar{\phi}_{j,\rho_j}(\bar{x}_{j,\rho_j}) + \varepsilon_{j,\rho_j} + g_{j,\rho_j}(\bar{x}_{j,\rho_j})u_j \\ &+ d_{j,\rho_j} - \dot{\chi}_{j,\rho_j}\end{aligned} \tag{36}$$

Consider the quadratic function

$$V_{z_{j,\rho_j}} = \frac{1}{2}z_{j,\rho_j}^2. \tag{37}$$

Similarly, we know that $g_{j,\rho_j}(\bar{x}_{j,\rho_j})$ can be rewritten as

$$g_{j,\rho_j}(\bar{x}_{j,\rho_j}) = \mu_{j,\rho_j}(\bar{z}_{j,\rho_j}, \bar{e}_{j,\rho_j}, \bar{\hat{\vartheta}}_{j,\rho_j-1}, \bar{\hat{\delta}}_{j,\rho_j-1}, y_{j,d}) \tag{38}$$

where $\mu_{j,\rho_j}(\cdot)$ is a continuous function.

In light of previous steps (Lemma 3), it can be seen that, for $\Omega_{j,\rho_j} \times \Omega_{j0}$ and $g_{j,\rho_j}(\bar{x}_{j,\rho_j})$, there exist positive constants $\underline{g}_{j,\rho_j}$ and $\bar{g}_{j,\rho_j}$ satisfying

$$\underline{g}_{j,\rho_j} \le g_{j,\rho_j}(\bar{x}_{j,\rho_j}) \le \bar{g}_{j,\rho_j},\ \bar{x}_{j,\rho_j} \in \Omega_{j,\rho_j} \times \Omega_{j0}. \tag{39}$$

Let us now design the actual control law u_j and adaptation laws $\hat{\vartheta}_{j,\rho_j}$ and $\hat{\delta}_{j,\rho_j}$ as

$$\begin{aligned}u_j =& -c_{j,\rho_j}z_{j,\rho_j} - \frac{\hat{\vartheta}_{j,\rho_j}z_{j,\rho_j}}{2a_{j,\rho_j}^2} - \hat{\delta}_{j,\rho_j}\tanh\left(\frac{z_{j,\rho_j}}{\nu_{j,\rho_j}}\right) \\ &- \xi_{j,\rho_j}\frac{e_{j,\rho_j}}{\tau_{j,\rho_j}}\tanh\left(\frac{z_{j,\rho_j}e_{j,\rho_j}}{\tau_{j,\rho_j}\nu_{j,rho_j}}\right)\end{aligned} \tag{40}$$

$$\dot{\hat{\vartheta}}_{j,\rho_j} = \frac{\beta_{j,\rho_j}z_{j,\rho_j}^2}{2a_{j,\rho_j}^2} - \sigma_{j,\rho_j}\beta_{j,\rho_j}\hat{\vartheta}_{j,\rho_j} \tag{41}$$

$$\dot{\hat{\delta}}_{j,\rho_j} = \gamma_{j,\rho_j}z_{j,\rho_j}\tanh\left(\frac{z_{j,\rho_j}}{\nu_{j,\rho_j}}\right) - \sigma_{j,\rho_j}\gamma_{j,\rho_j}\hat{\delta}_{j,\rho_j} \tag{42}$$

where the corresponding parameters are defined similarly to that of (26)∼(28).

Consider the following Lyapunov function candidate

$$V_{j,\rho_j} = V_{z_{j,\rho_j}} + \frac{\underline{g}_{j,\rho_j}\tilde{\delta}_{j,\rho_j}^2}{2\gamma_{j,\rho_j}} + \frac{\underline{g}_{j,\rho_j}\tilde{\vartheta}_{j,\rho_j}^2}{2\beta_{j,\rho_j}} \tag{43}$$

where $\tilde{\delta}_{j,\rho_j} = \delta_{j,\rho_j} - \hat{\delta}_{j,\rho_j}$ and $\tilde{\vartheta}_{j,\rho_j} = \vartheta_{j,\rho_j} - \hat{\vartheta}_{j,\rho_j}$.

Following the same way as the former steps, we have

$$\begin{aligned}\dot{V}_{j,\rho_j} \le & - c_{j,\rho_j}\underline{g}_{j,\rho_j} z_{j,\rho_j}^2 + 0.2785\nu_{j,\rho_j}\left(\varepsilon_{j,\rho_j}^* + d_{j,\rho_j}^* + 1\right) \\ & + \underline{g}_{j,\rho_j}\sigma_{j,\rho_j}\left(\hat{\delta}_{j,\rho_j}\tilde{\delta}_{j,\rho_j} + \hat{\vartheta}_{j,\rho_j}\tilde{\vartheta}_{j,\rho_j}\right) + \frac{a_{j,\rho_j}^2}{2}\end{aligned} \tag{44}$$

where a_{j,ρ_j} is a positive constant.

B. Stability analysis

Consider the following Lyapunov function candidate for the whole systems

$$V = \sum_{j=1}^{m} V_j \tag{45}$$

where V_j is the Lyapunov function for the jth subsystem

$$\begin{aligned}V_j = & \frac{1}{2}\sum_{i_j=1}^{\rho_j}\left(z_{j,i_j}^2 + \frac{\underline{g}_{j,i_j}}{\gamma_{j,i_j}}\tilde{\delta}_{j,i_j}^2 + \frac{\underline{g}_{j,i_j}}{\beta_{j,i_j}}\tilde{\vartheta}_{j,i_j}^2\right) \\ & + \frac{1}{2}\sum_{i_j=1}^{\rho_j-1} e_{j,i_j+1}^2.\end{aligned} \tag{46}$$

The main stability result of the proposed method is summarized in the Theorem 1.

Theorem 1: Consider the nonlinear MIMO non-strict-feedback system (1), and let Assumptions 1-3 hold. Consider the control design composed by the virtual control laws (13) and (26), the actual control law (40), filters (17) and (29), adaptation laws (14), (15), (27), (28), (41) and (42). For any $p > 0$ and bounded initial conditions satisfying $\hat{\vartheta}_{j,i_j}(0) \ge 0$, $\hat{\delta}_{j,i_j}(0) \ge 0$ and $V_j(0) \le p$, there exist design parameters c_{j,i_j}, a_{j,i_j}, ν_{j,i_j}, β_{j,i_j}, σ_{j,i_j}, γ_{j,i_j}, ξ_{j,ρ_j} and τ_{j,i_j} such that: (1) $\Omega_{j,\rho_j} \times \Omega_{j0}$ is an invariant set, namely, $V_j(t) \le p$ for $\forall t > 0$, and hence all the closed-loop signals are SGUUB; (2) the output tracking error $z_{j,1}$ is such that $\lim_{t\to\infty}|z_{j,1}(t)| \le \Delta_{j,1}$, where $\Delta_{j,1}$ is a positive constant depending on the design parameters. Furthermore, the whole system output tracking error $z_1 = [z_{1,1}, ..., z_{m,1}]^T$ satisfies $\lim_{t\to\infty}\|z_1(t)\| \le \Delta_1$ with Δ_1 a positive constant depending on the design parameters.

Proof: According to (22), (35) and (44), the time derivative of V_j is

$$\begin{aligned}\dot{V}_j \le & \sum_{i_j=1}^{\rho_j}\left[-c_{j,i_j}\underline{g}_{j,i_j} z_{j,i_j}^2\right] + \sum_{i_j=1}^{\rho_j-1}\left[\left|e_{j,i_j+1}\zeta_{j,i_j+1}(\cdot)\right|\right] \\ & + \sum_{i_j=1}^{\rho_j-1}\left[-\frac{e_{j,i_j+1}^2}{\tau_{j,i_j+1}} + \bar{g}_{j,i_j}\left(\left|z_{j,i_j+1}\right| + \left|e_{j,i_j+1}\right|\right)\left|z_{j,i_j}\right|\right] \\ & + \sum_{i_j=1}^{\rho_j}\left[\sigma_{j,i_j}\underline{g}_{j,i_j}\left(\tilde{\vartheta}_{j,i_j}\hat{\vartheta}_{j,i_j} + \tilde{\delta}_{j,i_j}\hat{\delta}_{j,i_j}\right) + b_{j,i_j}\right]\end{aligned} \tag{47}$$

where $b_{j,i_j} = 0.2785\nu_{j,i_j}\left(\varepsilon_{j,i_j}^* + d_{j,i_j}^* + 1\right) + \frac{a_{j,i_j}^2}{2}$.

By completion of squares, we have

$$\begin{aligned}\left|e_{j,i_j+1}\zeta_{j,i_j+1}(\cdot)\right| &\le \frac{e_{j,i_j+1}^2\zeta_{j,i_j+1}^2(\cdot)}{2k_{j,1}} + \frac{k_{j,1}}{2} \\ \bar{g}_{j,i_j}\left|z_{j,i_j+1}\right|\left|z_{j,i_j}\right| &\le \frac{\bar{g}_{j,i_j}z_{j,i_j+1}^2}{2} + \frac{\bar{g}_{j,i_j}z_{j,i_j}^2}{2} \\ \bar{g}_{j,i_j}\left|z_{j,i_j}\right|\left|e_{j,i_j+1}\right| &\le \frac{k_{j,2}\bar{g}_{j,i_j}^2 e_{j,i_j+1}^2}{2} + \frac{z_{j,i_j}^2}{2k_{j,2}}\end{aligned} \tag{48}$$

with $k_{j,1}$ and $k_{j,2}$ being positive constants.

Let $\frac{1}{\tau_{j,i_j+1}} \ge \frac{D_{j,i_j+1}^2(\cdot)}{2k_{j,1}} + \frac{k_{j,2}\bar{g}_{j,i_j}^2}{2} + \alpha_j$ with $\bar{G}_j = \max\{\bar{g}_{j,1}, ..., \bar{g}_{j,\rho_j}\}$ and α_j positive constant. Therefore, we obtain the time derivative of V_j as

$$\dot{V}_j \le -\lambda_j V_j + C_j \tag{49}$$

where $\lambda_j = \min\left\{2\alpha_j, \sigma_{j,i_j}\gamma_{j,i_j}, \sigma_{j,i_j}\beta_{j,i_j}\right\}$ and $C_j = \frac{1}{2}\sum_{i_j=1}^{\rho_j}\sigma_{j,i_j}\underline{g}_{j,i_j}\left(\vartheta_{j,i_j}^2 + \delta_{j,i_j}^2\right) + \sum_{i_j=1}^{\rho_j} b_{j,i_j} + \frac{(\rho_j-1)k_{j,1}}{2}$.

By solving (49), one has

$$V_j(t) \le \left[V_j(0) - \Sigma\right]e^{-\lambda_j t} + \Sigma \tag{50}$$

with $\Sigma = C_j/\lambda_j$ a positive constant. Thus we have

$$\lim_{t\to\infty}|z_{j,1}| \le \lim_{t\to\infty}\sqrt{2V_j(t)} \le \sqrt{2\Sigma} = \Delta_{j,1} \tag{51}$$

Now let us consider the Lyapunov function candidate for the whole systems as $V = \sum_{j=1}^{m} V_j$. From (50), it follows that

$$\dot{V} = \sum_{j=1}^{m}\dot{V}_j \le \sum_{j=1}^{m}\left[-\lambda_j V_j + C_j\right] \le -\kappa V + \Pi \tag{52}$$

with $\kappa = \min\{\lambda_1, ..., \lambda_m\}$ and $\Pi = \sum_{j=1}^{m} C_j$. Then, one has

$$V(t) \le \left[V(0) - \Gamma\right]e^{-\kappa t} + \Gamma \tag{53}$$

where $\Gamma = \frac{\Pi}{\kappa}$ is a positive constant.

Similarly, we have $\lim_{t\to\infty}V(t) \le \Gamma$, which leads to

$$\lim_{t\to\infty}\|z_1(t)\| \le \lim_{t\to\infty}\sqrt{2V(t)} \le \sqrt{2\Gamma} = \Delta_1 \tag{54}$$

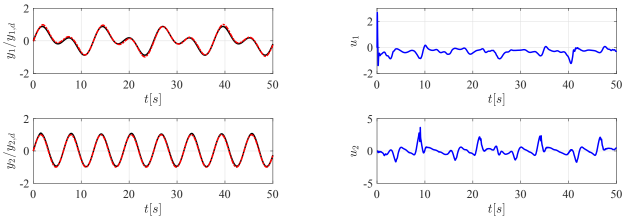
This completes the proof of Theorem 1. □

IV. SIMULATION RESULTS

Consider the nonlinear MIMO uncertain systems as follows:

$$\begin{cases}\dot{x}_{1,1} = x_{1,1}^3 e^{-0.3x_{1,2}^2} + \left(0.5 + e^{x_{2,1}^2}\right)x_{1,2} + d_{1,1}(t,x) \\ \dot{x}_{1,2} = \cos(x_{1,1}(x_{1,2}^2))x_{1,2}^2 + \left(1 + e^{x_{1,1}^2 x_{1,2}^3}\right)u_1 \\ \qquad + d_{1,2}(t,x) \\ \dot{x}_{2,1} = \left(1 + \sin(x_{1,2}x_{2,1})^2\right) + e^{x_{1,1}x_{2,1}}x_{2,2} + d_{2,1}(t,x) \\ \dot{x}_{2,2} = x_{2,1}x_{2,2}^2 + x_{1,1}x_{1,2}^2 + \left(1.5 + e^{x_{1,1}x_{2,1}x_{2,2}}\right)u_2 \\ \qquad + d_{2,2}(t,x) \\ y_1 = x_{1,1}, y_2 = x_{2,1}\end{cases} \tag{55}$$

where $d_{1,1} = 0.5\cos(x_{1,1}^2 x_{2,1}x_{2,2})\sin(0.2t)$, $d_{1,2} = 0.5\cos(x_{1,2}^2 + x_{1,2}x_{2,1})$, $d_{2,1} = 2\sin(x_{1,1}x_{2,1}x_{1,2}^2)$ and



(a) y_1 and y_2, and $y_{1,d}$ and $y_{2,d}$ (b) u_1 and u_2

Fig. 1: Simulation results

$d_{2,2} = \sin(x_{2,2}^2 + x_{2,1}^2)(\sin(t))^3$. The desired tracking trajectories are $y_{1,d} = 0.5(\sin(t) + \sin(0.5t))$ and $y_{2,d} = \sin(t)$. Note that the control gain functions $g_{1,1} = \left(0.5 + e^{x_{2,1}^2}\right)$, $g_{1,2} = \left(1 + e^{x_{1,1}^2 x_{1,2}^3}\right)$, $g_{2,1} = e^{x_{1,1}x_{2,1}}$ and $g_{2,2} = (1.5 + e^{x_{1,1}x_{2,1}x_{2,2}})$ cannot be bounded a priori, but they apparently satisfy Assumption 1. Thus, where existing methods cannot be applied, our scheme can be used to the nonlinear system (55).

The adaptation laws are given by (14), (15), (27) and (28) with design parameters $\beta_{1,1} = \beta_{1,2} = 1.5$, $\beta_{2,1} = \beta_{2,2} = 1$, $\sigma_{1,1} = \sigma_{1,2} = 0.2$, $\sigma_{2,1} = \sigma_{2,2} = 0.15$, $\gamma_{1,1} = 1$, $\gamma_{1,2} = \gamma_{2,2} = 1.5$ and $\gamma_{2,1} = 2$. Let the initial conditions be $[x_{1,1}(0), x_{1,2}(0), x_{2,1}(0), x_{2,2}(0)]^T = [0, 0,\ 0, 0]^T$, $\hat{\vartheta}_{1,1}(0) = \hat{\vartheta}_{1,2}(0) = \hat{\vartheta}_{2,1}(0) = \hat{\vartheta}_{2,2}(0) = 0$ and $\hat{\delta}_{1,1}(0) = \hat{\delta}_{1,2}(0) = \hat{\delta}_{2,1}(0) = \hat{\delta}_{2,2}(0) = 0$. The simulation results are provided in Fig. 1 (a) and (b).

From Fig. 1 (a), we can see that the outputs y_1 and y_2 track the desired trajectories $y_{1,d}$ and $y_{2,d}$ as closely as possible and excellent tracking performance has been achieved. Fig. 1 (b) shows that the proposed scheme works well with bounded system inputs even in the presence of possibly unbounded control gain functions.

V. Conclusion

A novel extended adaptive tracking control approach has been presented for a less restrictive class of nonlinear MIMO systems with possibly unbounded control gain functions and external disturbances. The restrictive assumption that the upper and lower bounds of control gain functions must be positive constants or coefficients has been removed by introducing appropriate compact sets where the maximums and minimums of continuous control gain functions are well defined and used in the control design. Stability of the closed-loop systems has been rigorously proved using Lyapunov theory in combination with invariant set theory.

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