Eight-vertex criticality in the interacting Kitaev chain

Chepiga, Natalia; Mila, Frédéric

DOI
10.1103/PhysRevB.107.L081106

Publication date
2023

Document Version
Final published version

Published in
Physical Review B

Letter

Eight-vertex criticality in the interacting Kitaev chain

Natalia Chepiga [1] and Frédéric Mila [2]
[1]Kavli Institute of Nanoscience, Delft University of Technology, Lorentzweg 1, 2628 CJ Delft, The Netherlands
[2]Institute of Physics, Ecole Polytechnique Fédérale de Lausanne (EPFL), CH-1015 Lausanne, Switzerland

(Received 19 June 2022; revised 1 February 2023; accepted 6 February 2023; published 13 February 2023)

We show that including pairing and repulsion into the description of one-dimensional spinless fermions, as in the domain wall theory of commensurate melting or the interacting Kitaev chain, leads, for strong enough repulsion, to a line of critical points in the eight-vertex universality class terminating floating phases with emergent $U(1)$ symmetry. For nearest-neighbor repulsion and pairing, the variation of the critical exponents along the line that can be extracted from Baxter's exact solution of the XYZ chain at $J_x = -J_z$ is fully confirmed by extensive density matrix renormalization group (DMRG) simulations of the entire phase diagram, and the qualitative features of the phase diagram are shown to be independent of the precise form of the interactions.

DOI: 10.1103/PhysRevB.107.L081106

Models of interacting spinless fermions in one dimension (1D) have appeared in many contexts over the years [1]. First used to reformulate and solve spin models in the 1970s thanks to a Jordan-Wigner transformation [2], they were introduced and further studied in the 1980s in the domain wall theory of commensurate melting in 2D, building on the equivalence of classical 2D systems and quantum 1D models [3]. In that context, the model is more naturally formulated in terms of hard-core bosons with a term creating p consecutive particles for the commensurate melting of a period-p phase, but for $p = 2$ the model is strictly equivalent to spinless fermions. In the early 2000s, Kitaev [4] revisited it as a model of a p-wave superconductor, and showed that it possesses Majorana edge states, triggering tremendous experimental activity [5–11] motivated by their potential use for qubits [12,13]. Later on, and quite logically since electrons experience repulsion, the interacting version of the Kitaev chain was studied [14–20]. Finally, the problem of commensurate melting recently resurfaced in the context of chains of Rydberg atoms, and 1D models of hard-core bosons including pairing and higher order creation terms have been investigated in that context [21–29].

In this Letter, we will first focus on a model with nearest-neighbor pairing and repulsion. In the context of the domain-wall theory in which it was first introduced, this model is usually written with the following terms:

$$H_{\rm NN} = \sum_i -t(d_i^\dagger d_{i+1} + {\rm H.c.}) - \mu n_i + \lambda(d_i^\dagger d_{i+1}^\dagger + {\rm H.c.}) + V n_i n_{i+1}, \qquad (1)$$

where t is the hopping amplitude, μ is the chemical potential that controls the band filling, λ is the amplitude of the terms that create pairs of domain walls, and V describes the nearest-neighbor repulsion, a term absent from the original Kitaev model [4]. Due to the pairing term, this model does not have $U(1)$ symmetry but only a Z_2 symmetry corresponding to the parity of the number of particles. At half filling ($\mu = V$), it also has particle-hole symmetry.

In the context of the interacting Kitaev model, slightly different notations are often used, with in particular an explicitly particle-hole symmetric form of the repulsion term, leading to the Hamiltonian:

$$H'_{\rm NN} = \sum_i -t(d_i^\dagger d_{i+1} + {\rm H.c.}) - \tilde{\mu} n_i + \Delta(d_i^\dagger d_{i+1}^\dagger + {\rm H.c.}) + U(2n_i - 1)(2n_{i+1} - 1). \qquad (2)$$

In that formulation, the particle-hole symmetric point always occurs at $\tilde{\mu} = 0$, but $\tilde{\mu}$ is strictly speaking no longer the chemical potential. Up to a constant, the two models map onto each other with $\lambda \equiv \Delta$, $\mu \equiv \tilde{\mu} + 4U$, and $V \equiv 4U$. We will mostly use the notations of Eq. (1), but whenever possible the results will also be shown using those of Eq. (2).

The phase diagram of the model without repulsion is well known (see Fig. 1, top panel). For $\lambda > 0$, it consists of three phases: Two disordered phases where Z_2 is unbroken for $\mu/t < -2$ and $\mu/t > 2$ (the number of particles in the ground state has a well defined parity), and a gapped phase with broken Z_2 symmetry for $-2 < \mu/t < 2$. Inside this phase, there is a disorder line defined by $4\lambda^2 + \mu^2 = 4t^2$ below which correlations are incommensurate [30]. The top of this line corresponds to the famous Kitaev point where the Majorana edge operators are completely decoupled from the bulk [4]. For $\lambda = 0$, the intermediate phase is a noninteracting Luttinger liquid ($K = 1$), and the transition into the disordered phase is Pokrovsky-Talapov [31]. When switching on λ, this transition immediately turns into an Ising phase transition.

The phase diagram remains qualitatively similar up to $V/t = 2$, the intermediate phase of the $\lambda = 0$ line becoming a Luttinger liquid with $1/2 \leqslant K \leqslant 1$. When $V/t > 2$ however, the phase diagram becomes much richer, as already pointed out by several authors [14,19,20], with three new phases: A period-2 phase in which the translation symmetry is broken,

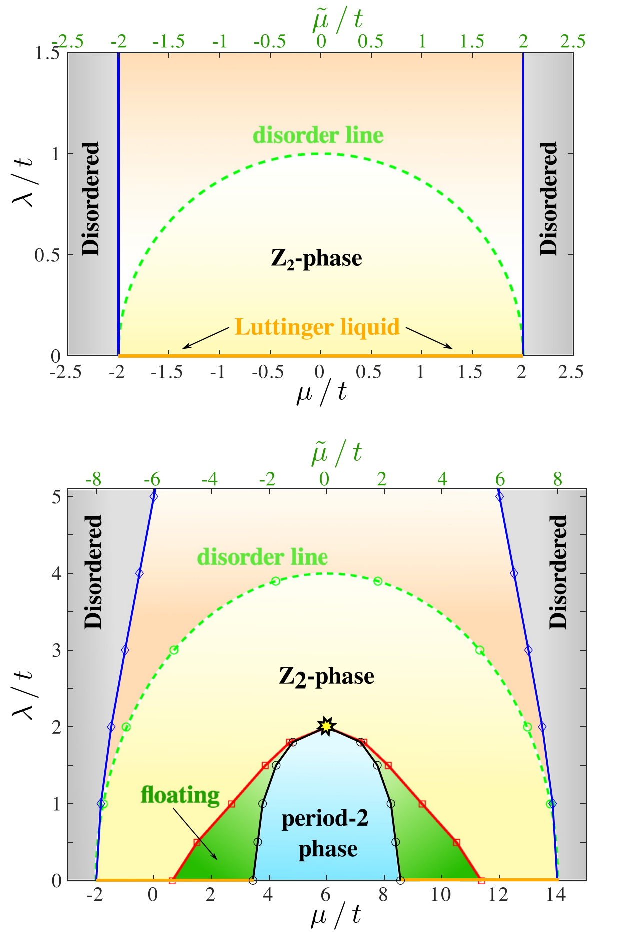

FIG. 1. Phase diagram of the Kitaev chain of Eq. (1) in the noninteracting case $V = 0$ (top) and with nearest-neighbor repulsion $V/t = 6$ (bottom). Orange lines at $\lambda = 0$ indicate the critical Luttinger liquid phase. Blue lines are Ising transitions. The $\mathbb{Z}_2$ phase has short-range incommensurate order below the frustration-free disorder line (dashed green). For $V/t > 2$ the phase diagram also contains a gapped period-2 phase with spontaneously broken translation symmetry and a floating phase that separates the period-2 and the $\mathbb{Z}_2$ phases everywhere except along the particle-hole symmetry line $\mu = V$ where the transition is direct in the eight-vertex universality class (yellow star). The floating phase is separated from the period-2 phase by a commensurate-incommensurate Pokrovsky-Talapov transition (black circles) and from the $\mathbb{Z}_2$ phase by the Kosterlitz-Thouless transition (red squares).

and two critical floating phases [20] that surround it and touch at a multicritical point (see Fig. 1, bottom panel). The appearance of a period-2 phase at $V/t = 2$ for the model without pairing is known from Bethe ansatz [32]. At that point, the Luttinger liquid exponent reaches the value $K = 1/2$, and Umklapp scattering becomes relevant. For $V/t > 2$, the Luttinger liquid exponent reaches the value $K = 1/4$ at the transition into the period-2 phase, and the transition is in the Pokrovsky-Talapov universality class [31]. Since the pairing term has a scaling dimension $1/K$, it is irrelevant as long as $K < 1/2$, and the Luttinger liquid phase gives rise to an extended floating phase when $1/4 < K < 1/2$. All the boundaries in Fig. 1, bottom panel, have been determined numerically with state-of-the-art density matrix renormalization group (DMRG) [33–36] simulations, except the disorder line that coincides with the frustration-free line [19], which is known to be given exactly by $4\lambda^2 + (\mu - V)^2 = (V + 2t)^2$, and the multicritical point marked as a star, which sits in the particle-hole plane at $\lambda = (V - 2t)/2$ (see below). The DMRG simulations were performed using a two-site routine with open boundary conditions on systems with up to 3001 sites keeping up to 2000 states and discarding all singular values below 10^{-8}. The boundary between the floating phase and the Z_2 phase was determined as the line $K = 1/2$, and that with the period-2 phase as the line where the wave-ector becomes equal to π (see Supplemental Material [37] for details). When scanning V/t from 2 to $+\infty$, the multicritical points at which the floating phases touch build a line. The universality class of this line of continuous phase transitions is the main open issue in the 3D (λ/t, μ/t, V/t) phase diagram.

In this Letter, we argue that this line of multicritical points is in the eight-vertex universality class, and that it is a generic feature of models with pairing and repulsion. For the model of Eqs. (1) and (2), this conclusion is based on a mapping on the integrable point $J_x = -J_z$ of the XYZ model defined by the Hamiltonian

$$H = \sum_i J_x\sigma_i^x\sigma_{i+1}^x + J_y\sigma_i^y\sigma_{i+1}^y + J_z\sigma_i^z\sigma_{i+1}^z - B\sigma_i^z, \quad (3)$$

where σ^x, σ^y, and σ^z are Pauli matrices; it was solved by Baxter [38,39] in the 1970s, and it is supported by extensive DMRG simulations that show that the behavior close to the critical point both in the period-2 phase and in the broken Z_2 phase is controlled by the critical exponents that can be extracted from Baxter's solution. We also study a hard-core boson model with a next-nearest neighbor pairing term for which there is no exact solution, and we provide strong numerical evidence that the point at which the floating phases meet is still in the eight-vertex universality class.

Let us start by discussing the nature of the critical point of the model of Eq. (1). The only piece of information so far was that its central charge $c = 1$, a result fully confirmed by fitting our results for the entanglement entropy [37] with the Calabrese-Cardy formula [40], hence that it is a Luttinger liquid. However, as we now explain, it is possible to fully identify the universality class of the transition. Using a Jordan-Wigner transformation, the model can be mapped on the model of Eq. (3), the XYZ chain in a field [3], with $J_x = -(t + \lambda)/2$, $J_y = -(t - \lambda)/2$, $J_z = V/4$, and $B = (V - \mu)/2$. In the particle-hole symmetric plane, the magnetic field vanishes, and the model reduces to an XYZ chain. This model is well known to be integrable when two of the coupling constants are equal, in which case it is usually referred to as the XXZ chain [32]. For our model, this is the case for $\lambda = 0$. It can also be solved when one of the coupling constants vanishes, which occurs for $\lambda = t$ (see Miao *et al.* [41]). A less well known result due to Baxter is that it is also integrable when two coupling constants are opposite, e.g., $J_x = -J_z$. For our model, this occurs when $\lambda = (V - 2t)/2$. Along this line the model can actually be mapped on the XXZ chain by rotating the spins by π around z ($\sigma_i^x \to -\sigma_i^x$, $\sigma_i^y \to -\sigma_i^y$, $\sigma_i^z \to \sigma_i^z$) on every other site, which leads to $J_x = J_z$. Since $|J_y| < J_z$, the model is critical (it is in the XY phase of the XXZ model).

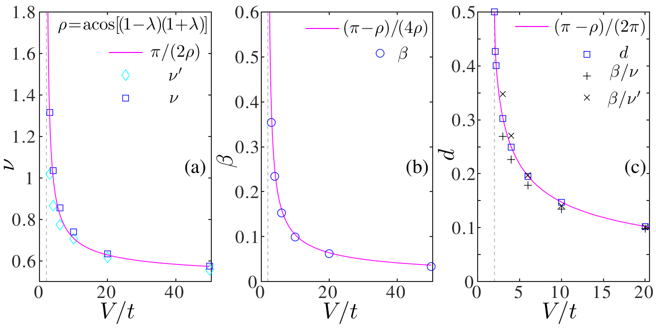

FIG. 2. Exponents in the vicinity of the multicritical point as a function of V/t: (a) Correlation length exponent extracted from density-density correlations in the period-2 phase (ν', light blue) and in the $\mathcal{Z}_2$ phase (ν, dark blue). (b) Critical exponent β of the amplitude if local density oscillations in the period-2 phase. (c) Scaling dimension d extracted from the slope of the separatrix of Friedel oscillations (blue squares), and estimated from the ratios β/ν and β/ν' (black pluses and crosses respectively). In all cases, the numerical results (symbols) are compared with the theory predictions of Eqs. (4) and (6) (magenta lines).

Away from this line the model does not map to a simple extension of the XXZ chain, but Baxter managed to show that the critical behavior in the vicinity of the critical line is governed by the universality class of the eight-vertex model [39]. More precisely, he showed that the critical exponents depend on a single parameter that he called μ, and to which we will refer to as ρ to avoid confusion with the chemical potential. For $|J_y| < |J_x|$, this parameter is given by $\cos\rho = J_y/J_x$. In terms of this parameter, the critical exponents of the correlation length and of the order parameter [42] are given by

$$\nu = \pi/(2\rho), \quad \beta = (\pi - \rho)/(4\rho), \tag{4}$$

This very special relation between these two critical exponents $4\beta = 2\nu - 1$ formally defines the eight-vertex universality class [43]. From the mapping of Eq. (1) to the XYZ model [3], J_y/J_x is of the form $(1-\lambda)/(1+\lambda)$, leading to

$$\rho = \text{acos}[(1-\lambda)/(1+\lambda)]. \tag{5}$$

In order to check these predictions, we have calculated the density-density correlation length both above and below the transition [37], from which we extracted the exponents ν and ν', and the dimerization in the period-2 phase as defined by the amplitude of the local density oscillations in the middle of a chain [37], from which we have extracted the exponent β. The results are plotted as a function of V/t and compared with Baxter's prediction in Figs. 2(b) and 2(c). The agreement with the analytical result is excellent, with only a slight deviation for ν' due to severe finite-size effects for small values of the repulsion V.

As a cross-check, we have also looked at the Friedel oscillations [37] which, in chains with open and fixed boundary conditions, have the profile $|n_j - n_{j+1}| \propto 1/[(N/\pi)\sin(\pi j/N)]^d$, where the scaling dimension d is equal to the ratio of the two critical exponents $d = \beta/\nu$. From Baxter's results, the scaling dimension d is thus expected to be given by

$$d = (\pi - \rho)/(2\pi), \tag{6}$$

The results are compared to this prediction in Fig. 2(a). The agreement is excellent. Note that Eq. (6) can also be obtained through the mapping on the XXZ chain. Indeed, the scaling dimension of the σ^z component in our model corresponds to the scaling dimension of one of the transverse components, say S^x, in the XXZ model. This scaling dimension is given by $d = 1/(4K)$, where K, the Luttinger liquid parameter of the XXZ chain, is known analytically from the Bethe ansatz and is given by $K = \pi/2(\pi - \rho)$ in terms of Baxter's parameter ρ, leading again to Eq. (6).

Note that, when going from $V/t = 2$ to $+\infty$, the parameter ρ changes from 0 to π, i.e., it describes all the possible interval of the eight-vertex model. Accordingly, the critical exponents change rather dramatically. This is most remarkable for β, which covers all the range from 0 to ∞. It becomes infinite at the opening of the period-2 phase, implying a very smooth development of the dimerization in that limit, while it goes to zero when $V \to \infty$, approaching a steplike behavior in that limit. This is logical since, when V is infinite, the pairing term cannot induce fluctuations in the ground state. ν is also infinite at the opening of the period-2 phase, in agreement with the Kosterlitz-Thouless [44] nature of the transition, and decreases to $1/2$ when $V \to \infty$, a value typical of mean field. But the transition is definitely not mean field since β goes to zero, and not $1/2$. In the limit $V/t = 2$, the Luttinger liquid parameter of the multicritical point takes the value $1/2$, as it should since, at that point, it must be equal to the value of the Luttinger liquid parameter at which the gap opens when $\lambda = 0$. However, away from that limit, the Luttinger liquid parameter of the multicritical point $K = \pi/2(\pi - \rho)$ is larger than $1/2$ while that of the adjacent floating phases is always between $1/4$ and $1/2$, demonstrating that this multicritical point is *not* controlled by the adjacent floating phases.

To investigate how universal this property might be, we look next at a model where the pairing term is between next-nearest neighbors, for which there is to the best of our knowledge no exact solution. In terms of hard-core bosons, this model is defined by the Hamiltonian

$$H_{\text{NNN}} = \sum_i -t(d_i^\dagger d_{i+1} + \text{H.c.}) - \mu n_i + \lambda_2(d_i^\dagger d_{i+2}^\dagger + \text{H.c.}) + V n_i n_{i+1}. \tag{7}$$

In terms of fermions, the pairing term would have an extra factor $(-1)^{n_{i+1}}$ due to the Jordan-Wigner transformation.

The phase diagram of this model is shown in Fig. 3 for $V/t = 10$. It is qualitatively similar to that of the nearest-neighbor pairing model, with the same phases and similar boundaries. The only qualitative difference appears for very large V, where the floating phase develops a re-entrant behavior upon approaching the $\lambda_2 = 0$ line [37].

As long as $V < +\infty$, there are two floating phases that are found numerically to end up at a multicritical point [45]. To study the properties of this multicritical point, we have again calculated the exponents ν, ν', β, and d. This time, we do not have any prediction for the dependence of ρ on the parameters of the model, so, in order to check if the multicritical point is still eight-vertex, we have eliminated ρ from Eqs. (4) and (6), leading to expressions for ν and β as a function of d. These expressions are checked in Fig. 4. The error bars are

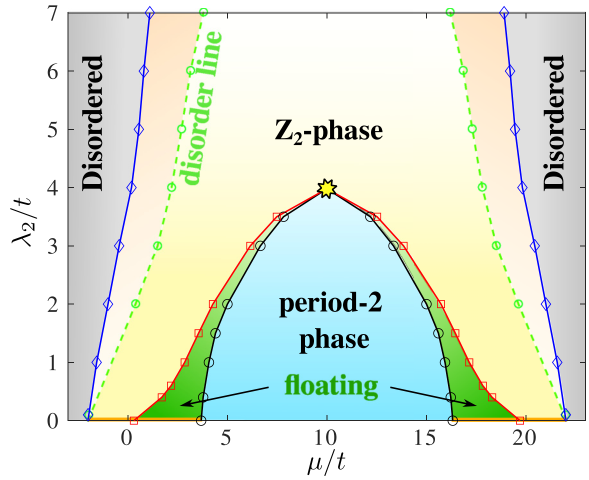

FIG. 3. Phase diagram of the model of Eq. (7) with nearest-neighbor repulsion $V/t = 10$ as a function of the next-nearest pairing term λ_2 and chemical potential μ. Red and black solid lines stand for Kosterlitz-Thouless and Pokrovsky-Talapov transitions respectively. The blue lines are Ising transitions to the disordered phases. The system has particle-hole symmetry along the $\mu = V(= 10t)$ line, and the phase diagram is mirror symmetric with respect to it. Along this line the transition between the period-2 and $\mathbb{Z}_2$ phases is direct through a multicritical point (yellow star). Inside the $\mathbb{Z}_2$ phase, short-range correlations are incommensurate between the two disorder lines.

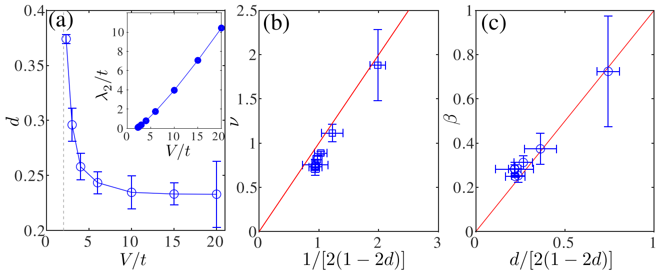

FIG. 4. Exponents of the model with next-nearest neighbor pairing [Eq. (7)] in the vicinity of the critical point: (a) Scaling dimension d as a function of the repulsion strength V/t. Inset: Location of the critical point as a function of V/t. (b)–(c) Exponents ν and β compared with their values predicted by the eight-vertex universality class in terms of the scaling dimension d of panel (a) (red lines).

larger than for the model with nearest-neighbor pairing, in part because the critical value of λ is not known exactly, but the results clearly support the eight-vertex universality class. Note that the values reached in the limit $V \to +\infty$ do no longer correspond to $\rho = \pi$. The exponents seem to saturate from above at $d \simeq 0.23$, $\nu \simeq 0.8$, and $\beta \simeq 0.22$, corresponding to $\rho \simeq 0.54\pi$. The difference regarding β with the nearest-neighbor model can be traced back to the possibility of inducing quantum fluctuations in the ground state with the next-nearest neighbor pairing term even in the limit $V \to +\infty$.

Let us now briefly compare our results with recent literature on the interacting Kitaev chain. Sela *et al.* [14] studied the full phase diagram, but they could not decide if the floating phases extend up to the particle-hole symmetric plane, and accordingly they did not discuss the multicritical line at which they touch. Their focus was the fate of the Majorana edge states. Miao *et al.* [41] also studied an integrable line in the particle-hole symmetric plane, but a different one given by $\lambda = t$ in our notation. For small V/t, this line is in the Z_2 phase. It crosses our line at the point where the period-2 phase opens, $V/t = 2$, and it lies in the period-2 phase for larger V/t, in full agreement with our phase diagram. Hassler and Schuricht [46] looked at another cut in the 3D parameter space $(\lambda/t, \mu/t, V/t)$, namely $\lambda = t$, and not $V/t = cst$, as we did. Again their results are fully consistent with ours. They spotted the multicritical point at $V/t = 4$ but did not identify its universality class beyond the fact that it has a central charge $c = 1$. More recently, Verresen *et al.* [20] revisited the $\lambda = t$ plane and emphasized the emergent $U(1)$ symmetry in the floating phase.

The present results also have strong connections with the physics of 2D classical models. The eight-vertex model has been introduced and solved in the context of 2D ice-type models where different Boltzmann weights are attributed to different arrow configurations around a vertex, and the paradigmatic model of 2D frustrated magnetism—the anisotropic next-nearest neighbor Ising (ANNNI) model [47–49]—has a phase diagram similar to ours, with a multicritical point in Baxter's eight-vertex universality class.

Finally, the standard model of Rydberg atoms is related to that of Eq. (1) by duality [28]. The period-2 phase of Rydberg chains corresponds to the Z_2 phase of $H_{\rm NN}$, and the Ising transition that surrounds it is equivalent to the Ising transition into the disordered phase. The equivalent of the period-2 phase of $H_{\rm NN}$ should be a Z_2 broken phase, but, in the standard setting, the model of Rydberg chains contains single-particle creation and annihilation operators and does not have Z_2 symmetry. However, it should be possible to directly program the models of Eq. (1) or Eq. (7) in optical cavities with individual control over trapped atoms. In any case, it will be rewarding to see if the eight-vertex universality class can be experimentally identified in 1D quantum systems.

The authors acknowledge Loïc Herviou, Samuel Nyckees and Dirk Schuricht for useful discussions, and an anonymous referee for pointing out the connection of the critical point to the XXZ chain. The work was supported by the Swiss National Science Foundation (F.M.) Grant No, 182179 and by the Delft Technology Fellowship (N.C.). Numerical simulations were performed on the Dutch national e-infrastructure with the support of the SURF Cooperative and the facilities of the Scientific IT and Application Support Center of EPFL.

[1] T. Giamarchi, *Quantum Physics in One Dimension*, International Series of Monographs on Physics (Clarendon, Oxford, 2004).
[2] P. Pfeuty, Ann. Phys. (NY) **57**, 79 (1970).
[3] M. den Nijs, Phase Transit. Crit. Phenom. **12**, 219 (1988).
[4] A. Y. Kitaev, Phys. Usp. **44**, 131 (2001).

[5] V. Mourik, K. Zuo, S. M. Frolov, S. R. Plissard, E. P. A. M. Bakkers, and L. P. Kouwenhoven, Science **336**, 1003 (2012).
[6] L. P. Rokhinson, X. Liu, and J. K. Furdyna, Nat. Phys. **8**, 795 (2012).
[7] M. T. Deng, C. L. Yu, G. Y. Huang, M. Larsson, P. Caroff, and H. Q. Xu, Nano Lett. **12**, 6414 (2012).
[8] A. Das, Y. Ronen, Y. Most, Y. Oreg, M. Heiblum, and H. Shtrikman, Nat. Phys. **8**, 887 (2012).
[9] R. Toskovic, R. van den Berg, A. Spinelli, I. S. Eliens, B. van den Toorn, B. Bryant, J.-S. Caux, and A. F. Otte, Nat. Phys. **12**, 656 (2016).
[10] C. Beenakker, Annu. Rev. Condens. Matter Phys. **4**, 113 (2013).
[11] J. Alicea, Rep. Prog. Phys. **75**, 076501 (2012).
[12] C. Nayak, S. H. Simon, A. Stern, M. Freedman, and S. Das Sarma, Rev. Mod. Phys. **80**, 1083 (2008).
[13] S. Das Sarma, M. Freedman, and C. Nayak, npj Quantum Inf. **1**, 15001 (2015).
[14] E. Sela, A. Altland, and A. Rosch, Phys. Rev. B **84**, 085114 (2011).
[15] A. Rahmani, X. Zhu, M. Franz, and I. Affleck, Phys. Rev. Lett. **115**, 166401 (2015).
[16] A. Rahmani, X. Zhu, M. Franz, and I. Affleck, Phys. Rev. B **92**, 235123 (2015).
[17] L. Fidkowski and A. Kitaev, Phys. Rev. B **81**, 134509 (2010).
[18] L. Fidkowski and A. Kitaev, Phys. Rev. B **83**, 075103 (2011).
[19] H. Katsura, D. Schuricht, and M. Takahashi, Phys. Rev. B **92**, 115137 (2015).
[20] R. Verresen, A. Vishwanath, and F. Pollmann, arXiv:1903.09179.
[21] A. Keesling, A. Omran, H. Levine, H. Bernien, H. Pichler, S. Choi, R. Samajdar, S. Schwartz, P. Silvi, S. Sachdev, P. Zoller, M. Endres, M. Greiner, V. Vuletić, and M. D. Lukin, Nature **568**, 207 (2019).
[22] P. Fendley, K. Sengupta, and S. Sachdev, Phys. Rev. B **69**, 075106 (2004).
[23] R. Samajdar, S. Choi, H. Pichler, M. D. Lukin, and S. Sachdev, Phys. Rev. A **98**, 023614 (2018).
[24] N. Chepiga and F. Mila, Phys. Rev. Lett. **122**, 017205 (2019).
[25] G. Giudici, A. Angelone, G. Magnifico, Z. Zeng, G. Giudice, T. Mendes-Santos, and M. Dalmonte, Phys. Rev. B **99**, 094434 (2019).
[26] N. Chepiga and F. Mila, Nat. Commun. **12**, 414 (2021).
[27] I. A. Maceira, N. Chepiga, and F. Mila, Phys. Rev. Res. **4**, 043102 (2022).
[28] S. Whitsitt, R. Samajdar, and S. Sachdev, Phys. Rev. B **98**, 205118 (2018).
[29] N. Chepiga and F. Mila, Phys. Rev. Res. **3**, 023049 (2021).
[30] E. Barouch and B. M. McCoy, Phys. Rev. A **3**, 786 (1971).
[31] V. L. Pokrovsky and A. L. Talapov, Phys. Rev. Lett. **42**, 65 (1979).
[32] C. N. Yang and C. P. Yang, Phys. Rev. **150**, 321 (1966).
[33] S. R. White, Phys. Rev. Lett. **69**, 2863 (1992).
[34] U. Schollwöck, Rev. Mod. Phys. **77**, 259 (2005).
[35] S. Östlund and S. Rommer, Phys. Rev. Lett. **75**, 3537 (1995).
[36] U. Schollwöck, Ann. Phys. (NY) **326**, 96 (2011).
[37] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevB.107.L081106 for additional information on how we extracted the critical exponents β and ν, and the scaling dimension d, for numerical results for the central charge, the profiles of the correlation length and of the Luttinger liquid parameter, and for the phase diagram of the model with nearest-neighbor blockade and next-nearest-neighbor pairing.
[38] R. J. Baxter, Ann. Phys. (NY) **70**, 193 (1972).
[39] R. J. Baxter, Ann. Phys. (NY) **70**, 323 (1972).
[40] P. Calabrese and J. Cardy, J. Phys. A: Math. Theor. **42**, 504005 (2009).
[41] J.-J. Miao, H.-K. Jin, F.-C. Zhang, and Y. Zhou, Phys. Rev. Lett. **118**, 267701 (2017).
[42] The exponent β that we quote corresponds to that of the polarization in Baxter's language, and it is denoted by β_e in his book.
[43] This should be for instance contrasted with the Ashkin-Teller universality class, another transition for which the critical point has a central charge $c = 1$, but for which β and ν are related by $\beta = \nu/8$. More generally, let us emphasize that additional information on top of $c = 1$ is needed to fully characterize a transition when the critical point is a Luttinger liquid.
[44] J. M. Kosterlitz and D. J. Thouless, J. Phys. C **6**, 1181 (1973).
[45] For infinite V/t, a limit known as the blockade model, the particle-hole symmetric point is sent to infinity.
[46] F. Hassler and D. Schuricht, New J. Phys. **14**, 125018 (2012).
[47] R. J. Elliott, Phys. Rev. **124**, 346 (1961).
[48] W. Selke and M. E. Fisher, Phys. Rev. B **20**, 257 (1979).
[49] M. P. M. den Nijs, Phys. Rev. B **23**, 6111 (1981).