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RESIDUAL STATICS ESTIMATION WITH QUANTUM ANNEALING

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Summary

Quantum computing could be a potential game-changer in industry sectors relying on the efficient solutions of large-scale global optimization problems. Exploration geoscience, is full of optimization problems and hence is a good candidate for application of quantum computing. It was recently suggested that quantum annealing, a form of adiabatic quantum computer, is a much better suited quantum computing platform for optimization problems than gate-based quantum computing. In this work, we show how the residual statics estimation problem can be solved on the quantum annealer and present our first results obtained on a quantum computer.

Residual statics estimation with quantum annealing

Introduction

Near-surface characterization is a crucial step in land seismic data processing and imaging. Kinematic distortions of seismic waves produced by shallow complex features can be mitigated by velocity model building, calculating and correcting the seismic data for the relative time delays (statics). However, limited resolution in the estimated model, results in remaining residual statics (RSs), which can be calculated and corrected for by applying a surface-consistent analysis (Taner and Koehler, 1981). Determining the source- and receiver-location-dependent RSs through cross-correlation and inversion suffers from cycle skips, therefore Ronen and Claerbout (1985) suggested to recast the problem as stack-power optimization. In practice, however, even the modern-day global optimization techniques might not always be capable of finding a global optimum solution in an affordable amount of time Pierini et al. (2019).

Quantum computing brings an expectation of solving some of the most complicated computational problems. Dukalski (2021a) focused on I/O limitations and discussed what appears to be achievable in popular geoscientific applications by the future programmable circuit model quantum computers. Dukalski (2021b) discussed the potential of quantum annealers, quantum computers specialized at solving certain types of optimization problems (McGeoch et al., 2020). These recently started being tested on first geophysical applications (e.g. O'Malley, 2018).

In this work we use the quantum annealer (McGeoch et al., 2020), to solve the stack-power optimization problem, with the objective of finding a better or, ideally, even global optimum, rather than searching for a computational speed-up, which is often the primary metric when discussing the potential advantage of quantum computers over classical technologies. To this end, we first formulate the Binary Quadratic Model (BQM) for one of two encodings (one-hot and binary) and test their performance. Lastly, we apply this approach to a small subset of the SEAM Arid Model dataset (Oristaglio, 2015), characterized by a complex near-surface geology, and we compare the result against a cross-correlational solver.

Theory

A quantum bit (qubit) is an elementary building block of a quantum computer, both as an actual quantum device and a theoretical construct. A qubitâĂŹs ability to be in a superposition $a_0 |0\rangle + a_1 |1\rangle$ of two states $|0\rangle$ and $|1\rangle$ with some user-controlled amplitudes a_0 and a_1, is one of the main reasons behind the expectation of computational speed-up or a better result. In a circuit model, computations are comprised of single- and two-qubit reversible operations, which alter the superposition state amplitudes. It is down to the sequence of the operations and quantum measurement, to determine what problem gets solved and how. Quantum annealing (QA) attempts to solve NP-Hard optimization problems, where the underlying objective function needs to be translated to a BQM. The problem graph of the latter is further translated to the energy level separation of individual qubits and to the interaction strengths between them. The system is then allowed to settle, and through quantum tunneling, the system converges to a global optimum. In both computational paradigms, accessing information encoded in quantum states requires an irreversible measurement, which collapses the qubits to one of the (multi-qubit) states in the superposition with some probability. The knowledge of such a probability, or simply, the verification of the solution on each run, is how we obtain the answer to the problem we are trying to solve.

The conceptual framework of QA is similar to that of classical simulated annealing (SA) (Kadowaki and Nishimori, 1998). In both methods, tuning of energetic fluctuations, which cause transitions of states, is essential. In QA, these fluctuations have a quantum mechanical nature as opposed to the classical thermal fluctuations in SA. Therefore, instead of gradually lowering the temperature of the system, the tunneling probability is lowered by modulating a transverse field coefficient which is directly related to the tunneling rate. As it is easier to overcome steep and narrow energy barriers by means of tunneling than by thermal jumps, this method is more efficient at traversing the solution landscape and locating the global optimum.

Residual Statics Estimation using Quantum Annealing

Stack power optimization for Residual Statics Estimation (RSE) aims to find a set of static corrections τ_i for the traces $\mathbf{d}_i(t_a)(i = 1,,M)$ in a common mid-point (CMP) gather, such that $\left|\sum_{i=1}^{M} \mathbf{d}_i(\tau_i)\right|$is maximized. This is a complicated multi-modal objective function whose maximization can be re-written as a combinatorial optimization problem (in particular a BQM), which is interpretable by the quantum annealer. A BQM is an unconstrained optimization problem in which the variables are binary and the objective function is a multivariate quadratic polynomial. The stack power objective function variables τ_i are not binary and the objective function is not a quadratic polynomial in τ_i. To rewrite the stack power objective function to a BQM, we define the search space S_K of possible values that $\tau_1 \leq i \leq M$ can take. Given some choice of some maximum number K of discrete possible shifts t_a we let $S_K\,\{t_1,\ldots,t_K\}$. The discretized set is then encoded using either the one-hot encoding or the standard binary encoding. More details on this and other aspects of this work can be found in van der Linde (2021). For example, for the one-hot encoding one gets the following BQM formulation:

$$\max_{\mathbf{x}\in B^{MK}} \quad \sum_{i=1}^{M} \|\mathbf{d}_i\|_2^2 + 2\sum_{i=1}^{M}\sum_{j=i+1}^{M}\sum_{a=1}^{K}\sum_{b=1}^{K} x_{ia}x_{jb}\langle \mathbf{d}_i(t_a), \mathbf{d}_j(t_b)\rangle - p\left(2\sum_{i=1}^{M}\sum_{a=1}^{K}\sum_{b=a+1}^{K} x_{ia}x_{ib} - \sum_{i=1}^{M}\sum_{a=1}^{K} x_{ia} + M\right) \tag{1}$$

where x_{ia} is a binary variable, which is responsible for applying a t_a shift to the i^{th} trace, and the last term is a penalty term with weight p. Without this penalty term (i.e. for $p = 0$), the optimum could correspond to an instance were multiple statics are applied to a single trace. The penalty term ensures that the objective function favors the cases were each trace has only one static applied. The binary encoding objective function is given a more complicated higher order (than quadratic) model, which needs to be converted to a BQM by means of a quadratization, which also introduces penalty terms. For larger problems, i.e. $MK \gg 100$, this polynomial degree reduction step can become a computational bottleneck.

The BQMs of the one-hot and binary encoding describe problem graphs, which are mapped to the Quantum Processing Unit (QPU) which solves only one BQM given by the hardware graph. Using so-called minor-embedding the hardware graph variables (the qubit states) effectively represent those of the problem graph, which requires introducing additional variables and hence using additional qubits. Even though the state-of-the-art quantum annealers have over 5000 qubits, the number of variables the QPU can solve for the RSE problem is about one hundred âĂŞ roughly the maximum number of binary variables on a totally connected problem graph. This means, that we are able to load onto the quantum annealer RSE problems where roughly $MK < 100$. For problems with $MK > 100$ one can use hybrid quantum- classical solvers, combining QA with classical algorithms to efficiently explore the search space. Further research is required to determine whether hybrid solver can guarantee finding the global optimum. For problems of this size, we only considered the one-hot encoding.

Numerical Examples

We ran RSE on the QPU using two synthetic datasets. First, we built a simple synthetic model comprised of the trace in Fig. 1a, copied 16 times and scrambled with 4 possible shifts τ_i. This is the approximate maximum size problem that âĂIJfitsâĂİ on the 5000-qubits QPU used here and it has a known global optimum. This test was used to establish the optimal parameter setting and to test if the quantum-only approach finds a global maximum and whether it needs additional post-processing steps. Secondly, encouraged by the results, we have tried a hybrid solver on a subset of the SEAM Arid model dataset. For the simple synthetic dataset, the residual statics were estimated with both the one-hot encoding and the binary encoding. With 500 annealing attempts the one-hot encoding did not find the global optimum, whereas the binary did. A histogram of the results of the stack power objective function in equation 1 is shown in Fig. 1b, with the lower energies corresponding to âĂIJbetterâĂİ results. We see, however, that most of these results violate the penalty term, whereas only very few (barely visible green histogram) meet the constraints. Upon closer investigation, the majority of these results were just one or two bit flips away from a global optimum, Hence, by definition the BQM could not have been in a local optimum. Therefore, we have decided to apply a simple (binary) steepest descent as a post-processing step. This

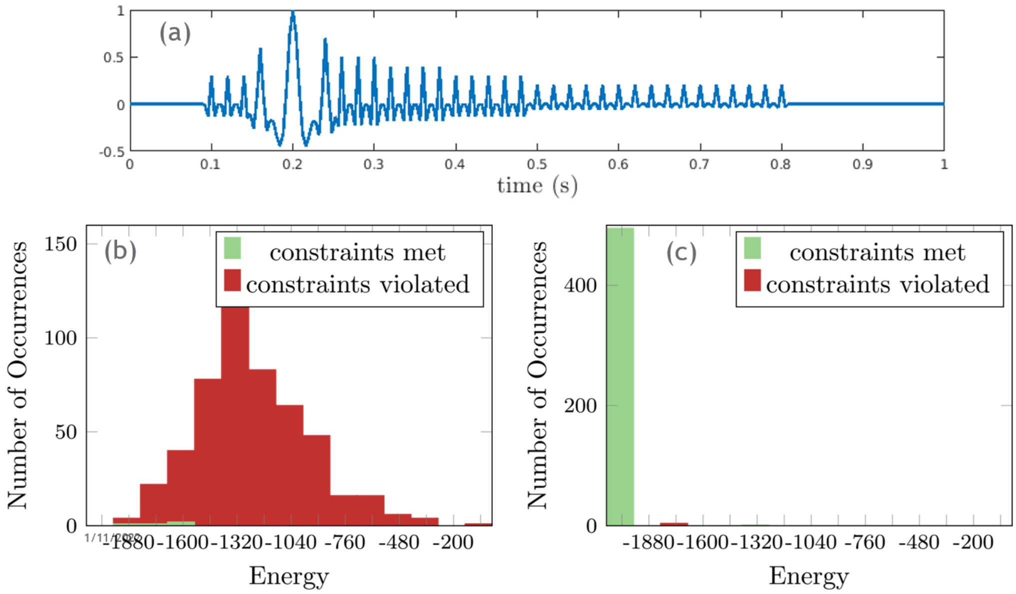

Figure 1 *(a) input trace for the simple synthetic, (b) QPU output histogram without and (c) with post-processing.*

shifts the solution energy distribution histogram, reducing the energy of some of the results by as much as 2000 energy units, to that shown in Fig. 1c. With the post-processing a perfect solution is found 99% of the time.

We also tested the hybrid solver on a 108-trace CMP gather from the SEAM Arid model dataset, with 15 possible statics giving rise to possible combinations. In this case the hybrid solver partitions the problem and uses a set of proprietary quantum and classical tools to try and find the global optimum.

In Fig 2a we show the input (misaligned traces), in Fig. 2b the result of applying a standard cross-correlation solver, and in Fig 2c the result of alignment using the quantum-classical hybrid solver. In the latter, the traces are notably better aligned compared to the gather in Fig. 2b. Furthermore, the stack power of gather Fig. 2c was 10.7% higher compared to the stack power of gather Fig. 2b, suggesting that the hybrid solver may be capable of performing at least just as well as the standard cross-correlation solver on a problem of near-industrial relevance.

Conclusion and Outlook

We have shown that the RSE objective function can be mapped onto a BQM and further run on a quantum annealer. We have analyzed the performance of two different problem encodings, their pre-processing requirements, the need for additional constraints and the advantages stemming from classical post-processing. We have also used a classical-quantum hybrid solver to estimate RSs on a subset of the SEAM Arid model dataset. Future research should look at (1) ways to reduce the number of auxiliary variables for example through other encodings with the purpose of running larger problems on the same QPU, (2) encoding the initial guess via advanced annealing settings, which could increase the probability of finding the optimal solution for larger problems, and (3) problem-specific hybrid solvers.

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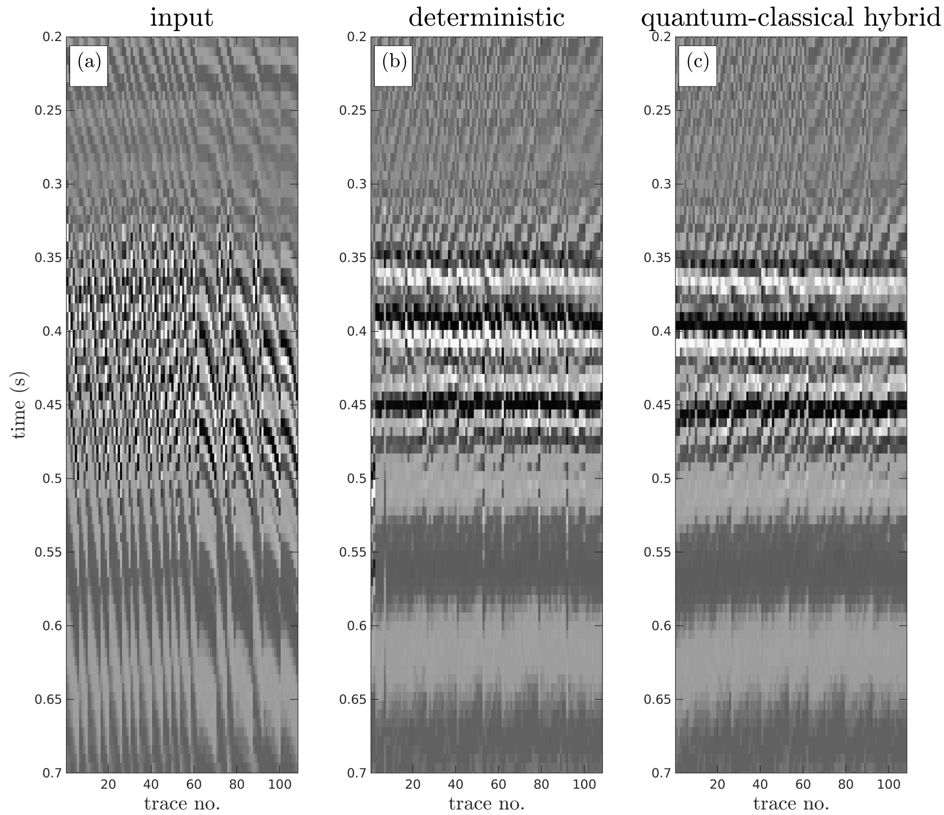

Figure 2 *Results of 108 traces of the SEAM Arid model dataset.*

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