CHAPTER 5

WATER WAVES GENERATED BY UNDERWATER EXPLOSIONS

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ABSTRACT

Explosion waves were simulated by plunging a parabolic surface of revolution in and out of an indoor test basin 3 feet deep by 92 feet square. These impulsively-generated water waves are found to be dispersive as evident from the following properties: The period and the celerity of the individual waves increase with travel time, and the amplitude of each wave group decreases. It is shown that the eventual shoaling in approaching the coast limits the dispersion process, and that the waves eventually amplify like other gravity waves. The run-up on the beach is a function of the wave steepness (H/L) at the beginning of the slope and the steepness of the slope itself. The run-up of any one wave is also affected by the preceding and following waves in the group. Experimental results on dispersive-wave attenuation in water of constant depth are in fairly close agreement with theory. The results on amplification due to shoaling indicate that a modification of Green's Law is necessary for describing these waves.

INTRODUCTION

Water wave systems generated by an impulsive release of energy, such as an underwater explosion, have been studied analytically, Kranzer and Keller (1959), Van Dorn (1961), and experimentally, Kaplan (1953). Results are presented, describing the deep water wave properties, based on the theory by Lamb (1932) and Penney (1945). The experimental simulation facility used was a wave basin equipped with a paraboloidal plunger reproducing the initial crater shape following the collapse of the explosion bubble.

The properties of dispersive wave motion in the space-time field have been derived for general conditions and certain practical applications. Maximum wave heights, periods, lengths, velocities, travel times, envelopes, group velocities, and the modification by a shoaling bottom can be uniquely expressed in terms of the initiating disturbance, Jordaan (1964). The waves are tractable up to the point of breaking and a substantive derivation and experimental confirmation of the run-up caused on beaches is presented.
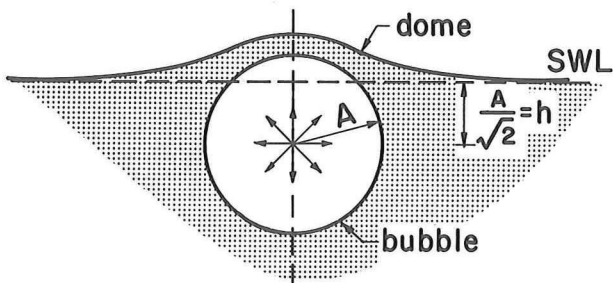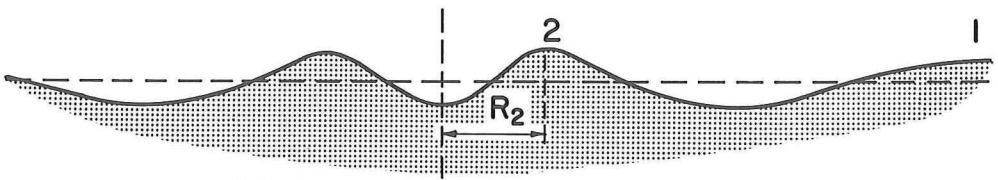
(a) Formation of Bubble and Dome
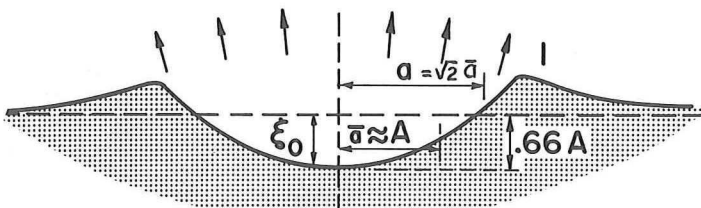
(b) Formation of Crater
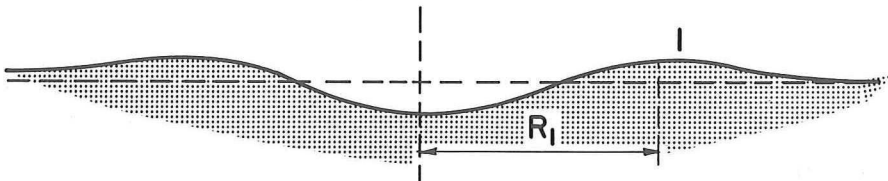
(c) Formation of First Wave

(d) Formation of Second Wave

Figure 1   Wave generation by an underwater explosion – schematic.

This paper is based on work carried out by the author for the U. S. Naval Civil Engineering Laboratory, Port Hueneme, during his tenure of duty there.

THE INITIAL STAGES OF WAVE GENERATION

An underwater nuclear blast can originate a system of locally very high water waves. The distant effects, such as run-up on exposed shorelines will vary greatly, depending on the ocean topography traversed as well as on the magnitude, depth and distance of the blast.

Consider a nuclear explosion at the optimum depth for making the largest possible waves at a desired location. Based on the work of Penney (1945), the detonation depth h for maximum wave-making efficiency should be equal to $A/\sqrt{2}$, figure 1(a), where A is the bubble radius at first maximum.

Upon penetrating the water surface the bubble momentarily transforms into a crater with an essentially parabolic section of water line radius a, and effective mean radius $\bar{a}$, approximately equal to A, figure 1(b). This crater is the effective initial displacement, giving rise to the dispersive wave train, figures 1(c) and 1(d). As time proceeds these impulsively-generated dispersive waves progress outward at an accelerating rate and the center becomes relatively quiet, new waves being formed continuously at the rear of the advancing wave group, Lamb, (1932).

The period of the waves of maximum height has the value:

$$\tau_{max} = 2\sqrt{2\pi\bar{a}/g}$$

which is also the pulsation period of the initial crater. The wave length at maximum height has the value:

$$\lambda_{max} = 4\bar{a}$$

The first wave length is $4R_1$, the second wave length is $\frac{4}{5}R_2$, the third, $\frac{4}{9}R_3$ etc, where R_1, R_2, R_3, denote the distance traveled from surface zero.

Figure 2 shows the distance vs. time graph for successive waves emanating from an explosion near the water surface, in deep water. Each successive crest accelerates uniformly outward and becomes the maximum of the wave train at successive uniform increments of distance. The phase velocity, group velocity, and distance-time relationship of the group maxima, are dependent on the crater's effective radius $\bar{a}$, whereas the individual phase accelerations are independent of $\bar{a}$.
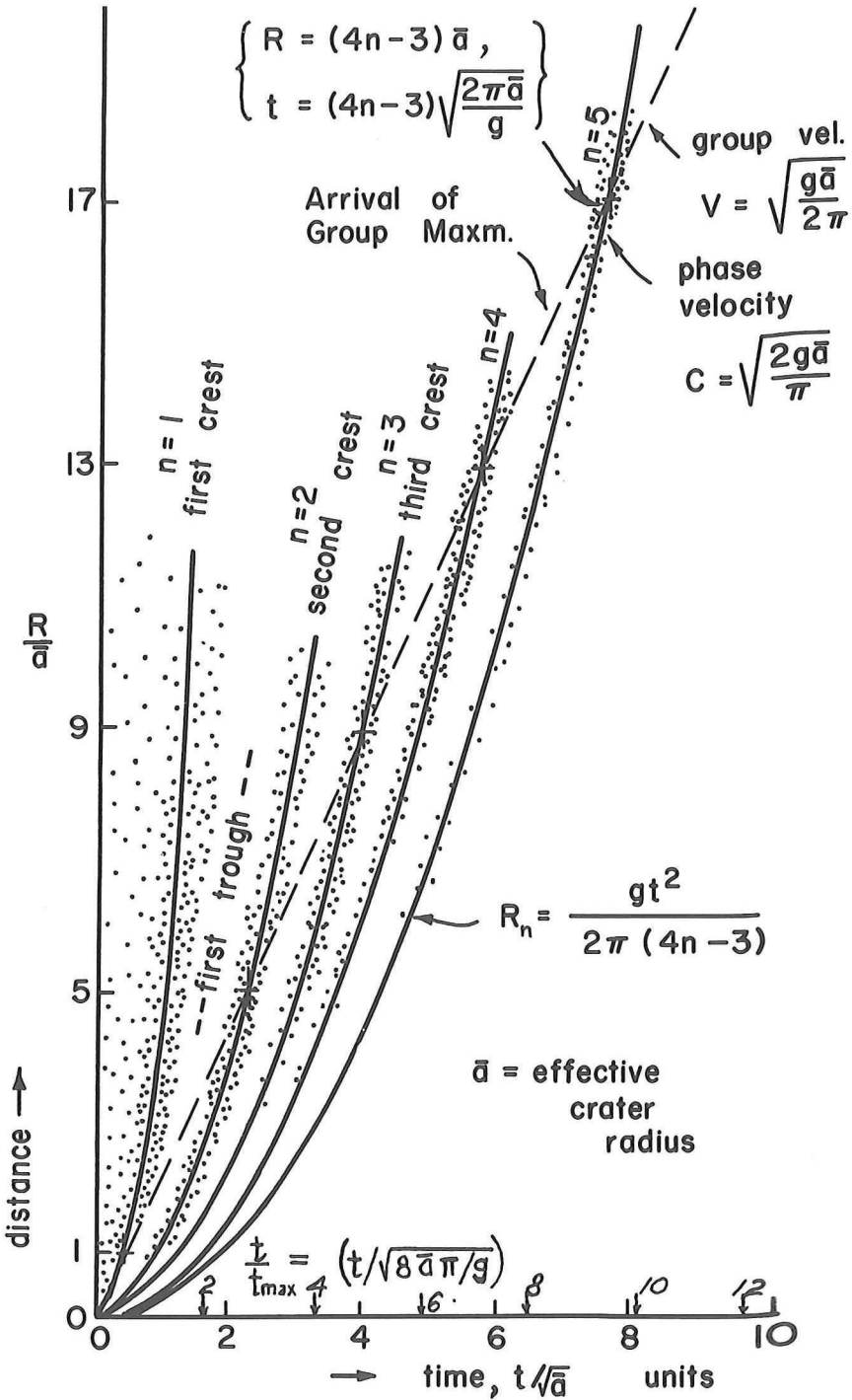
Figure 2   Water wave propagation characteristics due to
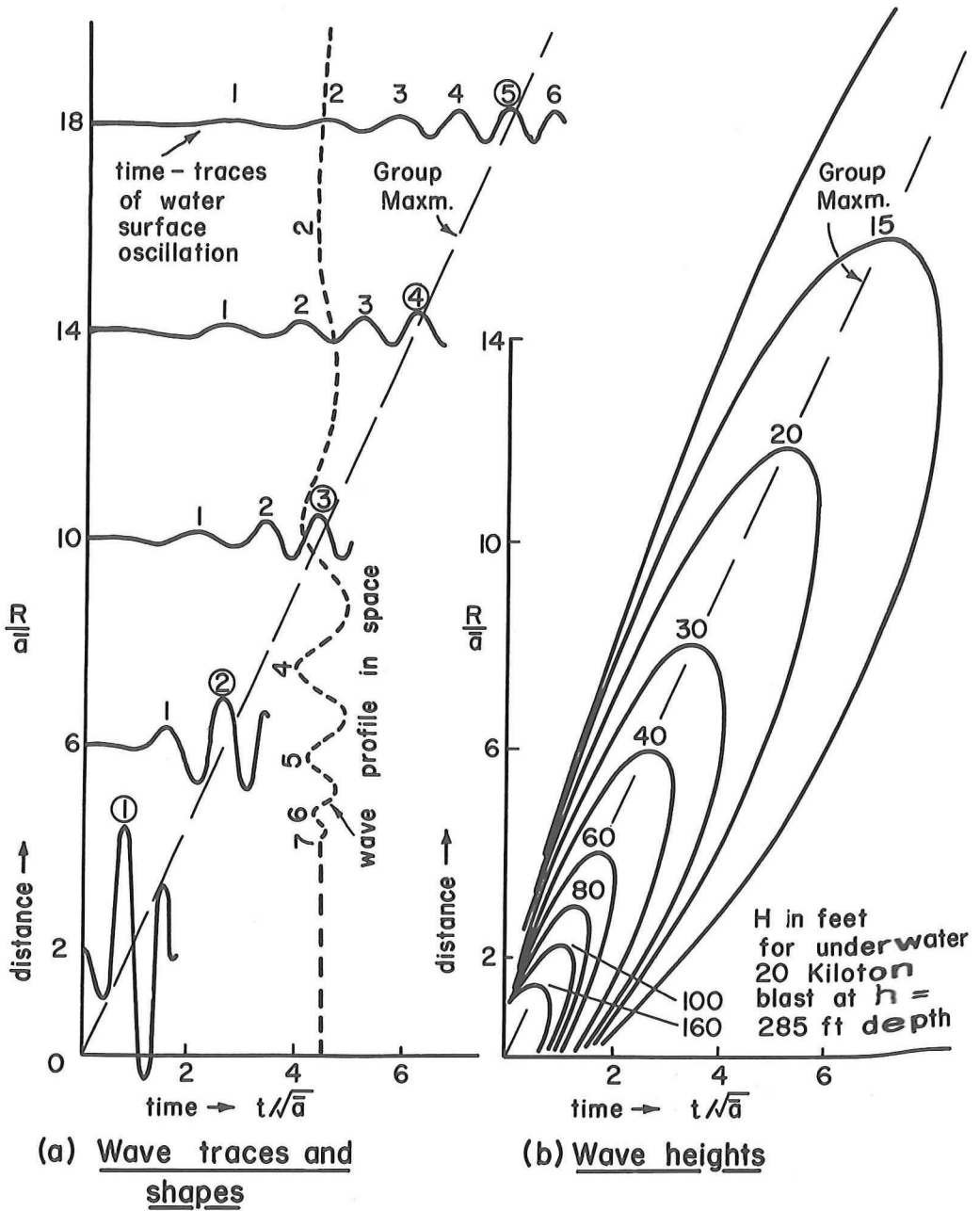an explosion - deep water.

Figure 3   Water wave traces, profiles and heights for underwater
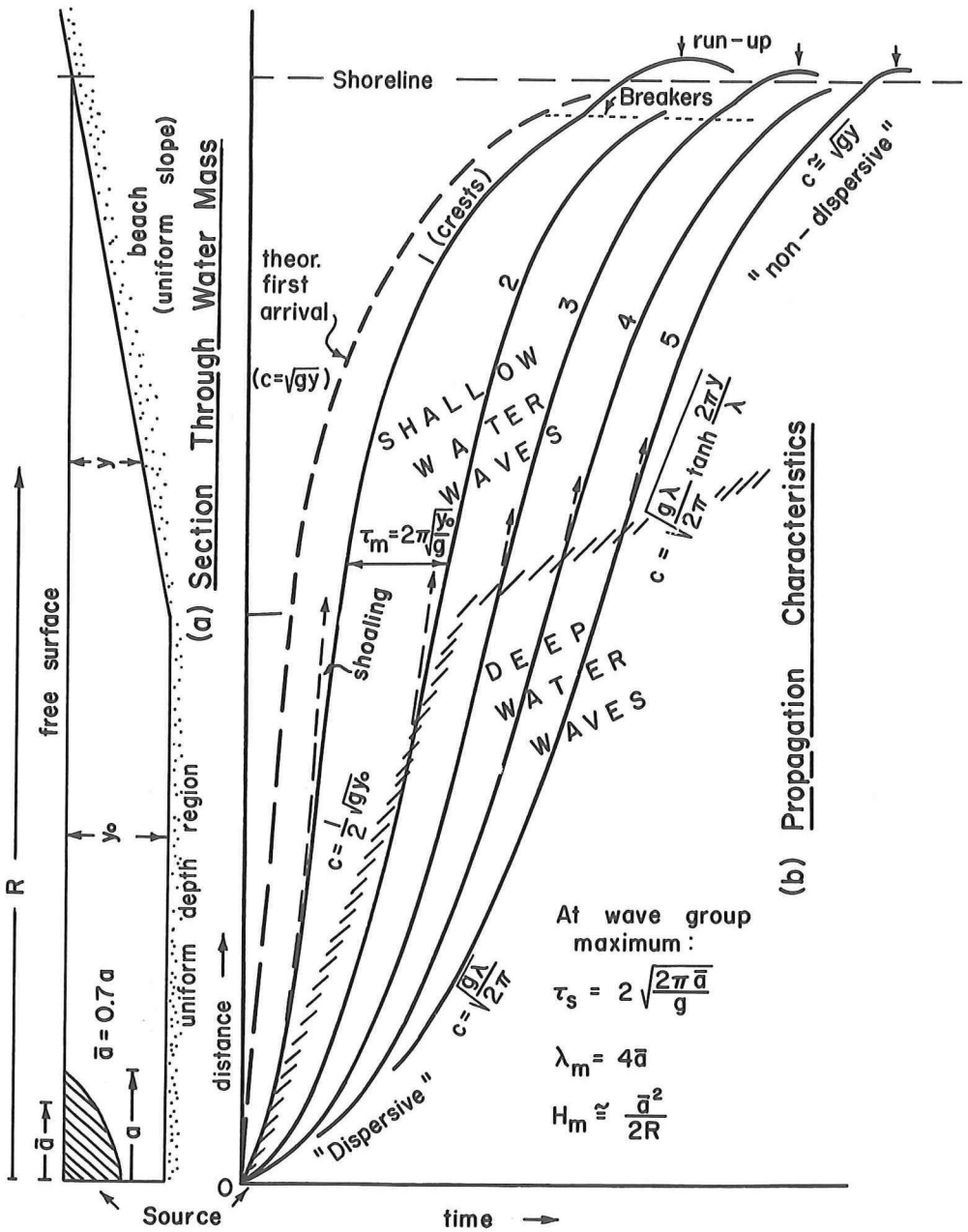explosion - deep water.   Computed, Penney-theory.

Figure 4   Water wave propagation characteristics due to a shallow depth underwater explosion - schematic.

Figure 3a shows the wave time traces and profiles in space following an explosion and figure 3b the contours of equal envelope height. The group maxima are inversely proportional to the travel distance. The wave heights at maximum are given by

$$H_{max} \propto \frac{(\bar{a})^2}{R}$$

Finite Depth and Shoaling Effects - The above case presumed an effectively infinite depth, i. e. a depth at least greater than 2A. Since the wavelengths become longer continuously, the wavelength will eventually become greater than twice the depth and the bottom will limit the dispersion. If subsequently the sloping bed to the shoreline is encountered, the waves will be slowed down by shoaling and their properties will change as in the case of oscillatory waves. Figure 4 illustrates first the finite depth effect, indicated by the short dashed limit, and then the shoaling effect. It is seen that the finite depth affects mostly the leading waves whereas shoaling affects all the waves. Values for the maximum wave properties under the limiting conditions are given.

Generation Depth Effect - Figure 5 shows the effect of shallow depth generation ($y < 0.64\bar{a}$) versus deep water generation ($y > 2\bar{a}$) as experimentally obtained in the NCEL wave basin, Jordaan (1964), indicated by solid line, and as theoretically obtained by calculations based on equations of Kranzer and Keller (1959), figure 5(a) for shallow water, and equations of Penney (1945), figure 5(b) for deep water.

The experiments show that the group maximum shifts to the earlier waves in the train when the generation depth is small comparable to the bubble diameter.

WAVE MAKING EFFECT OF VARIOUS YIELDS AT VARIOUS RANGES

Figure 6 shows the general wave making effects at various ranges of various explosive yields, according to extrapolations based on the Penney theory. The main results are:

(i) The effective crater radius is proportional to one-fourth root of charge yield:

$$\bar{a} \propto B^{1/4}$$

(ii) At constant range R, wave height at maximum of envelope is proportional to square root of charge yield:
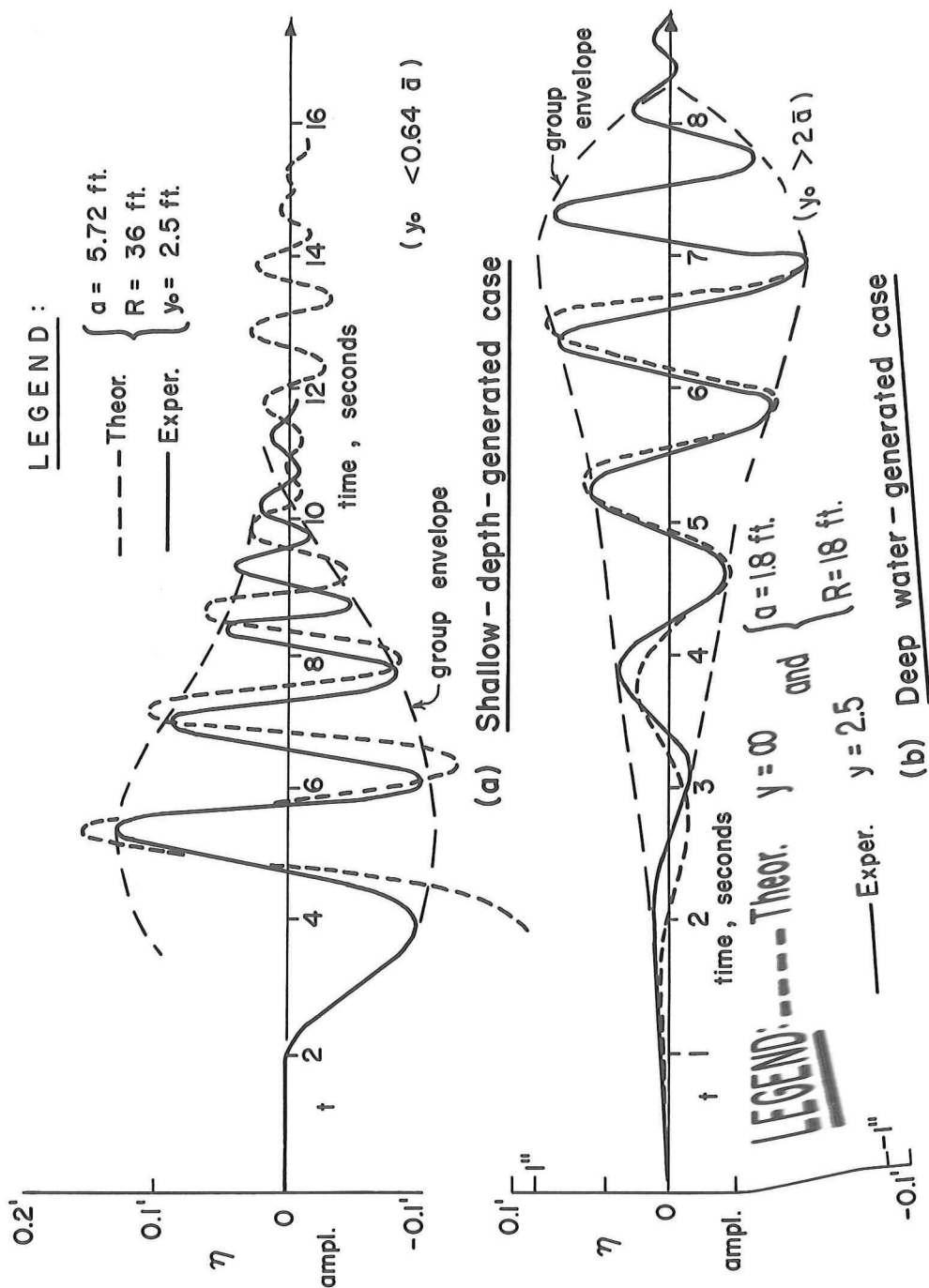
$$H_{max} \propto B^{1/2}$$

Figure 5   Dispersive wave (time trace) due to simulated underw̶ater
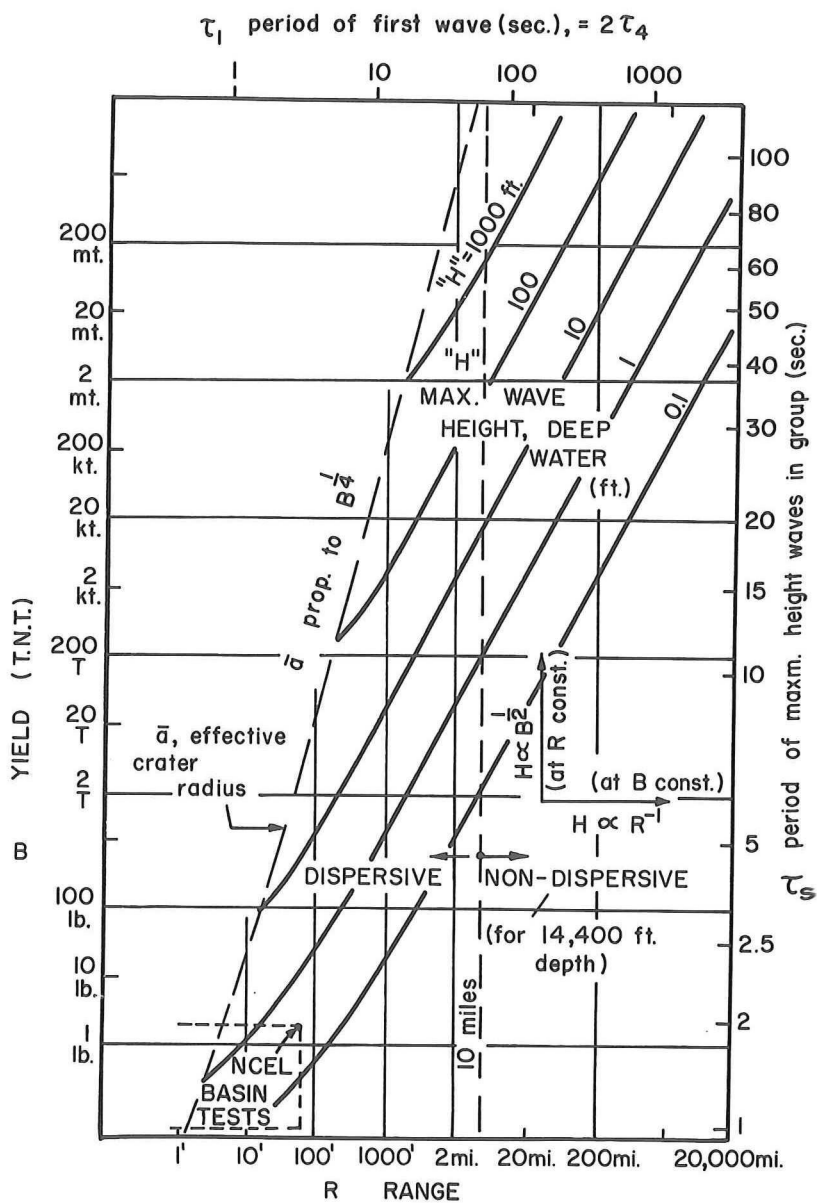explosion - (paraboloid) experimental.

τ_1 period of first wave (sec.), $= 2\tau_4$

1   10   100   1000

"H"=1000 ft.
100
"H"
MAX. WAVE
HEIGHT, DEEP
WATER
(ft.)

$\bar{a}$ prop. to $B^{\frac{1}{4}}$

$\bar{a}$

$H \propto B^{-\frac{1}{2}}$ (at R const.)

(at B const.)

$H \propto R^{-1}$

$\bar{a}$, effective
crater
radius

DISPERSIVE   NON-DISPERSIVE

(for 14,400 ft.
depth)

10 miles

NCEL
BASIN
TESTS

100   80   60   50   40   30   20   15   10   5   2.5   2   1

period of maxm. height waves in group (sec.)

τ_s

YIELD (T.N.T.)

200 mt.
20 mt.
2 mt.
200 kt.
20 kt.
2 kt.
200 T
20 T
2 T
B
100 lb.
10 lb.
1 lb.

1'   10'   100'   1000'   2mi.   20mi.   200mi.   20,000mi.
R   RANGE

Figure 6   Wave generating capacity of underwater explosions:
in deep water at critical depth = bubble diameter
(extrapolation by Penney's theory).

(iii) For constant yield B, wave height at maximum is proportional to inverse of range:

$$H_{max} \propto R^{-1}$$

(iv) Period of leading waves (whether sensibly high or not) increases as the square root of the range:

$$\tau_1 \propto R^{1/2}$$

(v) Period of waves of maximum height is independent of range and is proportional to $(\bar{a})^{1/2}$ or $B^{1/8}$

$$\tau_{max} \propto B^{1/8}$$

The diagram also shows that beyond a range of about 10 miles the leading waves from an explosion in deep ocean become non-dispersive, or similar in propagation characteristics to tsunami, due to the limiting depth effect.

COMPARISON BETWEEN EXPLOSION-GENERATED WAVES AND OSCILLATORY WAVES

Figure 7 indicates that simulated explosion waves, generated in the laboratory, follow closely the relationship $H \propto R^{-1}$ up to the point of encounter with the toe of the beach slope, whereas uniform oscillatory waves would follow the horizontal dashed line. From that point on the dispersive waves deviate from $H \propto R^{-1}$, yet still decrease to a minimum some distance up the slope. (Oscillatory waves would have increased according to Green's Law (dashed curve) $H \propto b^{-1/2} y^{-1/4}$, where b is the horizontal spacing between rays or orthogonals.) Experimentally (chain-dotted curve) it is obtained that in the shoaling of these dispersive waves until breaking a relationship $H \propto b^{-1/2} y^{-1/4} R^{-1/2}$ is followed (dotted curve), Jordaan (1965).

Figure 8a shows again the comparison of the shoaling of oscillatory waves, increasing according to Green's Law, as compared to Figure 8b showing the measured dispersive waves generated by sudden withdrawal of the para-boloidal plunger. For comparable wave heights and forms just before reaching the shoreline (figure 8c), it is seen that the prior history of the dis-persive waves is entirely different from that of the oscillatory waves. The loci of the highest crest and lowest trough show local minima and maxima over the sloping portion while the oscillatory waves are monotonically increasing in amplitude.

LIMITATIONS ON THE BREAKING HEIGHT AND RUN-UP OF DISPERSIVE WAVES

Figure 9 shows the relationship for dispersive waves between relative run-up RU/H, and deep water wave steepness H_0/L_0 of impulsive waves, as
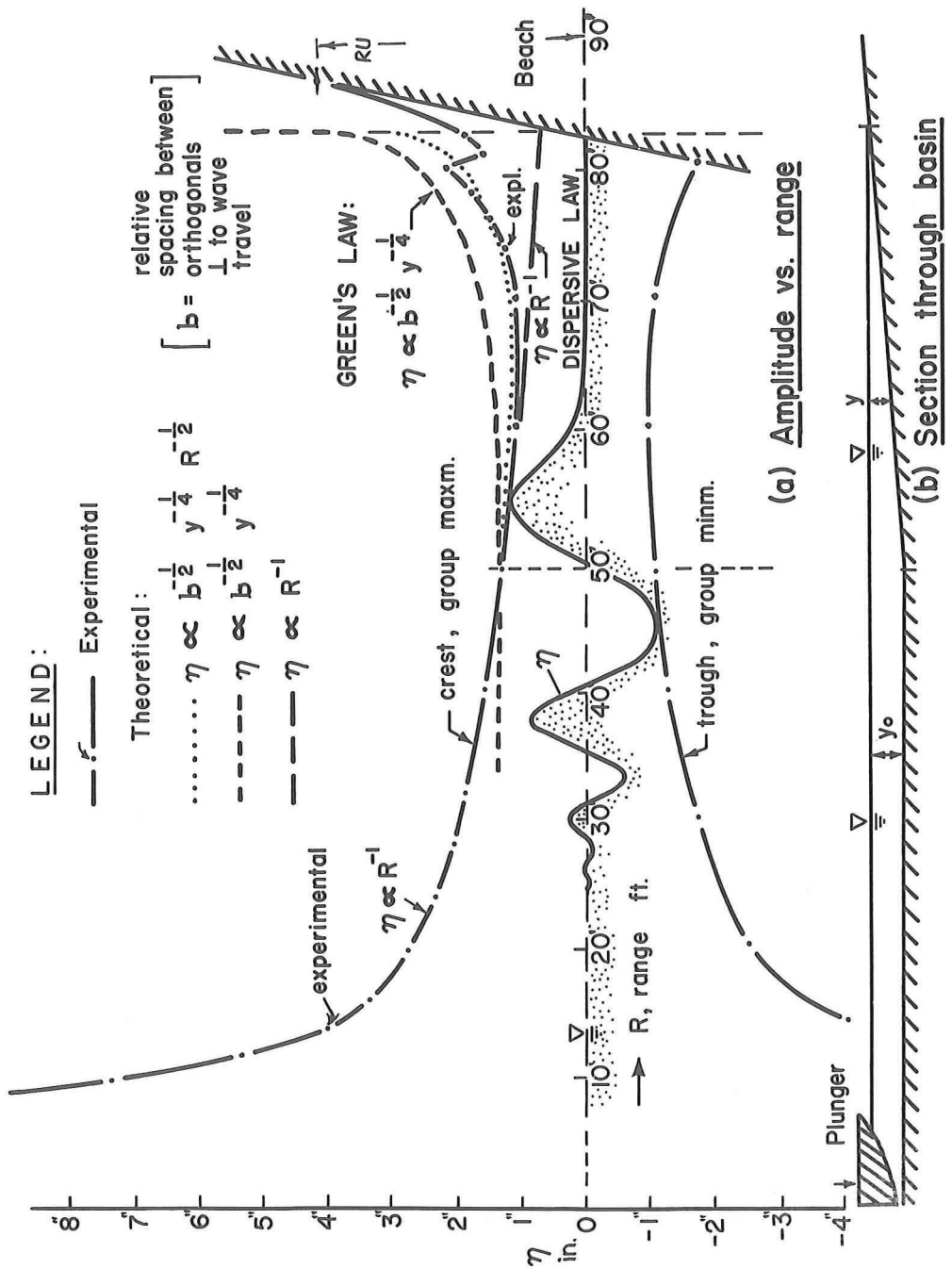
Figure 7 Impulsively-generated wave: amplitude vs. range,
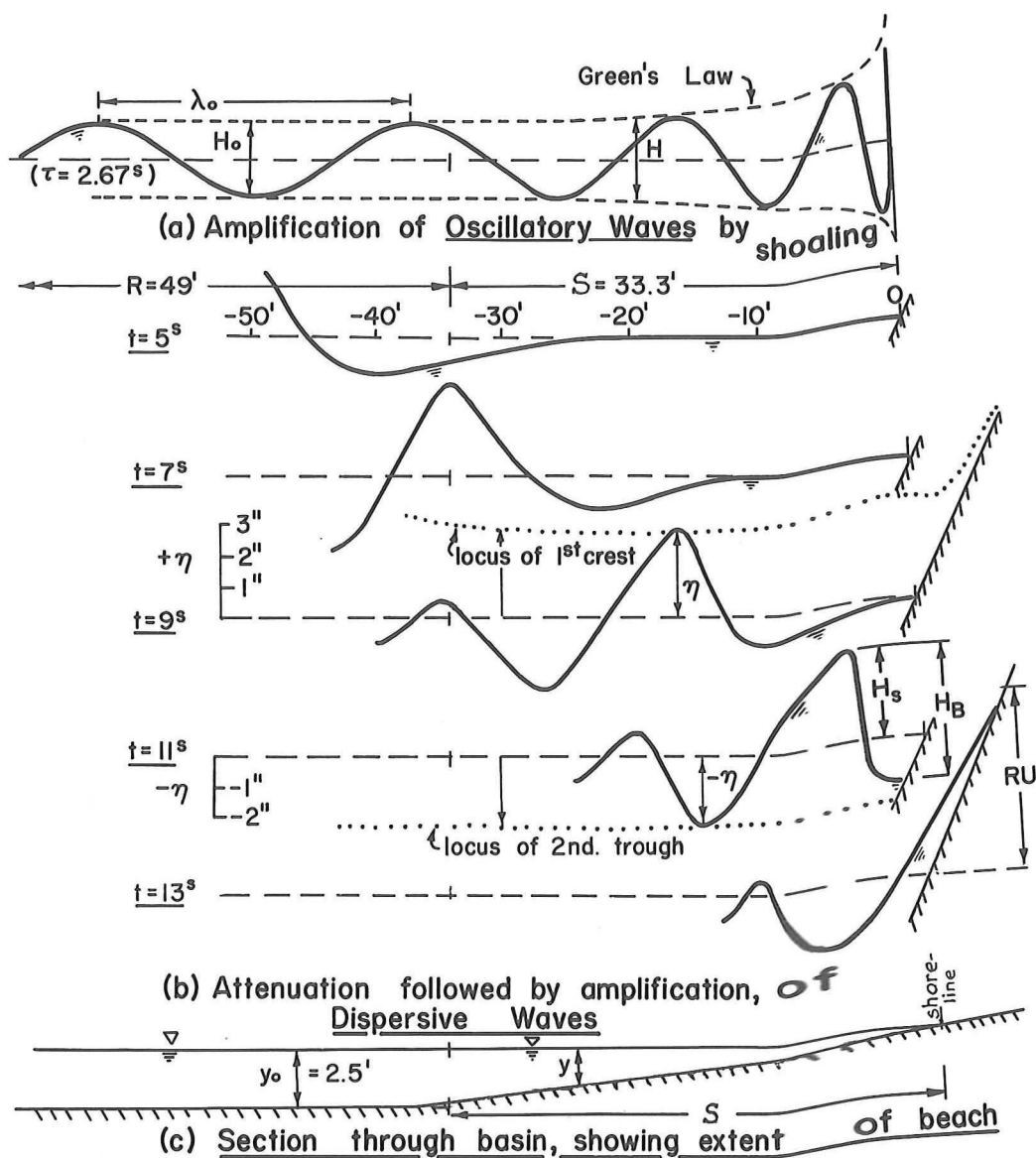showing increase due to beach, experimental.

Figure 8   Wave profiles and attenuation or amplification. Comparison
between oscillatory (computed) vs. dispersive waves (ex-
perimental) generated by impulse.

measured at NCEL (1965). Kaplan's (1955) results for impulsive waves are shown for comparison. The run-up relationship obtained

$$\frac{R.U.}{H_o} = 0.34\left(\frac{H_o}{L_o}\right)^{-1/3}$$

has the same exponent in the NCEL and Kaplan data, for the beach slopes used, 1:15 and 1:30 respectively. The effect of charge in beach slope on the run-up of dispersive waves is seen in the plot to involve a change in the proportionality constant principally.

Finally the run-up of tsunami-type explosive waves (i.e. non-dispersive waves such as would be generated by very large underwater nuclear explosions where $A \geq y/2$) is illustrated in Figure 10.

A first approximation is obtainable for the breaker height of waves originating as long waves; as follows:

By Green's law, if $\quad H_B = H_o\left(\frac{y_B}{y_o}\right)^{-1/4}\quad$ where

H_B is the breaker height, and H_o the wave height in deep water, and

if $\quad H_B = y_B \quad$ then it is obtained that:

$$H_B = H_o^{4/5}\, y_o^{1/5}$$

For example, for $H_O = 1$ ft, in 14,500-ft depth, Green's Law is satisfied up to $H_B/H_o = 6$ and hence $\quad H_B = y_B = 6\,ft.$

For a value $H_O = 1$ ft, in 600-ft depth on the other hand the value

$H_B = y_B = 3.5\,ft$ is obtained.

Data from Kaplan (1955) for H_B vs H_o/L_o and from Jordaan (1965) for RU vs H_B indicate that the run-up of tsunami type explosive waves is between two to three times the breaker height H_B. Hence from the data in Figures 9 and 10, (taking the upper envelope rather than the line through the mean of the points) two meaningful estimates of the upper limit of run-up can be obtained:

$$(1)\qquad \frac{RU}{H_o} \not> 0.5\left(\frac{H_o}{L_o}\right)^{-1/3}$$

and $\quad(2)\qquad \dfrac{RU}{H_o} \not> 3\left(\dfrac{y_B}{y_o}\right)^{-1/4}$
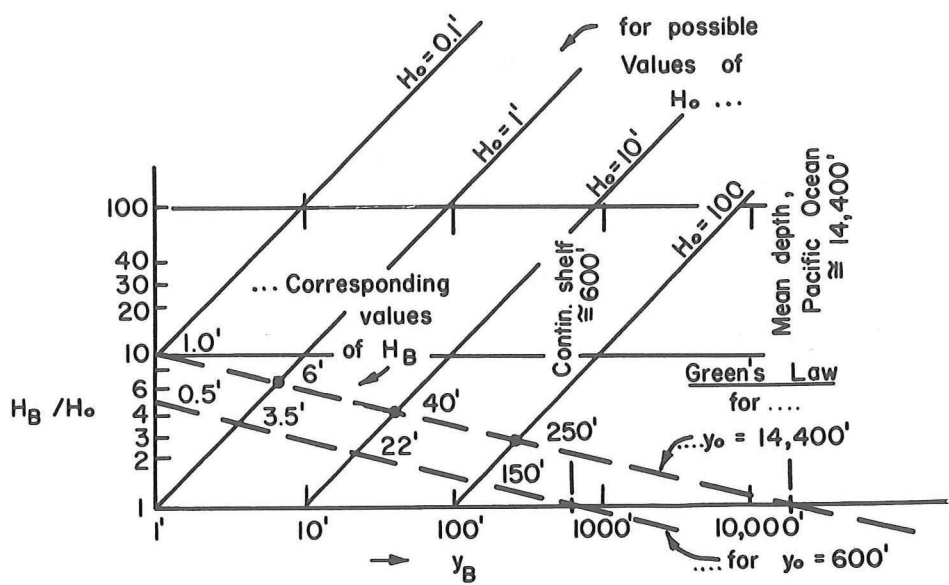
Figure 9   Relative run-up $\frac{RU}{H_o}$ vs. wave steepness $\frac{H_o}{L_o}$, experimental.

Figure 10   Deep water wave height H_O producing various values of shore-
line (breaker) wave height H_B for very large underwater
nuclear explosions ($A \geq y/2$).

With $\quad H_B \cong y_B \quad$ and $\quad RU \not> 3H_B$

there results, (3) $\qquad \dfrac{RU}{H_o} \not> 3\left(\dfrac{y_o}{H_o}\right)^{1/5}$

CONCLUSION

In summary, the dominant wave properties are determined based on the generating source magnitude and crater dimensions. The dispersive wave attenuation and the subsequent shoaling amplification is determined from a modified Green's Law relationship for two-dimensional radiating dispersive waves; or by the conventional Green's Law relationship for waves that are already shallow water waves at their generating area (tsunami type). The maximum breaker height and maximum run-up on a uniform ideal slope for the latter case is found to be directly expressible in terms of the deep water wave height, length or generating depth.

ACKNOWLEDGMENT

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