Delft University of Technology

Comment on ”Faulty assumptions: Groundwater modeling through anisotropic fault zones” by Jun-Hong Lin and Ying-Fan Lin

Anderson, Erik I.; Bakker, Mark

DOI
10.1016/j.jhydrol.2025.133728

Publication date
2025

Document Version
Final published version

Published in
Journal of Hydrology

Citation (APA)
Anderson, E. I., & Bakker, M. (2025). Comment on ”Faulty assumptions: Groundwater modeling through anisotropic fault zones” by Jun-Hong Lin and Ying-Fan Lin. *Journal of Hydrology*, *662*, Article 133728. https://doi.org/10.1016/j.jhydrol.2025.133728

Important note
To cite this publication, please use the final published version (if applicable).
Please check the document version above.

Copyright
Other than for strictly personal use, it is not permitted to download, forward or distribute the text or part of it, without the consent of the author(s) and/or copyright holder(s), unless the work is under an open content license such as Creative Commons.

Takedown policy
Please contact us and provide details if you believe this document breaches copyrights.
We will remove access to the work immediately and investigate your claim.

This work is downloaded from Delft University of Technology.
For technical reasons the number of authors shown on this cover page is limited to a maximum of 10.

Green Open Access added to TU Delft Institutional Repository
as part of the Taverne amendment.

More information about this copyright law amendment
can be found at https://www.openaccess.nl.

Otherwise as indicated in the copyright section:
the publisher is the copyright holder of this work and the
author uses the Dutch legislation to make this work public.

Journal of Hydrology 662 (2025) 133728

Contents lists available at ScienceDirect

Journal of Hydrology

journal homepage: www.elsevier.com/locate/jhydrol

Discussion

Comment on "Faulty assumptions: Groundwater modeling through anisotropic fault zones" by Jun-Hong Lin and Ying-Fan Lin

Erik I. Anderson [a,*], Mark Bakker [b]

[a] INTERA, 3123 Molly Brown Lane, Green Bay, 54313, WI, USA

[b] Water Management Department, Civil Engineering and Geosciences, Delft University of Technology, Stevinweg 1 2628 CN, Delft, Netherlands

ARTICLE INFO

This manuscript was handled by Renato Morbidelli, Editor-in-Chief.

Keywords:
Groundwater
Fault zone
Boundary conditions

ABSTRACT

Lin and Lin (*Faulty assumptions: Groundwater modeling through anisotropic fault zones, Journal of Hydrology 653(2025)*) make unfounded claims about earlier studies of fault aquifer interaction: they state that the standard boundary conditions for a conductive fault used in earlier studies are flawed or based on faulty assumptions and that the earlier studies did not consider the discontinuities in both the normal component of flow and hydraulic head across a general fault. Further, they are unclear about the approximations in their own analysis which attempts to replace the two-dimensional flow field within a thin, anisotropic fault zone with two, one-dimensional, internal boundary conditions. Their claims about earlier studies are refuted and the approximations in their analysis are examined. Despite the limitations of their analysis, the work of Lin and Lin has value.

1. Introduction

In their recent paper, Lin and Lin (2025) (L&L, briefly) apply the theory of imperfect interfaces to the problem of groundwater flow to a well through a fault. They present, without derivation, interface boundary conditions to replace a thin, anisotropic fault zone embedded in an aquifer of infinite extent. The authors state that their result *"suggests that the mathematical formulas used for fault-aquifer interfaces in the existing analytical studies...may be fundamentally flawed or based on oversimplified assumptions"*. They refer directly to the work of Anderson (2006) and Anderson and Bakker (2008), and explicitly mention the boundary condition for a conductive fault as being in error. They present a condition that reflects the effects of fault conductance (equation 4.b in L&L), which includes a surface Laplacian of the drawdown. Conditions used in previously published analytical studies do not include this term. They attribute the difference to an incorrect assumption, made in the earlier studies, of one-dimensional flow along the fault. The condition that they question is standard, presented in textbooks (i.e., Strack (1989), p. 422), and has been used by many researchers to describe the effects of conductive faults or fractures on groundwater flow. If the authors are correct, a large body of published work is flawed. Fortunately, this is not the case. We show here that the boundary condition used by Anderson (2006) and others is, after differentiation, the same condition as presented by L&L.

Further, when applying the theory of imperfect interfaces to a general fault, L&L fail to state that the presented boundary conditions are approximate, developed by truncating asymptotic expansions. In fact, the main approximation made by L&L related to the conductance of a general fault is an incorrect assumption of one-dimensional flow within the fault. We discuss both the exact boundary conditions for a conductive fault and the approximate boundary conditions proposed by L&L to represent a general fault.

2. The conductive fault

We begin with the internal boundary conditions that represent a conductive fault, as presented by Anderson (2006) and Anderson and Bakker (2008), for steady flow in an isotropic aquifer. The conditions are expressed in the coordinate system of L&L where the fault is aligned with the y-axis. The first condition ensures that the heads are continuous across the fault:

$$\Phi^- = \Phi^+ \tag{1}$$

where Φ [L^3/T] is the discharge potential equal to Th, where T [L^2/T] is the aquifer transmissivity and h [L] is the hydraulic head. The superscripts "+" and "−" indicate evaluation along the y-axis in the right and left aquifer domains, respectively. The second condition reflects a jump in the normal component of flow across the fault:

$$\Psi^+ - \Psi^- = C_f \frac{Q_y^-}{T} \tag{2}$$

* Corresponding author.
E-mail address: erik.isaac.anderson@gmail.com (E.I. Anderson).

https://doi.org/10.1016/j.jhydrol.2025.133728
Received 9 March 2025; Received in revised form 13 June 2025; Accepted 16 June 2025
Available online 10 July 2025
0022-1694/© 2025 Elsevier B.V. All rights are reserved, including those for text and data mining, AI training, and similar technologies.

where Ψ [L^3/T] is the stream function, C_f [L^3/T] is the fault conductance, and Q_y [L^2/T] is the y-component of the discharge vector. Note that L&L use the symbol C_f to represent the storage in the fault rather than the fault conductance.

The right-hand side of Eq. (2) is the total discharge [L^3/T] within the fault at any value of y. The first condition (1) ensures that the component of flow along the fault is the same on both sides of the fault, and therefore,

$$C_f \frac{Q_y^-}{T} = C_f \frac{Q_y^+}{T} \tag{3}$$

We note that the internal boundary conditions (1) and (2) exactly represent the effects of an anisotropic fault zone with an infinite hydraulic conductivity normal to the fault, and a finite hydraulic conductivity tangential to the fault axis (Anderson, 2006).

Condition (2) is disputed by L&L. They use aquifer drawdown, s [L], as the dependent variable where $s = h_o - h$ and h_o is the initial constant head in the aquifer. The components of the discharge vector are, in the notation of L&L

$$Q_x = T\frac{\partial s}{\partial x} \qquad Q_y = T\frac{\partial s}{\partial y} \tag{4}$$

On substitution of (4) into (2) and (3), we obtain

$$\Psi^+ - \Psi^- = C_f \left(\frac{\partial s}{\partial y}\right)^- = C_f \left(\frac{\partial s}{\partial y}\right)^+ \tag{5}$$

Fig. 1 shows a sketch of the relationship between the component of the drawdown gradient vector tangential to the fault on either side of the fault (gray arrows) and the drawdown gradient within the fault. The profile of the y-components of the gradient within the fault (black arrows) is uniform; the discharge within the fault varies with y as groundwater enters and leaves the fault, but the discharge profile, like the gradient profile, remains uniform for all y.

The discharge within the fault, $Q_y(y)$, is one-dimensional. This is not an assumption, as suggested by L&L, but rather a consequence of the infinite hydraulic conductivity of the fault in the x-direction, which is reflected in condition (1).

The fault conductance, C_f, defined by Anderson, translated into the notation of L&L, is

$$C_f = b b_f K_{f,y} \tag{6}$$

where b [L] is the aquifer thickness, b_f [L] is the width of the fault, and $K_{f,y}$ [L/T] is the hydraulic conductivity of the fault in the y-direction.

Finally, from the definition of the stream function,

$$\frac{\partial \Psi}{\partial y} = -Q_x = -T\frac{\partial s}{\partial x} \tag{7}$$

As the problem considered by L&L is transient, the stream function does not exist. We differentiate both sides of (5) with respect to y and substitute (6) and (7) into the results to obtain

$$\frac{T}{b}\left[\left(\frac{\partial s}{\partial x}\right)^+ - \left(\frac{\partial s}{\partial x}\right)^-\right] = -K_{f,y} b_f \left(\frac{\partial^2 s}{\partial y^2}\right)^- = -K_{f,y} b_f \left(\frac{\partial^2 s}{\partial y^2}\right)^+ \tag{8}$$

The expression prior to the second equal sign is identical to the condition proposed by L&L for an isotropic aquifer when no water is stored in the fault (equation 4.b in L&L).

In the case of the conductive fault, the surface Laplacians on either side of the fault ($(\partial^2 s/\partial y^2)^+$ and $(\partial^2 s/\partial y^2)^-$) that appear in (8) are equal, a consequence of (1). This is important for the discussion of the general fault that follows: the drawdown, the y-component of the drawdown gradient, and the surface Laplacian on either side of a general fault are not equal, as (1) is no longer valid.

3. The general fault

A general fault, as described by Anderson (2006), is a thin anisotropic domain embedded within an aquifer. The principal directions of the hydraulic conductivity tensor within the fault domain are aligned with the x- and y-axes in the coordinate system of L&L. The principal components of the hydraulic conductivity tensor in the fault domain are both finite and non-zero.

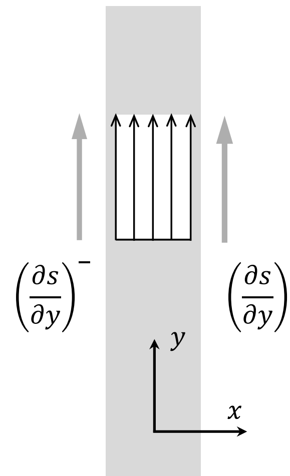

Fig. 1. The exact uniform profile of the y-component of the gradient vector (black arrows) within a conductive fault. The aquifer drawdown is the same on both sides of the fault, as are the gradient vectors (gray arrows) and the second derivatives of drawdown with respect to y (surface Laplacians).

The work of L&L focuses on solving an initial value problem with internal boundary conditions representing a general fault. They motivate their work by claiming that earlier work by Anderson (2006), and Anderson and Bakker (2008) did not account for discontinuities in both the head and flux across a general fault. In fact, Anderson (2006) developed an analytical solution for steady flow to a well through a general fault. From the results, the jumps in both the head and normal component of flow were identified (see Figures 4 and 6 in Anderson (2006)).

Further, Anderson (2006) demonstrated that for the two limiting cases of anisotropy – the conductive fault and the leaky fault – the fault domain may be replaced exactly by internal boundary conditions (see Anderson, 2006, Figure 7). This is possible as in the limiting cases, the flow within the fault becomes one-dimensional. For the case of the conductive fault the discharge within the fault in the y-direction depends only on y ($Q_y(y)$). For the case of the leaky fault the discharge within the fault in the x-direction depends only on y ($Q_x(y)$). For the general fault, however, the two-dimensional flow field within the anisotropic fault domain cannot be replaced in an exact fashion by internal boundary conditions, even when the physical width of the fault becomes infinitesimal. Any attempt to do so is, necessarily, an approximation.

The internal boundary conditions proposed by L&L attempt to define the flow field within a general fault by two one-dimensional conditions. This is expressed symbolically as:

within the fault: $Q_x = Q_x(y)$ (eq. 4.a of L&L) (9)

within the fault: $Q_y = Q_y(y)$ (eq. 4.b of L&L) (10)

where Q_x and Q_y represent the components of the discharge vector in the fault domain. The true two-dimensional flow field within a general fault must be expressed as

within the fault: $Q_x = Q_x(x, y)$ (11)

within the fault: $Q_y = Q_y(x, y)$ (12)

We consider the conductive condition (equation 4.b of L&L), as expressed in Eq. (5) above. Fig. 2.a shows a non-uniform gradient

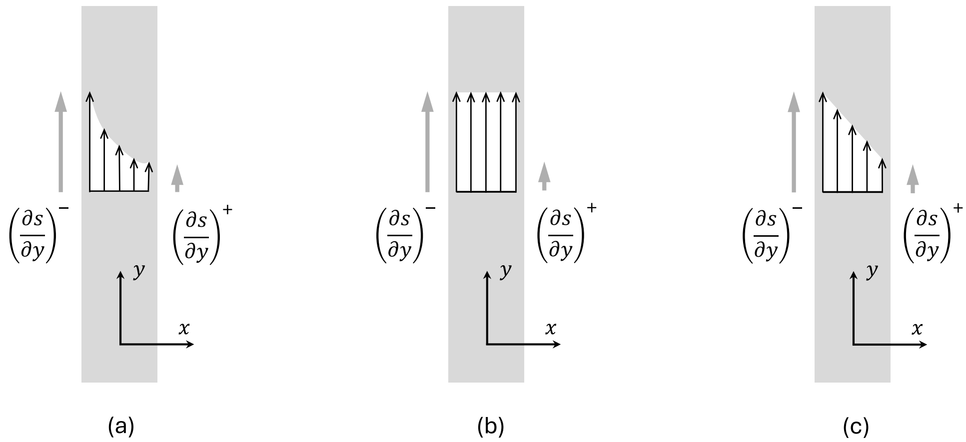

Fig. 2. The profile of the y-component of the gradient vector within a general fault: (a) the actual non-uniform profile; (b) the approximation applied by Lin and Lin; (c) a higher order approximation.

profile within a general fault, which results in a non-uniform discharge profile within the fault. Both the total discharge and the discharge profile along the fault axis vary with y as groundwater enters and leaves the fault. For all y, the profile is bounded by, but cannot be fully described by, the y-components of the drawdown gradients (or in the condition presented by L&L, the surface Laplacians) on either side of the fault.

Fig. 2.b shows the one-dimensional approximation made by L&L, with the total discharge within the fault based on the gradient on the left side of the fault; the jump in the y-component of the drawdown gradient across the fault is ignored when estimating the flow along the fault. The jump is created by the discontinuous heads and flux across the general fault. This feature, which is the motivating factor for the work of L&L, is not incorporated into their boundary conditions.

4. Conclusions and discussion

Lin and Lin (2025) make false claims about earlier studies of aquifer fault interaction and are unclear about their own analysis:

- Boundary conditions used in earlier studies to represent the conductive fault are not based on “faulty assumptions”, but are identical to the condition proposed by L&L. The confusion appears to be related to the use of the stream function in the earlier studies, and a misunderstanding of the relationship between the conductive fault and the general fault.
- The jumps in both the head and normal component of flow across a general fault have been identified and investigated in earlier studies. An exact solution for steady flow to a well through a general fault is available (Anderson, 2006).
- The boundary conditions proposed by L&L for a general fault are approximate. The conductive condition (equation 4.b in L&L) is based on an approximation of one-dimensional flow along the fault axis within the fault, with no consideration of the jump in the tangential component of the gradient across the fault. No discussion or analysis of the approximation or error is provided. The error could be directly evaluated by comparison of the approximate model of L&L to the exact model of Anderson (2006), for steady state discharges.

Despite these issues, the efforts of L&L have value. Accurately representing a thin, highly anisotropic fault domain embedded in a regional model is difficult. If the fault domain can be replaced, even approximately, by internal boundary conditions, the difficulties can be reduced.

The conditions proposed by L&L may reasonably approximate the effects of a fault on flow in the aquifer, and this should be tested. It is encouraging that the approximate conditions used by L&L include the exact boundary conditions for a conductive fault and a leaky fault as special cases (for $K_{f,x} \to \infty$ and $K_{f,y} \to 0$, respectively).

Higher order approximations can improve the representation of the general fault. For example, Fig. 2.c illustrates an alternate approximation of the flow in the fault which makes use of the drawdown gradients on both sides of the fault:

$$\Psi^+ - \Psi^- = \frac{C_f}{2}\left[\left(\frac{\partial s}{\partial y}\right)^- + \left(\frac{\partial s}{\partial y}\right)^+\right] \quad (13)$$

In a similar fashion, higher order approximations of the leaky condition (equation 4.a in L&L) should be investigated.

The form of the general boundary conditions presented by Lin and Lin (equations 4.a and 4.b), as well as higher order approximations, are well suited for use in analytic element models, such as presented by Toller (2022) to simulate fracture flow. Toller simulates flow through intersecting conductive or leaky fractures using line dipoles with real or imaginary strengths, respectively. The general fault can be represented efficiently in the model as line dipoles with complex strengths.

CRediT authorship contribution statement

Erik I. Anderson: Writing – original draft, Formal analysis, Conceptualization. **Mark Bakker:** Writing – review & editing, Formal analysis.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

References

Anderson, E., 2006. Analytical solutions for flow to a well through a fault. Adv. Water Resour. 29, 1790–1803.

Anderson, E., Bakker, M., 2008. Groundwater flow through anisotropic fault zones in multiaquifer systesm. Water Resour. Res. 44.

Lin, J.-H., Lin, Y.-F., 2025. Faulty assumptions: Groundwater modeling through anisotropic fault zones. J. Hydrol. 653.

Strack, O., 1989. Groundwater Mechanics. Prentice Hall.

Toller, E., 2022. An analytic element model for intersecting and heterogeneous fractures in groundwater flow. Water Resour. Res. 48.