Geotechnical Safety and Risk V
T. Schweckendiek et al. (Eds.)

doi:10.3233/978-1-61499-580-7-328

Probabilistic Analysis at Ultimate Limit State of Strip Footings Resting on a Spatially Varying Soil Using Subset Simulation Approach

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Abstract. The failure probability of geotechnical structures with spatially varying soil properties is generally computed using Monte Carlo simulation (MCS) methodology. This approach is well known to be very time-consuming when dealing with small failure probabilities. One alternative to MCS is the subset simulation approach. This approach was mainly used in the literature in cases where the uncertain parameters are modelled by random variables. In this paper, it is employed in the case where the uncertain parameters are modelled by random fields, because the spatial variability of the soil properties has proven to greatly affect the behavior of geotechnical structures and to induce a significant change in the variability of their responses. This is illustrated through the probabilistic analysis at the ultimate limit state (ULS) of a strip footing resting on a one- and two-layer purely cohesive soil with a spatially varying cohesion. The soil cohesion parameter was modeled as an anisotropic non-Gaussian (log-normal) random field using a square exponential autocorrelation function. The Expansion Optimal Linear Estimation (EOLE) method was used to discretize this random field. The deterministic model was based on numerical simulations using the finite difference software FLAC^{3D}.

Keywords. Strip footing, ultimate limit state, subset simulation approach, Monte Carlo simulation, random field, spatial variability, Expansion optimal linear estimation method (EOLE)

1. Introduction

Traditionally, the analysis of geotechnical structures is based on deterministic approaches. In these approaches, the soil input parameters and the system responses are considered deterministic. During recent years, much effort has been paid to the probabilistic analysis of geotechnical structures. Some simplified methods have modelled the different uncertain parameters by random variables where the soil is considered as a uniform material. However, in nature, the soil parameters (shear strength parameters, elastic properties, etc.) vary spatially in both the horizontal and vertical directions as a result of depositional and post-depositional processes. This leads to the necessity of representing the soil parameters by random fields characterized not only by their marginal probability density functions (as is the case of random variables), but also by their autocorrelation functions. In this regard, more advanced probabilistic approaches were proposed in the literature. These approaches are generally based on the finite element or the finite difference method. In these approaches, one needs to discretize the random field into a finite number of random variables. Once the random field is discretized into a finite number of random variables, the failure probability can be determined. In the framework of these approaches, Monte Carlo simulation (MCS) is generally used to perform the probabilistic analyses. Notice that MCS methodology is not suitable for the computation of a small failure probability because the number of simulations required becomes very large in this case. Au and Beck (2001) proposed an alternative efficient approach (called subset simulation 'SS') to calculate the small failure probabilities in cases where the uncertain parameters are modelled by random variables. In this approach, the failure probability is expressed as a product of conditional probabilities of some chosen intermediate failure events. Thus, the problem of evaluating a small failure probability in the original probability space is replaced by a sequence of events in the conditional probability space. Later on, Ahmed and Soubra (2011, 2012) extended the SS approach to the case of random fields.

In this paper, the subset simulation method was employed to perform a probabilistic analysis at the ultimate limit state of a strip footing resting on a one- and two-layer purely cohesive soil with a two-dimensional spatially varying cohesion. As for the random field discretization method, the Expansion Optimal Linear Estimation (EOLE) methodology proposed by Li and Der Kiureghian (1993) and extended by Vořechovsky (2008) was used to discretize the random field. The deterministic model employed for the computation of the system response was based on numerical simulations using $FLAC^{3D}$ software. After a brief description of the deterministic model, the EOLE method and SS approach are presented. Then, the probabilistic analysis and the corresponding results are presented and discussed. This paper ends with a conclusion.

2. Deterministic Model

The deterministic model used for the computation of the ultimate footing load was based on numerical simulations using the finite difference code $FLAC^{3D}$. For this calculation, a footing of width B that rests on a soil domain of width $7.5B$ and depth 3B was considered in the analysis. For the displacement boundary conditions, the bottom boundary was assumed to be fixed and the vertical boundaries were constrained in motion in the horizontal direction.

The undrained soil behavior was modeled using a conventional elastic-perfectly plastic model based on the Tresca failure criterion. On the other hand, an associative flow rule was considered in this study. Notice that the soil Young modulus E and Poisson ratio υ were assumed as follows: E = 60MPa and υ= 0.3. The footing of breadth equal to 2m and a depth equal to 0.5m was simulated by a weightless elastic material. Its elastic properties are the Young's modulus E=25GPa and the Poisson's ratio υ=0.4. The connection between the footing and the soil mass was modeled by interface elements having the same mean values of the soil shear strength parameters in order to simulate a perfectly rough soil-footing interface. Concerning the elastic properties of the interface, their values were as follows: Ks =1GPa*,* Kn =1GPa*,* where Ks and Kn are the shear and normal stiffnesses of the interface.

It should be noted that the size of a given element in the deterministic mesh depends on the autocorrelation distances of the soil properties. Der Kiureghian and Ke (1988) have suggested that the length of the largest element of the deterministic mesh in a given direction (horizontal or vertical) should not exceed 0.5 times the autocorrelation distance in that direction. This condition was satisfied in this paper.

For the computation of the ultimate bearing capacity of a rigid rough strip footing subjected to a central vertical load using $FLAC^{3D}$, the following method was adopted: an optimal controlled downward vertical velocity of 5×10^{-6} m/time step (i.e. displacement per time step) was applied to the bottom central node of the footing. Damping of the system was introduced by running several cycles until a steady state of plastic flow is developed in the soil underneath the footing. At each cycle, the vertical footing load was obtained by computing the integral of the normal stress components for all elements in contact with the footing. The value of the vertical footing load at the plastic steady state is the ultimate footing load.

3. EOLE Methodology

The expansion optimal linear estimation (EOLE) method originally proposed by Li and Der Kiureghian (1993) and then extended by Vořechovsky (2008) to cover the case of correlated non-Gaussian fields is used herein to discretize the non-Gaussian (log-normal) random field of the soil cohesion. For a non-Gaussian random field $Z_C^{NG}(x,y)$ of the soil cohesion which is described by: (i) constant mean μ_C and standard deviation σ_C, (ii) non-Gaussian marginal cumulative distribution function F_C, and (iii) a square exponential autocorrelation function $\rho_z^{NG}[(x,y),(x',y')]$, the value of the correlation between two arbitrary points (x, y) and (x', y') is given as follows:

$$\rho_z^{NG}[(x,y),(x\acute{},\acute{y})] = exp\left(-\left(\frac{x-\acute{x}}{a_x}\right)^2 - \left(\frac{y-\acute{y}}{a_y}\right)^2\right) \quad (1)$$

where a_x and a_y are the autocorrelation distances along x and y respectively. In the EOLE method, one should first define a stochastic grid composed of q grid points (or nodes) obtained from the different combination of H points in the x (or horizontal) direction, and V points in the y (or vertical) direction assembled in a vector $Q = \{Q_n = (x_h, y_v)\}$ where h=1, …, H, v=1, …, V and n=1, …, q. Notice that for the vector Q composed of q elements, the values of the field are assembled in a vector $\chi = \{\chi_n = Z(x_h, y_v)\}$ where h=1, …, H, v=1, …, V and n=1, …, q. Then, one should determine the correlation matrix for which each element $(\Sigma_{\chi;\chi}^{NG})_{i,j}$ is calculated using Eq. (1) as follows:

$$(\Sigma_{\chi;\chi}^{NG})_{i,j} = \rho_z^{NG}[Q_i, Q_j] \quad (2)$$

where i=1, …, q and j=1, …, q. Notice that the matrix $\Sigma_{\chi;\chi}^{NG}$ in Eq. (2) provides the correlation between each point in the vector χ and all the other points of the same vector. The non-Gaussian autocorrelation matrix $\Sigma_{\chi;\chi}^{NG}$ should be transformed into the Gaussian space using the Nataf transformation. As a result, one obtains a Gaussian autocorrelation matrix $\Sigma_{\chi;\chi}^{G}$ that can be used to discretize the Gaussian random field of the soil cohesion as follows:

$$\tilde{Z}_C(x,y) = \mu_C + \sigma_C \sum_{j=1}^{N} \frac{\xi_j}{\sqrt{\lambda_j}} \cdot \phi_j \cdot \Sigma_{z(x,y);\chi} \quad (3)$$

where (λ_j, ϕ_j) are the eigenvalues and eigenvectors of the Gaussian autocorrelation matrix $\Sigma_{\chi;\chi}^{G}$, $\Sigma_{z(x,y);\chi}$ is the correlation vector between each point in the vector χ and an arbitrary point (x, y) of the field, ξ_j is a standard normal random variable, and N is the number of terms (expansion order) retained in the EOLE method. Once the Gaussian random field is obtained, it should be transformed into non-Gaussian space by applying the following formula:

$$\tilde{Z}_C^{NG}(x,y) = F_C^{-1}\{\Phi[\tilde{Z}_C(x,y)]\} \quad (4)$$

where $\Phi(.)$ is the standard normal cumulative density function.

4. Subset Simulation Approach

4.1. Basic Idea of Subset Simulation

Consider a failure region F defined by the condition G<0 where G is the performance function and let (s_1, …, s_k, ..., s_{Nt}) be N_t samples located in the space of the uncertain variables where 's' is a vector of random variables. It is possible to define a sequence of nested failure regions F_1, …, F_j, ..., F_m of decreasing size where $F_1 \supset ... \supset F_j \supset ... \supset F_m = F$ (cf. Figure 1). An intermediate failure region F_j can be defined by $G_j < C_j$ where C_j>0. Thus, there is a decreasing sequence of positive numbers C_1, …, C_j, ..., C_m corresponding respectively to F_1,…, F_j,…, F_m where C_1>…>C_j>...> C_m=0. The N_t samples (s_1,…, s_k, ..., s_{Nt}) will be divided into groups with equal number N_s of samples (s_1, …, s_k, ..., s_{Ns}). Thus, N_t=m x N_s where m is the number of failure regions. The first N_s samples are generated according to MCS methodology following a target PDF (P_t). The next N_s samples of the different subsequent failure regions are obtained using Markov chain method based on the modified Metropolis-Hastings (M-H) algorithm according to a proposal PDF (P_p) as explained in Ahmed and Soubra (2012) among others.

The conditional failure probability corresponding to an intermediate failure region F_j is calculated as follows:

$$P(F_j|F_{j-1}) = \frac{1}{N_s} \sum_{k=1}^{Ns} I_{F_j}(s_k) \quad (5)$$

where $I_{F_j} = 1$ if s ∈ F_j and $I_{F_j} = 0$ otherwise. The failure probability P(F)=P(F_m) of the failure region F can be calculated from the sequence of conditional failure probabilities as follows:

$$P(F) = P(F_m) = P(F_m|F_{m-1}) \times P(F_{m-1}|F_{m-2}) \times P(F_{m-2}|F_{m-3}) \times ... \times P(F_2|F_1) \times P(F_1) \quad (6)$$

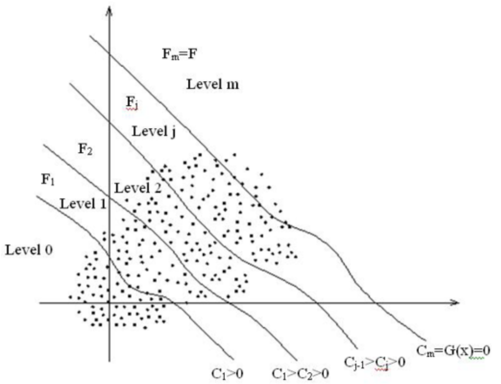


Figure 1. Nested Failure domain

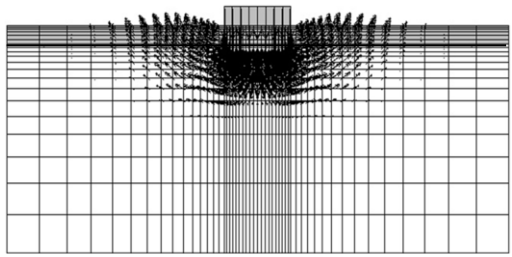
4.2. Implementation of SS Approach in Case of a Spatially Varying Soil Property

In order to employ the SS methodology in the case of a spatially varying soil property, a link between the SS approach and EOLE methodology was performed. It should be emphasized here that EOLE includes two types of parameters (deterministic and stochastic). The deterministic parameters are the eigenvalues and eigenvectors of the covariance function. The role of these parameters is to ensure the correlation between the values of the random field at the different points in the space. On the other hand, the stochastic parameters are represented by the vector of the standard normal random variables $\{\xi_i\}_{i=1,\ldots,M}$. The role of these parameters is to ensure the random nature of the uncertain parameter. The link between the SS approach and EOLE was performed through the vector $\{\xi_i\}_{i=1,\ldots,M}$. The basic idea of the link is that for a given random field realization obtained by EOLE, the vector $\{\xi_i\}_{i=1,\ldots,M}$ represents a sample 's' of the subset simulation method for which the system response is calculated in two steps. The first step is to substitute the vector$\{\xi_i\}_{i=1,\ldots,M}$ in Eq. (3) to calculate the value of the random field at each point in space according to its coordinates. The second step is to use the deterministic model to calculate the corresponding system response. A detailed algorithm explaining the application of the SS approach in the case of a spatially varying soil may be found in Ahmed and Soubra (2012).

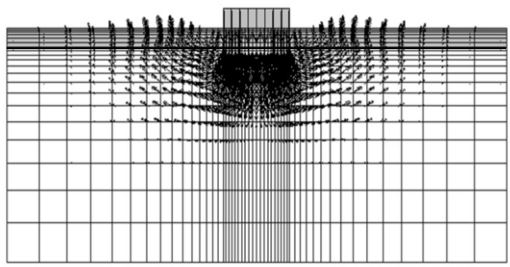
5. Numerical Results

5.1. Deterministic Numerical Results

Only the results of a two-layer medium are represented in this section. The velocity fields for two cases of strong over soft clay (Cu1/Cu2=2 and Cu1/Cu2=5) when H/B=0.25 are shown in Figure 2. Notice that Cu1 and Cu2 are the soil cohesion of the upper and lower layers respectively. Notice also that H is the depth of the upper layer. Figure 2 shows that the displacement field becomes larger and deeper with increasing Cu1/Cu2. This indicates that the small soil strength of the bottom layer (with respect to the upper layer) has a significant influence on the displacement field, and it leads to a reduction in the bearing capacity. To investigate the influence of the depth of soft clay relative to that of the underlying stronger clay, the ratios H/B=0.25 and 0.5 were considered with Cu1/Cu2=0.25. The bearing capacity decreases with the H/B increase (cf. Figure 4). Also the failure mechanism is likely to occur (for both cases) entirely in the soft clay.

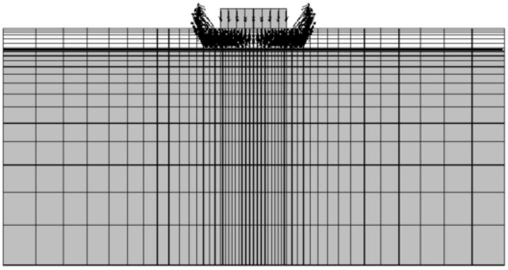


H/B=0.25 and Cu1/Cu2=2 (q_{ult}=173.48 kPa)

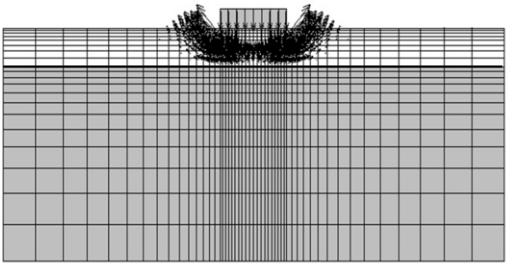


H/B=0.25 and Cu1/Cu2=5 (q_{ult}=92.51 kPa)

Figure 2. Displacement fields for strong over soft clay layers when H/B=0.25



H/B= 0.25 and Cu1/Cu2 = 0.25 (q_{ult}=351.5 kPa)



H/B= 0.5 and Cu1/Cu2= 0.25 (q_{ult}=295.64 kPa)

Figure 3. Displacement fields for soft over strong clay layers when Cu1/Cu2=0.25

5.2. Probabilistic Numerical Results

Figure 4 presents one realization of the random field for the case of a soft layer overlying a strong layer (Cu1/Cu2=0.25) when H/B=0.25.

Figure 4. Typical realization of the cohesion random field where H/B=0.25: (ax=1m, ay=1m for top layer ; ax=5000m, ay=1m for bottom layer)

The following values were adopted for the horizontal and vertical autocorrelation distances of the two layers: ax=1m and ay=1m for the top layer and ax=5000m and ay=1m for the bottom layer. This realization clearly shows the ability of EOLE method to accurately provide the correlation structure of the random field. Indeed, for the top layer, the soil cohesion is perfectly correlated in the vertical direction and it exhibits some fluctuations in the horizontal direction. The inverse occurs for the bottom layer.

5.2.1. Selection of the Optimal Number of Realizations Ns per Level of Subset Simulation

The number of realizations Ns to be used per level of subset simulation should be sufficient to accurately calculate the Pf value. The case of a single clay layer was investigated where the mean value and the coefficient of variation of the undrained shear strength Cu were respectively μc=50kPa and COVc=30%. A square exponential covariance function with a_x=10m and a_y=1m was considered herein. Notice that the failure thresholds Cj of the different levels of SS were calculated and presented in Table 1 for different values of Ns. This table indicates that the failure threshold decreases with successive levels until reaching a negative value at the last level, which means that the realizations generated by SS successfully progress towards the limit state surface G=0. It should be mentioned here that P(Fj) was chosen to be equal to 0.1. The Pf values and the corresponding values of the coefficient of variation for the different values of Ns are presented in Table 2. As expected, the coefficient of variation of Pf decreases with the increase of Ns. Notice that a smaller value of the coefficient of variation corresponds to a more accurate value of the estimated failure probability. This means that the number of realizations Ns to be used per level of subset simulation depends on the value of the coefficient of variation adopted in the analysis. For a target coefficient of variation of 0.3129, the required number of realizations per level is equal to Ns =1500 which corresponds to a total number of realizations of 7500. The corresponding value of P_f is equal to 1.44×10^{-5} as may be seen from Table 2.

Table 1. Evolution of the failure threshold Cj with the different levels j of the subset simulation and with the number of realizations (Ns) per level

Ns per level	Failure threshold Cj for each level j					
	C1	C2	C3	C4	C5	C6
100	0.973	0.610	0.417	0.269	0.107	-0.059
500	1.004	0.568	0.290	0.121	-0.01	
1000	1.006	0.544	0.312	0.145	0.019	-0.085
1500	1.017	0.586	0.316	0.125	-0.02	
2000	1.028	0.550	0.273	0.076	-0.06	
2500	1.041	0.579	0.299	0.118	-0.03	
3000	1.022	0.552	0.293	0.118	-0.03	

Table 2. Values of P_f and $COVP_f$ versus the number Ns of realizations per level

Ns per level	P_f x (10^{-4})	COV_{Pf}
100	0.034	1.0264
500	0.120	0.5438
1000	0.068	0.3794
1500	0.144	0.3129
2000	0.315	0.2689
2500	0.158	0.2387
3000	0.157	0.2152

6. Conclusion

This paper aims at presenting a probabilistic analysis at the ultimate limit state of a strip footing resting on a one- and two-layer purely cohesive soil with a spatially varying cohesion using the subset simulation approach. The main findings of the paper can be summarized as follows:

1- The deterministic results of a two clay layer medium have shown that at the same strength ratio, the ultimate bearing capacity decreases as the thickness of the top layer increases for a soft-over-strong clay profile, whereas an inverse trend occurs for a strong-over-soft clay profile. Also, the small soil strength of the bottom layer (with respect to the upper layer) has a significant influence on the displacement field, and it leads to a reduction in the ultimate bearing capacity.

2- The probabilistic analysis of a single clay layer with a spatially varying soil cohesion has shown that the number of realizations required by the subset simulation approach (to calculate the failure probability Pf) was relatively large. As expected, this number becomes smaller for a greater value of the coefficient of variation. The on-going work will focus on the probabilistic analysis of a two clay layer medium.

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