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DEPARTMENT OF AERONAUTICAL ENGINEERING

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Summary

This report gives the results of an investigation of the flow field on the expansion side of a delta wing with supersonic leading edges at supersonic speed. Pitot pressure measurements were made on a flat delta wing with a leading-edge sweep angle of 45° and an incidence of 14.2° at a free stream Mach number of 2.96. The measurements were concentrated on the determination of the location and the shape of the inboard shock wave. It was found that the inboard shock wave is conical to a good approximation. Near the wing surface the shock wave starts perpendicular to the wing surface; when passing the boundary characteristic of the central region of the flow field it bends strongly towards the wing symmetry plane.

A theoretical discussion is given and some shortcomings in previous theoretical considerations and numerical calculations are clarified by the results of the present measurements.

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1. Introduction

The problem of non-linear supersonic flow over a delta wing with supersonic leading edges has attracted considerable attention from theoreticians, because it represents a fundamental configuration in the theory of three-dimensional non-linear supersonic flow. Solutions have been presented by Maslen (ref. 1), Fowell (ref. 2) and Babaev (ref. 3) and discussions were given by Bulakh (ref. 4,5,6) and Reyn (ref. 7,8,9). Also a second-order solution to the problem was given by Clarke and Wallace (ref. 10). However, the problem has not been solved in a completely satisfactory way, yet. Especially, the shape and location of the inboard shock wave on the expansion side of the delta wing has been a point of controversy. In order to obtain supplementary information particular on this point the present measurements were made. Experimental evidence of the existence of the inboard shock wave was given by Fowell (ref. 2). Recently, Pierce and Treadgold (ref. 11) succeeded in visualising this shock wave by means of a conical shadowgraph technique and showed that near the wing surface the inboard shock was of the lambda type*).

The experiments described in this report concern the location and strength of the inboard shock in the flow on the expansion side over a delta wing having a leading edge sweep angle of 45° at an angle of incidence of 14.2° and a Mach number of 2.96. They were carried out in the 6 x 6 inch supersonic wind tunnel of the Department of Aeronautical Engineering, Technological University Delft.

2. Description of apparatus

2.1. Supersonic wind tunnel

The supersonic wind tunnel used for the experiments is a blow-down tunnel with a 6 x 6 inch test section having interchangeable nozzle blocks and a free jet chamber followed by a fixed diffuser. The wind tunnel has no heating equipment. The settling chamber pressure can be kept constant within 0.02 kg/cm^2 (0.3 p.s.i.). For the present tests the nozzle blocks of design Mach number 3 were used. The flow calibration

*) An indication of the same phenomenon is also found during experiments done by the first two authors.

at this Mach number is reported in ref. 13. This calibration has been done in the region of the test section occupied by the delta wing. The dimensions of this region were 100 mm, 80 mm and 20 mm in spanwise-, flow- and vertical direction, respectively.
Since no boundary layer correction has been applied to the nozzle blocks, the Mach number in the calibrated region decreases in flow direction with an amount of 0.004 per cm, the mean value being 2.96. There is no gradient in spanwise direction. The local variations in Mach number in the test region are less than 0.7°/o.

2. Model

The model is a triangular wing having a leading edge sweep angle of 45^{o} and a root chord of 60 mm. The wing has a flat upper surface and a wedge type cross-section, the apex angle perpendicular to the leading edge is 8^{o}. The leading edge thickness is less than 0.05 mm.
The model is mounted on a fixed support at an angle of incidence of 14.2^{o} to the undisturbed stream. The model is sketched in fig. 1.

Description of the tests

Pitot pressure measurements were made on the expansion side of the delta wing at a settling chamber pressure of 6.81 kg/cm^2 (96.86 p.s.i.). The settling chamber pressure was measured with a Bourdon manometer having an accuracy of 0.01 kg/cm^2 (0.15 p.s.i.). The pitot tube used had an inner diameter of 0.6 mm and an outer diameter of 1.0 mm. The pressures were measured in planes perpendicular to the wing surface at 30,33.4, 34.6,49.2 mm from the apex, respectively, at heights differing by 0.2 mm approximately. The measurements were made in two different ways. Firstly, to obtain a continuous picture of the pressure distribution, the pitot tube was traversed in spanwise direction at a speed of 1 mm/sec. The pressure was measured with a Statham pressure transducer (type PA 208TC, accuracy 0.75°/o) and recorded on an ACB Ultra Violet recorder (accuracy 2°/o). The position of the pitot tube was indicated by a 10 turns potentiometer (Beckman Helipot 200Ω , linearity 0.1°/o). Secondly, to determine more precisely the location and strength of the inboard shock wave, additional measurements were made with the pitot tube fixed during each individual run. In this case the data at various heights above the wing surface were obtained from points at intervals of 0.2 mm in spanwise direction. The pitot tube could be adjusted within 0.1 mm. This time the pressures were read from a mercury manometer. The data reproduced within 2°/o in most cases when measuring with the traversing pitot tube and within 0.5°/o for the data with the fixed pitot tube.

4. Results of the tests

Typical results of the pitot pressure distribution measured with a continuously traversing pitot tube at various stations from the apex and at various heights above the wing surface are plotted in fig. 2. Results found with the pitot tube held at a fixed position during a run are given in fig. 3. The position of the inboard shock wave as indicated in figs 4 and 5 were determined from fig. 3. Fig. 6 gives the differences in pitot pressure measured across the inboard shock wave.

5. Discussion of results of the measurements

The pitot pressures as given in fig. 2 all show the same trend. Going from the leading edge to the wing symmetry plane the following regions and phenomena can be distinguished: a leading edge shock wave; a Prandtl-Meyer expansion region; a small region having a weaker expansion due to a re-attachment of the flow near the leading edge; a region of nearly constant pitot pressure followed by a transition through a shock wave and/or a gradual compression to a second region of nearly constant pitot pressure. The leading edge shock wave appeared just upstream of the first theoretical straight characteristic of the Prandtl-Meyer expansion as shown in fig. 4. In ref. 12 Bardsley shows this shock wave to be generated from a small separation bubble at the leading edge of the wing; the re-attachment of the flow causes another shock wave. In this report the location of a re-attachment shock wave is assumed to be given by the beginning of the reduced expansion region. In fig. 4 the re-attachment shock wave practically coincides with the last characteristic of the Prandtl-Meyer expansion, as calculated from inviscid theory. If the quantities in the uniform flow of the outboard region of the wing are calculated from inviscid theory, it appears from comparison with the experimental data that the shock waves associated with the leading edge separation can be assumed to be isentropic.

The measurements indicated that the expansion region did not extend to the region in between the characteristics, as calculated from inviscid theory. It started immediately downstream of the leading edge shock wave and continued somewhat downstream of the re-attachment shock wave. This phenomenon can be explained by the occurrence of the leading edge separation bubble, which extends over about $5^{o}/o$ of half the local span. Also given in fig. 2 are the pitot pressures in the Prandtl-Meyer expansion region and the uniform flow, as calculated from inviscid theory. Towards the wing symmetry plane the region of nearly constant pitot pressure passed into a region of higher pitot pressures. As is shown in figs 2 and 3 this occurs in various ways, depending on the height above the wing surface. For a value of h/b_1 less than approximately 0.12 and not to close to the wing surface the compression took place almost exclusively in a shock wave; for values of h/b_1 approximately in between 0.12 and 0.25 a gradual compression was found in front of a shock wave, whereas above a value of h/b_1 equal to 0.25 no shock wave could be traced and only a gradual compression was found. Also indicated in fig. 4 is the location on the wing surface of the shock wave, obtained by two-dimensional shock relations from the condition that the stream direction immediately downstream of the shock is parallel to the wing center line.

The conical structure of the inboard shock wave emerges quite clearly from figs 4 and 5. In between values of h/b_1 of 0.05 and 0.10 the shock wave has a direction perpendicular to the wing surface. Starting from the value of $h/b_1 = 0.10$ the shock wave bends towards the wing symmetry plane. In fig. 6 the variation of the inboard shock strength with height above the wing surface is given. Its strength firstly increases, when going away from the wing surface, reaches a maximum at about $h/b_1 = 0.15$ and then decreases again. The latter phenomenon may be explained by the fact that the compression takes place more and more as an isentropic compression when moving away from the wing surface.

An explanation of the part of the curve for values of h/b_1 near the wing surface is difficult to give without going into the details of the vortex formation in that region.

6. Theoretical discussion

At this point the theoretical considerations given to describe the flow field may be recalled and the results of the numerical calculations may be reviewed. The first numerical calculation of the non-linear flow on the expansion side of the delta wing was given by Maslen (ref. 1), but no inboard shock wave was introduced. Fowell (ref.2) pointed out, that there was such an inboard shock wave and gave experimental evidence to this fact. Once the existence of the shock wave was accepted an explanation of its occurrence was given in various ways. Depending on the type of explanation also various suggestions were made on the location and shape of the shock wave.
It is argued by Reyn (ref. 7,8 and 9), with the help of local properties of the flow field, obtained by means of the hodograph transformation, that if a potential-theoretical solution for the flow on the expansion side of the delta wing exists, a consideration of the wave pattern leads to the introduction of limit-line singularities. The straight characteristic MD (fig. 7) and part of the characteristic MB would then be a conical limit line (of the second type). Since limit lines do not exist in real flow, an alternative solution involving a shock wave would have to be thought for and he expects the shock wave to be somewhat upstream of that part of the characteristic BMD, which possibly could be a limit line. The considerations leading to the impossibility of a shock free flow, as given by Bulakh (ref. 6) refer to the region downstream of the straight characteristic MD. This region should be a conical simple wave flow, since the characteristic MD is straight. Bulakh however argues that such a flow cannot be terminated by a conical-sonic line and that it is very unlikely that this may be accomplished by curved characteristics, since then all curved characteristics would have to converge to one point. As a result, he introduces a shock wave upstream of BMD and points out that also a

shock wave upstream of BE may be introduced; in the numerical calculation procedure the strength of the shock wave would automatically vanish at those points where no shock wave is present. The numerical calculations following such a procedure were made by Babayev (ref.3) and the shock wave was found to vanish along BE, the characteristic BM and some part of the characteristic MD. A straight shock wave perpendicular to the wing surface and extending to the characteristic MD was found. However, the flow field thus calculated by Babayev (ref.3), remains open to criticism. Firstly, if the shock wave would end at the characteristic MD, it should have zero strength there and be tangent to the characteristic. Since these things were not found in the numerical calculations it may be expected, that the shock wave extends beyond MD into the inboard region. Secondly, in ref. 3 the simple-wave flow downstream of MK is bordered by a conical-sonic line, which cannot occur as has been shown by Bulakh. An extension of the shock wave into the interior region, however, would provide an acceptable means to end the simple wave region. Also, a wave pattern may be envisaged, following the thoughts developed in ref. 8 and 9, which may fit in with the occurrence of the inboard shock wave in the interior region. In this pattern the expansion waves traveling along the straight characteristics in the Prandtl-Meyer flow continue along the extensions of these characteristics downstream of BM and are reflected as compression waves at the conical-sonic line. In the region in between this conical-sonic line and BM there are thus expansion waves along one family of characteristics and compression waves along the other family. The compression waves then continue along the straight characteristics in the conical simple-wave flow and are eventually absorbed by the shock wave. The shock wave would probably end at the conical-sonic line. A sketch of the possible flow pattern is given in fig. 7.

The present theoretical description fits in nicely with the results of the experiments reported on in this report. However, in order to make a real comparison between theory and experiments, it would be interesting to have a solution as given by Babayev, but starting with a zeroth approximation containing a shock wave extending into the interior region. The latter could possible best be obtained using the second-order theory given by Clarke (ref. 10).

7. Conclusions

1. Pitot pressure measurements on the expansion side of a triangular wing with supersonic leading edges confirm the existence of an inboard shock wave.
2. The inboard shock wave has a conical structure to a good approximation.
3. The inboard shock wave starts perpendicular to the wing surface and extends into the central region; in this region it bends strongly towards the wing symmetry plane.
4. The results of the present measurements can be used to clarify some shortcomings in previous theoretical considerations and numerical calculations.

Acknowledgement

The authors wish to thank Mr. J. Boeker of the Department of Aeronautical Engineering, Technological University Delft for his helpful advices concerning the experiments.

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9. Symbols

b : spanwise dimension measured from the central chord.

b_1 : half the local span.

h : dimension perpendicular to the wing surface.

p_p : pitot pressure.

p_{p_∞} : free stream pitot pressure.

$\Delta\left(\frac{p_p}{p_{p_\infty}}\right)$: difference in $\left(\frac{p_p}{p_{p_\infty}}\right)$ across the inboard shock wave.

x : chord wise dimension measured from the apex of the wing.

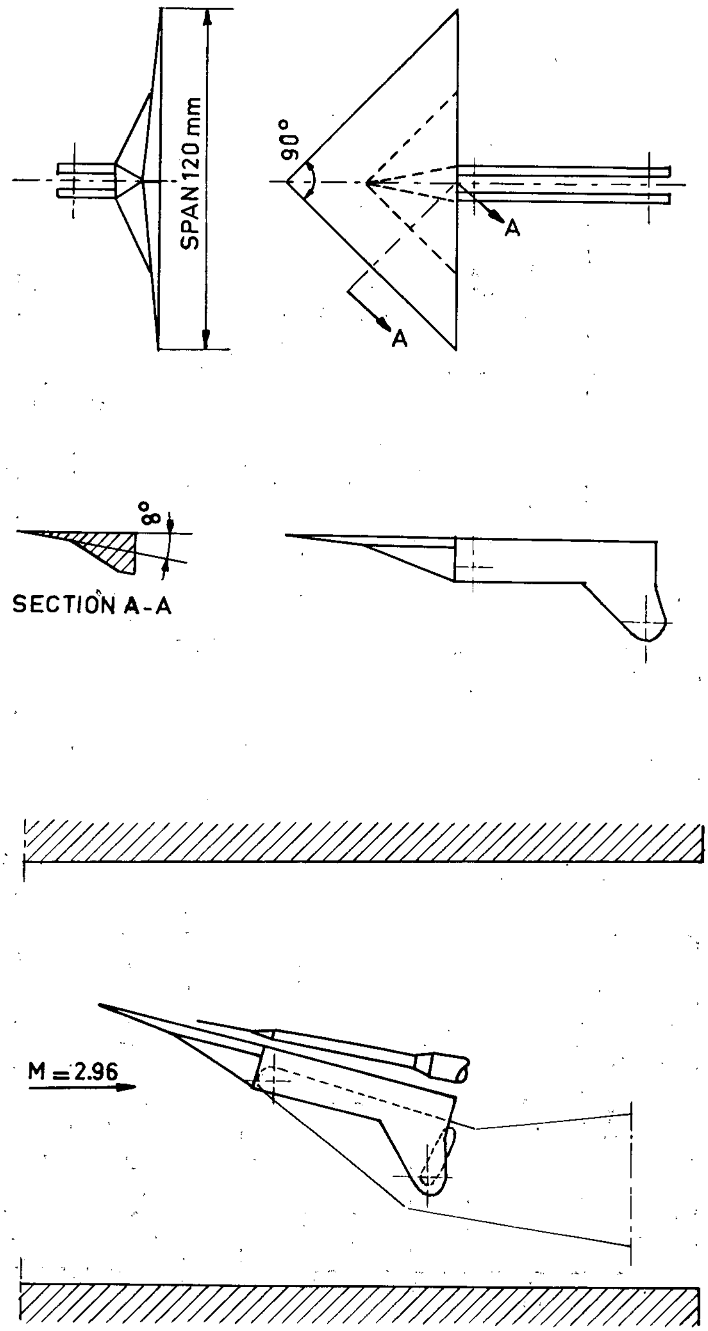

FIG.1 MODEL AND ARRANGEMENT

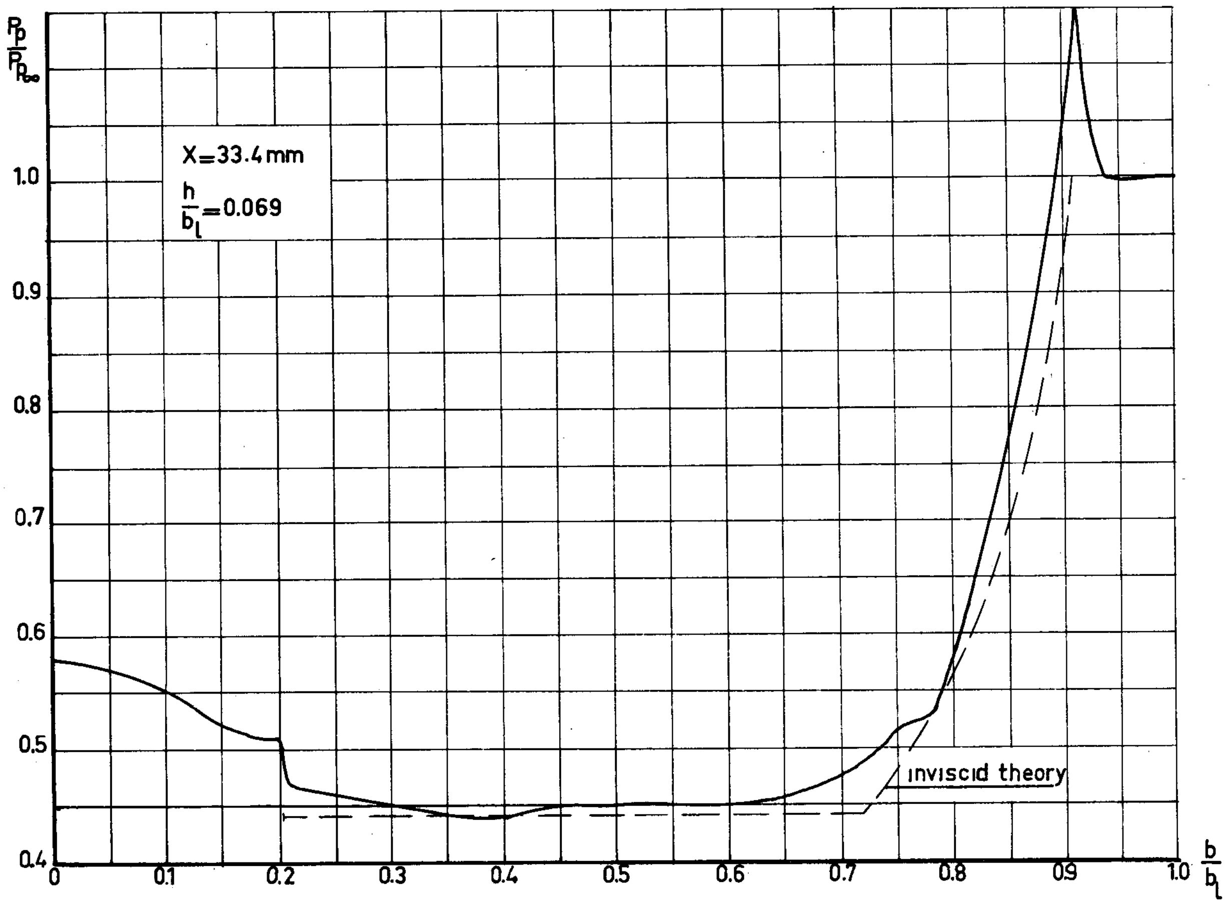

FIG.2a PITOT PRESSURE DISTRIBUTION IN SPANWISE DIRECTION; CONTINUOUS TRAVERSE

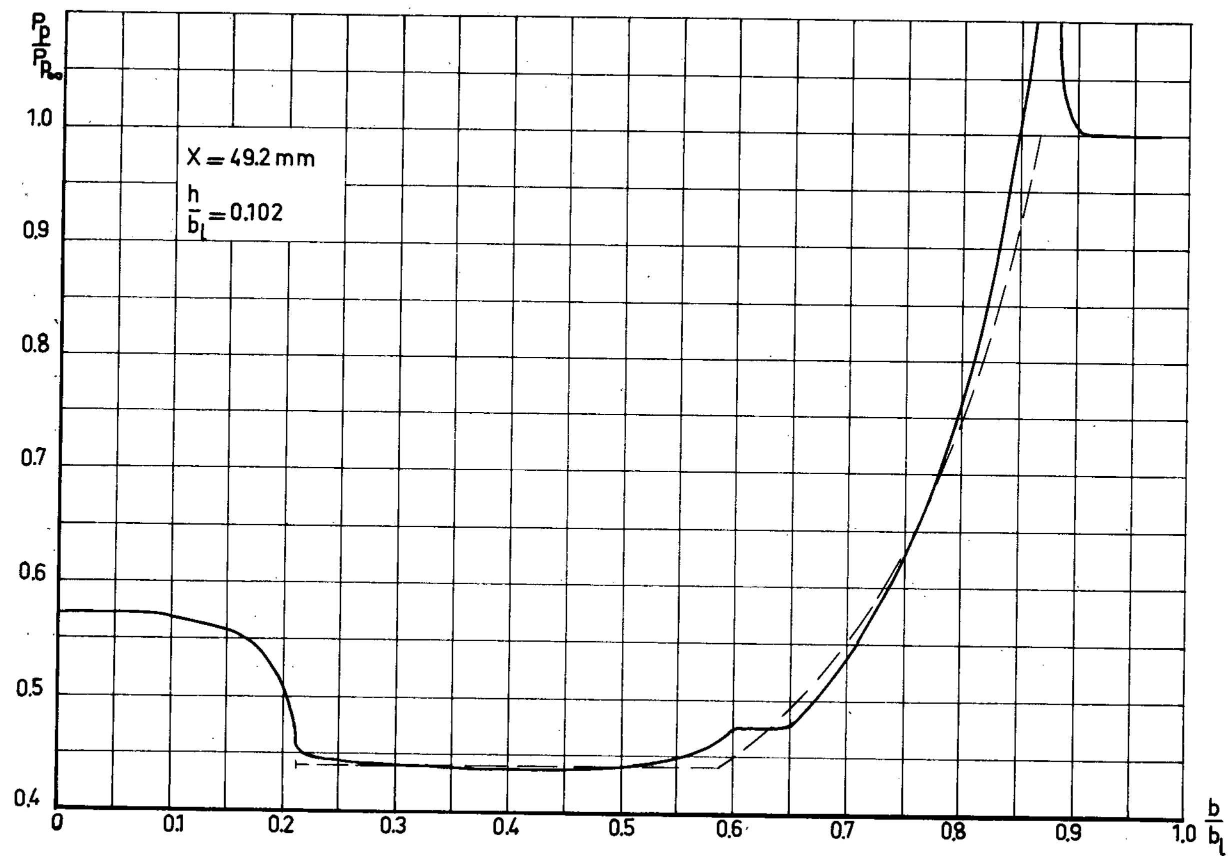

FIG. 2b CONTINUED

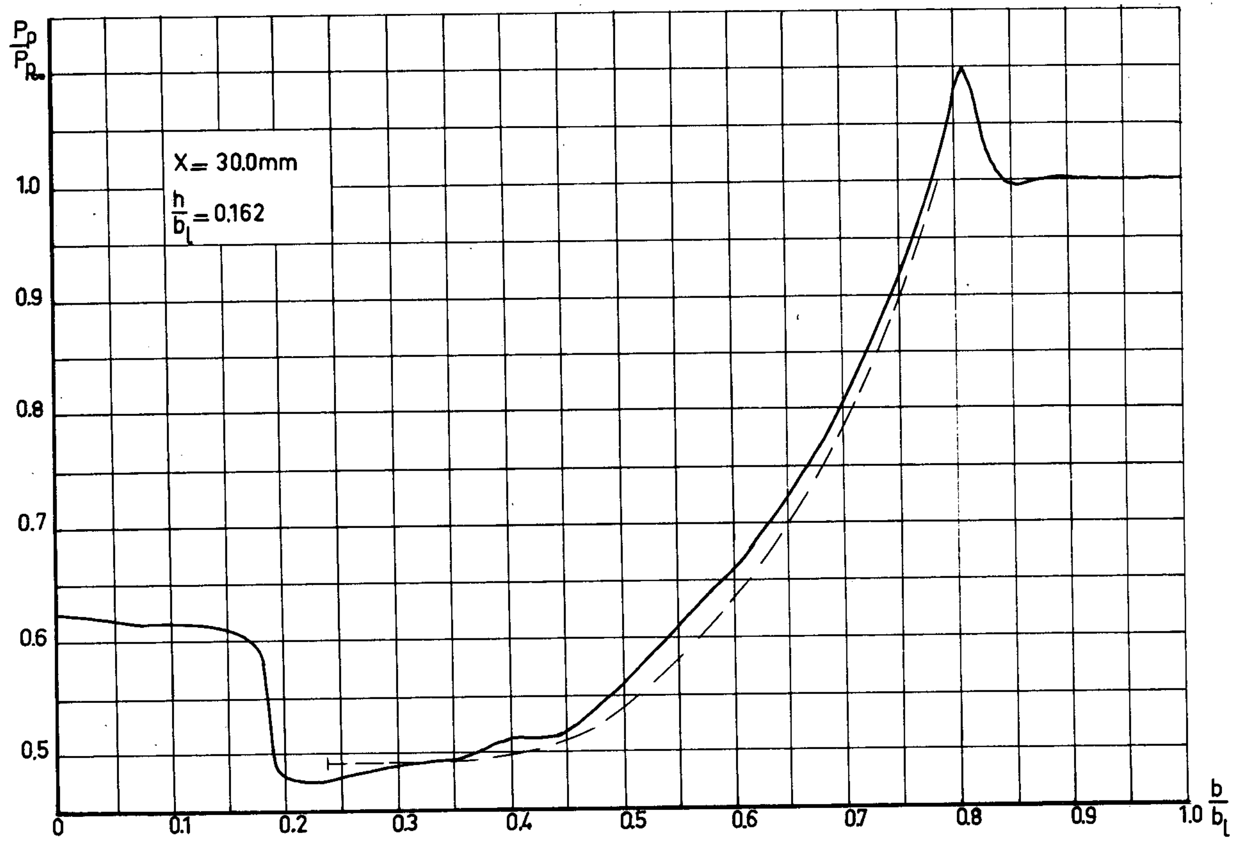

FIG.2c CONTINUED

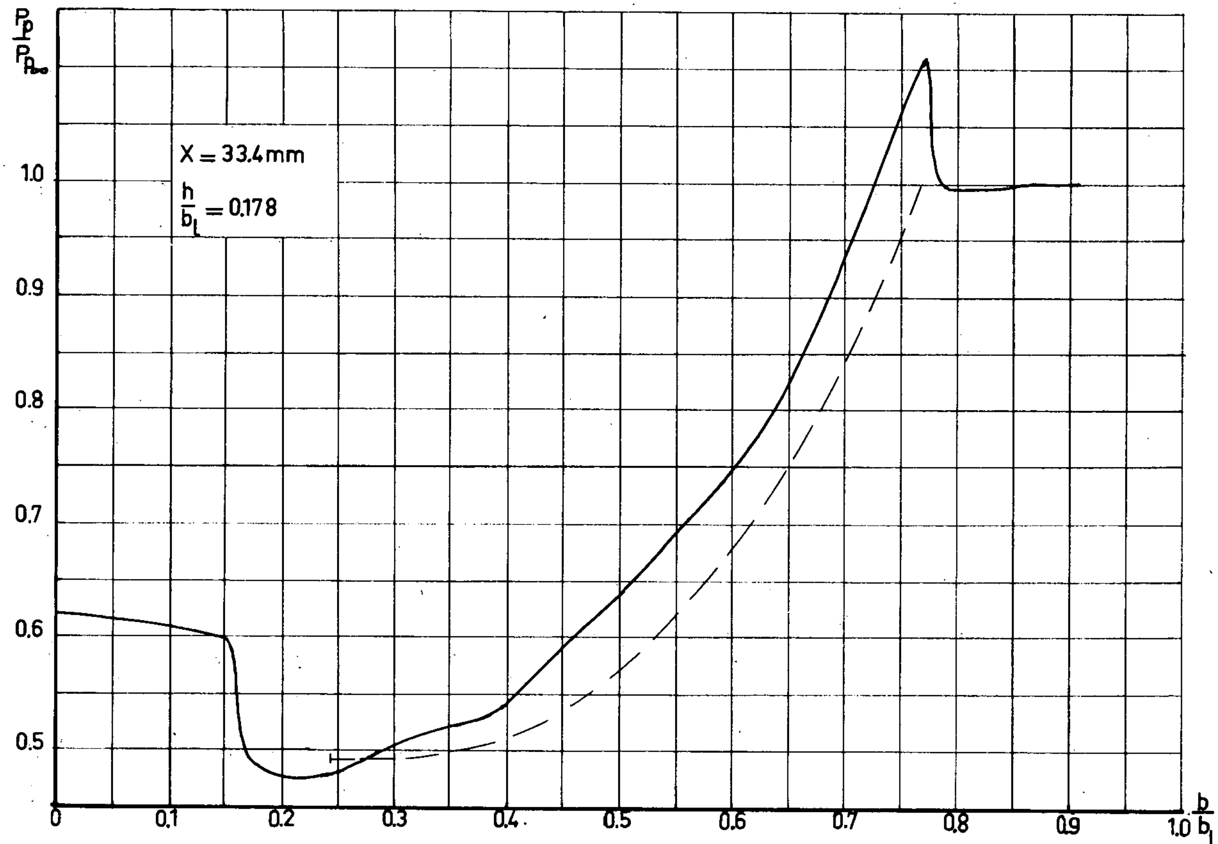

FIG.2d CONTINUED

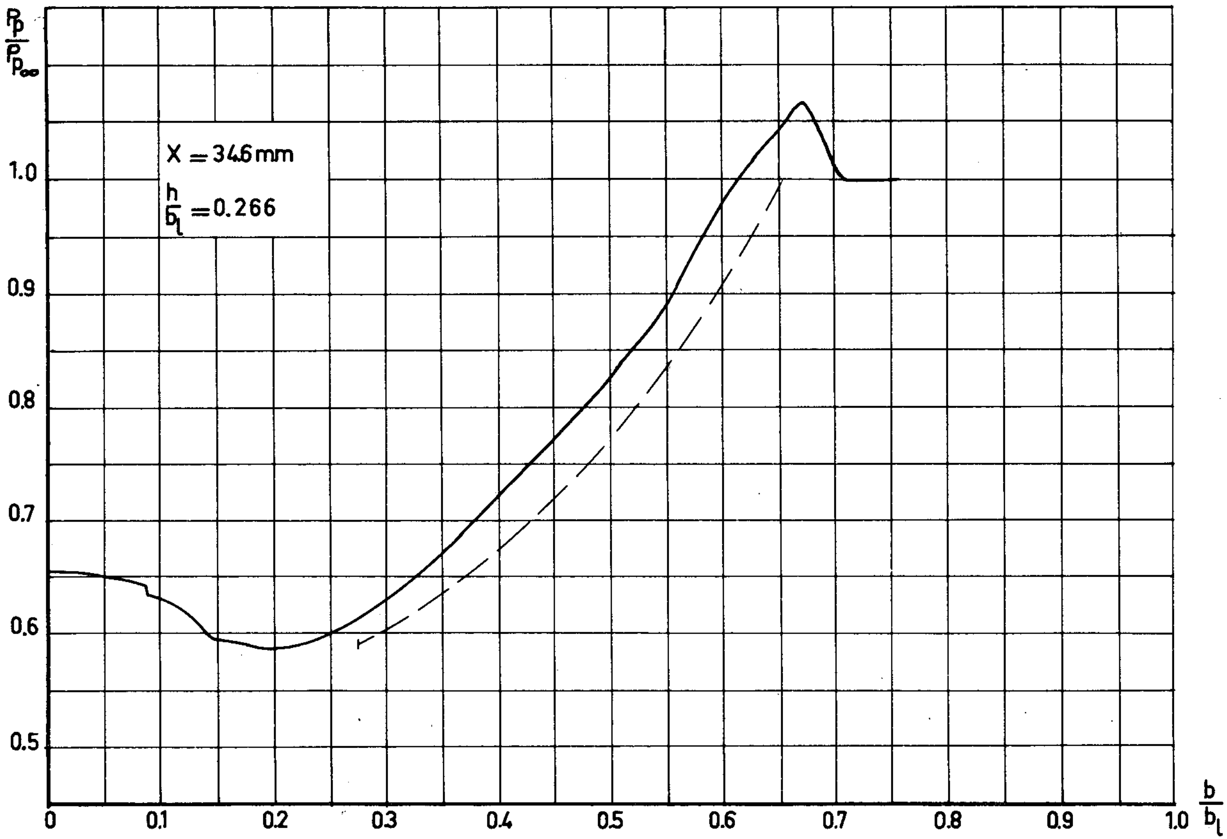

FIG. 2e CONCLUDED

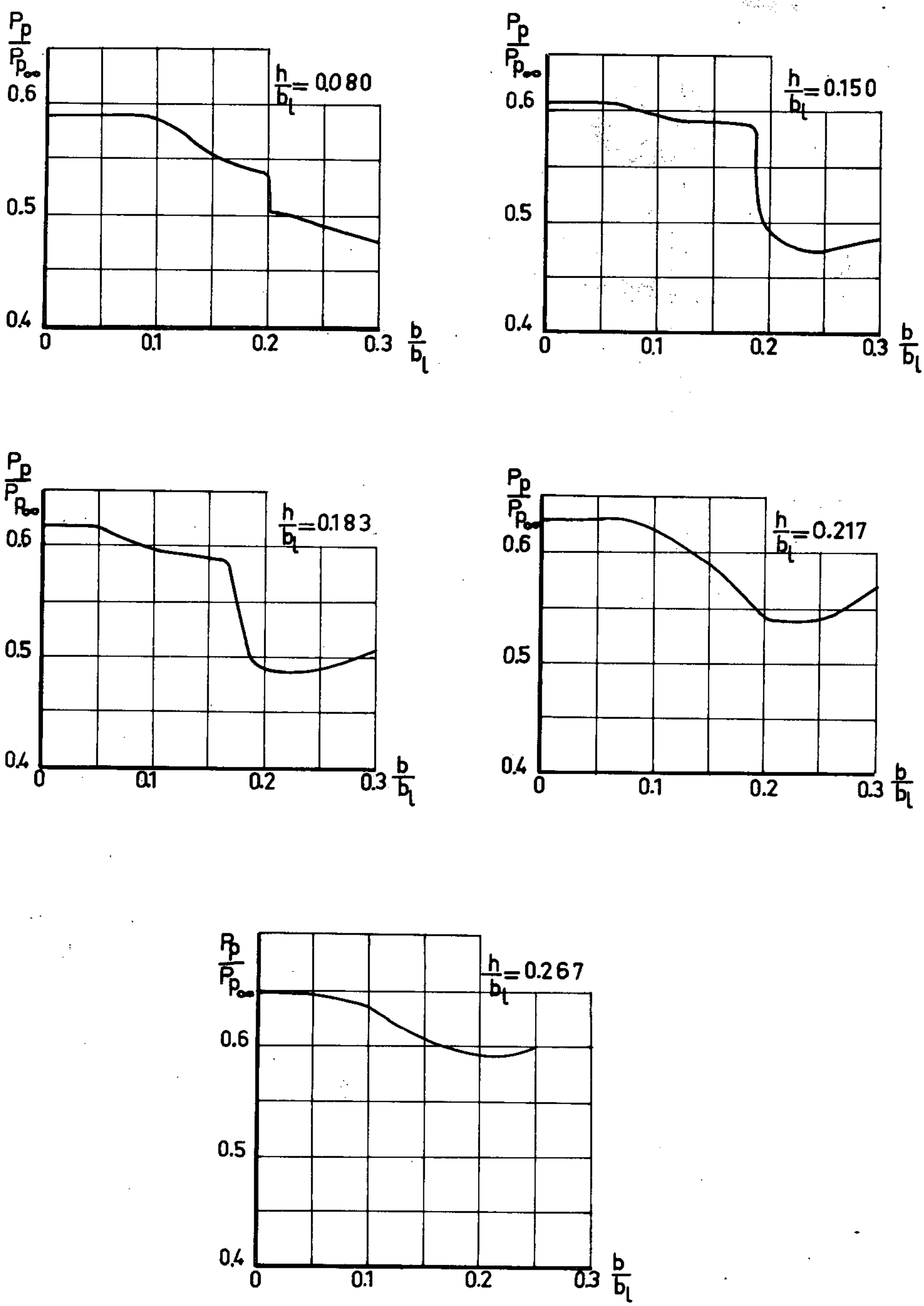

FIG.3a PITOT PRESSURE DISTRIBUTION IN SPANWISE DIRECTION; INTERMITTENT TRAVERSE; X = 30.0mm

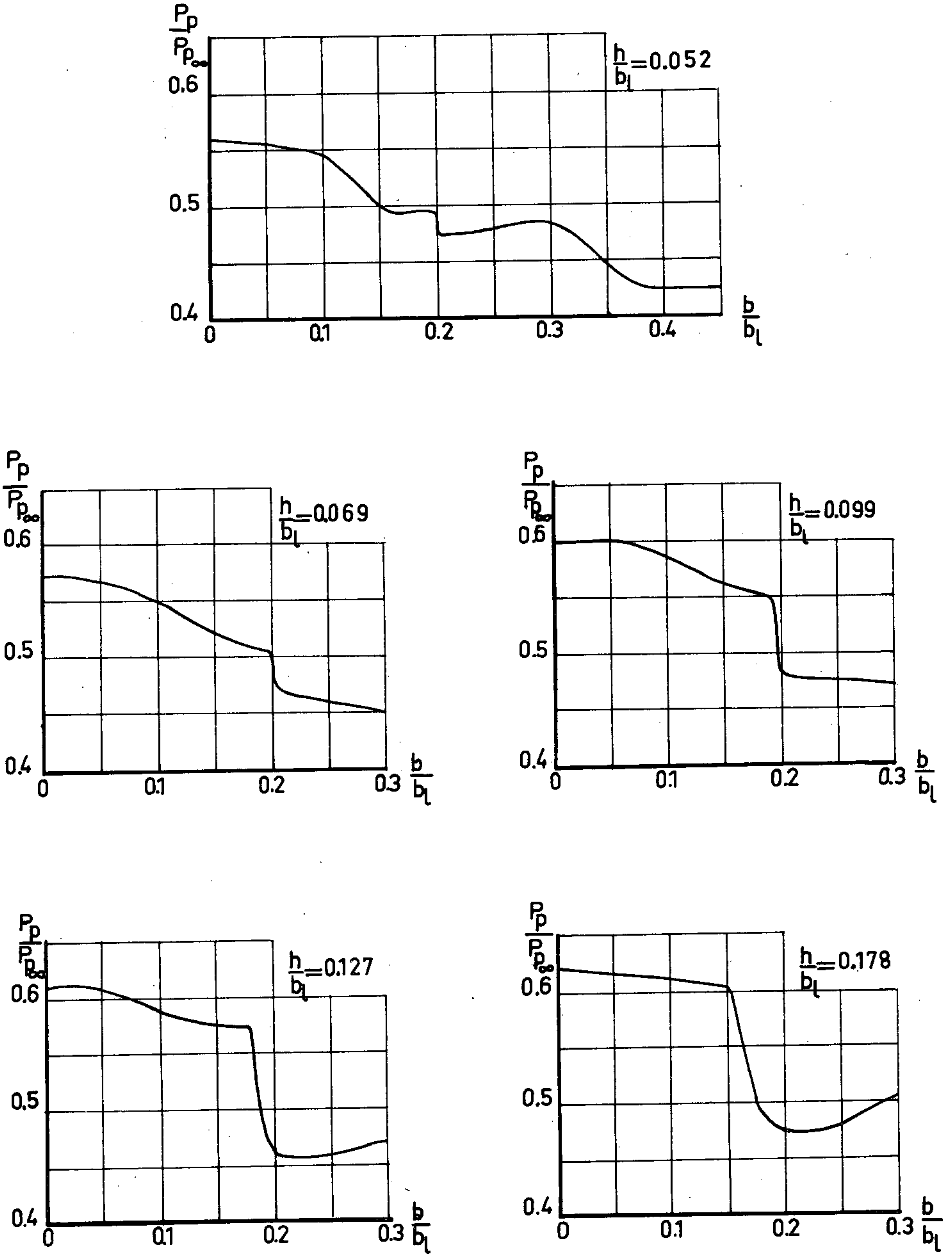

FIG.3b CONTINUED; $X = 33.4$ mm

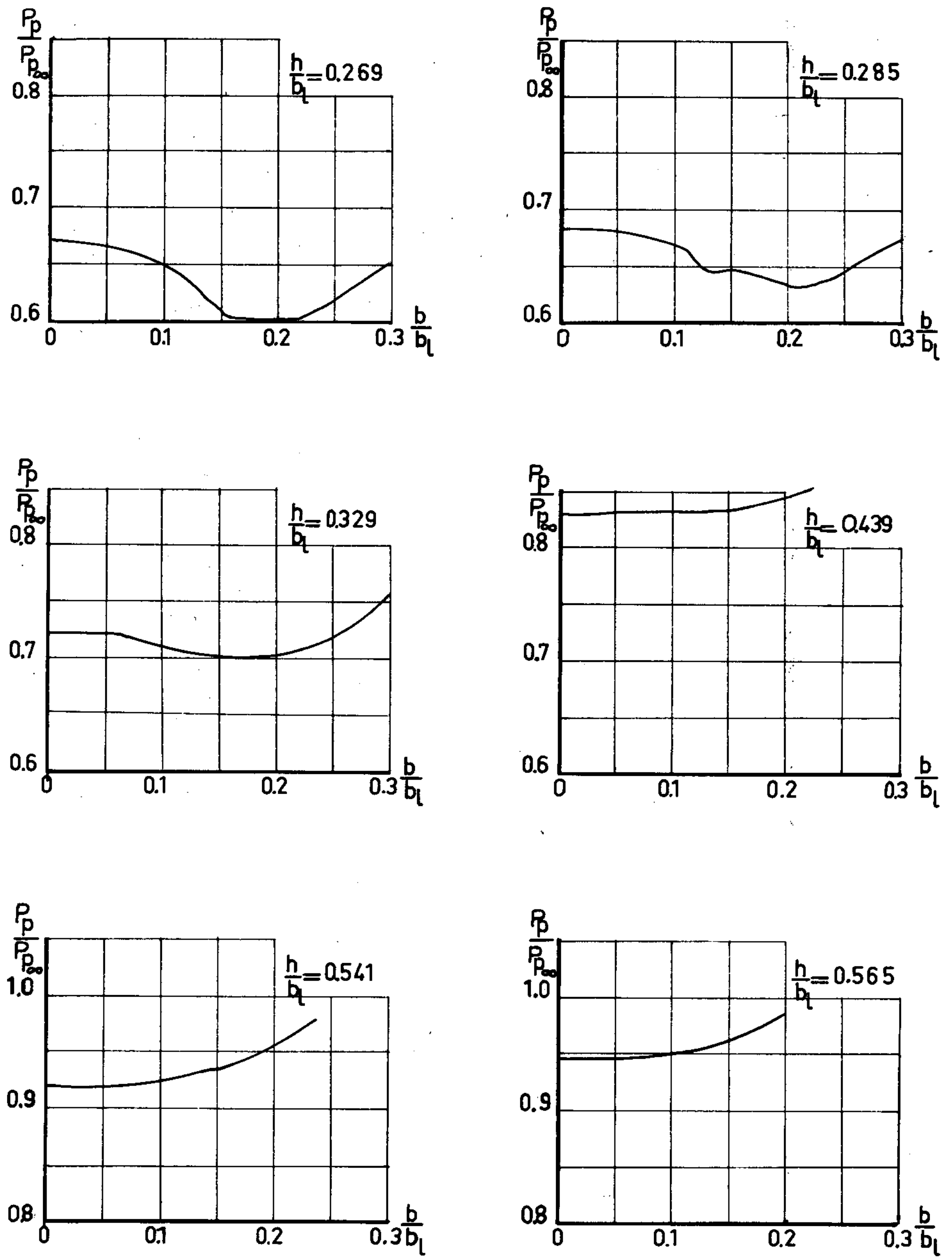

FIG.3c CONTINUED; X=33.4mm

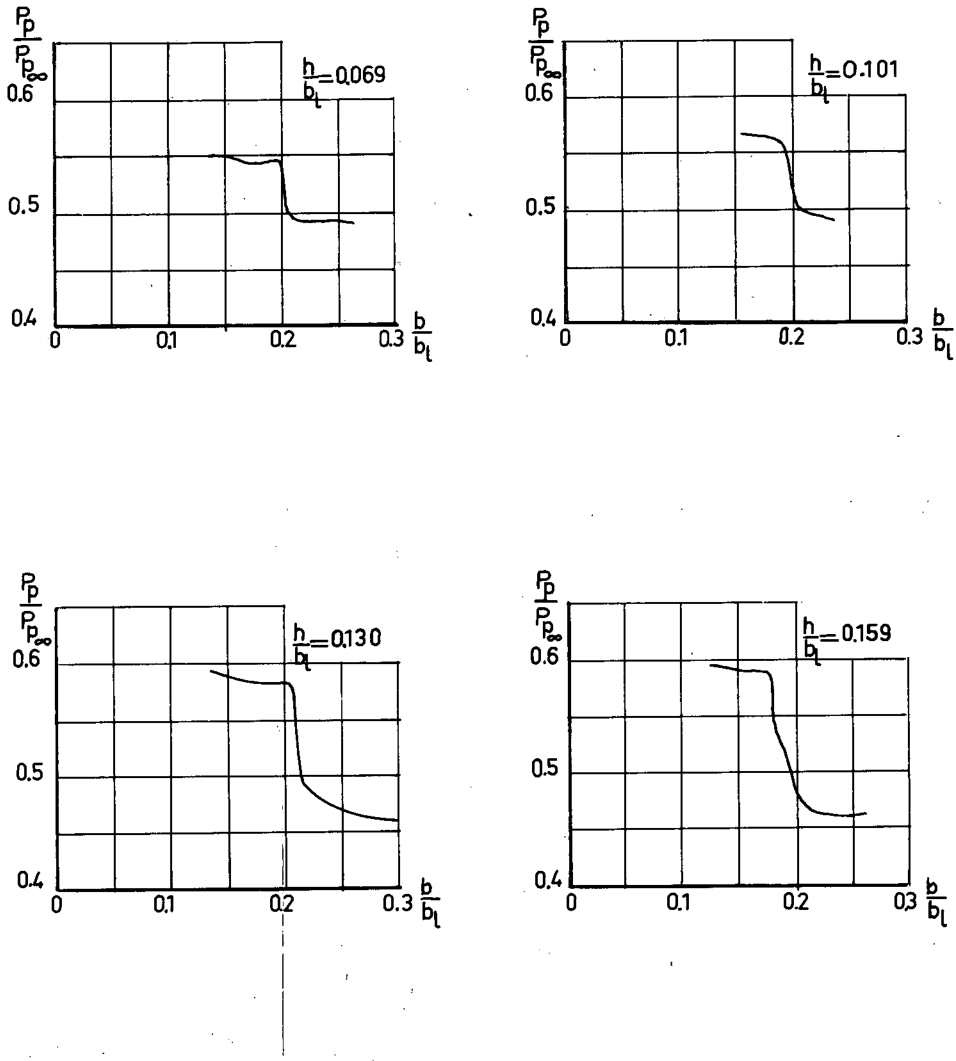

FIG. 3d CONTINUED X=34.6 mm

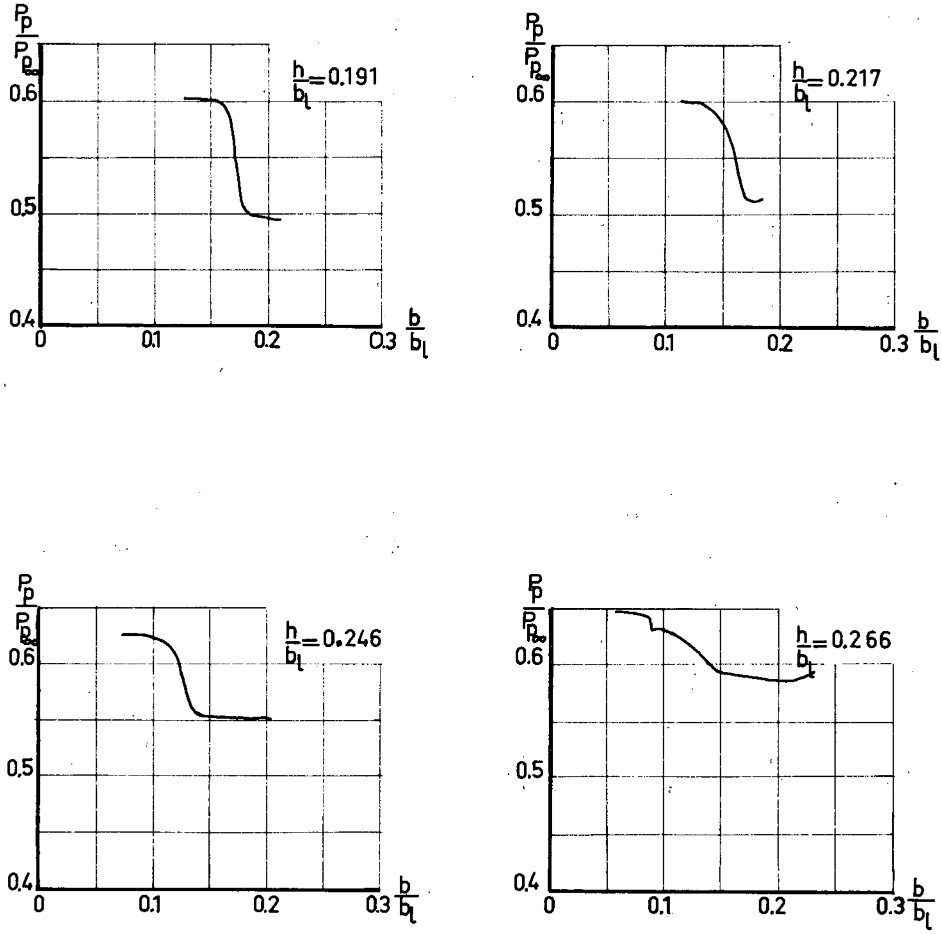

FIG. 3e CONTINUED; X=34.6mm

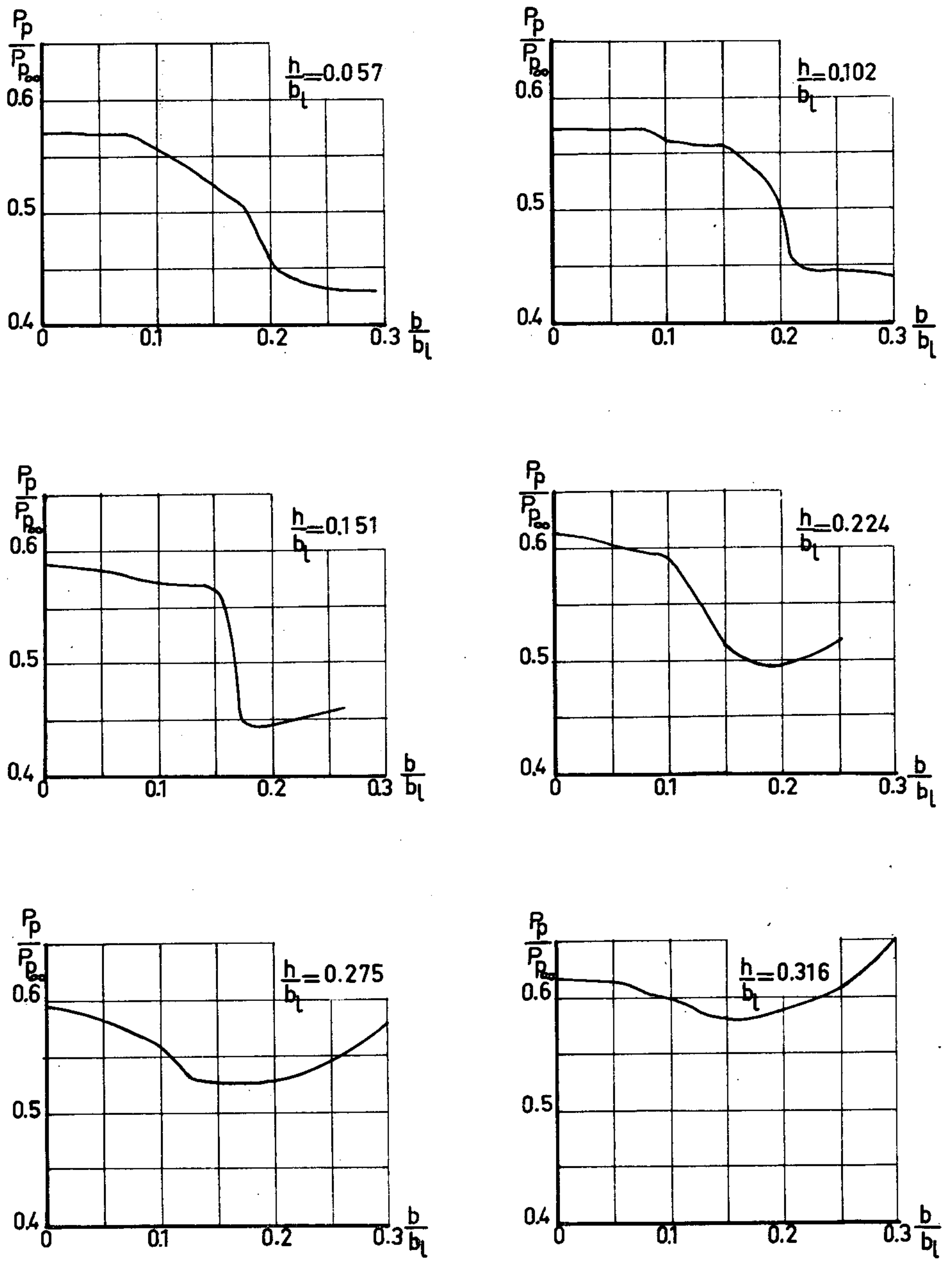

FIG. 3f CONCLUDED; X=49.2mm

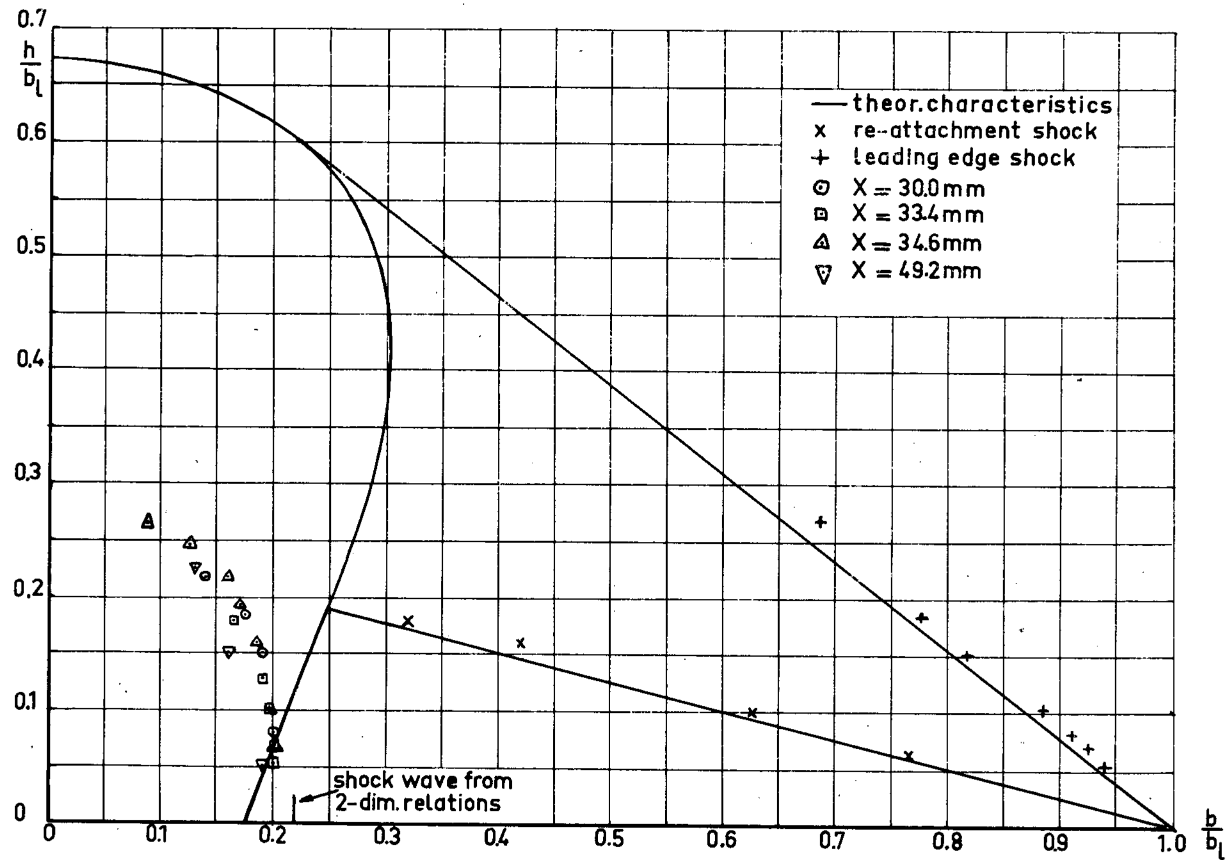

FIG.4 LOCATION OF THE INBOARD SHOCK IN A PLANE PERPENDICULAR TO THE CENTRAL CHORD OF THE WING

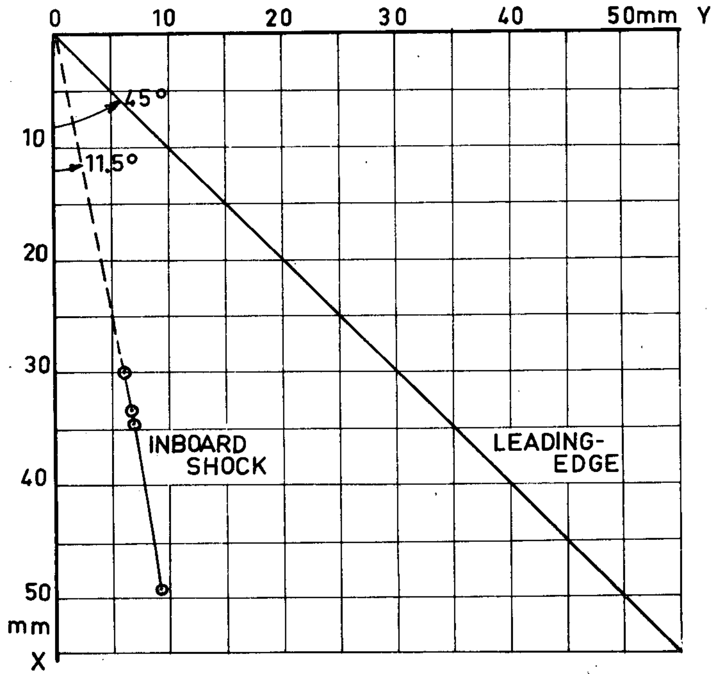

FIG.5 LOCATION OF THE INBOARD SHOCK NEAR THE WING SURFACE

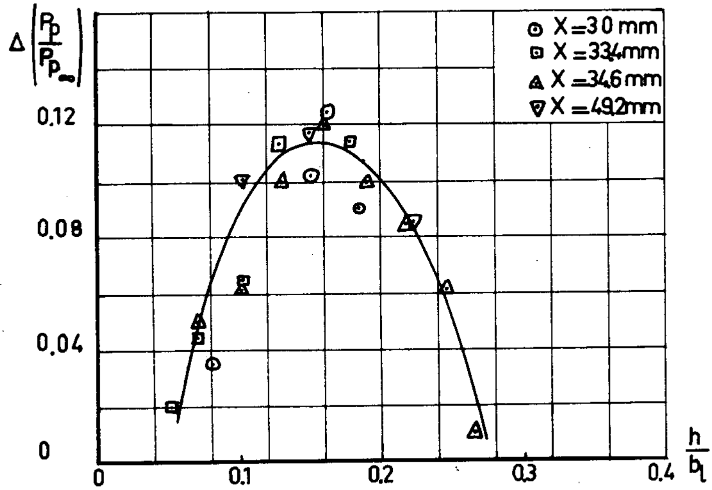

FIG.6 VARIATION OF THE STRENGTH OF THE INBOARD SHOCK WITH HEIGHT ABOVE THE WING SURFACE

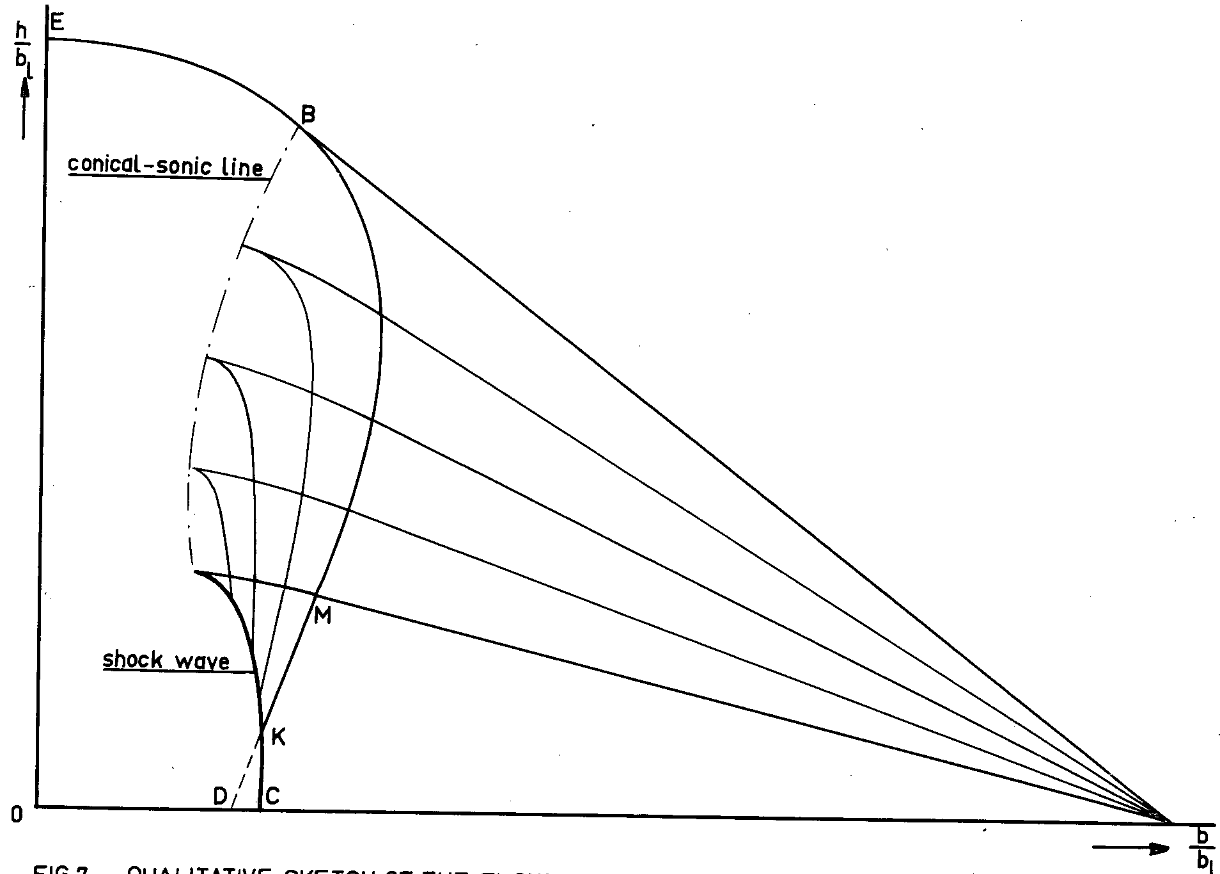

FIG.7 QUALITATIVE SKETCH OF THE FLOW PATTERN