# MULTI-OBJECTIVE OPTIMIZATION OF A BLDC DRIVE

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***Abstract:***

Many applications can not be realised with standard sized motors, so there is a need for a application specific motors. In most of the applications there are stringent conditions on the motor, viz. efficiency, cost, size etc. As a result of this the designer is faced with a task to design a motor that meets most these requirements. Thus the motor design becomes a typical case of multi-objective optimisation. When a multi-objective problem is treated, quite often objective conflict with each other and, unlike a single objective optimisation, the solution to this problem is not a single point, rather it has a family of solutions known as Pareto optimal set. Among these solutions, the designer selects the best compromise taking into account higher level information available. In this article we discuss the multi-objective optimisation of permanent magnet brushless DC (BLDC) motors using genetic algorithms.

## 1. Introduction

Since the emergence of new high field permanent magnet materials (Chalmers et al., 1997) brushless DC motors (BLDC) have become increasingly attractive in a wide range of applications. They have smaller volume compared with equivalent wound field machines, operate at higher speed, dissipate heat better, require less maintenance, and are more efficient and reliable than conventional motors. Many researchers have made efforts to improve motor performance in terms of efficiency, maximum torque, back EMF, power/ weight ratio, and minimum losses in iron, coils, friction, and windage. A scheme for optimization of a three phase electric motor based on genetic algorithms (GA) was presented by Bianchi et. al. [1]. As a demonstration of this technique the authors took a surface mounted permanent magnet motor as an example and applied genetic algorithm to minimise the permanent magnet weight. Similarly an optimal design of Interior Permanent Magnet Synchronous Motor using genetic algorithms was performed by Sim *et. al* [2]. In this case the efficiency of the motor was taken as the objective function. Usually there are many conflicting design objectives in the optimal design of electrical machines. So multiobjective optimization (MOOP) technique is required to meet design purposes The presence of several conflicting objectives is typical for engineering design problems. In many cases where optimization techniques are utilised, the multiple objectives are aggregated into one single objective function. Optimization is then conducted with one optimal design. Another approach to handle multiobjective design problems is to employ the concept of Pareto optimality. Pareto optimality was introduced in the late eighteen hundreds by the economist Vilfredo Pareto, and is defined as follows: A solution is said to be Pareto optimal if there exists no other solution that is better in all attributes. This implies that in order to achieve a better value in one objective at least one of the other objectives is going to deteriorate if the solution is Pareto optimal. Thus, the outcome of a Pareto optimization is not one optimal point, but a set of Pareto optimal solutions that visualise the trade-off between the objectives. In recent years research has been pursued in the area of multiobjective optimization of PM motors. Multiobjective optimization of PM motor using genetic algorithms was performed by Yamada *et. al* [3]. A surface mounted PM synchronous motor was taken for optimization and ε-constraint method was used to obtain the solution. The objective functions that were considered for optimization were motor weight and material cost. The authors used a two step method for optimization. First a preliminary design was carried out in which the design is formulated as a constraint non-linear programming problem by using space harmonic analysis. Then the motor configuration was optimised using a procedure which combined the finite element method (FEM) with the optimization algorithm. Sim *et. al* [4] implemented multiobjective optimization for a permanent magnet motor design using a modified genetic algorithm. The genetic algorithm used in this case was adjusted to the vector optimization problem. Multiobjective optimization of an interior permanent magnet synchronous motor was carried out again by Sim *et.al* [5]. In both cases the authors chose weight of the motor and the loss as objective functions. In the present work the MOOP of PM motors is taken a step further. The optimization of the motor so far laid focus mainly on the magnetic circuit of the motor. Here we have included the power supply, namely a H-bridge voltage source inverter, along with the magnetic model of the motor. The advantage of this procedure is that it always ensures that the optimised motor will deliver the required torque under steady state operation. In the next section a formal definition of multiobjective optimization problem is presented. Section 3 gives an overview of the genetic algorithms. The motor and the inverter model is not decribed in the paper but the results obtained are discussed in section 4. Finally conclusions are drawn in section 5.

## 2.The multiobjective optimization

The multiobjective optimization problem deals with simultaneously finding optima of $m$ objectives:

$$y_i = f_i(\vec{x}), \quad i = 1...m \tag{1}$$

Where each objective is a function of vector $\vec{x} \in \boldsymbol{X}$ of $n$ decision variables and $\boldsymbol{X}$ is the search space. The parameters of the problem may also be subjected to $p$ inequality and $q$ equality constraints:

$$h_j(\vec{x}) \geq 0, \quad j = 1...p \tag{2}$$

$$g_j(\vec{x}) = 0, \quad j = 1...q \tag{3}$$

Without loss of generality it may be assumed that all the objectives are to be minimised, hence the multi-objective optimization problem can be stated as:

$$\text{minimise } \vec{y} = \boldsymbol{f}(\vec{x}) = (f_1(\vec{x}), f_2(\vec{x}), ..., f_m(\vec{x})) \tag{4}$$

$$\text{subject to } \boldsymbol{h}(\vec{x}) = (h_1(\vec{x}), h_2(\vec{x}), ..., h_p(\vec{x})) \geq 0$$

$$\text{and} \quad \boldsymbol{g}(\vec{x}) = (g_1(\vec{x}), g_2(\vec{x}), ..., g_q(\vec{x})) = 0 \tag{5}$$

where $\vec{x} = (x_1, ..., x_n)$

The principles of multi-objective optimization are different from that of a single-objective optimization. When faced with only a single objective an optimal solution is one that minimises the objective subject to the constraints. However, in a multi-objective optimisation problem there are more than one objective function and each of them may have a different individual optimal solution, hence it is clear that many solutions exist for such problems. The reason for a set of optimal solutions is that no single solution can be considered better than the other with respect to all the objectives. Such solutions are said to be Pareto optimal and the set of all Pareto optimal solutions forms the Pareto front.

## 3. Genetic Algorithms

John Holland first developed the concept of genetic algorithm in 1960s. The genetic algorithm (GA) relies on the Darwin's concept of survival of the fittest with sexual reproduction, where stronger individuals in the population have a higher chance of creating an offspring. The GA begins with random initialisation of the population. The transition of population from one generation to the next takes place via the application of the genetic operators like *selection*, *crossover* and *mutation*. The *selection* operator selects the individuals from the population for reproduction. The *crossover* operator randomly chooses a locus and exchanges the subsequences before and after that locus between two chromosomes to create two offspring. For example, the strings 1000100 and 11111111 may be crossed over at fourth locus to yield two offspring 10001111 and 1111100. The crossover operator roughly mimics biological recombination between two single chromosomes. The *mutation* operator randomly flips some of the bits in a chromosome. For example the string 11110011 may be mutated in its fifth position to yield 11100011. Mutation can occur at each bit position in a string with some probability. The genetic algorithms (GAs) have the following features:

- GAs operate with a population of possible solutions instead of single individual. Thus the search is carried out in a parallel form.
- GAs are able to find optimal or sub-optimal solutions in complex and large search spaces. The GAs can be modified to solve multiobjective optimisation problems.

GAs examine many possible solutions at the same time, hence they have a high probability to converge to a global optimum. The flowchart of simple genetic algorithm is shown below in Fig.1.

Although GAs are simple but to describe their behaviour can be complicated and many questions exist about how they work. To explain the working of the genetic algorithms Holland proposed the Schemata Theory [6].

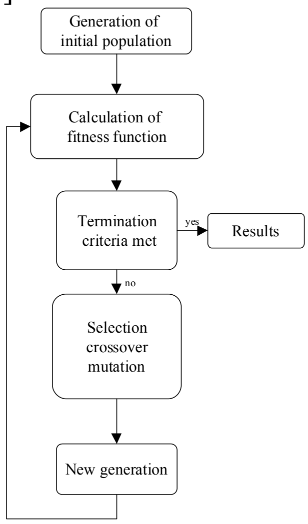


**Fig. 1:** Flow chart of simple Genetic Algorithm

In contrast to single objective optimization, where objective function and fitness functions are often identical, both fitness assignment and selection must allow for several objectives with multi objective optimization problems. To solve this problem the genetic algorithms use the concept of *Pareto dominance* as proposed by Goldberg [7].

The second problem involved in multi objective genetic algorithms, i.e. to maintain a diverse population in order to prevent premature convergence and achieve a well distributed wide spread trade-off front, is tackled by the use of *Elitism [10,11]*. Srinivas and Deb [8] developed Gas for multiobjective optimization using Nondominated Sorting Genetic algorithm (NSGA). An advanced version of NSGA was developed by Deb *et al.*, that incorporated the concept of elitism in it, is Non Dominated Sorting Genetic Algorithm II (NSGA-II) [9].

In our present problem of multi objective optimization of a permanent magnet brushless DC motor (BLDC), a modified version of NSGA-II was used. The modelling aspects of a BLDC motor as well as of 3 phase H-bridge voltage source inverter (VSI) are omitted here.

## 4. Results of Optimization

Many unknown parameters are involved in the design of a BLDC. Therefore it is necessary to fix some of the parameters and then determine the others by optimization. Table 1 below describes the parameters involved in the design process.

**Table 1**

List of variables used in the present work

| Parameter Name | Description |
|---|---|
| T | Power or rated torque |
| $N_r$ | Rated Speed |
| $N_{ph}$ | Number of phases |
| g | Air gap |
| $R_{ro}$ | Outer radius of rotor |
| $K_{cu}$ | Copper fill factor |
| ρ | Conductor resistivity |
| Br | Reminance fill factor |
| $\mu_r$ | Relative permeability of permanent magnet |
| $B_{max}$ | Max. steel flux density |
| $C_h$ | Hysteresis loss coefficient |
| $C_e$ | Eddy current loss factor |
| $N_s$ | Number of slots |
| $N_m$ | Number of magnets |
| $L_{turns}$ | Length of the stack |
| $N_{turns}$ | Number of turns |
| I | Current per phase |
| $\alpha_d$ | Ratio of inner diameter to outer diameter |
| $\alpha_m$ | Ratio of pole pitch |

For a typical situation it is usually required to design a motor subject to certain boundary conditions and certain parameters of the motor are to be optimised. In this work a scenario is considered where the mass and the loss of the BLDC motor are to be minimised. Besides that it is also required that the motor produces a minimum torque of 1Nm and the power supply unit is a 3 phase H-bridge inverter with 120° block commutation. It is further required that the motor should fit in a certain volume. Hence it becomes very important in this case to perform optimization of machine taking into account the power supply. If optimization of the machine is done purely on the basis of magnetic circuit of the motor then we will reach erroneous result as will be evident from the results of the different cases discussed below.

***Case I:***

In this case the multi-objective optimization of the BLDC motor is done without taking into account the power supply. However the voltage of the power supply was fixed. The parameters of the motor that are held constant and the parameters that are varied are listed in table 2 and table 3 respectively. The objectives that are to be optimised in this case are the losses (iron and copper) and the mass of the motor. Mathematically the present optimization problem can be stated as follows:

$$\text{minimise} \begin{cases} f_1(\vec{x}) = P_{cu} + P_{hys} + P_{eddy} \\ f_2(\vec{x}) = M_{iron} + M_{magnet} \end{cases} \qquad (6)$$

where $P_{cu}, P_{hys}$ and $P_{eddy}$ are the copper loss, hysteresis loss in the stator yoke and the eddy current loss in the statot yoke respectively and $M_{iron}$ and $M_{magnet}$ are the mass of yoke (stator and rotor) and mass of permanent magnets respectively.

subject to $\boldsymbol{h}(\vec{x}) = T_{motor} \geq 1$ Nm (7)

where $\vec{x} = (L_{motor}, \alpha_m, \alpha_d, N_m, N_s, N_{turns})$ (60)

$$\text{and } 1 \leq L_{motor} \leq 100, \quad 0.1 \leq \alpha_m \leq 1,\ 0.1 \leq \alpha_d \leq 0.7, \quad 2 \leq N_m \leq 20,\ 3 \leq N_s \leq 30, \quad 1 \leq N_{turns} \leq 100 \qquad (8)$$

**Table 2**

CONSTANT PARAMETERS FOR CASE 1

| Parameter | Value | Units |
|---|---|---|
| Speed | 1000 | rpm |
| Reminance field of magnets | 1.2 | T |
| Density of iron | 7700 | $Kg/m^3$ |
| Copper fill factor | 0.5 | |
| Density of magnets | 5000 | $Kg/m^3$ |
| Outer radius of the stator | 20 | mm |
| Resistivity of copper | $1.68 \cdot 10^{-8}$ | Ohm m |
| Air gap length | 0.5 | mm |

**Table 3**

PARAMETERS THAT ARE VARIED IN CASE 1

| Parameter | Description | Units |
|---|---|---|
| $L_{motor}$ | Length of the motor | mm |
| $\alpha_m$ | Ratio of magnet angle to pole pitch | |
| $\alpha_d$ | Ratio of inner motor diameter to outer diameter | |
| $N_m$ | Number of magnets | |
| $N_s$ | Number of slots | |
| $N_{turns}$ | Number of turns per coil | |

The results obtained are shown in Fig.2.

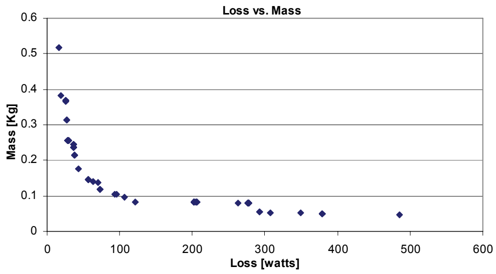


**Fig. 2:** Pareto optimal solutions for loss vs. mass for case 1

In table 4 the parameters of two sample motors from the above Pareto Front are shown.

**Table 4**

Values of the motor parameters for the analysis for case 1

| Variable | Motor 1 | Motor2 | Variable | Motor 1 | Motor2 |
|---|---|---|---|---|---|
| Ns | 21 | 30 | Losses | 306 | 63.166 |
| Np | 14 | 20 | Mass | 0.05 | 0.1407 |
| $I_{ph}$ | 20.66 | 10.835 | $\eta$ | 0.38 | 0.7571 |
| $N_{turns}$ | 22 | 8 | $L_{phph}$ | 0.0005 | 0.0003 |
| $L_{stack}$ | 7.62 | 25.772 | $R_{phph}$ | 0.72 | 0.5312 |
| $\alpha_d$ | 0.58 | 0.6613 | Torque | 1.07 | 1.1676 |
| $\alpha_m$ | 0.93 | 0.9985 | $Vb_{phph}$ | 5.43 | 11.285 |

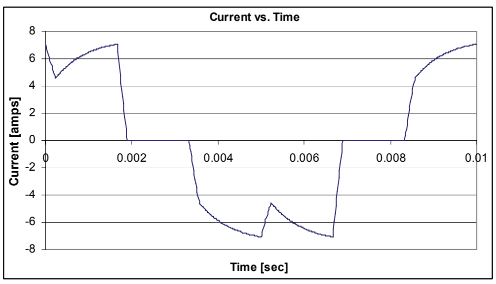


**Fig.3a** Current for phase A for the first motor in table 4

When the motors with parameters listed in table 4 are fed with a H-bridge voltage source inverter the motors do not produce the required torque. The first motor requires an rms current of 20.66 amps to produce a torque of 1.02 Nm, the actual current and torque produced by this motor is shown in Fig.3a and Fig.3b respectively.

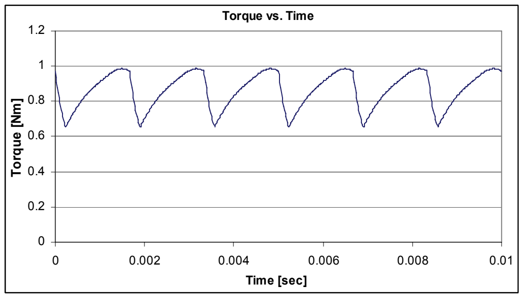


**Fig.3b** Torque produced by the first motor in table 4

The second motor requires an rms current of 10.83 amps to produce a torque of 1.17 Nm, the actual current and torque produced by this motor is shown in Fig.4a and Fig.4b respectively.

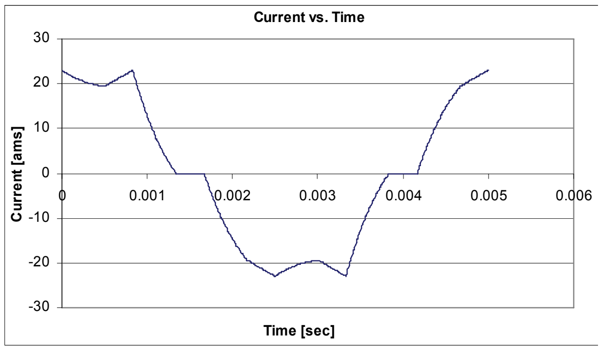


**Fig.4a** Current for phase A for the second motor in table4

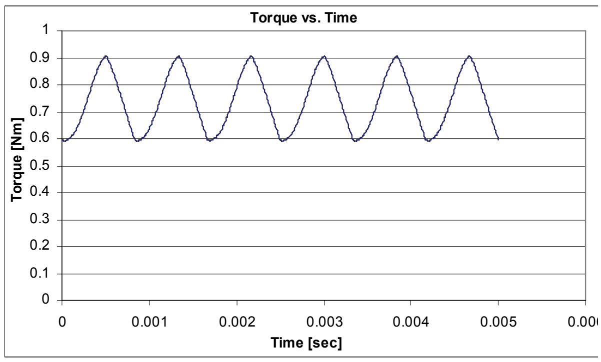


**Fig.4b** Torque produced by the second motor in table 4

From these results it can be seen that both the motors do not produce the required torque when they are operated with the voltage source inverter (VSI). In the next case we consider the optimization of the BLDC motor considering that the motor is fed with the VSI.

***Case 2:***

In this case 6 parameters were taken as variables and VSI inverter is considered too. These parameters are listed in table 5 and the fixed parameters are listed in table 6. The objectives that are to be optimised in this case are losses (iron and copper) and the mass of the motor and during the optimization process the VSI is also taken into account. Mathematically the present optimization problem can be mathematically written as follows:

$$\text{minimise} \begin{cases} f_1(\vec{x}) = P_{cu} + P_{hys} + P_{eddy} \\ f_2(\vec{x}) = M_{iron} + M_{magnet} \end{cases} \quad (9)$$

$$\text{subject to } \boldsymbol{h}(\vec{x}) = T_{motor} \geq 1 \text{ Nm} \quad (10)$$

$$\text{where } \vec{x} = (L_{motor},\ \alpha_m,\ \alpha_d,\ N_m,\ N_s,\ N_{turns})$$

(64)

$$\text{and } 1 \leq L_{motor} \leq 100,\ 0.1 \leq \alpha_m \leq 1,$$
$$0.1 \leq \alpha_d \leq 0.7,\ 2 \leq N_m \leq 20, \quad (11)$$
$$3 \leq N_s \leq 30,\ 1 \leq N_{turns} \leq 100$$

**Table 5**

PARAMETERS THAT ARE VARIED FOR CASE 2

| Parameter | Description | Units |
|---|---|---|
| $L_{motor}$ | Length of the motor | mm |
| $\alpha_m$ | Ratio of magnet angle to pole pitch | |
| $\alpha_d$ | Ratio of inner motor diameter to outer diameter | |
| $N_m$ | Number of magnets | |
| $N_s$ | Number of slots | |
| $N_{turns}$ | Number of turns per coil | |

The results obtained are shown if Fig.5.

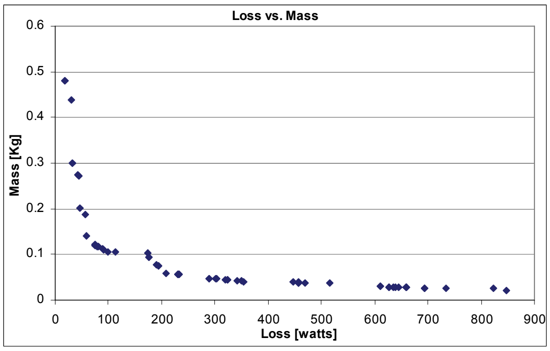


**Fig.5** Pareto optimal solutions for loss vs. mass for case 2

In table 7 the parameters of two sample motors from the above Pareto Front are shown. The current and torque profiles for the first motor for case 2 are shown in Fig.6a and Fig.6b respectively. Similarly the current and torque profile for the second motor are shown in Fig.7a and Fig.7b respectively.

**Table 7**

Values of the motors for analysis for case 2

| Variable | Motor 1 | Motor2 | Variable | Motor 1 | Motor2 |
|---|---|---|---|---|---|
| Ns | 15 | 27 | Mass | 43.24 | 88.59 |
| Np | 10 | 18 | h | 0.72 | 0.61 |
| $N_{turns}$ | 12 | 11 | $L_{phph}$ | 5.90E-04 | 3.74E-04 |
| $L_{stack}$ | 47.72 | 21.1 | $R_{phph}$ | 1.02 | 0.98 |
| $a_d$ | 0.69 | 0.7 | Torque | 1 | 1.09 |
| $a_m$ | 1 | 0.99 | $Vb_{phph}$ | 16.8 | 12.3 |
| Losses | 6.45 | 9.49 | | | |

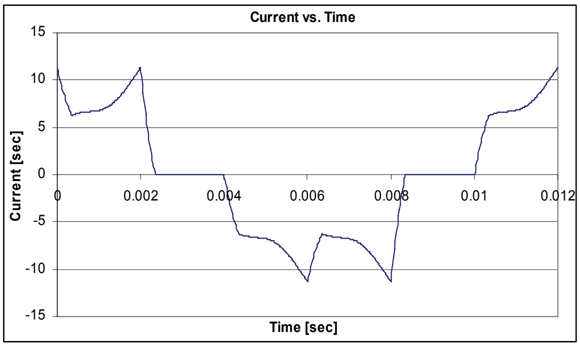


**Fig.6a** Current for phase A for the first motor in Table 7

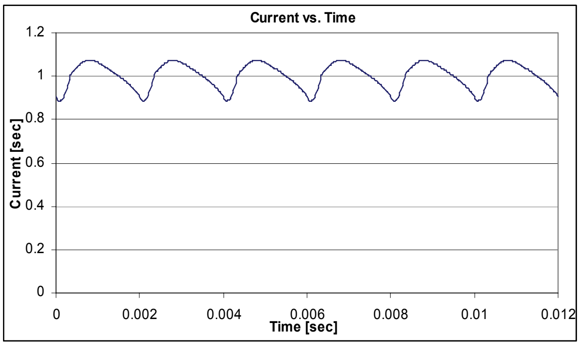


**Fig.6b** Torque for the first motor in table 7

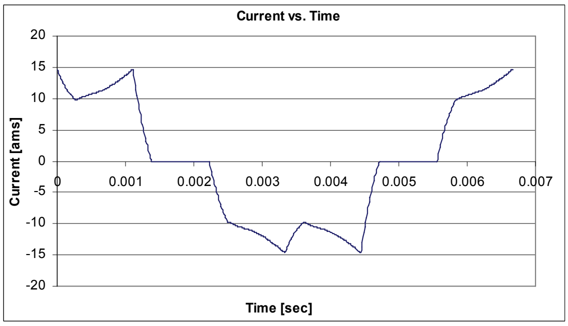


**Fig.7a** Current for phase A for the second motor in table 7

From the above figures it is seen that both the motors meet the torque constraints. Thus when the optimization of the motor is done together with the VSI we reach more realistic solutions.

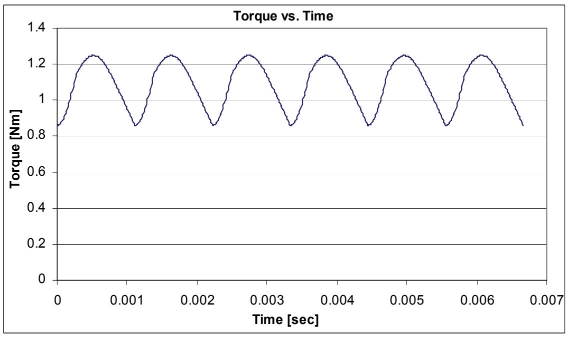


**Fig.7b** Torque for the second motor in table 7

## 5. Conclusions

In this work the utility of genetic algorithms for multi-objective optimization is shown. Besides a very important issue on the modelling itself has been discussed. It is of paramount importance that when a motor is to be optimised for certain objectives and torque constraint the power supply unit should be taken into account. The results obtained by this method is more realistic and the compatibility of power supply unit and motor is guaranteed.

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