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Creating the Green's Response to a Virtual Source Inside a Medium Using Reflection Data with Internal Multiples

F. Broggini* (Colorado School of Mines), R. Snieder (Colorado School of Mines), K. Wapenaar (Delft University of Technology) & J. Thorbecke (Delft University of Technology)

SUMMARY

Seismic interferometry is a technique that allows one to reconstruct the full wavefield originating from a virtual source inside a medium, assuming a receiver is present at the virtual source location. We discuss a method that creates a virtual source inside a medium from reflection data measured at the surface, without needing a receiver inside the medium and, hence, presenting an advantage over seismic interferometry. An estimate of the direct arriving wavefront is required in addition to the reflection data. However, no information about the medium is needed. We illustrate the method with numerical examples in a lossless acoustic medium with laterally-varying velocity and density. We examine the reconstructed wavefield when a macro model is used to estimate the direct arrivals and we take into consideration finite acquisition aperture. Additionally, a variant of the iterative scheme allows us to decompose the reconstructed wave field into downgoing and upgoing fields. These wave fields are then used to create an image of the medium with either crosscorrelation or multidimensional deconvolution.

Introduction

We present and discuss a new approach to retrieve the full response from a virtual source $\mathbf{x}_{VS}$ inside a medium and, consequently, to focus the wavefield at the virtual source location. Conventional methods for seismic interferometry (Curtis et al., 2006; Bakulin and Calvert, 2006; Schuster, 2009) allow one to reconstruct such a response without knowing the medium parameters, but these methods require a receiver in the subsurface at the location of the virtual source and assume that sources surround the medium. The approach that we propose removes the constraint of having a receiver at the virtual source location and is based on a development of the 1D theory previously proposed by Broggini et al. (2011, 2012) and Broggini and Snieder (2012). Given the reflection response of a 1D medium, they show that it is possible to reconstruct the response originating from a virtual source inside the medium, without the presence of a receiver at the virtual source location and without knowing the medium.

Wapenaar et al. (2013) generalize the 1D method to three-dimensional media. They propose an iterative scheme that transforms the reflection response of a 3D medium (measured at the $z = 0$) into the response to a virtual source located inside the unknown medium. Additionally, the proposed method requires an estimate of the direct arrivals propagating between the virtual source location and the acquisition surface (besides the reflection data measured at the surface). These direct arrivals represent a key element of the method because they specify the location and the spatial extent of the virtual source in the subsurface. Due to this reason, the proposed method is not fully model-independent. A model that relates the direct arrivals to a virtual source position is, however, simpler than a model that correctly handles the internal multiples. In our proposed approach, the reflection data contributes to the reconstruction of the multiple-scattering part of the virtual-source response.

Our objective is to retrieve the response originating from a virtual source inside an unknown medium, removing the imprint of a complex subsurface, as in seismic interferometry (Wapenaar et al., 2005; Curtis et al., 2006; Schuster, 2009). This is valuable in situations where waves have traveled inside a strongly inhomogeneous overburden, like a salt body (e.g., in subsalt imaging, Sava and Biondi, 2004). In this paper, we demonstrate that the requirement of having an actual receiver inside the medium can be circumvented, going beyond seismic interferometry.

We present numerical examples in a lossless acoustic two-dimensional medium. We discuss the influence of errors in the estimate of the first arrivals on the reconstructed wavefield. Such errors arise when a macro model (a routine product of velocity analysis) is used to compute the first arriving waveforms when such data are not available with other approaches, e.g., check shots or microseismic events.
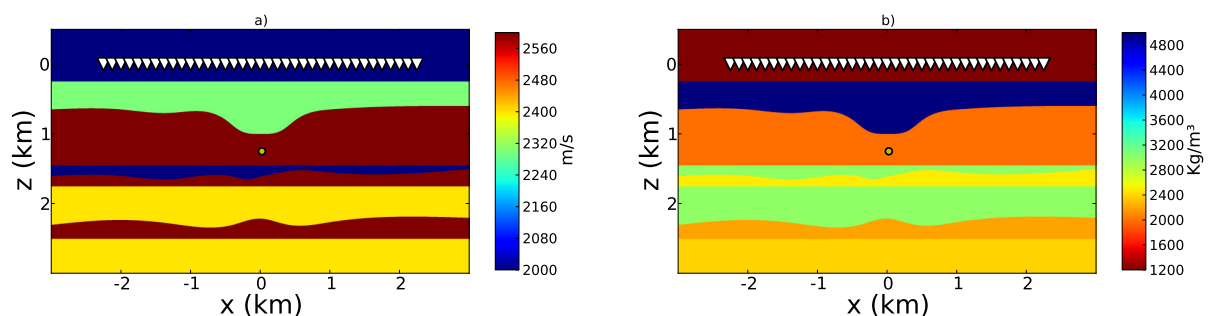

Figure 1 *(a) Velocity model. (b) Density model. The white triangles are the receivers. In both panels, the yellow dot represents the location $\mathbf{x}_{VS}$ of the virtual source.*
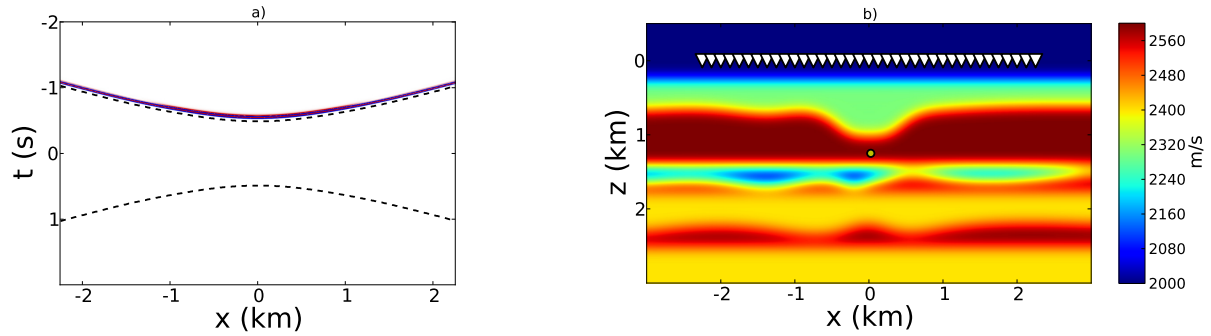
Figure 2 *(a) Initial incident wavefield $p_0^+(\mathbf{x},t)$. (b) Smooth velocity model used to model the first arrivals between the virtual source location $\mathbf{x}_{VS}$ (yellow dot) and the acquisition surface at $z = 0$ (white triangles). The smooth density model is not shown.*

Iterative process and numerical examples

We require that the wavefield focuses at a specific location, hence the proposed method is not totally independent of knowledge about the medium. The iterative scheme requires the reflection response of the medium measured at the surface, complemented with independent information about the primary arrivals originated from the focusing location, to focus the acoustic wavefield inside the medium. The primary arrival wavefront can be estimated or measured in various ways: by forward modeling using a macro model, directly from the data by the Common Focusing Point method (CFP) (Thorbecke, 1997) when the virtual source is located at an interface, from microseismic events (Artman et al., 2010), or from borehole check shots. We denote the 2D spatial coordinates as $\mathbf{x} = (x, z)$. We assume that the reflection response does not include any multiples due to the free surface. Hence, $R(\mathbf{x}_R, \mathbf{x}_S, t) * s(t)$ can be obtained from reflection data measured at the recording surface $z = 0$ after a surface-related multiple elimination processing (Verschuur et al., 1992), where $s(t)$ is a zero-phase wavelet.

We examine a configuration whose velocity and density are shown in Figure 1. To start the iterative scheme, we compute the direct arrivals originating from the virtual source using the macro model of Figure 2b. This is a smooth version of the velocity model of Figure 1a. We define the initial incident downgoing wavefield $p_0^+(\mathbf{x},t)$ at $z = 0$ as the time-reversed version of the direct arrivals at the recording surface excited by the virtual source $\mathbf{x}_{VS}$. The initial incident wavefield is shown in Figure 2a. The subscript 0 in $p_0^+(\mathbf{x},t)$ denotes the 0th iteration (initial) of the incident wavefield. Note that, due to the smoothing, the triplications are not present in this field (i.e., the time-reversed version of the direct arrivals). In Figure 2a, we also define two traveltime curves, indicated by the dashed black lines. The upper curve follows directly after the initial incident wavefield $p_0^+(\mathbf{x},t)$ and the lower curve is the time-reversed version of the upper curve. These curves define a key component of the iterative scheme: the window function defined as

$$w(\mathbf{x},t) = 1 \quad \text{between the dashed black lines of Figure 2a}$$
$$w(\mathbf{x},t) = 0 \quad \text{elsewhere.} \tag{1}$$

The upgoing reflection response $p_0^-(\mathbf{x},t)$ is obtained either by injecting the downgoing incident wavefield $p_0^+(\mathbf{x},t)$ into the actual medium or by convolving the downgoing incident wavefield $p_0^+(\mathbf{x},t)$ with the deconvolved reflection response and integrating over the source positions:

$$p_k^-(\mathbf{x}_R,t) = \int_{-\infty}^{\infty} \left[R(\mathbf{x}_R,\mathbf{x},t) * p_k^+(\mathbf{x},t) \right]_{z=0} dx, \tag{2}$$

for $\mathbf{x}$ and $\mathbf{x}_R$ at $z = 0$, and $k = 0$.

We discuss an iterative scheme that uses the $(k\text{-}1)$th iteration of the reflected wavefield $p_{k-1}^-(\mathbf{x},t)$ to
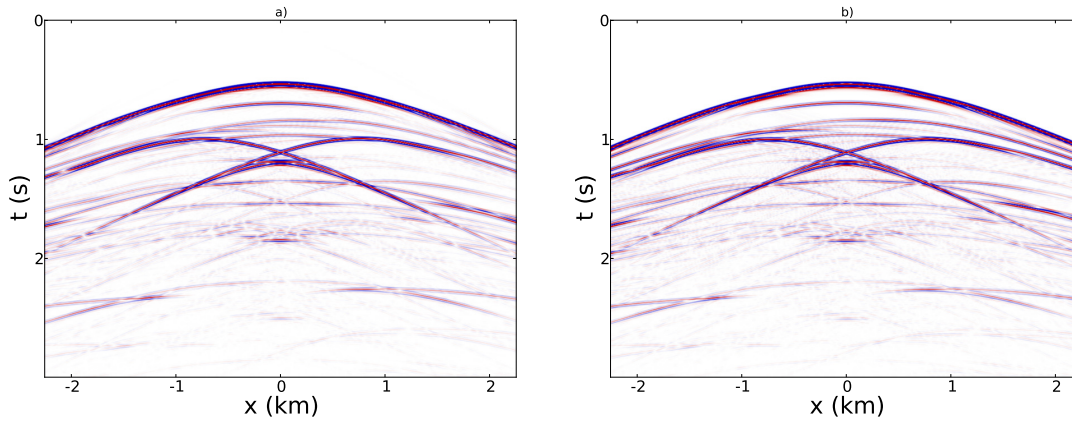
Figure 3 *(a) Causal part of the superposition of the total field and its time-reversed version, $p(\mathbf{x},t) + p(\mathbf{x},-t)$. (b) Directly-modeled full response to the virtual source (yellow dot) shown in Figure 1a.*

construct the kth iteration of the downgoing incident field $p_k^+(\mathbf{x},t)$. The proposed method uses a combination of time reversal and time windowing to construct the next iteration of the incident field. The kth iteration of the incident field $p_k^+(\mathbf{x},t)$ is specified by

$$p_k^+(\mathbf{x},t) = p_0^+(\mathbf{x},t) - w(\mathbf{x},t)p_{k-1}^-(\mathbf{x},-t), \quad \text{for } \mathbf{x} \text{ at } z = 0, \tag{3}$$

where the time window $w(\mathbf{x},t)$ is defined by equation (1).

We define the superposition of the kth version of the incident (downgoing) and reflected (upgoing) wavefields as $p_k(\mathbf{x},t) = p_k^+(\mathbf{x},t) + p_k^-(\mathbf{x},t)$. Also, we define $p(\mathbf{x},t)$ as the final result of the iterative process. We form the field $p(\mathbf{x},t) + p(\mathbf{x},-t)$ to reconstruct the response originating from the virtual source location. The causal part of this field is shown in Figure 3a. The amplitudes of the data shown in Figure 3 are clipped to 70% of the maximum amplitude. According to our theory (Wapenaar et al., 2013), the result in Figure 3a should be equal to the Green's function with its source point at $\mathbf{x}_{VS}$. This can be understood with the following heuristic derivation. The first event in Figure 3a has the same arrival time as the direct arrival of the response to the virtual source at $\mathbf{x}_{VS}$. If we combine this last reasoning with the fact that the causal part propagates upward at $z = 0$, and that the total field is symmetric and obeys the wave equation in the inhomogeneous medium, it is reasonable that the total field in Figure 3a is proportional to the response due to a real source placed at $\mathbf{x}_{VS}$, as shown in Figure 3b.

The comparison between the two panels of Figure 3 shows that it is possible to reconstruct the full response to a virtual source inside the medium, including all multiples, using the reflection data at the surface and the direct arrivals computed using a smooth model. Note that this procedure is expected to converge because in each iteration the reflected energy is smaller than the incident energy. We interpret the proposed iterative method as a correction scheme that minimizes the energy of the wavefield $p(\mathbf{x},t) + p(\mathbf{x},-t)$ inside the time window $w(\mathbf{x},t)$.

Additionally, a variant of the iterative scheme, in which the subtraction in equation (3) is replaced by an addition, allows us to decompose the reconstructed wavefield at $\mathbf{x}_{VS}$ into downgoing and upgoing fields. These fields can be used to create an image of the subsurface with multi-dimensional deconvolution (Wapenaar et al., 2011)

Conclusions

We discussed a generalization to two dimensions of the model-independent wavefield focusing and reconstruction method of Broggini et al. (2011, 2012) and Broggini and Snieder (2012). Unlike the 1D method, which uses the reflection response only, the proposed multi-dimensional extension requires, in

addition to the reflection response, independent information about the first arrivals.

The proposed data-driven procedure yields the response to a virtual source (Figures 3a), removes the imprint of the subsurface (as the virtual source method), and reconstructs internal multiples, without needing a receiver at the virtual source location and without needing detailed knowledge of the medium. The method requires (1) the direct arriving wave front at the surface originated from a virtual source in the subsurface, and (2) the reflection impulse responses for all source and receiver positions at the surface. The direct arriving wave front can be obtained by modeling in a macro model, directly from the data by the CFP method (Berkhout, 1997) when the virtual source is located at an interface, from microseismic events (Artman et al., 2010), or from borehole check shots. A variant of the iterative scheme allows to decompose the reconstructed wavefield into downgoing and upgoing fields. These fields can be used to create and image of the subsurface with multi-dimensional deconvolution The required reflection impulse responses are obtained from seismic reflection data after surface-related multiple elimination (Verschuur et al., 1992) and deconvolution for the source wavelet.

Errors in the estimated first arrivals (due to a smooth macro model) cause defocusing and a mislocalization of the virtual source (similar as in standard imaging algorithms). Such errors, however, do not affect the handling of the internal multiples and do not deteriorate their reconstruction, which is handled by the actual medium through the reflection data measured at the surface (that includes all the information about the medium itself). Furthermore, because the proposed method is non-recursive, the reconstruction of internal multiples will not suffer from error propagation, unlike other internal multiple suppression techniques used in seismic imaging.

Acknowledgments

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