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Zero stiffness composite shells using thermal prestress

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Zero stiffness composite shells

Using thermal prestress

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Preface

This thesis is the final part of my study of Mechanical Engineering at the TU Delft. I want to thank Giuseppe Radealli for his continuing support during the writing of my thesis. I also want to thank the companies Laevo and Yumen Bionics for giving me the opportunity to manufacture my prototypes. And I finally want to thank my friends and family for their support and friendship throughout the years that I studied here in Delft.

*B. Doornenbal
Delft, September 2018*

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Introduction

Compliant shell mechanisms are thin walled structures that achieve their motion through deformation. Shell mechanisms are of recent interest for designing exoskeletons that are inconspicuous. One of the challenges in designing shell mechanisms is getting as much compliance as possible in certain directions while keeping the other directions sufficiently stiff.

Prestressing is a technique that is used to change the stiffness of compliant mechanisms and it makes it possible to generate compliant mechanisms that have zero or negative stiffness. Seffen et al. have generated a shell with zero stiffness by rolling a metal plate in two perpendicular directions showing that it is possible to generate shell mechanisms without stiffness.

The main disadvantage of this technique is how it limits the different shapes that are possible. The goal of this thesis is to create a compliant shell with more freedom in shape that used the same physical principle to generate a degree of freedom with a much lower stiffness.

1.1. Research objectives

The research objectives of this thesis are the following:

- Identification of viable methods to generate a zero stiffness shell.
- Manufacturing of a zero stiffness shell using composites and thermal prestress.
- Investigation of visualization methods to describe non-linear behaviour of compliant shells.

1.2. Thesis outline

The thesis consist out of three papers. Each paper answers one of the research objectives.

The first paper describes how prestress can be used to remove the stiffness of a shell. The paper gives a summary of literature on the different methods that can be used to prestress different materials. Most forms of prestress are side effects of the manufacturing process but there are also processes that purposefully add prestress. The viability of different methods is compared.

The second paper is the central part of the thesis as it describes how thermal stresses left behind by the curing process can be used to prestress shells in such a way that it reduces its stiffness. An analytical and a FEM model have been made to describe the behaviour and the models have been experimentally verified.

The third paper describes Three methods to describe the non-linear behaviour compliant mechanisms. The three methods are based on fields that describe the behaviour of one point on the mechanism as it is moved through space. Each method is based on a different physical quantity that can be used to describe the behaviour of the mechanism: energy, force and stiffness.

The appendix contains extra information about the production process of the prototypes and a description of different methods that can be used to describe the linear behaviour of compliant mechanisms.

I

Review of prestressing techniques to obtain
compliant shell mechanisms with zero and
negative stiffness

Review of prestressing techniques to obtain compliant shell mechanisms with zero and negative stiffness

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Abstract

Compliant shell mechanisms are mechanisms that use flexibility to achieve motion. Shells are interesting because they can, for instance, follow the body and can therefore be used to make exoskeletons and support devices that are less intrusive than the state of the art alternatives.

One of the challenges in designing compliant shell mechanisms is making the flexible directions flexible enough. The stiffness can be changed by prestressing the shell. This allows the designer to reduce the stiffness in specific directions. It is possible to stress a shell in such a way that it has zero or negative stiffness in a certain direction.

Most of current research is focused on flat plates with an uniform stress distribution while most applications require more complex shapes and a more complex stress distribution to get the right stiffness behaviour. New manufacturing methods are necessary to generate these shells. The techniques that are currently used are: cold rolling, curing stresses in composites and prestressing fibres in composites. There are other promising techniques like casting, laser forming or using viscoelastic fibres but their effectiveness needs to be verified.

1. INTRODUCTION

Classical mechanisms consist of parts that are joined using hinges, axles and sliders to allow for motion. Connections are never perfect and have, for instance, friction and play which will cause wear, energy loss and noise. Play is a large problem for precision applications because it limits the maximum accuracy that can be achieved.

Compliant mechanisms use the flexibility within a single part instead of a combination of parts to achieve the desired motion. Friction and play are removed because there are no longer separate parts. The reduction

in the number of parts that is needed can lead to large cost reductions because assembly can be an expensive part of the manufacturing process [15].

Using flexibility also has its disadvantages: deforming an object will cause a reaction force and the maximum deformation is limited by the maximum strain that a material can handle before breaking. Compliant mechanisms for large deformations can therefore become very large in comparison with their ‘classical’ counterparts.

A shell is a spatially curved thin walled structure. Compliant shells are both the frame and the ‘deformable part’ at the same time reducing the total space required. Shells are especially efficient for exoskeletons and supportive devices because they can follow the body.

Any deformation costs energy unless the necessary energy is already stored in structure. Energy can be stored in the form of strain energy by deforming the structure. The fact that every compliant mechanism can already store energy in the form of strain energy makes strain energy a good form of energy storage. Storing strain energy in such a way that it is released as the structure deforms will reduce the amount of energy needed to deform the structure and therefore the amount of force needed to deform the structure and the stiffness of the structure.

Strain energy can be added by buckling a part of the structure which generates a second stable state in which the strain energy is not zero. Many manufacturing processes like, for instance, rolling [20] and casting [6] also lead to residual stresses and strains within the material. Strain energy can be added by prestressing the material during the manufacturing process, this is often done with reinforced concrete [7] but the same principle can be used in composites [4, 19].

The energy needs to be stored in such a way that it is released when the structure deforms in order to decrease the stiffness and generate zero or negative stiffness.

The goal of this study is to describe and compare

techniques that can be used to prestress shell structures in order to generate zero or negative stiffness. First the physical principles behind the influence of prestress on the stiffness of shells will be explained and the possible manufacturing methods for such shells will be explained and compared thereafter.

2. METHOD

2.1. Search method

Hyer was the first to describe bistability in a plate in 1982 [12], his work is the basis for nearly all the subsequent publications on zero/negative stiffness shells. All the articles that refer back to Hyer have therefore been investigated. A search string has been used to be certain that other methods that do not refer back to Hyer were missed:

Plate OR shell AND neutrally stable OR neutral stability OR bistable OR bistability OR multistable OR multistability OR compliant OR static balancing OR constant force OR buckling

The search for manufacturing methods has been split into two parts. First the methods used in the articles about zero/negative stiffness shells were investigated and afterwards all methods that leave residual stresses either intentionally or as a by-product. The search strings which are used are:

1: *Stress or pre-stress AND residual OR thermal OR cured OR curing OR plastic deformation OR rolling OR chemical OR casting OR molding*

2: *PPMC OR pre-stressed polymeric matrix composites*

2.2. Comparison method

The different manufacturing methods are compared using a table. the measurable criteria are rated from - - to ++. The other features are rated - or + if the property is present.

Maximum deformation is determined by the maximum strain that the material that can be used with the manufacturing method can take before it breaks or deforms plastically. Composites and engineering plastics can, for instance, take larger strains than metals and are therefore rated higher.

Equipment costs are related to the purchase of the equipment, not the cost per product in the case of mass production. Values without a source are estimations because not all equipment can be bought of the shelf and no data is therefore available.

Production time is determined by the total time that is required to make a prestressed shell using the

manufacturing method. This is mainly determined by curing time.

Gaussian curvature is the fact that the final shape can have curvatures in multiple directions. Methods that do not allow for gaussian curvature are limited to ruled surfaces.

Local stress distribution allows the user to vary the stiffness of the shell locally which is important for more complex designs. With some methods it is easier to vary stiffness locally than with others but this is very difficult to quantify especially because with most methods nobody has tried to do this yet.

Sensitivity to environment is determined by looking at the sensitivity of the prestress to variations in temperature and humidity.

3. RESULTS

Nearly all research that has been done looks into flat plates, Researchers [5, 14] have looked at composite shells with a cylindrical curvature and Seffen [21] predicted that a spherical shell under certain circumstances would exhibit both neutrally and bistable behaviour.

3.1. Working principle of zero and negative stiffness shells

Stiffness is the second derivative of the energy with respect to the spatial coordinates, it is therefore possible to determine the stiffness by looking at the change of energy in the shell. Let us assume a plate which is completely unstressed and has therefore no strain energy stored in itself. Deforming this plate in any way will increase the strain energy stored in the plate. The force will increase, because it is zero in the unstressed configuration, and the stiffness, which is the derivative of the force with respect to the deformation, has therefore to be positive.

Storing energy in the structure is essential if you want to create zero or negative stiffness. The energy needs to be stored in such a way that it is released when the shell deforms.

One way of achieving this is by changing the curvature of the shell. The curvature at every point on the surface of the shell can be described by three components: the longitudinal curvature k_x , the transversal curvature k_y and the twist k_{xy} . The three midplane strains that correspond to these curvatures are ϵ_x , ϵ_y and ϵ_{xy} [3, 8].

The Gaussian curvature is the product of k_x and k_y . Changing the Gaussian curvature will cause a large amount of strain which often causes the material to fail.

It is, for instance, not possible to wrap a flat sheet of paper, which has no Gaussian curvature, over a sphere,

Table 1: Comparison criteria manufacturing processes

	-	-	+-	+	++
Maximum deformation	<1%	1<2%	2<5%	<10%	>10%
Equipment costs (euro)	>100,000	>10,000	>1000	>100	<100
Production time	>3 hours	<3 hours	<1 hours	<30 minutes	<10 minutes
Gaussian curvature		No		Yes	
Local stress distribution		No		Yes	
Sensitive to environment		Yes		No	

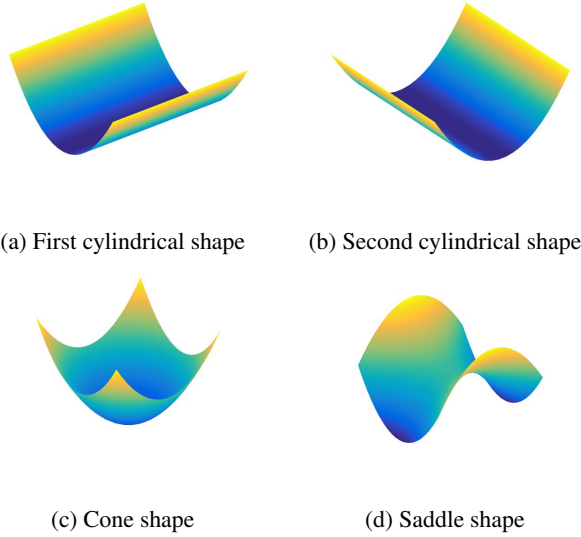


Figure 1: Basic curved shapes shell

which has positive Gaussian curvature, without causing it to wrinkle. The paper stays in a shape that has the minimum amount of strain energy which is the shape that has the least amount of Gaussian curvature.

Adding midplane strains ϵ_x and ϵ_y that try to curve a plate around two axis will lead to a mismatch between the curvatures because the shell cannot curve around two axes at the same time without introducing a large amount of Gaussian curvature and therefore a large amount of strain energy. A shell that has initially no Gaussian curvature will have the lowest strain energy in states where the amount of Gaussian curvature is small [8].

There is no longer a configuration in which the midplane strains are zero and this technique can therefore be used to store strain energy in the shell.

If we take, for instance, a flat plate and apply a stress distribution ϵ_x and ϵ_y so that it wants to curve around the x and y axis (Fig. 1a and 1b). The Gaussian curvature, which is the product of both curvatures, will be very small for small deformations. Linear theory applies in this case and both curvatures can simply be added using superposition which leads to either a saddle or cone shape (Fig. 1c and 1d) depending on whether or not the centres of curvature are on the same side of the plate ϵ_x and ϵ_y have the same sign).

This theory does not hold for larger deformations because the Gaussian curvature can no longer be neglected. Bending a plate around an axis increases the bending stiffness about the perpendicular in-plane axis. This means that it is not possible to have two large perpendicular curvatures at the same time if you start with a flat plate without introducing a large amount of strain. The saddle and cone shape will become unstable which means that in practice one of the two cylindrical shapes is obtained [12]. It is possible to snap between both shapes by applying an external force. This snap-through force will disappear near the critical length at which the saddle shape becomes unstable [12], which means that under these circumstances zero-stiffness can be obtained.

Plates can be prestressed in two different ways: Opposite-sense prestressing, when the centres of curvature are on opposite sides of the plate and same-sense prestressing, when the centres of curvature are on the same side of the plate.

3.1.1. same-sense prestressed plate. A same-sense pre-stressed plate without initial Gaussian curvature has a cone shape for small deformations which turns into a cylindrical shape for larger deformations (fig. 2a). If the level of prestress is equal in both principal directions than the direction of the principal curvature is undeter-

mined: every direction has the same amount of energy. Changing the direction of the principal curvature does not cost energy and the stiffness of this transformation is therefore zero [9, 21].

3.1.2. opposite-sense pre-stressed plate. An opposite-sense pre-stressed plate without initial Gaussian curvature has a saddle shape for small deformations which turns into one of two cylindrical shapes for larger deformations (fig. 2b). The difference with a same-sense pre-stressed plate is that there is an energy barrier between these shapes and the plate has therefore become bistable. The strain energy in the shell needs to be increased to switch from one stable configuration to another.

The bifurcation point, where the saddle shape transforms into one of the two cylindrical shapes, is predicted to have zero-stiffness [12]. The bifurcation takes place at small deformations which limits the range of motion.

3.2. possible techniques to create zero and negative stiffness shells

The stress distribution needed for zero/negative stiffness shells is very specific. Current experiments make use of flat plates with an uniform stress distribution which are relatively easy to manufacture. More complex designs require different manufacturing methods that allow for more control over shape and stress distribution.

3.2.1. Rolling. Rolling the plate in multiple directions leads to a stress distribution with two different principal curvature components but no Gaussian curvature. The amount of curvature can accurately be controlled and both opposite and same sense pre-stressed plates

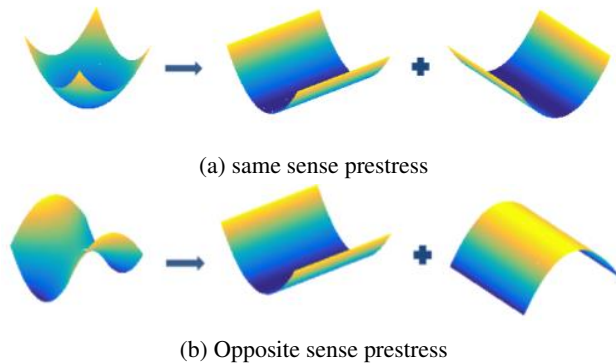


Figure 2: transformation from double curved to single curved surface

can therefore be made using this technique.

Rolling limits the freedom that the designer has because only flat plates can be rolled. This technique has already successfully been used to generate negative and zero stiffness mechanisms [21].

3.2.2. Casting and injection moulding. Heating a material causes it to expand and when it cools down it will shrink again but the material around it can prevent from going back to a stress-free shape. Processes like casting and injection moulding leave complex stress distributions behind [6]. controlling these stresses and the deformations they cause is difficult, further research is therefore necessary to determine if this is a valid method to manufacture multi or neutrally stable shells.

3.2.3. Laser forming. Heating a plate locally with a laser leads to non-uniform stress distribution along the scanning line of the laser which causes the plate to bend around that line. [23]. There are three different mechanisms that cause three different deformations [1]. The first mechanism is based on a steep temperature gradient over the thickness of the plate (small Fourier number), this causes the plate to bend toward the laser beam. The second method is based on an equal temperature distribution over the thickness of the plate, this causes the plate to shorten locally what can lead to different deformations depending on the geometry of the undeformed material. The third method is based on local buckling and causes the plate to bend away from the laser beam. Combining these methods can lead to different shapes and stress distributions. There is a lot of freedom in stress distributions that can be generated with this method and buckling is always a sign of negative stiffness. This technique is promising but no research has been done that uses this technique to generate negative or zero stiffness mechanisms.

3.2.4. Tempering with thermal gradient. Tempering a material removes all the stresses within a material. Tempering a material at different temperatures leads to residual stresses after the material cools down. Using a thermal gradient over the thickness of the material leads to same-sense pre-stress that is equal in all directions. This method has not yet been experimentally validated but Mansfield [16] did calculations for a plate that was heated with a thermal gradient over the thickness of the plate and predicted neutrally stable behaviour at elevated temperatures which is equivalent to neutrally stable behaviour at room temperature after tempering.

3.2.5. Curing effects in composites. Composites consist out of fibres that are held together by a matrix material. The matrix material has a larger thermal expansion

coefficient than the fibres which causes residual stresses in the composite when the material cools down after the curing process. Each layer shrinks the most in the direction perpendicular to the fibres. An asymmetrical layup leads to shear stresses that try to bend the plate around two axes but does not introduce Gaussian curvature. The pre-stress of such a lay-up will be equal and opposite direction [11]. The final shape and stress distribution is sensitive to variations in temperature, expansion coefficients, Young's moduli and ply-thickness [2].

A possible technique to generate a shell that is same-sense prestressed is using a pre-curved plate. Lee et al. [14] have looked into the effects of initial curvature on the snap-through load, but only looked at plates that were pre-curved in the same direction as the direction in which the thermal load would bend them (Fig. 3a). They noticed an decrease in the snap-through force toward the pre-curved cylindrical shape and an increase in the reverse snap-through force.

The initial curvature and the curvature caused by the thermal stresses can be summed. Precurving the plate with a strain that is twice as large and in the opposite direction as the thermal strain will lead to a plate that has the same strain in both directions (Fig 3b) and would therefore be neutrally stable. True neutral stability cannot be obtained because the material is not isotropic [source] but a very low stiffness should be possible

This technique could possibly be used to generate more complex shapes by varying the initial shape and the local stress distribution. Accurately varying the direction of the fibres is difficult to do accurately by hand but there are ply-steering machines that can achieve good geometrical accuracy [22]. Panesar et al. [18] have used ply-steering in combination with curing stresses to generate a bistable morphing wing.

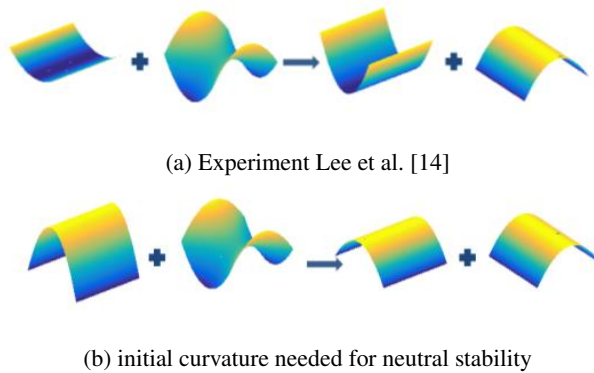


Figure 3: effects of using a precurved composite plates on the stable shapes after curing

3.2.6. Chemical. Chemical reactions could cause material to locally shrink or expand and in that way generate prestress. There is little research into this area yet but there is a lot of potential because chemical reactions are not dependent on the shape of the shell and the composition of the material could be varied locally to change the amount of prestress locally which gives the designer a lot of freedom in the final shape.

A well documented case is the effects of humidity on asymmetric composites. As opposed to the shrinking stresses that is caused by curing, humidity causes the matrix to swell. Stresses caused by humidity are much smaller than the stresses caused by thermal expansion during curing [11]. Humidity is so sensitive to changes in the environment to be an efficient way of creating zero or negative stiffness shells but can have a large influence on the performance of shells that use asymmetric composites and another method for prestressing.

3.2.7. Mechanically stretched fibres. Mechanically stretching fibres during the curing process of composites leads to residual stresses in the final product. This method is used to increase the strength of the composite by breaking the weak fibres, this minimizes the dynamic overstress on the neighbouring fibres when the weaker fibres fail, and compensating for the thermal stresses induced during curing [10]. Daynes et al. use this method to generate a bistable shell [4] and this technique could therefore in theory be used to generate neutrally stable shells as well.

Prestressed polymeric matrix composites (PPMCs) are made by clamping the ends of the fibres and stretching them during the curing process. This method allows for accurate control over the amount of prestress but it limits the freedom in the shape because the fibres need to be clamped and stretched during the curing process.

Both same-sense and opposite sense stress distributions can be obtained using this technique and it is possible to use symmetric lay-ups which are much less sensitive to hydrothermal effects [11].

3.2.8. Viscoelastically stretched fibres. Viscoelastic fibres have an extra stage of deformation between elastic and plastic deformation: the viscoelastic state. Fibres slowly return to their original length in this stage which can be used to prestress the plate. The fibres are stretched before they are placed in the mould, the composite is cured without the need of clamping the fibres and the prestress is introduced after the curing process when the fibres shrink back to their original length. It is difficult to accurately predict the stresses because part of the deformation is plastic. [19]. Viscoelastic fibres

are interesting because they give the same amount of control of the stresses as PPMCs without the need to clamp the fibres during the curing process and they give the designer more freedom in choosing a shape.

No experiments have been done to look into the possibility to generate multistable om neutrally stable shells Experiments have however be done to look at the aging effects and did not reveal any significant effect after they were aged for an equivalent of 100 years at 20°C [19].

3.2.9. Combining layers. Combining layers with different material properties or different stress distributions allows for more complex designs with more control over the local stress distributions. Kebabze [13] welded strips of pre-bend steel together to show that the snap-through behaviour of opposite-sense pre-curved plates can be reproduced by combining two layers of single curved material (fig.??).

The curvatures of the final plate will be much smaller than that of the individual plates because the combined plate is much stiffer and the stresses remain the same.

3.3. comparison manufacturing methods

Every manufacturing method has advantages and disadvantages which are compared in table 2.

- Rolling is the simplest process but also gives the least freedom in both shape and stress distribution.
- The heat controlled methods: Casting/injection molding, laser forming and tempering with gradient show promise but not enough research has be done to know whether or not it is possible to generate the desired stress distribution.
- Curing stresses in composites have been proved to be able to generate bistable shells and could be used in a lot of situations if combined with ply-steering.
- It is proven that stretched fibres can be used to generate bistable shells [4] further research is needed

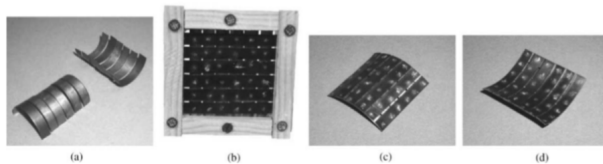


Figure 4: bistable structure made of half circular strips [13]

to determine whether or not this could also be done by using viscoelastic fibres and to see if it possible to generate neutrally stable shells as well.

- Chemical processes have not been used in this context and their performance can therefore not be rated.
- Combining layers uses layers that are produced using other methods and their performance depends therefore mostly on the performance of the layers.

4. DISCUSSION

Prestressing shells to change their stiffness is a relatively new endeavour within the field of compliant mechanisms. Most knowledge about their behaviour is focused on flat rectangular plates. The ability to apply the principles found for these simple geometries to more complex geometries is needed to make progress toward practical applications.

One of the largest problems with prestressed shells is manufacturing. All manufacturing methods have limitations in both freedom of shape and stress distribution. The more versatile techniques like ply-steering and laser forming require expensive equipment.

A lot is known about the stresses that are left behind by most manufacturing methods but most this knowledge has not yet been used to see whether or not the stiffness of the shell can be changed using this technique. The lack of experimental data that show the performance of the different methods in this context makes it difficult to compare the different techniques. Different methods could be combined to generate more complex designs, but this also requires more knowledge about the manufacturing methods in the context of stiffness.

Most of the current research is focussed on prestressing plates with two bending stresses but this is not necessarily the only method of prestressing that changes the stiffness. Daynes [4] used prestressed buckled laminates to generate bistable behaviour. This is a completely different method that uses a different physical principle to generate bistability. There might be more unexplored methods and more fundamental research into the behaviour of prestressed shells is therefore necessary.

5. CONCLUSIONS

Zero and negative stiffness shells can be made by prestressing a shell beyond the bifurcation point where

Table 2: Comparison manufacturing methods

	Rolling	Casting/Injection moulding	Laser forming	Tempering with gradient	Curing	Curing + ply-steering	Chemical	Stretched fibres	Viscoelastic fibres	Combining layers
Maximum deformation	-	+	-	-	+	+	?	+	++	?
Equipment costs	++	?	-	-	+	-- [17]	?	+	+	?
Production time	+	+	?	?	-	-	?	-	--	?
Gaussian curvature	-	+	+	+	+	+	+	+	+	+
Local stress distribution	-	+	+	+	+	+	+	+	+	+
Sensitivity to environment	+	-	+	+	-	-	?	-	-	+

the saddle or cone shape turns into one of two cylindrical shapes. The level and direction of the prestress determines whether the shell is neutrally stable or bistable. Current research focusses on flat plates with an uniform stress distribution. There is research that indicates that the same principles hold for more complex shapes and stress distributions.

New manufacturing methods are necessary to generate more complex shells. A lot is known about the effects of manufacturing on stresses left in the material but most of these methods have not yet been used to change the stiffness of shells. Cold rolling is the simplest technique but is very limited in freedom of shape or stress distribution. Curing stress is currently the most used technique for adding prestress. This technique allows for more complex geometries and stress distributions and has therefore more potential than rolling. There are other promising techniques like casting, laser forming or using viscoelastic fibres but their effectiveness for generating zero or negative stiffness shells needs to be verified.

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II

Using thermal prestress to reduce stiffness of
composite shells

Using thermal prestress to reduce stiffness of composite shells

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Abstract

Compliant shell mechanisms are shells that achieve their motion through deformation. Deforming a material requires energy which gives the shell its stiffness. The amount of stiffness depends on how much the force changes as the shell deforms. This change in energy can be tailored by storing energy in the shells before the deformation.

Curing a composite at an elevated temperature stores energy in the shell and can therefore change its stiffness. The effects of this stored energy on the behaviour of the shell have been modelled using both an analytical model and a finite element model and have been verified experimentally.

It is possible to tailor the stiffness in specific directions by choosing the right combination of geometry and layup. It is possible to generate shells with zero stiffness or negative stiffness, which is impossible without prestress.

1. introduction

Compliant shell mechanisms are mechanisms that use flexibility to achieve motion. Shells are interesting because they can, for instance, follow the body and can therefore be used to make exoskeletons and support devices that are less intrusive than the state of the art alternatives [6]. The challenge is to make a compliant shell mechanism that follows the body and has the right stiffness in all directions. A mechanism is difficult to deform in the direction in which it has a high stiffness. Having little or no stiffness in certain directions allows for new degrees of freedom in a similar way that hinges create new degrees of freedom in classical mechanisms.

Deforming a shell introduces strain in a material which increases its potential energy. This change in energy determines the amount of force needed to deform. The change of the force determines the stiffness of the shell. Zero stiffness is desirable if you want to keep the forces small. Prestressing the shell is one way

in which this can be achieved. Such a shell has internal stresses present in absence of external loads. These stresses can be generated by manufacturing processes such as rolling and thermal processes.

Stresses are in this case already present in the shell in the initial shape. Some of this strain is released as the shell deforms while other strains are introduced. The change of the total energy of the shell determines the direction and magnitude of the reaction force and stiffness.

There are different ways to prestress a shell. Guest et al. [3] generated a neutrally stable shell by adding curvature in two directions by rolling a plate in two directions.

In this paper the residual thermal stresses left behind by the curing process of composites are proposed as an alternative way to generate a similar stress condition, needed to obtain neutral stability. Composites are often manufactured at an elevated temperature. This process leaves thermal stresses behind [5] which can lead to the warping of the shell. This warping usually can be prevented by using a symmetrical layup [1] in which case the thermal stresses on both sides of the shell balance each other out. An antisymmetric layup, on the other hand, maximises the effect of these stresses.

The goal of this paper is to investigate the possibility to use thermal prestress in composite shells to obtain low stiffness shells.

The first chapter describes the two different cases that have been used to test the theory. The first case is a cylindrical shell and is similar to the shell used by Guest et al. except that it is made of carbon fibre instead of metal and it is prestressed by thermal stresses instead of rolling. The second case is a saddle shaped shell which has Gaussian curvature which cannot be produced by rolling.

The second chapter describes an expansion of the analytical model used by Guest et al. [2] to describe their neutrally stable shell. In the third chapter describes the finite element model. The fourth chapter describes the experiment that is used to verify the predictions

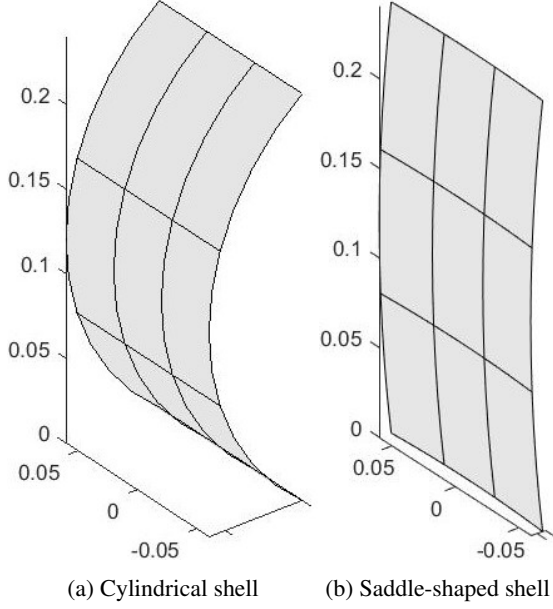


Figure 1: The two shells used in the paper.

made by the model.

2. The two cases

Two different shells are used throughout the paper. The first shell is cylindrical and is therefore similar to the neutrally stable shell described by Guest et al. [2]. The main differences are the way the shell is prestressed and that the material is not isotropic. The initial radius of the single curved shell is 0.15 m. (figure 1a)

The second shell is saddle shaped which means that the shape bends down in one direction while it bends up in the perpendicular direction. This shape has Gaussian curvature and can therefore not be produced by rolling. This represents therefore a range of new shapes that can be produced by this technique. The radius in the principal direction is 1 m and in the secondary direction is -1 m (figure 1b).

The shell is made of unidirectional sheets of pre-impregnated fibres. This technique allows for good consistency with respect to the fibre/matrix ratio and the orientation of the fibres which is important to get accurate results from the experiment. The material properties are given in table 1.

Two different load-cases are considered for each shape. The load cases are different than the load case for the analytical model because the FEM needs other boundary conditions. In the case of the analytical model the shell is free in all directions and the energy is given as a function of the direction and magnitude of the prin-

ciple curvature.

in the case of the FEM and the experiment the shell is clamped at the bottom and actuated at top (fig. 2). There are two different load-cases in the first case a twist is applied around the vertical axis, while the other five DOFs of the top are left unconstrained. In the second case a displacement along the y-axis is applied while the other 5 DOFs at the top are left unconstrained. These load-cases have been chosen because they are the easiest to measure accurately.

Figure 2: Boundary conditions FEM; blue lines are beams that constrain the edges. Black arrows show both loadcases (not applied at the same time)

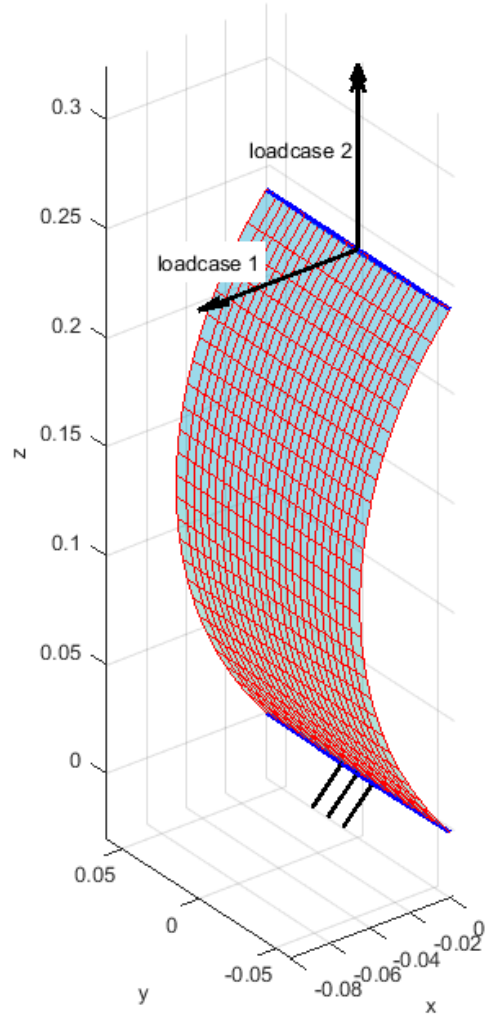


Table 1: material properties and dimensions of the analysed shells

Material	
E1	170 GPa
E2	8.4 GPa
ν_{12}	0.3
G12	6.34 GPa
α_1	0 T^{-1}
α_2	$26 \times 10^{-6} \text{ T}^{-1}$
Dimensions	
Thickness	0.7 mm
Length	24 cm
Width	12 cm
Layup	[0 90]

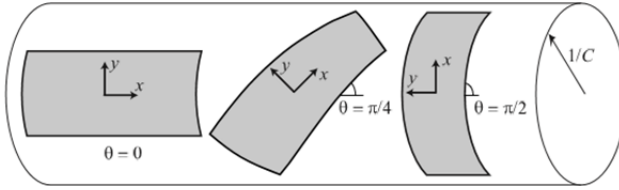


Figure 3: Definition of the geometry shell in terms of an underlying cylinder with curvature C . The angle Θ specifies the orientation of the shell with respect to the cylinder. Figure taken from Guest [2]

3. Analytical model

The analytical model is based on the model that Guest and Pellegrino [3] use to describe a prestressed cylindrical metal plate that is neutrally stable.

The model describes the energy of the shell as a function of its shape. The energy of the shell stays constant as long as the principle radius stays the same but is independent of the orientation of this radius θ (figure 3). A number of assumptions are made to make it possible to solve the equations analytically.

- The amount of energy that is needed to stretch the shell is much larger than the energy needed to bend the shell. It is therefore possible to assume that the stretching strains can be ignored and the shell can therefore be considered inextensional. The relation between the generalized strains (ϵ for the stretching strains and κ for the bending strains) and the generalized stresses (N for the stretching stresses and M for the bending stresses) of a shell are described by the ABD-matrix eq. 1.

$$\begin{bmatrix} N \\ M \end{bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{bmatrix} \epsilon \\ \kappa \end{bmatrix}. \quad (1)$$

A , B and D are 3 by 3 stiffness tensors that give the relation between generalized strains and stresses

In the case of the inextensional model only the D -part of the stiffness-matrix which relates bending moments with the curvatures of the plate is important which makes it possible to simplify the equation to

$$M = D\kappa. \quad (2)$$

- The second important assumption is based on the observation that a cylindrical shape will stay cylindrical during the deformation. This is because a cylinder is a deployable surface (a surface that can be flattened onto a plane without distorting the shape) and has therefore no Gaussian curvature. Changing the Gaussian curvature of a shell will cause a large amount of strain and will therefore take a large amount of energy.

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$$\begin{bmatrix} N \\ M \end{bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{bmatrix} \epsilon \\ \kappa \end{bmatrix}. \quad (3)$$

In the case of the in-extensional model only the D -part of the matrix which relates bending moments with the curvatures of the plate is important which makes it possible to simplify the equation

$$M = D\kappa. \quad (4)$$

The second important assumption is based on the observation that a cylindrical shape will stay cylindrical during the deformation. This is because a cylinder is a deployable surface (a surface that can be flattened onto a plane without distorting the shape) and has therefore no Gaussian curvature. Changing the Gaussian curvature of a shell will cause a large amount of strain and will therefore take a lot of energy.

These assumptions are only valid if the changes in the total strain energy of the shell are small, moreover the model gives no longer accurate predictions when there is a large external load on the shell.

In this model the direction θ and magnitude C of the curvature of the cylindrical shell can change (figure 3). The curvature of the shell can be expressed by using only two parameters: θ and C

$$\kappa = \frac{C}{2} \begin{bmatrix} 1 - \cos 2\theta \\ \cos 2\theta + 1 \\ 2 \sin 2\theta \end{bmatrix}. \quad (5)$$

While the initial curvature κ_0

$$\kappa_0 = \begin{bmatrix} 0 \\ C_0 \\ 0 \end{bmatrix}. \quad (6)$$

In which C_0 is the magnitude of the initial curvature.

The change of curvature $\Delta\kappa$ can be found by taking the difference between the initial curvature κ_0 and the current curvature κ

$$\Delta\kappa = \frac{C}{2} \begin{bmatrix} 1 - \cos 2\theta \\ \cos 2\theta + 1 - \frac{2C_0}{C} \\ 2 \sin 2\theta \end{bmatrix}. \quad (7)$$

The change in strain energy can now be calculated as a function of this curvature change

$$\Delta U = \frac{1}{2} \Delta\kappa^T D \Delta\kappa. \quad (8)$$

The change in energy caused by adding prestress is equal to the prestress M_0 multiplied by the deformation $\Delta\kappa$

$$\Delta U_{prestressed} = \Delta\kappa^T M_0. \quad (9)$$

The total change in energy can be calculated by combining eq.8 and eq.9

$$\Delta U_{prestressed} = \frac{1}{2} \Delta\kappa^T D \Delta\kappa + \Delta\kappa^T M_0. \quad (10)$$

3.1. Isotropic

Guest et al. [3] describe a neutrally stable shell made of metal. Metal is isotropic and has therefore a specific stiffness matrix

$$D_{iso} = \frac{Et^3}{12(1-\nu^2)} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix}. \quad (11)$$

The prestress M_0 is introduced by rolling the shell perpendicular to its initial curvature.

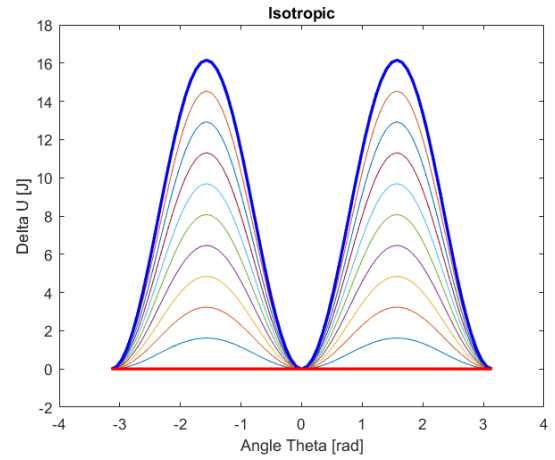
$$M_0 = \begin{bmatrix} m \\ 0 \\ 0 \end{bmatrix} \quad (12)$$

The shell is neutrally stable if the change in energy eq. 10 is independent of the direction of the curvature

θ . Combining eq.10 with eq.11 and eq.12 gives one magnitude of prestress m for which this is the case

$$m = \frac{-Et^3 C_0}{12(1-\nu)} \quad (13)$$

Figure 4: Change in internal energy of an isotropic cylindrical shell as a function of the direction of the curvature of the shell. The blue line represents the shell without prestress and the red line represents the prestressed shell which is neutrally stable. The intermediate lines show the behaviour for intermediate levels of prestress



In this case $\Delta U = 0$ and $C = C_0$ for all values of θ fig.(4).

3.2. Anisotropic

Two adjustments to Guest's model [2] are made to make it suitable for composites and for thermal prestress.

- The stiffness matrix for an isotropic material needs to be replaced by the stiffness matrix of a composite material.
- the prestress is no longer generated by rolling but by the difference in thermal expansion coefficients of the fibres and the matrix material.

3.2.1. calculating material properties composite shell. Both the prestress and the stiffness of the composite are dependent on the layout of the composite.

Both the prestress and the stiffness are calculated by first calculating the properties of each ply and then adding all the values up. The stiffness of each ply is given by the Q-matrix

$$Q_{ij} = \begin{bmatrix} \frac{E_{11}^2}{(E_{11}-v_{12}^2)E_{22}} & \frac{v_{12}E_{11}E_{22}}{(E_{11}-v_{12}^2)E_{22}} & 0 \\ \frac{v_{12}E_{11}E_{22}}{(E_{11}-v_{12}^2)E_{22}} & \frac{E_{11}E_{22}}{(E_{11}-v_{12}^2)E_{22}} & 0 \\ 0 & 0 & G_{12} \end{bmatrix}. \quad (14)$$

the stiffness matrix needs to be rotated to get the stiffness in the orientation of the ply by using the rotation matrix R in which θ is the angle of the ply relative to the length direction of the shell (vertical direction in figures).

$$R = \begin{bmatrix} c^2 & s^2 & -2sc \\ s^2 & c^2 & 2sc \\ sc & -sc & c^2 - s^2 \end{bmatrix} \quad (15)$$

with

$$c = \cos(\theta) \quad s = \sin(\theta). \quad (16)$$

The stiffness of the oriented layer is

$$\bar{Q} = RQR^T \quad (17)$$

and the thermal expansion coefficient is

$$\bar{\alpha} = R\alpha \quad (18)$$

The material properties of the composite can be calculated by combining the properties of each ply

$$D_{ij} = \sum_{k=1}^n [\bar{Q}_{ij}]_k [z_k^3 - z_{k-1}^3] \quad (19)$$

$$M_i^T = \frac{\Delta T}{2} \sum_{k=1}^n [\bar{Q}_{ij}\bar{\alpha}_j]_k [z_k^2 - z_{k-1}^2] \quad (20)$$

3.2.2. 0/90 layup. The layup needs to be either symmetric or anti-symmetric to ensure that the B-part, which couples bending and stretching, of the stiffness matrix is zero [Paradies]. The assumption that only the D-part of the stiffness matrix is necessary is invalid if the B-part of the stiffness matrix is not zero. Thermal stresses balance each other out for symmetric matrices [?] and the layup has therefore to be anti-symmetric.

The simplest anti-symmetric layup is the 0/90 layup which consists of two perpendicular layers. The stiffness matrix is the same as for an isotropic material except for the shear modulus (term [3,3])

$$D_{0/90} = \frac{t^3}{24} \begin{bmatrix} \frac{E_1+E_2}{1-v_{12}v_{21}} & \frac{v_{12}E_2}{1-v_{12}v_{21}} & 0 \\ \frac{v_{12}E_2}{1-v_{12}v_{21}} & \frac{E_1+E_2}{1-v_{12}v_{21}} & 0 \\ 0 & 0 & 2G_{12} \end{bmatrix} \quad (21)$$

The thermal prestress $M_{0/90}$ of a 0/90 layup composite is different from that of rolling because it has two

components which are perpendicular and have the opposite magnitude.

$$M_{0/90} = \begin{bmatrix} p_0 p_1 \Delta T \\ -p_0 p_1 \Delta T \\ 0 \end{bmatrix} \quad (22)$$

Where

$$p_0 = \frac{E_{11}t_2^2}{8(-E_{22}v_{12}^2 + E_{11})} \quad (23)$$

and

$$p_1 = (E_{11}\alpha_{11} - E_{22}\alpha_{22} + E_{22}\alpha_{11}v_{12} - E_{22}\alpha_{22}v_{12}). \quad (24)$$

Combining eq.10 with eq.21 and eq.22 gives a neutrally stable shell (ΔU is independent of θ) in the initial shape if the prestress balances the shear modulus

$$p_0 p_1 \Delta T = \frac{t^3}{6} C_0 G_{12} \quad (25)$$

The shell would only be neutrally stable over the entire range of motion if the shear modulus is the same in every direction. the material would behave isotropically in this case

$$G_{12} = \frac{E_1 + (1 - 2v_{12})E_2}{4(1 + v_{12}v_{21})}. \quad (26)$$

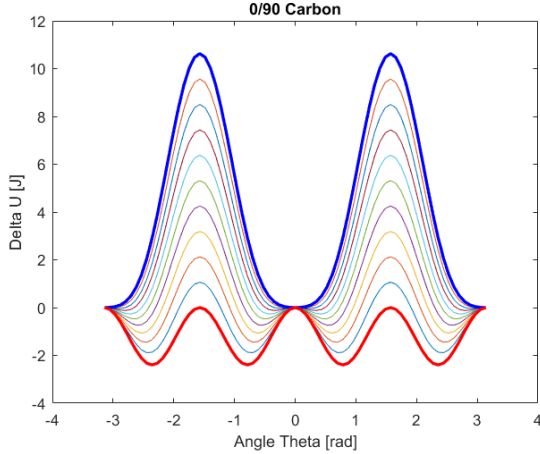
The curvature C is in this case $\frac{1}{2}C_0$ for all values of θ . The curvature of the mould needs to be twice as large as the curvature of the final product.

3.2.3. Effects shear modulus. The shear modulus of an unidirectional layer of a composite is direction dependent. The fibres slide relatively easily alongside each other the shear modulus is therefore smallest in the direction of the fibres and perpendicular to the fibres. The shear modulus is maximal at an angle of 45° relative to the fibre direction. This direction dependence of the shear modulus prevents the 0/90 layup to become neutrally stable over the entire range of motion unless the layers themselves are already isotropic. The 0/90 layup is the only 2 layer layup that has the same stiffness in both x and y direction but it also has the largest effect of anisotropy of the shear modulus. Filling in the material properties of the carbon fibre used in the experiment shows that there is no level of prestress that leads to neutral stability (fig. 5). A layup with more layers is needed to obtain a shell with a shear modulus that is independent of the angle.

3.3. quasi-isotropic

The anisotropy of the composite prevents the shell from being neutrally stable over a range of motion. It

Figure 5: Change in internal energy of a carbon fibre cylindrical shell with a 0/90 layup as a function of the of the direction of the curvature of the shell. The blue line represents the shell without prestress and the red line represents the fully prestressed shell. The intermediate lines show the behaviour for intermediate levels of prestress.



is however possible to make fully isotropic composite shells by using more layers. It is always possible to make fully isotropic shell using 36 layers independent of the material properties of the plies [8] but these layups are symmetric and therefore not useful for generating prestress. In this case it is only necessary to have isotropy in bending and not in extension, therefore only the D-part of the stiffness matrix needs to be isotropic. This can be achieved with 16 plies independently of the properties of the plies and with less plies if the plies may vary in thickness or material properties [7]. These quasi-isotropic layups are antisymmetric but have a high degree of internal symmetry which balances all internal stresses which makes it impossible to use these layups to generate the required prestress.

4. Finite element model

A finite element model is used to verify the analytical model and to see how the effects of prestress would translate to more complex geometries and layups.

There are two beams one along the top edge and one along the bottom edge to make it possible to apply the different load cases. Both beams have nodes at their centres where the loads and constraints are applied, see fig. (2).

The shell is clamped at the bottom node and is free at the top when the thermal load is applied. Each load is applied sequentially in 20 equal steps to obtain con-

vergence. There are two different load cases; in the first case a rotation around the z-axis is applied at the top of the shell, in the second case a translation along the x-axis is applied. All other degrees of freedom at the top node are left unconstrained (fig. 2).

4.1. Results FEM

The analytical model shows the importance of the shear modulus for obtaining zero stiffness over a large range of motion. The shell will show bistable behaviour instead of neutrally stable behaviour this is also the case for the FEM. A series of simulations has been done to determine the effects of the level of prestress, determined by the difference in temperature ΔT , and the shear modulus G_{12} . The initial curvature has been varied to compensate for the change in shape caused by the temperature difference so that the shape after prestressing was always the same and equal to that of the cylindrical shell used in the experiment. The shear modulus determines how much prestress is needed to obtain zero stiffness. The lower the shear modulus the less prestress is needed to obtain zero stiffness (fig. 6). The composite does not show neural stability for larger deformations. The range in which the shell behaves neutrally stable can be increased by increasing the shear modulus but cannot completely be removed as was possible in the analytical model. This is caused by the different boundary conditions and load case.

The analytical model and the finite element model have different boundary conditions and can therefore not be directly compared. The degree of freedom for analytical model is the orientation of the radius of curvature. The upper and lower edges deform as this orientation changes while the upper and lower edges in the finite element model and the experiment are constrained to stay in the same shape.

The analytical model (figure 7) shows the behaviour of the cylindrical shell as used in the experiment both with (red line) and without prestress (blue line). The red line shows two minima in the internal energy which correspond to the two stable shapes of the shell and the maximum shows the unstable equilibrium which were also found in the experiment.

5. Experiment

The material used in the experiment is SP-High Modulus' SE 84LV Prepreg. A prepreg has been chosen because they give a consistent fibre matrix ratio which is important to get a consistent level of prestress. There is a large difference between the stiffness of the fibres and matrix material and this material will therefore not

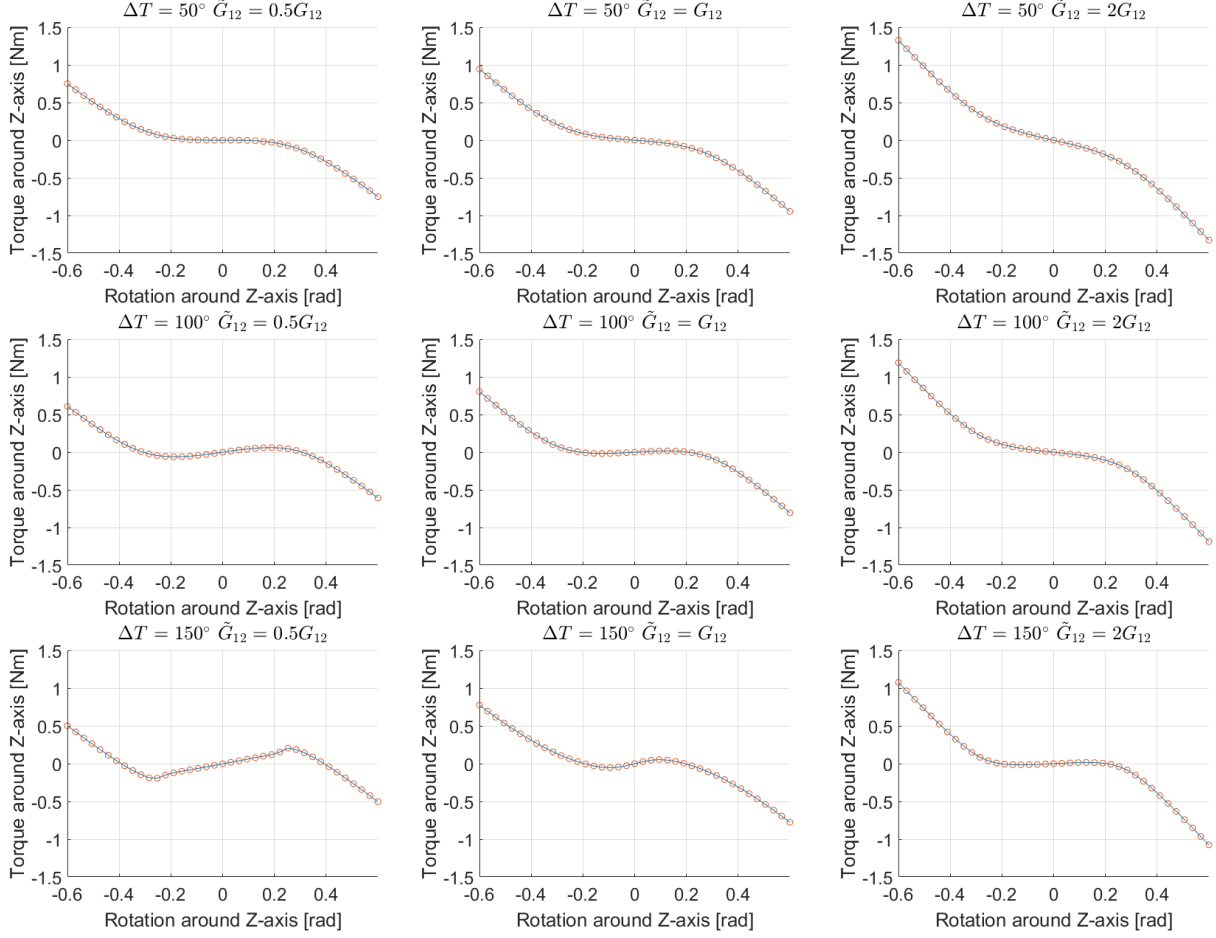


Figure 6: Comparison of torsion of the cylindrical shell to show the relation between the level of prestress (caused by ΔT) and the modified shear modulus of the individual layers $\tilde{G}_{12}$. The higher the shear modulus the more prestress is needed to obtain bistability. It is possible to generate a small plateau of zero moment by varying these two parameters.

give zero-stiffness behaviour over a significant range. The experiment is set-up to verify the model rather than generate a zero stiffness shell.

The material is cured at 120°C which is the highest temperature at which this composite can be cured and gives therefore the highest amount of prestress which makes measuring the effects of prestress easier. This specific prepreg is chosen because it does not require an autoclave to cure. The only pressure difference is generated by a vacuum bag.

There are at least two layers needed to generate a stress distribution that has a component that has a moment. This gives a thickness 0.7 mm. A thinner shell has a larger range of motion because bending causes smaller strains. The larger range of motion makes the measurements more accurate.

The shear modulus of the composite is not given in the data sheet and is therefore calculated. The shear

moduli of both the matrix and the fibres are calculated using the isotropic equation for the shear modulus.

$$G_{iso} = \frac{E}{2(1+\nu)} \quad (27)$$

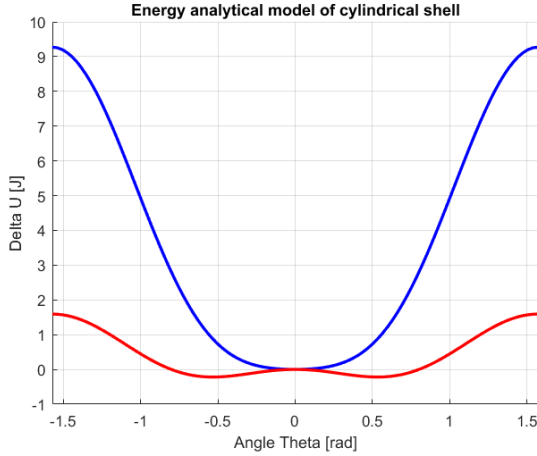
These moduli are used to calculate the shear modulus of a single layer using the equation for unidirectional composites from Hashin and Rosen [4].

$$\frac{G}{G_m} = \frac{(G_f + G_m) + (G_f - G_m)\nu_f}{(G_f + G_m) - (G_f - G_m)\nu_f} \quad (28)$$

with G_f is the shear modulus of the fibres G_m is the shear modulus of the matrix ν_f is the volume fraction of fibres.

The thermal expansion coefficients are also not mentioned in the datasheet and have been determined experimentally by manufacturing a flat sheet of 0/90 carbon fibre and measuring the curvature after the sheet

Figure 7: Change in internal energy of the cylindrical shell, with the same parameters as used in the experiment, as a function of the of the direction of the curvature of the shell. The blue line represents the shell without prestress and the red line represents the shell with the thermal prestress.



has cooled down. This technique can only be used to determine the difference between the coefficients in both directions but this is also this difference is the only important property in this case. α_2 is much larger than α_1 in unidirectional composites because they are much stiffer in the direction of the fibres compared to the direction perpendicular to the fibres. α_1 has therefore been set to 0.

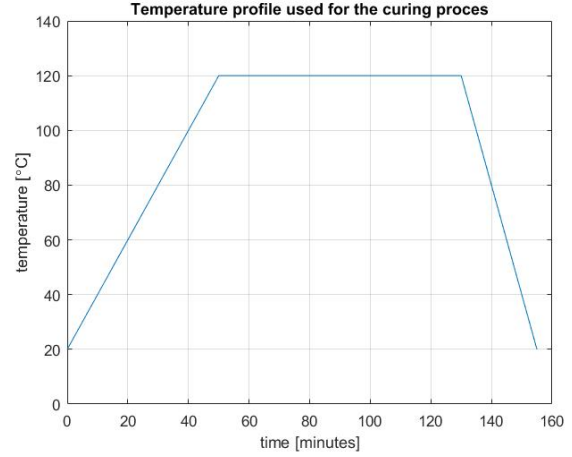
Two different shells have been made to verify the FEM model. The first shell is cylindrical and has a radius of 15 cm. The second shell is double curved and has radii of 1 and -1 m respectively.

The shells have been manufactured using a vacuum bag and a heating blanket (Briskheat SR Heating Blanket 16"×16") and temperature controller (Briskheat TT Table Top controller). The prescribed temperature envelope for the prepreg has been followed see fig. 8.

5.1. Measurement setup

Measuring the behaviour of the shell is challenging because the rotation axis changes as the shell deforms. Constraining the rotation to be about a fixed axis will increase the stiffness and make it more difficult to see the change in stiffness caused by the prestress. Motion capture has been used to determine the angle of the top edge of the shell while the basis is clamped (fig. 9). The torques and forces have been manually applied and been measured by a 6-DOF force sensor which has been

Figure 8: Temperature profile used during the curing process



connected via an DAC to a computer.

The motion capturing system consists of two cameras which tracks a set of red markers. The direction of the rotation axis changes while the shell deforms and the rotation axes of the force sensor and axes of the motion capturing system will misalign. The forces and torques measured by the force sensor are therefore transformed to fit the global coordinate system that is already used by the finite element model and the motion capturing system.

Applying a torque around the X-axis to bend the shell without accidentally applying a force in the y-direction was not possible. Applying a force in Y-direction without applying a torque around the X-axis is much easier and is therefore used to compare the stiffness of shell in bending between the FEM and the experiment.

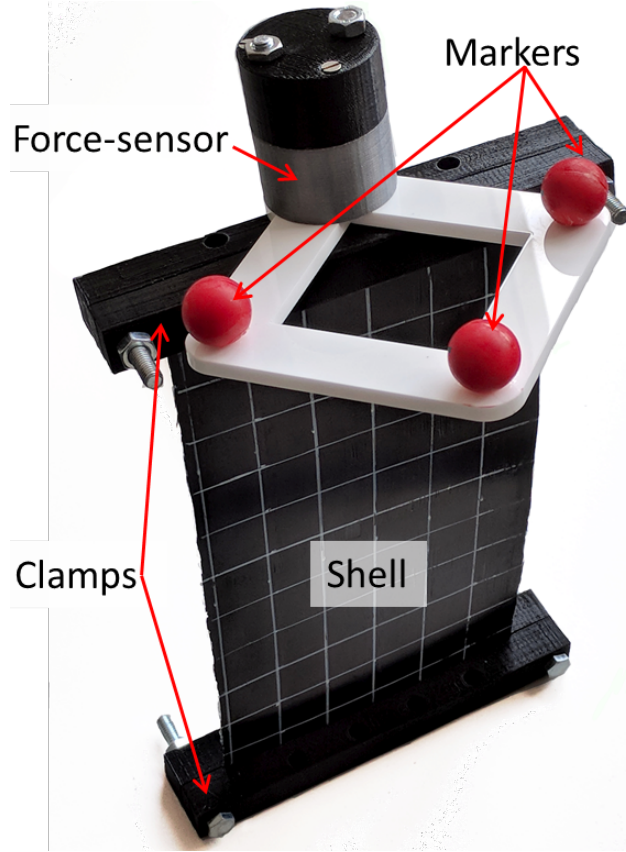
5.2. Results

The results of the finite element model and the measurements are compared with each other in figure 10. The finite element model is represented by the blue line and the measurements by red dots. The figures show the behaviour in bending and torsion of both the cylindrical shell and the saddle-shaped shell.

6. Discussion

This paper shows a new way of creating neutrally stable shells. The main differences with earlier techniques that have been used are the anisotropy of the material the way in which it is prestressed and that shapes with initial curvatures are used.

Figure 9: Prototype of shell with equipment necessary for measurements



Anisotropy makes composite shells fundamentally different from the isotropic shells from Guest et al. The shear modulus of a unidirectional layer of composite material is dependent on the direction because the fibres can slide relatively easily alongside each other. The shear modulus needs to be independent of the direction to obtain the neutral stability over a range of motion which means that the material needs to be isotropic in bending. There are a number of layups with this behaviour but these layups have a high amount of internal symmetry and generate therefore no bending prestress and are therefore not suited for in this case. It is not certain whether or not it is possible to generate a layup that is both isotropic in bending and generates the right stress distribution for neutral stability.

It is also possible to use isotropic layers which can be made by using chopped fibres. The required stress distribution with isotropic layers can be obtained by using layers with a different stiffness or thermal expansion coefficient which can be obtained by varying the density or stiffness of the fibres or by curing the shell with a temperature gradient over its thickness. This causes

one side of the composite to shrink more than the other side which leads to the stress distribution

$$M_0 = \begin{bmatrix} m \\ m \\ 0 \end{bmatrix} \quad (29)$$

which in combination with a flat plate leads to the same neutrally stable behaviour as the steel shell made by Guest et al.

The FEM shows the importance of geometry, the boundary conditions and load case for the behaviour of the shell (fig. 3).

Simulation with the finite element model showed that it was not possible to generate significant changes in stiffness by varying the layup and the level of the prestress for every possible geometry. This is caused by the fact that geometries with a large amount of Gaussian curvature are too stiff for the prestress to have any significant effect. It would be interesting for further research to investigate which geometries are suitable for prestressing and which are not.

The effects of the boundary conditions and load case are dominant over the effect of anisotropy in the FEM. It is possible to obtain zero stiffness for a single point but it was not possible to obtain neutral stability over a larger range of motion (fig. 3). Further investigation is needed to determine whether there is a combination of geometry, layup, boundary conditions and load case that leads to neutral stability.

There is a significant difference between the solution of the finite element model and the measurements of cylindrical shell in bending: The model predicts that the shape will stay cylindrical, while the experiment shows that the shell bulges when the principal curvature approaches zero. This bulging first reduces the stiffness, it is a lower energy configuration that is not found by the FEM, but blocks at a certain point any further motion. Refining the model and increasing the number of load steps does not change the solution.

The differences between the model and the measurements can be explained by looking at the production process. The layers are laid down by hand which limits the accuracy with which the layers can be positioned. The vacuum bagging method used is asymmetric and the amount of epoxy on one side of the shell is therefore not perfectly symmetric. The temperature of the heating blanket that was used had no local temperature control and the homogeneity could only be controlled by adding insulation around the blanket which led to variation of $\pm 3^\circ\text{C}$. One of the challenges that occurred during the manufacturing of the shells is the loss of prestress over time. The matrix rearranges itself over time ($\sim 5-10$ days) to remove the thermal stresses that

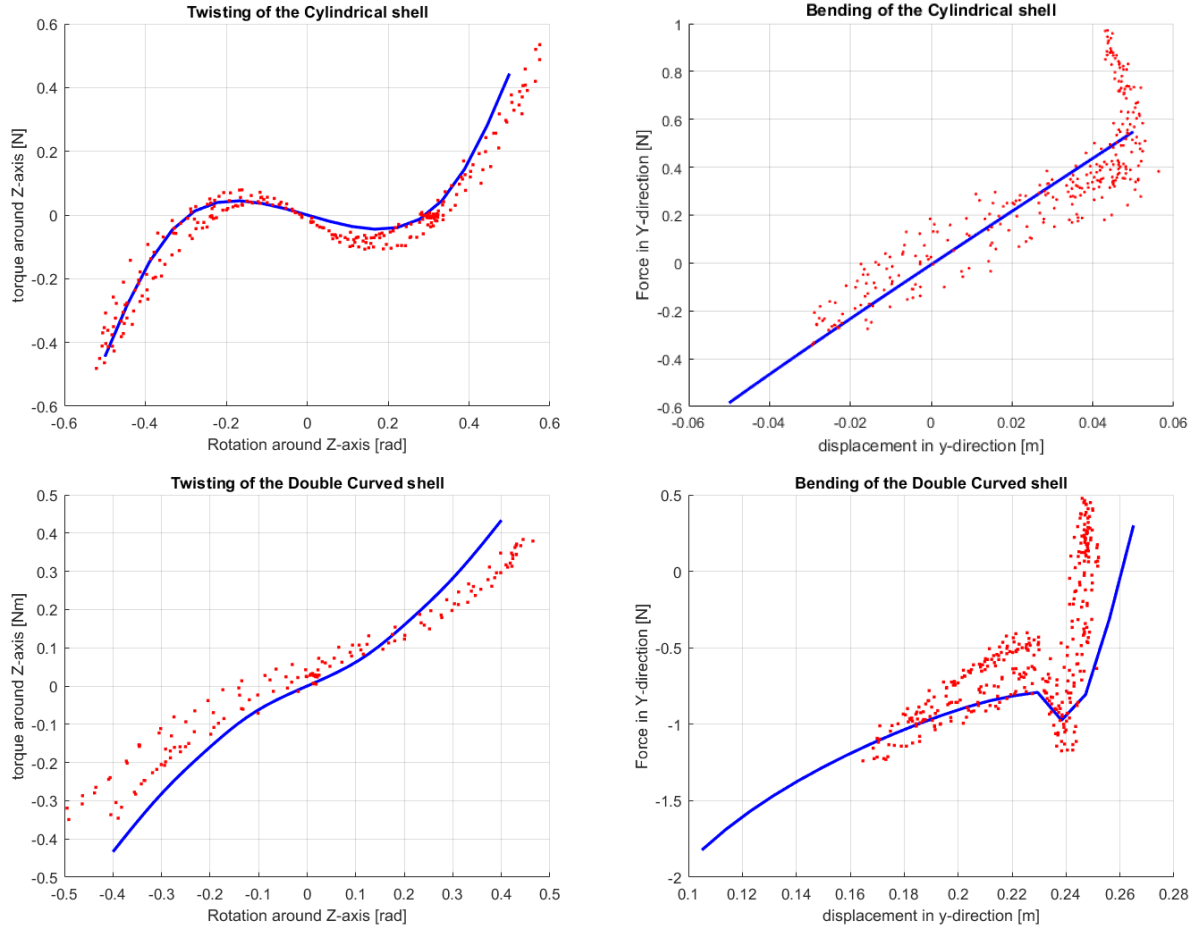


Figure 10: Comparison between FEM (blue line) and measurements (red dots) of the twisting and bending of both shells

are present in the material, further research should look into materials that do not show this behaviour at room temperature.

7. Conclusion

The analytical model of Guest et al. has been expanded to include the case where the material is anisotropic and prestressed using thermal prestress. It is possible to generate both shells with zero and negative stiffness using this technique. Shells made of composite materials show different behaviour as the steel shells made by Guest et al. [3] because the anisotropy of the material which causes the shear modulus to be dependent on the direction within the plane of the shell.

It was found that the material has to be at least isotropic in bending to obtain neutral stability over a range of motion. There are layups that show this behaviour but the required amount of internal symmetry

appears to prevent generating the required bending prestress.

It is shown that the stiffness of composites can be changed by the usage of thermal prestress. It is possible to tailor the stiffness of composites to generate shells that have very little stiffness or have a negative stiffness. The shapes of the shells do not necessarily need to be cylindrical but other shapes are possible as well as long as the Gaussian curvature remain small enough.

The FEM and the experiments show the importance of the boundary condition and the load case to the stiffness of the shell. No combination of geometry, layup, boundary conditions and load has been found that displays neutral stability over a large range of motion, further investigation could show whether or not such a combination exists.

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III

Paper: Representing the elasto-kinematic
behaviour of compliant mechanisms using
fields

Representing the elasto-kinematic behaviour of compliant mechanisms using fields

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Abstract

Forces and displacements are coupled in compliant mechanisms because of the finite stiffness. The stiffness is dependent on the shape of the structure which changes as it deforms which makes the relation between forces and displacements nonlinear for large deformations.

Fields show the behaviour of one chosen point in the structure as it is moved through space. The elasto-kinematic behaviour of this point can be described on the basis of three different but related physical quantities: energy, force and stiffness, each of which has its own field.

Energy fields show the amount of energy that is needed to move the chosen point from one position to another. Force fields show either how much force that it takes to keep the structure in a certain configuration or what the generated reaction force in that configuration is. Stiffness fields show configurations in which the structure becomes unstable and can for instance be used to predict when the structure will buckle.

1. Introduction

Most conventional mechanisms consist of rigid bodies joined by hinges and sliders. These joints are imperfect because they cause friction, play and they wear down over the course of time. Compliant mechanisms solve this problem by removing all joints and achieving their motion through the deformation of the structure itself instead.

Using deformation brings its own challenges, because deformation links forces, torques, translations and rotations together. It is therefore no longer possible to examine them separately. This is made even more difficult by the fact that the structure does not necessarily deform in the same direction as the applied forces. The stiffness of the structure changes in the case of large

deformations which means that the stiffness needs to be shown at all points within the working area.

The deformation of compliant mechanisms is different for every point within the structure. This generates a large amount of information which needs to be represented. Most of this information is not of interest for the designer and the elasto-kinematic behaviour of the entire structure is therefore often reduced to the elasto-kinematic behaviour of a single point where the structure is actuated the response of which is measured.

Even simple shapes show complex behaviour. This makes designing compliant mechanisms more difficult, but it also provides new opportunities. Simple shapes could be used as solutions for complex problems, as long as we understand these shapes and know how to tailor their behaviour. The coupling between rotations and translations is, for instance, interesting for making structures that can rotate around a virtual point that is outside the structure itself [3, 5] and there is also increasing interest in neutrally stable and bistable structures [2] that can only be described using nonlinear stiffness.

For many years, it has been possible to generate structures that show this behaviour but the tools required to efficiently generate designs for practical applications using this behaviour are lacking. The major challenge attached to creating such tools is the difficulty of creating a visual representation of the elasto-kinematic behaviour of these structures. Good representation methods are necessary to quickly compare and iterate different designs. The elasto-kinematic behaviour can be described either by the stiffness, the reaction force or with the energy of the structure in different configurations. Which of the three physical quantities is most important to the designer depends on the problem that needs to be solved.

The current representation methods make use of stiffness and compliance ellipsoids [6] [Spatial Concept Synthesis of Compliant Mechanisms utilizing Non Linear Eigentwist Characterization, not yet published], en-

ergy fields [8], and force fields [8].

Ellipsoids only show the linearised behaviour of the chosen point, but they can be used to represent either the rotational or the translational stiffness.

Fields are currently the only way to represent the nonlinear elasto-kinematic behaviour in multiple dimensions. Every point in the energy fields corresponds to the total potential energy of the system when the chosen point is moved to that position [8], while force fields show instead the force for each point.

The aim of this paper is to compare existing fields and describe new fields that could be used to represent the nonlinear elasto-kinematic behaviour of compliant mechanisms. Two example structures are used to compare the different fields. The first example is monostable and the second example is bistable. The examples and the techniques used to generate the fields are described in the methods and the fields themselves are shown in the results.

2. Methods

In this paper two different examples are used to show the different types of fields. The first example is based on a single beam and is monostable. The second example combines two instances of the beam given in the first example that are prestressed to generate a bistable structure.

The potential energy, the reaction force and the stiffness of the structure change when the structure deforms. A field represents one of these quantities when the chosen point is moved through field.

A field is either a plane or a volume through which the chosen point is moved. The field is made by calculating the aforementioned quantities for a grid of points. The strain energy, reaction force and stiffness for each point on the grid can be calculated by constraining the chosen point to that point on the grid.

Certain points on the grid have multiple configurations, buckling modes, that are in equilibrium when the chosen point is constrained to that position in space.

The field can only represent one buckling mode at a single point within the field and the choice of the mode that is represented is therefore important. The mode with the lowest potential energy is chosen in this paper but it is also possible to chose a specific mode.

The beam used in both examples is clamped at its base and is free to rotate at its tip (Fig. 1). The tip is also the chosen point for which the fields are calculated. The technical data of the beam and the grid can be found in table 1. A course grid has been used for the representation methods that use arrows or glyphs because they become to illegibly small if the grid is to fine. All the other

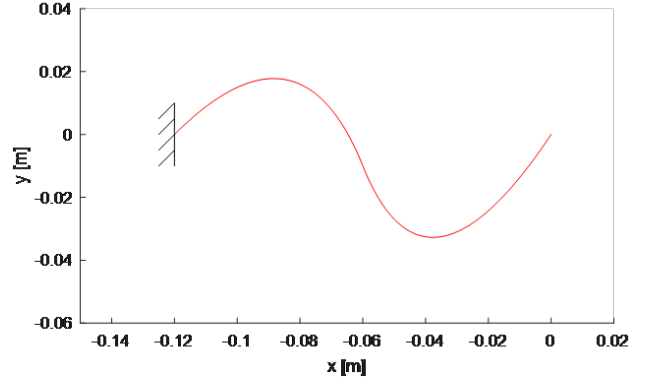


Figure 1: The beam which is used to generate the energy, force and stiffness fields for the first example.

Table 1: Technical data of the beam and the grid

Crossection beam	
Shape	rectangular
Height	2 mm
Width	20 mm
Material beam	
Young's modulus	70 GPa
Poisson's ratio	0.3
Grid	
X-range	[-80 50] mm
Y-range	[-100 100] mm
Resolution fine	1 mm
Resolution course	10 mm

methods use a fine grid to represent the elasto-kinematic behaviour of the structure as accurate as possible.

The stiffness of the beam increases exponentially as soon as the beam is fully straightened and all the points with an energy level above 12 J or a maximum stiffness above the 35 KN/m² were therefore removed.

The second example is used to show how the representation of a multistable field would look. The example consists of two beams which are connected to each other by a pin joint (Fig. 2). The structure is made bistable by prestressing it. This is done by moving the bases of the two beams 10 cm towards each other. The pin joint connects the translational degrees of freedom of both beams to each other without connecting the rotational degrees of freedom. This is important because the fields do not incorporate the rotational stiffness of the tip which is necessary to connect the rotational degrees of freedom. The fields of the combined system can be obtained by summing the fields of the parts.

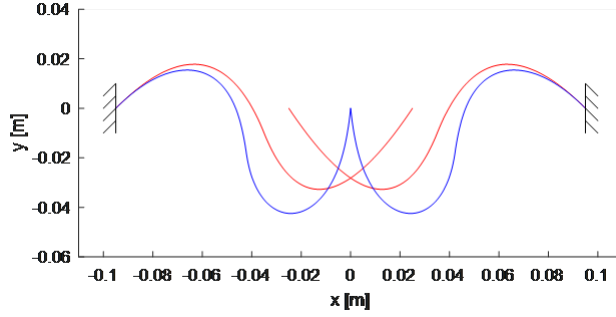


Figure 2: Double beams used to generate the energy, force and stiffness fields for the second example. The red beams show the shape of the beams when they are not connected; the blue beams show the shape of the beams when they are connected via a pin joint.

3. Results

3.1. The potential energy field

Potential energy fields are defined as a collection of points in space for which the energy value is known. The value corresponds to the total potential energy in the system when a chosen point is displaced to that point in space [8]. The energy field can only be generated when all the forces are conservative and the energy it takes to move the chosen point from one position to another is equal to the energy difference between those positions.

The energy at each point can be shown by colour or by isopotential lines (lines of constant energy) (Fig. 3). Force is the spatial derivative of the energy and it is therefore perpendicular to the isopotential lines and proportional to the distance between the isopotential lines.

3.2. The force field

The force field is a collection of points for which the force is known. The force can either be defined as the force necessary to keep the chosen point stationary or as the reaction force of the structure when the chosen point is kept stationary. The only difference between both definitions is that the forces are opposite in direction to each other. The fields in this paper show the reaction force of the structure.

A force is a vector which can be represented by arrows or streamlines.

3.2.1. Arrows. Vector fields are usually represented by plotting arrows for a grid of selected points (Fig. 4). Each arrow shows the magnitude and direction of the vector at the base of the arrow.

The size of the largest arrow is determined by the spacing of the grid. It becomes difficult to determine what the directions of the small arrows are if there is a large variation in the magnitude of the arrows.

The arrows only show what the force at their base is and not for every point within the field. It will therefore be difficult to use interpolation to find the force for the intermediate points if the grid is coarse and the arrows will become small and difficult to read if the grid is too fine.

3.2.2. Streamlines. Streamlines (Fig. 5) are lines which are instantaneously tangent to the vector field and follow the vector field for every point they pass through. Streamlines do not inherently show the magnitude of the force but this information can be added to the image by colouring and varying the thickness of the lines (Fig. 5 a, c) or by the adding isoforce (lines with a constant magnitude of force) in the background (Fig. 5 b, d).

3.3. The stiffness field

Stiffness gives the change of the reaction force stiffness is a tensor because it maps one vector, the displacement, to another, the force.

The stiffness field is a collection of points for which the stiffness is known. At every point within the stiffness field, the tensor corresponds to the tangent stiffness of the chosen point when the chosen point is moved to that point within the field.

The translational stiffness in two dimensions is a two-by-two tensor. This tensor can be decomposed into two principal stiffness components, which are perpendicular each with their own magnitude. Tensor fields can be represented by glyphs [9] or hyper streamlines [1].

3.3.1. Glyphs. Glyphs are shapes that can be used to represent tensors [9]. They can be used in the same way as arrows to represent a vector field. The glyphs show the stiffness for a grid of points. The tensors will also become very small and difficult to read if the grid is fine and the field becomes difficult to interpolate when the grid is coarse and the variations in the field are large. Using fixed spacing leads to a large space between the glyphs when the stiffness becomes small. There are algorithms that use variable spacing of the glyphs to remove this white space [7] but these algorithms have not been implemented in this paper.

There are different glyphs that can be used to describe the stiffness field. The first choice that needs to be made is whether the compliance or the stiffness is used as the basis for the glyph. The compliance will go to infinity at a transition between positive and negative stiff-

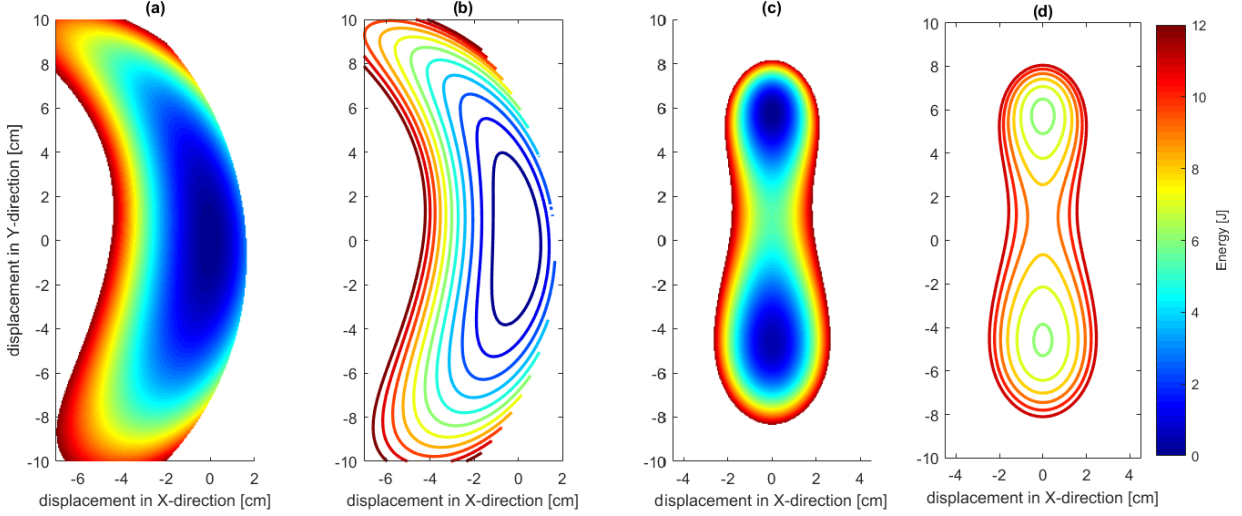


Figure 3: Energy fields of the single beam (a, b) and the double beam (c, d) visualized using colour (a, c) and isopotential lines (b, d).

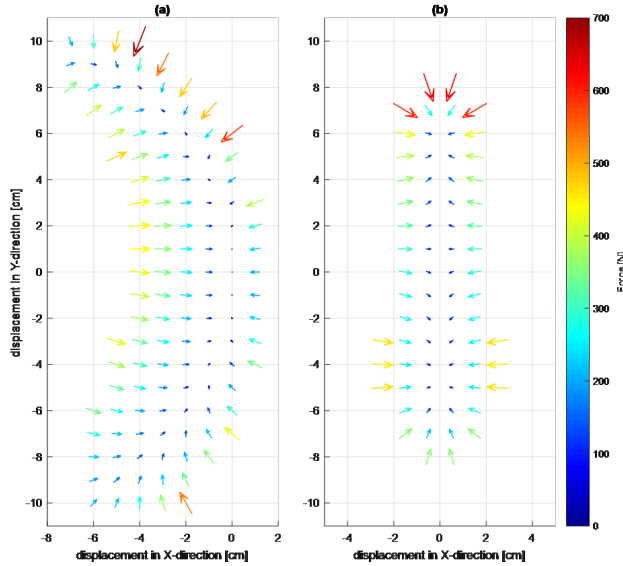


Figure 4: Force fields of the single beam (a) and the double beam (b) represented by the means of arrows. The colour and length of the arrows depends on the magnitude of the force.

ness and the radius of the glyph will go to infinity at this point if the size of the glyph is based on the compliance. The glyph therefore needs to be based on the stiffness.

The second choice is the shape of the glyph. This paper shows the fields for both the stiffness ellipse [4] and the stiffness Reynolds glyph.

The radius along the main axes of both the stiffness ellipse and the stiffness Reynolds glyph is equal to the absolute value of the principal stiffness. The sign of the stiffness is indicated by the colour: blue is positive and red is negative.

The edge of a stiffness ellipse is an isoforce line when the system is in equilibrium at the centre of the glyph. And the edge of the Reynolds glyph is equal to the component of the force in the same direction as the displacement when the system is in equilibrium at the centre of the glyph. The system is not in equilibrium for most points within the field and there is therefore no immediate physical interpretation of the shape for the intermediate directions.

3.3.2. Hyper streamlines. Hyper streamlines are an extension of normal streamlines that make it possible to represent tensor fields. Hyper streamlines are lines which are instantaneously tangent to the principal directions of the stiffness field. The two principal components of the stiffness tensor are always perpendicular and the hyper streamlines intersect with each other at right angles. Hyper streamlines do not inherently show the magnitude of the stiffness but this information can be added to the image by colouring and by varying the thickness of the lines (Fig. 7).

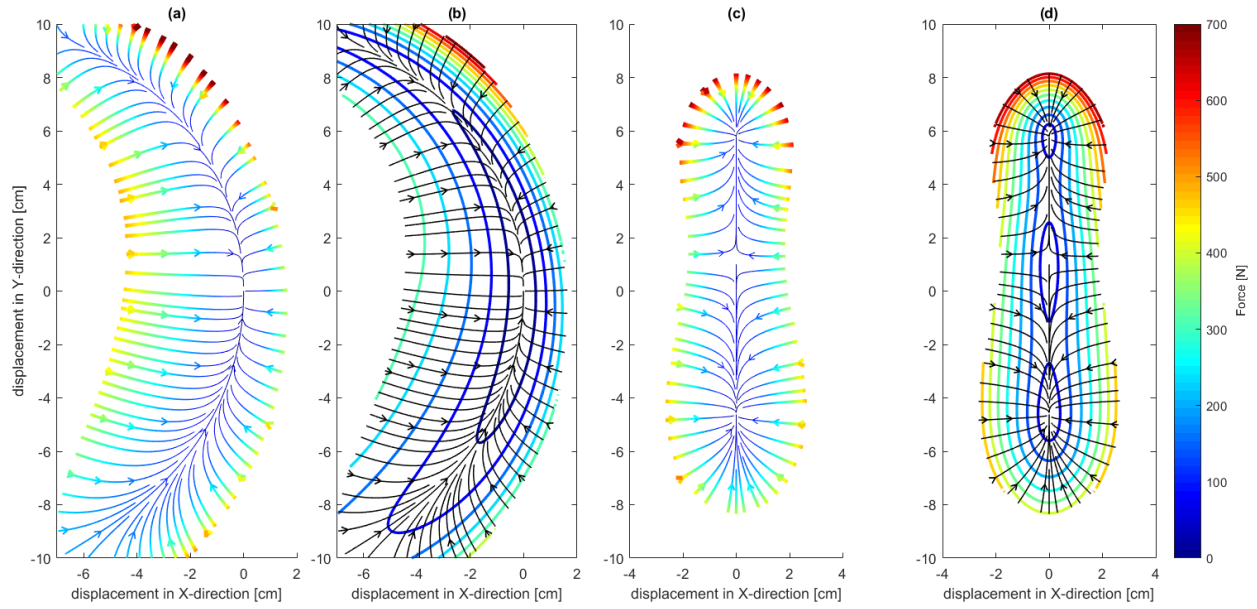


Figure 5: Force fields of the single beam (a, b) and double beam (c, d) visualized using streamlines. The magnitude of the force is shown by the colour and line thickness (a, c) or by the isoforce lines (b, d)

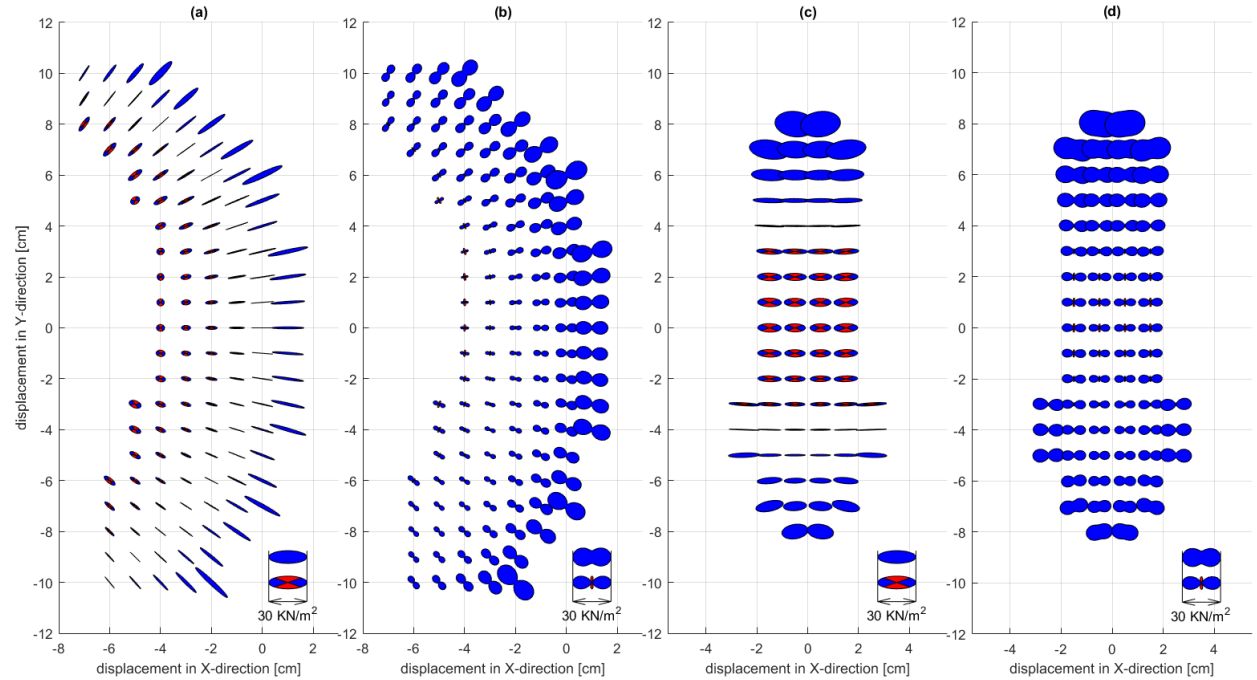


Figure 6: Stiffness field of single beam (a, b) and double beam (c, d) is represented using compliance ellipses (a, c), compliance Reynolds glyphs (b, d). The colour shows the sign of the stiffness: blue is positive, red is negative.

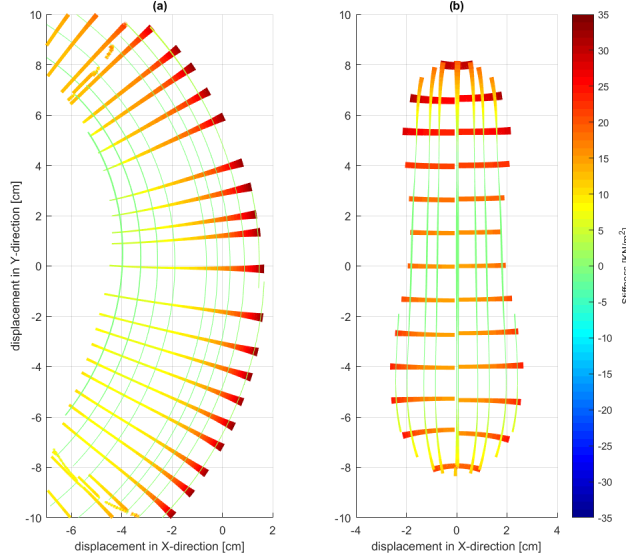


Figure 7: Stiffness fields of single beam (a) and double beam (b) represented using hyper streamlines

4. Discussion

It is not usual within the field of compliant mechanisms to represent the elasto-kinematic behaviour using fields because fields can only represent the behaviour of a single point of the structure. They are, however, the only representation method that is currently able to represent the nonlinear elasto-kinematic behaviour in multiple dimensions. Fields are therefore most suited for designing mechanisms for which the nonlinear behaviour is important as is the case with neutrally stable and multistable compliant mechanisms.

Neutrally stable mechanisms decouple force and motion because they can deform without changing the force. This is only the case if the stiffness in the direction of the motion is zero. Fields can be used to show the range of motion for which this is the case.

Multistable mechanisms have multiple stable equilibrium points which are separated by unstable regions. The stiffness in the unstable region is negative and a small change in force will therefore cause the structure to snap to a stable region.

The snapping through of the structure to another configuration often causes the structure to have multiple stable configurations for a single point within the field. Which configuration the structure has depends on the path of the chosen point. It is no longer possible to represent all configurations in a single field and it is therefore important to know which configuration the structure has.

Monostable structures can also buckle as long as

the applied force is large enough to move the chosen point into a region where at least one of the principal stiffness components becomes negative. The stiffness field therefore shows how much a structure can deform before it buckles. The force needed to cause the structure to buckle can be found by combining the force field with the stiffness field.

Fields can be used to predict the behaviour of a combination of different parts using superposition. The current limitation is that fields do not incorporate the rotational degrees of freedom. The different parts therefore cannot be connected using a rigid connection which is the most common way to connect different parts of compliant mechanisms. It is possible to work around this problem by constraining all the rotations at the connection or by connecting the parts with pin joints which separate the rotations of the different parts.

5. Conclusion

This paper describes the different fields that can be used to describe the nonlinear elasto-kinematic behaviour of compliant mechanisms. These fields can be used to represent three different physical quantities: energy, force and stiffness. It is shown that each quantity can be visualized in multiple ways. Which method is most useful depends on the case. They are most useful in cases where there are large deformations, e.g. compliant mechanisms, and cases where the non-linearities in the stiffness are important e.g. buckling.

The disadvantage of fields is that they only show the behaviour of a single point of the structure. They are, however, at this moment the only way to create a visual representation of the nonlinear elasto-kinematic behaviour in multiple dimensions. Fields are most useful in cases where the mechanism has multiple degrees of freedom and the nonlinearity of the stiffness is important. This is, for instance, the case for neutrally stable and multistable structures.

Fields can also be used to predict the behaviour of a system that consists of a combination of parts as long as the rotational degrees of freedom of the different parts are decoupled.

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Appendices

A. Ellipsoids

A.1. Introduction

This part of the appendix contains some work I have done to represent the stiffness of a single point for cases where the stiffness in one of the directions is negative. This was relevant because I had to deal with negative stiffness the design of the FEM of the shells I used in the experiment and I could not find any method to represent the results. Joep Nijssen has used compliance ellipsoids in his work to represent the behaviour of the shells he used and this is an expansion that includes the possibility to represent mechanisms with negative stiffness.

There are two reasons why I stopped with this approach. The first reason is that none of the different glyphs (shapes) I found gave much useful information outside the principal components of the stiffness, which were given by each shape. The displacement and the force can have a completely different direction once one of the principal stiffness components is negative and none of the shapes show this accurately. The second reason is that the non-linear aspect of the the stiffness becomes very important when you get a structure with a stiffness that is close to zero or even negative. I therefore focused on representing the non-linear behaviour which resulted in the paper about fields.

A.2. Method

The stiffness of a structure is different for every point within the structure and changes as the structure deforms. This leads to a large amount information which is often not of interest of the user. The system is often simplified by only looking at a single point of interest and linearized around a working point. This behaviour is described by the six-by-six stiffness tensor in three dimensions and by the three-by-three stiffness tensor in two dimensions. The tensor is symmetric and consist of three subtensors: A , B and D

$$K = \begin{bmatrix} A & B \\ B^T & D \end{bmatrix}. \quad (1)$$

- The A tensor show the relation between forces and translations: the translational stiffness.
- The D tensor shows the relation between torques and rotations: the rotational stiffness.
- The B tensor shows the relation between forces and rotation and the relation between torques and translations: The coupling stiffness. The compliance ellipsoid is the inverse of the stiffness tensor and the sub tensors are denoted with small letters

$$C = K^{-1} = \begin{bmatrix} a & b \\ b^T & d \end{bmatrix}. \quad (2)$$

- The a tensor show the relation between forces and translations: the translational compliance.
- The d tensor shows the relation between torques and rotations: the rotational compliance.
- The b tensor shows the relation between forces and rotation and the relation between torques and translations: The coupling compliance.

The translational stiffness A and the translational compliance a are three-by-three tensors in the three dimensional case and two-by-two tensors in the two dimensional case. $a \neq A^{-1}$ if the coupling stiffness B is unequal to zero. The translational stiffness is coupled to the rotational stiffness via B and it is therefore necessary to know the rotational constraints if you want to visualize the translational behaviour of a system. The stiffness shows the change in force for an applied displacement:

$$\Delta F = K \Delta U \quad (3)$$

The compliance shows the change in displacement for an applied force:

$$\Delta U = C \Delta F \quad (4)$$

These relations only apply if the system is in equilibrium. Applying a force in a direction where the stiffness is negative leads to an infinite amount of displacement. Compliance is in this case better described as the amount of displacement necessary to generate a unit reaction force.

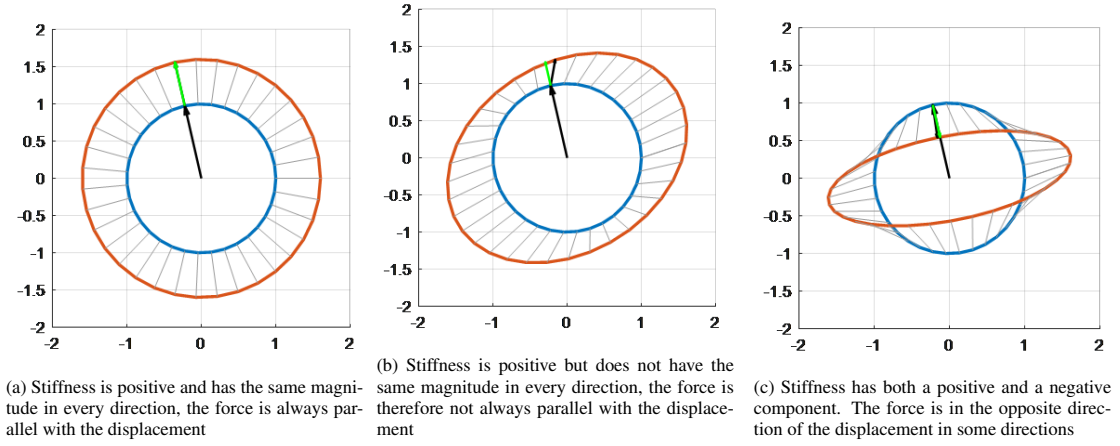


Figure 1: The applied displacement is represented by a circle with radius of unit magnitude. The corresponding force is represented by a grey line that starts on the unit circle and ends on an ellipse. The radius of the ellipse itself does not contain any information.

A.3. Results

Stiffness is a tensor that maps displacements to forces. Figure 1 shows this relation for the two dimensional case. The figures are generated by plotting a circle with unit radius that represents the displacement. The force that corresponds to the displacement in each direction is drawn starting on the edge of the unit circle. Applying a unit displacement in the direction will generally correspond to a force in a different direction (fig. 1).

The displacement and force are parallel in the principal directions where the stiffness is positive and antiparallel in the principal directions where the stiffness is negative. The stiffness tensor is underdetermined if two or more principal stiffnesses are equal to each other. The forces are either parallel or antiparallel depending on the sign of the stiffness in the subspace described by these principal stiffness components (fig. 1).

Ellipses

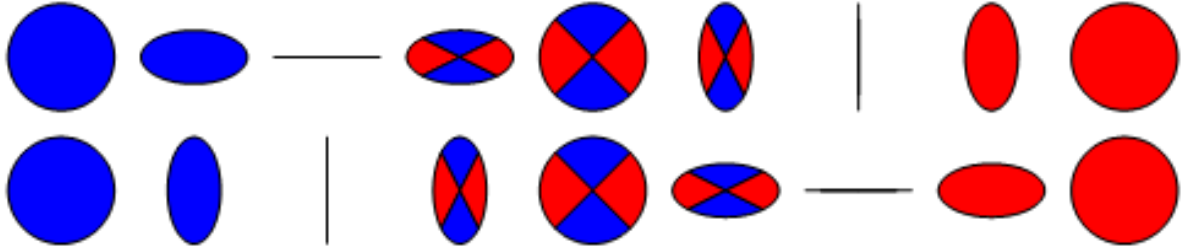


Figure 2: Stiffness ellipses (top) and compliance ellipses (bottom). Colour shows the sign of the stiffness: blue is positive, red is negative.

Compliance and stiffness ellipsoids were first used by Kim et al. who used them to describe and combine compliant building blocks. A stiffness ellipse is generated by mapping the unit displacement circle to an ellipse using the tangent stiffness matrix of the point of interest (fig. 1).

An ellipse can be made by reducing the radius of the unit circle to zero. The directional information of the actuation is lost and the radius is equal to the force that corresponds to a displacement of which the magnitude is unknown. The reaction force of the system is equal in magnitude and in opposite direction of the actuation force if the system when in equilibrium; applying a force in a direction where the stiffness is negative will never lead to a new equilibrium if the system is linearized. This means that actuation and reaction force cannot be used interchangeably when the stiffness is negative. The stiffness gives always the relation between the change of the reaction force for a displacement but not necessarily the displacement caused by a certain actuation force. Stiffness tells what the change in reaction force is for a certain displacement. The initial reaction force is zero if the system is linearized in its equilibrium position. The reaction force is proportional to the stiffness in that case and a visualization of the stiffness is automatically also a visualization of the reaction force of the system for a unit displacement and the visualization of compliance the displacement for a certain unit force. A common mistake is to assume that the displacement and the reaction force are in the same direction. This is only the

case in the principal directions of the stiffness tensor or if the stiffness is the same in all directions because the stiffness tensor is underdetermined in that case.

The radius of the ellipse is equal to the absolute value of the stiffness in that direction which is the magnitude of the reaction force caused by a displacement of unit length.

$$r(\sigma) = |K\sigma| \quad (5)$$

The inverse of the stiffness ellipse is the compliance ellipse and it is made the same way as the stiffness ellipse but it maps the unit force circle to an ellipse.

$$r(\sigma) = |K^{-1}\sigma| \quad (6)$$

It shows the absolute value of the compliance in all directions which is the amount of the displacement needed to generate a reaction force of unit force.

Only the magnitude of the stiffness or compliance is shown which means that the directional information is lost. An inexperienced user might think that the force is always in the same direction as the displacement while in reality they might be perpendicular. The force and displacement are only parallel, if the stiffness is positive, or antiparallel, if the stiffness is negative, in the principal directions of the stiffness. The force and displacement will not be parallel in all other directions (fig. 3) unless the stiffness tensor is underdetermined.

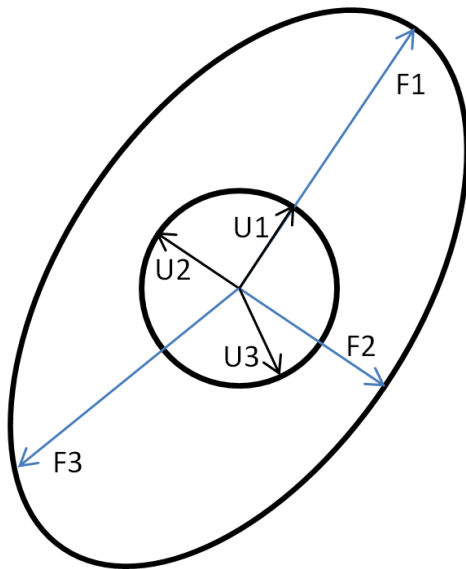


Figure 3: the stiffness ellipse shows the projection of the unit displacement circle onto a ellipse using the tangent stiffness matrix. The reaction force is either parallel, if the stiffness is positive (U_1, F_1), or antiparallel, if the stiffness is negative (U_2, F_2), to the displacement for the principal directions of the stiffness, but will not be parallel for all other directions (U_3, F_3)

It is impossible to distinguish between definite and indefinite tensors because the sign of the eigenvalue is lost when the absolute value is taken. The sign of the stiffness can be shown by adding colour. (fig. 2)

Reynolds glyphs

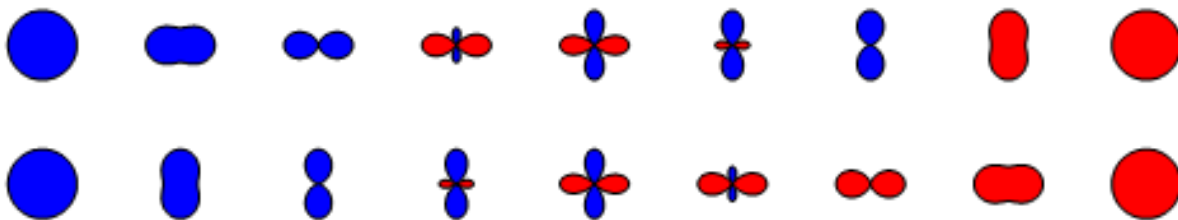


Figure 4: Reynolds glyphs based on stiffness (top) and compliance (bottom). Colour is based on sign of the stiffness: blue is positive, red is negative.

Reynolds glyphs show the component of the force and displacement that are parallel with each other. This is of interest because this is the component that determines the amount of energy that is needed for a certain deformation. The shape of definite and indefinite stiffness tensors is very different (fig. 4) but colour is still needed to show which directions have a positive stiffness and which directions have a negative stiffness. There are two different Reynolds glyphs. The radius of the stiffness Reynolds glyph is equal to the component of the force parallel to a unit displacement in that direction and shows the principal stiffnesses in the principal directions.

$$r(\theta) = \theta^T K \theta \quad (7)$$

The radius of the compliance Reynolds glyph shows the component of the displacement parallel to a unit force in that direction and shows the principal compliances in the principal directions.

$$r(\theta) = \theta^T K^{-1} \theta \quad (8)$$

The difference between Reynold glyphs and ellipses becomes most apparent when the stiffness matrix becomes indefinite (one positive and one negative principal stiffness). The amplitude for the Reynolds glyph will go to zero for certain directions which means that the force and displacement are perpendicular to each other in that direction. This does not mean that the stiffness in that direction is zero, the component of the stiffness that is perpendicular to that direction still exist.

Quadrics

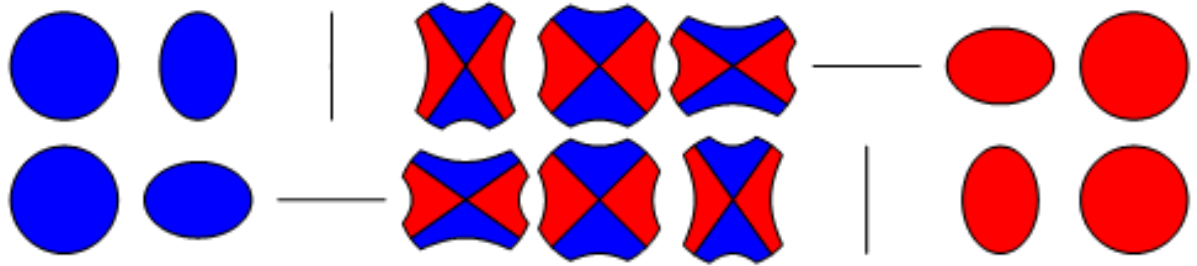


Figure 5: Quadrics based on displacement (top) or force (bottom). Colour is based on sign of the stiffness: blue is positive, red is negative.

Quadrics show the amount of displacement or force that corresponds to a certain energy level. Quadrics only show the energy level if the system is in equilibrium when it is linearized. The radius of the displacement driven quadric is equal to the amount of displacement that corresponds to a certain energy level.

$$r(\theta) = (\theta^T K \theta)^{-1/2} \quad (9)$$

The radius of the force driven quadric is equal to the amount of reaction force that that corresponds to a certain energy level.

$$r(\theta) = (\theta^T K^{-1} \theta)^{-1/2} \quad (10)$$

The structure has the same amount of elastic energy for all points on the edge of the glyph if the stiffness tensor is positive definite, these edges are therefore called isopotential lines. The reaction, force for the displacement driven quadric and displacement for the force driven quadric are perpendicular to the isopotential line and are pointed inwards.

The radius of the quadric becomes negative when the stiffness in that direction becomes negative, which means that energy would be released if you would move in that direction and not that you can reach the same energy level by moving in the opposite direction. The edge of the glyph shows the amount of actuation, force or displacement, that is needed to release a unit of energy, the reaction, displacement or force respectively, will still be perpendicular but will be pointed outwards instead of inwards.

Quadrics have a very different shape for indefinite stiffness tensors compared to the shape they have for definite stiffness tensors. Indefinite tensors have a direction in which it costs no energy to deform because the force and the displacement are perpendicular to each other in that direction. The radius becomes infinitely large in these directions which makes quadrics open shapes for indefinite tensors. The glyphs need to be capped at a maximum radius to make drawing the glyph possible. The reaction is not perpendicular to these capped edges. Colour can be used to show in which directions energy is absorbed and in which directions energy is released.

A.4. Conclusion

Stiffness and compliance give the relationship between displacement and force. It is important to realize that these two vector quantities are not necessary in the same direction. Ellipsoids are here for misleading; their shape suggest that it gives the stiffness every direction while it only gives the stiffness in the principal directions because that are the only directions where the force and displacement are in the same direction.

Reynolds glyphs and quadrics also give useful information in the other directions but they do not give a complete view of the stiffness behaviour of a point. A Reynold glyph show the parallel component of the force and displacement while quadrics give the energy as a function of the force or displacement. An experienced engineer can find the same information in each shape. The understanding of how stiffness works, especially if the stiffness in certain directions becomes negative, is much more important than the shape that is chosen.

B. Production methods

B.1. Introduction

I used a number of techniques to produce the shells I needed for the measurements. These are some of the experiences I had during the production process and might be of use to anyone who wants to repeat the process. This list is by no means exhaustive.

B.2. Open mould

The simplest process that can be used to produce a composite is open moulding this uses an open mould and the resin is applied with a roller or brush. The composite is cured at atmospheric pressure and the temperature will increase up to 60°C due to the endothermic nature of the curing process. This method has too much variability in the resin concentration and the number of air bubbles in the resin. This method is not accurate enough obtain reliable results.

Advantages:

- Simple
- No special equipment needed

Disadvantages:

- Difficult to get uniform resin/fibre ratio
- Air bubbles in the resin

B.3. Vacuum infusion

This can be solved by using vacuum infusion. Vacuum infusion uses a vacuum bag to pull the resin through the fibres.

Advantages:

- Unlimited amount of time align the different layers
- Pressure difference pushes out air bubbles

Disadvantages:

- Very limited choice of resins, need to have a low viscosity
- Even the smallest leak will cause air bubbles to form in the resin

This was the first technique I tried. It proved difficult to produce a shell without any air bubbles. It is important to limit the rate at which resin is pulled in to make sure that all the air can escape. You need to be absolutely certain that there are no leaks before you start pumping in the resin. It is not possible to remove any air bubbles once the mould is filled. Even a very small leak that does not have any effect on the level of vacuum that the pump can pull will lead to air bubbles in the final result. The best way to check is to shut of the pump and to look at the rate at which the vacuum disappears.

Vacuum infusion leads to a very uniform resin distribution and the temperature does not increase during the curing. This is probably caused by the thickness of the composite (0.7 mm) and the slow rate at which the curing takes place (14 hours). Extra insulation did not help in increasing the temperature and an external heat source

is therefore necessary. First I used an electric stove to heat the composite but this did not lead to a uniform distribution; it burned a ring into the composite while the rest stayed more or less at room temperature, adding an 2 cm thick aluminium slab did not help to improve the diffusion.

Electric heating blankets are used for the repair of composite aeroplane parts. The blanket is mounted on the mould using an airtight seal and is also pulled vacuum to improve thermal conduction. The vacuum was lost during one of the attempts which removed the direct contact between the blanket and the thermocouple which lead to the blanket overheating and being permanently damaged. The big disadvantage of these blankets is that it is only possible to control the temperature of the blanket as a whole. This means that the only way I could be certain that the temperature is uniform is to manually measure the temperature at different points using an infra-red thermometer and locally varying the amount of insulation.

The composite did not show any effects of thermal prestress. This is caused by the resin, that even after curing for twice the time that is necessary according to the specifications, stays viscous. Resins that have a low enough viscosity to be used for vacuum infusion and can be cured at a high enough temperature to obtain enough prestress are very rare.

B.4. Prepreg fabrics

Prepregs consist out of layers of fibres which have be pre-impregnated with a special resin. Prepregs need to be cured at an elevated temperature and under high pressure. An autoclave used to be necessary to produce prepregs, but new prepregs can be produced by only using vacuum pressure. I used one of those prepregs in a test and it immediately showed strong effects of prestress. The thermal expansion coefficient was not given in the data sheet but is easy to calculate by measuring the curvature of a initially flat shell after curing.

Advantages:

- Uniformity of the resin/fibre ratio
- Strength and stiffness of the final product

Disadvantages:

- Expensive equipment needed for production
- Prepregs can only be stored for a limited time and needs to be stored in a cooled environment