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INCORPORATING SPATIAL VARIABILITY INTO FEM ANALYSES OF ANCHORED RETAINING WALLS

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Abstracts: In a finite element-based reliability analysis, distributions of geotechnical input parameters from soil properties measured at the site may overlook spatial variability, especially across sparsely investigated areas.In doing so, soil parametersfrom weak zones are often considered as representative values across entire soil layers, potentially leading to overly conservative safety assessments. This issue becomes critical when analysing geotechnical failures in anchors, where their tensile resistance relies heavily on the averaged soil properties along the anchors. To address these challenges, this paperpresents a method that incorporates spatial averaging into a reliability analysis. To demonstrate its application, a quay wall at the Port of Rotterdam is presentedwhere the parameter distributionsand their spatial variability were derived from the Cone Penetration Test (CPT). Comparing the reliability index before and after the inclusion of spatial variability shows that ignoring spatial variability may be overly conservative when assessing shaft capacity of piles or anchors.
Keywords: Parameter uncertainty;Reliability analysis; Spatial variability;Quay walls; Cone penetration testing.

1. Introduction

Finite element (FE)methodscan model geotechnicalstructures with complex soil-structure interactions and non-linear soil behaviour.Nevertheless, this method is a deterministic analysis that can only input a representative geotechnical parameter, ignoring uncertainty in that parameter. To address this problem, FE models are often coupled with a probabilistic assessment such as an FE-based reliability analysis which can quantify the failure probability(Brinkgreve et al, 2017). In doing so, statistical distributions are constructed for the input parameters and then many FE simulations are run, with each simulation taking a new set of input parameters from the distributions. The results are then used to determine the probability of failure or reliability of the structure.

Measurements or correlations to different soil properties are typically used to develop these distributions, often from specific depths across sparsely investigated areas. However, the tails of these distributions are sometimes used as representative values for entire soil layers, which can be overly sensitive to extreme values and ignore spatial variability. This limitation becomes particularly significant when analyzingthe shaft resistance of anchors or piles. Unlike the pile base resistance, where thin weak layers dominate the ultimate response (White and Bolton, 2005), thefailure surface for the shaft resistance of anchors and piles is constrained to a narrow zone along the pile shaft and is generally dictated by the average soil response.

Spatial averaginghas been implemented in various geotechnical applications (Fenton and Griffiths, 2005; Ching et al., 2016), helping to simplify spatially variable soil parametersto a homogeneous spatial averageover a prescribed domain. Using this spatial average can then help with reducing implementation complexity in conventional, homogeneous finite element models. This method is also ideal for developing a distribution of geotechnical parameters depending on the average soil properties.This papershows howto develop a representative input distribution the shaft resistance of piles or anchors by incorporating spatial variability into an FEM-based reliability analysis. To demonstrate the application of this method, a quay wall case study was taken from the Port of Rotterdam, showing the influence of the method on the quay wall's reliability.

2. Methodology

Using the following steps, the input distribution of a geotechnical parameter (referred to as x) can incorporate the spatial variability in the same parameter, even with limit test points:

(i) Determine the mean trend with depth$z$in$x$, whereby:

$$x(\mathrm{z})_{original} \; = (b + az)\varepsilon_z \# \tag{1}$$

where$x(\mathrm{z})_{original}$ arethe soil properties measured at different depths atone testing location and $\varepsilon_z$isthe spatial variability around the trend, assumed to follow a normal distribution.

(ii) Estimate the scale of fluctuation θ by minimizing the error (Eq.4)between the experimental correlation function (Eq.3) and the theoretical correlation function(Eq. 2):

$$\rho(\Delta z) = \exp\left(-\frac{2|\Delta z|}{\theta}\right) \quad \#(2)$$

$$\hat{\rho}(\Delta z) = \frac{1}{\hat{\sigma}_{\varepsilon}{}^{2} n\Delta z}\sum_{i=1}^{n\Delta z}(\varepsilon_{z_i} - \hat{\mu}_{\varepsilon})(\varepsilon_{z_i+\Delta z} - \hat{\mu}_{\varepsilon}) \quad \#(3)$$

$$E(\theta) = \sum_{i}[\rho(\Delta z) - \hat{\rho}(\Delta z)]^2 \quad \#(4)$$

where $\hat{\mu}_{\varepsilon}$ and $\hat{\sigma}_{\varepsilon}$ are the mean and standard deviation of ε_z.

(iii) Using these descriptive statistics, generate numerous random realizations of x by random processes (Prendergast et al., 2018).

(iv) Calculate the averageof each realization andconstructa histogram based on theseaverages to describe the distribution of x_{avg} .

The procedure above canbe further simplified by applying the Central Limit Theorem(Kwak and Kim, 2017) and the Delta method (Ver Hoef, 2012).The basic equation of the Central Limit Theorem is:

$$\sqrt{n}(\bar{X} - \mu) \xrightarrow{d} \mathcal{N}(0, \sigma)\#(5)$$

where $\bar{X}$ is the expected value of $X$from a sample of size n,$\mu$is the true mean of the random variable X and$\sigma$is the standard deviation of X .According to this theorem, x_{avg} invariably converges to a normal random variable,regardless of the underlying distribution of the original x. Thisis becauseeach point that makes up the histogram of x_{avg} isa weighted average of each realization based on step (iv), satisfying the conditions of the Central Limit Theorem.The Delta method can extend Eq.5 from X to $g(X)$, which is shown as below:

$$\sqrt{n}(\overline{g(X)} - \mu_{g(X)}) \xrightarrow{d} \mathcal{N}(0, g^{'}(X)\sigma)\#(6)$$

where $g(\cdot)$ is a differentiable transformation function and$g^{'}(X)$ is the first derivative of $g(\cdot)$. In the proposed methodwhere $n = \sqrt{\frac{L}{\theta}}$ and $g(\varepsilon) \ = (b + a\frac{z}{2})\varepsilon$, the mean, standard deviation and the distribution of x_{avg} canbe easily deduced:

$$x_{avg} \xrightarrow{d} \mathcal{N}\left(\mu_{avg}, \sigma_{avg}\right)\#(7)$$

where,

$$\mu_{avg} \ = b + a\frac{z}{2}$$

$$\sigma_{avg} \ = (b + a\frac{z}{2})\ \hat{\sigma}_{\varepsilon_z}\sqrt{\frac{\theta}{L}}$$

where a and $b$are parameters in the mean trend derived fromStep (i), $z/2$ is the average soil depth,L is the total length of the soil, θ is the scale of fluctuationderived from Step (ii) and$\hat{\sigma}_{\varepsilon_z}$ is an estimate of the standard deviation of spatial variabilityε_z.With this equation, this method canbe easily applied toreal-world engineering scenarios in the industry.

3. Application to a quay wall

3.1.*Background and reliability interface*

To show the model in practice, a deep-sea quay wall from the port of Rotterdamwas taken. Measuring 2.2 km in length, the quay wall was constructed in 2018 and has a total retaining height of 42m. Prior to construction, three hundred CPTs were performed alongthe quay wall. Considering a single section of the quay wall (Fig. 1), the subsoil consists of primarily sand, with the exception of a three-meter-thick stiff clay layer at -22m. Dense to very dense sand underlies this clay layer, with CPT cone resistances q_c of around 35 MPa on average. The retaining wall is tied in by Müller Verpress (MV) anchors: large steel I-beams (0.6 m in height, 0.3 m in width) that were installed at a spacing of 3.8meters by both driving and grout injection through to the dense to very dense sand. The superstructure also includes a concrete slab known as a relieving platform, which uses the weight of the overlying soil to resist overturning moments in the retaining wall. Compression piles underneath the relieving platform provide additional support.

Plaxis 2D was used to build a finite element model of the quay wall. Monitoring during construction also meant that thisFEM model could be validated based on measured wall displacements and anchor loads during construction (Duffy et al., 2024). The reliability interface was then developed in Python by connectingthis model withOpenTURNS. This interface generated values from the input distributionsand transmittedthem to the FEM model for simulation. OpenTURNS then used the outputs of the FEM model to calculate the limit state functions (LSFs).With these calculations, the reliability indices could then be determined usingLatin Hypercube Sampling Monte Carlo (Song and Kawai, 2023).

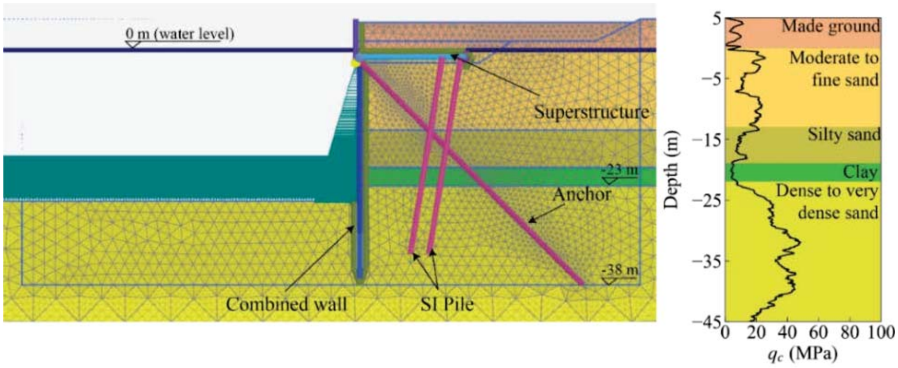

Fig. 1.(a)FE modelsimulation and (b)CPT profile

3.2.*Applying the method to generate thedistribution of the representative cone resistance*q_c

The limit state function for geotechnical tensile failure in the anchors is represented by:

$$LSF_{pile,geo} = \alpha_t O_A L_A q_c - F_{anc\,hor} \#(8)$$

where α_t is a factor that depends on the soil type and installation method (CUR,2008),O_A is the anchor's circumference,L_A is the length of the grout body,q_c is the CPT cone tip resistance and$F_{anc\,hor}$ is the axial force on the anchor from the finite element model.

The anchor's tensile capacity depends on theintegrated effect of the soil strength along its entire length, rather than by thin layers with substantially different strengths.This effect could be captured by the proposed method, where x was replaced by the cone resistance q_c. Using a CPT taken at the cross-section (Fig. 1b), the $q_c$values followed a normal distribution$\mathcal{N}(36, 6.8)$, as shown in Fig. 2c.The mean trend of $q_c$was$0.0314 - 1.1421z$, where ε_z followeda normal distribution $\mathcal{N}(1.0, 0.13)$. The vertical scale of fluctuation θ_y of 2.5m was then calculated from the detrended $q_c$profile (Fig. 2a), according to the minimum error calculated by Eq.4 (the errors of $\theta = 1.0m, 2.5\ m, 4.0m$are4.04, 2.48,2.64 respectively). Using all these descriptive statistics, 10,000 realizations were performed.The average q_c value was calculated from each realization.These 10,000 corresponding average valueswere collated intoone histogramfollowing $\mathcal{N}(34.9,1.9)$,from which a generalised spatial average$q_{c,avg}$ could be obtained. Likewise, when this process was simplified down to Eq. 7, an identical normal distribution of $\mathcal{N}(34.9,1.9)$ was obtained. This result validates Eq. 7.

Overall, this process prevents the tails of the original distribution being inputted into the FEM model as the representative q_c value for the entire layer—something which is not representative of actual site conditions.

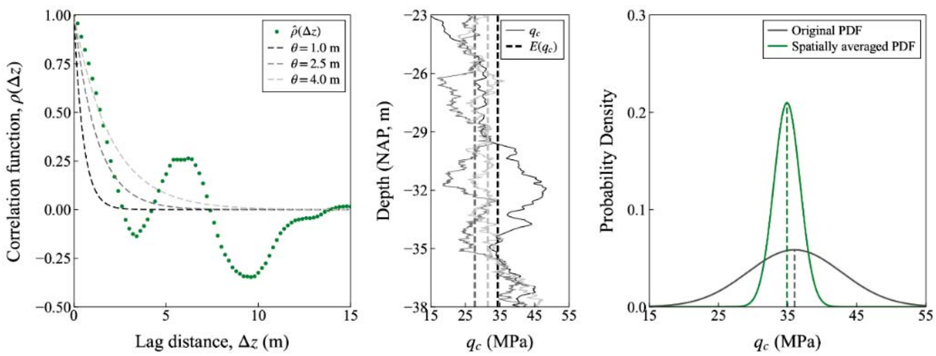

Fig. 2.Calculating$q_{c,avg}$ for the anchor's tensile resistance: (a) Step1: obtain the descriptive statistics of q_c;(b) Step2:generate random realizations of q_c (using n=3 as an example); (c) Step 3: Derive the probability density function (PDF) of$q_{c,avg}$; and comparison with that of the original q_c

3.3.*Reliability indices*

To show the influence of the method on the reliability index for the anchor, three different methods for deriving the input $q_c$distribution were considered: $q_{c,avg}$ determined by Eq. 7, which followeda normal distribution of $\mathcal{N}(34.9,1.9)$; $q_{c,original}$ determined by generating point statistics from the original CPT alongside the anchor, which followeda normal distribution of $\mathcal{N}(36,6.8)$; $q_{c,soil}$ determined bygenerating point statistics from all the

CPTs across the entire deep sand layer, which followeda log-normal distribution of $Lognormal(3.526, 0.347)$.Fig.3 shows the reliability indices calculated by the reliability interface developed in Section 2.1.The three reliability indices varied significantly, showing that the method of derivingq_c had a significant impact on reliability.The target reliability index of quay walls is 3.8, corresponding to a Consequence Class of CC2—according to both Eurocode (CEN, 2002) and quay wall guidelines in the Netherlands (de Gijt and Broeken, 2014).Comparing the three different input types (Fig. 3), only the reliability index derived using $q_{c,avg}$ exceeded the target reliability indexβ_{target} of 3.8, in-line with measurement observations at the quay wall which did not show excessive deformations or inclinations in the quay wall. The other two reliability indices, based on differentrepresentativeq_c,were at least half that of the spatially averaged $q_{c,avg}$ and fell below the target reliability index of3.8,indicatinga potentiallyunsafe situation.

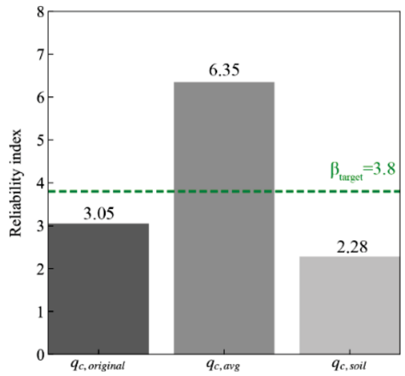

Fig. 3.Reliability indices calculated using three different methods for selecting cone resistance.

4. Conclusion

This study presentsa method to characterizethe shaft capacity of a pile or anchor for finite element-based reliability analyses of anchored retaining walls. By deriving the mean, standard deviation and scale of fluctuations from the soil properties measured at the site, numerous realizationsof this geotechnical parameterwere generated.Input distributions for the FE-based reliability analysis were then obtained by averaging each realization. As an illustrative example, this method was considered for an actual quay wall in the Port of Rotterdam, focussing on the reliability of the anchors in the primary load-bearing layer. Comparing the derived reliability indices before and after incorporation of the spatially averaged anchor capacityshowed that standard statistical approaches can be overly conservative in how they estimate the reliability and that spatially averaging the input parameters is appropriate for the shaft resistance of piles or anchors.This method can also be extended to applications beyond anchored retaining walls in sand, including the shaft capacity of piles and anchors in other soil conditions.

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