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On the use of a horizontal  $k$ - $\epsilon$  model for  
shallow-water flow

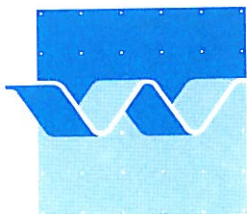
Literature Study

November 1999

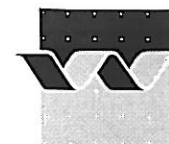
# On the use of a horizontal $k$ - $\varepsilon$ model for shallow-water flow

Literature Study

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**wL | delft hydraulics**



CLIENT: Rijkswaterstaat, Rijksinstituut voor Kust en Zee (RIKZ)

TITLE: On the use of a horizontal k-ε model for shallow-water flow

ABSTRACT:

For an accurate modelling of the vertical structure of flows and water related processes a so-called two-equation model is essential. Such a model is based on the eddy-viscosity concept in which the vertical eddy-viscosity is the product of the length and velocity scales of the 3D small-scale turbulence. These scales need to be modelled by means of two quantities that are the solution of the two-equation model.

In TRIWAQ the horizontal eddy-viscosity is supposed to be a constant, which can be specified by the user. For applications such as the simulation of flows near barriers, groynes and sluices the approach with a constant value for the horizontal eddy-viscosity is considered to be unsuitable. Therefore, RIKZ has commissioned WL | Delft Hydraulics to implement in this project a horizontal turbulence model for the computation of the 2D eddy-viscosity in TRIWAQ. In particular, the two-length-scale model, in which both 2D and 3D k-ε models occur, developed in (Bijvelds, 1997) will be employed.

The report describes the first activity of the project, being a literature study. An overview is given of horizontal k-ε turbulence models that are known in literature.

REFERENCES: Contract number RKZ-745, part of NAUTILUS 10.3 project

VER.	ORIGINATOR	DATE	REMARKS	REVIEW	APPROVED BY			
0.1		12-10-99	first draft version					
0.2		3-11-99	second draft version					
1.1		17-11-99	first final version					
1.2	M. Bijvelds, E. de Goede	22-11-99	second final version	R.E. Uittenbogaard	J.A.Th.M. van Kester			
PROJECT IDENTIFICATION:		Z2747.10						
KEYWORDS:		Horizontal k-ε model, TRIWAQ						
CONTENTS:	TEXT PAGES	15	TABLE	2	FIGURES	1	APPENDICES	0
STATUS:		<input type="checkbox"/> PRELIMINARY		<input type="checkbox"/> DRAFT		<input checked="" type="checkbox"/> FINAL		

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# I Introduction

For an accurate modelling of the vertical structure of flows and water related processes a so-called two-equation model is essential. Such a model is based on the eddy-viscosity concept in which the vertical eddy-viscosity is the product of the length and velocity scales of the 3D small-scale turbulence. These scales need to be modelled by means of two quantities that are the solution of the two-equation model. A well-known example of such a model is the standard k-ε model of Launder and Spalding (1974). This model has been implemented in TRIWAQ (Zijlema, 1998) and Delft3D-FLOW (Delft Hydraulics, 1999).

The horizontal eddy-viscosity is assumed to be a superposition of two eddy-viscosities, representing the 2D large-scale and 3D small-scale turbulence, respectively. Usually, the eddy-viscosity corresponding to the large-scale turbulence is assumed to be a both in space and time constant value, whereas the 3D eddy-viscosity is computed by the standard k-ε model. For example, in TRIWAQ the horizontal eddy-viscosity is supposed to be a constant, which can be specified by the user.

For applications such as the simulation of flows near barriers, groynes and sluices the approach with a constant value for the horizontal eddy-viscosity is considered to be unsuitable. Therefore, RIKZ has commissioned WL | Delft Hydraulics to implement in this project a horizontal turbulence model for the computation of the 2D eddy-viscosity in TRIWAQ. In particular, the two-length-scale model, in which both 2D and 3D k-ε models occur, developed in (Bijvelds, 1997) will be employed. For three realistic applications this method will be tested, viz. a shallow water mixing layer model (see (Tukker, 1997)), model “t Steel” (Bijvelds, 1997) and the MOHA (Monding Haringvliet) model (Soerdjballi, 1999). These test models have been specified by RIKZ.

The report describes the first activity of the project, being a literature study. In Chapter 2 an introduction is given of shallow-water turbulence, whereas Chapter 3 contains a discussion about horizontal turbulence modelling. An overview of approaches known in literature are presented.

This project is carried out by order of RIKZ as part of the NAUTILUS 10.3 project (contract number RKZ-745). At RIKZ this project is supported by M. Zijlema.



## 2 Physics of shallow-water turbulence

### 2.1 Introduction

Turbulence plays an important role in hydraulic engineering practice since it highly determines the water movement. Transport of contaminants, flow around hydraulic structures and the erosion and the sedimentation of beds are examples in which turbulent processes play a main role. For predicting the global effects of the turbulence on the flow, it is necessary to use a proper turbulence model.

Most civil engineering flows are shallow-water flows, which means that horizontal length scales of the flow are much larger than vertical length scales. Under such conditions the Reynolds-averaged<sup>1</sup> Navier-Stokes equations may be reduced to the shallow-water equations. Induced by the shallowness of the flow as well as bed friction, the turbulence characteristics may differ significantly from the turbulence observed in unbounded flows. In shallow-water flows turbulence structures with both a horizontal length scale of several times the water depth and a vertical length scale smaller than the water depth can be present under certain conditions. Such a strongly non-isotropic nature of turbulence may produce a large difference between vertical and horizontal diffusion coefficients and should therefore be taken into account when modelling shallow-water flow. Figure 2.1 shows the presence of the 2D coherent structures of a mixing layer in a harbour entrance, in which four developing quasi-2D vortices and a gyre can be discerned. The width of the harbour entrance is approximately ten times the average water depth. The next section is concerned with the theoretical and experimental observations of the (quasi-) two-dimensional turbulence.

### 2.2 Theory and experimental results

Research on two-dimensional turbulence has already been carried out by e.g. (Kraichnan, 1967), (Batchelor, 1969). These studies on strictly two-dimensional turbulence were based on conservation of enstrophy as well as of turbulent kinetic energy. Therefore, effects of energy loss due to bed friction were not considered. Under such conditions, turbulent kinetic energy is transferred from small wave numbers to large wave numbers as expressed by Fjørtoft's theorem (Fjørtoft, 1953). A totally disorganized system may then become organized by self-organization of the system. The large coherent structures that develop, are mainly formed by pairing of vortices of the same vorticity sign.

Recently, experiments which include the effect of bed friction have been initiated. The large-scale turbulence occurring in these type of flows is generally indicated by quasi-two-dimensional turbulence, because there is some interaction with three-dimensional turbulence and there is a depth dependence in contrary to strictly two-dimensional

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<sup>1</sup> Because the Reynolds number may exceed values of  $10^6$  in large-scale free surface flows, it is unfeasible to use 3D direct numerical simulation or 3D large eddy simulation for numerical computations.

turbulence. Detailed experiments on large turbulent structures in jets and wakes (Dracos et al., 1992), (Jirka, 1998) and mixing layers (Uijttewaal & Tukker, 1998), (Uijttewaal & Booij, 1999) were performed. Both the jet/wake experiments and the mixing layer experiments revealed that under certain conditions large coherent structures may be present in the flow with a characteristic length scale that is (significantly) larger than the water depth. The presence of these structures is governed by the bed-friction or wake parameter (Ingram & Chu, 1987), (Chu et al., 1991). This number expresses the stabilizing effect of the bottom friction on the large structures that are fed by horizontal velocity gradients. It is defined as the ratio of the loss of large-scale kinetic energy due to bottom friction and the production of large-scale turbulent kinetic energy due to transverse velocity gradients, see e.g. (Tukker, 1997). This ratio can be simplified to

$$S = \frac{c_f}{\lambda} \frac{\delta}{H} \quad (2.1)$$

that depends only on mean-flow quantities. In Eq. (2.1)  $H$  represents the water depth,  $\lambda$  is a dimensionless measure for the velocity difference,  $\delta$  is a characteristic length scale for the mixing region and  $c_f$  is a bottom friction coefficient. By stability analysis it can be shown that there exists a critical value for  $S$ , above which no large scale turbulent structures can occur (Chu et al. 1991). For a horizontal mixing layer, the critical value  $S_c$  is found to lie between 0.09 and 0.12 (Chu et al. 1991), (Alavian & Chu, 1985). However, this condition seems not to be sufficient for the existence of large coherent structures. From experiments Uijttewaal & Booij (1999) conclude that for flow configurations with different depths, but the same bed-friction number, smaller than the critical value, large structures do not necessarily exist.

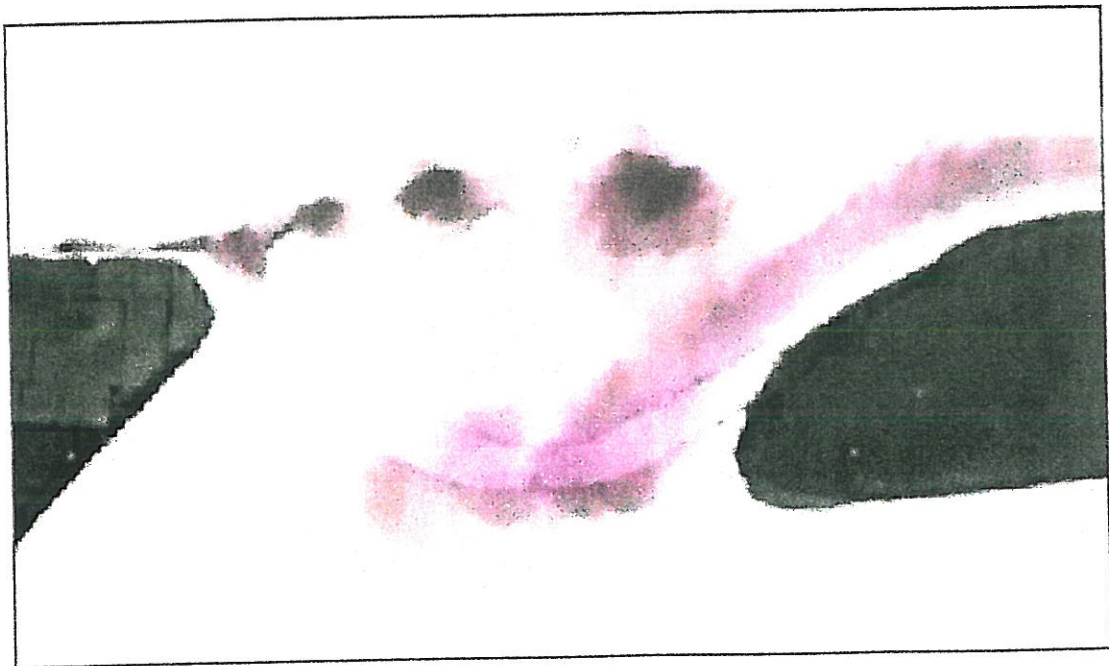


Figure 2.1 Top view of a shallow-water mixing layer and flow pattern in a harbour entrance at laboratory scale. Taken from Bijvelds (1997).

Chen & Jirka (1998) included the effect of viscosity in their stability analysis and found that the combined effect of the viscosity and bed-friction reduce flow instabilities. This, however, appears to be in contrast with the observations of (Uijttewaal & Booij, 1999), who observe a more stable flow for smaller eddy-viscosity. It is noted that in the analysis of (Chen & Jirka, 1998) assumptions on the flow characteristics are made, such as linearity and the neglect of the bottom boundary layer. These assumptions may influence the analytical results.

It may be argued that for a certain value of  $S$ , instabilities in vertical direction may develop that counteract the large coherent structures. Therefore, it is likely that an upper limit for  $S$  exists above which large structures are less dominant or even absent.

Three regions can be discerned in the development of large structures. In the near-field the shearing flow contains 3D small-scale turbulence with a length scale of the water depth typically. The power spectrum of the turbulent kinetic energy in the near field is similar to power spectra measured in three-dimensional turbulence, showing for sufficiently large Reynolds numbers an inertial subrange with a  $-5/3$  wave number dependence. Somewhat further downstream (intermediate-field), there is a strong interaction between the mean flow and the turbulent flow with the vertical boundaries, i.e. the bottom and the free surface. In this region large structures start to develop. The spectral energy density at small wave numbers increases significantly faster than in the near field and at the same time the peak around the maximum becomes more pronounced. In addition, a range develops in which the energy transfer follows a  $-3$  wave number dependence, something which is also observed for strictly 2D turbulence. In the far-field the turbulent structures have grown to sizes (significantly) larger than the water depth. Because of the kinematic constraints, these vortical structures have a dominant vertical vorticity component and they may grow in size by pairing. These large-scale structures are hardly effected by the small-scale turbulence. Moreover, since vortex stretching of the large-scale structures is impeded and the large structures are well organized with respect to each other, these large-scale structures are long-lived. This is characterized by the quasi-periodic behaviour of the autocorrelation function of the turbulent part of the transverse velocity component (Dracos et al., 1992), (Uijttewaal & Tukker, 1998).

For the generation of large 2D coherent turbulent structures, three mechanisms can be discerned (Jirka, 1998). First of all, topographic features, such as the presence of islands, jetties and groynes for example, may lead to local flow separation and may cause an intense shear layer. Secondly, internal transverse shear instabilities can give rise to gradual growth of large coherent structures. These shear instabilities are induced by velocity differences that are caused by topographic changes or roughness distribution (e.g., compound channel flow) or due to fluxes of momentum excess or deficit (mixing layers, jets, wakes). Finally, secondary instabilities of the base flow are believed to be a driving mechanism for the development of 2D coherent structures, although experimental evidence is still limited. Slight imbalance of the base flow may lead to a redistribution of the momentum exchange processes at the bottom boundary, causing the vortex lines to be distorted, which ultimately leads to 2D large coherent structures. It is clear from above mentioned examples that two-dimensional turbulence in presence of bed friction can occur in many geophysical and environmental engineering problems.



In the remainder of this report attention is focussed on modelling quasi-two-dimensional turbulence. Thereafter, it is argued that this model together with the approach for modelling the 3D small-scale turbulence forms an appropriate basis for the application of detailed-flow models, such as RIJMAMO (Van Dijk, 1999). In absence of large horizontal turbulent structures, the use of a standard single-length-scale turbulence models, see e.g. (Uittenbogaard et al., 1992), (Zijlema, 1996b), has shown to yield satisfactory results in numerous applications.

## 3 The modelling of quasi-2D-turbulence

### 3.1 Reynolds-averaging and splitting of turbulence

Presently, two different conceptual approaches are used for simulating shallow-water. The first method is essentially based on Reynolds-averaged flows, or in case of tidal motions, phase-averaged flows, which still allow for the variation of tidal motions but averaging out 3D and quasi-2D turbulence. This approach is followed in this project. Another approach is using LHES, i.e. simulation of the Large Horizontal Eddies at scales  $\Delta x$  grid size. The resolved eddies are now random in time and space. The latter approach is based on low-pass filtering the Navier-Stokes equations rather than phase-averaging or Reynolds-averaging in the former case.

The Reynolds stresses that are obtained by Reynolds-averaging the Navier-Stokes equations, are in general modelled by using the Boussinesq hypothesis or eddy-viscosity concept:

$$\tau_{ij} = -2\rho \nu_t S_{ij} \quad (3.1)$$

where

$$S_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \quad (3.2)$$

is the Cartesian deformation tensor of the flow field. The eddy-viscosity appearing in Eq. (3.1), viz.  $\nu_t$ , is a scalar, which means that any directional dependency is omitted. However, this is only true in a few cases of free turbulent flows and cannot be expected to be correct as a general rule. In general, the Reynolds stress in a given plane may depend on mean-velocity gradients in other planes so that the eddy-viscosity is a fourth order tensor (Hinze, 1975). Owing to its complexity, this tensor is unusable in practice and therefore a scalar is mostly used in engineering practice. However, in case of highly non-isotropic turbulence, more than one characteristic turbulent length scale may exist. Therefore the use of the eddy-viscosity as a scalar to model these types of flow appears to be inadequate. Nonetheless, progress has been made to develop a relative cheap turbulence model for accounting the non-isotropic nature of turbulence. For example, in (Zijlema, 1996a) a nonlinear k-ε model (Speziale, 1987), which is a special case of the algebraic Reynolds-stress modelling, has been successfully applied to 3D industrial flows. In this project another approach is adopted which is sufficiently suitable for 3D shallow-flow applications.

To develop a turbulence model with disparate length scales, we first define the splitting of turbulence into 3D small-scale turbulence and the quasi-2D large-scale turbulence. The small-scale turbulence represents fluctuations with wavelengths of the order of the water

depth and hardly the situation in which the large-scale turbulence is related to fluctuations with wavelengths larger than the water depth. In analogy with fully 3D turbulence we use

$$u^{2D} + u^{3D} = u - \langle u \rangle \quad (3.3)$$

where  $u$  is the instantaneous velocity and  $\langle u \rangle$  is ensemble averaged in any point  $\bar{x}$ . In civil engineering, beds have gradual slopes and three-dimensional turbulence properties change only gradually in horizontal direction. In other words, most 3D turbulence in civil engineering is of a boundary layer type with exceptions such as near sills and gates. We assume that 3D turbulence gradually varies in horizontal direction and at scales significantly larger than the water depth  $H$ . On the other hand, 3D turbulence does strongly depend on the vertical direction by bed friction, wind and density-stratification.

For separating quasi-2D turbulent motions from 3D turbulence, we therefore introduce a splitting operator which is dedicated to the 3D boundary layer type of turbulence. This operator is averaging over a horizontal area  $A(\bar{x})$  at some depth  $z$ :

$$\bar{u}^{A(\bar{x})} = \frac{1}{A(x, y, z)} \int_{A(\bar{x})} u(x', y', z') dx' y' \quad (3.4)$$

with  $A(\bar{x}) = O(H^2)$ . This operator removes 3D turbulence, because it averages over many mutually independent points on  $A(\bar{x})$  of 3D turbulence that we assume to be (sufficiently) homogeneous on  $A(\bar{x})$  so that the area averaging corresponds to ensemble averaging. This averaging may be performed along  $\sigma$ -planes or  $z$ -coordinates. Note that  $\bar{u}^{A(\bar{x})}$  may become zero in case of turbulence homogeneous in vertical direction. However, this is not the case in the presence of a bottom boundary layer in quasi-2D turbulence. Using the above averaging of Eq. (3.4), we obtain

$$u^{2D} = \bar{u}^{A(\bar{x})} - \langle u \rangle \quad (3.5)$$

and hence

$$u^{3D} = u - \bar{u}^{A(\bar{x})} \quad (3.6)$$

The necessity to separately model the effects of the large-scale turbulent structures on the one hand and the small-scale turbulence on the other hand has been recognized for a long time (Leendertse & Liu, 1977). Within the eddy-viscosity concept, the two eddy-viscosities have been introduced, viz.  $\nu_t^{2D}$  for the horizontal viscosity and  $\nu_t^{3D}$  ( $= \nu_t^v$ ) for the vertical viscosity, respectively. The vertical eddy-viscosity  $\nu_t^v$  is related to the small-scale turbulent fluctuations  $u^{3D}$  and it is used for the computation of the vertical Reynolds stresses and  $\nu_t^{2D}$  is the eddy-viscosity related to the large-scale turbulent fluctuations  $u^{2D}$  and is used for the computation of the horizontal Reynolds stresses. We note that in TRIWAQ and in Delft3D-FLOW two different approaches are followed for the computation of  $\nu_t^{2D}$ . For TRIWAQ holds that

$$\nu_t^{2D} = [\nu_t^h] \quad (3.7a)$$

whereas for Delft3D-FLOW we have, see (Uittenbogaard et al., 1992)

$$\nu_t^{2D} = [\nu_t^h] + \nu_t^v \quad (3.7b)$$

with  $[\nu_t^h]$  the constant value specified by the user. Finally, we remark that the use of the eddy-viscosity concept for quasi-2D turbulence is not trivial since the eddy-viscosity may become negative in this situation (Kraichnan, 1976). The next section discuss the calculation of  $\nu_t^{2D}$  by means of a horizontal turbulence model. Details on the modelling of  $\nu_t^v$  based on the standard k-ε model can be found in (Zijlema, 1998) and (Delft Hydraulics, 1999).

### 3.2 Horizontal turbulence models

Many numerical models use a value for the horizontal eddy-viscosity  $\nu_t^{2D}$ , which is constant in time (and sometimes even constant in space), see e.g. (Delft Hydraulics, 1981), (Zijlema, 1998) and (Delft Hydraulics, 1999). This means that the value of  $\nu_t^{2D}$  does not depend on local flow characteristics, which makes it physically unsound. The horizontal eddy-viscosity is mainly applied for model calibration and numerical stabilization of the computations. For large-scale tidal applications, in which detailed flow information is not of interest, such a crude approximation has proven to be sufficiently accurate. However, for more detailed calculations, such as RIJMAMO (Van Dijk 1999), a more sophisticated approach needs to be applied.

The Smagorinsky model (Smagorinsky, 1963) is a sub-grid model that uses an eddy-viscosity for the energy dissipation in Large Eddy calculations. However, as a closure model for Reynolds-averaged equations it is sometimes used beyond its scope in large-scale oceanographic and atmospheric applications, see e.g. (Blumberg & Mellor, 1985), (Delft Hydraulics, 1981). This model uses a mixing-length concept in which the eddy-viscosity is assumed to be proportional to the subgrid characteristic length scale equal to the mesh width  $\Delta x$  and to a characteristic turbulent velocity  $\Delta x \sqrt{(2S_y S_{yy})}$ . The eddy-viscosity then reads:

$$\nu_t^{2D} = (C_s \Delta x)^2 \sqrt{(2S_y S_{yy})} \quad i, j = 1, 2 \quad (3.8)$$

in which  $C_s$  is a constant and is often set to 0.1. This model is based on a local balance between production and dissipation. No history effects are present in this model, which are likely to be important given the long-liveliness of the large-scale turbulence.

In (Bijvelds et al., 1999) a two-length-scale approach is proposed that uses the splitting in Eq. (3.7b). The method consists of two distinct k-ε models that independently simulate the 3D and quasi-2D turbulence. The eddy-viscosity  $\nu_t^v$  is computed with a 3D k-ε turbulence



model, in which the production of turbulent kinetic energy results from vertical shear only, i.e. bottom friction. Note that  $\nu_t^v$  is used for the computation of the vertical Reynolds stresses. The horizontal eddy-viscosity  $\nu_t^h$  is calculated by a 2D depth-averaged k-ε model, in which the production of turbulent kinetic energy results from horizontal velocity gradients, based on depth-averaged velocities only. The coefficients in both the 3D and 2D model, are taken equal to the standard k-ε model (Launder and Spalding, 1974). No experimental results are available for tuning of these coefficients. Direct interaction between the two turbulence models, by means of energy transfer, is neglected. However, interaction via the mean-flow equations still exists. The neglect of the interaction is based on the estimate of the production term of the small-scale turbulence and the dissipation of the large-scale turbulence, which is a source term for the small-scale turbulence (Bijvelds, 1997). The production of turbulent kinetic energy for the small, 3D eddies caused by the presence of the bottom, can be estimated by

$$P = u_*^2 \frac{\partial u}{\partial z} = c_f u_*^2 \frac{\partial u}{\partial z} \approx c_f \frac{U^3}{H} \quad (3.9)$$

where  $u_*$  is the friction velocity and  $U$  the depth-averaged velocity. The single major loss of energy for the large-scale structures, is due to bottom friction, generating bed-friction induced, small-scale, turbulence. Assuming that the mean, depth-averaged transverse velocity  $V$  and the large-scale depth-averaged fluctuations  $U'$  and  $V'$  are much smaller than the depth-averaged longitudinal velocity  $U$ , the energy loss  $F$ , from which the small-scale turbulence benefits, can be estimated by (Babarutsi & Chu, 1991):

$$F = \frac{c_f}{2H} U (\overline{2U'^2} + \overline{2V'^2}) \quad (3.10)$$

Taking into account that the characteristic turbulent velocities  $U'_i$  are at least an order of magnitude smaller than the mean velocities  $U_i$ , directly shows that this term is small compared to that in Eq. (3.9), and can therefore be neglected. Note that the upcascading of energy from small vortices to larger scales is also neglected in this model. According to (Bijvelds, 1997) modelling this transfer term for simple flows is likely to be unfeasible, in particular for complex flows.

In (Bijvelds, 1997) it is noted that the use of the k-ε turbulence model for quasi-2D turbulence is beyond its fundamental limits of applicability. In contrast with the physics of 2D-turbulence, this model is based on the energy cascade process. Using this concept is therefore formally incorrect in such a situation. However, this model has been applied for steady flow in two different harbours in which shallow mixing layers are present. In both situations the two-length-scale turbulence model has shown to perform better than the standard k-ε model without an additional horizontal eddy-viscosity, i.e. both horizontal and vertical stresses are computed with a single-scale eddy-viscosity (Launder & Spalding, 1974). Also the computational effort hardly increases. For one situation the turbulence model has also been compared to the standard 3D k-ε model with an additional horizontal eddy-viscosity, which has been taken constant in time and space. Results are comparable for both models, which is caused by the fact that the computed eddy-viscosity

in the two-length-scale turbulence model is almost constant in the area of interest. In (Bijvelds et al., 1999) it is stressed that for non-stationary flow in combination with complex geometry of the domain, the determination of a constant horizontal eddy-viscosity is difficult and therefore the use of a turbulence model for the determination of  $v_i^{2D}$  is likely to be advantageous.

By using a two step averaging procedure (Babarutsi & Chu, 1991) have derived a depth-averaged transport equation for the Reynolds stresses that results from the large-scale turbulent structures. First they derive a momentum equation for the small-scale average of the horizontal velocity. This velocity is averaged over a period small compared to large-scale turbulence structures but large compared to the small-scale turbulence. Moreover, it is averaged over depth. Fluctuations on this average velocity are defined as a small-scale fluctuating part of the velocity vector and are related to small-scale 3D turbulence. By splitting this horizontal average velocity into a mean and a fluctuating part, which is related to the large-scale turbulent structures, they arrive at the Reynolds stress equation for the large-scale turbulence. In this procedure the Boussinesq approximation is used for the small-scale turbulence. Beside the usual diffusion, production, dissipation and pressure strain terms, the equation contains a term that is associated with the negative work done by the large-scale turbulent motion against the bed friction. From the Reynolds stress equation, they derive a depth-averaged equation for the turbulent kinetic energy of the large eddies, which contains a loss term due to bed friction that is given by Eq. (3.10). Together with an equation for the dissipation rate of the turbulent kinetic energy for the large-scale structures, which is identical to the  $\epsilon$  equation used by (Rodi, 1985), it forms a depth-averaged turbulence model for the large-scale structures. Again, it is noted that the k-ε model is used improperly. The eddy-viscosity related to the 3D turbulence is modelled by means of the well-known local equilibrium assumption and is proportional to  $u_* H$  (Babarutsi & Chu, 1998).

	3D turbulence	2D turbulence	remarks
<b>2DH LES</b>	subgrid: $\Delta x > H$	resolved $> 2\Delta x$	
	Smagorinsky (1963)		2DH, dynamics of 2D eddies resolved $> 2\Delta x$
<b>RANS</b>			
	3D k-ε algebraic	constant eddy visc. $\nu^h$ depth-averaged k-ε	3D TRIWAQ/DELFT3D-FLOW 2DH Babarutsi & Chu (1998)
	3D k-ε	depth-averaged k-ε	3D Bijvelds et al. (1999)

Table 3.1. Quasi-2D turbulence models for modelling grids characterized by  $\Delta x > H$

In (Babarutsi & Chu, 1998) the model is compared with the standard depth-averaged k-ε model of (Rastogi & Rodi, 1978) and with an improved version of this model (Booij, 1989), which removes the bias of the single-length-scale turbulence model towards the small-scale, in case of a transverse mixing layer. The three models initially exhibit the same behaviour regarding the development of the mixing layer. However, at some point the single-length-

scale turbulence models become dominated by bed friction and from this point the growth of the mixing layer deviates significantly from the one obtained with the two-length-scale model, which yields a better agreement with observations. (Babarutsi & Chu, 1998) note that the results of the two-length-scale model strongly depend on the friction coefficient  $c_f$ , appearing in the loss term due to bed friction. Table 3.1 contains an overview of quasi-2D turbulence models discussed in this section.

### 3.3 Stratified flow

The turbulence models described in the previous section are based upon a depth-averaged velocity. Under stratified conditions, flows are mainly of a three-dimensional nature, which means that the velocity can vary significantly in vertical direction. In circulating flows the depth-averaged velocity may tend to zero, whereas the root mean square value can deviate significantly from zero. This implies that in the above mentioned turbulence models production of turbulent kinetic energy related to large-scale structures may vanish and no turbulence will be generated in the model. It is clear that modifications of the quasi-two-dimensional turbulence models in order to justify the application of the models under stratified conditions is imperative, since the 2D eddy-viscosity is not a function of depth.

## 4 Summary

In civil engineering turbulent flow with disparate turbulent length scales may occur. This strongly non-isotropic nature of turbulence may produce a large difference between vertical and horizontal diffusion coefficients and should therefore be taken into account when modelling shallow-water flow.

Only a few experiments have been carried out on quasi-2D turbulence in the presence of bed friction. Hence only a few models for closure of the Reynolds averaged equations are available from the literature for this type of flows. An overview of such models is given in Table 3.2. Note that the Smagorinsky model is sometimes incorrectly used for closure of the Reynolds-averaged equations and shallow-water flows. The use of a constant eddy-viscosity (that may vary in space) does not depend on the flow characteristics and has no history effects. Since the large turbulent structures are long-lived, this approach is likely to be inadequate for the modelling of quasi-2D turbulence.

Models for quasi-2D turbulence		
single-length-scale turbulence models	standard 2D depth-averaged k-ε	Rastogi & Rodi (1978)
	adjusted Rastogi & Rodi model; removed bias towards small scale turbulence	Booij (1989)
	standard 3D k-ε	Launder&Spalding (1974)
two-length-scale turbulence models	3D k-ε+ constant eddy viscosity	TRIWAQ / DELFT3D-FLOW
	algebraic + depth-averaged 2D k-ε	Babarutsi & Chu (1998)
	3D k-ε+ depth-averaged 2D k-ε	Bijvelds et al. (1999)

Table 3.2. Overview of turbulence models discussed in this report.

It appears that single-length-scale turbulence models are inadequate for modelling anisotropic turbulence. To our knowledge, the only two-length scale model applied for large-scale 3D free surface flows is the model developed by (Bijvelds et al., 1999). This model is based on the standard 2D and 3D k-ε model and has shown to yield improved results in comparison to the standard single-length-scale turbulence model. However, some remarks must be made with respect to the theoretical background of the model:

- **The model is based on the k-ε model.** The k-ε model has been developed for three-dimensional turbulent flow characterized by the energy cascade process. The vortex stretching related to the cascade process is impeded in case of quasi-2D turbulence and therefore the model is applied beyond its limits of applicability.



- **Standard coefficients of the depth-averaged k-ε model are used.** These coefficients have been derived for flows that deviate strongly from the ones described in this report.
- **Transfer of energy neglected.** The transfer of energy from the large scale turbulent structures to the small scale turbulent structures and vice versa have been neglected. The neglect of transfer of energy from the large scale turbulent structures to the small scale turbulent structures is justified, based on comparison with the production term in the turbulence model. Interaction via the mean flow equations exists.
- **Use of the eddy-viscosity.** The use of the eddy viscosity concept for quasi-2D turbulence meets difficulties due to the possible reverse flow of energy from small scales to large scales.
- **Not applicable for stratified flows.** The model has been developed for unstratified flows. Since the averaging operator (see Eq. (3.4)) is defined as an averaging over a horizontal plane it is formally possible to apply the two-length-scale model. However, extension of the model for application in stratified flows is not trivial and requires additional research.

Notwithstanding these shortcomings, the model has proven efficient and useful for practical applications. Therefore it has been decided to implement this turbulence model in TRIWAQ. Future research on turbulence models that are physically more sound is, however, recommended as well as additional laboratory experiments to validate the turbulence models.

## 5 References

- Alavian, V. and Chu, V.H., 1985: Turbulent exchange flow in shallow compound channel, Proc. 21th I.A.H.R. congress, 19-23.
- Babarutsi, S. and Chu, V. H., 1991: A two-length-scale model for quasi-two-dimensional turbulent shear flows, Refined flows. Modelling, Vol. C, IAHR, 53-60.
- Babarutsi, S. and Chu, V.H., 1998: Modelling transverse mixing layer in shallow open-channel flows, J. Hydr. Eng., Vol.124, No. 7, 719-727.
- Batchelor, G.K., 1969: Computation of the energy spectra in homogeneous two-dimensional turbulence, Phys. Fluids Suppl. II, 233-239.
- Bijvelds, M.D.J.P., 1997: Recirculating steady flow in harbours: comparison of numerical computations to laboratory measurements, Fac. of Civil Eng., Delft University of Technology, Tech. Report 1-97.
- Bijvelds, M.D.J.P. and Kranenburg, C. and Stelling, G.S., 1999: 3-D numerical simulation of turbulent shallow-water flow in square harbor, J. Hydr. Eng., Vol. 125, No. 1, 26-31.
- Blumberg, A.F. and Mellor, G.L., 1995: A description of a three-dimensional coastal ocean circulation model, Three-dimensional shelf models, coastal and estuarine sciences, Ed. Heaps, N., Amer. Geophys. Union.
- Booij, R., 1989: Depth-averaged k-ε model in ODYSSEE, Delft University of Technology, No. 1-89.
- Chen, D. and Jirka, G.H., 1998: Linear stability analysis of turbulent mixing layers and jets in shallow water layers, J. Hydr. Res., Vol. 36, No. 5, 815-830.
- Chu, V.H. and Wu, J.-H. and Khayat, R.E., 1991: Stability of transverse shear flows in shallow open channels, J. Hydr. Eng., Vol. 117, No. 10, 1370-1388.
- Delft Hydraulics, 1981: Aspects of modelling horizontal momentum transfer in shallow flow, Delft Hydraulics Laboratory Report number R1150.
- Delft Hydraulics, 1999: Delft3D-FLOW manual, Release 3.05, September 1999.
- Dijk, R. van, 1999: Gereduceerd Zeedelta model; bouw en afregeling, Rijkswaterstaat/Rijksinstituut voor Kust en Zee, Werkdocument RIKZ/OS-99.133X (in dutch).
- Dracos, T. and Giger, M. and Jirka, G.H., 1992: Plane turbulent jets in a bounded fluid layer, J. Fluid Mech., Vol. 241, 587-614.
- Fjørtoft, R., 1953: On the changes in the spectral distribution of kinetic energy for two-dimensional, nondivergent flow, Tellus, Vol. 5, No. 3, 225-230.
- Hinze, J., 1975: Turbulence, Mc-Graw Hill, New York.
- Ingram, R.G. and Chu, V.H., 1987: Flow around islands in Rupert Bay: An investigation of the bottom friction effect, J. Geoph. Res., Vol. 92, No. 13, 14521-14533.

- Jirka, G.H., 1998: Large scale flow structures and mixing processes in shallow flows, Second International Symposium on Environmental Hydraulics.
- Kraichnan, R.H., 1967: Inertial ranges in two-dimensional turbulence, *Phys. Fluids*, Vol. 10, 1417-1423.
- Kraichnan, R.H., 1976: Eddy viscosity in two and three dimensions, *J. of Atm. Sci.*, Vol. 33, 1521-1536.
- Lauder, B.E., and Spalding D.B., 1974: The numerical computation of turbulent flows, *Comput. Meth. Appl. Mech. Engng.*, Vol 3., 269-289.
- Leendertse, J.J. and Liu, S.K., 1977: A three-dimensional model for estuaries and coastal seas. IV turbulent energy computation, Rand Corporation Santa Monica, No. R-2187-OWRT.
- Rastogi, A.K. and Rodi, W., 1978: Predictions of heat and mass transfer in open channels, *J. Hydr. Eng.*, Vol. 104, No. 3, 397-420.
- Rodi, W., 1985: Calculation of stable stratified shear-layer flows with a buoyancy-extended k-ε turbulence model, Ed. Hunt, J.C.R., *Turbulence and diffusion in stable environments*, Clarendon, Oxford, 111-140.
- Soerdjballi, M., 1999: Detailmodel Mond Haringvliet, Werkdocument RIKZ/OS-99.117X (in Dutch).
- Smagorinsky, J., 1963: General circulation model experiments with the primitive equations, *Monthly Weather Review*, Vol. 91, No. 3, 99-164.
- Speziale, C.G., 1987: On nonlinear k-l and k-ε models of turbulence, *J. Fluid Mech.*, Vol. 178, 459-475.
- Tukker, J., 1997: Turbulence structures in shallow free-surface mixing layers, Ph. D. Theseis, Delft University of Technology.
- Uijttewaal, W.S.J. and Booij, R., 1999: Effects of shallowness on the development of free-surface mixing layers, Submitted to *Phys. Fluids*.
- Uijttewaal, W.S.J. and Tukker, J., 1998: Development of quasi two-dimensional structures in a shallow free-surface mixing layer, *Exp. in Fluids*, Vol. 24, 192-200.
- Uittenbogaard, R.E. and van Kester, J.A.Th.M. and Stelling, G.S., 1992: Implementation of three turbulence models in TRISULA for rectangular horizontal grids, Delft Hydraulics Laboratory, Report Z162.
- Zijlema, M., 1996a: Evaluation of the anisotropic two-equation models for the calculation of channel and duct flows, *Fac. of Tech. Math. and Inf.*, Delft University of Technology, Tech. Report 96-125.
- Zijlema, M., 1996b: Computational modeling of turbulent flow in general coordinates, Delft University of Technology.
- Zijlema, M., 1998: TRIWAQ-three-dimensional shallow water flow model, technical documentation, Rijkswaterstaat/RIKZ, May 1998, Version 1.1.



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