# Incorporating AM-specific Manufacturing Constraints into Topology Optimization

Final presentation

Reuben Serphos
June 13, 2014

Coaches : Dr.ir. M. Langelaar (TU Delft)
: Dr. J. Fatemi, MSc (Dutch Space)
Professor : Prof.dr.ir. F. van Keulen (TU Delft)

TU Delft

Dutch Space
an EADS Astrium company

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* **Introduction**
* Problem
* Goal
* Approach
* Results
* Conclusion
* Future work

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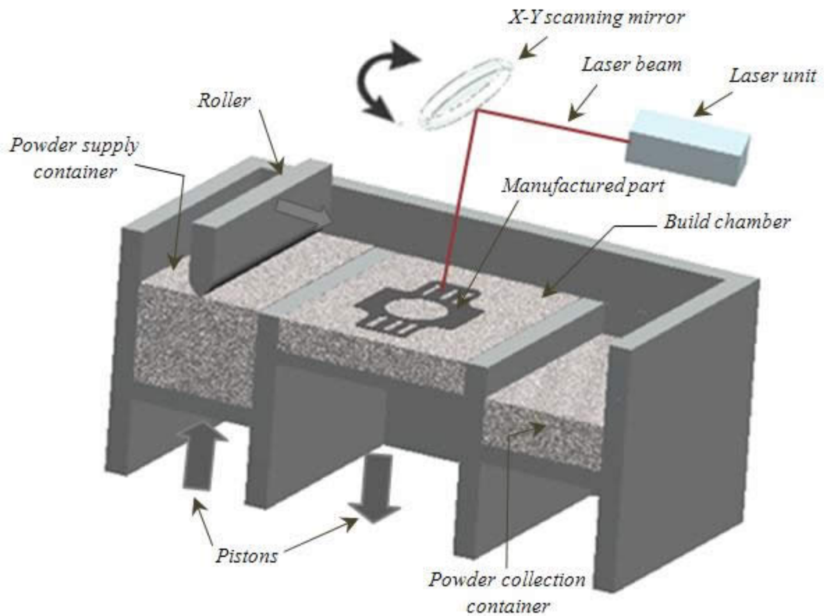
# Additive Manufacturing

- Commonly known as 3D-printing
- Focus on Selective Laser Melting (SLM)



# Additive Manufacturing

- Benefitting from AM
  - Complex geometry
- Topology optimization
  - Structural optimization method

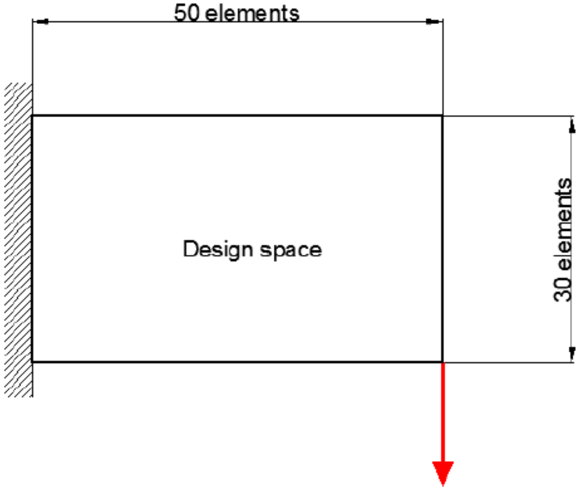
# Topology Optimization

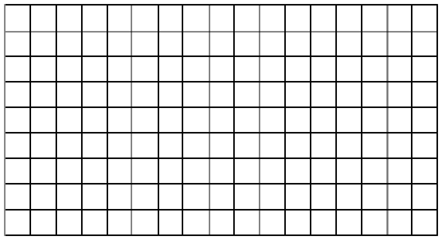
$$\min_{\boldsymbol{\rho}} \quad c(\boldsymbol{\rho}) = \mathbf{U}^{\mathrm{T}}\mathbf{K}\mathbf{U} = \sum_{e=1}^{N} (\rho_e)^p \mathbf{u}_{\mathrm{e}}^{\mathrm{T}} \mathbf{k}_{\mathrm{e}} \mathbf{u}_{\mathrm{e}}$$

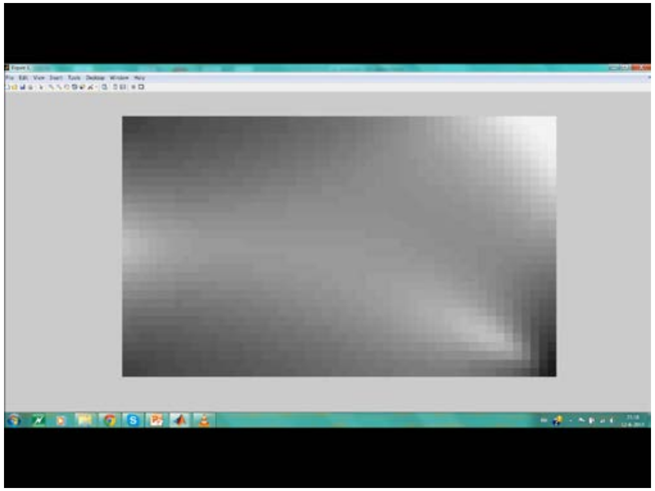
$$\text{subject to} \quad \mathbf{K}\mathbf{U} = \mathbf{F}$$

$$\frac{V(\boldsymbol{\rho})}{V_0} = f$$

$$\mathbf{0} < \boldsymbol{\rho}_{min} \leq \boldsymbol{\rho} \leq \mathbf{1}$$

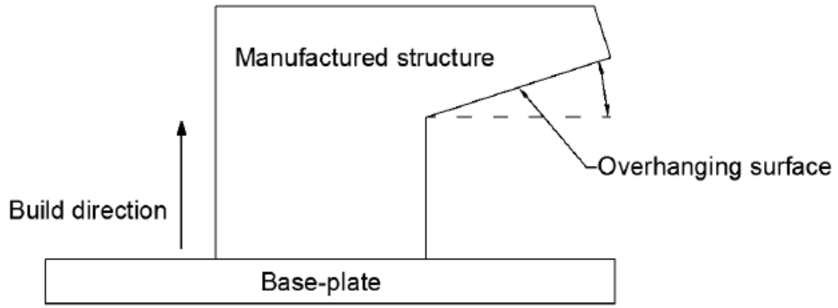




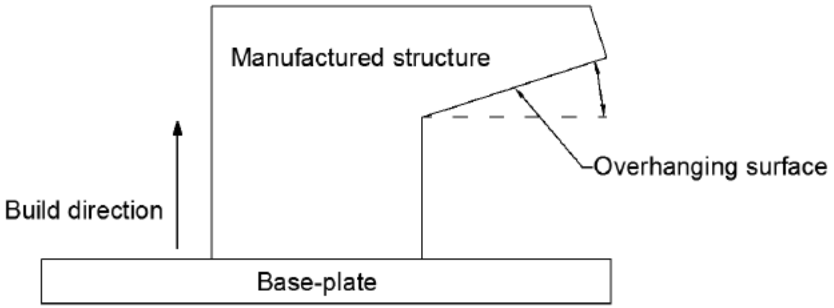
# Manufacturing constraints

- Minimum feature size
- Minimum slot/hole size
- Overhang≥ 45 degrees
- Orientation



# Manufacturing constraints

- Minimum feature size
- Minimum slot/hole size
- **Overhang ≥ 45 degrees**
- Orientation



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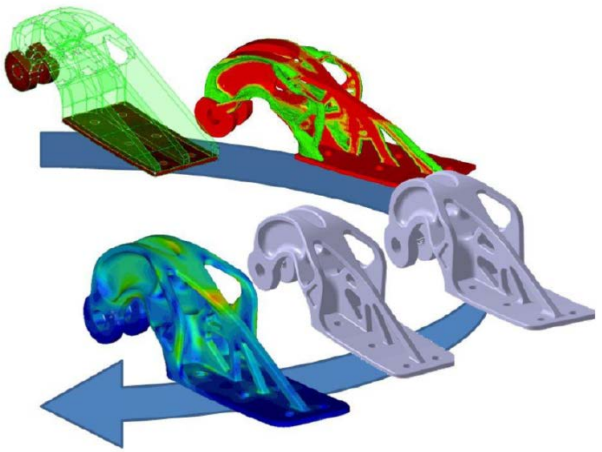
# Problem

- The design obtained from TO does not comply with manufacturing constraints
- Modification is necessary
- Maybe a reduction in optimality



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# Goal

- Extend capabilities of existing program to include AM-specific manufacturing constraint: 45 degree overhang

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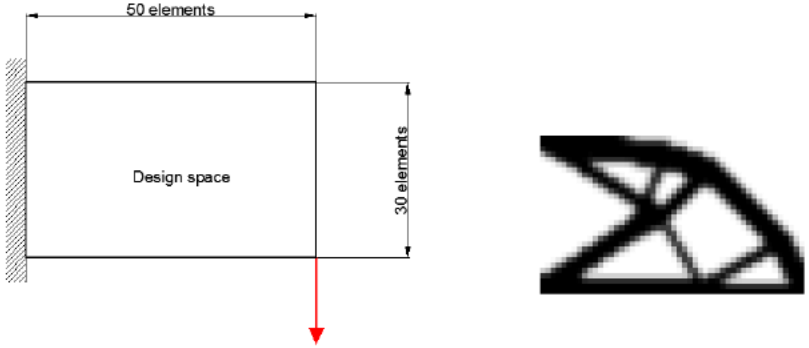
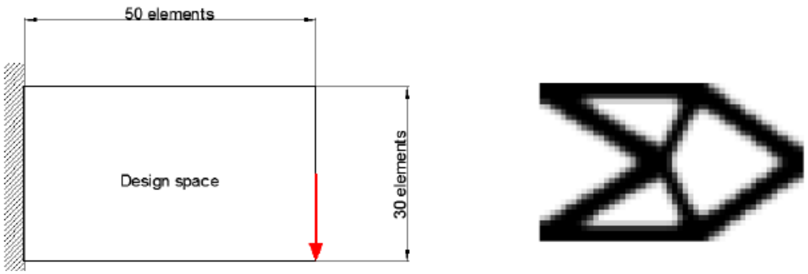
# Approach
## Method and test models

- Use publicly available MATLAB code to test approach
    - Solves 2D problem
- Cantilever beams
- Tension beam

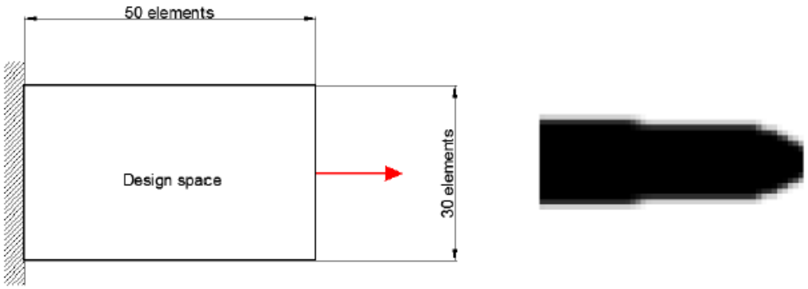
Test case 1




Test case 2

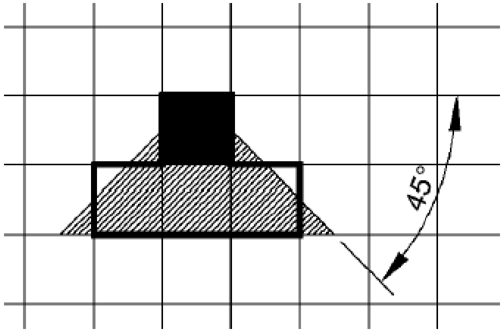



Test case 3



# Approach
## Detecting overhang

- Using discrete finite element nature of optimization problem
- Element must be supported by candidate support elements
- Find maximum density of candidate support elements
- Approximate maximum using P-norm




P-norm:

$$\rho_{max} = \lim_{p_n \to \infty} \sqrt[p_n]{\sum_{a=1}^{k} \rho_a^{p_n}}$$

# Approach
## Three methods

1. Multiple objective
2. Global constraint
3. Filter

# Approach
## 1. Multiple objective

- New optimization problem
  - Minimize overhang along with compliance

$$\min_{\boldsymbol{\rho}} \quad \mathbf{U^T K U} + \Omega_{fac}\Omega_{tot} = \sum_{e=1}^{N} (x_e)^p \boldsymbol{u}_e^T \boldsymbol{k}_e \boldsymbol{u}_e + \Omega_{fac}\Omega_{tot}$$

$$\text{subject to} \quad \mathbf{KU} = \mathbf{F}$$

$$\frac{V(\boldsymbol{\rho})}{V_0} = f$$

$$\mathbf{0} < \boldsymbol{\rho}_{min} \leq \boldsymbol{\rho} \leq \mathbf{1}$$

Where:
$\Omega_{tot}$ = A function that defines a value for the level of overhang
$\Omega_{fac}$ = A weight factor for the additional overhang term

# My approach
# 2. Global constraint

- New optimization problem
  - Original problem but with added constraint

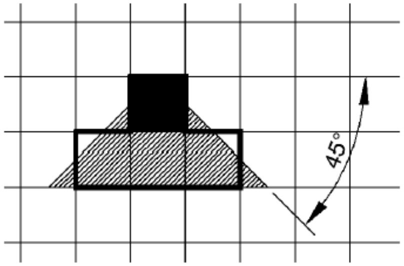
$$
\begin{aligned}
\min_{\boldsymbol{\rho}} \quad & c(\boldsymbol{\rho}) = \mathbf{U}^{\mathbf{T}}\mathbf{K}\mathbf{U} = \sum_{e=1}^{N} (\rho_e)^p \boldsymbol{u}_e^T \boldsymbol{k}_e \boldsymbol{u}_e \\
\text{subject to} \quad & \mathbf{K}\mathbf{U} = \mathbf{F} \\
& \frac{V(\boldsymbol{\rho})}{V_0} = f \\
& \Omega_{fac}\Omega_{tot} = 0 \\
& \mathbf{0} < \boldsymbol{\rho}_{min} \leq \boldsymbol{\rho} \leq \mathbf{1}
\end{aligned}
$$

# My approach
# Global measure for overhang

- Based on difference between element under inspection and maximum density of candidate support element
- No overhang when: $\rho_\Delta \leq 0$

$$\boldsymbol{\rho}_\Delta = \boldsymbol{\rho}_i - \boldsymbol{\rho}_{max}$$

$$\Omega = \begin{cases} 1 & \text{if} \quad \rho_\Delta > 0 \\ 0 & \text{if} \quad \rho_\Delta \leq 0 \end{cases}$$



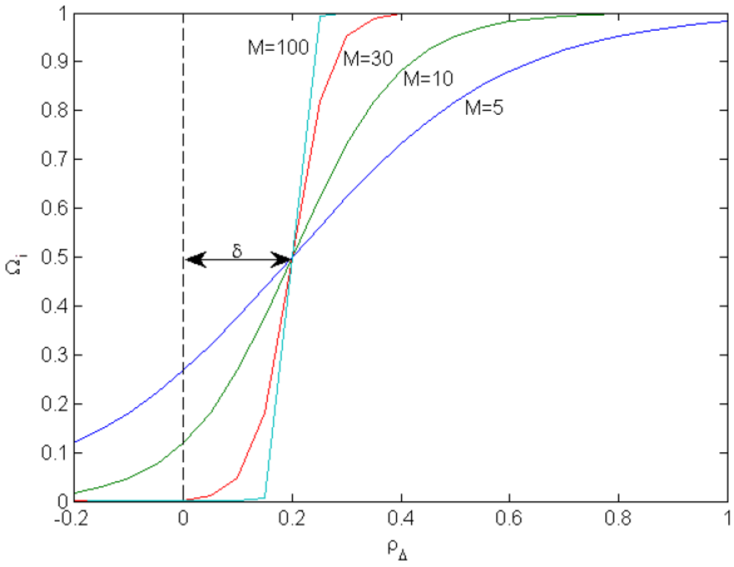
# My approach
# Global measure for overhang

- Overhang of element
  - Approximate step using logistic function
  - Normalized by number of elements in design space

$$\Omega = \begin{cases} 1 & \text{if} \quad \rho_\Delta > 0 \\ 0 & \text{if} \quad \rho_\Delta \leq 0 \end{cases}$$

$$\Omega_i = \frac{1}{1 + e^{-M(\rho_\Delta - \delta)}}$$

$$\Omega_i^* = \frac{1}{1 + e^{-M(\rho_\Delta - \delta)}} \frac{1}{N}$$

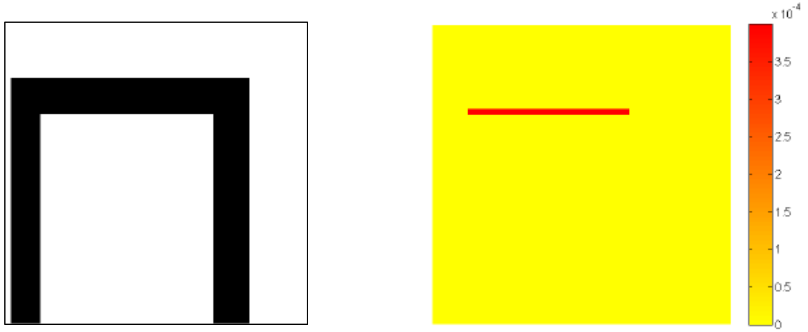


# My approach
## Global measure for overhang

- Overhang coefficient
  - Sum of normalized overhang of each element

$$\Omega_i^* = \frac{1}{1+e^{-M(\rho_\Delta-\delta)}}\frac{1}{N}$$

$$\Omega_{tot} = \sum_{i=1}^{N} \Omega_i^*$$




$$\Omega_{tot} = 0.011$$

# My approach
## 3. Filter

- Original problem
  - Densities are modified through filtering scheme to comply with restriction

$$\tilde{\rho}_i = \begin{cases} \rho_i & \text{if} \quad \rho_i \leq \rho_{max} \\ \rho_{max} & \text{if} \quad \rho_i > \rho_{max} \end{cases}$$

$$\tilde{\rho}_i = 1 - \sqrt[p_n]{(1-\rho_i)^{p_n} + (1-\rho_{max})^{p_n}}$$

(a) Original topology

(b) Filtered topology

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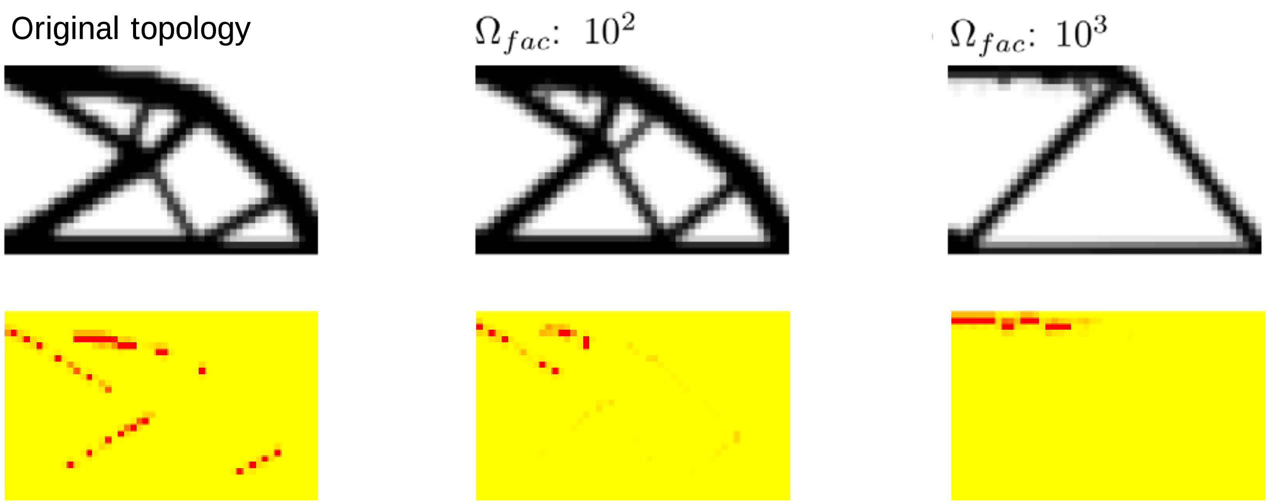
# Results

## 1. Multiple objective

Test case 1

- Drastic design change
- Stalactite formation
- Low volume fraction
- High compliance
- Significant decrease in overhang coefficient



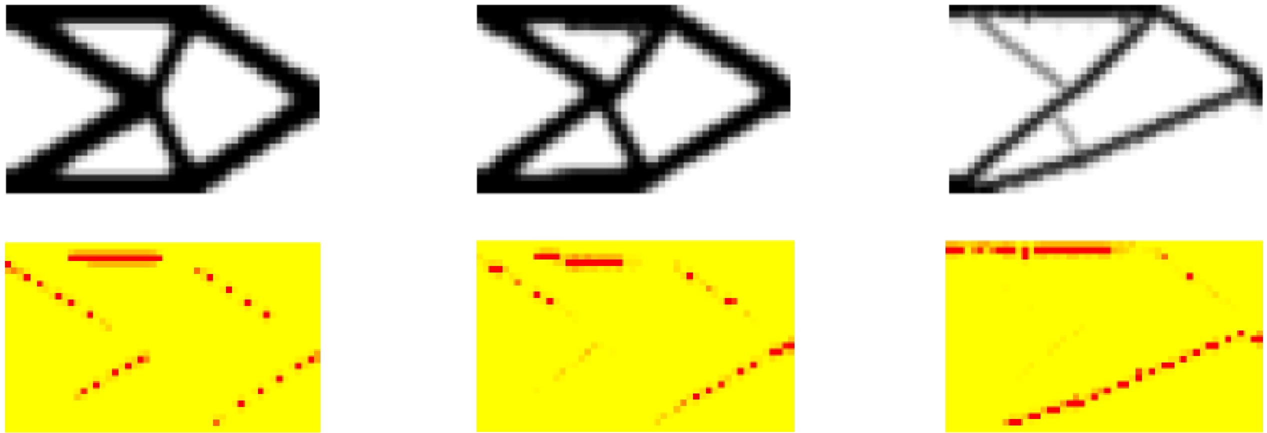
# Results

## 1. Multiple objective

Test case 2

- Some struts re-oriented
- Stalactite formation
- Low volume fraction
- High compliance
- Overhang coefficient decreases then increases

Original topology | $\Omega_{fac}$: $10^2$ | $\Omega_{fac}$: $10^3$



# Results

## 1. Multiple objective

Test case 3

- Stalactite formation
- Low volume fraction
- High compliance
- Only slightly lower overhang coefficient

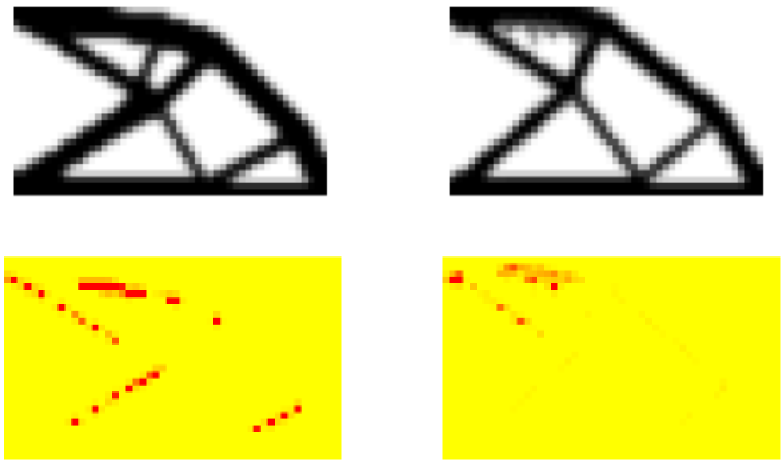
Original topology | $\Omega_{fac}$: 10 | $\Omega_{fac}$: $10^2$

# Results
## 2. Global constraint

Test case 1

- Some struts re-oriented
- Stalactite formation
- Low volume fraction
- High compliance
- Lower overhang coefficient

Original topology | $\Omega_{fac}$: 10



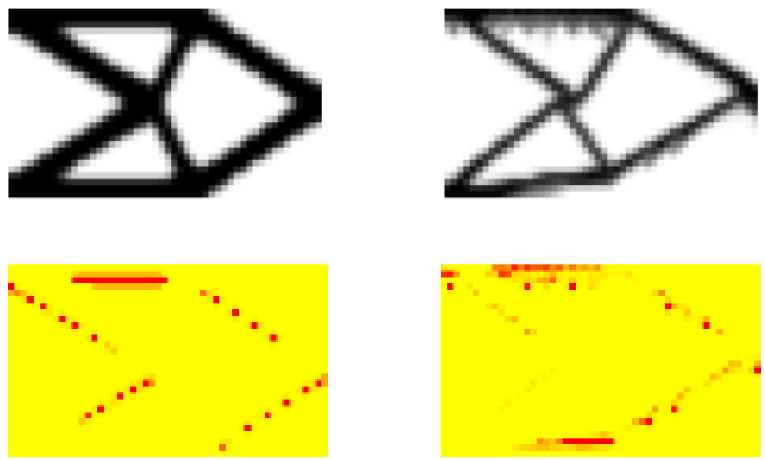
# Results

## 2. Global constraint

Test case 2

- Some struts re-oriented
- Stalactite formation
- Low volume fraction
- High compliance
- Only slighlty lower overhang coefficient

Original topology | $\Omega_{fac}$: 10



# Results

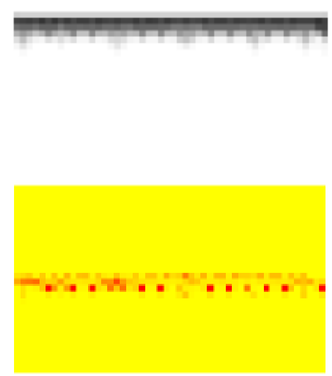
## 2. Global constraint

Test case 3

- Low volume fraction
- High compliance
- Stalactite formation

Original topology

$\Omega_{fac}$: 10



# Results

## 3. Filter

- High cost
- Negative densities
- Low volume fraction
- High compliance

- **OVERHANG NEGLIGIBLE!**

Original topology | Result | Overhang

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# Conclusions

- Proof of concept achieved
- At this point the filter has shown the most promising results
- Further development still necessary

| | Multi. Ob. | Glob. Cons. | Filter |
|---|---|---|---|
| Design altered | Yes | Yes | Yes |
| Overhang | Yes | Yes | No! |
| Compliance | Higher | Higher | Higher |
| Computational cost | Acceptable | Acceptable | High |

# Conclusions

- Proof of concept achieved
- At this point the filter has shown the most promising results
- Further development still necessary

| | Multi. Ob. | Glob. Cons. | Filter |
|---|---|---|---|
| Design altered | Yes | Yes | Yes |
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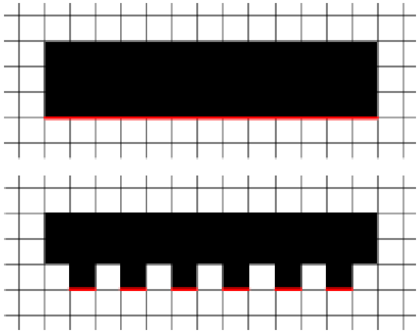
# Future work

- Further investigation of parameters for multiple objective/global constraint
- Further refine the filtering scheme
- Speed-up the process for filter method
- Extend to 3D
- Include part orientation in the optimization process

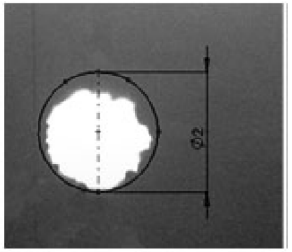
# Extra slides
# Stalactite formation

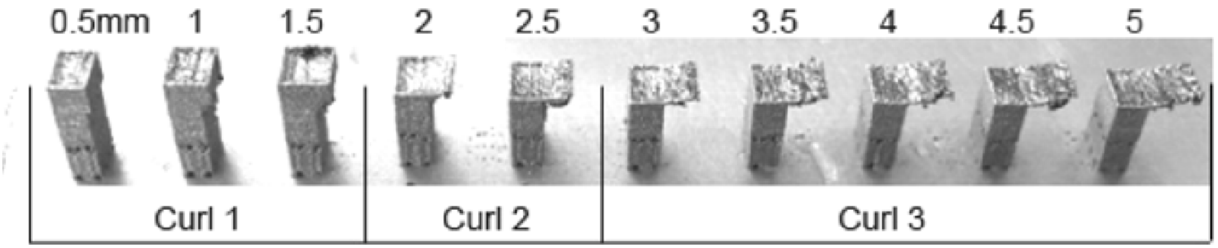
- Provides work-around
- Numerically efficient way to decrease overhang
- Not a practical solution



# Extra slides
## Why overhang ≥ 45 degrees?

- Sagging, not enough support from powder
- Residual stress leads to curl





# Extra slides
## Identifying a maximum

- The P-norm
- Approximates maximum

$$\rho_{max} = \lim_{p_n \to \infty} \sqrt[p_n]{\sum_{a=1}^{k} \rho_a^{p_n}}$$

Where:

$\rho_{max}$ = the maximum value of the included density values

$p_n$ = a parameter that influences how well the true maximum value is approximated

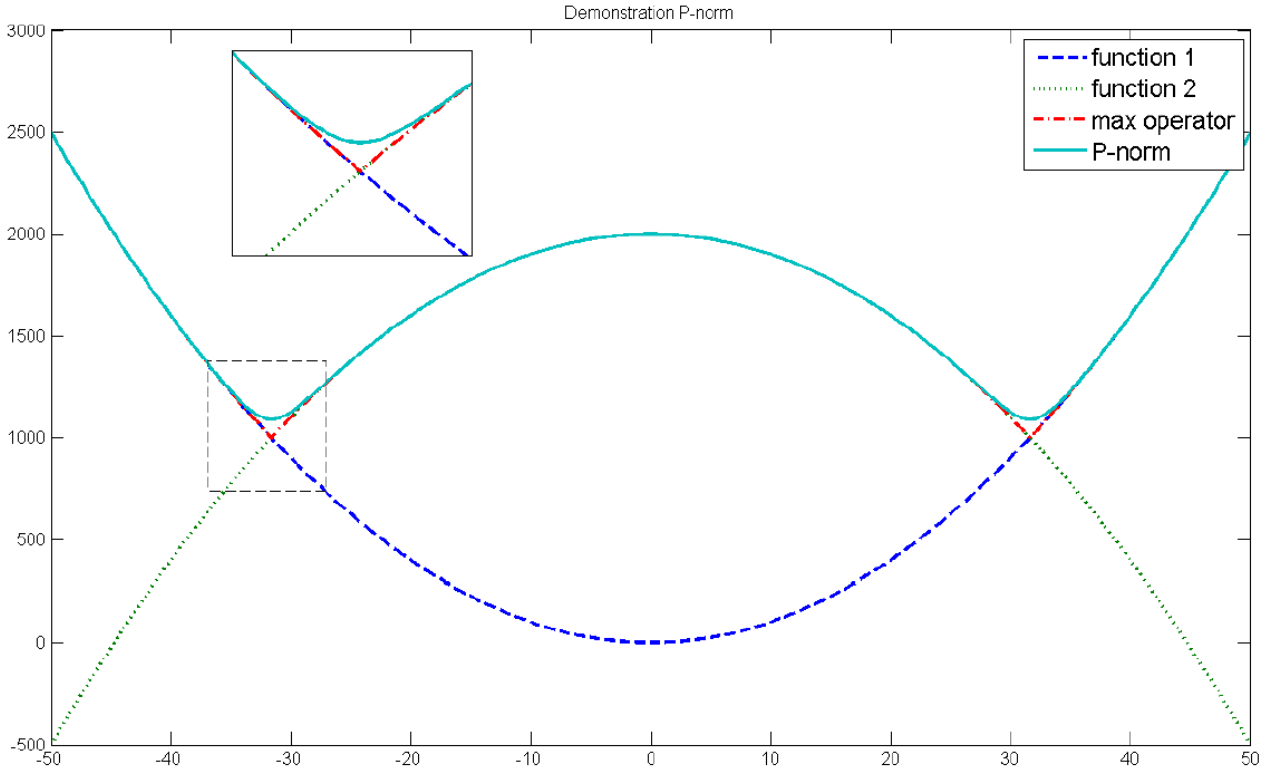
$a$ = the element number for $a = 1 \ldots k$

$k$ = the number of elements being evaluated

$\rho_a$ = the density value of element a

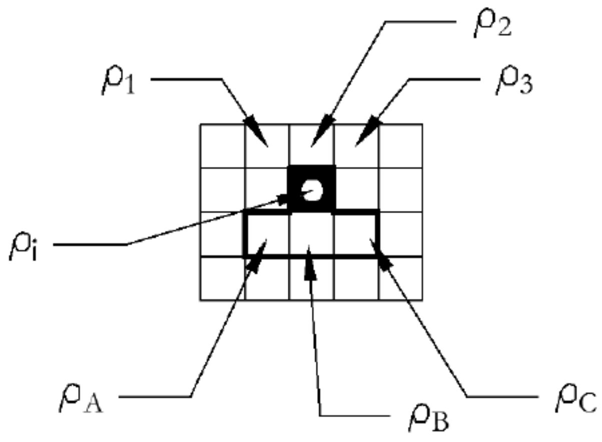
# Extra slides
## Identifying a maximum

- The P-norm



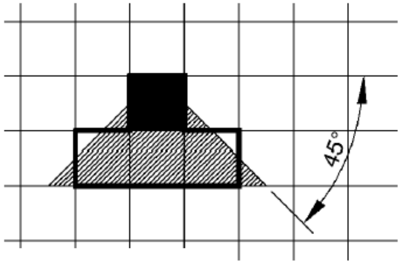
# Extra slides
# P-norm and density difference

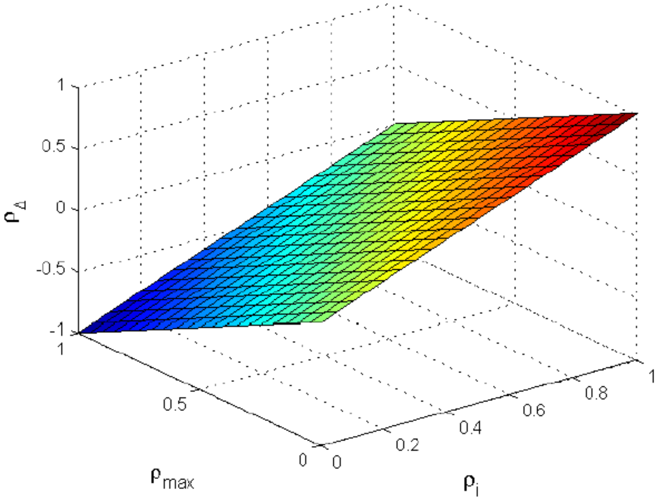
$$\rho_\Delta = \rho_i - \sqrt[pn]{\rho_A^{pn} + \rho_B^{pn} + \rho_C^{pn}}$$



# Extra slides
# Why a step function?

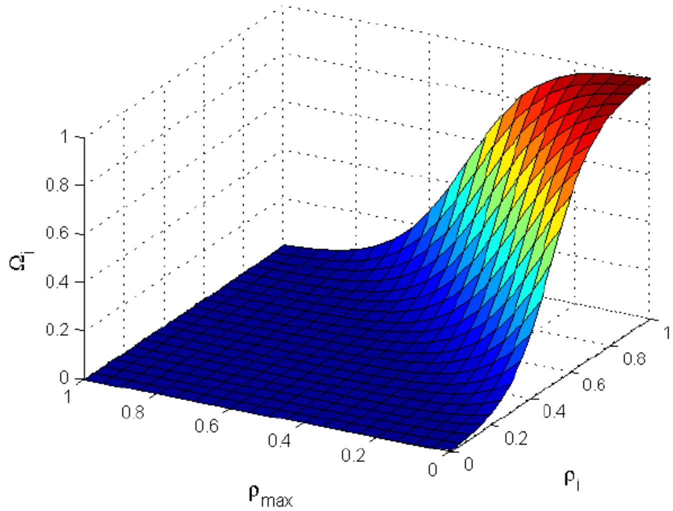
- Negative overhang for negative values of $\rho_\Delta$
- Linear function means: overhang is more acceptable for lower density values



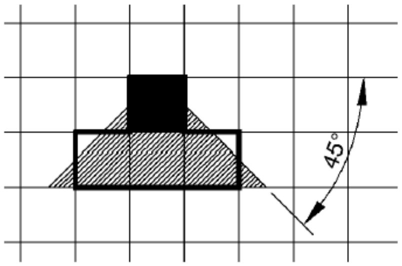



# Extra slides

## Why a step function?

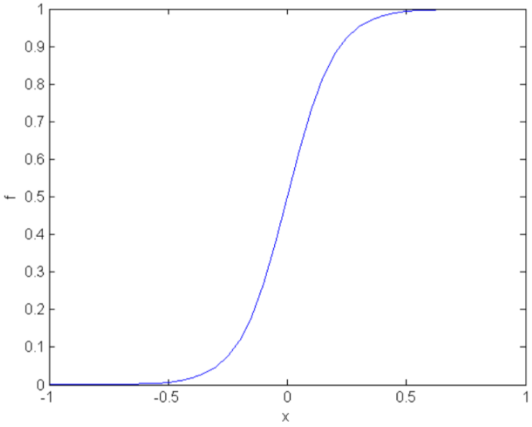



Overhang with $M = 10$ and $\delta = 0,5$



# Extra slides

## What is the logistic function?




$$f = \frac{1}{1 + e^{-M \cdot x}}$$

Where:

$M$ = A parameter that controls the steepness of the curve (i.e. higher values result in a steeper curve)

$x$ = The independent variable in the function

# Extra slides
## Potential of multiple objective

Cantilever beam corner-load with parameters $\delta = 0.2$, $M = 8$, $p_n = 8$ and $\Omega_{fac} = 10^{-3}$

# Extra slides
## Potential of filtering

**(a)** Result from filtering approach for mid loaded cantilever beam with compliance value: 474.55; Volume fraction: 0.24; Number of iterations: 98

**(b)** Threshold applied for elements above 0.3

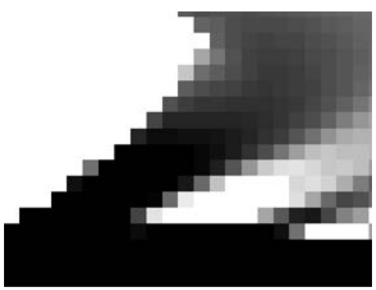
**(c)** Filtered black-white structure with compliance: 53.88; Volume fraction: 0.5

Extracting a black-white topology from the filter result

# My approach
## Continuous vs. discrete

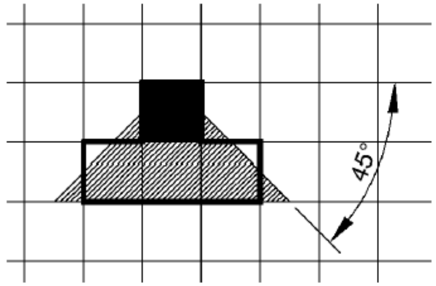
### Continuous

- Use gradient
- Needs interpetation step
- Competes with penalization



### Discrete

- Must indentify maximum locally
- Straightforward



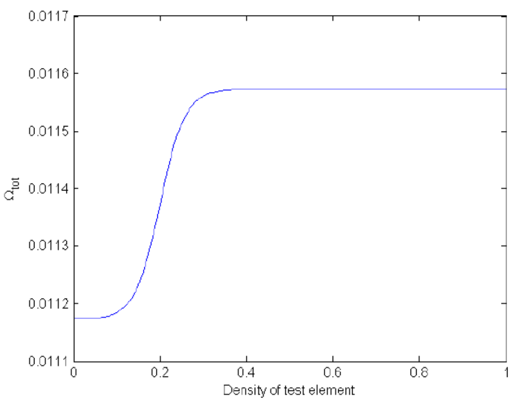
# Extra slides
## Why should the functions be continuous?

- Optimization method is gradient based
- Makes use of sensitivities to choose new values
- Discrete optimization methods not suitable for many variables

# Extra slides
## Why is the volume fraction low?

- Overhang is zero when density of element is zero
  - Turns out this is always an option locally
- Gradients in approximations still high
  - Makes it difficult for the optimizer to find other solutions
- In some cases overhang increases before it decreases due to errors

Overhang coefficient




Sensitivity