Using Frequency Response Coherent Structures for Model-Order Reduction in Microwave Applications

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***Abstract*—An effective practical technique for model-order reduction of large-scale linear time-invariant problems in microwave applications is presented. The reduction is performed as the Galerkin projection onto a subspace of frequency response coherent structures that contain the spectrum of the system multiinput impulse response. The subspace basis is created by the proper orthogonal decomposition of the system transfer characteristics sampled at discrete frequency points. A reduced-order modeling of an integrated planar spiral transformer is used for practical verification and comparison to the standard moment matching subspace approach.**

***Index Terms*—Coherent structures, Galerkin method, integrated transformer, proper orthogonal decomposition (POD), reduced-order systems.**

I. Introduction

MICROWAVE designers often use predictive multidimensional field solvers to analyze and simulate passive devices and interconnections, as well as substrate and thermal coupling effects. However, the models derived from first principles could be prohibitively large for direct incorporation into the circuit- or system-level design. The model-order reduction (MOR) provides an attractive way to deal with this problem. The main idea of MOR is to capture the most important features of the detailed model internal states by a similar model having a state space of significantly smaller size. The hierarchical relationship between internal states of the original and reduced-order model clearly distinguish MOR from the generic modeling approaches based on the parameter identification from measured or simulated external characteristics.

Following the early ideas of [1], it is widely accepted today [2] to develop and analyze MOR using the formalism of *subspace projection*. In that framework, MOR is a projection of the system internal states and governing equations onto corresponding lower-dimensional interpolating subspaces. The orthogonal subspace projections that preserve stability and passivity of the original model in its reduced-order formulation are of special importance. To this end, the Galerkin method that employs a single orthonormal basis for the MOR projection is particularly appealing. In principle, it is a discrete version of the Galerkin method used to replace infinite-dimensional continuous systems by finite-dimensional ones [3].

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The existing techniques for the creation of MOR projection subspaces could be quite roughly classified as the local approximation (LA) methods, like *Krylov subspaces* [4]–[7], and global approximation (GA) methods, like *balancing transformations* [8]–[10]. LA methods exploit the idea of matching the moments of the system transfer function around the localized complex frequency points. On the other hand, GA methods, in principle, perform eigendecomposition of the model governing equations and use the resulting eigenfunctions as a subspace basis. For the same reduction order, the GA methods could provide MOR in a wider frequency range. However, they are applicable only to relatively small problems due to the large computational costs of the eigensystem analysis. There have been attempts to ameliorate the situation by introducing multiple moment-matching points and more efficient balancing transformations [2], as well as combining the effects of LA and GA methods [11].

An efficient alternative to the operator-based eigensystem analysis is the method of proper orthogonal decomposition (POD) [12]. In principle, POD exploits the correlations in the dynamics of the system states under different excitations to determine dominant modes, or *coherent structures*, governing the system behavior. As a MOR tool, POD has been mainly applied to autonomous dynamic systems in time domain. In that case, snapshots of the system states at discrete time points are used to create coherent structures [13]. The application of POD-based MOR in the frequency domain is quite sparse [14]. It is based on the straightforward Fourier transformation of the corresponding time-domain POD subspaces. The main goal of this paper is to propose a concept of *frequency response coherent structures*, a new class of subspaces for MOR obtained in a systematic way from the sinusoidal steady-state frequency responses.

This paper is organized as follows. Section II describes the method of Galerkin projection for MOR of the linear time-invariant systems. A concept of frequency response coherent structures is introduced in Section III. Case study experimental results are given in Section IV and conclusions are presented in Section V.

II. MOR by Galerkin Subspace Projection

The dynamics of linear microwave components and circuits with m inputs and p outputs may be, after spatial discretization or circuit-equation formulation [15], expressed as a time-invariant state-space system

$$C\frac{dx(t)}{dt} = -Gx(t) + Bu(t)$$
$$y(t) = L^T x(t) \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is a vector of internal system states at time t, $u(t) \in \mathbb{R}^m$ and $y(t) \in \mathbb{R}^p$ are vector-valued functions of input and output signals, while $C, G \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$ and $L \in \mathbb{R}^{n\times p}$ are given matrices. It should be emphasized that the model description (1) may be also obtained by linearizitaion of the equations governing the nonlinear microwave components and circuits.

The state vector $x(t)$ in (1) belongs to an n-dimensional linear space or system state space. Let us assume that $x(t)$ could be approximated with sufficient accuracy in a lower dimensional subspace

$$\mathcal{V} = \text{span}\{v_1, v_2, \ldots, v_q\} \tag{2}$$

where $v_i \in \mathbb{R}^n$, $1 \le i \le q$ are linearly independent subspace basis vectors, and $q \ll n$. The state vector $x(t)$ is approximated in the subspace $\mathcal{V}$ by

$$x_{\mathcal{V}}(t) = V x_R(t) \tag{3}$$

where $x_R(t) \in \mathbb{R}^q$ are the coordinates of state vector approximation $x_{\mathcal{V}}(t)$ and

$$V = [v_1\, v_2\, \ldots\, v_q] \in \mathbb{R}^{n\times q} \tag{4}$$

is the *basis matrix* whose columns are the subspace basis vectors.

The Galerkin projection onto the subspace $\mathcal{V}$ requires that after substitution of $x(t)$ by its subspace approximation $x_{\mathcal{V}}(t)$, the residuals of the state-space equation in (1) are orthogonal to the basis matrix V. The result is a reduced-order model

$$\begin{aligned}\underbrace{V^T C V}_{C_R}\frac{dx_R(t)}{dt} &= -\underbrace{V^T G V}_{G_R} x_R(t) + \underbrace{V^T B}_{B_R} u(t)\\ y(t) &= \underbrace{L^T V}_{L_R^T} x_R(t)\end{aligned} \tag{5}$$

having the same structure as (1), but with matrices $C_R, G_R \in \mathbb{R}^{q\times q}$, $B_R \in \mathbb{R}^{q\times m}$, and $L_R \in \mathbb{R}^{q\times p}$, which are reduced in size.

In order to secure the full rank of V, the subspace basis is commonly generated as orthonormal. It should be emphasized that the reduced-order model (5) inherits important numerical range properties of the original model formulation (1) by virtue of the Galerkin projection symmetry. Namely, for $G \ge 0$, $C \ge 0$, and the full rank basis matrix V, the Galerkin projection guarantees $G_R \ge 0$ and $C_R \ge 0$, which is a sufficient condition for passivity (and stability) of the reduced-order model [5].

III. Frequency Response Coherent Structures

The state-space system (1) can also be fully characterized in the real frequency domain by the transfer characteristics

$$Y(j\omega) = L^T (G + j\omega C)^{-1} B \tag{6}$$

where ω is the angular frequency. The most important for MOR is the system impulse response since it uniquely determines any other input waveform. Notice that the transfer characteristics $Y(j\omega) \in \mathbb{C}^{p\times m}$ actually represents the spectrum of the unit impulse response. This spectrum is indirectly defined as

$$Y(j\omega) = L^T X(j\omega) \tag{7}$$

where the complex matrix $X(j\omega) \in \mathbb{C}^{n\times m}$ is the solution of the steady-state frequency response problem

$$(G + j\omega C)\, X(j\omega) = B. \tag{8}$$

Separating $X(j\omega)$ into the real and imaginary part as

$$X(j\omega) = P(\omega) + jQ(\omega) \tag{9}$$

it is obvious that an orthonormal projection basis containing the columns of $P(\omega) \in \mathbb{R}^{n\times m}$ and $Q(\omega) \in \mathbb{R}^{n\times m}$ will preserve the spectrum $Y(j\omega)$ in the reduced-order model formulation. However, it should be emphasized that the state response for a single input impulse excitation is [16]

$$\begin{aligned}x(t) &= \frac{2}{\pi}\int_0^\infty P(\omega)\cos(\omega t)d\omega\\ &= -\frac{2}{\pi}\int_0^\infty Q(\omega)\sin(\omega t)d\omega, \qquad t > 0.\end{aligned} \tag{10}$$

In other words, the state impulse response can be determined by the help of either $P(\omega)$ or $Q(\omega)$. Since $P(\omega)$ also include the static case $\omega = 0$, it is sufficient to look for the orthogonal subspace basis matrix V that contain only the columns of $P(\omega)$, i.e.,

$$P(\omega) \in \text{colspan}\,(V) \tag{11}$$

in the whole frequency range of interest.

The method of POD [12] provides a powerful and nearly optimal way to express the distributed function space $P(\omega)$ by an orthonormal subspace basis. For computational efficiency reasons, POD is applied in practice to the finite set of the function space samples instead of the continuous space. To this end, $P(\omega)$ is represented by a data ensemble

$$P = [P(\omega_1)\, P(\omega_2)\, \ldots\, P(\omega_r)] \in \mathbb{R}^{n\times mr} \tag{12}$$

which is obtained by sampling $P(\omega)$ at r selected frequencies. Given the data ensemble P, POD aims at creating an orthonormal subspace basis $\mathcal{V}$ that minimize the error estimate

$$e_{\mathcal{V}} = \sum_{k=1}^{r} \|P(\omega_k) - P_{\mathcal{V}}(\omega_k)\|^2 \tag{13}$$

being the least square measure of the distance between the data ensemble P and its approximation $P_{\mathcal{V}}$ in the subspace $\mathcal{V}$. The data ensemble P should be created to capture as much of the transfer characteristic frequency dependence as is required to represent it in the reduced-order model.

It should be emphasized that there are three different ways to realize POD [17], which are: 1) Karlhunen–Loève decomposition (KLD); 2) principal component analysis (PCA); and 3) singular value decomposition (SVD). Due to simplicity and wide availability, SVD has been adopted in this paper to evaluate the POD subspaces from data ensemble P. The SVD–POD procedure steams from the observation that the POD basis vectors are also the leading eigenfunctions of the correlation matrix

$$R = PP^T \in \mathbb{R}^{n\times n}. \tag{14}$$

In practice, it is not necessary to evaluate the correlation matrix R and its eigenfunctions. Instead, it is sufficient to subject the data ensemble P to the economic SVD [18]

$$P = U\Sigma W^T \tag{15}$$

where Σ is a diagonal matrix

$$\Sigma = \mathrm{diag}(\sigma_1, \sigma_2, \ldots, \sigma_{mr}) \tag{16}$$

with singular values $\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_{mr} \geq 0$, while $U \in \mathbb{R}^{n\times mr}$ and $W \in \mathbb{R}^{mr\times mr}$ are orthogonal matrices ($U^T U = I$ and $W^T W = I$, where I is the identity matrix). The subspace basis of frequency response coherent structures is simply obtained as

$$V = U(:, 1:q) \tag{17}$$

i.e., taking the first q ($q < mr$) columns of matrix U as the subspace basis matrix V.

The two most important features of the resulting subspace basis are optimality and controllability. Namely, the subspace (17) provides the best q-dimensional approximation to the ensemble P [19]. Moreover, the minimum value of $e_{\mathcal{V}}$ over the q-dimensional subspace is bounded as [20]

$$e_{\mathcal{V}} \geq \sum_{i=q+1}^{mr} \sigma_i^2 \tag{18}$$

which may be employed as an estimate of the achievable accuracy in approximating a data ensemble P in the subspace of frequency response coherent structures. Nevertheless, the fidelity of the resulting reduced-order model still essentially depends on the selection of the data ensemble P and its ability to capture dominant modes of the system behavior.

With a large variety of MOR techniques proposed in various fields, it is worthwhile to point out here some differences and links of the frequency response coherent structures to other projection subspaces. Notice that the complex matrix $X(j\omega)$ in (8) also defines zeroth-order moments of the system transfer function in the real frequency domain. In that sense, the proposed MOR technique may be interpreted as a trivial (zero-moment) multipoint moment-matching method that employs a POD–SVD orthogonalization procedure to create the nearly optimal subspace basis. The SVD has been used before in connection with the POD-based MOR in the time domain [13] and also, outside of the POD framework, to improve orthogonalization of the block Krylov subspaces [21]. The current method should also be distinguished from the frequency-domain POD [14] that employs the complex data ensemble (obtained by the Fourier transformation) and KLD-based eigenvalue analysis to obtain the subspace basis. One of the benefits of the harmonic frequency response framework is the possibility to apply the economic SVD only to the real part of the impulse response spectrum.

IV. Case Study

As a practical example, the proposed MOR technique is applied here to create and test reduced-order models for the resistive and inductive behavior of an integrated planar spiral transformer. The geometry and the detailed state-space system of the integrated planar transformer are created by the three-dimensional (3-D) simulator FastHenry [22], a program for extraction of frequency-dependent inductances and resistances among conductors of different shapes. The conductors of complex shapes are represented in FastHenry as a sequence of piecewise-straight segments connected together at common nodes. Fig. 1 shows such a skeleton geometry for the integrated planar transformer positioned over the ground plane that is used in this case study. Three turns of 8-μm-wide metal lines are used to create the primary and secondary coils of the planar integrated transformer covering the total area of 200×200 μm^2.

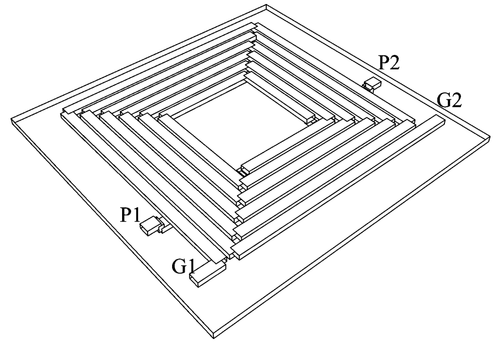

Fig. 1. Two sequences of straight metal segments, i.e., a primary coil P1-G1 and secondary coil P2-G2, that from a two-port planar transformer over the ground plane.

The program FastHenry employs an integral formulation of the equations governing magnetoquasi-static coupling among the metal filaments [22]. In order to account for the variations of the current density, all metal line segments, including those used to represent the ground plane, are partitioned along the length and cross section. To this end, the metal lines of the transformer in Fig. 1 has been partitioned into 2824 discrete filaments. The system state variables are mesh currents where the mesh is any independent closed loop of the filaments. The system matrix is obtained by enforcing the Kirchhoff's voltage law to the meshes. The dimensions of the resulting state-space system (1) for the integrated planar transformer in Fig. 1 are $m = 2$, $p = 2$, and $n = 2582$. A quite convenient access to the FastHenry internal data structures has been used to assemble system matrices C, G, B, and L in (1) for the purpose of MOR. The linear solvers for the frequency-response analysis, as well as the economic SVD procedure required to create the subspace of frequency response coherent structures, are provided externally. The SVD–POD procedure is based on $r = 25$ discrete frequency points with logarithmical distribution in the frequency range of 10^6–10^{11} Hz. The resulting data ensemble P has $mr = 50$ samples.

It should be emphasized that FastHenry is itself equipped with the state-of-the-art MOR procedure. It belongs to the class of moment-matching methods based on the orthogonal Krylov subspaces generated via the Arnoldi process [23]. The resulting Krylov subspace with q basis vectors in principle ensures that $q - 1$ moments of the reduced-order transfer function match $q - 1$ moments of the original problem transfer function around the selected single complex frequency. The MOR method implemented in FastHenry provides the moment matching around

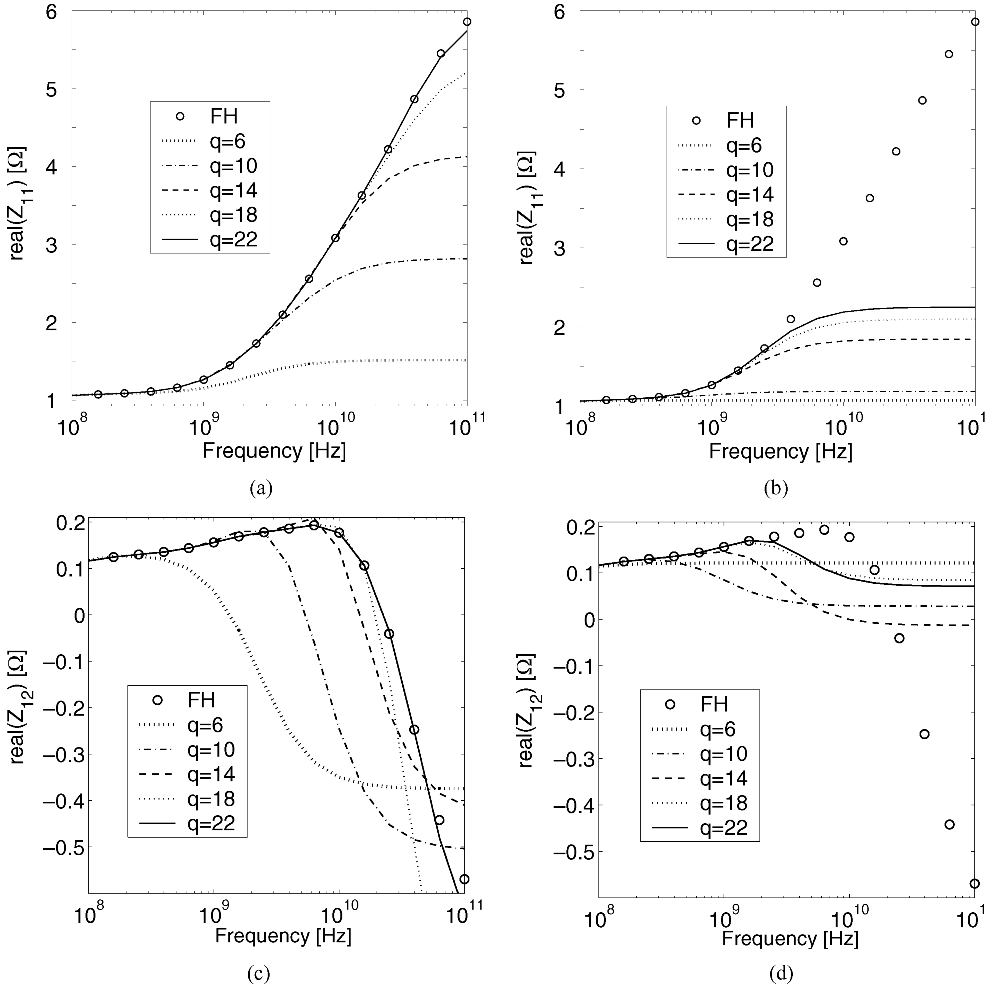

Fig. 2. Frequency dependence for the real part of the planar transformer Z_{11}- and Z_{12}-parameters. (a) Frequency response coherent structures. (b) Krylov subspace. (c) Frequency response coherent structures. (d) Krylov subspace.

frequency $\omega = 0$ and it is used here without modifications for comparison. The choice $\omega > 0$ (or certain complex frequency) might result in Krylov subspaces with better accuracy at high frequencies. However, in the microwave problems involving mutual inductances, the system matrix C is typically dense. For an arbitrary selected complex frequency, the Krylov subspace procedure would require *LU* factorization of the dense matrix instead of just matrix-vector products used for the expansion around $\omega = 0$ [11]. The reduced-order models generated by FastHenry are exported in the form of SPICE compatible equivalent circuits and verified by the general-purpose circuit simulator WinSpice3.[1]

Fig. 2 shows the frequency dependence for the real parts of the planar inductor Z_{11}- and Z_{12}-parameters obtained by FastHenry, as well as by reduced-order models of different complexity. The circle symbols in Fig. 2 denote the FastHenry results (denoted here as FH), but also the sampling frequencies used for the generation of the frequency response coherent structures. Similarly, Fig. 3 shows a frequency dependence of the primary coil inductance L_{11} (very similar results are also obtained for the secondary coil) and the mutual inductance L_{12} obtained by FastHenry and by the reduced-order models of different complexity. Notice that, for clarity, only the results in the range of 10^8–10^{11} Hz, where significant deviations of the reduced models occurs, are shown in Figs. 2 and 3.

The principle difficulty in the reduced-order modeling of planar transformers (and inductors) is to accurately represent the energy dissipation at high frequencies. In the detailed simulation by FastHenry, these losses are physically attributed to ohmic and eddy currents in metal lines having nonzero resistivity, as well as eddy currents in the underlying substrate. The current crowding in the conductors due to the skin and proximity effects makes the planar transformer losses highly frequency dependent. The effects of the various transformer losses are best visible in the real values of its Z-parameters. Notice from Fig. 2(a) and (c) that reduced-order models obtained using frequency response coherent structures are converging quite fast toward the FastHenry results with the increase of the subspace size. On the other hand, Fig. 2(b) and (d) again demonstrates the well-known problem of reduced-order models based on moment matching around single complex frequency point (including classical Krylov subspace projection) to capture current crowding effects at microwave frequencies [11]. Namely, these methods in principle tend to capture moments near expansion frequency $\omega = 0$. Very similar results are also obtained for the real part of the planar transformer's Z_{22}-pa-

[1]WinSpice3, 2002. [Online]. Available: http://winspice.co.uk

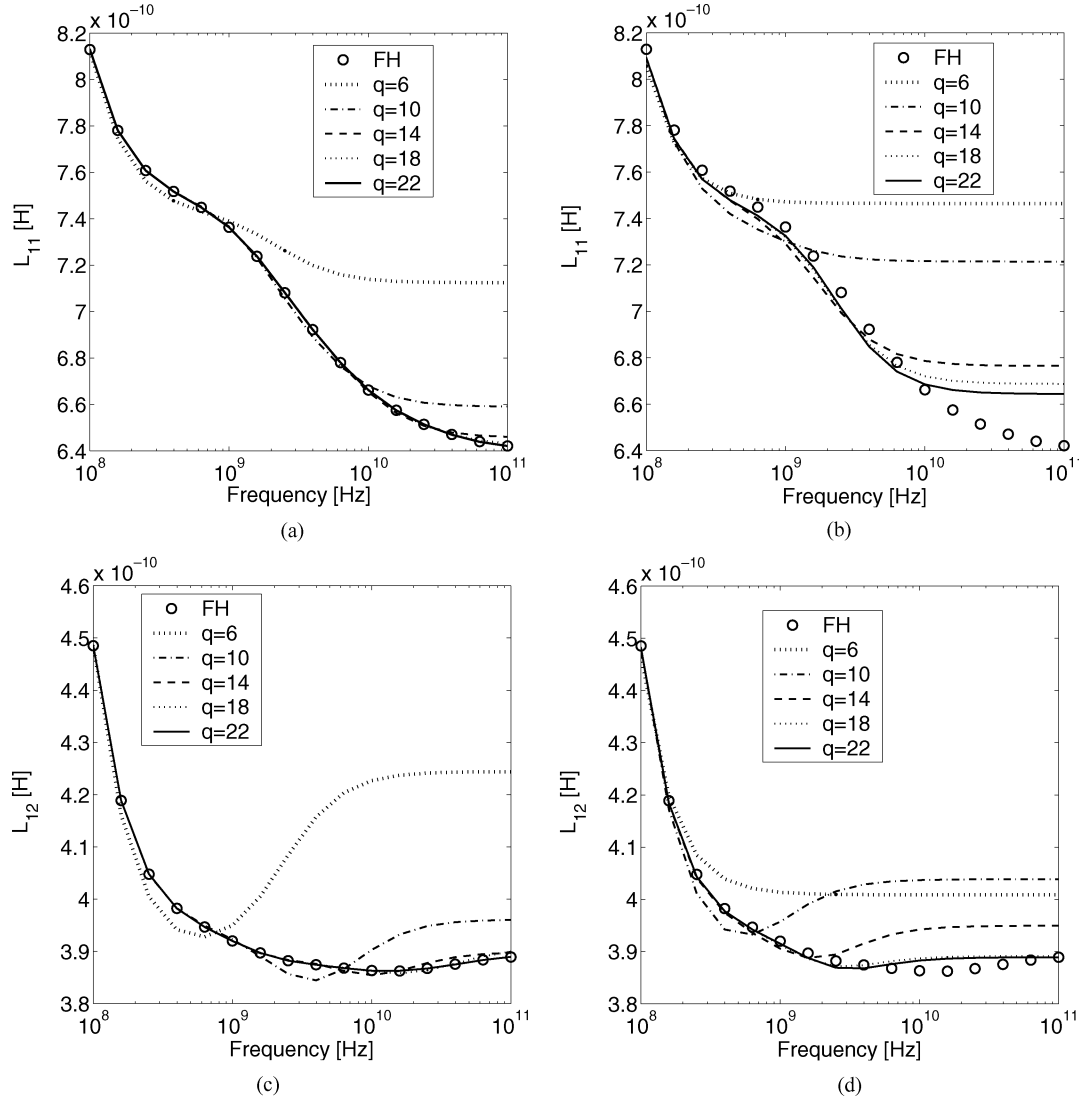

Fig. 3. Frequency dependence for the primary coil inductance L_{11} and mutual inductance L_{12} of the planar transformer. (a) Frequency response coherent structures. (b) Krylov subspace. (c) Frequency response coherent structures. (d) Krylov subspace.

rameter. Generally, the self-inductance and mutual inductance of the planar transformers are less sensitive to the current crowding effects, and reduced-order models typically capture frequency dependence of the inductances with much smaller relative error then the corresponding resistive components. Nevertheless, it is again possible to observe monotonous and quite fast convergence of the reduced-order models based on the frequency response coherent structures.

V. Conclusions

A practical approach to the creation of effective subspaces for stable and accurate MOR in the wide frequency range has been presented. It has been introduced in the framework of the Galerkin subspace projection. The projection subspaces are created by an SVD-based POD procedure in the from of frequency response coherent structures.

The proposed MOR approach is very simple to implement. It requires only a set of state vector phasors from the sinusoidal steady-state frequency response and the standard linear algebra economic SVD procedure. The computational cost of a single frequency sweep is certainly higher than the cost to generate Krylov subspaces (that involves only matrix-vector products). However, it is still significantly smaller then the cost of balanced truncation methods. Using an advanced preconditioned iterative method for the solution of the detailed model equations, the cost of the frequency sweep could be only linearly proportional to the problem size n. The selection of the test real frequency points is the one-dimensional problem and is more straightforward in comparison to the two-dimensional selection of the multiple expansion complex frequency in the Krylov subspace methods. However, the optimal distribution of the sampling frequency points is left as an open problem for further research. It has been practically demonstrated that the frequency response coherent structures provide faster convergence then the standard Krylov subspace methods with single expansion complex frequency.

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