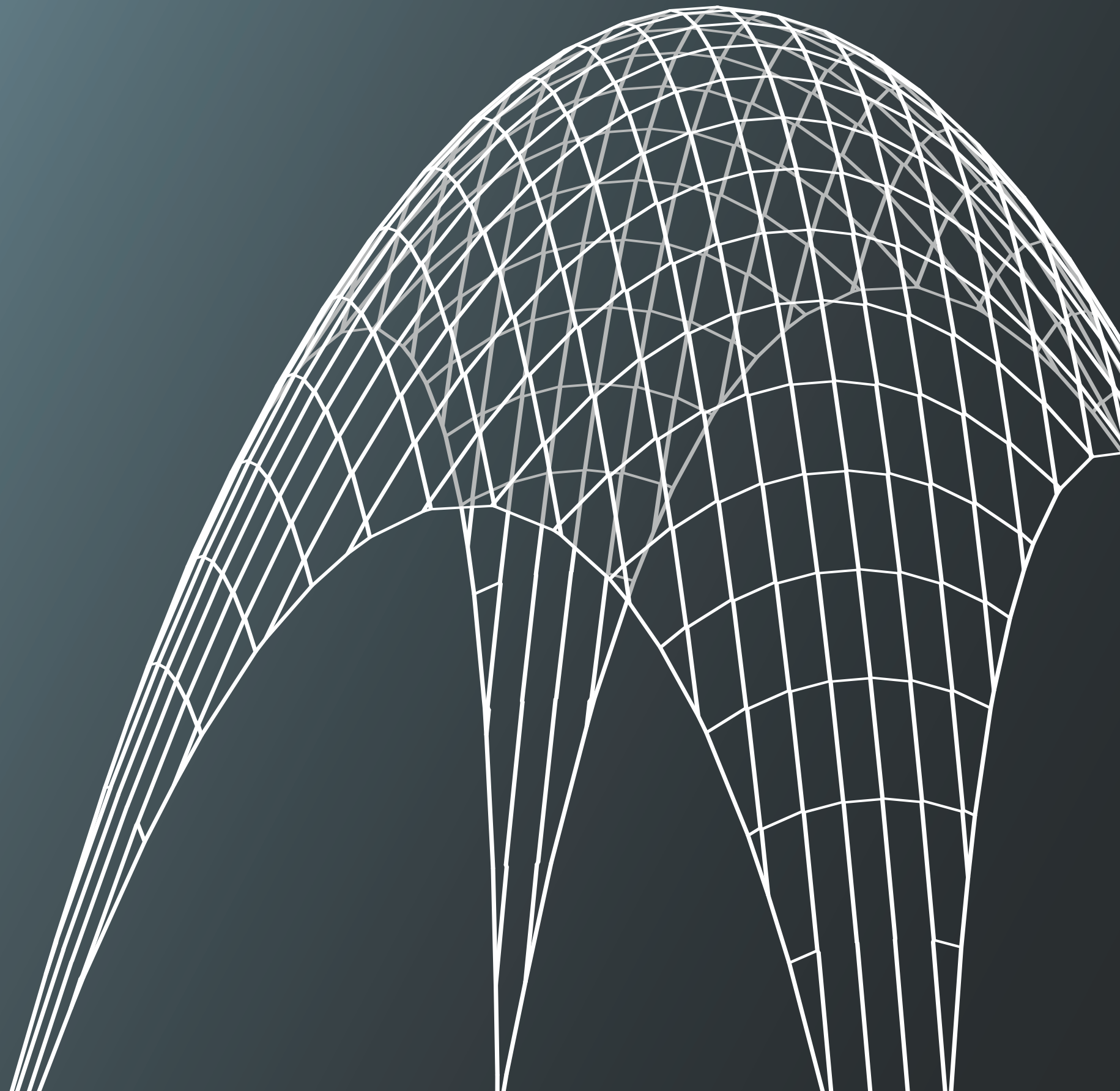


GRADUATION PRESENTATION

Pim Buskermolen, 29 January 2019

First mentor: **Ir. A. Borgart** (Structural Mechanics)
Second mentor: **Dr. M. Turrin** (Design Informatics)
Delegate Board of Examiners: **Drs. A. Mulder**

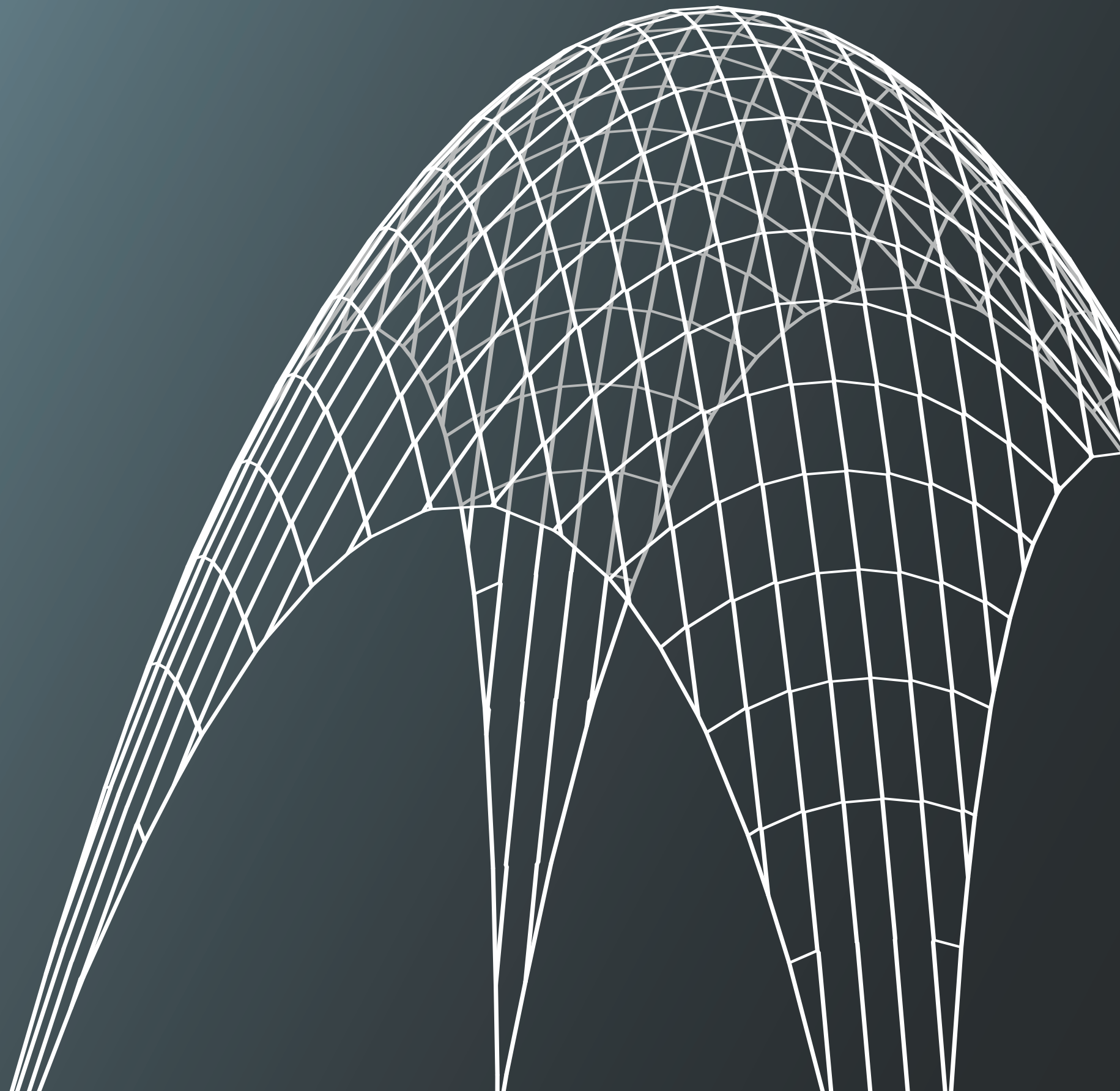
*Master track Building Technology
Faculty of Architecture and the Built Environment
Delft University of Technology*



SHELL STRUCTURES

First mentor: **Ir. A. Borgart** (Structural Mechanics)
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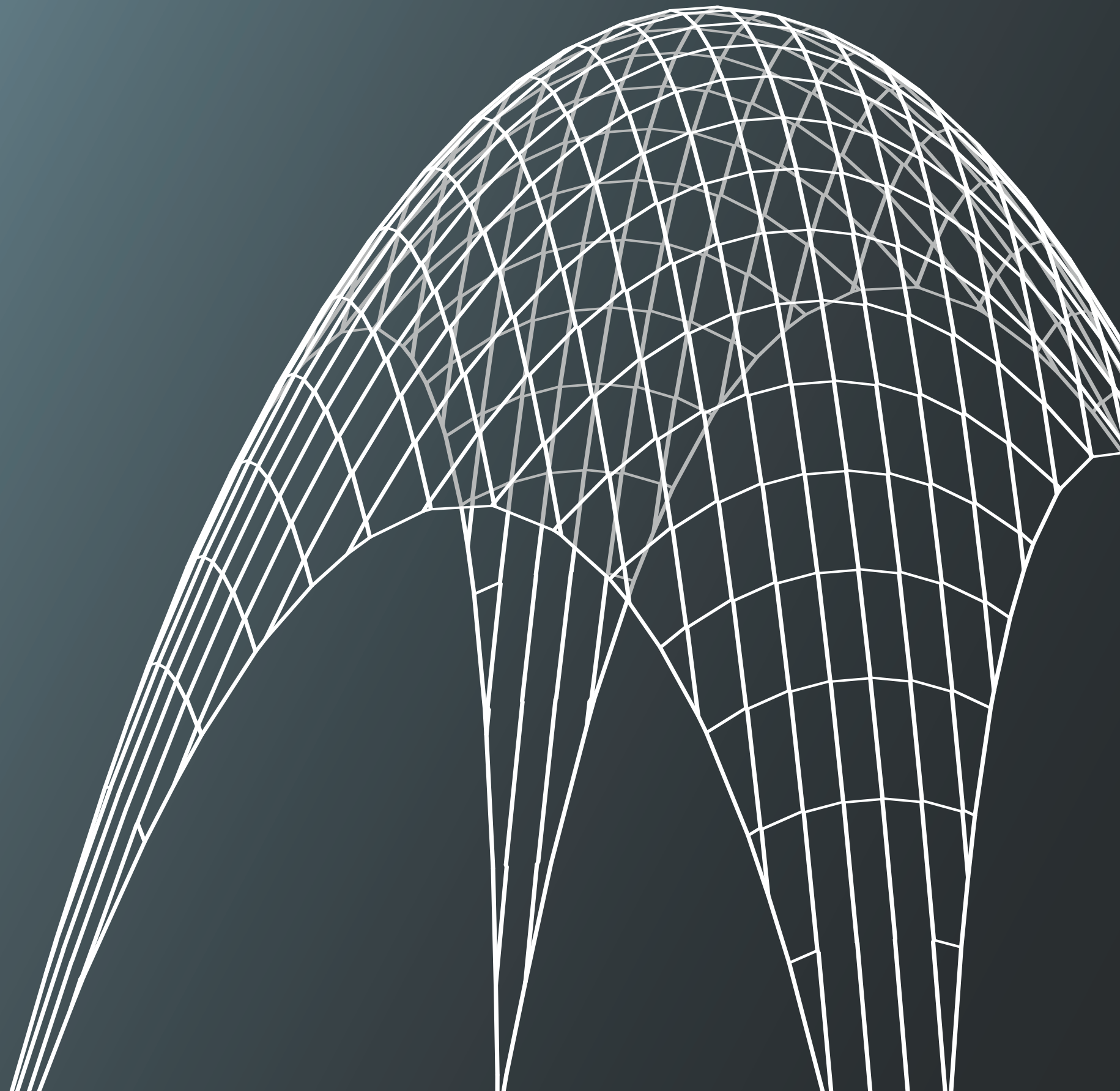


SHELL STRUCTURES

On the relationship between
moment hills, stress functions
and thrust surfaces in the
design of shell structures

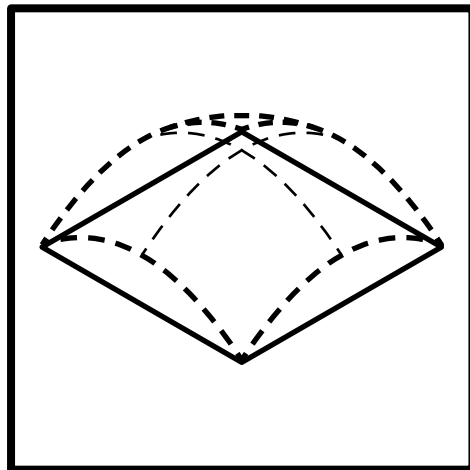
First mentor: **Ir. A. Borgart** (Structural Mechanics)
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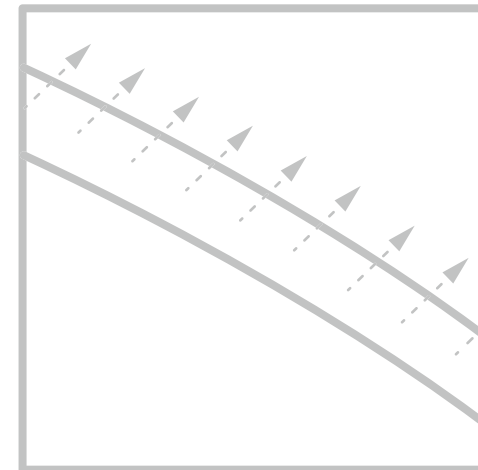
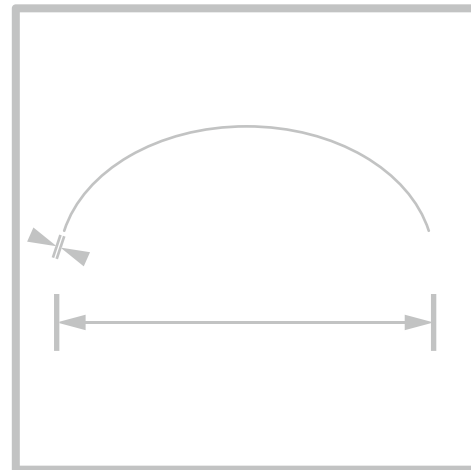


WHAT ARE SHELL STRUCTURES?

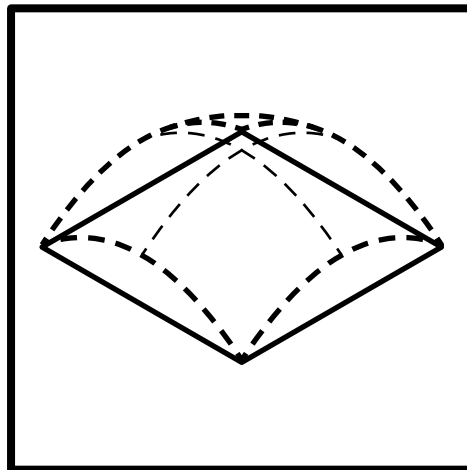
WHAT ARE SHELL STRUCTURES?



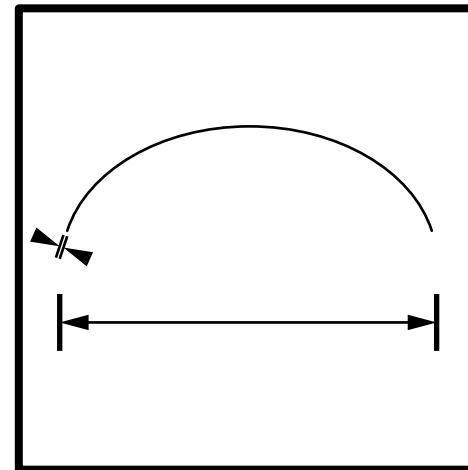
Curved plates



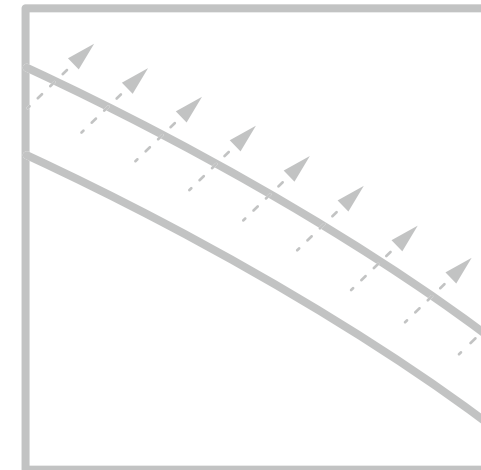
WHAT ARE SHELL STRUCTURES?



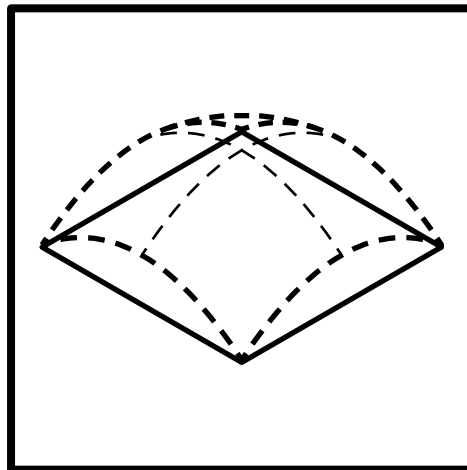
Curved plates



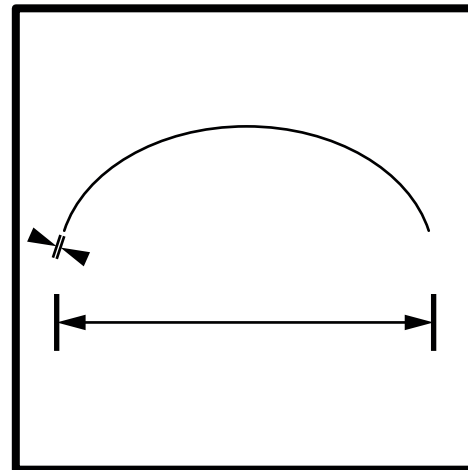
Large span to thickness ratio



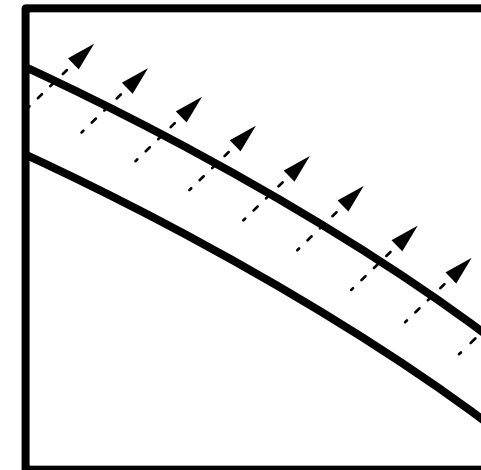
WHAT ARE SHELL STRUCTURES?



Curved plates



Large span to thickness ratio



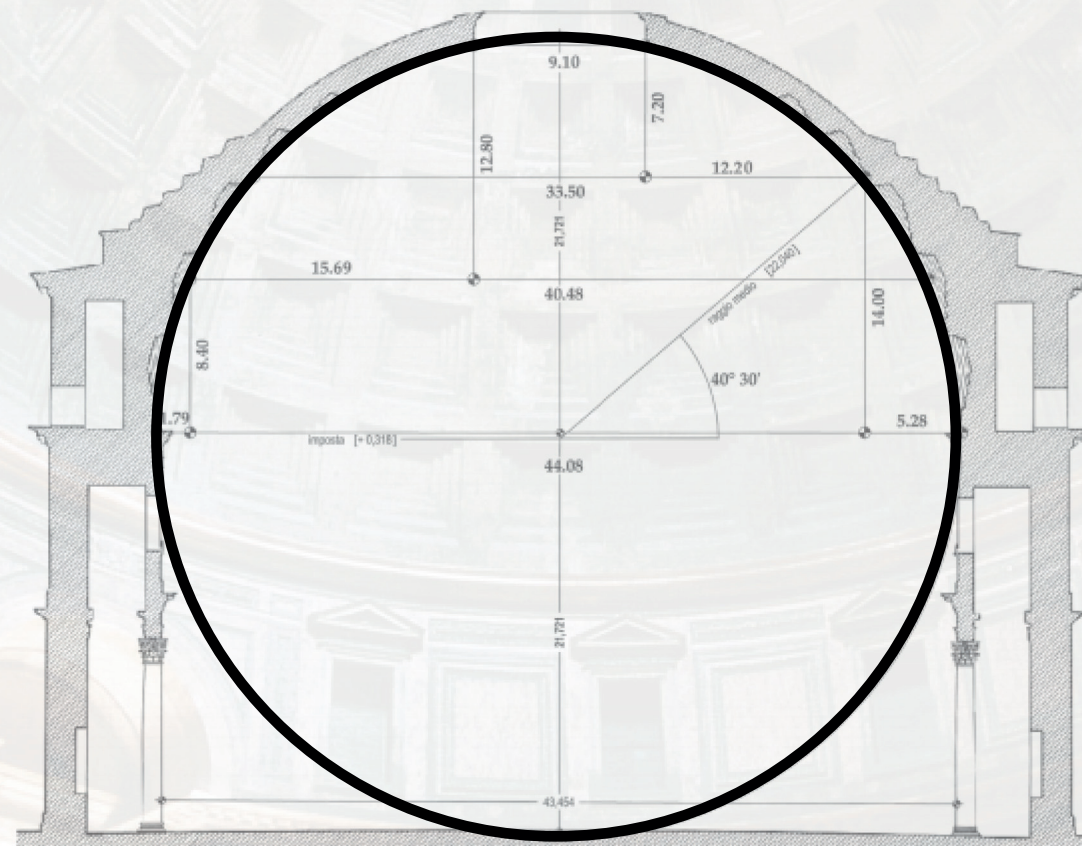
Primarily membrane forces



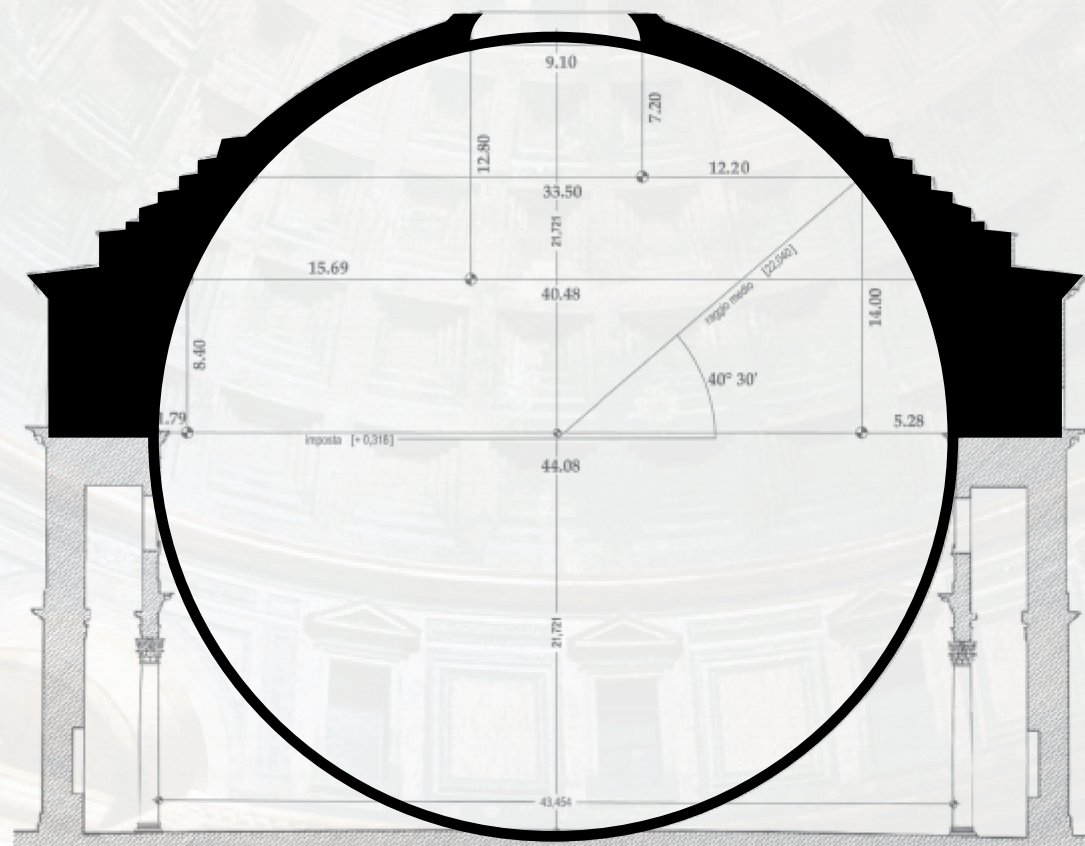
Pantheon // Apollodorus of Damascus
Rome, Italy (125 A.D.)



Pantheon // Apollodorus of Damascus
Rome, Italy (125 A.D.)



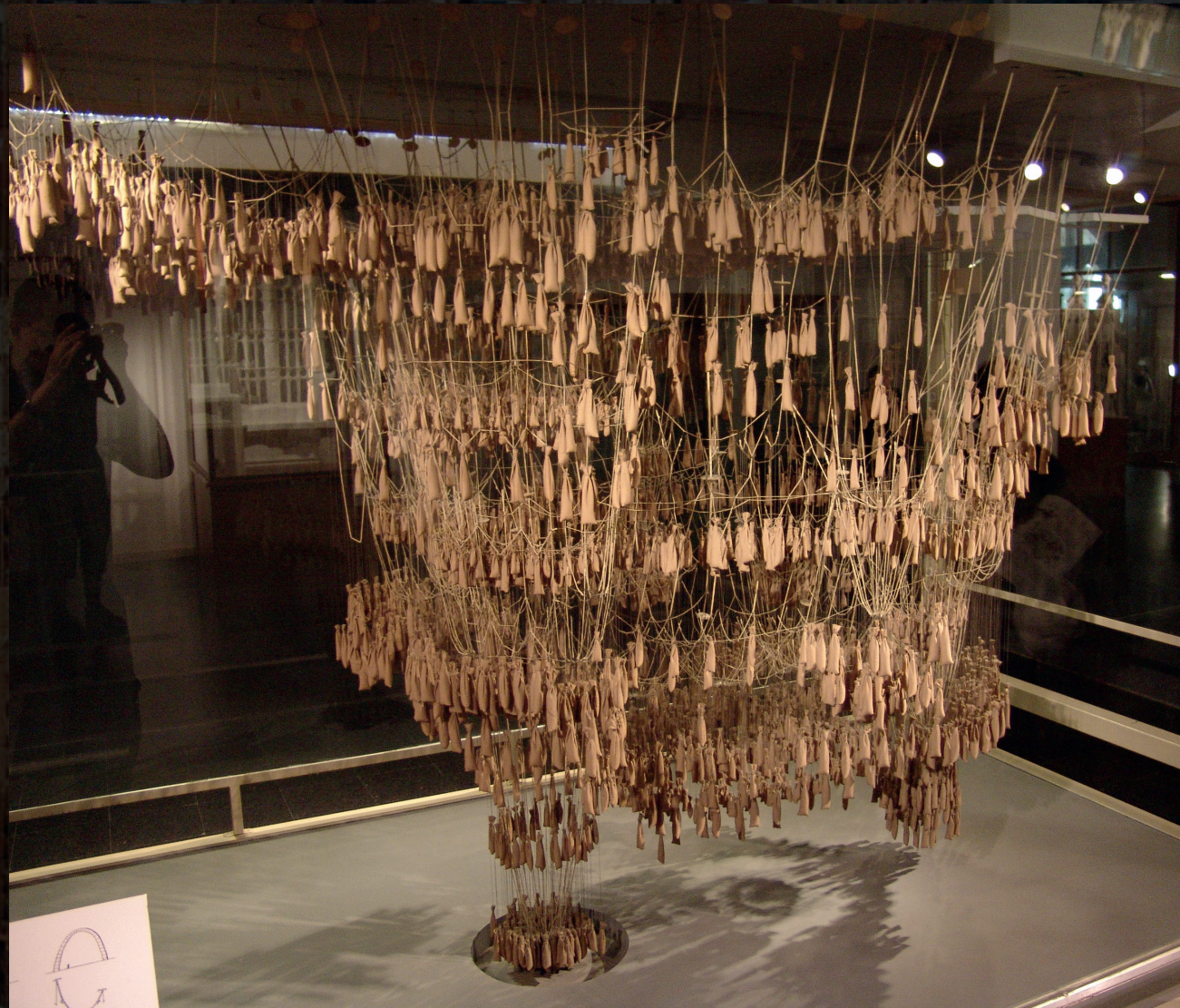
circular section



circular section



Sagrada Família // Antoni Gaudí
Barcelona, Spain (1882 - start of construction)



hanging chain models

Sagrada Família // Antoni Gaudí
Barcelona, Spain (1882 - start of construction)



Norwich Sports Centre // Heinz Isler
Norwich, UK (1991)



hanging fabric model

Norwich Sports Centre // Heinz Isler
Norwich, UK (1991)

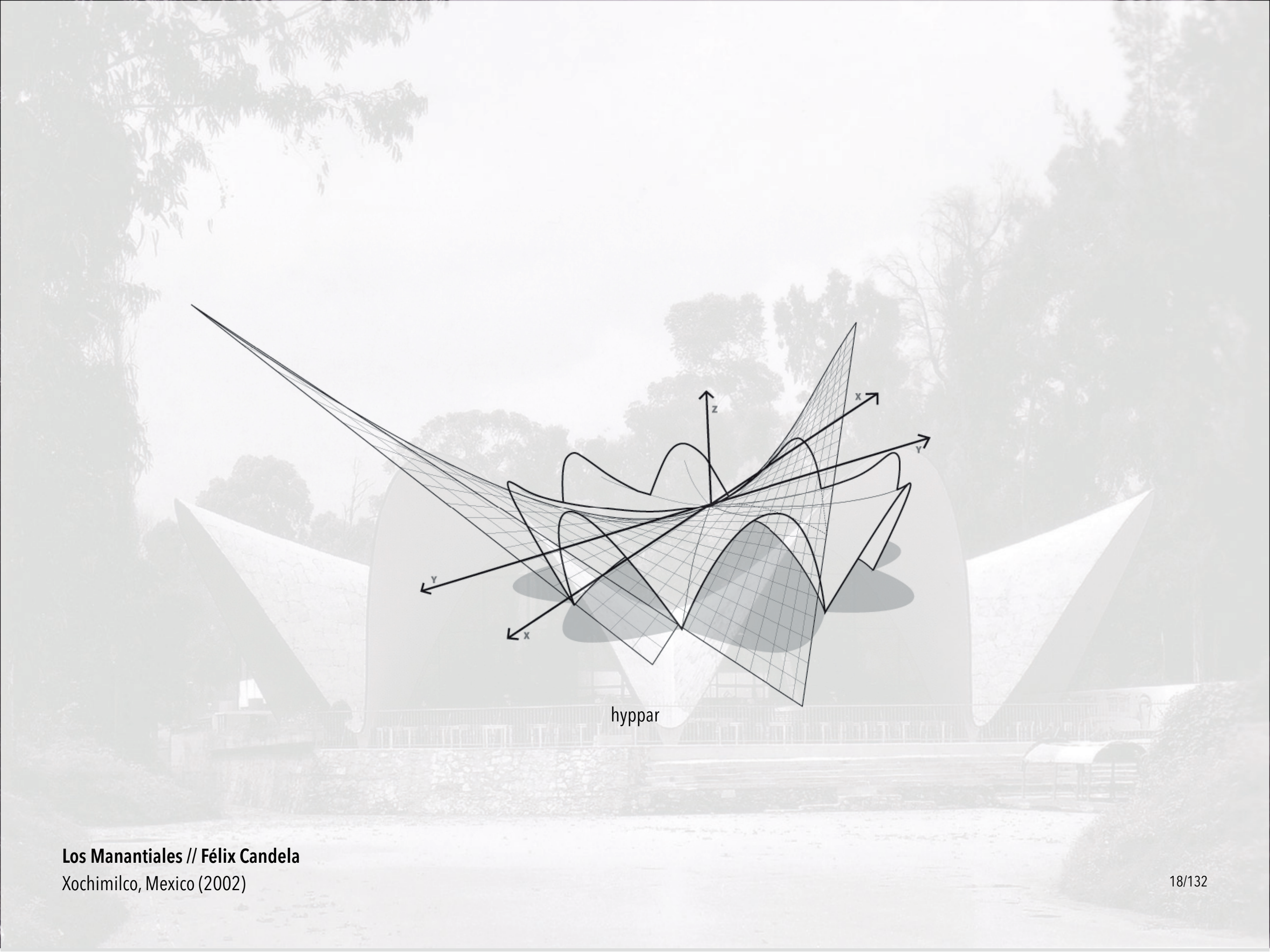


Restaurante Submarino

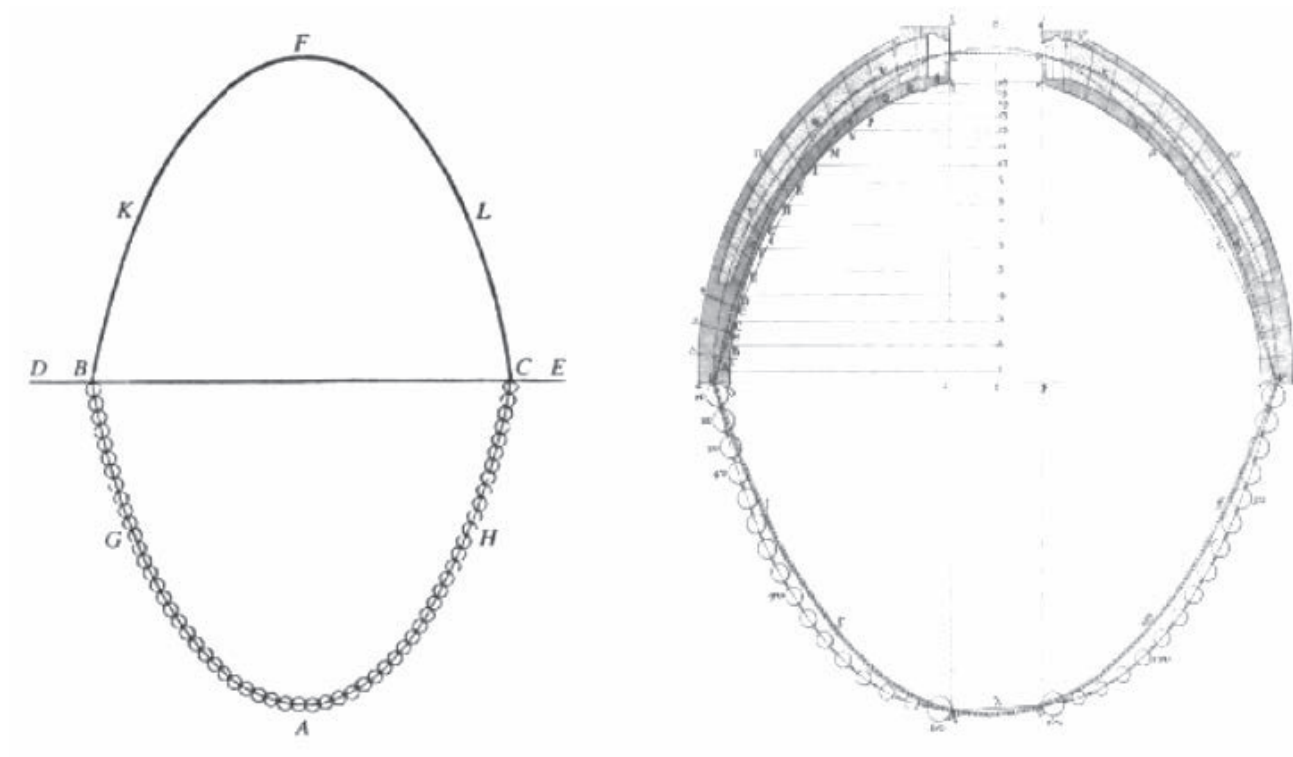
L'Oceanogràfic, Félix Candela
Valencia, Spain (2002)

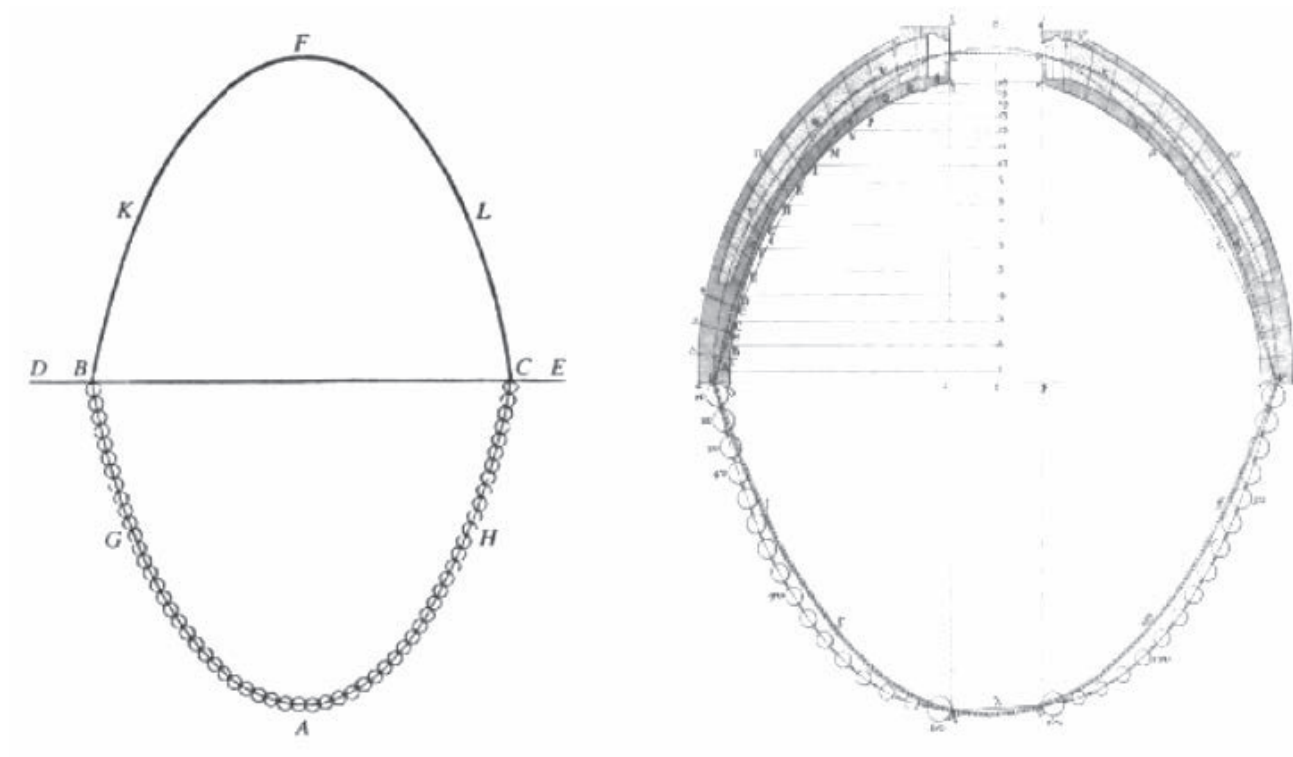


Los Manantiales // Félix Candela
Xochimilco, Mexico (2002)

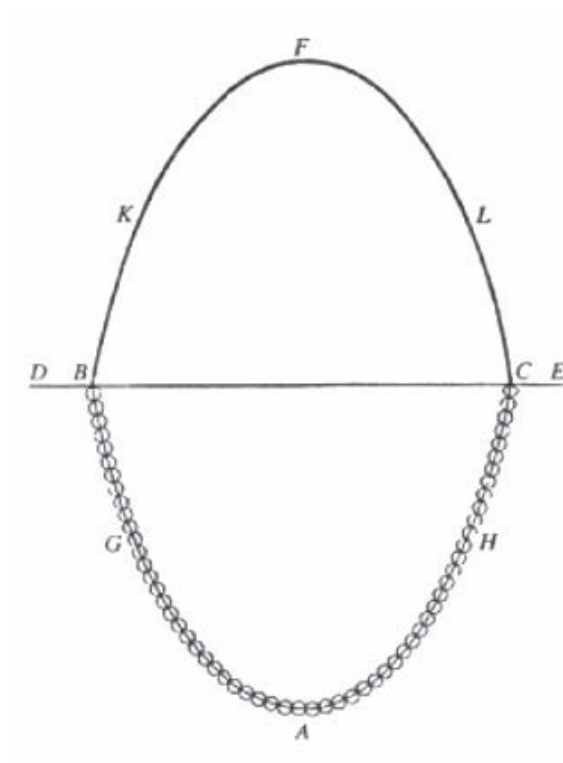


hyppar

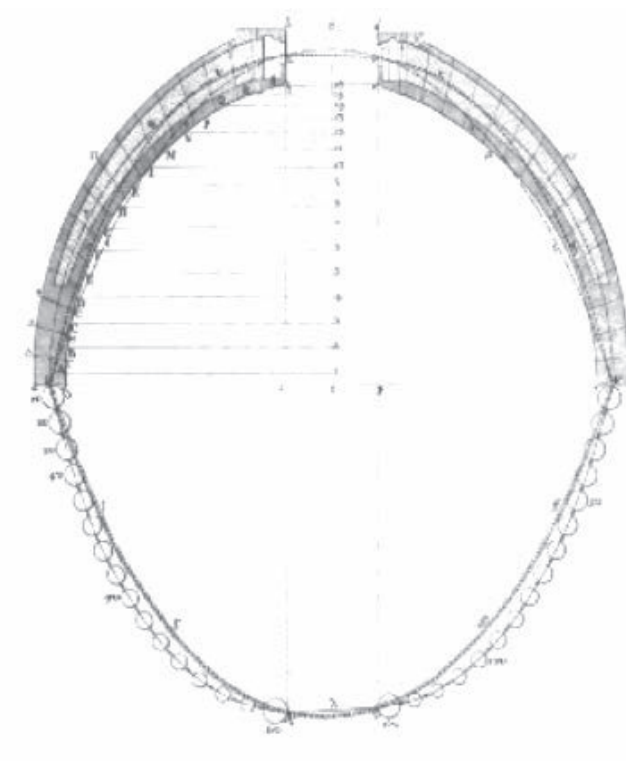




↓
hanging catenary



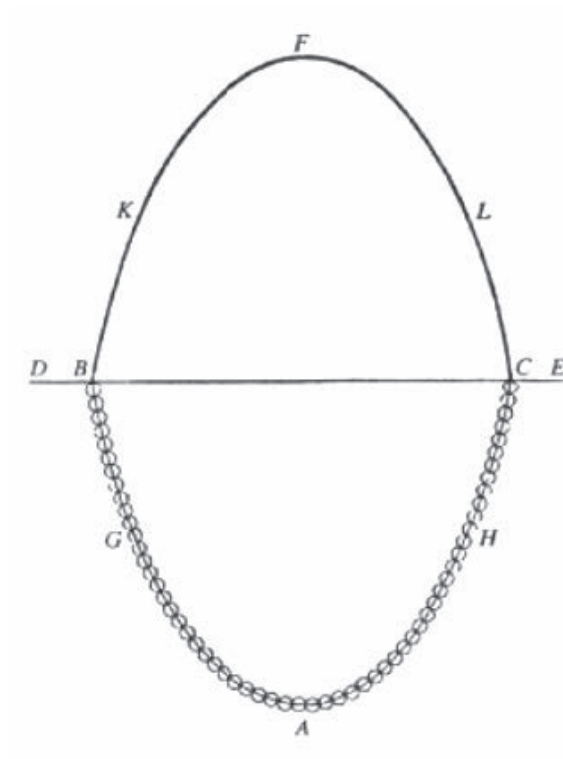
hanging catenary



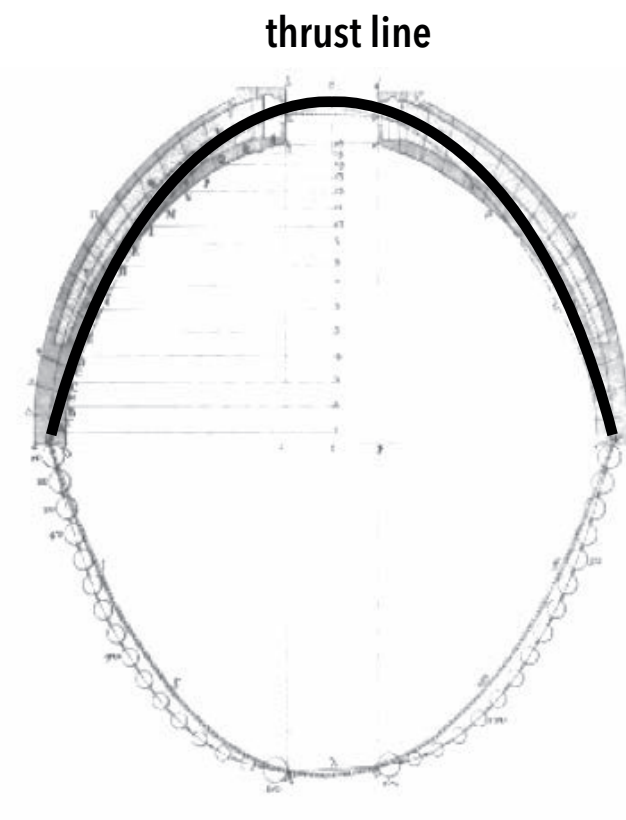
standing arch

*"As hangs the flexible
line, so but inverted will
stand the rigid arch"*

- Hooke, 1675



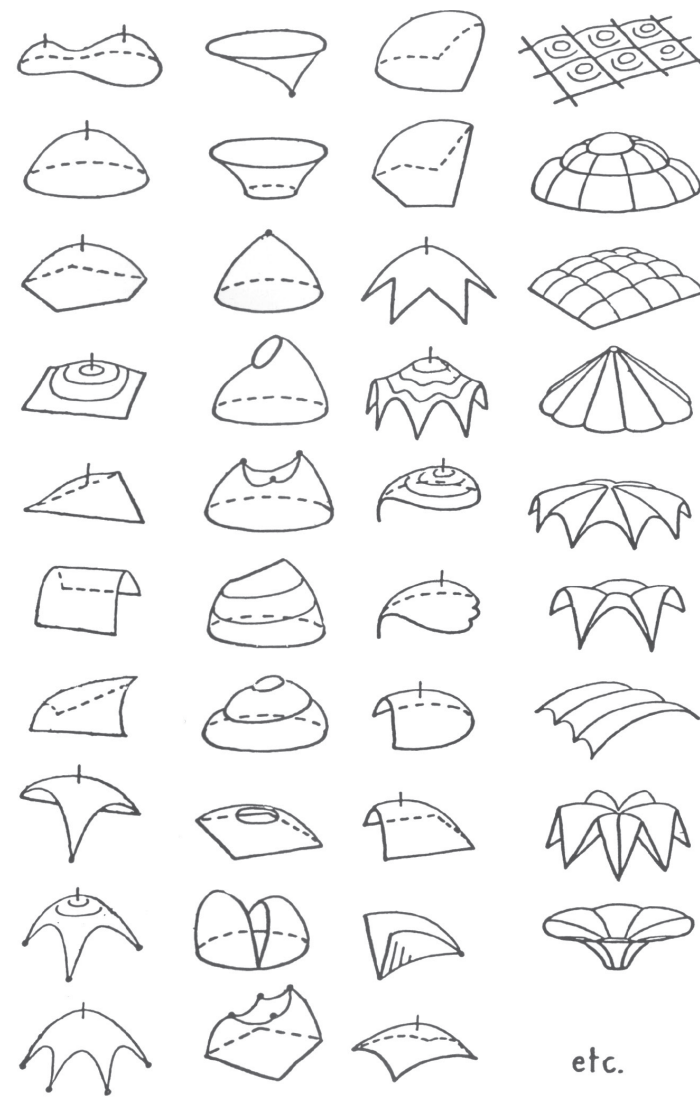
hanging catenary
(tension)



standing arch
(compression)

*"As hangs the flexible
line, so but inverted will
stand the rigid arch"*

- Hooke, 1675



thrust surface

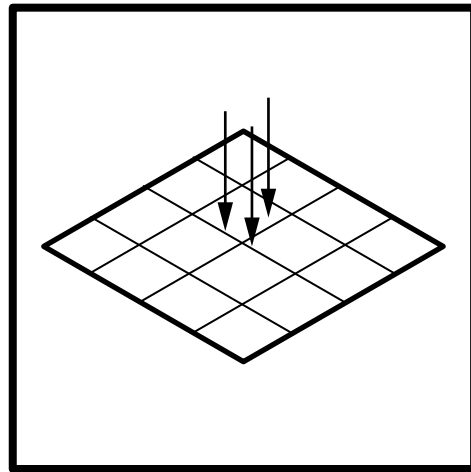
RESEARCH QUESTION

'How can the relation between the **geometrical** and **mechanical** properties of shell structures be described and result in an intuitive and graphical design tool for (structural) designers?'

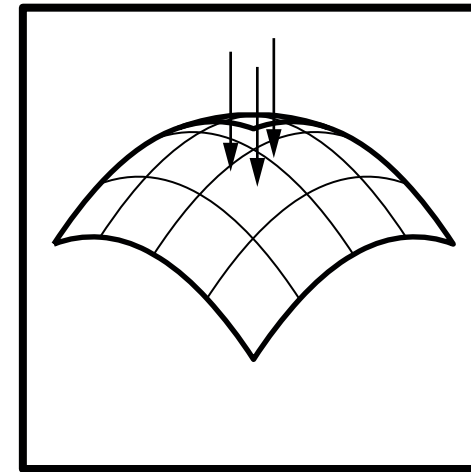
CONTENTS

- 1 Properties of shells
- 2 Existing methods
- 3 Plate and shell analogies
- 4 Parametric design tools
- 5 Conclusions

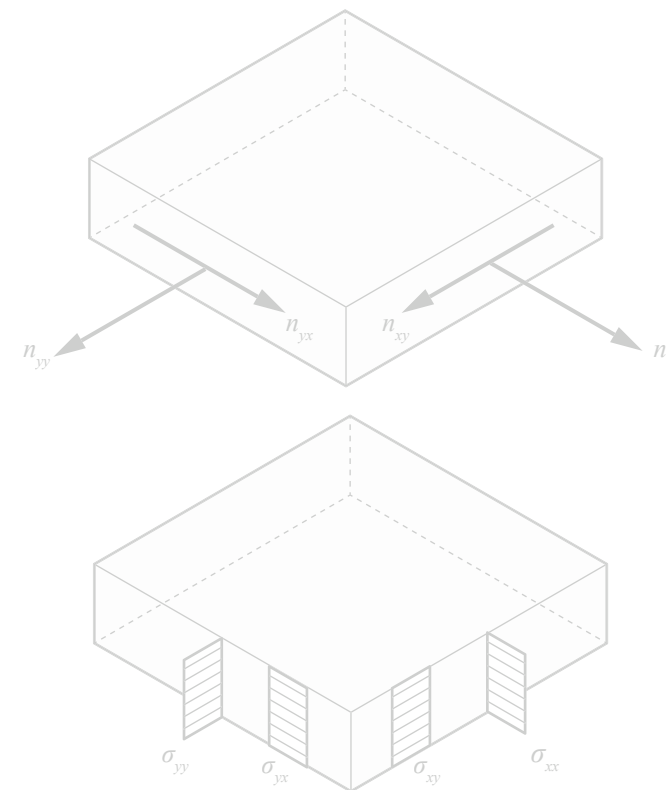
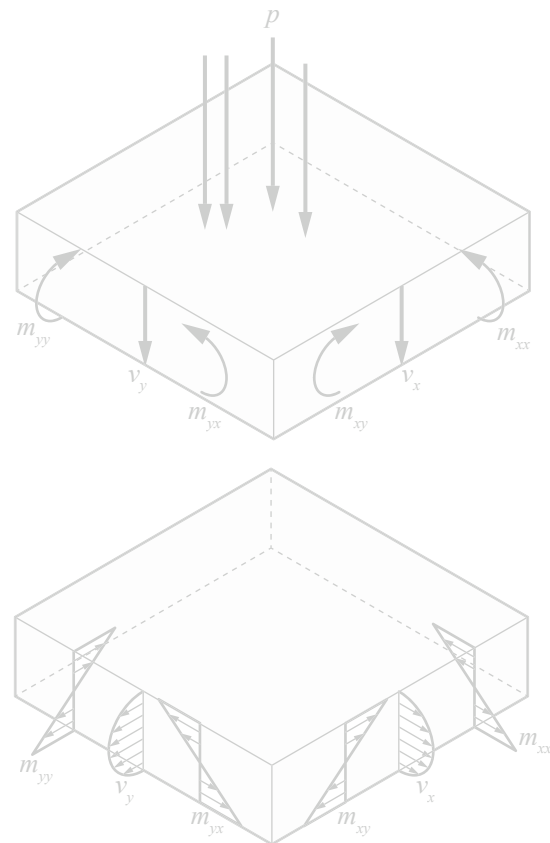
PROPERTIES OF SHELLS

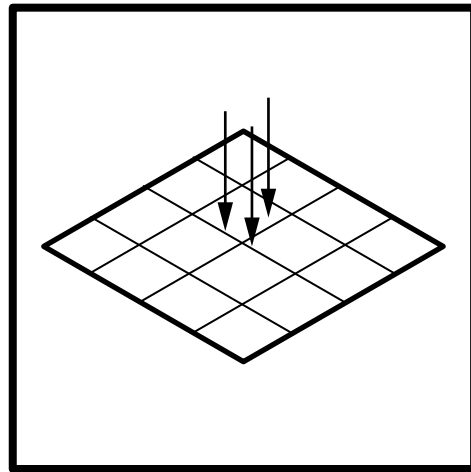


plate

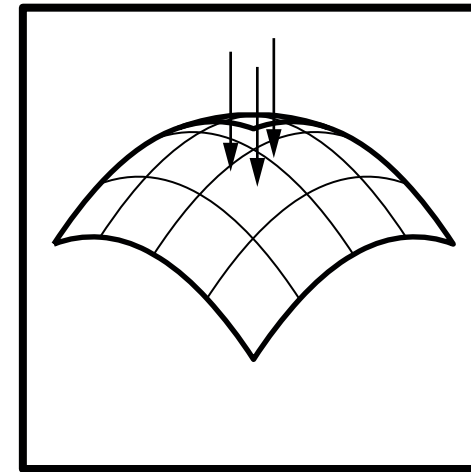


shell



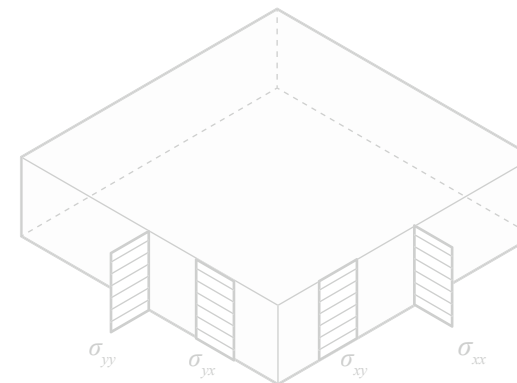
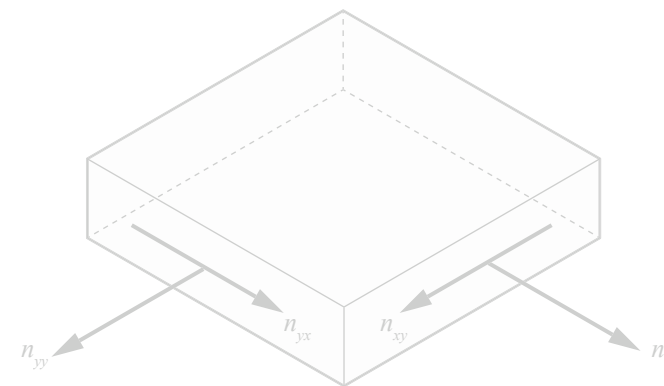
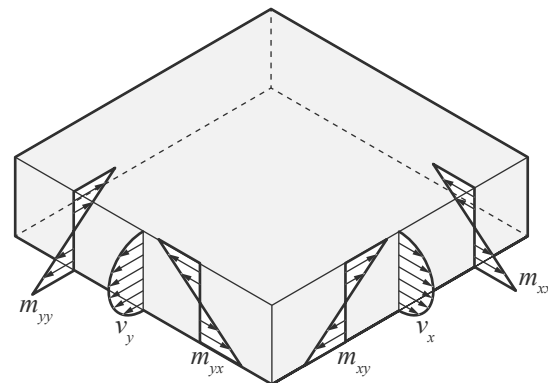
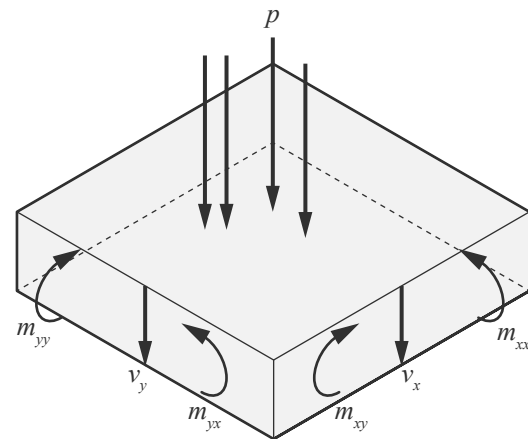


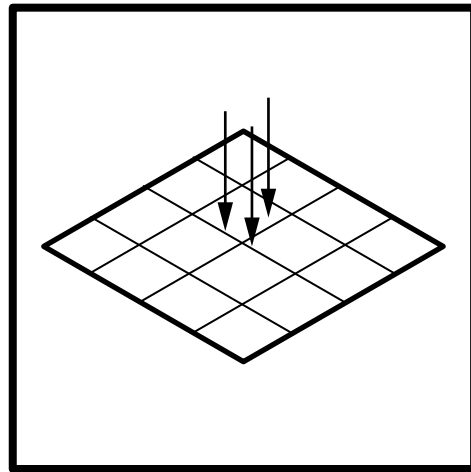
plate



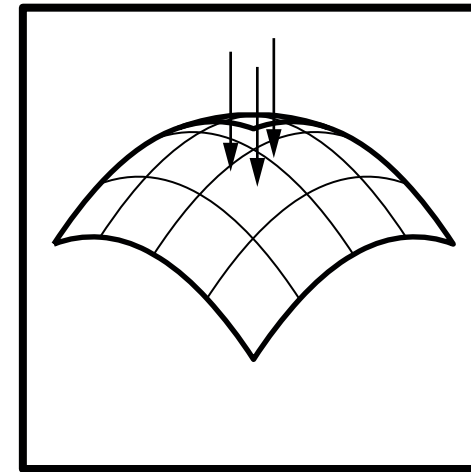
shell

shear forces that
cause bending/
torsional moments



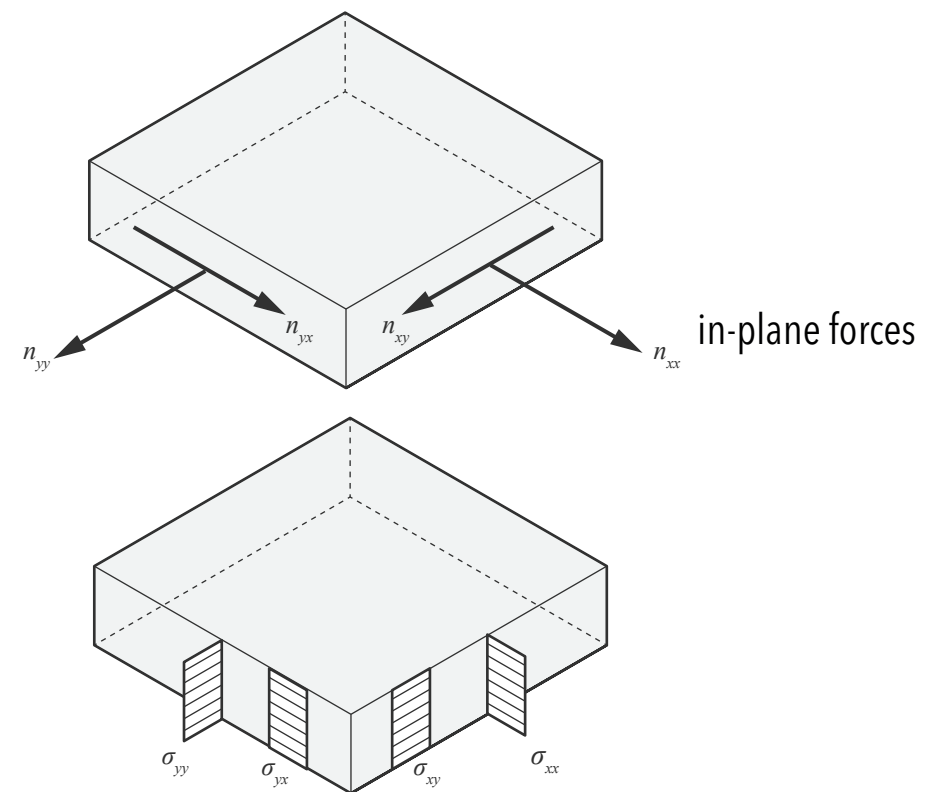
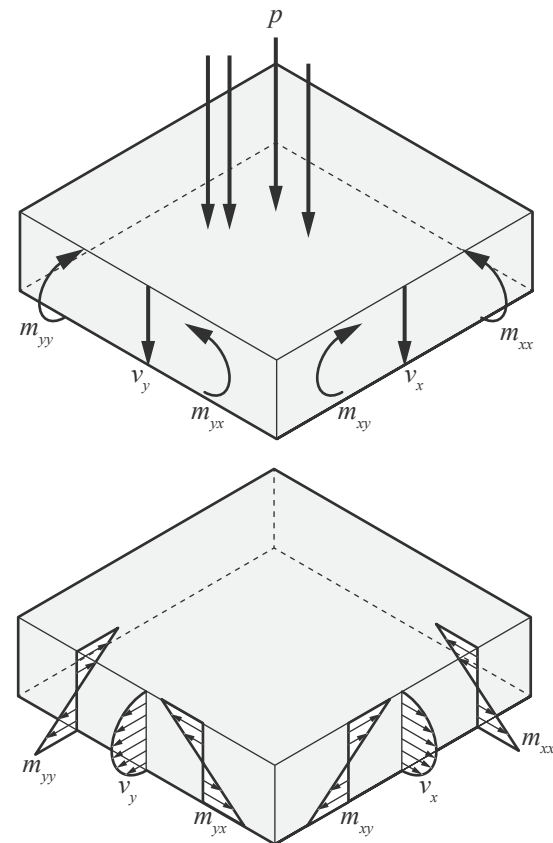


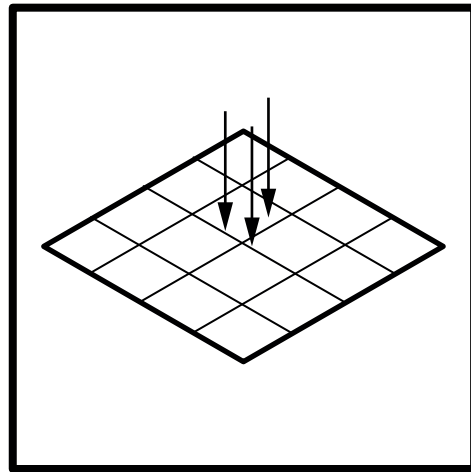
plate



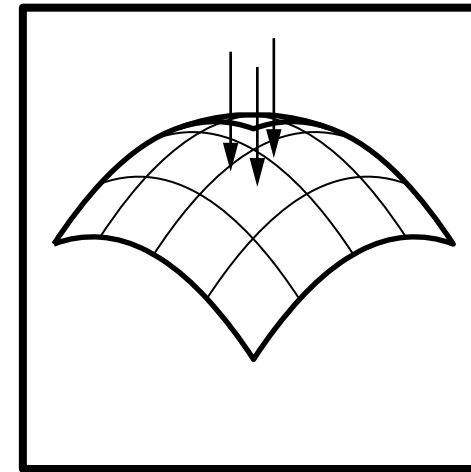
shell

shear forces that
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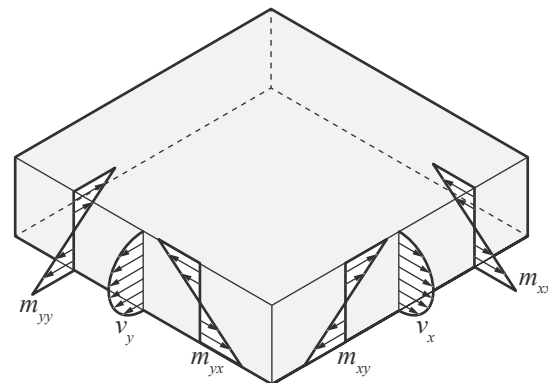
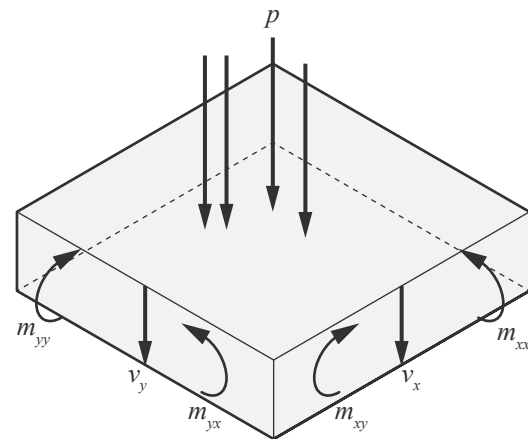


plate

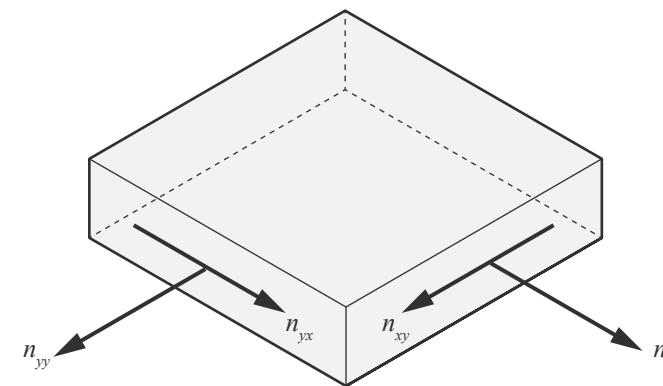


shell

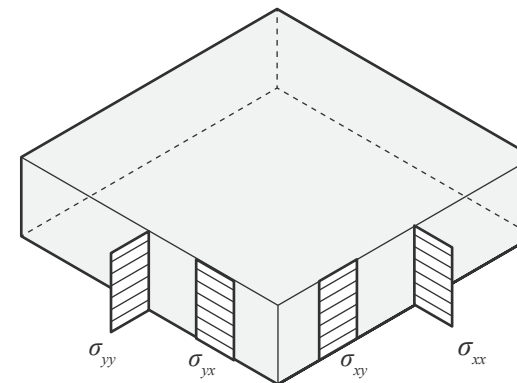
shear forces that
cause bending/
torsional moments



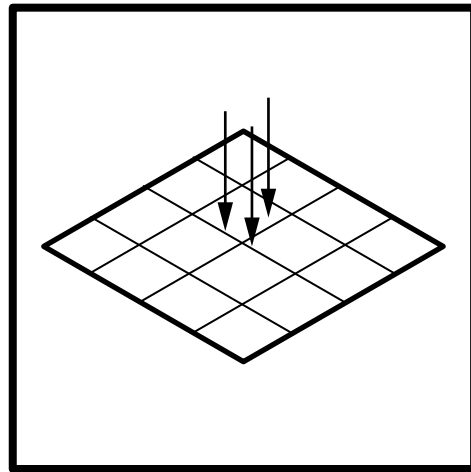
compression + tension



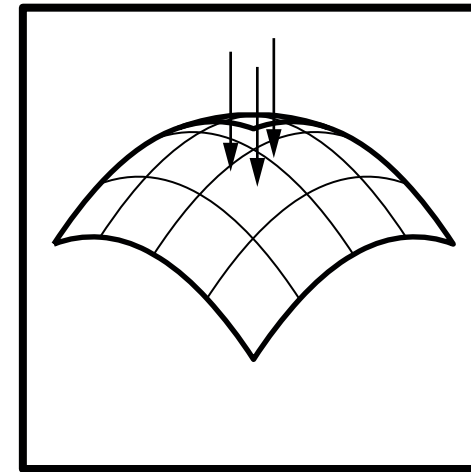
in-plane forces



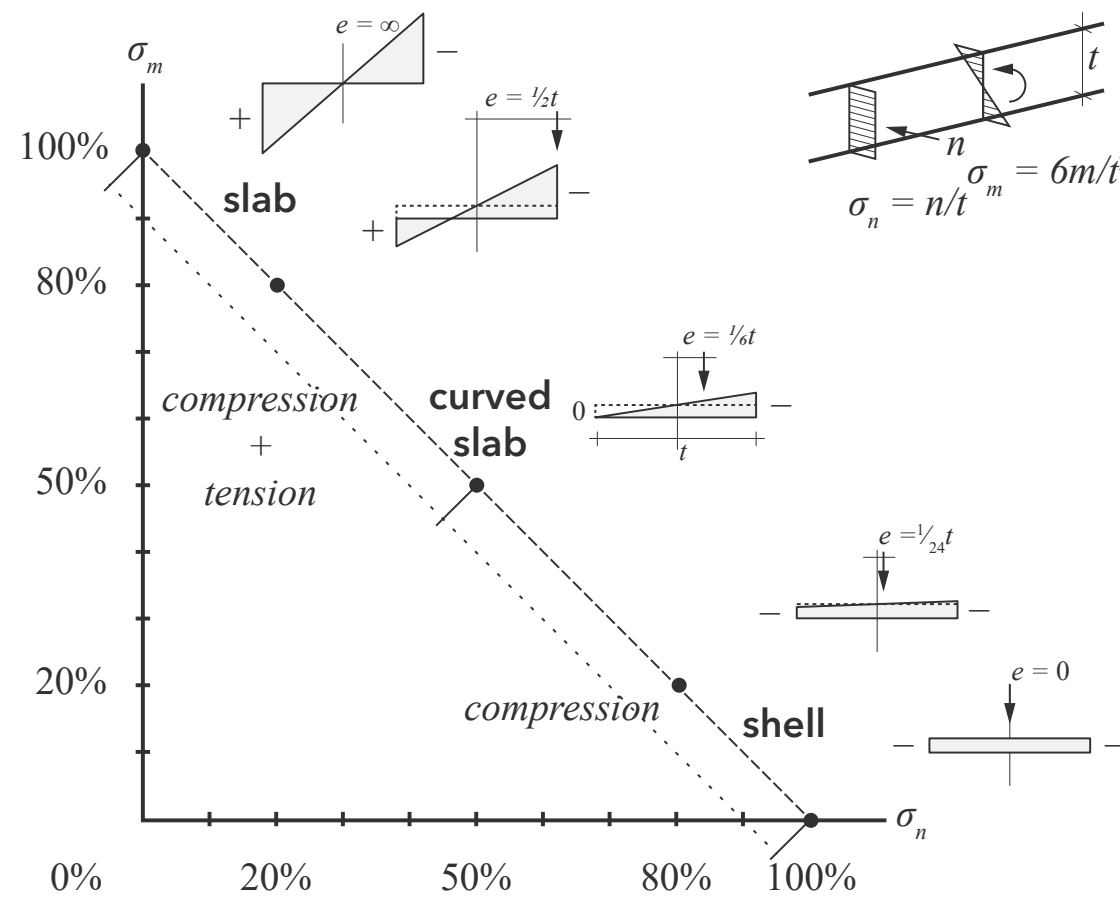
compression

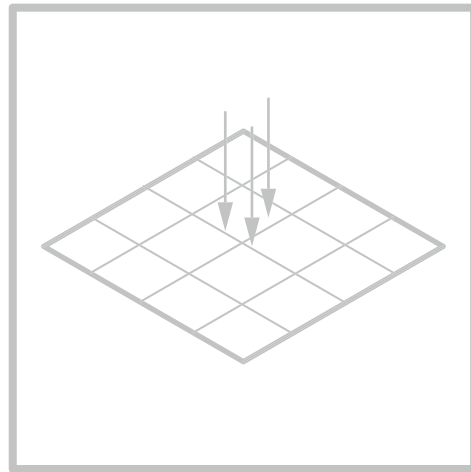


plate

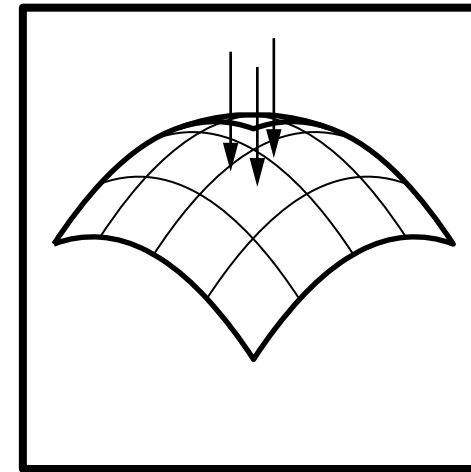


shell

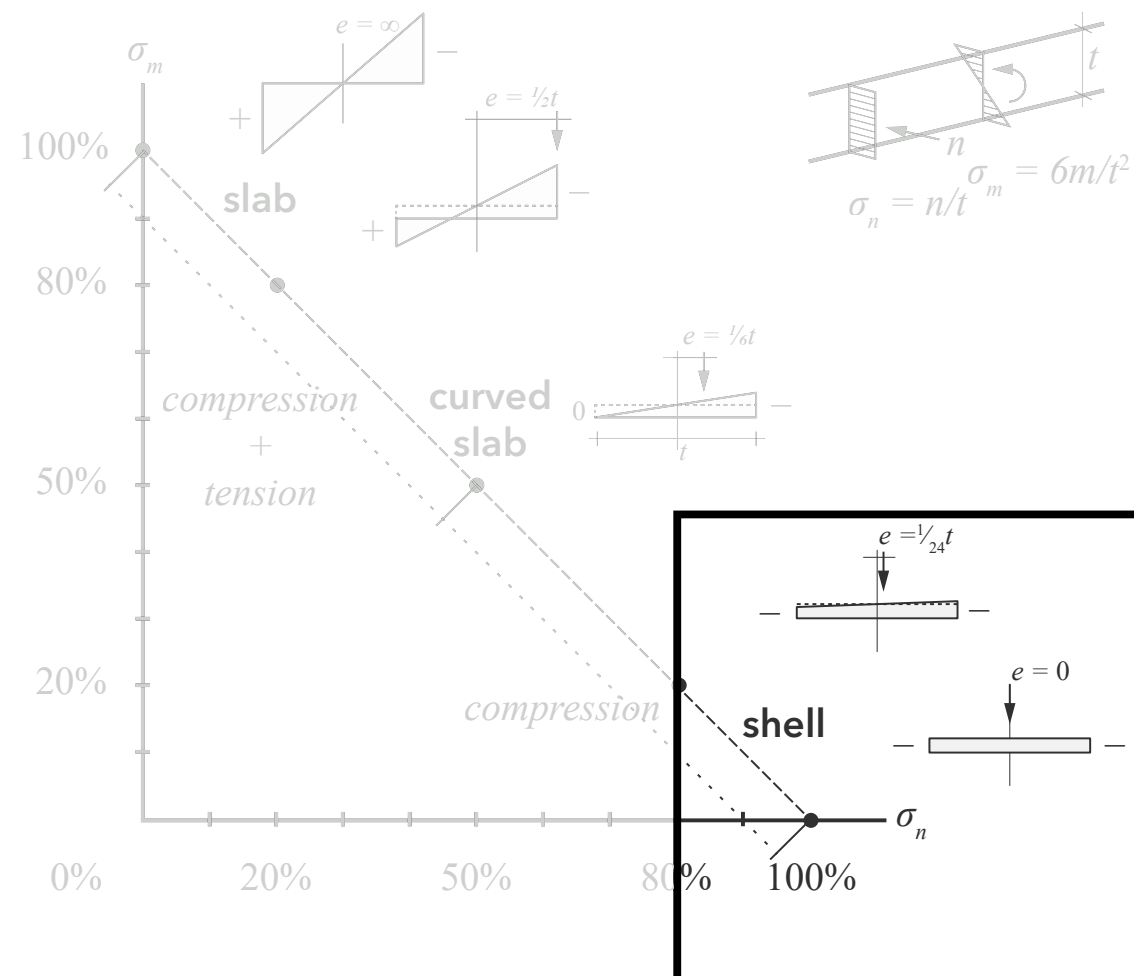




plate

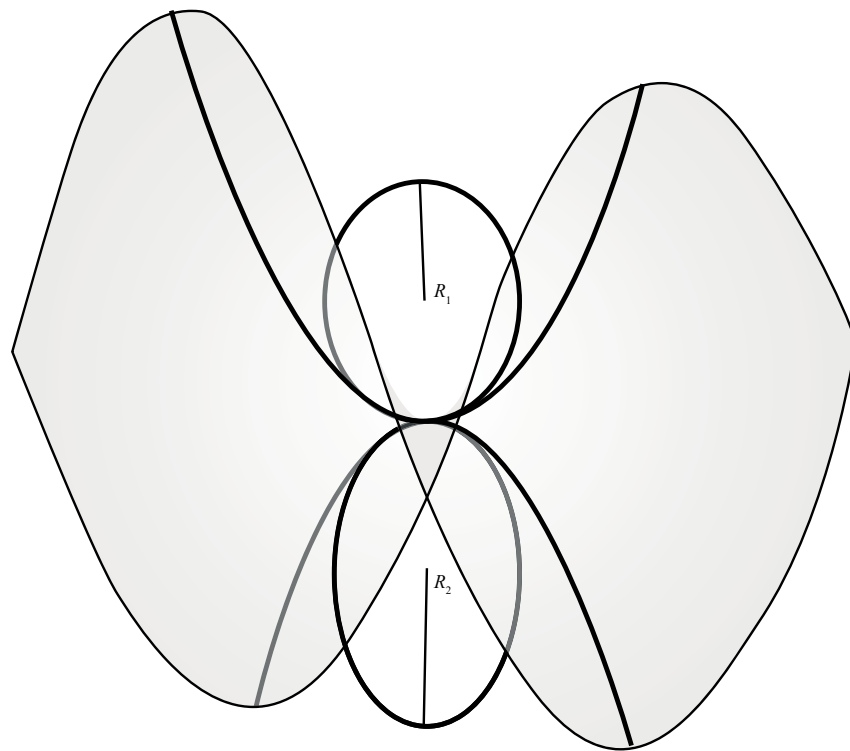


shell



GAUSSIAN CURVATURE

$$K = \kappa_1 \kappa_2 = \frac{1}{R_1 R_2}$$



K = Gaussian curvature

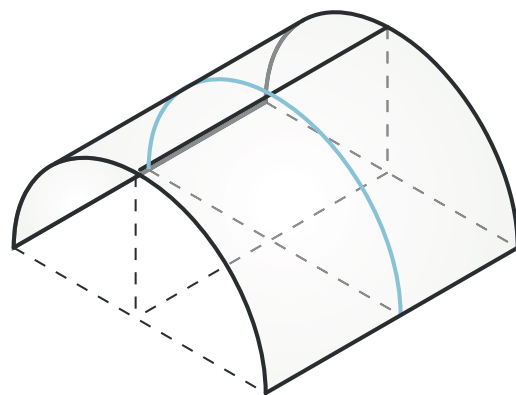
κ = principal curvature (minimum & maximum curvature)

R = principal radius

GAUSSIAN CURVATURE

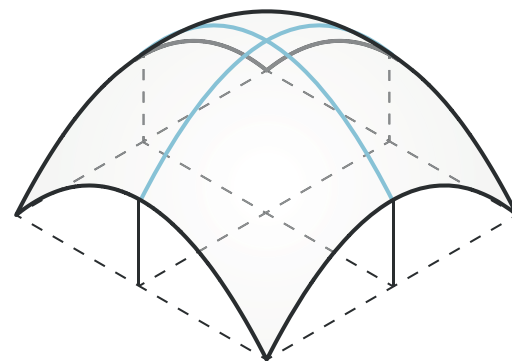
$$K = \kappa_1 \kappa_2 = \frac{1}{R_1 R_2}$$

monoclastic
(parabolic)



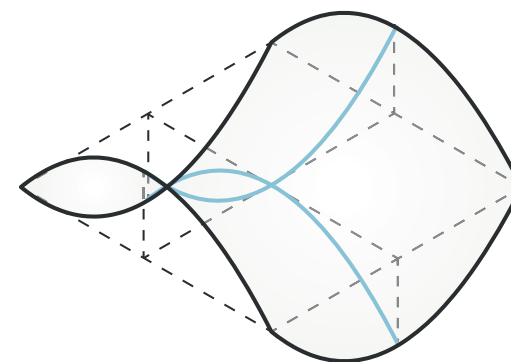
K = 0

synclastic
(elliptic)



K > 0

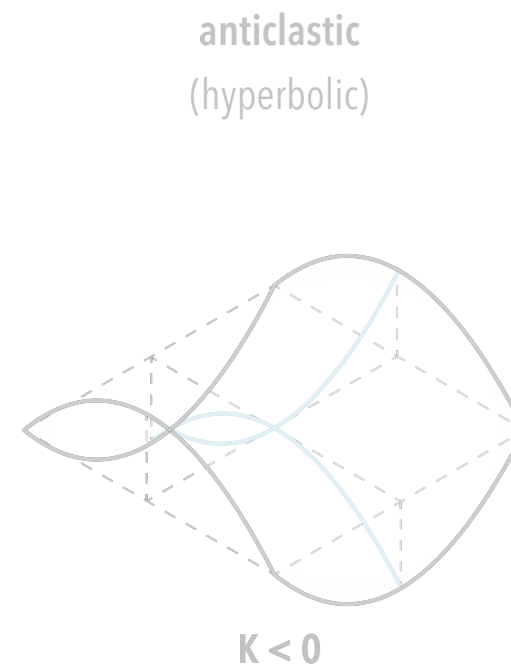
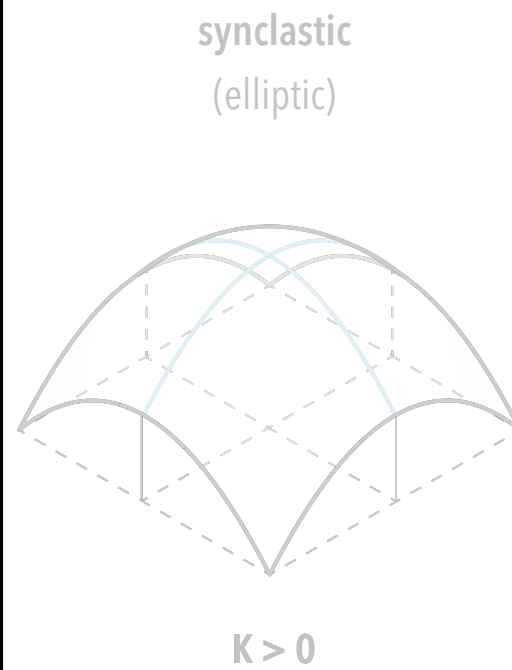
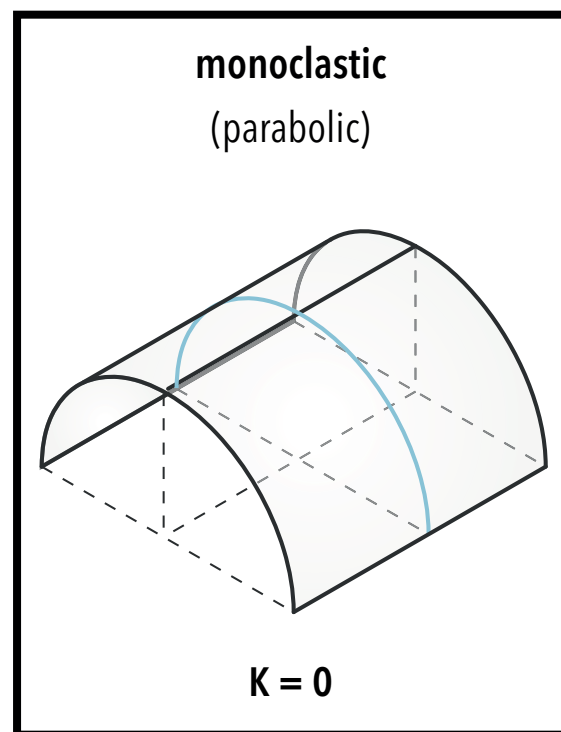
anticlastic
(hyperbolic)



K < 0

GAUSSIAN CURVATURE

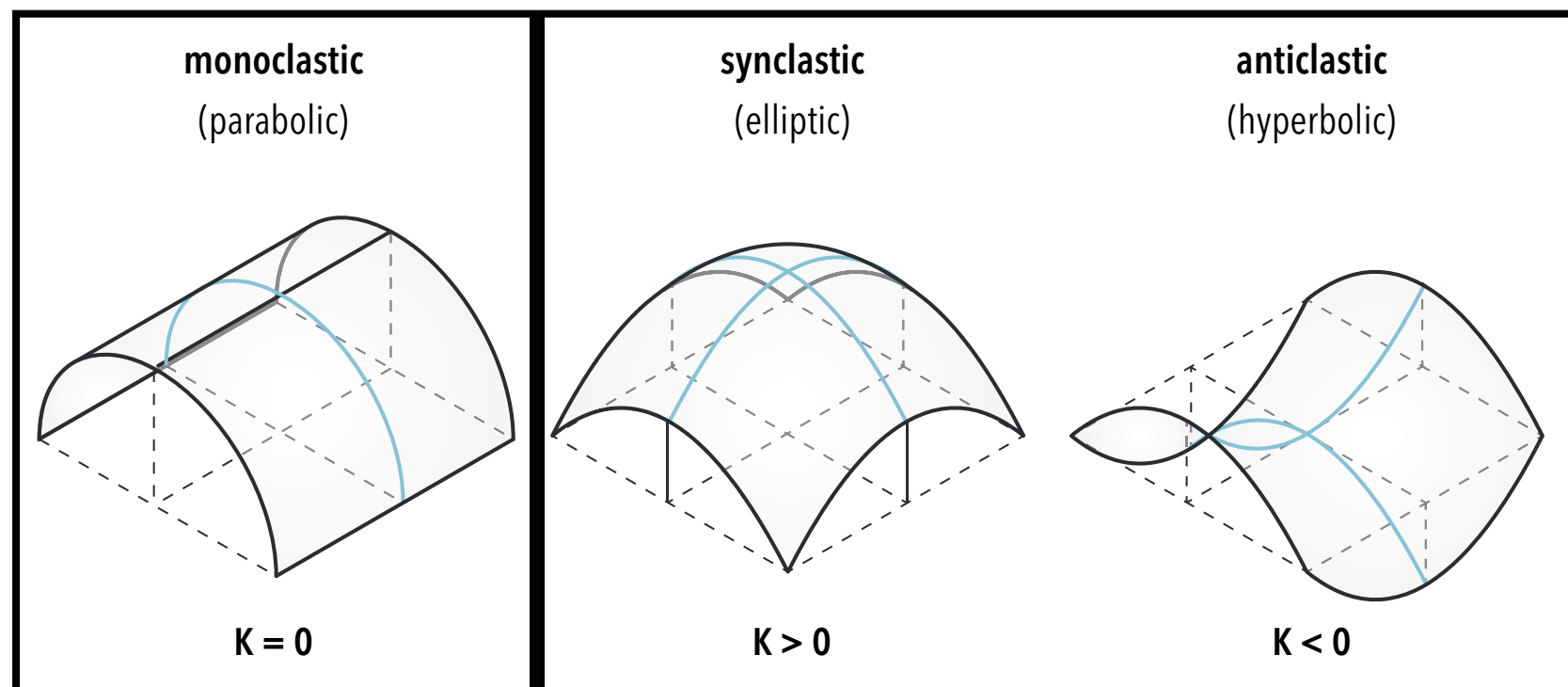
$$K = \kappa_1 \kappa_2 = \frac{1}{R_1 R_2}$$



developable

GAUSSIAN CURVATURE

$$K = \kappa_1 \kappa_2 = \frac{1}{R_1 R_2}$$

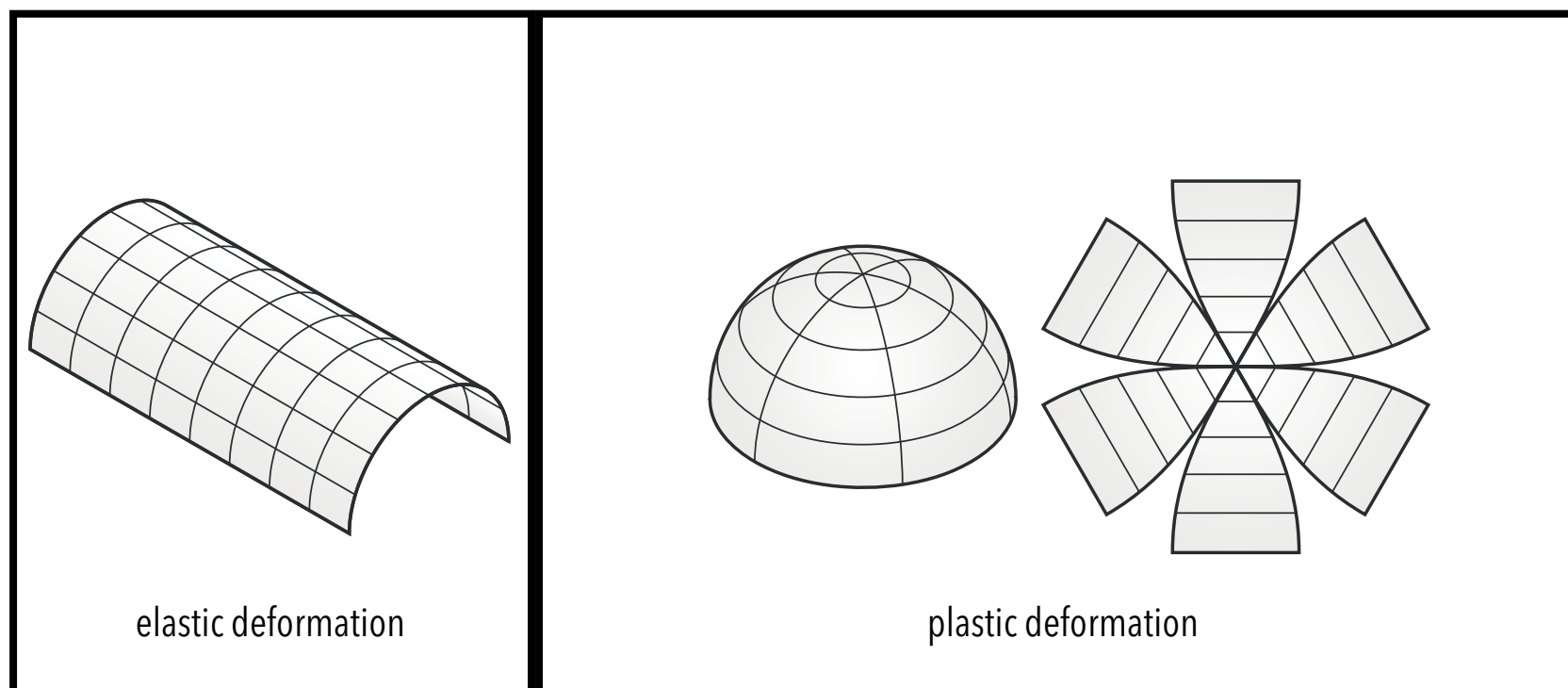


developable

undevelopable

GAUSSIAN CURVATURE

$$K = \kappa_1 \kappa_2 = \frac{1}{R_1 R_2}$$

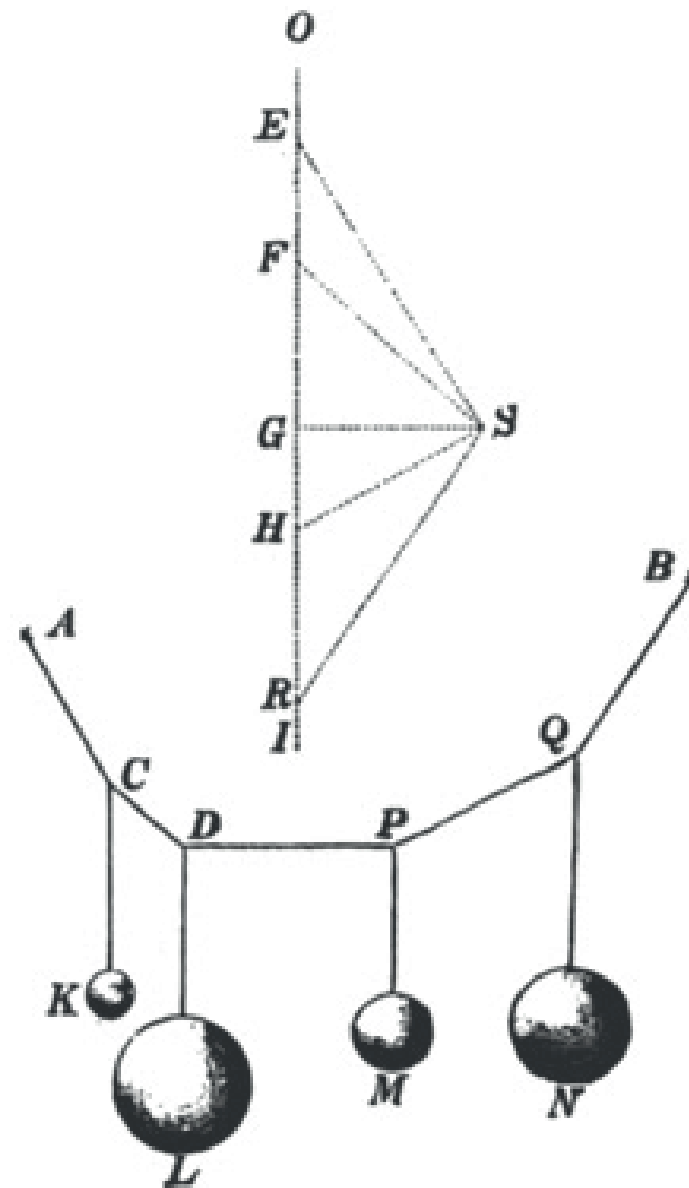


developable

undevelopable

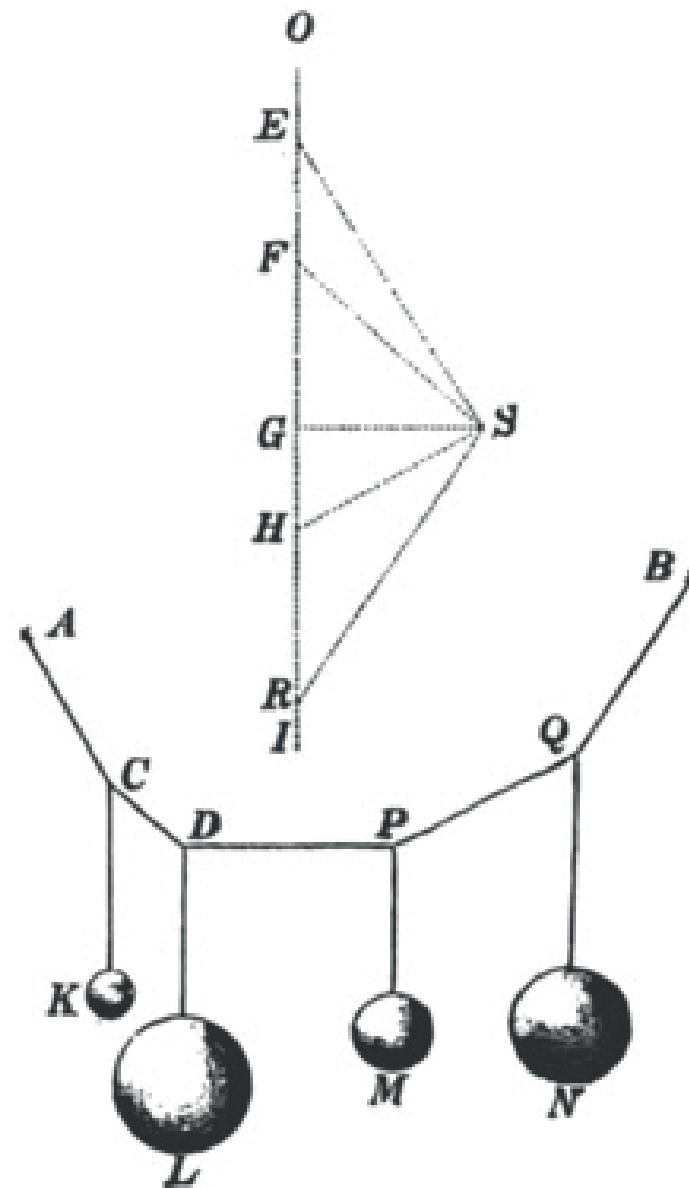
EXISTING METHODS

GRAPHIC STATICS



(Varignon, 1725)

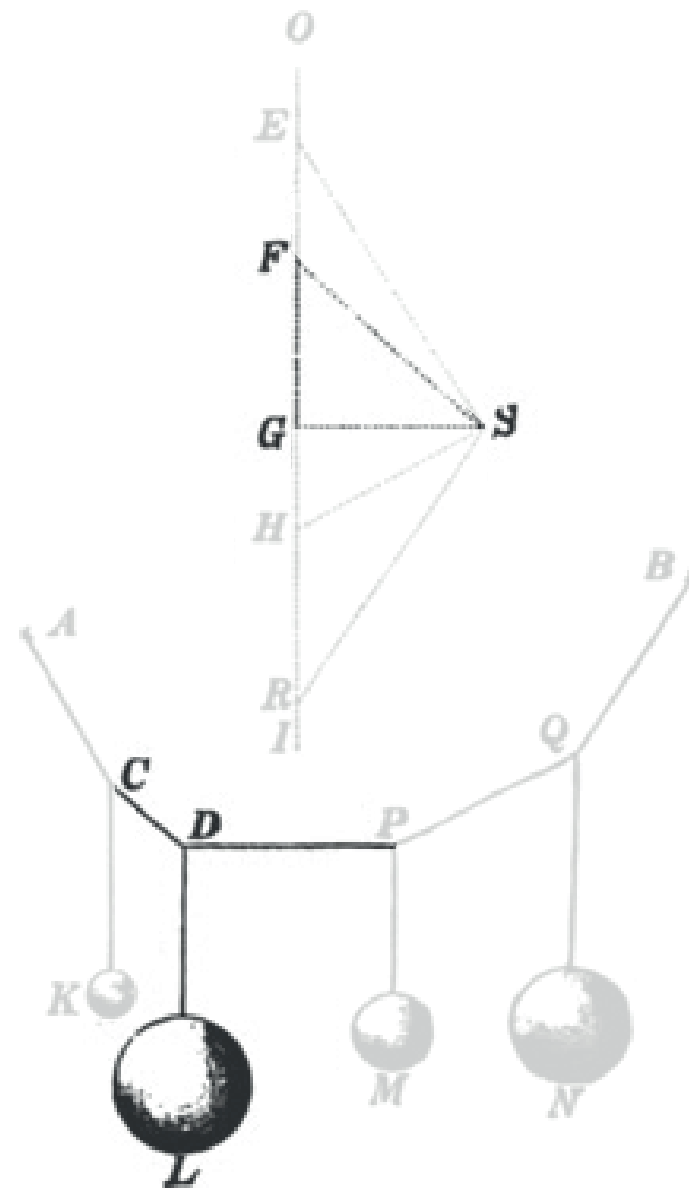
GRAPHIC STATICS



forces correspond with vectors

(Varignon, 1725)

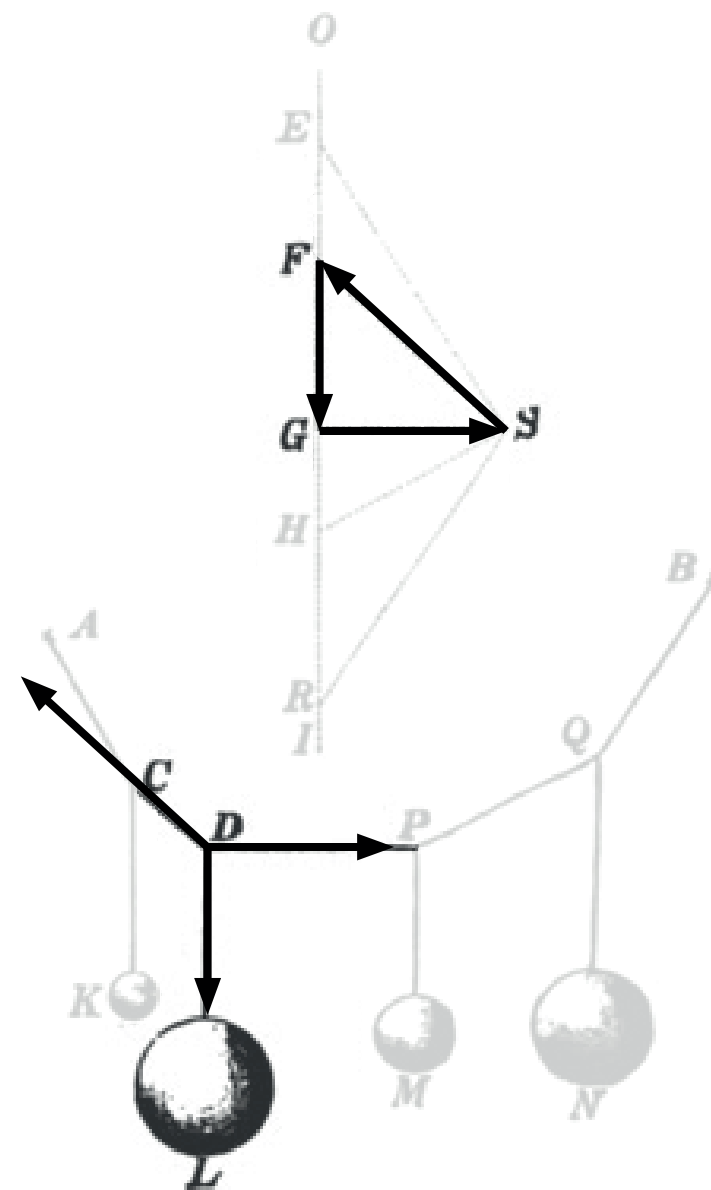
GRAPHIC STATICS



forces correspond with vectors

(Varignon, 1725)

GRAPHIC STATICS



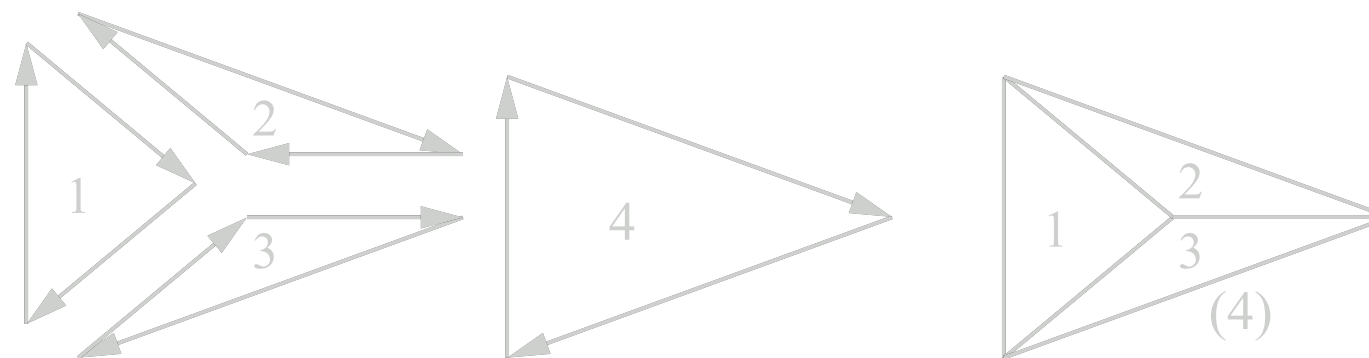
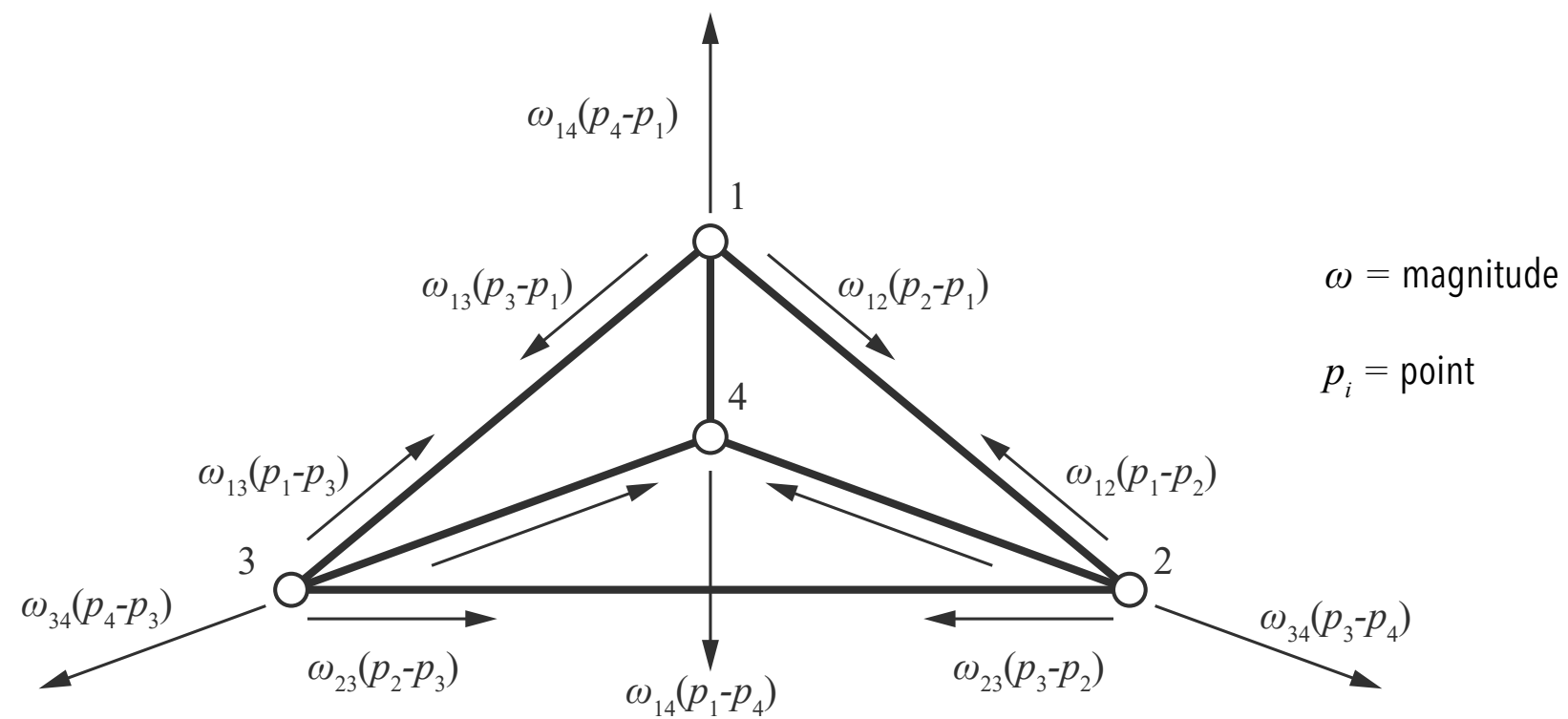
force polygon

form diagram

forces in equilibrium form a closed polygon

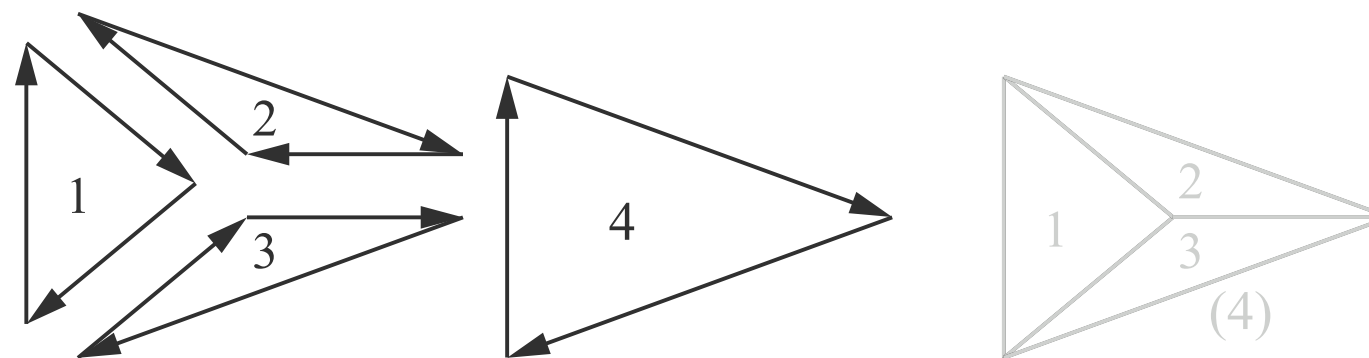
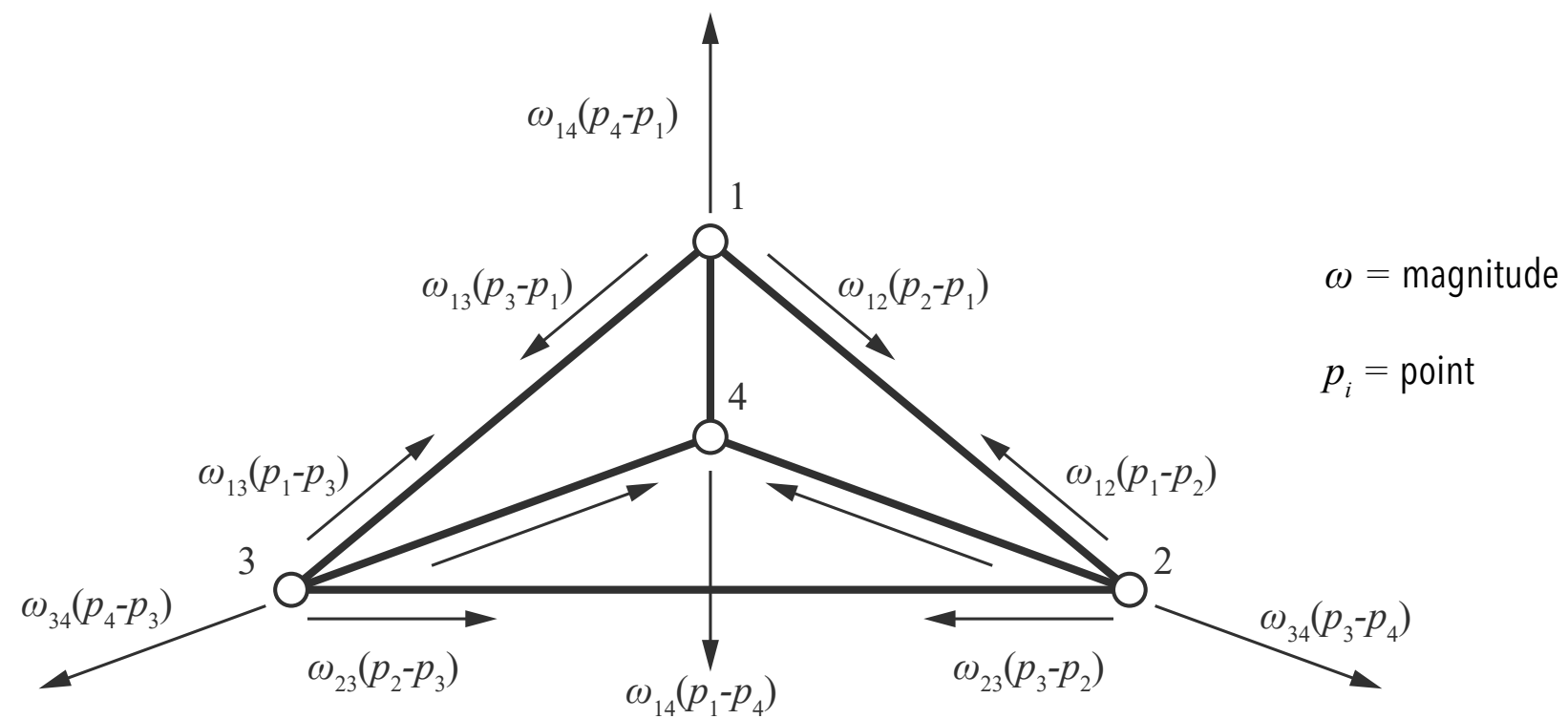
(Varignon, 1725)

GRAPHIC STATICS



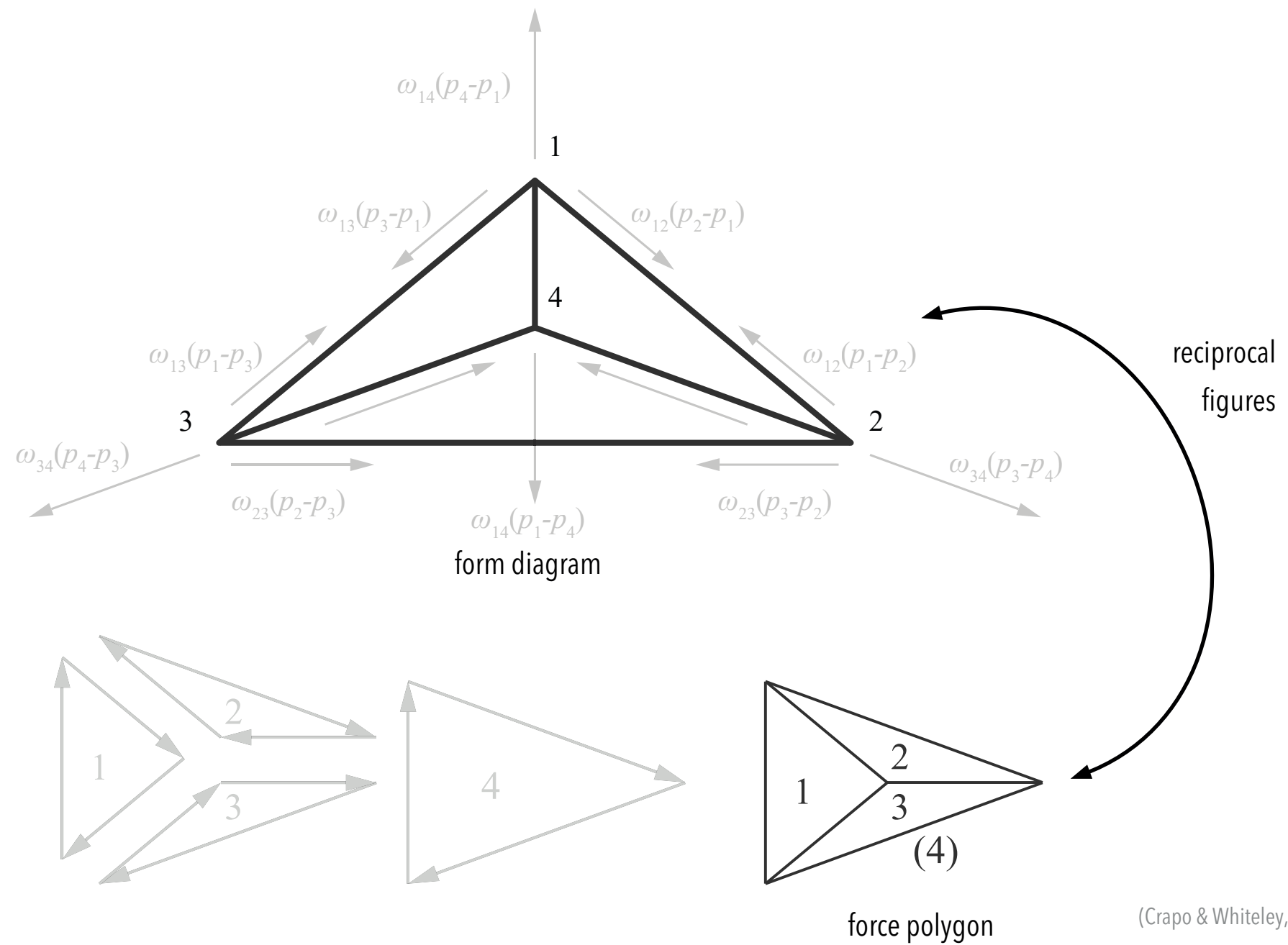
(Crapo & Whiteley, 1993)

GRAPHIC STATICS

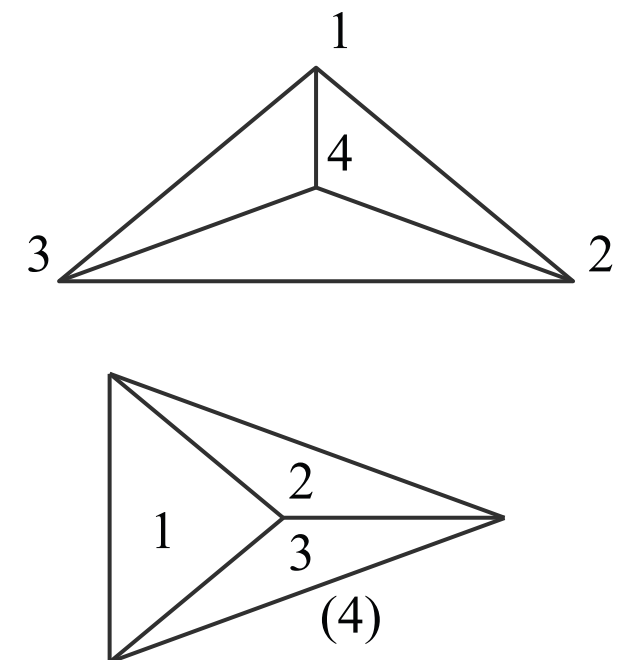
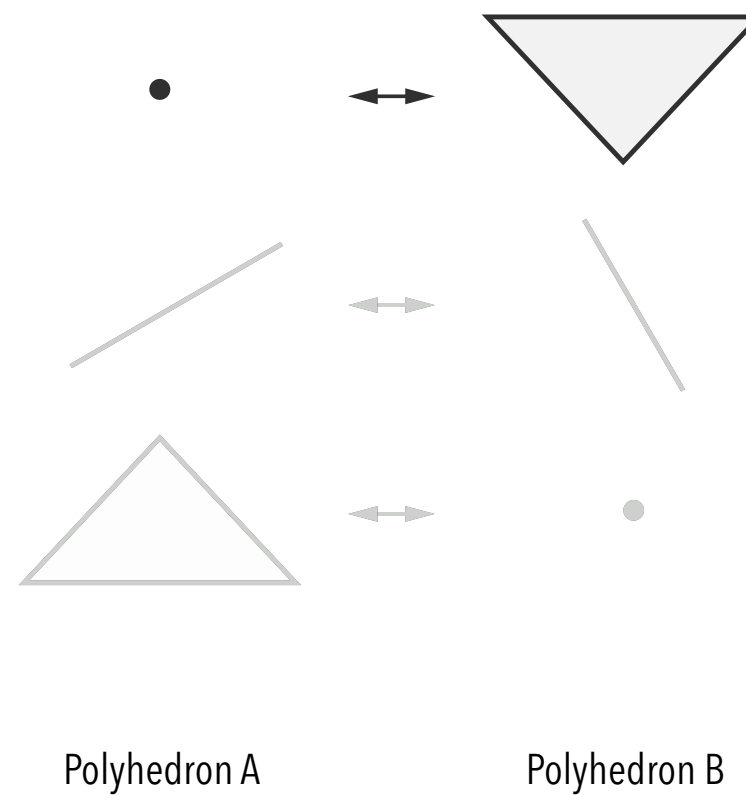


(Crapo & Whiteley, 1993)

GRAPHIC STATICS

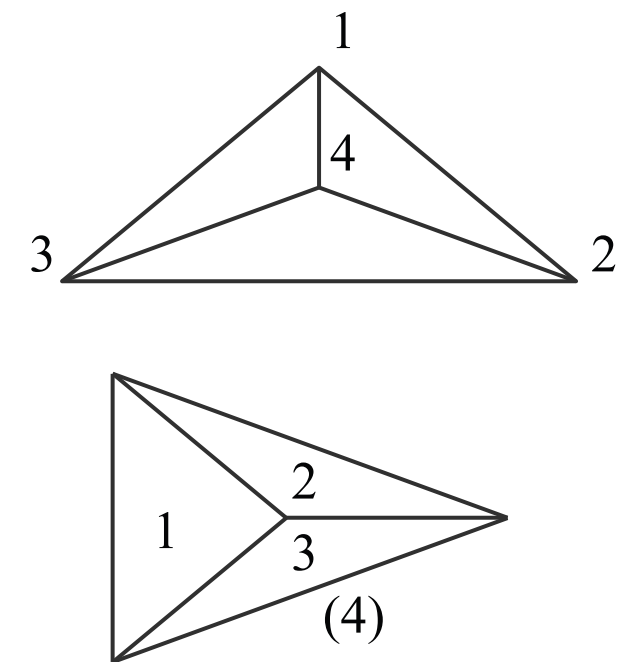
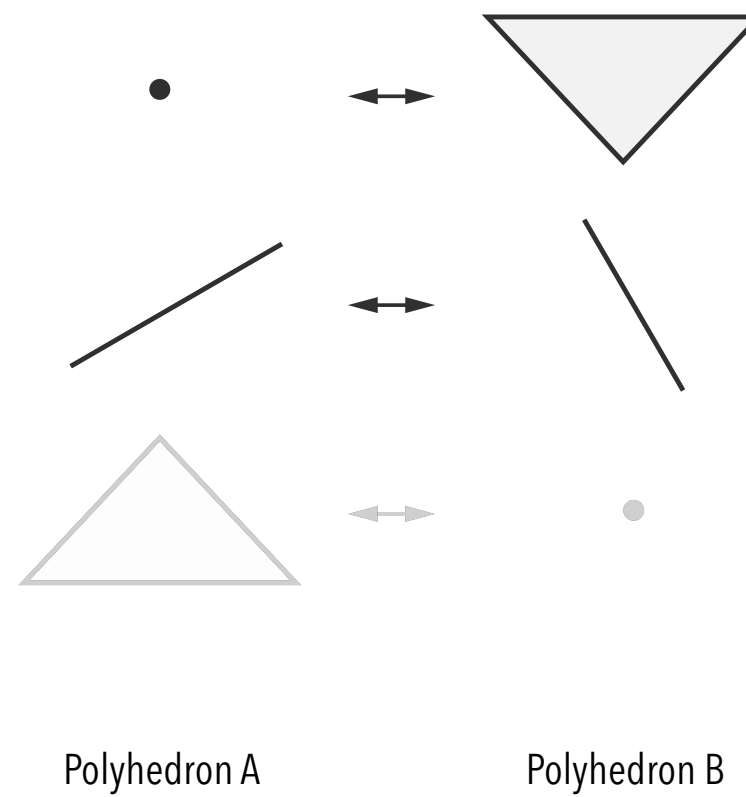


RECIPROCAL FIGURES



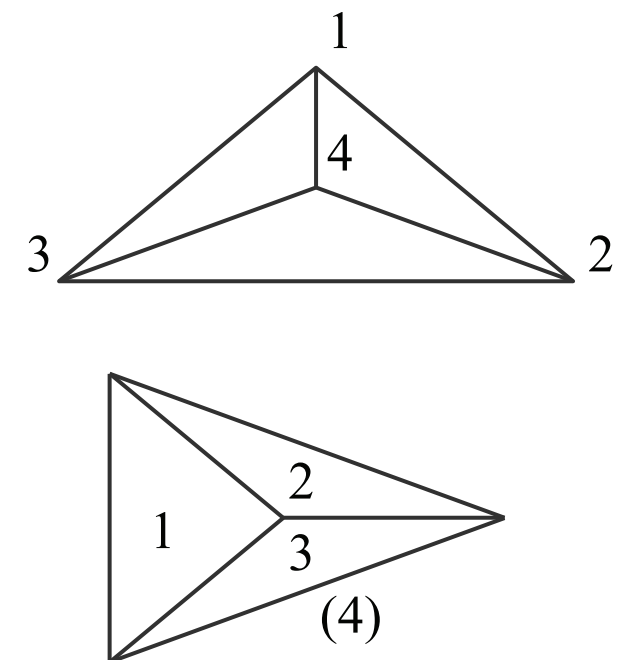
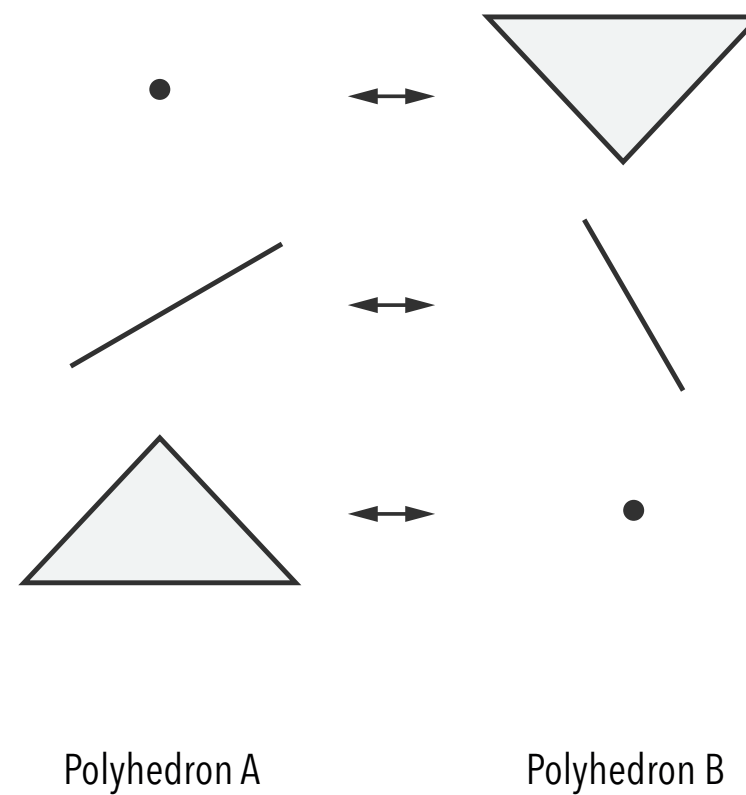
(Constantatou & McRobie, 2016)

RECIPROCAL FIGURES



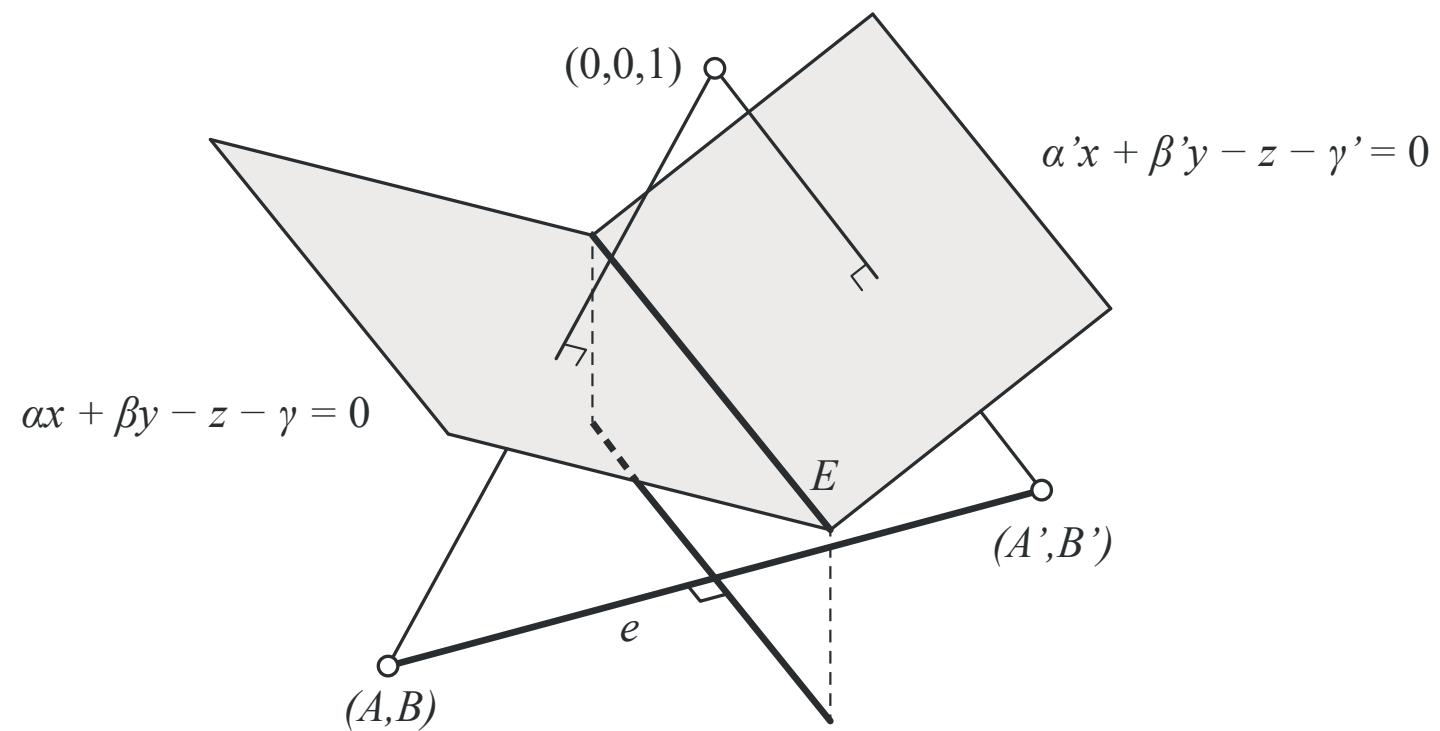
(Constantatou & McRobie, 2016)

RECIPROCAL FIGURES



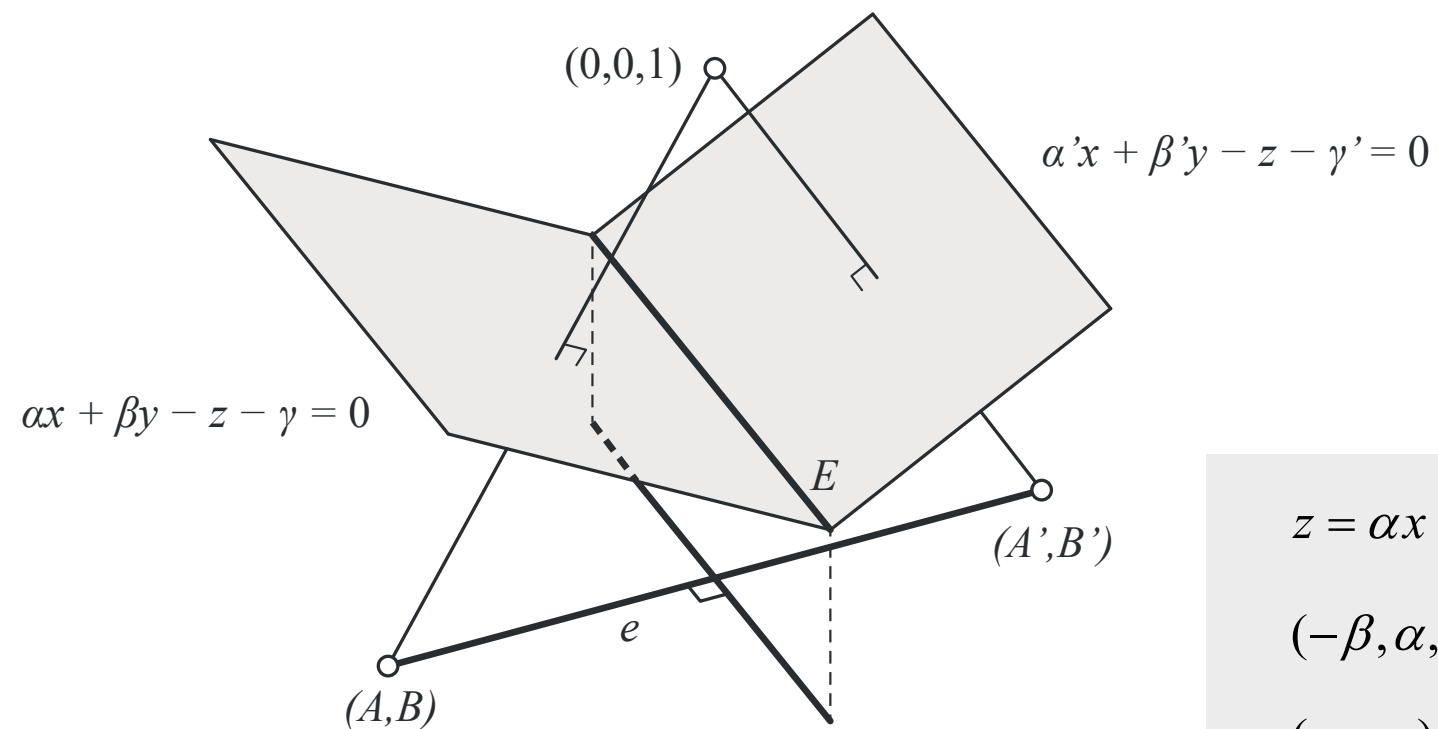
(Constantatou & McRobie, 2016)

RECIPROCAL FIGURES



(Whiteley, Ash, Bolker & Crapo, 2013)

RECIPROCAL FIGURES



$$z = \alpha x + \beta y + \gamma \quad (\text{Face of Polyhedron A})$$

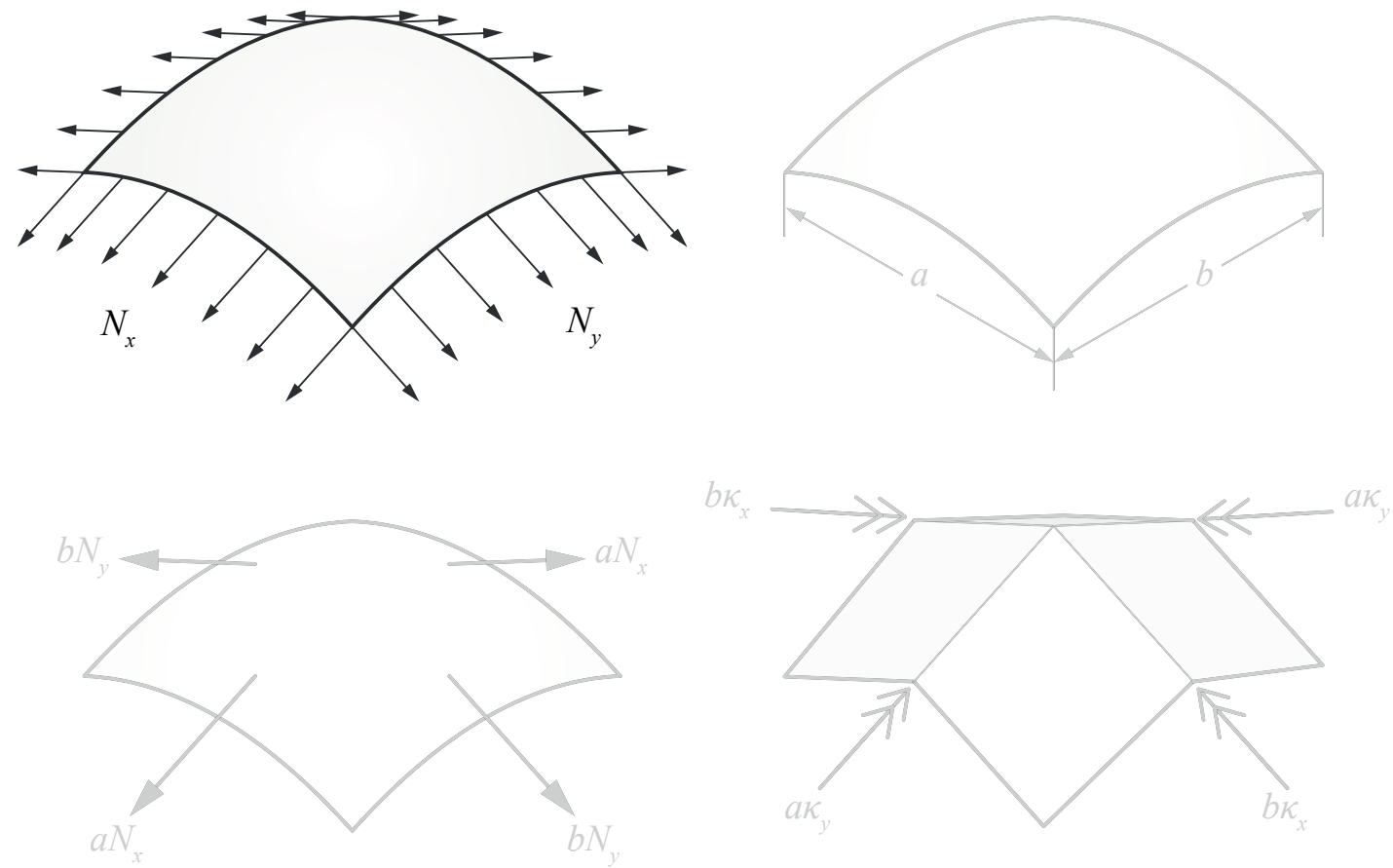
$$(-\beta, \alpha, \gamma) \quad (\text{Vertex of Polyhedron B})$$

$$(q, r, s) \quad (\text{Vertices of Polyhedron A})$$

$$z = rx - qy + s \quad (\text{Face of Polyhedron B})$$

(Whiteley, Ash, Bolker & Crapo, 2013)

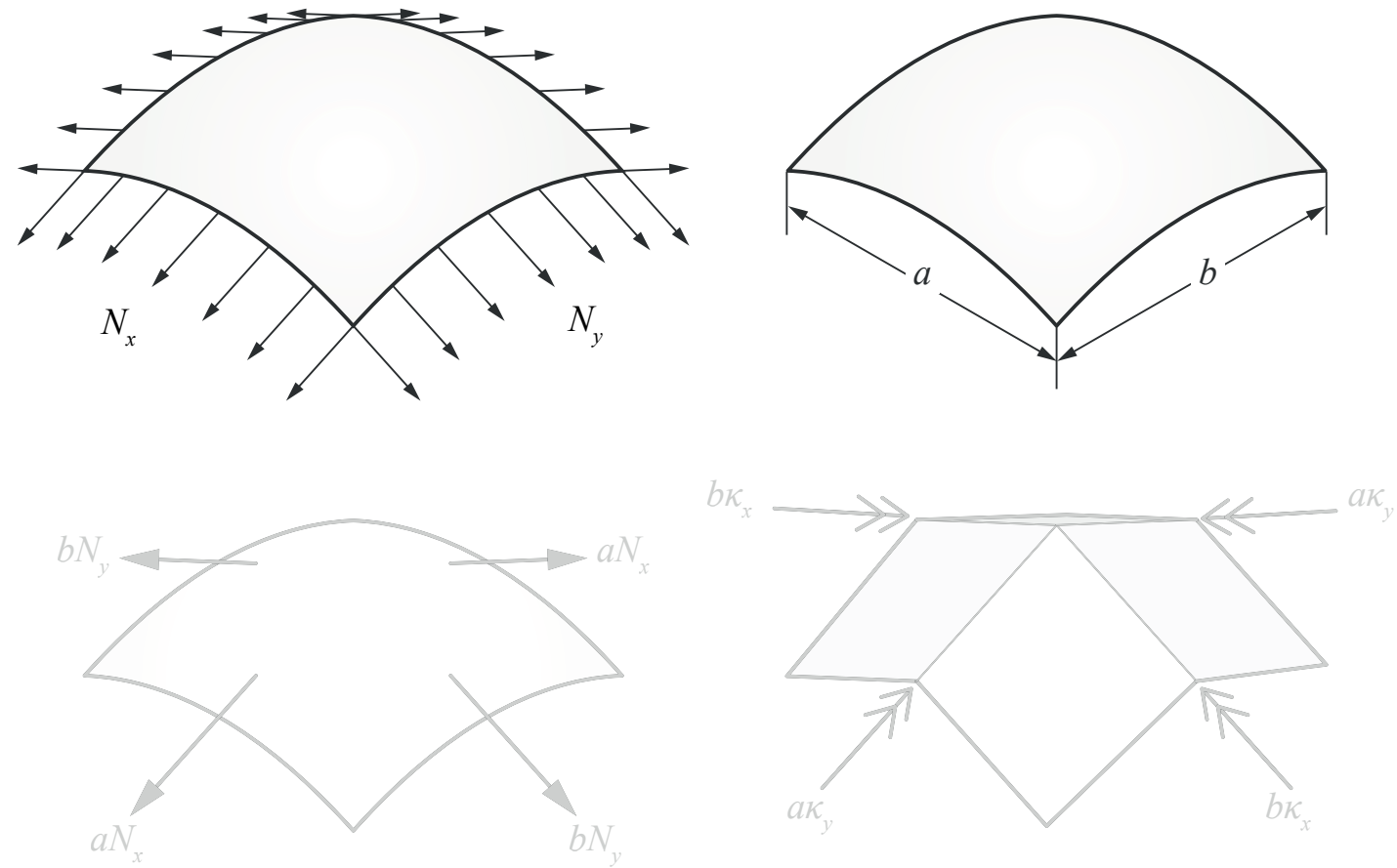
ANGULAR DEFECT



Internal forces

(Calladine, 1977)

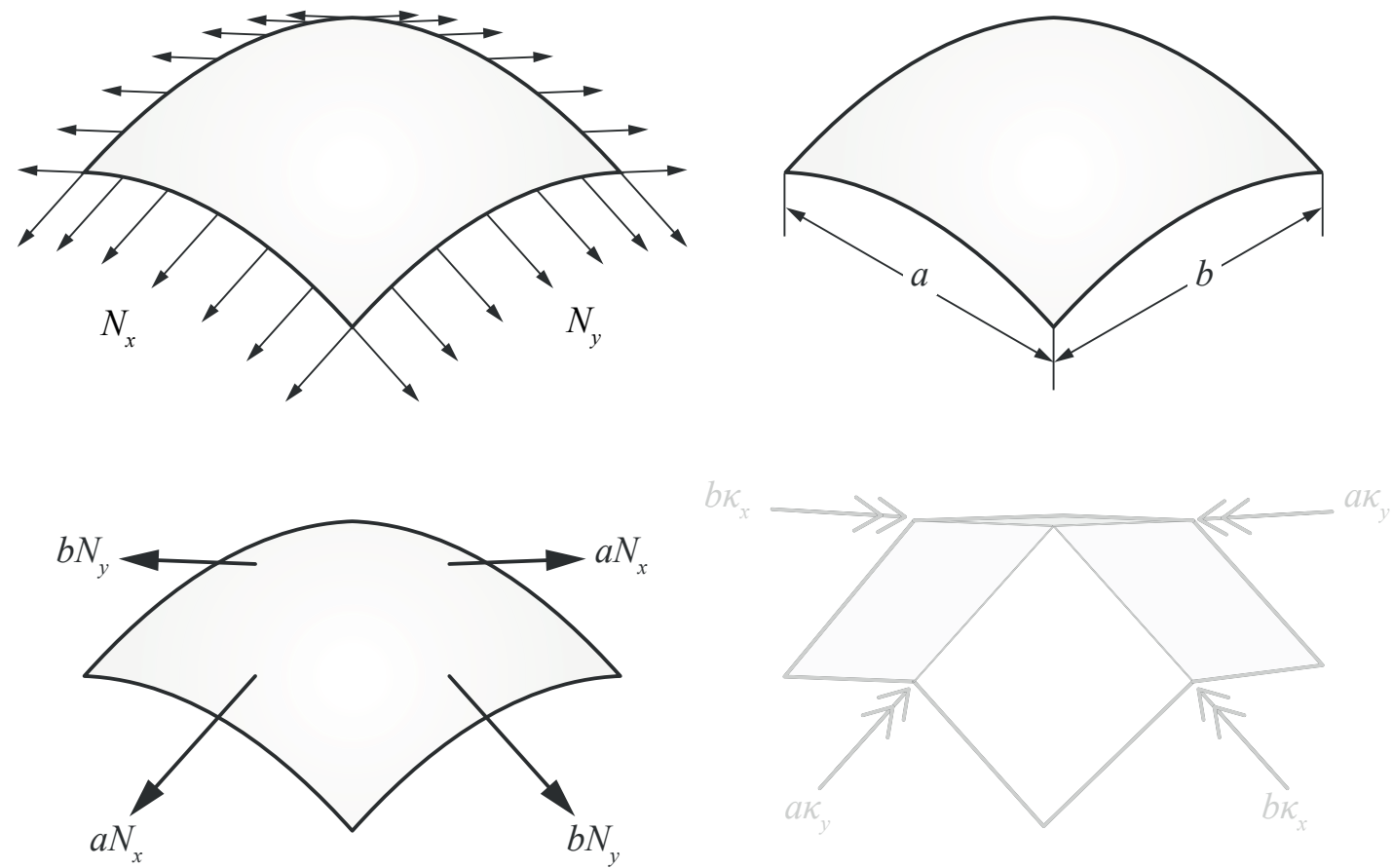
ANGULAR DEFECT



Internal forces

(Calladine, 1977)

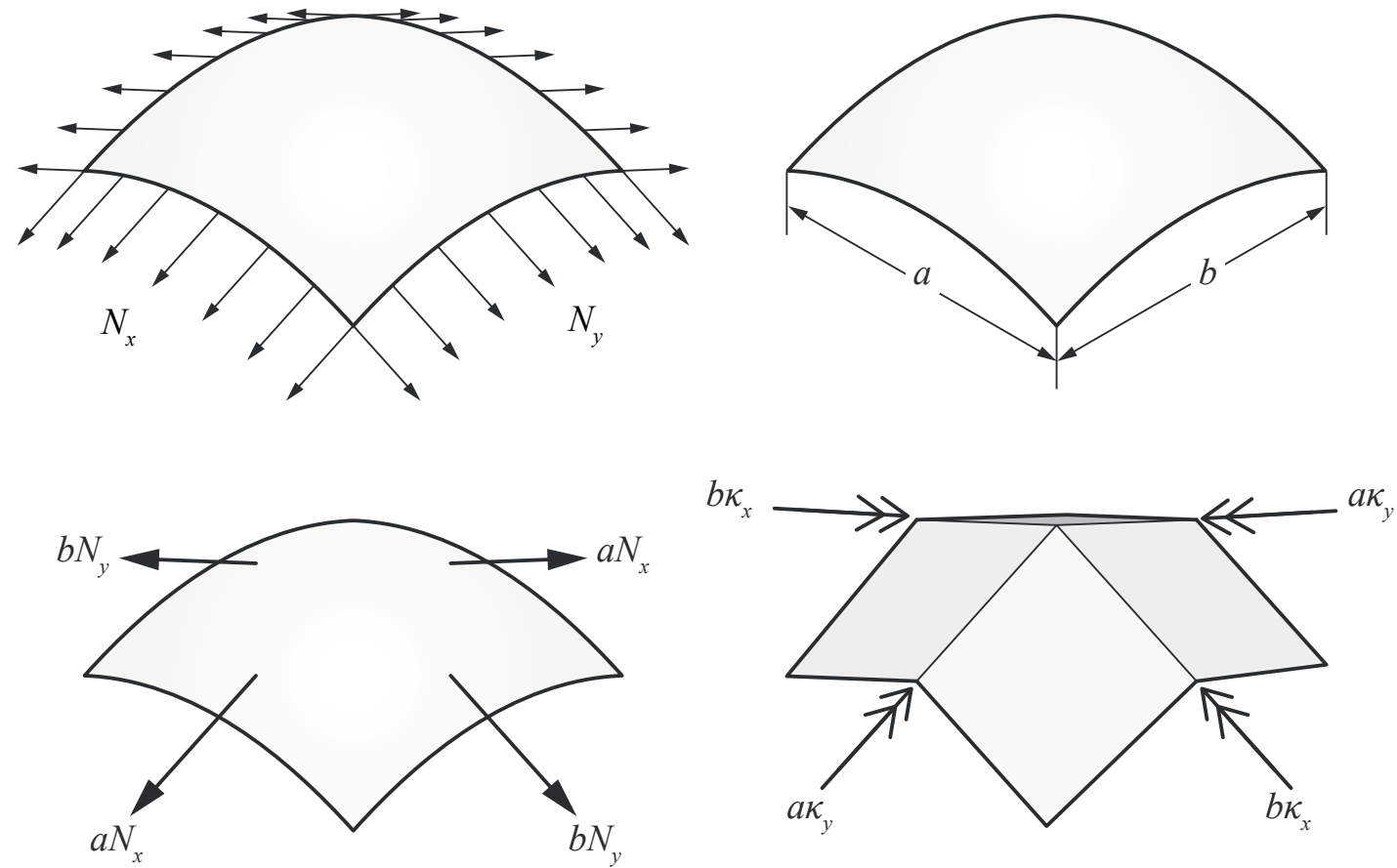
ANGULAR DEFECT



Converted into single force vectors

(Calladine, 1977)

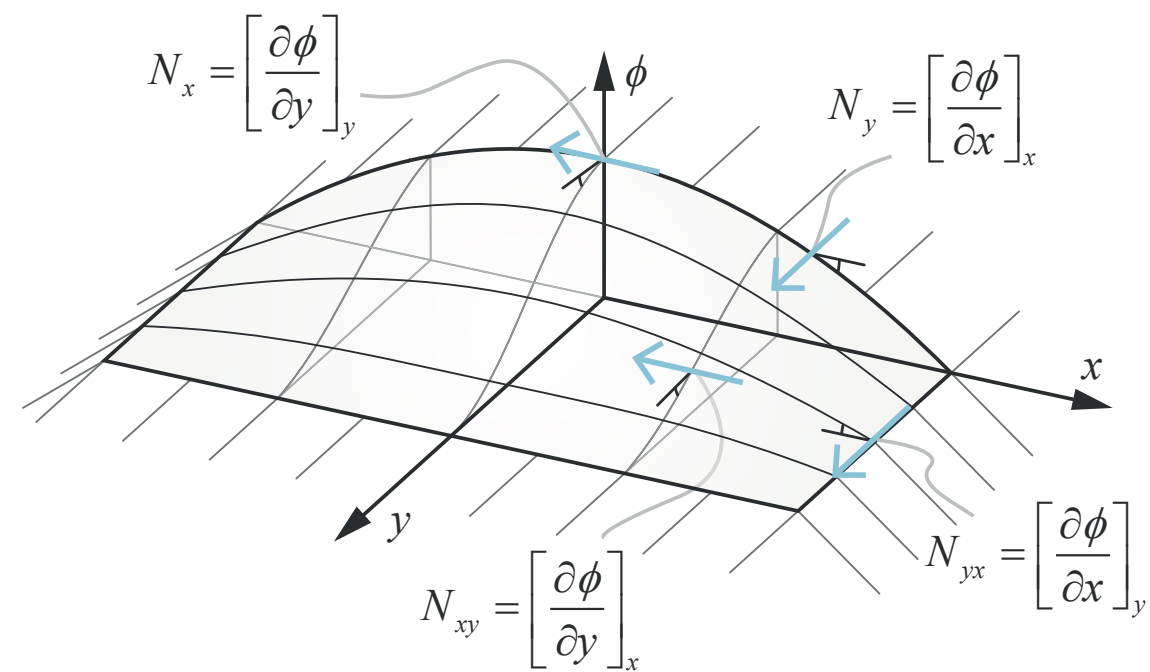
ANGULAR DEFECT



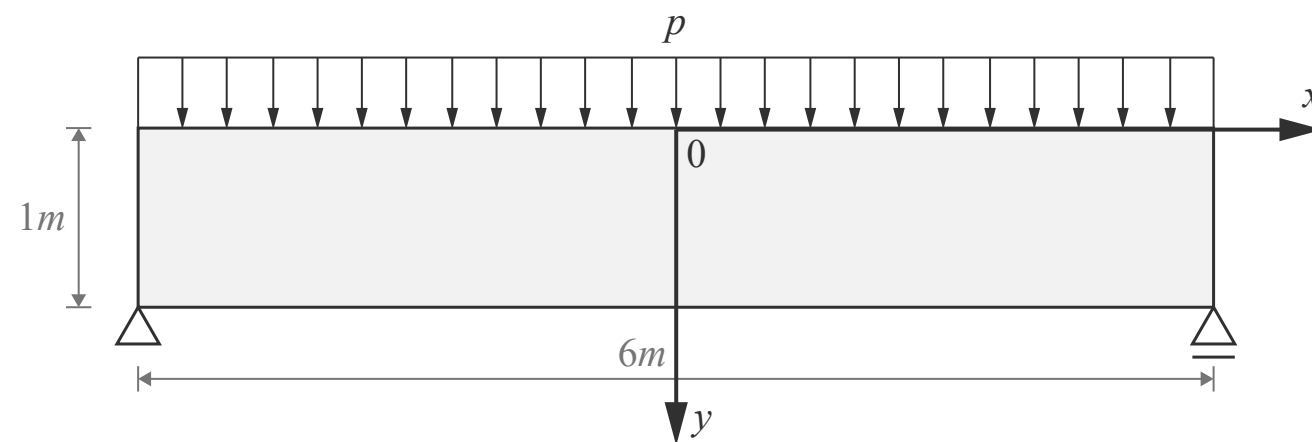
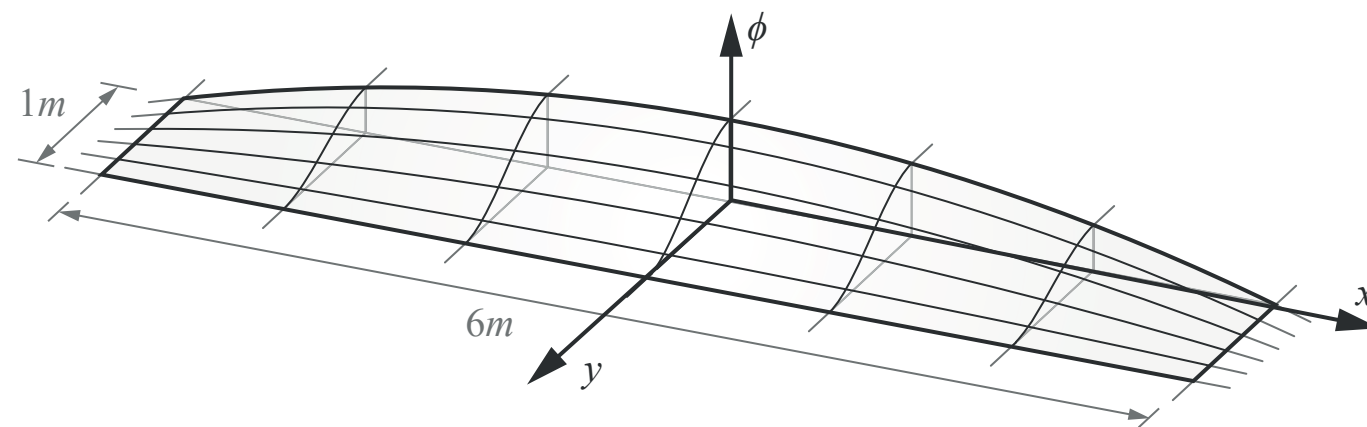
Force vectors across fold into angular defect κ

(Calladine, 1977)

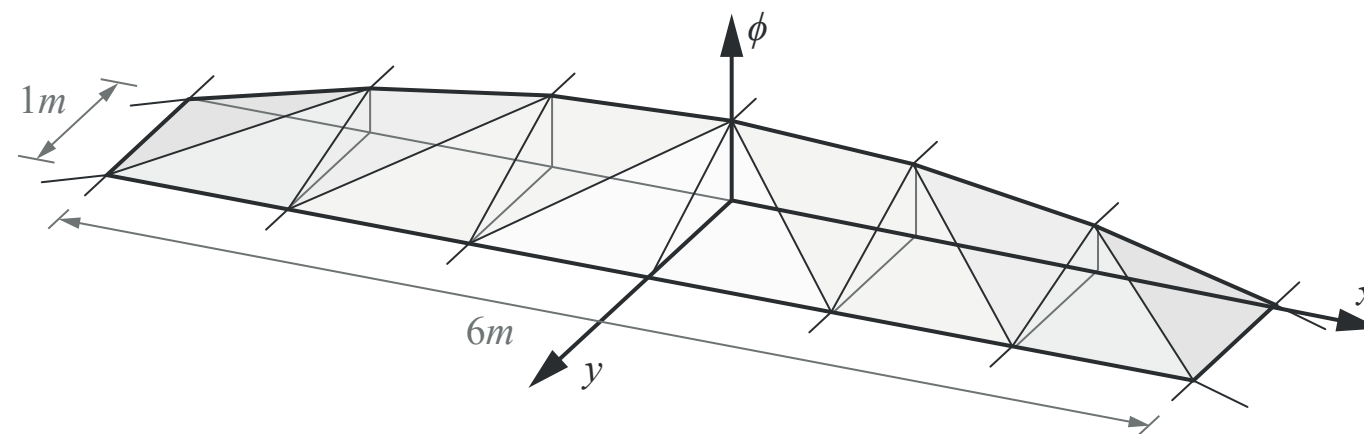
AIRY STRESS FUNCTION



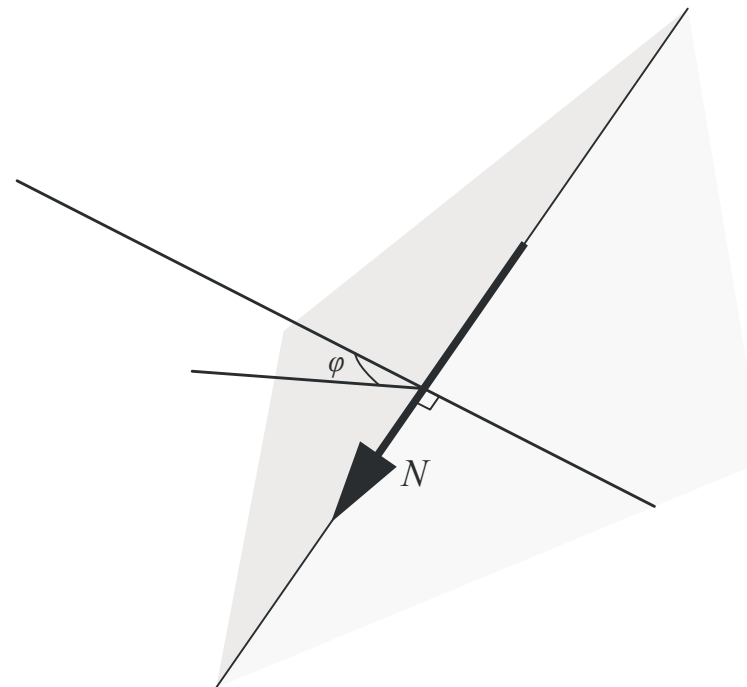
AIRY STRESS FUNCTION



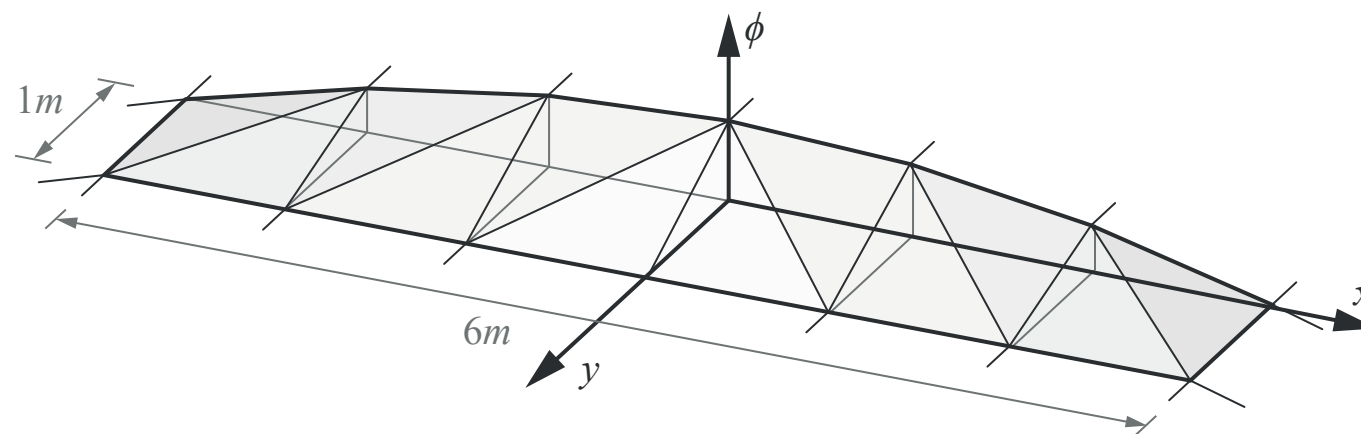
AIRY STRESS FUNCTION



$$N_{BE;i} = \varphi_{AE;i}$$



AIRY STRESS FUNCTION



$$N_{BE;i} = \phi_{AE;i}$$

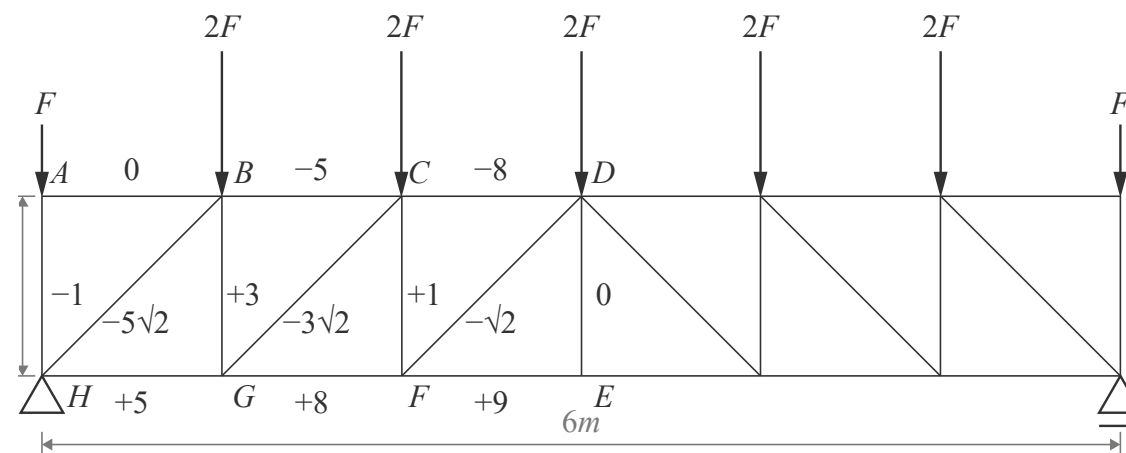
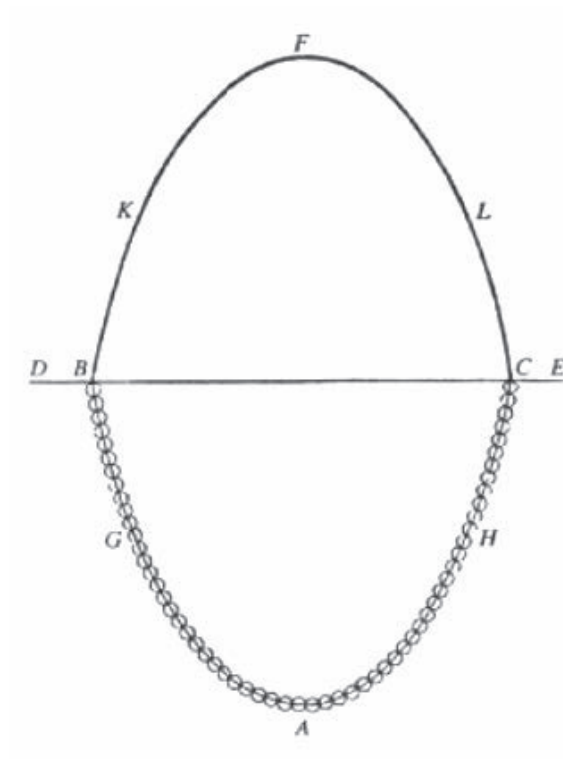
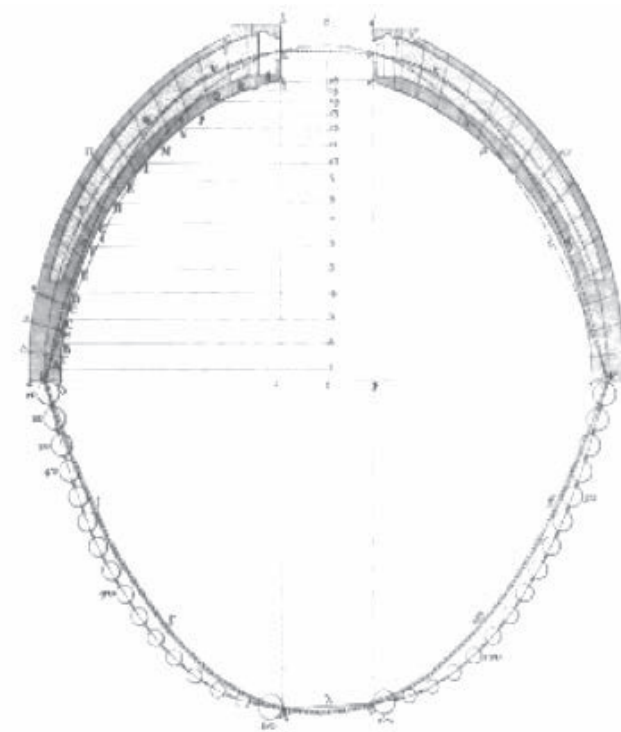


PLATE AND SHELL ANALOGIES

ANALOGY MOMENT LINE & THRUST LINE

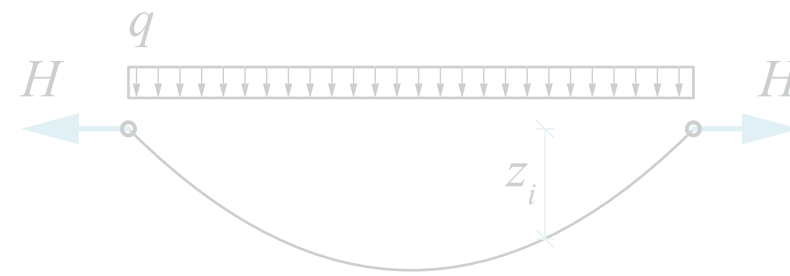
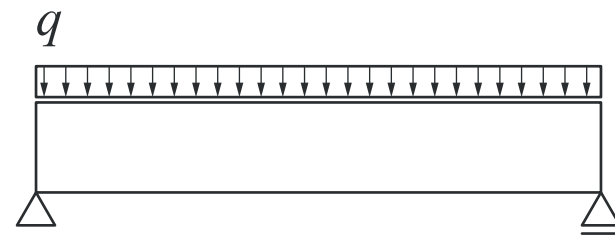


hanging catenary



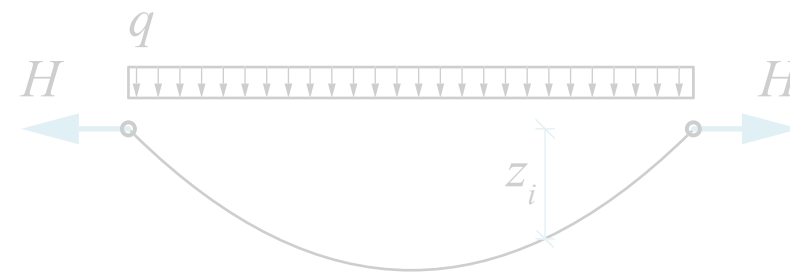
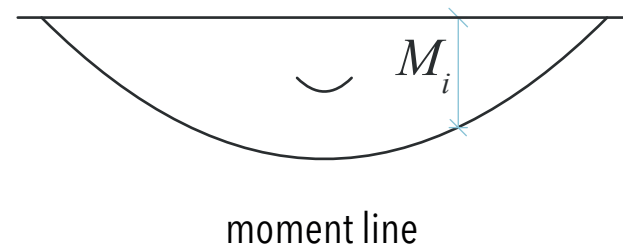
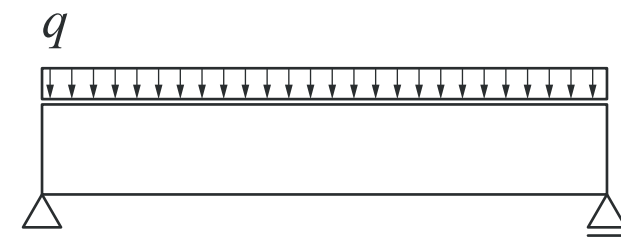
standing arch

ANALOGY MOMENT LINE & THRUST LINE



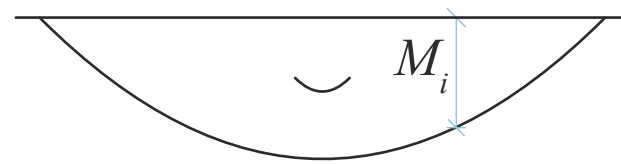
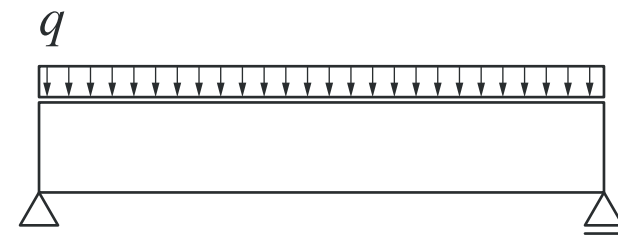
$q =$ evenly distributed load $[N/m]$

ANALOGY MOMENT LINE & THRUST LINE

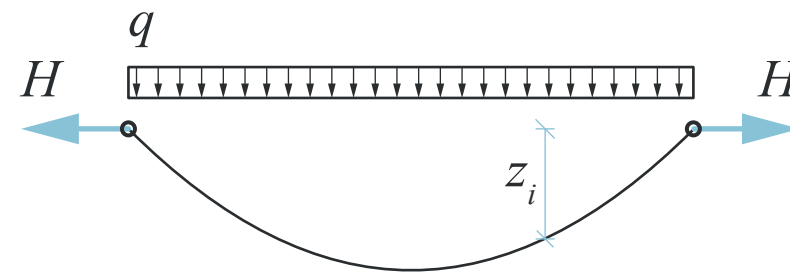


$q =$ evenly distributed load $[N/m]$
 $M =$ bending moment $[Nm]$

ANALOGY MOMENT LINE & THRUST LINE



moment line



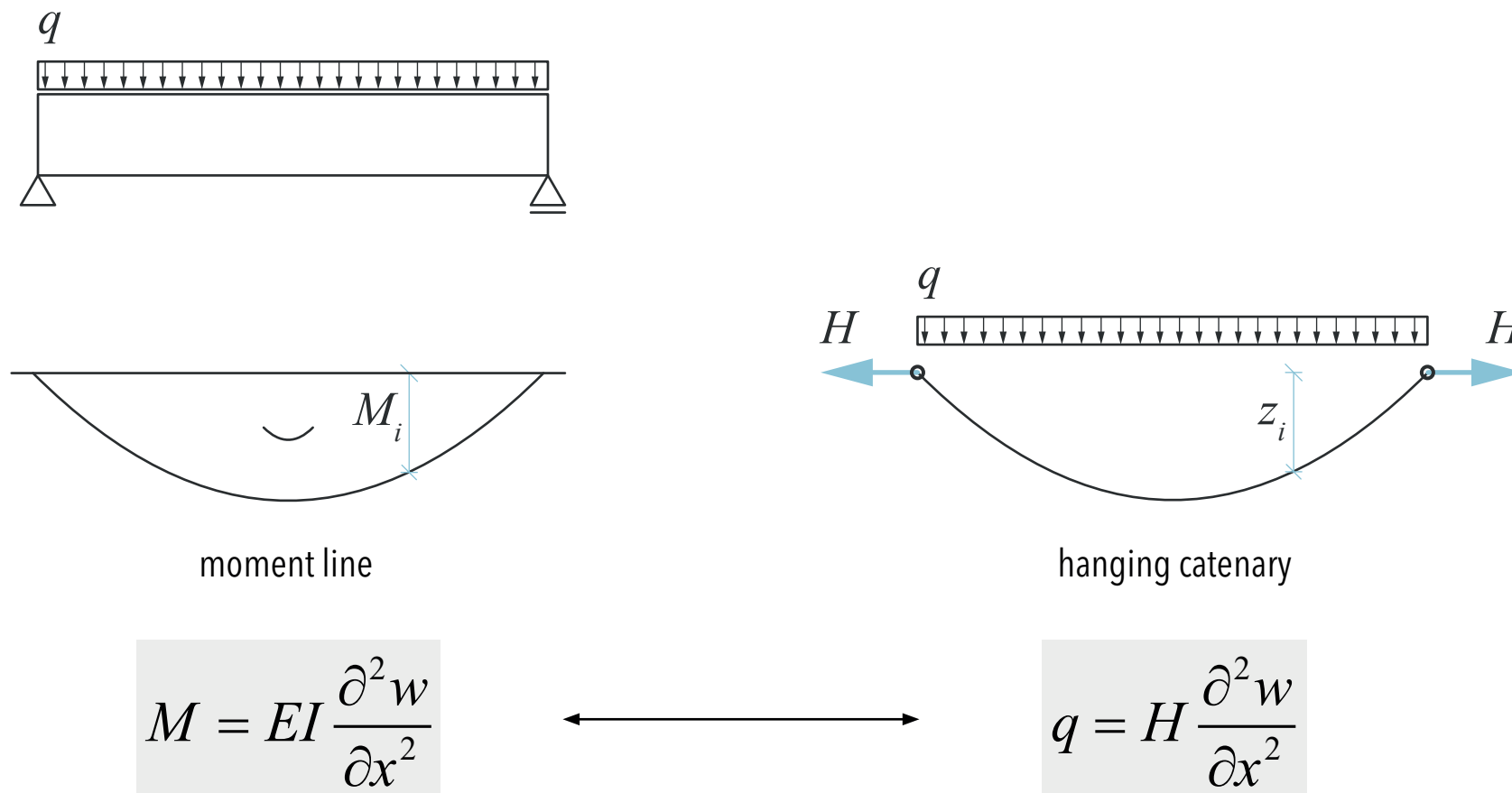
hanging catenary

q = evenly distributed load $[N/m]$

M = bending moment $[Nm]$

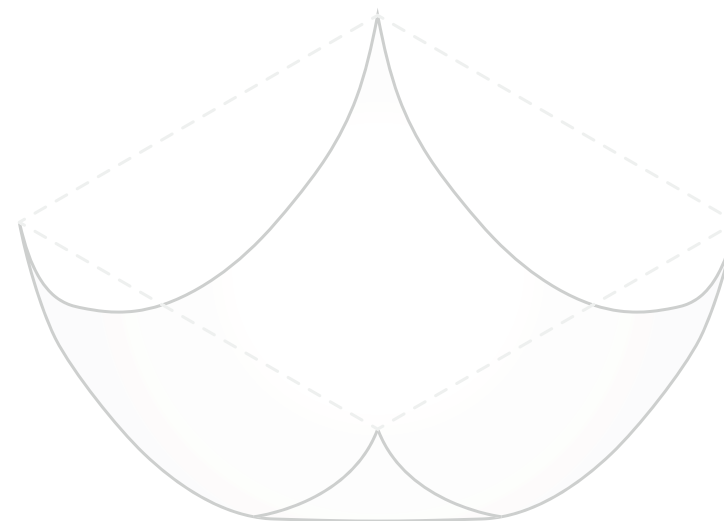
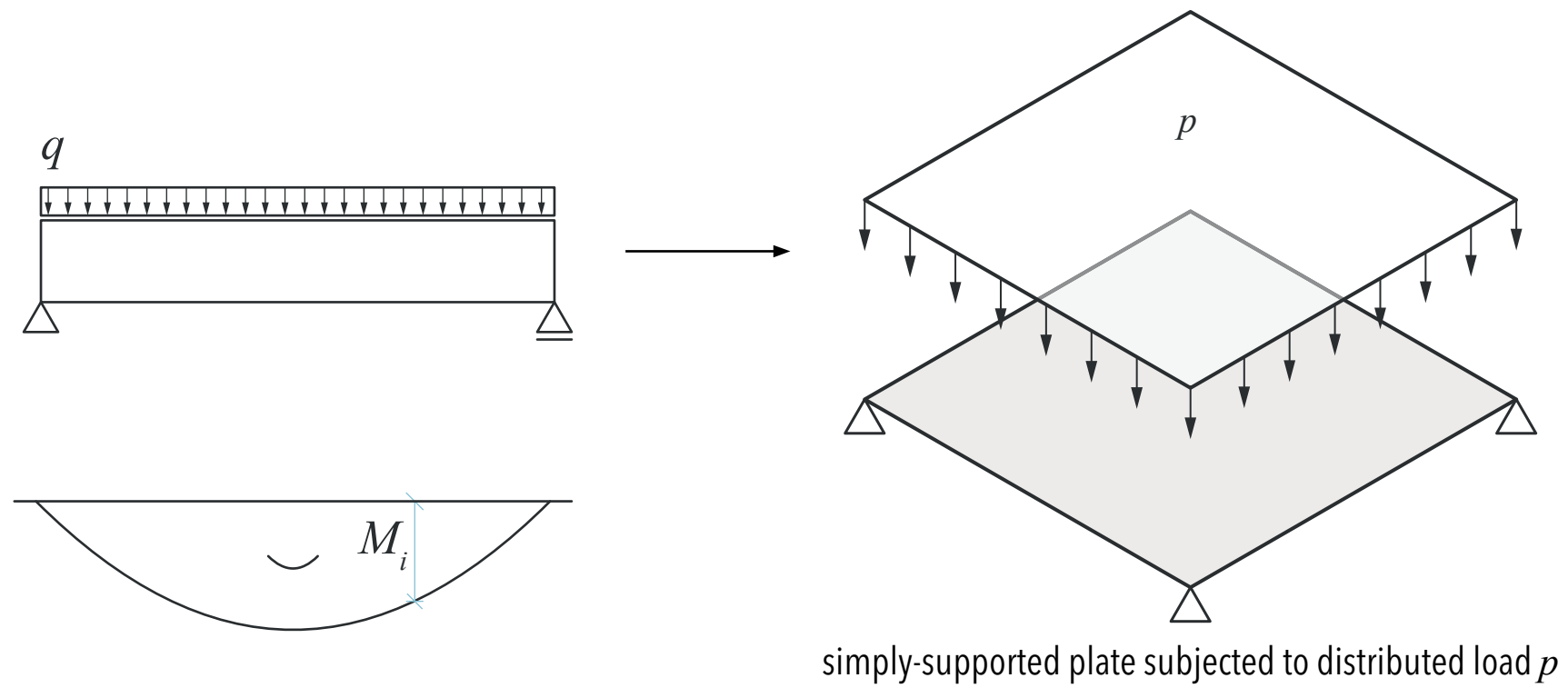
H = horizontal support reaction $[N]$

ANALOGY MOMENT LINE & THRUST LINE

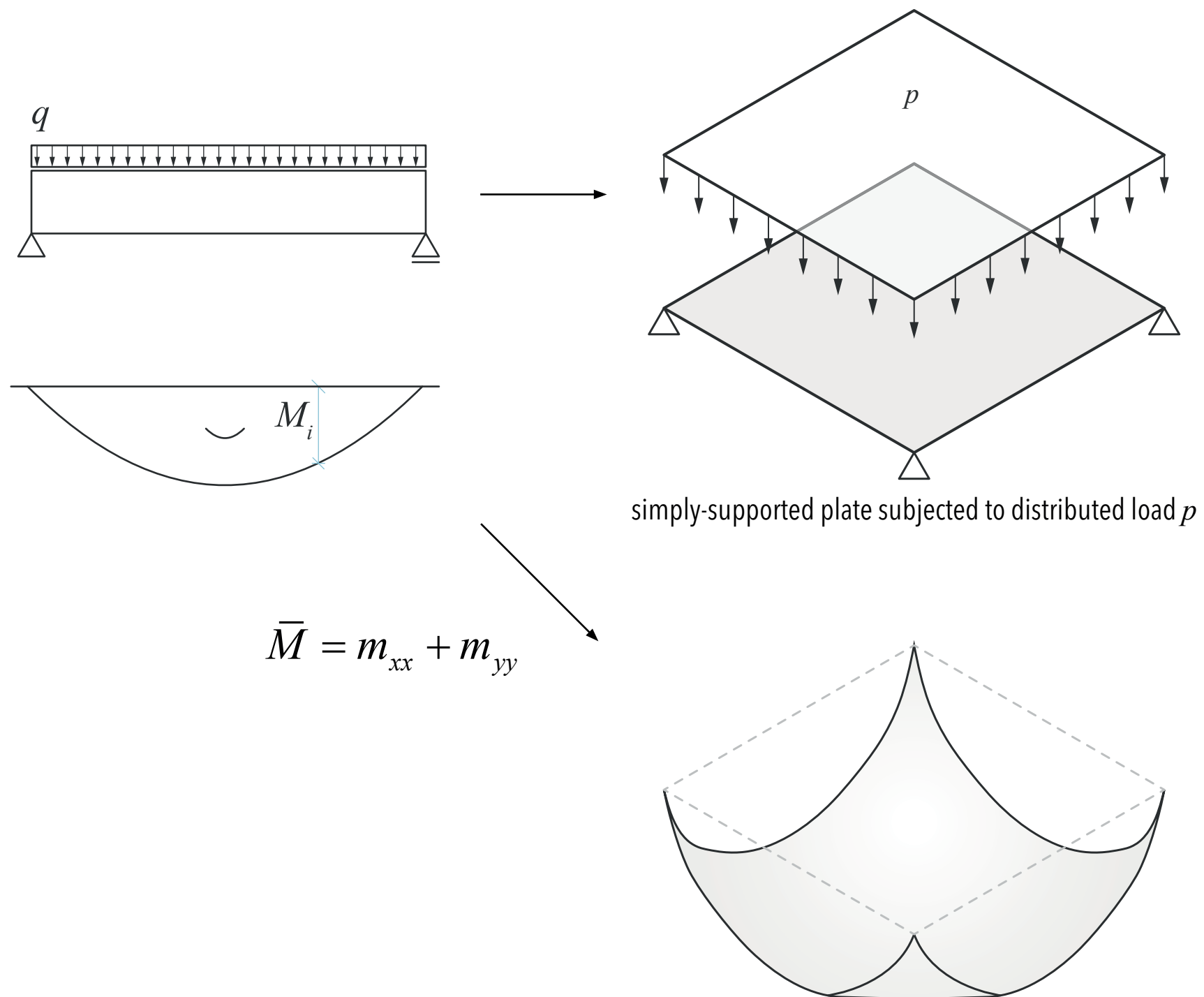


q = evenly distributed load [N/m]
 M = bending moment [Nm]
 H = horizontal support reaction [N]
 w = displacement [m]
 EI = flexural rigidity [Nm²]

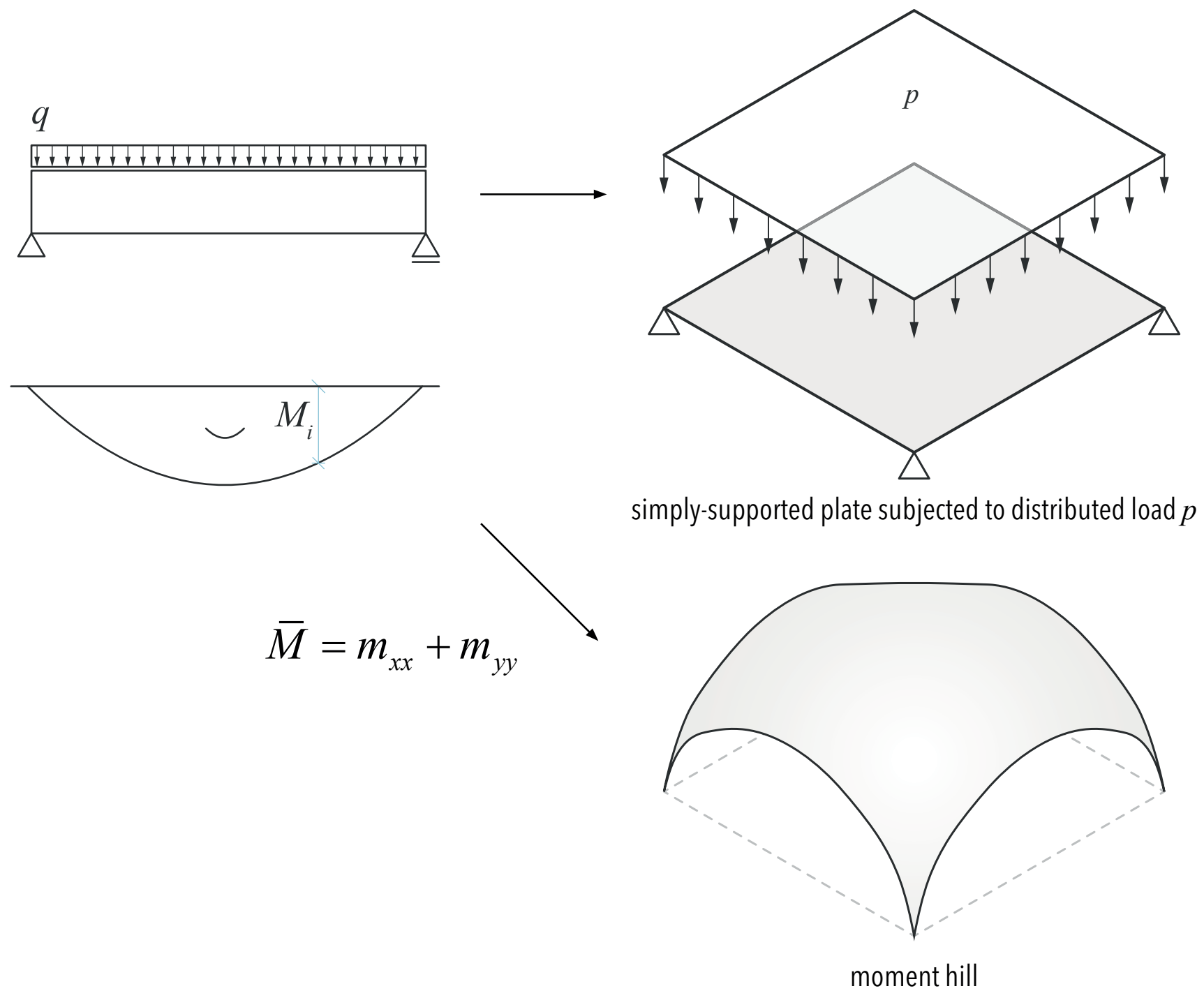
THRUST SURFACE (3D)



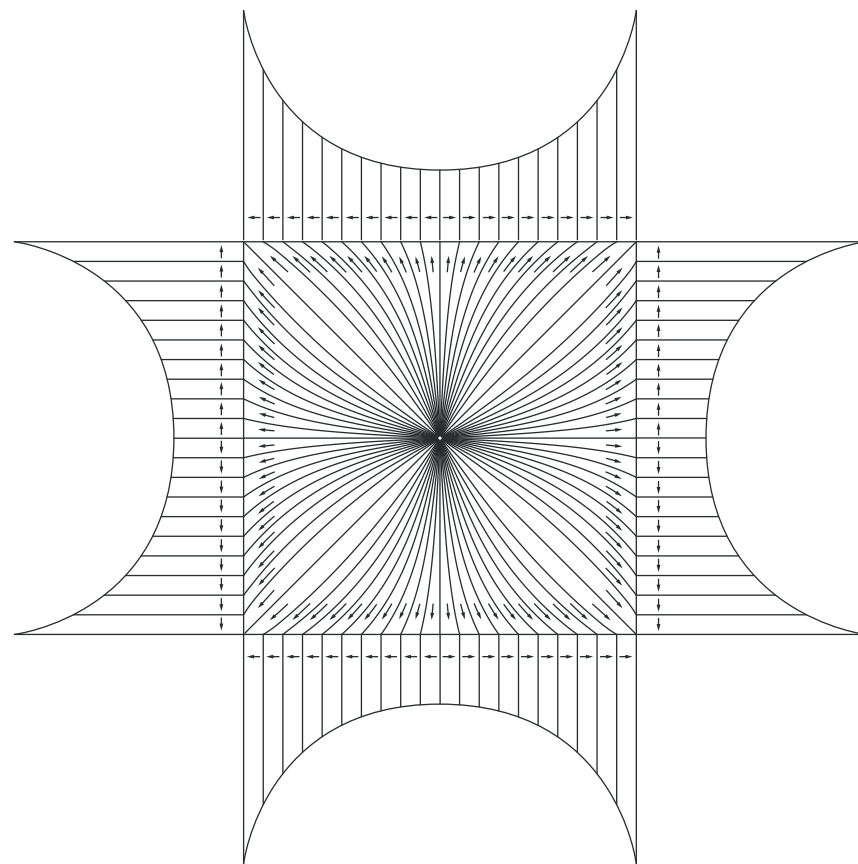
THRUST SURFACE (3D)



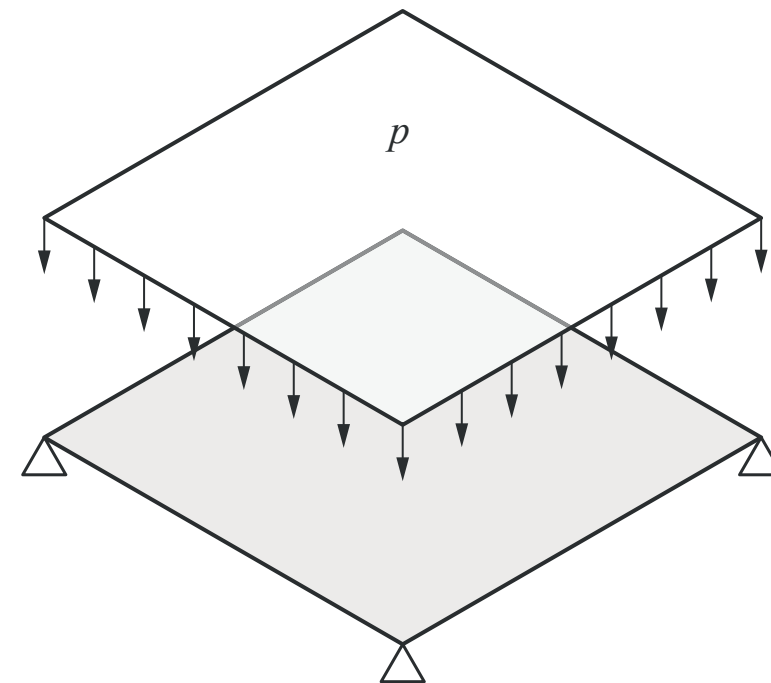
THRUST SURFACE (3D)



THRUST SURFACE (3D)

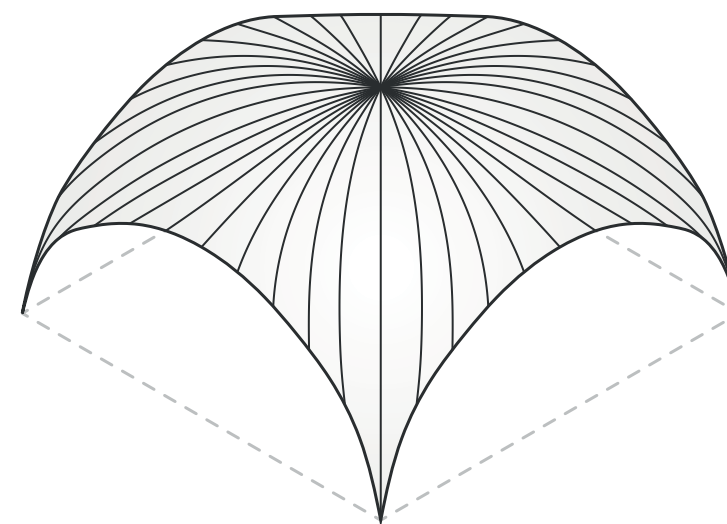


rainflow analogy



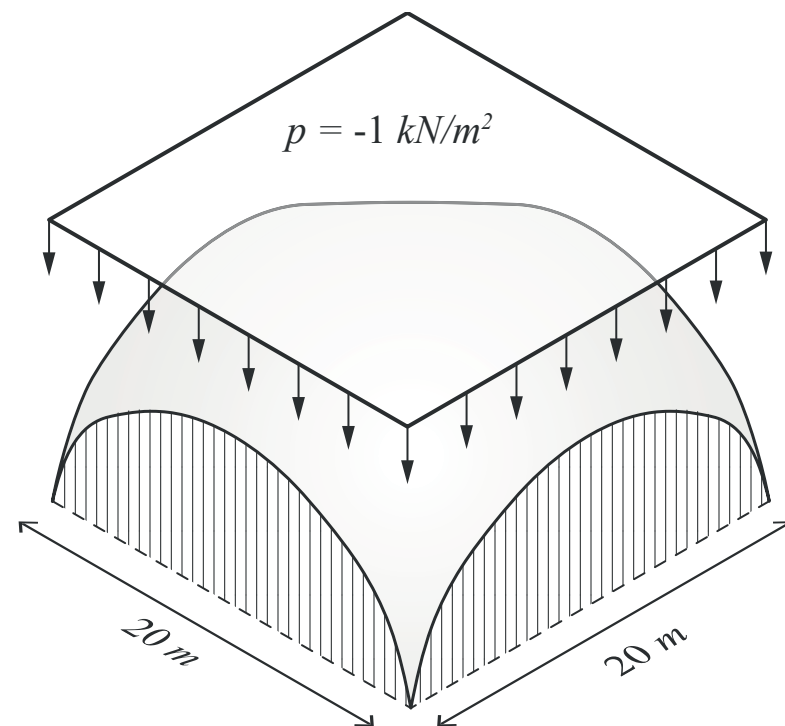
simply-supported plate subjected to distributed load p

$$\bar{M} = m_{xx} + m_{yy}$$



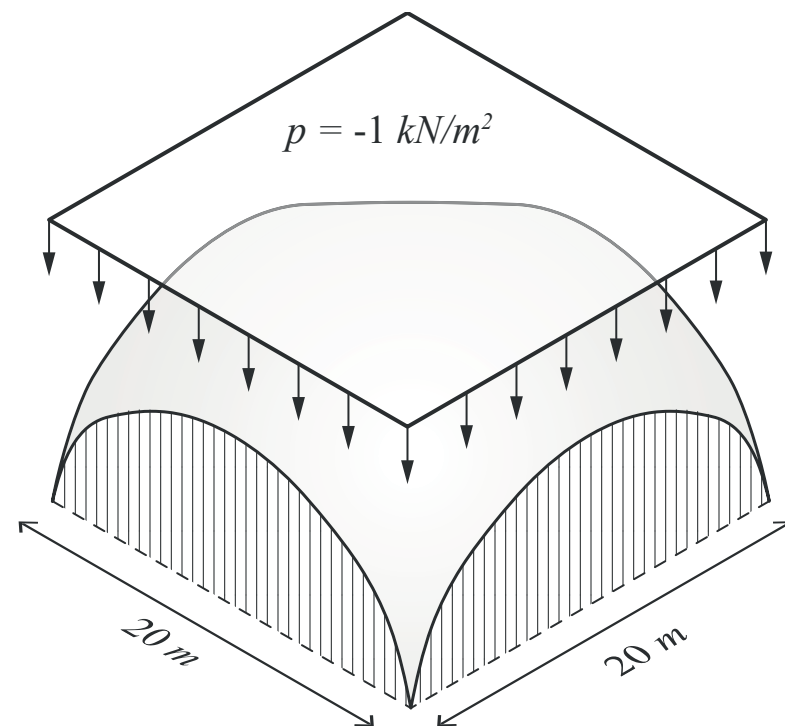
moment hill

THRUST SURFACE (3D)

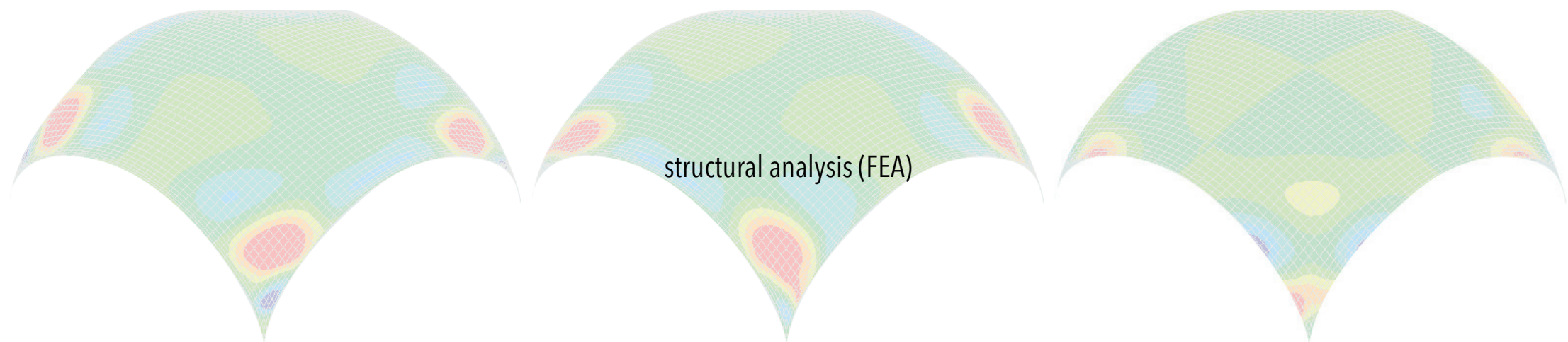


shell structure on semi-rigid edges

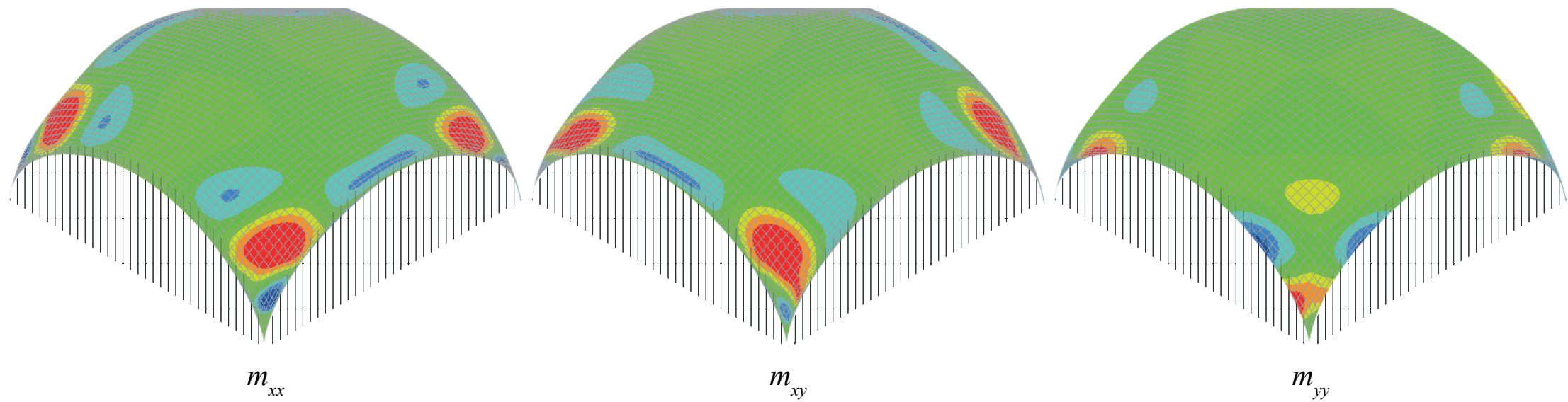
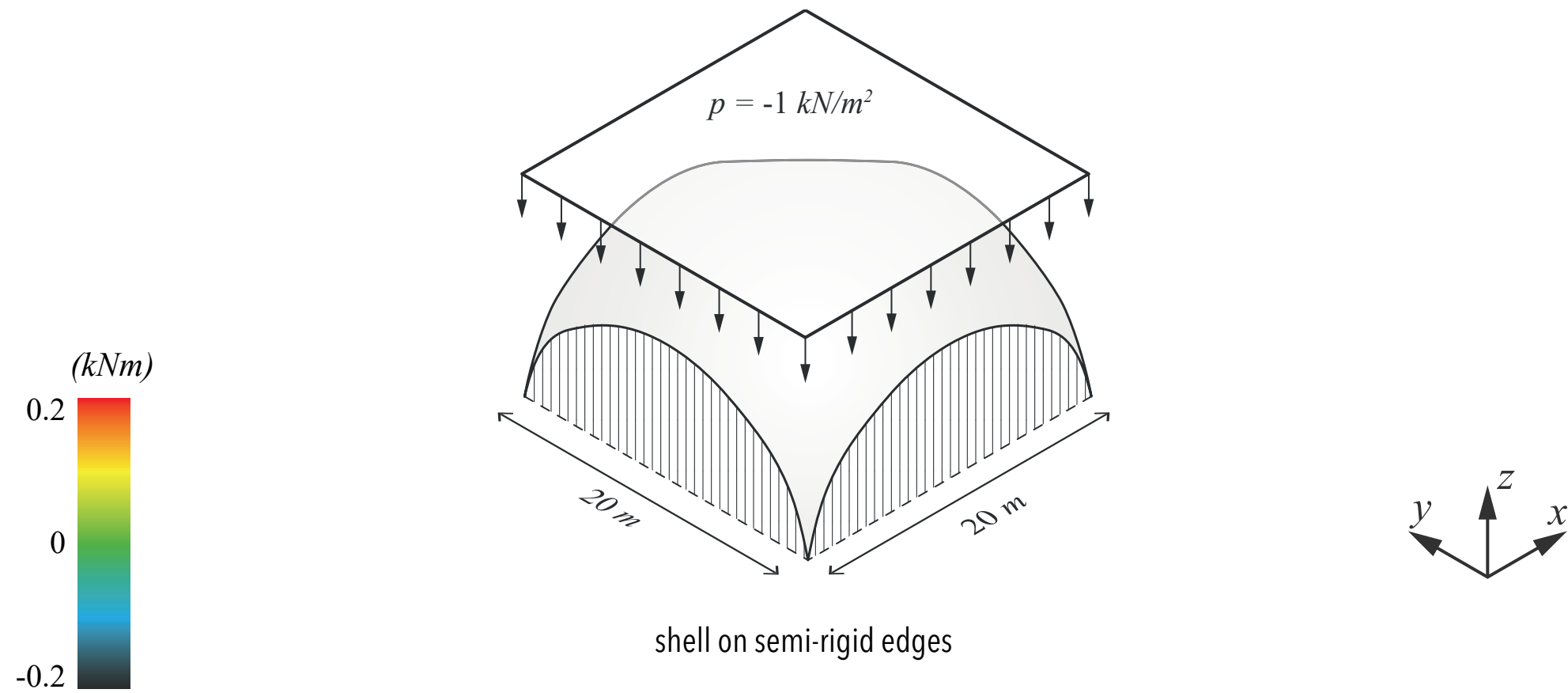
THRUST SURFACE (3D)



shell structure on semi-rigid edges

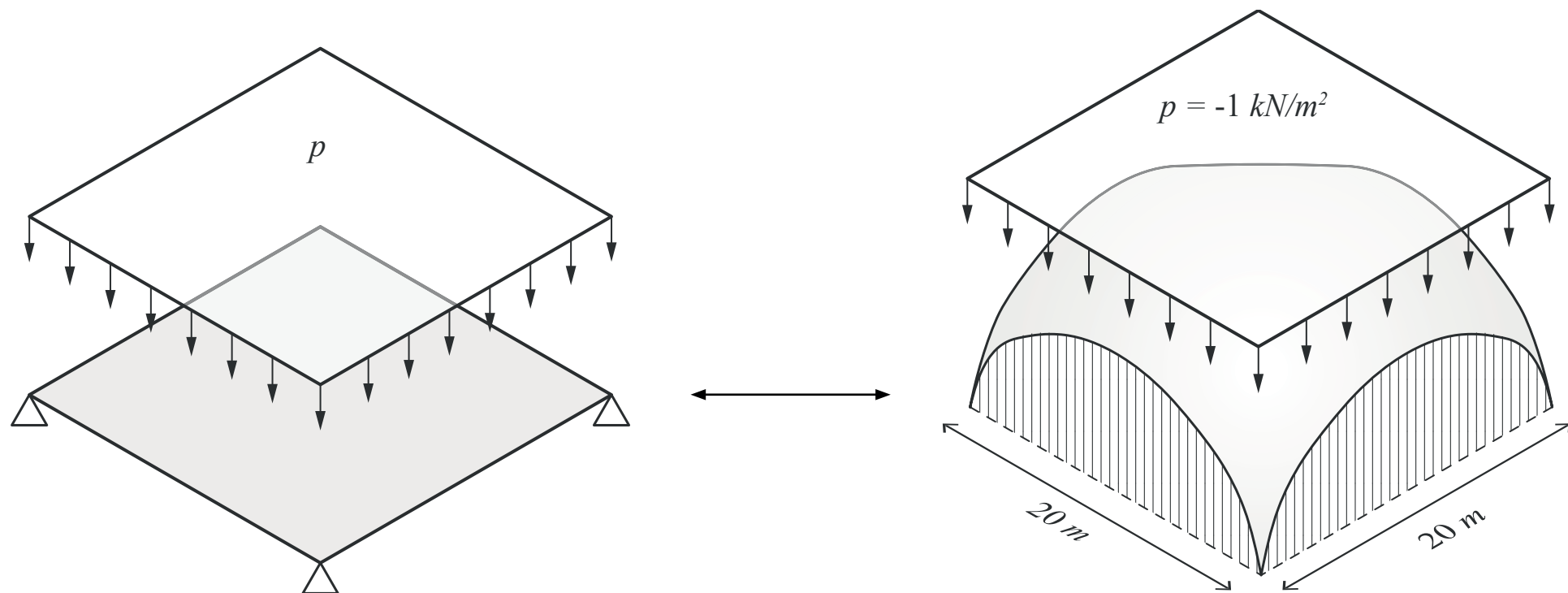


THRUST SURFACE (3D)



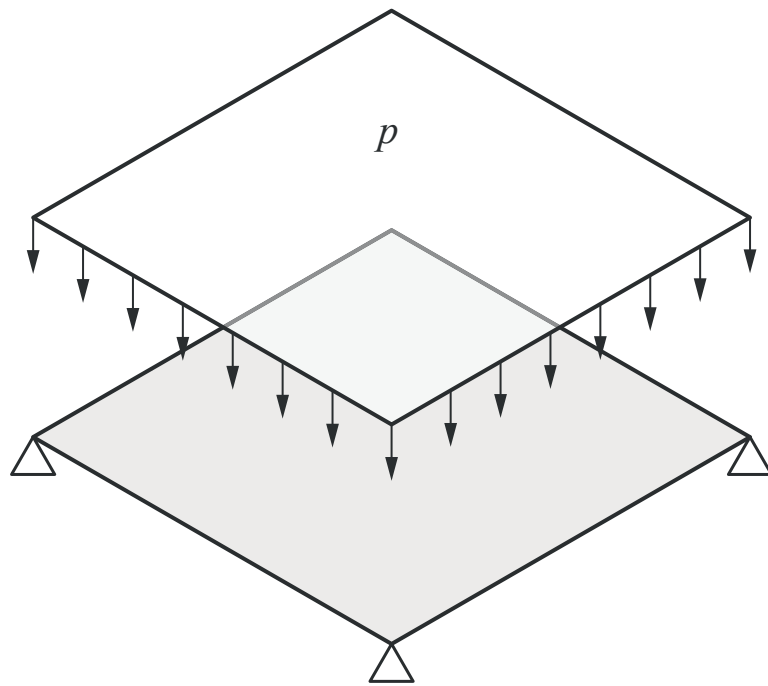
MOMENT HILL OF A PLATE AS THRUST SURFACE?

ANALOGY MOMENT HILL & THRUST SURFACE



$$p = D \left(\frac{\partial^4 w}{\partial^4 x} + 2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial^4 y} \right)$$

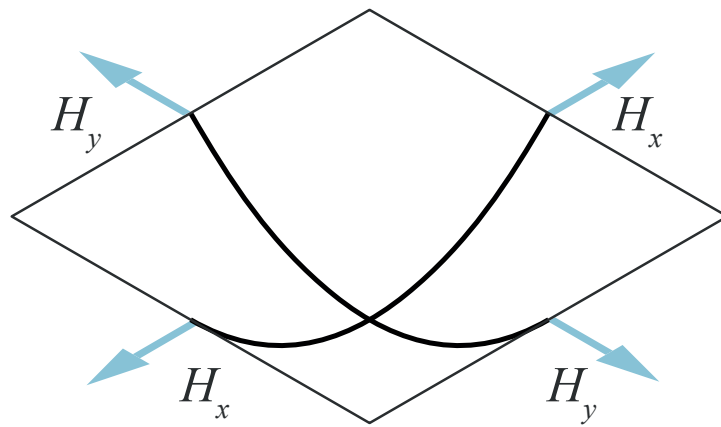
ANALOGY MOMENT HILL & THRUST SURFACE



$$p = D \left(\frac{\partial^4 w}{\partial^4 x} + 2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial^4 y} \right)$$

displacement due to shear

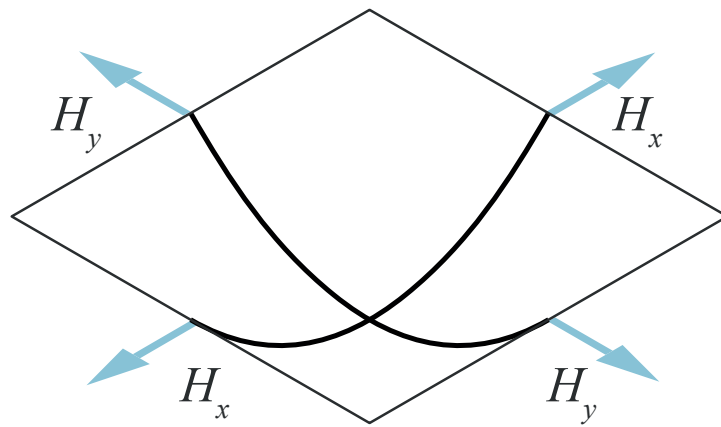
ANALOGY MOMENT HILL & THRUST SURFACE



Two individual systems in x- and y-direction

$$p = D \left(\frac{\partial^4 w}{\partial^4 x} + 2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial^4 y} \right)$$

ANALOGY MOMENT HILL & THRUST SURFACE

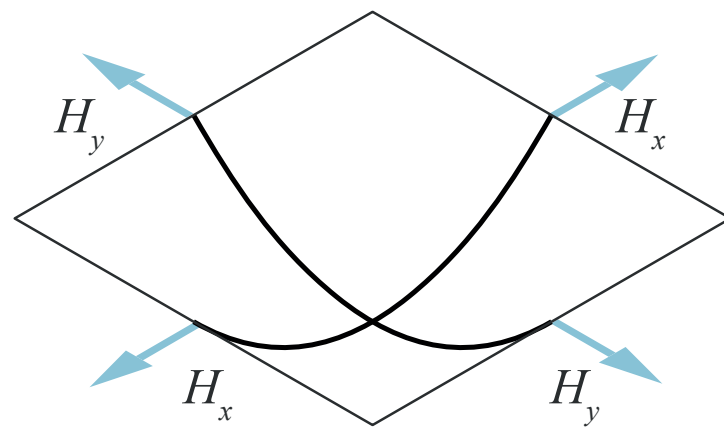


Two individual systems in x- and y-direction

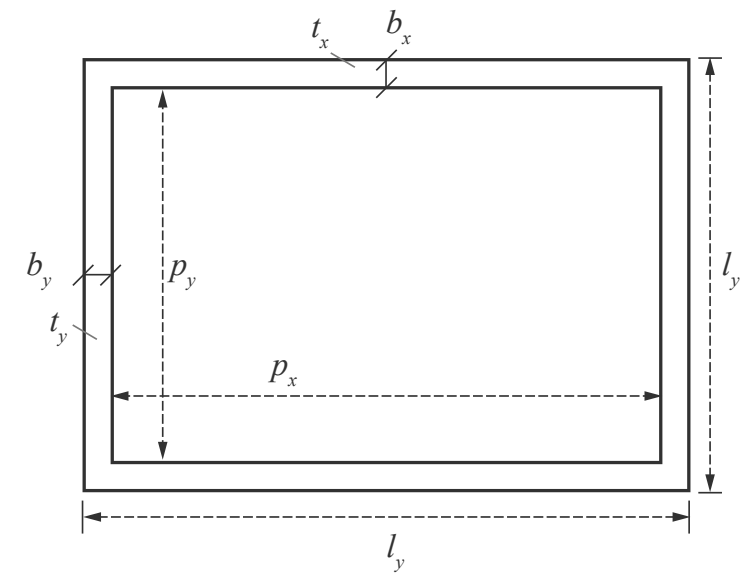
$$p = D \left(\frac{\partial^4 w}{\partial^4 x} + 2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial^4 y} \right)$$

$$p = H_x \frac{\partial^2 w}{\partial x^2} + H_y \frac{\partial^2 w}{\partial y^2}$$

ANALOGY MOMENT HILL & THRUST SURFACE



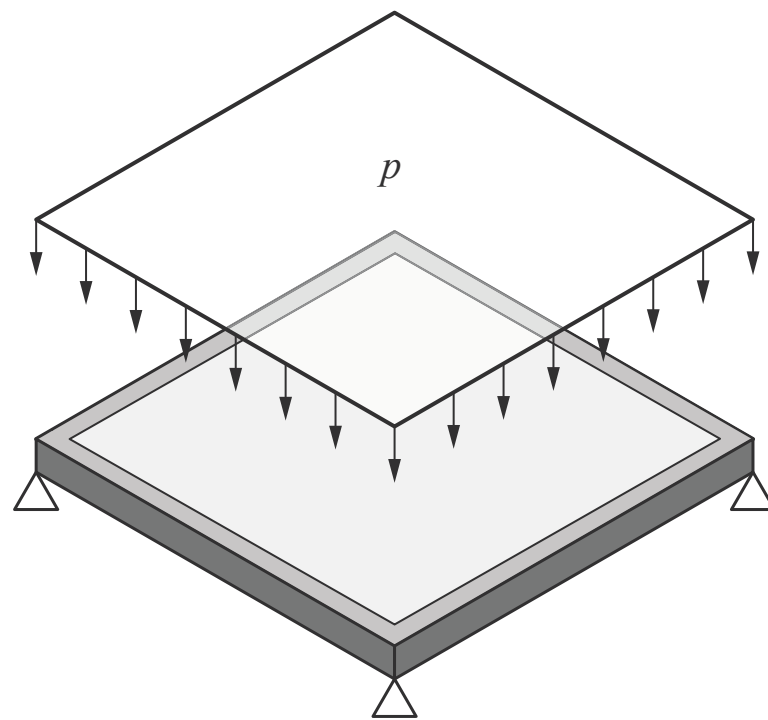
Two individual systems in x- and y-direction



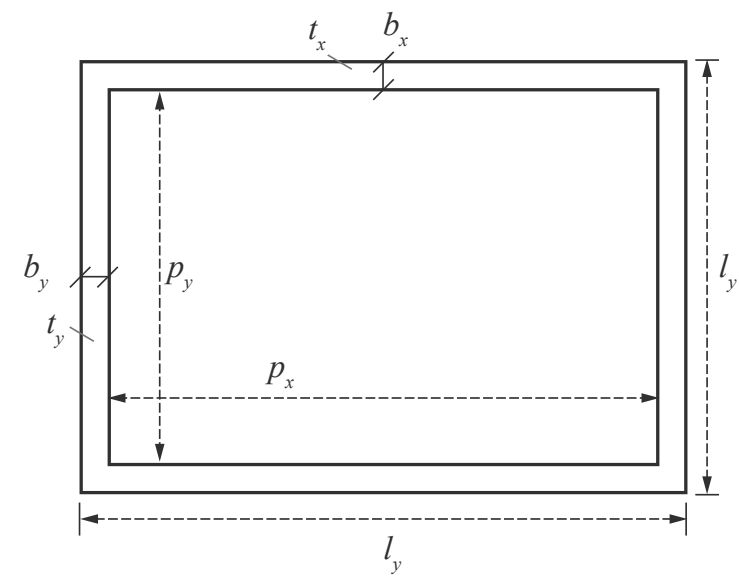
'twistless case'

Beranek & Hobbelman, 1984

ANALOGY MOMENT HILL & THRUST SURFACE



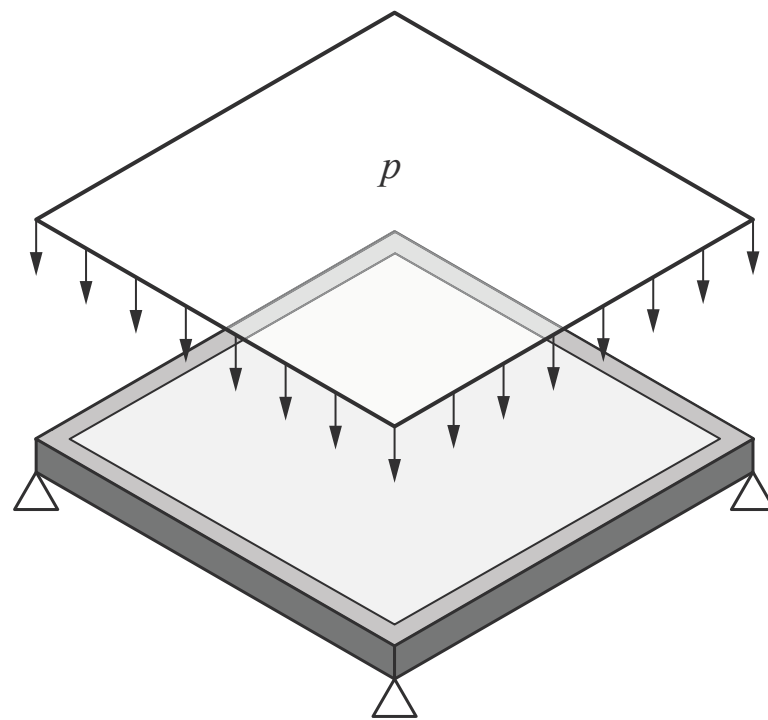
orthotropic plate
isotropic rim



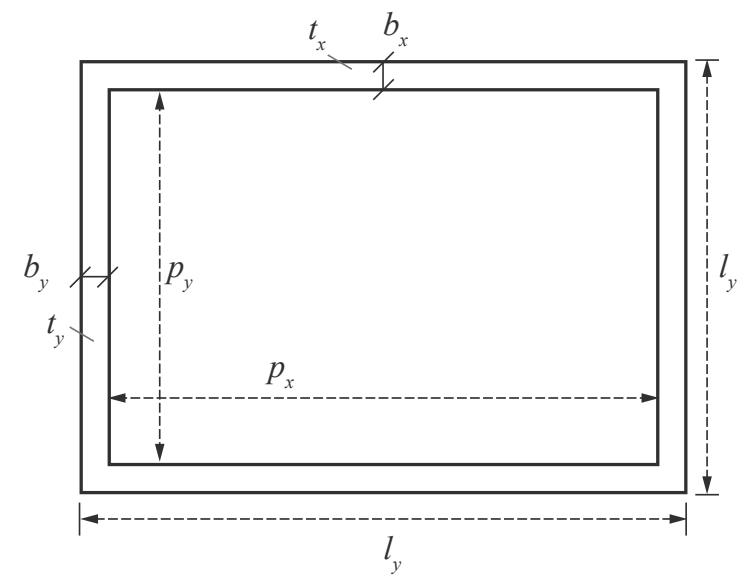
'twistless case'

Beranek & Hobbelman, 1984

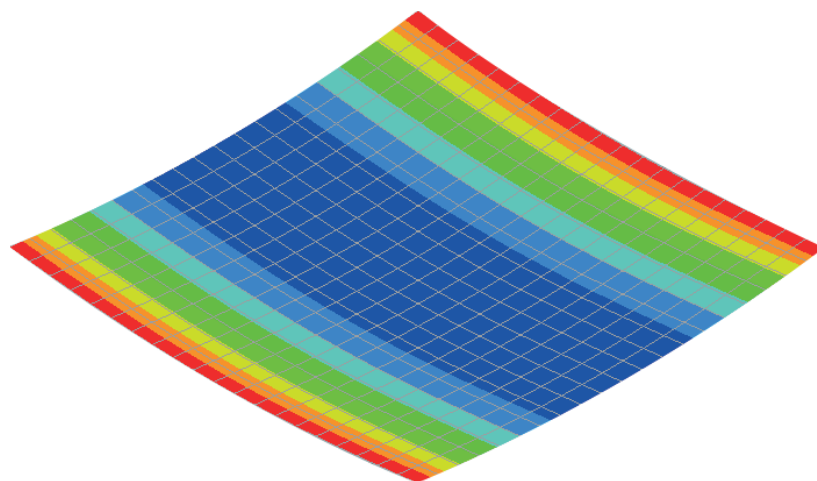
ANALOGY MOMENT HILL & THRUST SURFACE



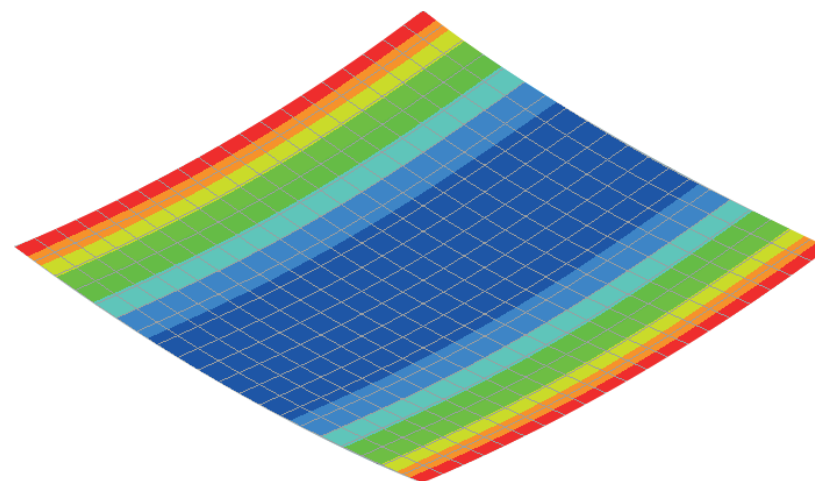
orthotropic plate
isotropic rim



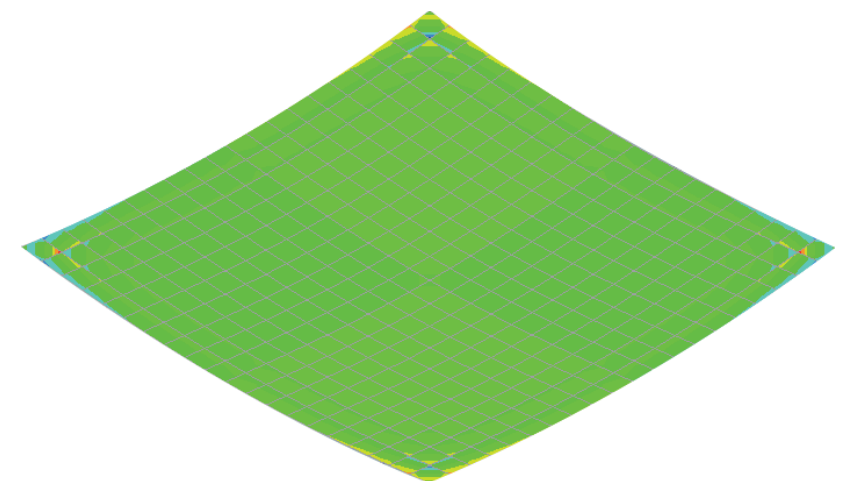
'twistless case'



m_{xx}

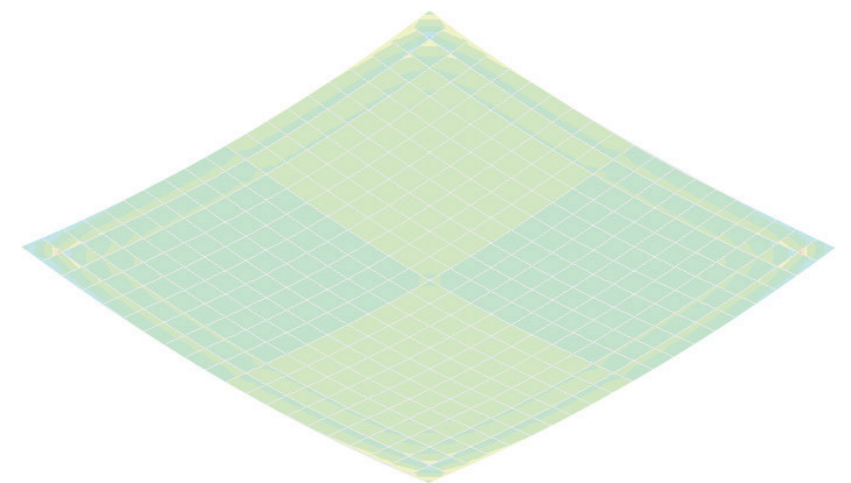
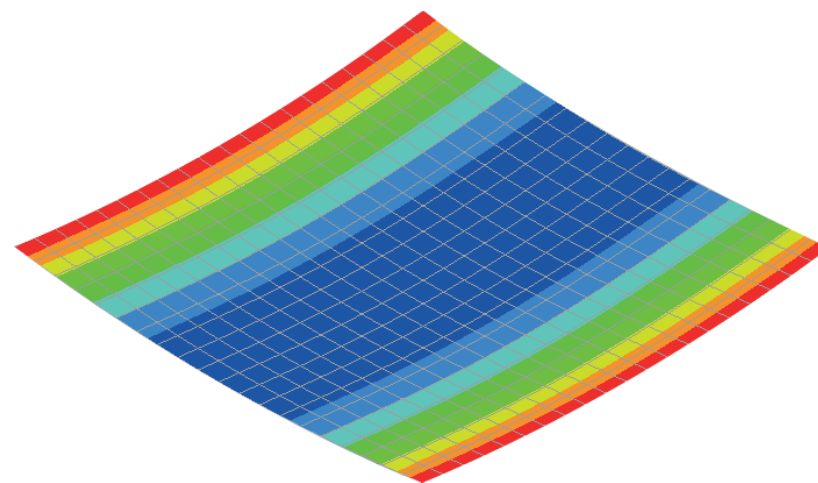
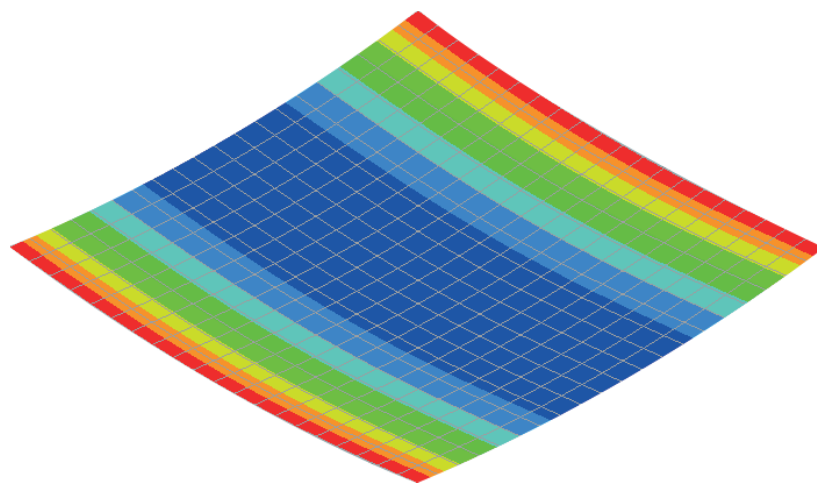
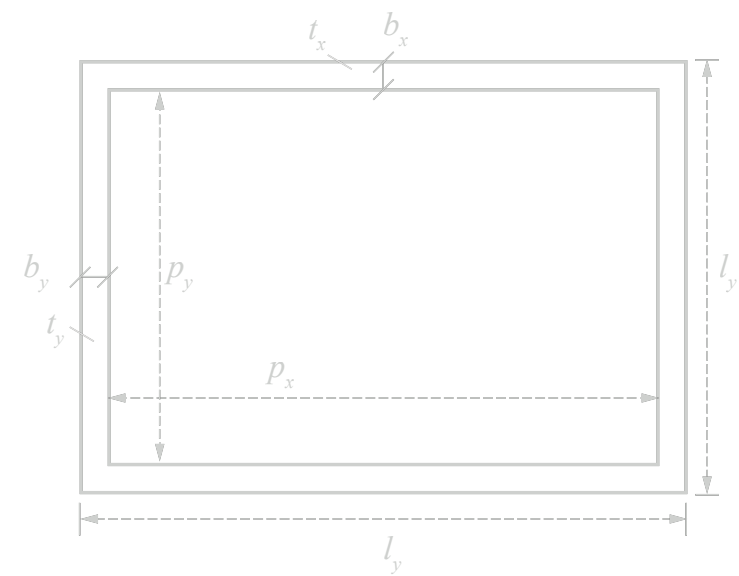
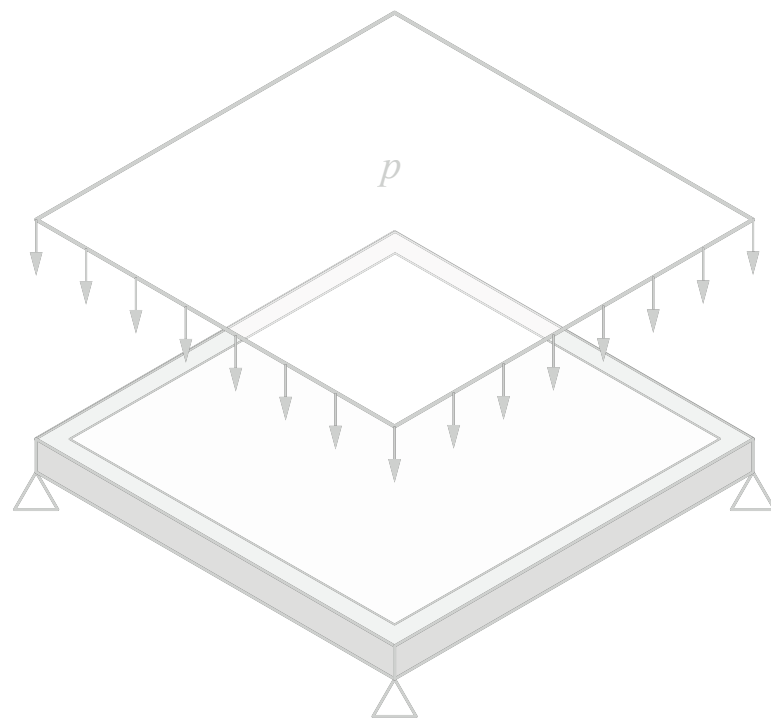


m_{yy}

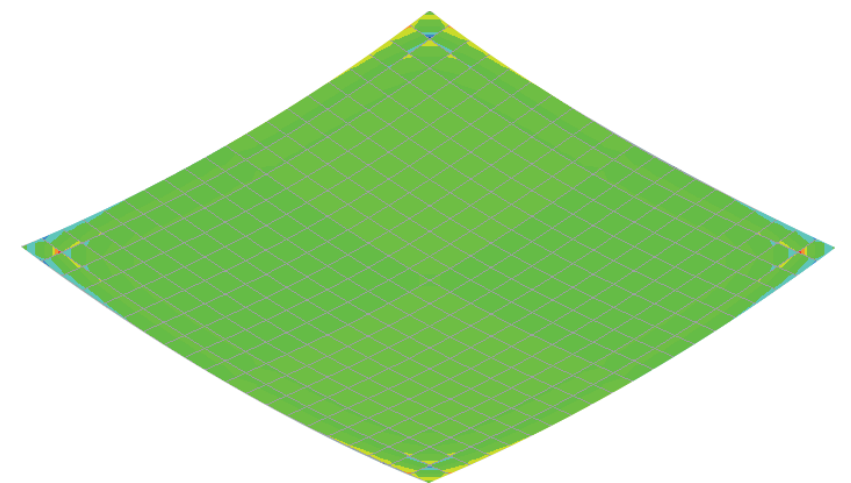
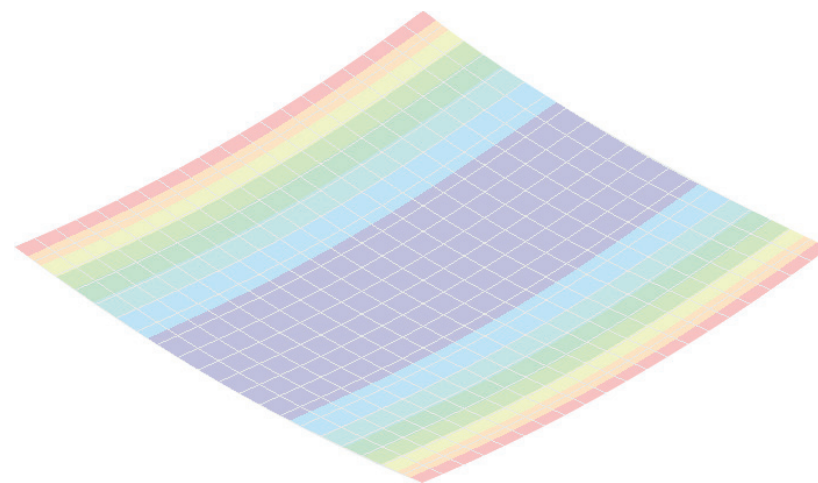
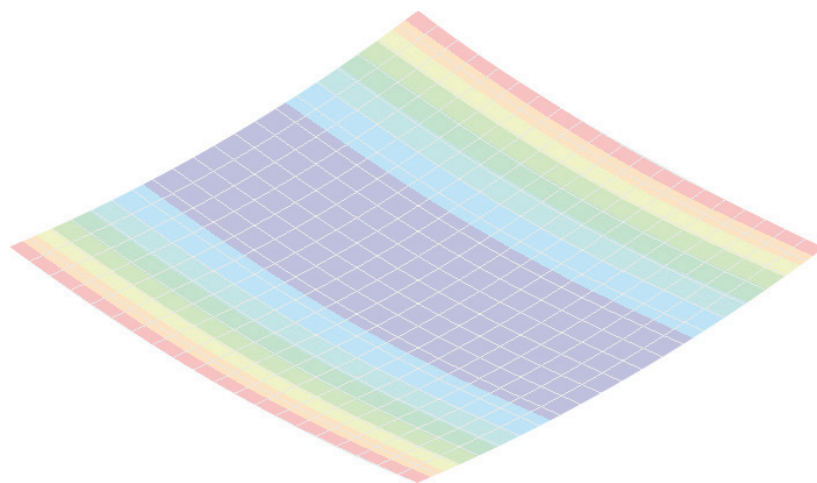
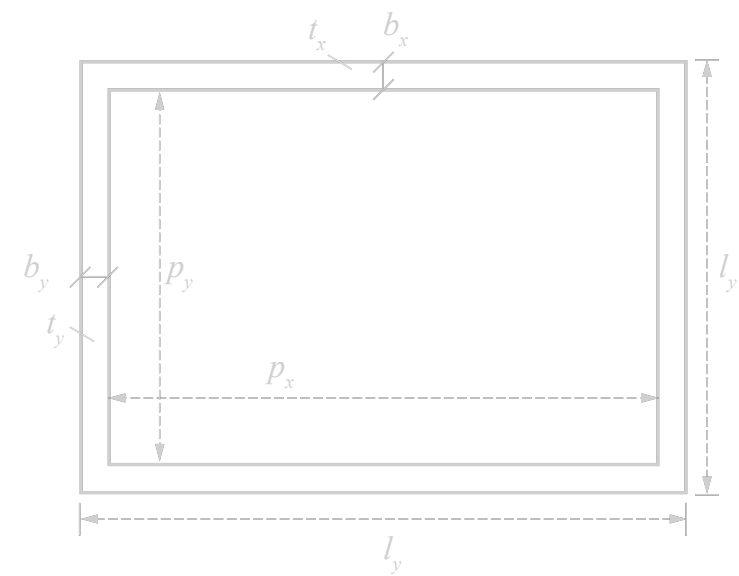
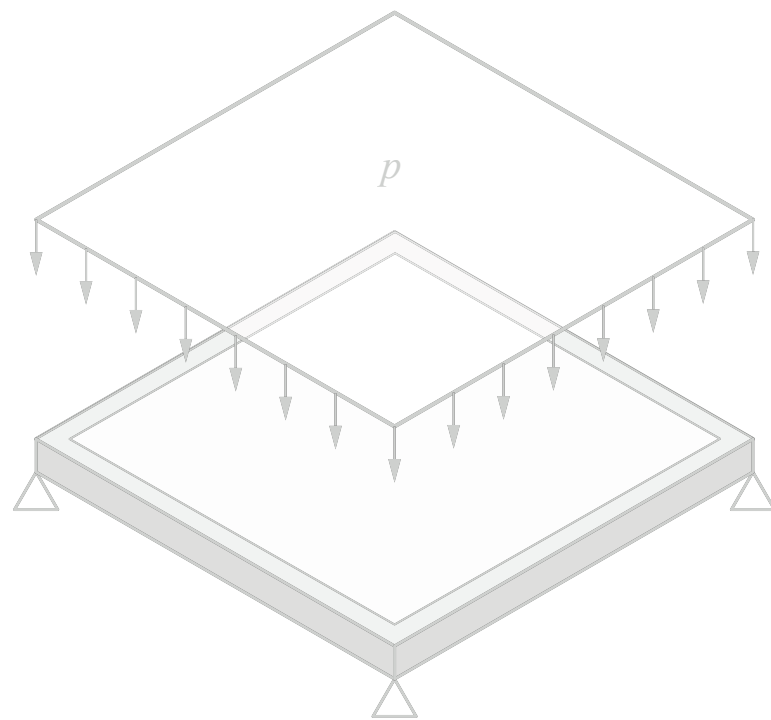


m_{xy}

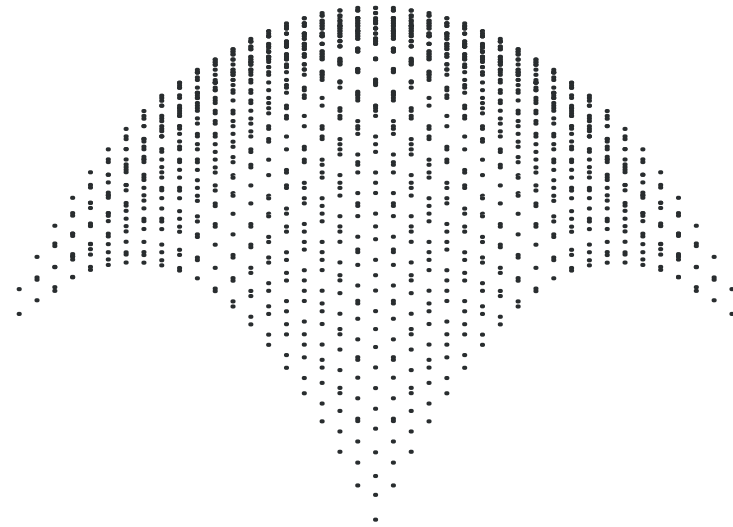
ANALOGY MOMENT HILL & THRUST SURFACE



ANALOGY MOMENT HILL & THRUST SURFACE

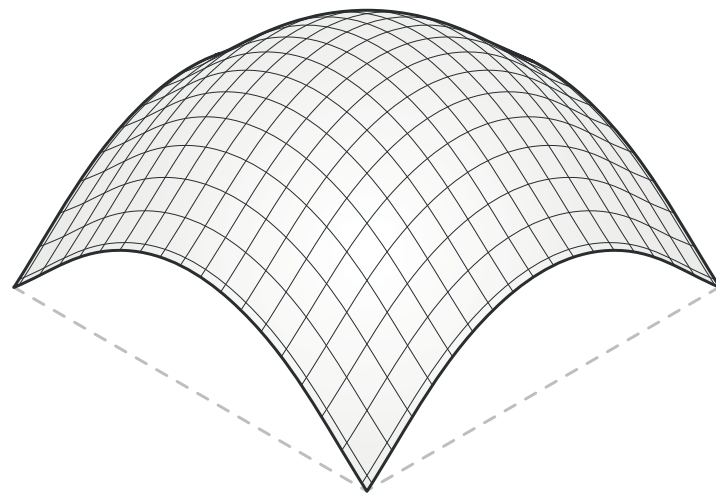


ANALOGY MOMENT HILL & THRUST SURFACE

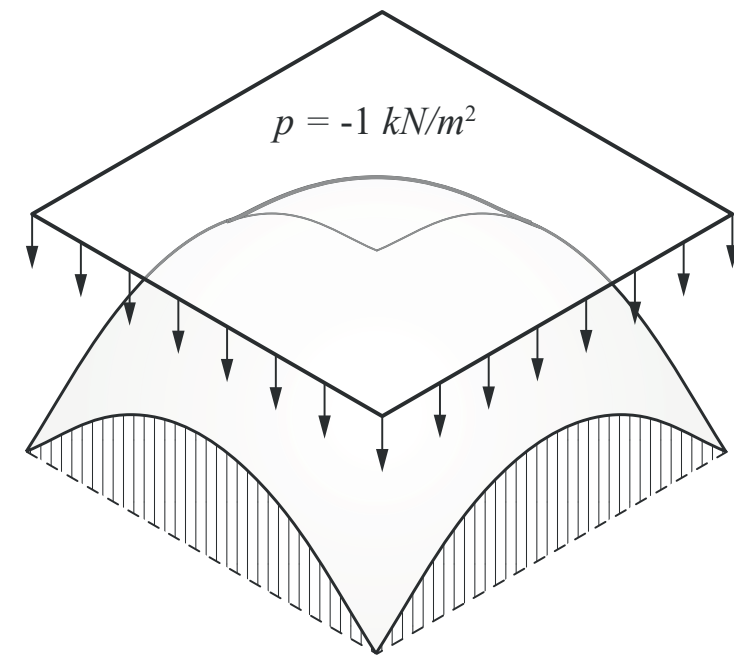


values of m_{xx} and m_{yy} plotted

ANALOGY MOMENT HILL & THRUST SURFACE

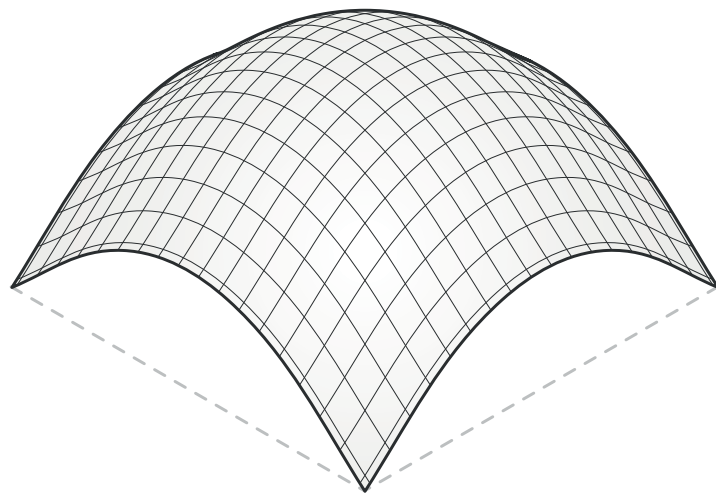


moment hill 'twistless case'

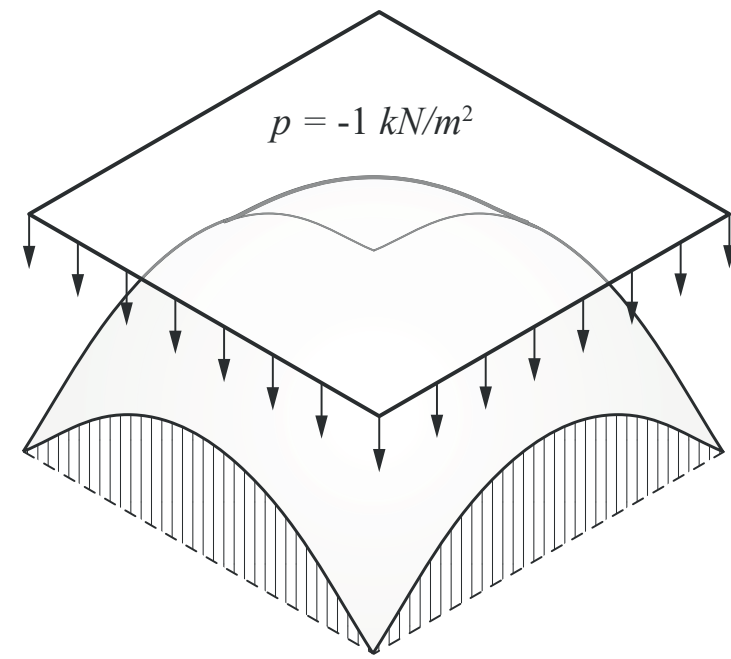


shell supported on semi-rigid edges

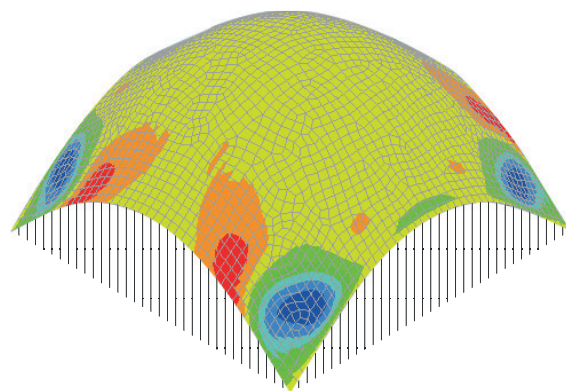
ANALOGY MOMENT HILL & THRUST SURFACE



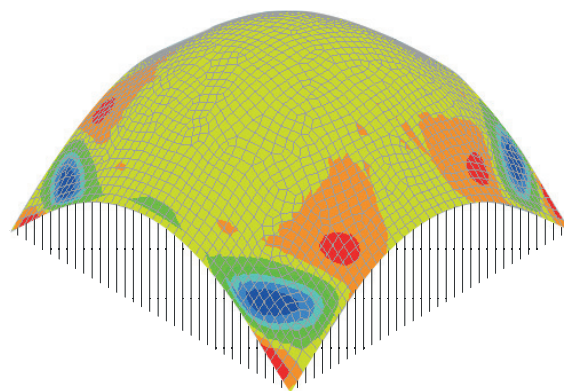
moment hill 'twistless case'



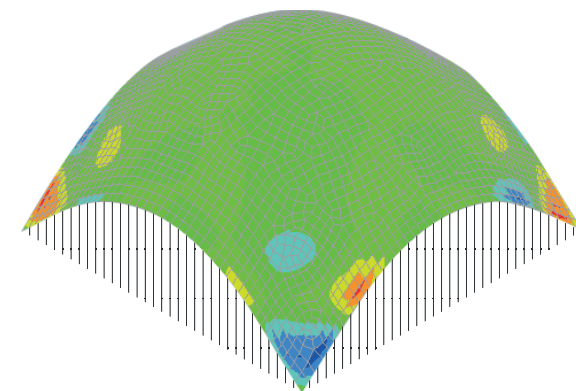
shell supported on semi-rigid edges



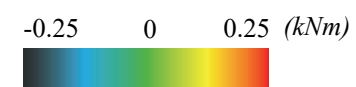
m_{xx}

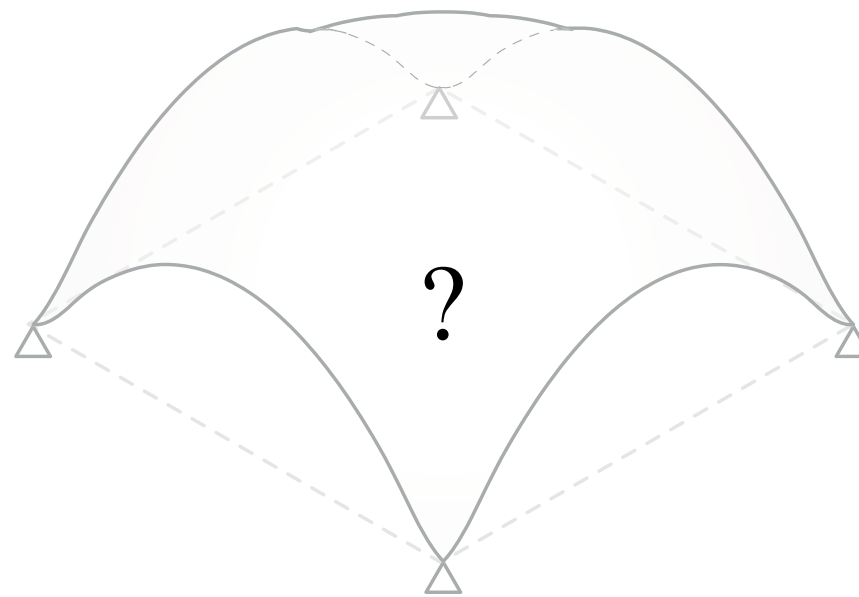


m_{yy}



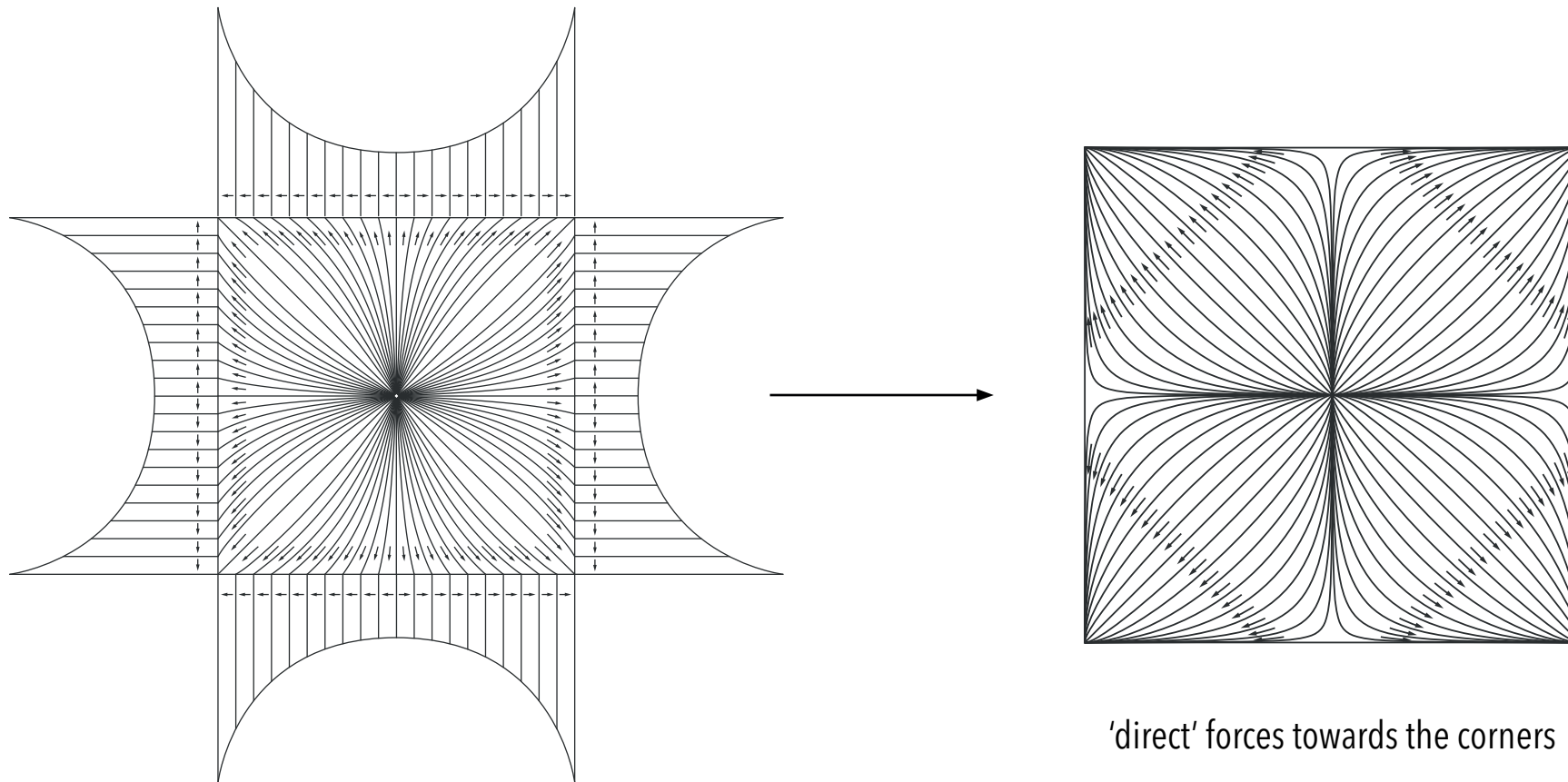
m_{xy}

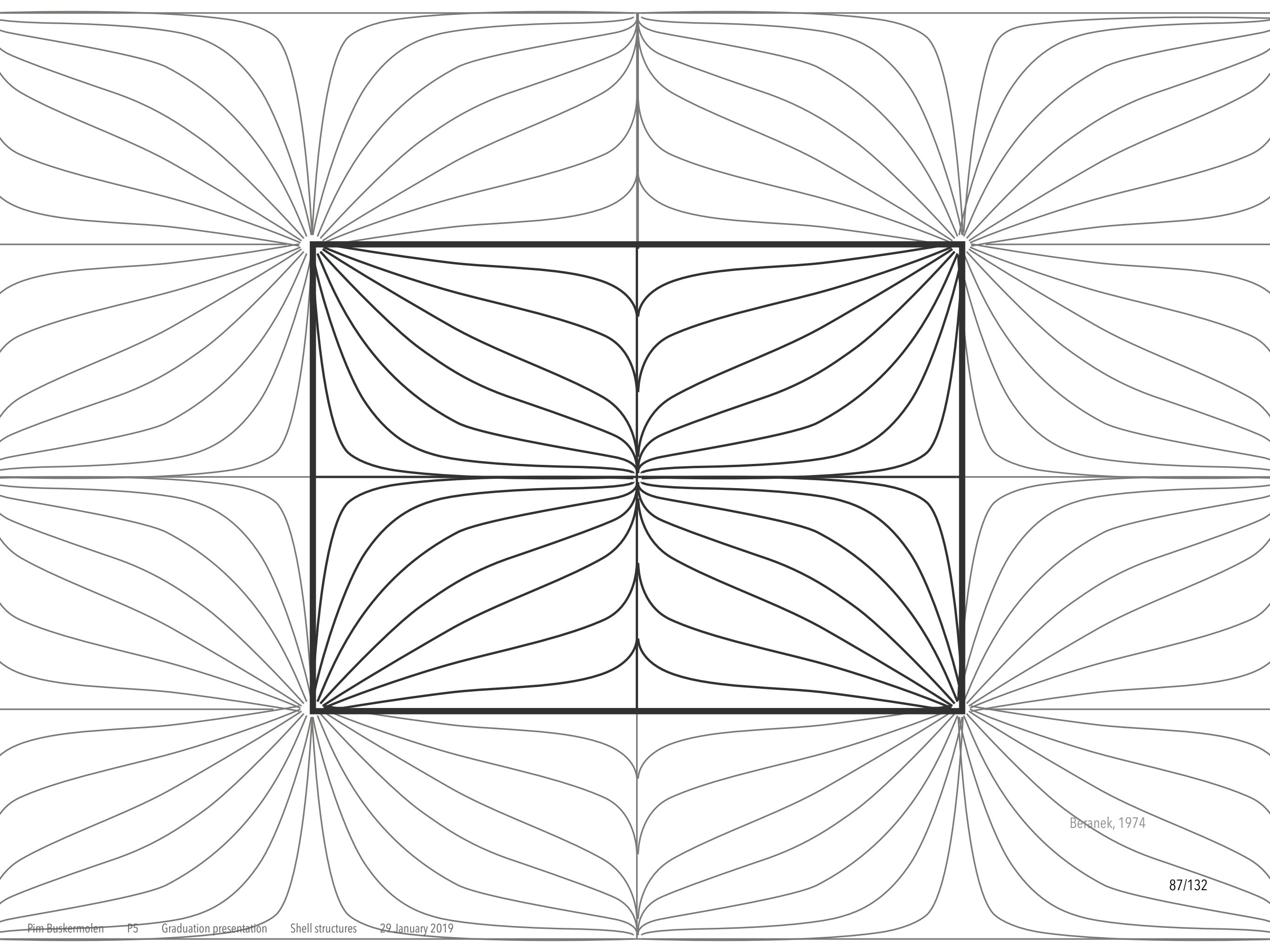




shell structure with free edges?

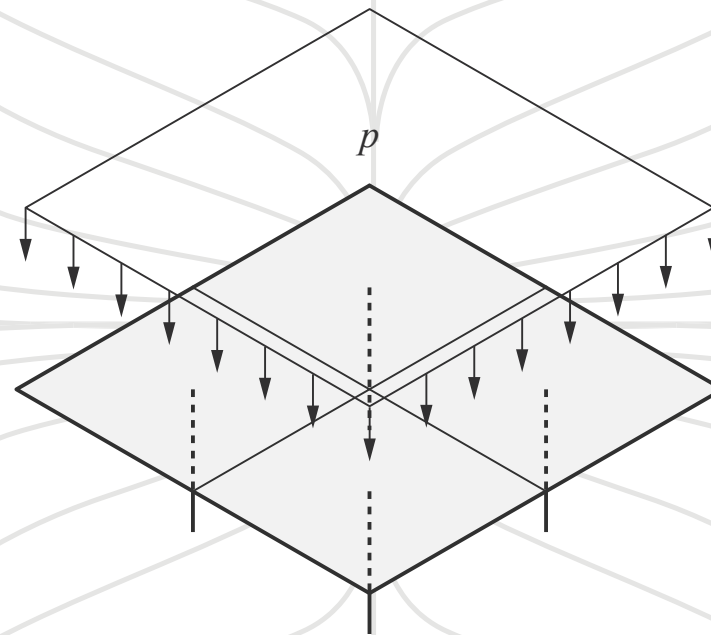
SHELL WITH FREE EDGES





Beranek, 1974

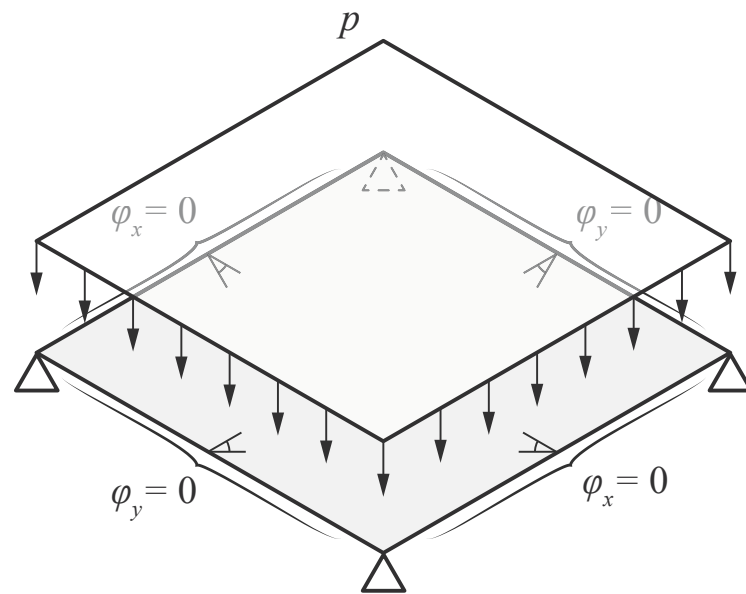
87/132



continuous floor on four supports

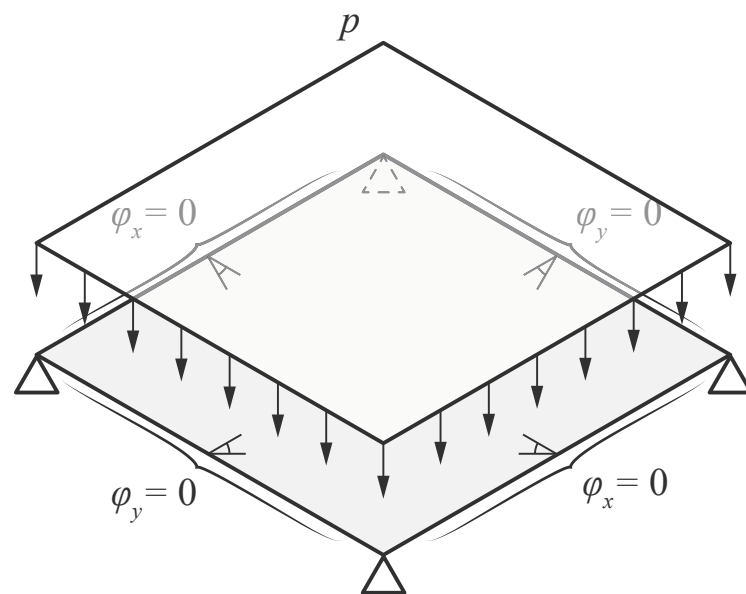
Beranek, 1974

SHELL WITH FREE EDGES

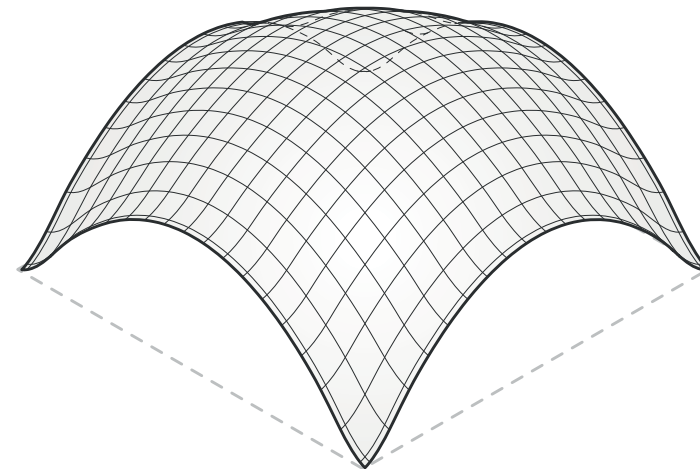


rotation $\varphi = 0$ (symmetry edge)

SHELL WITH FREE EDGES

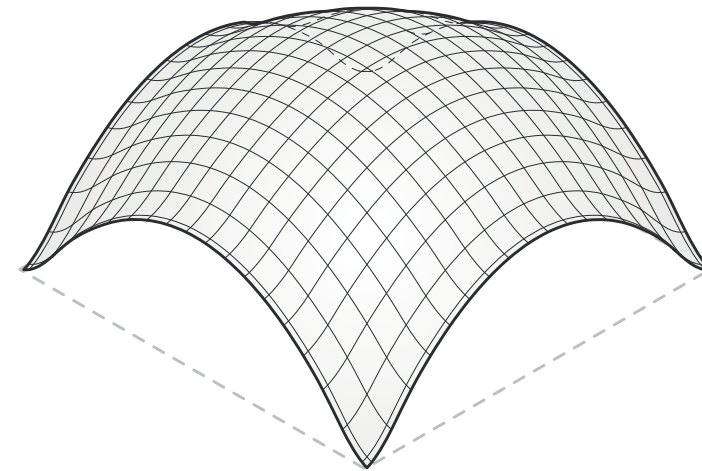
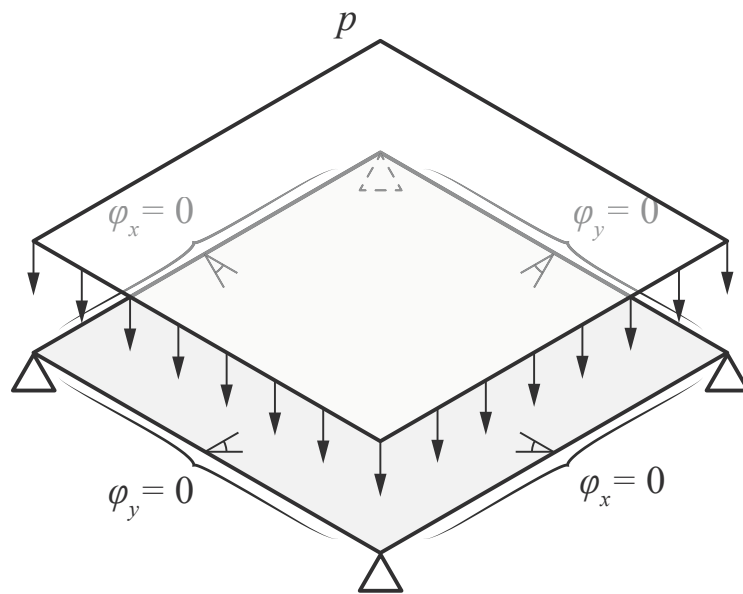


rotation $\varphi = 0$ (symmetry edge)



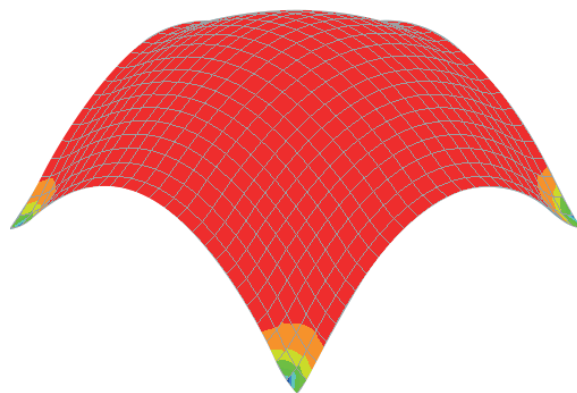
moment hill (of twistless plate)

SHELL WITH FREE EDGES

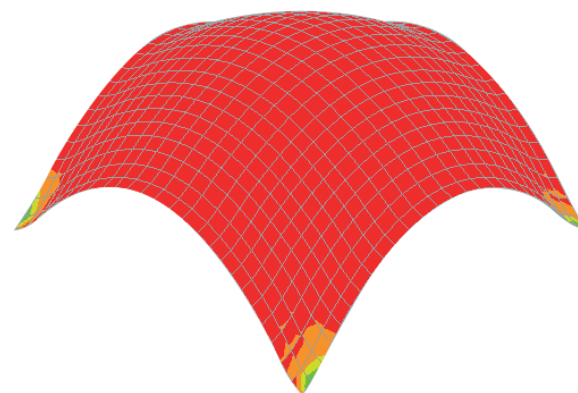


moment hill (of twistless plate)

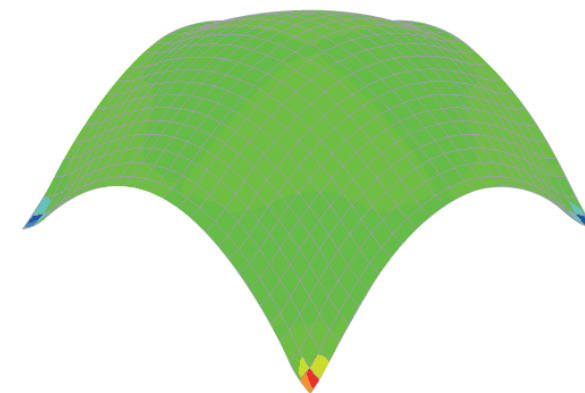
rotation $\varphi = 0$ (symmetry edge)



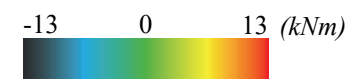
m_{xx}



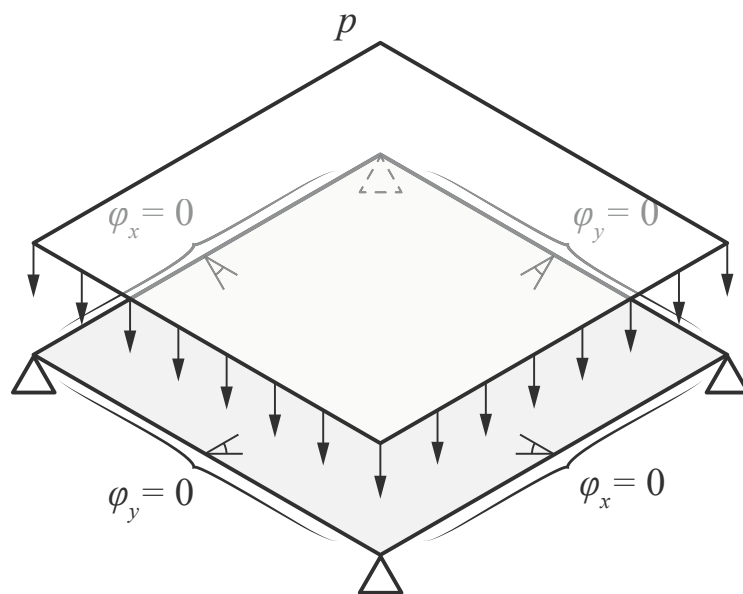
m_{yy}



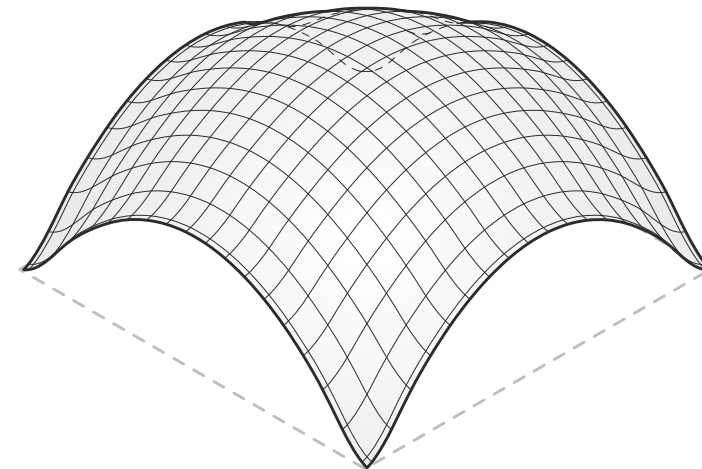
m_{xy}



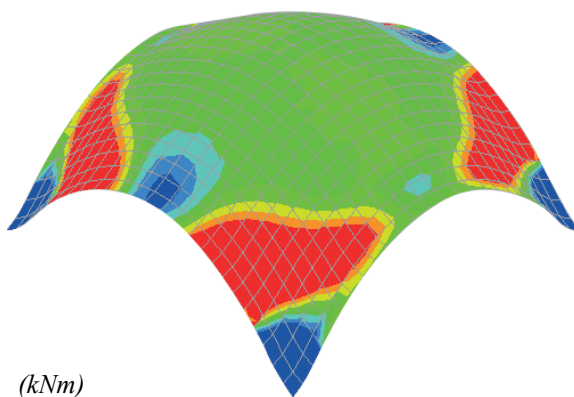
SHELL WITH FREE EDGES



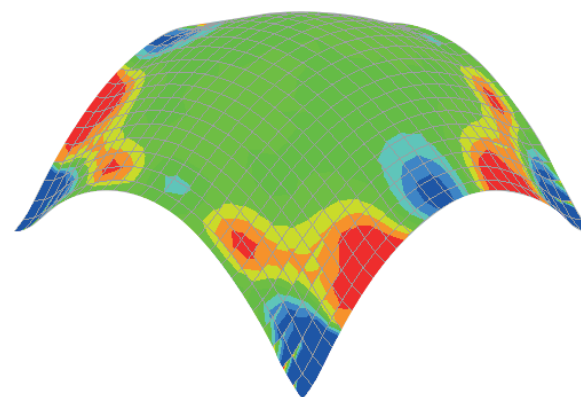
rotation $\varphi = 0$ (symmetry edge)



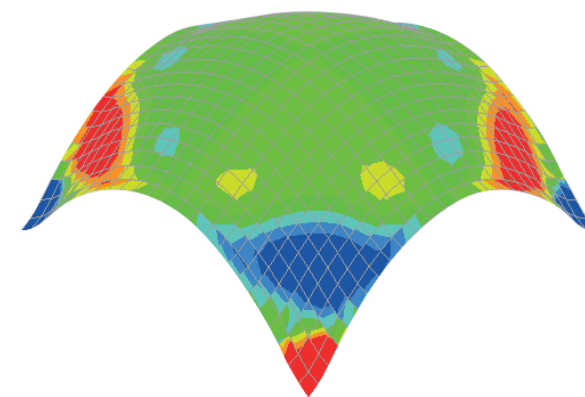
moment hill (of twistless plate)



m_{xx}



m_{yy}



m_{xy}

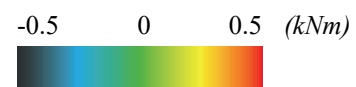
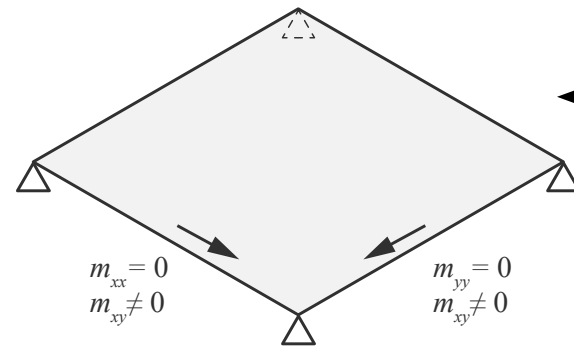
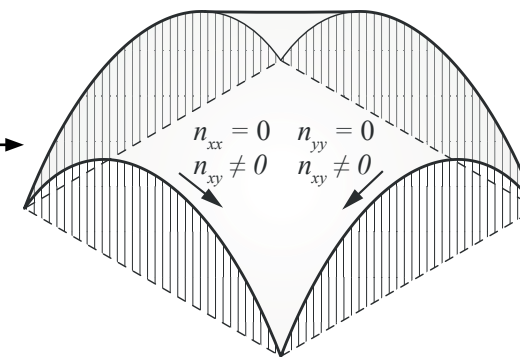


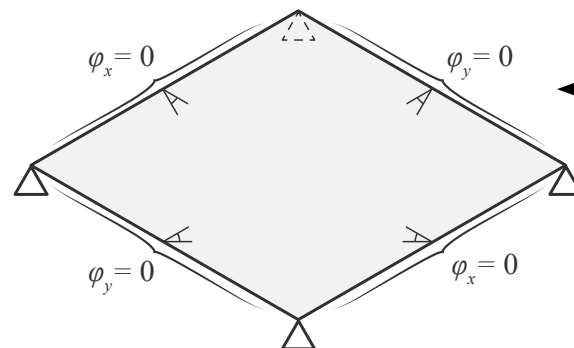
PLATE AND SHELL ANALOGIES



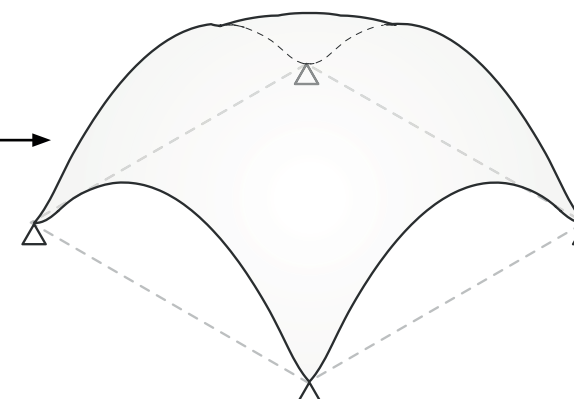
Simply-supported plate
(twistless)



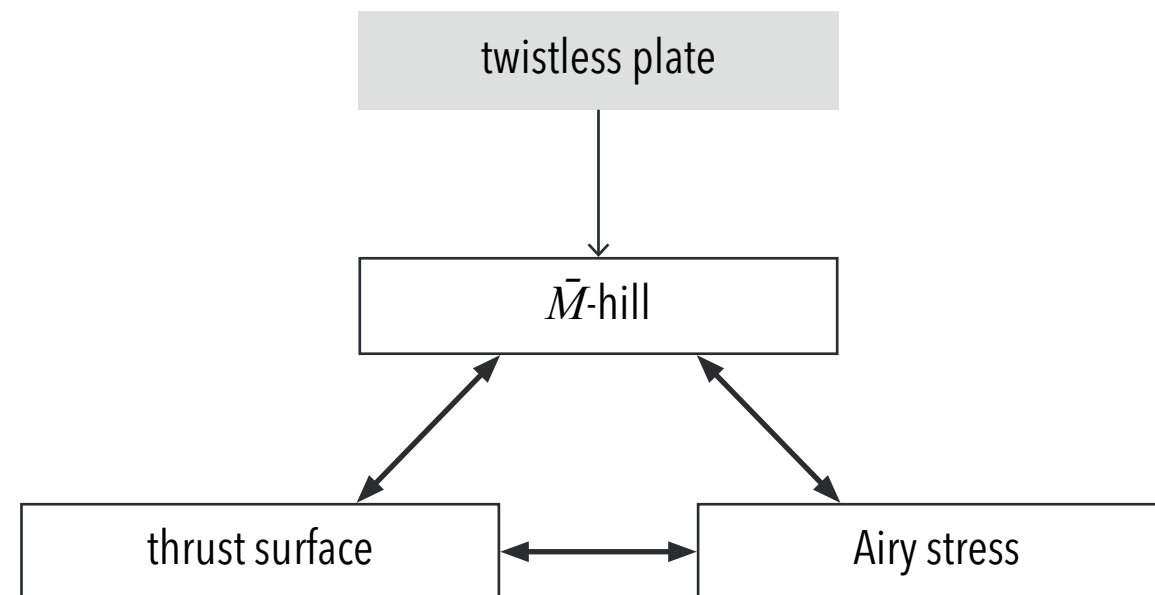
Shell on semi-rigid edge supports



Simply-supported plate
(twistless & no rotation)

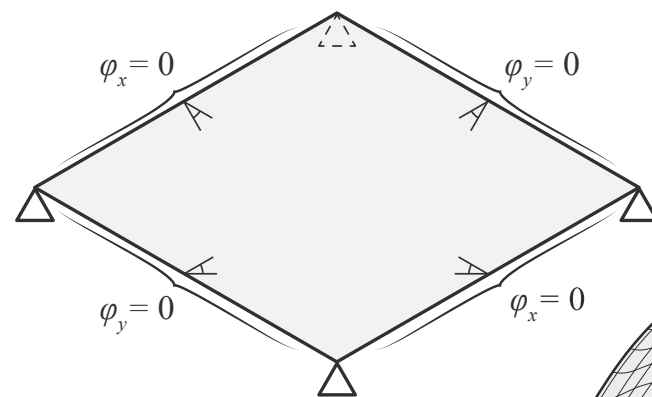


Shell with free edges

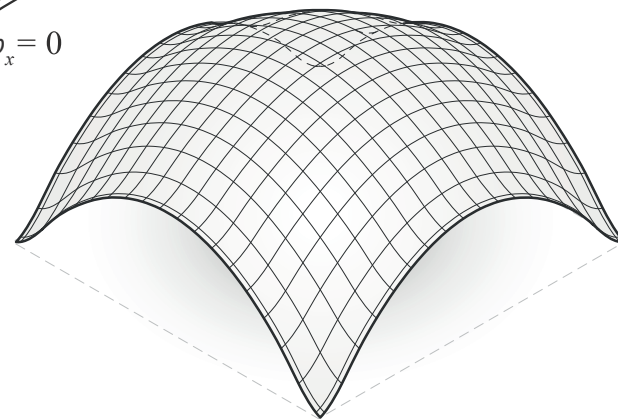


PARAMETRIC DESIGN TOOLS

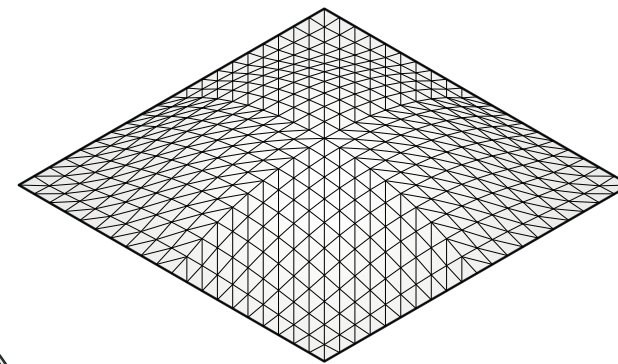
CONNECTING THE ELEMENTS



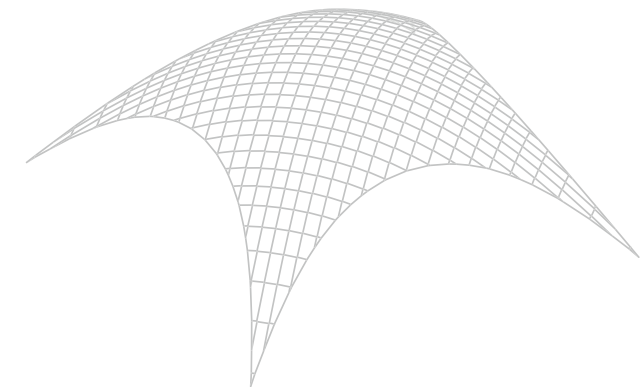
twistless flat plate



moment hill

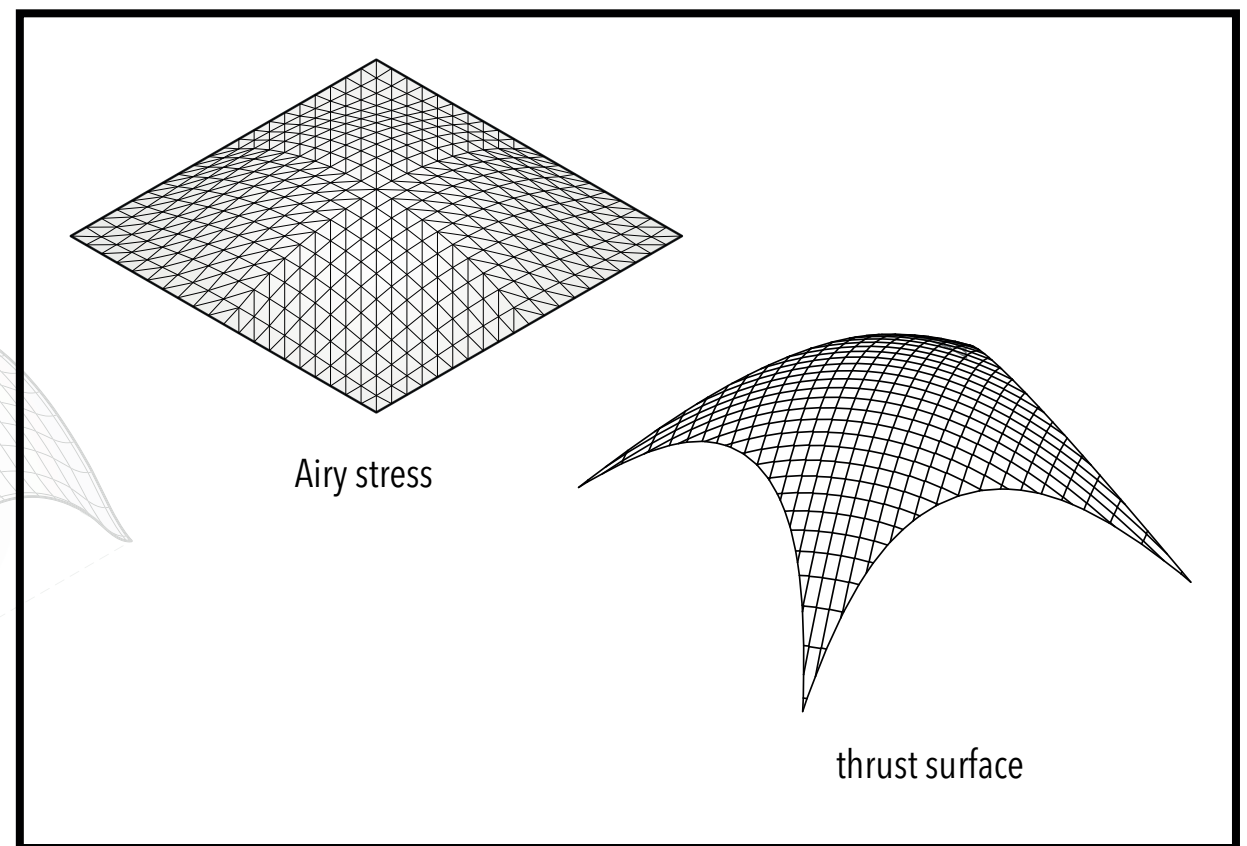
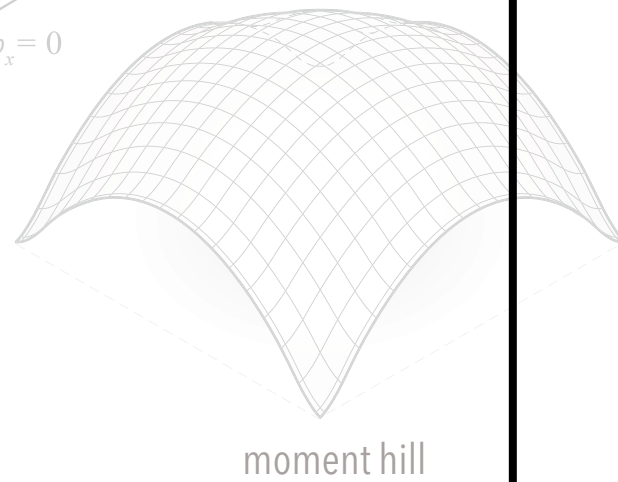
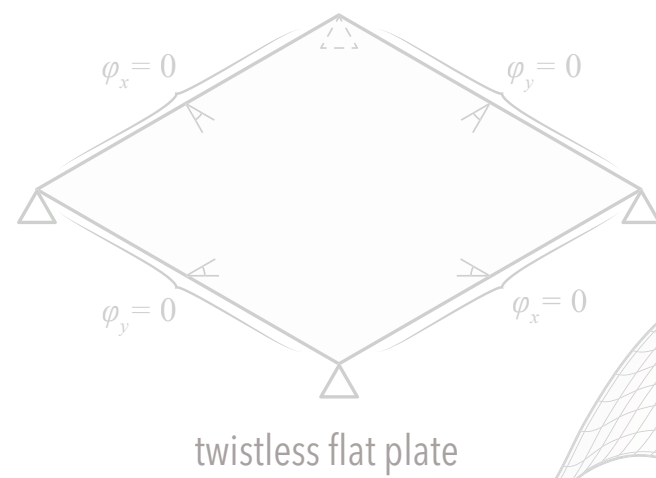


Airy stress



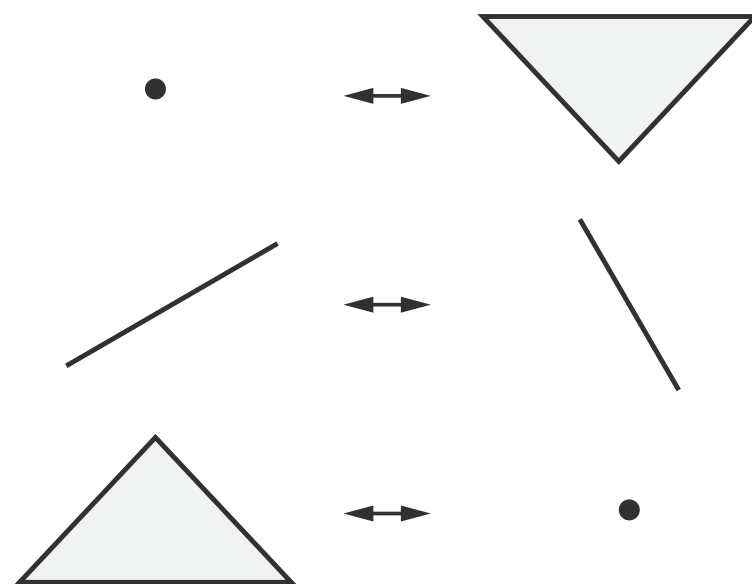
thrust surface

CONNECTING THE ELEMENTS

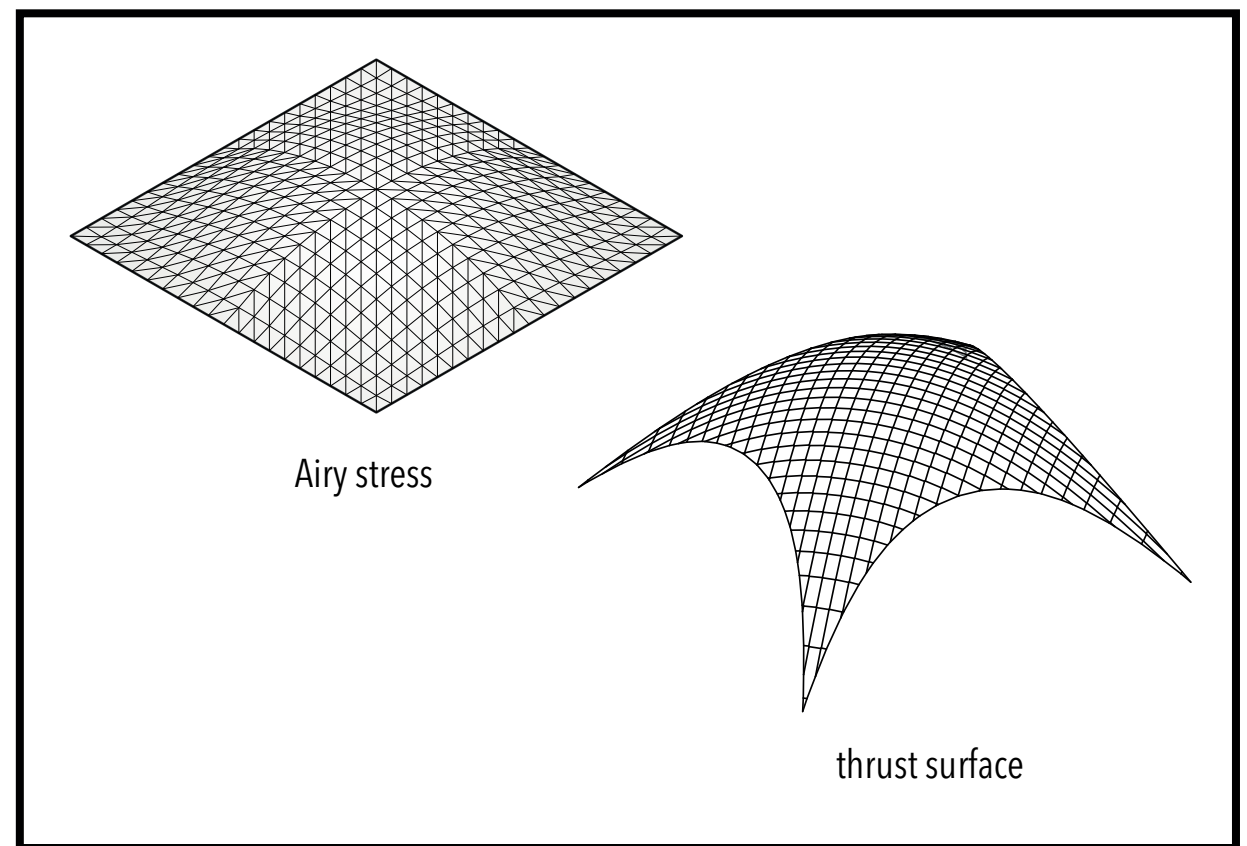


parametric design tool

PARAMETRIC DESIGN TOOL

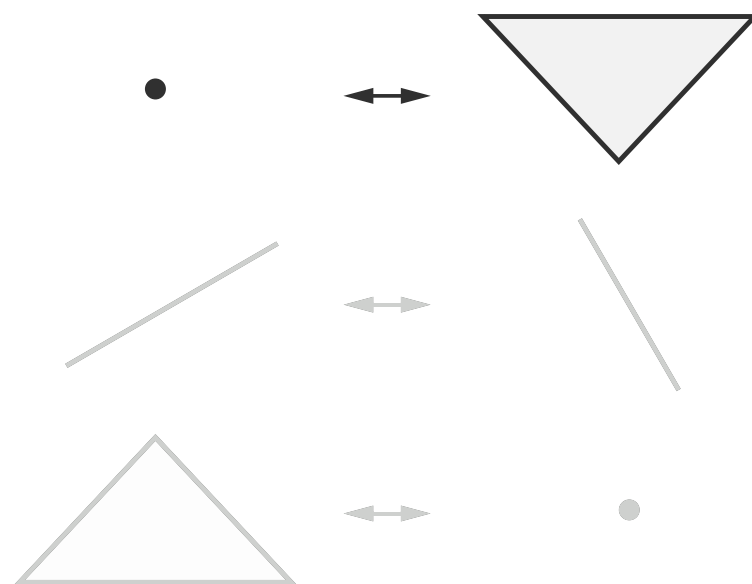


reciprocal figures



parametric design tool

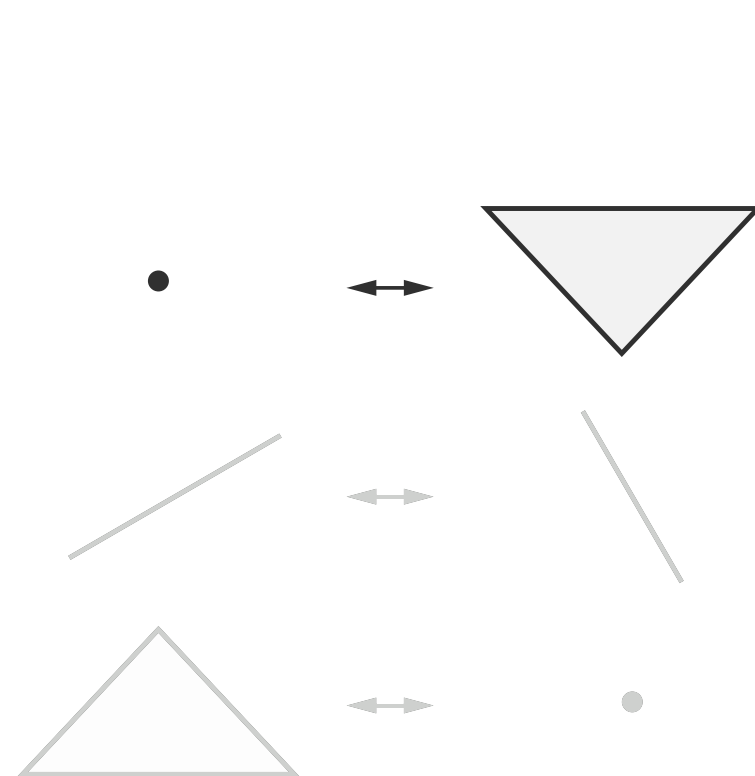
PARAMETRIC DESIGN TOOL



Airy stress mesh

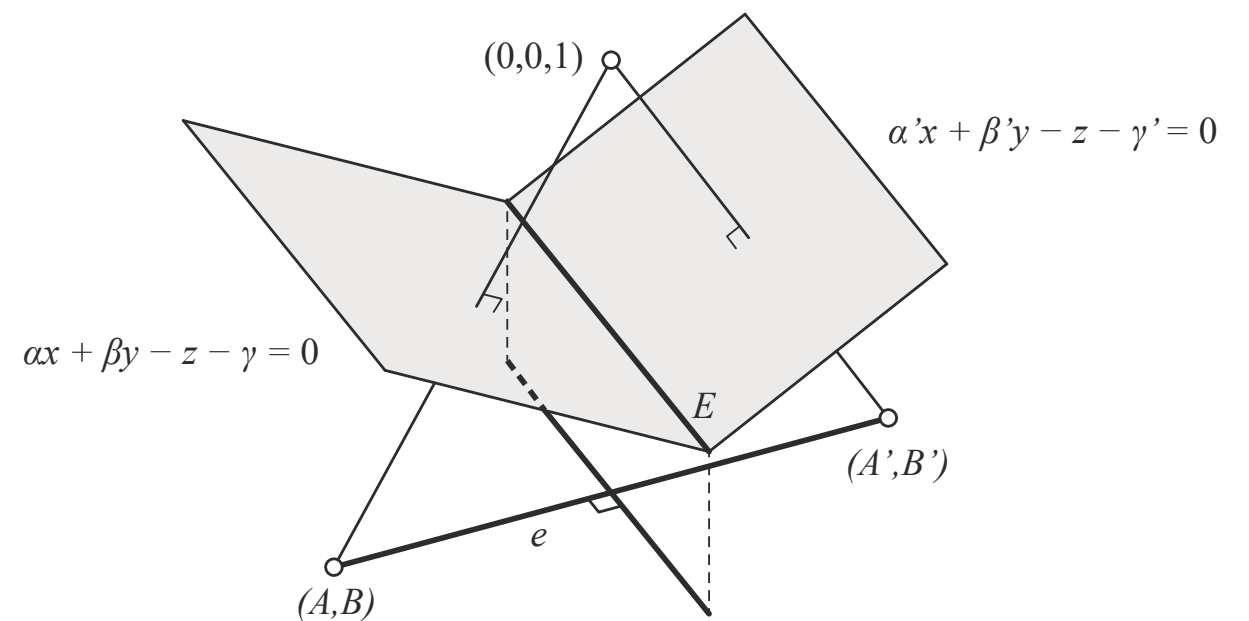
Thrust surface

PARAMETRIC DESIGN TOOL



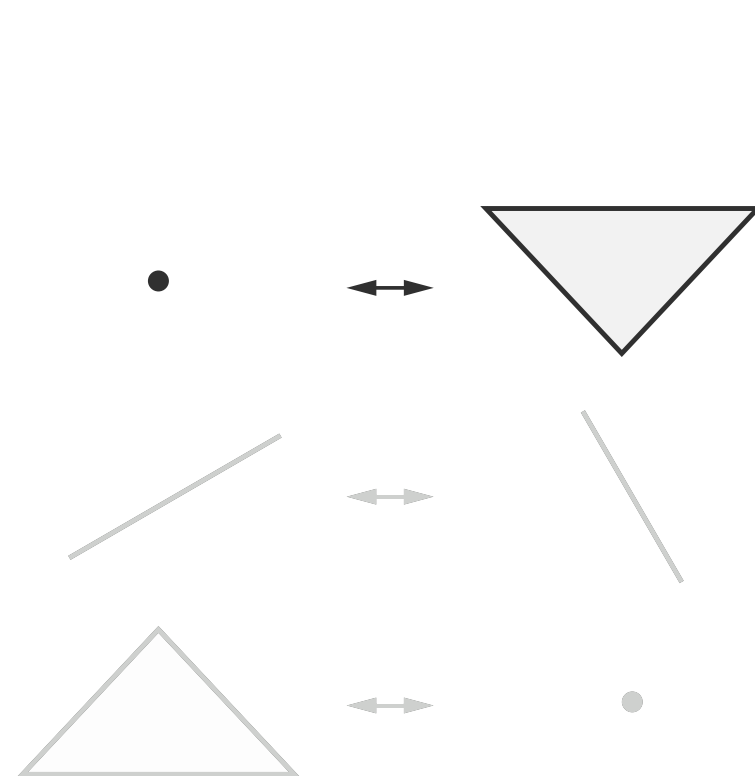
Airy stress mesh

Thrust surface



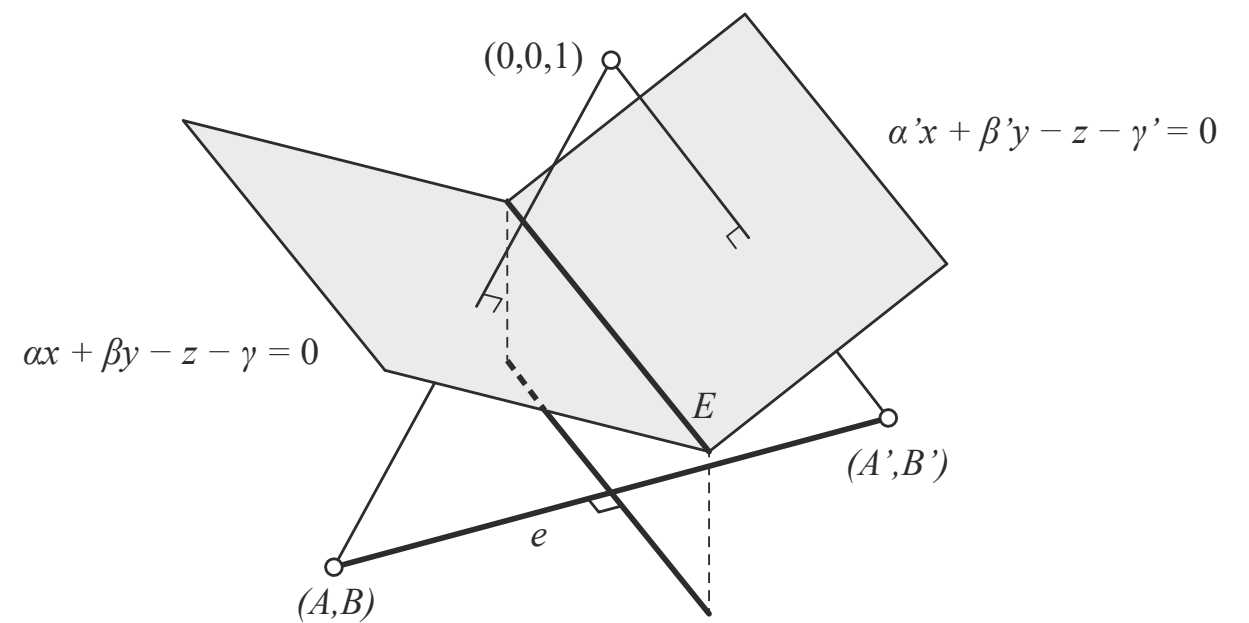
$z = \alpha x + \beta y + \gamma$	(Face of Airy stress mesh)
$(-\beta, \alpha, \gamma)$	(Vertex of thrust surface)
(q, r, s)	(Vertices of Airy stress mesh)
$z = rx - qy + s$	(Face of thrust surface)

PARAMETRIC DESIGN TOOL



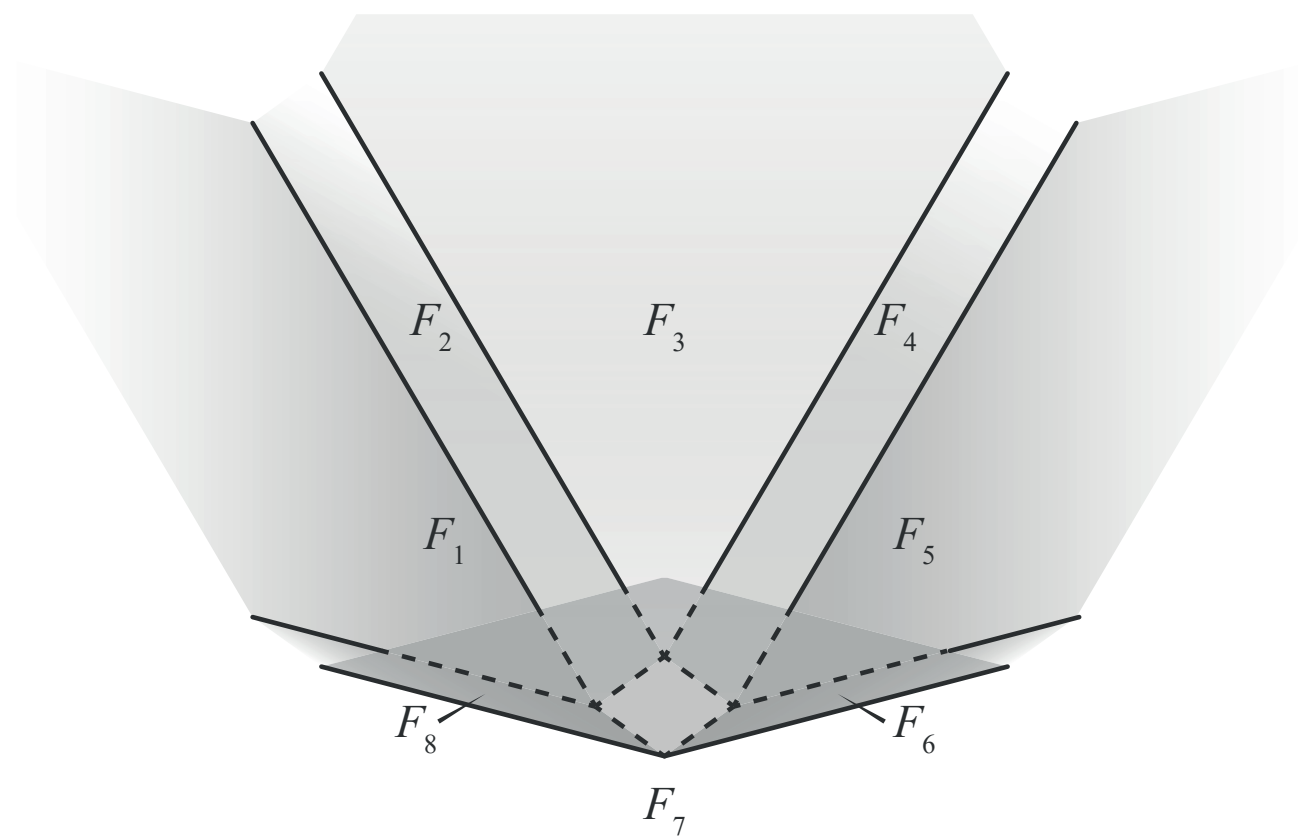
Airy stress mesh

Thrust surface



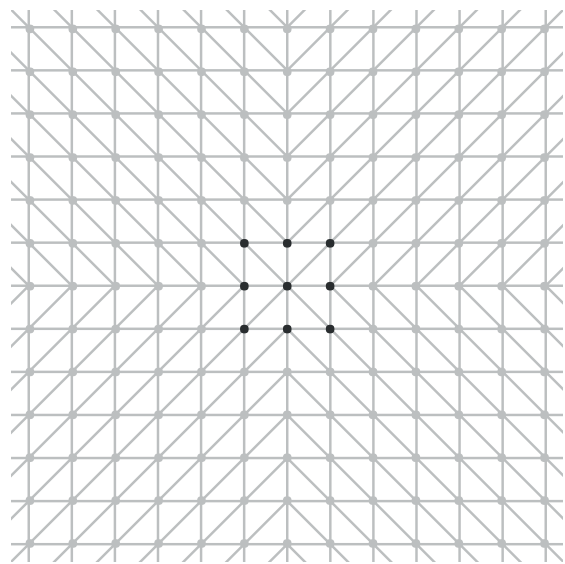
$z = \alpha x + \beta y + \gamma$	(Face of Airy stress mesh)
$(-\beta, \alpha, \gamma)$	(Vertex of thrust surface)
(q, r, s)	(Vertices of Airy stress mesh)
$z = rx - qy + s$	(Face of thrust surface)

PARAMETRIC DESIGN TOOL

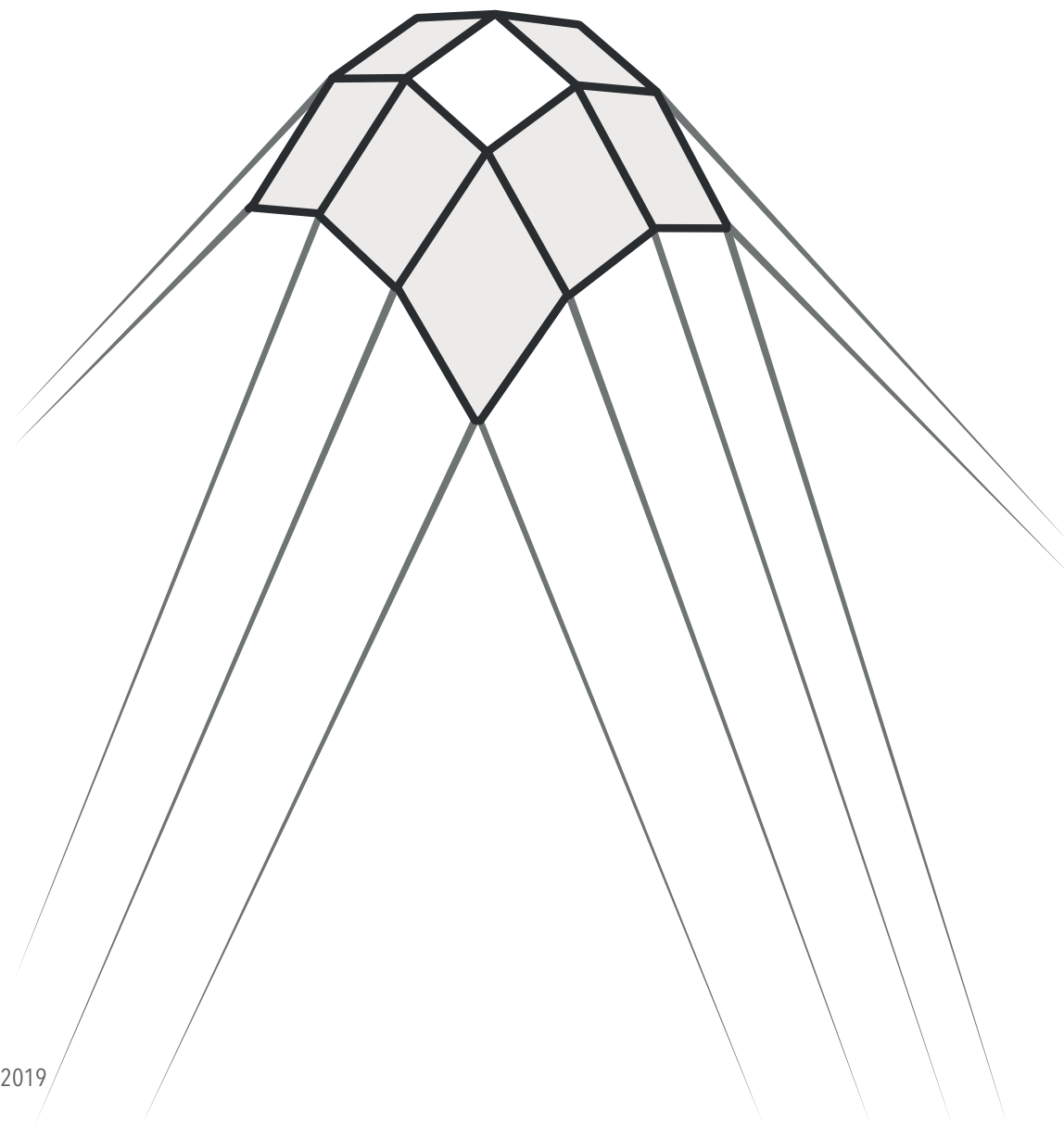


intersecting planes

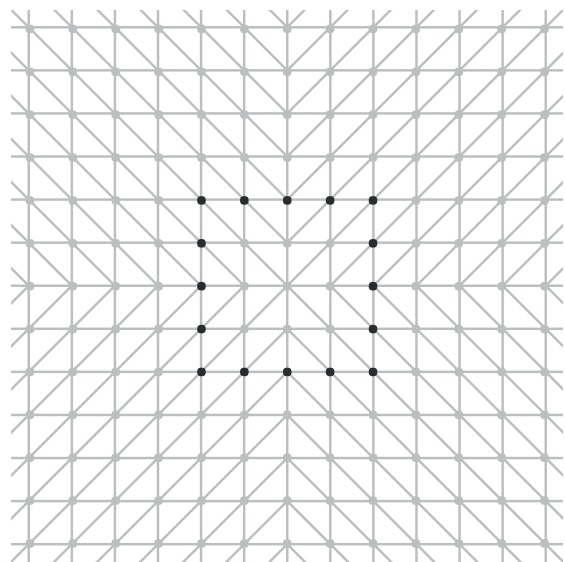
PARAMETRIC DESIGN TOOL



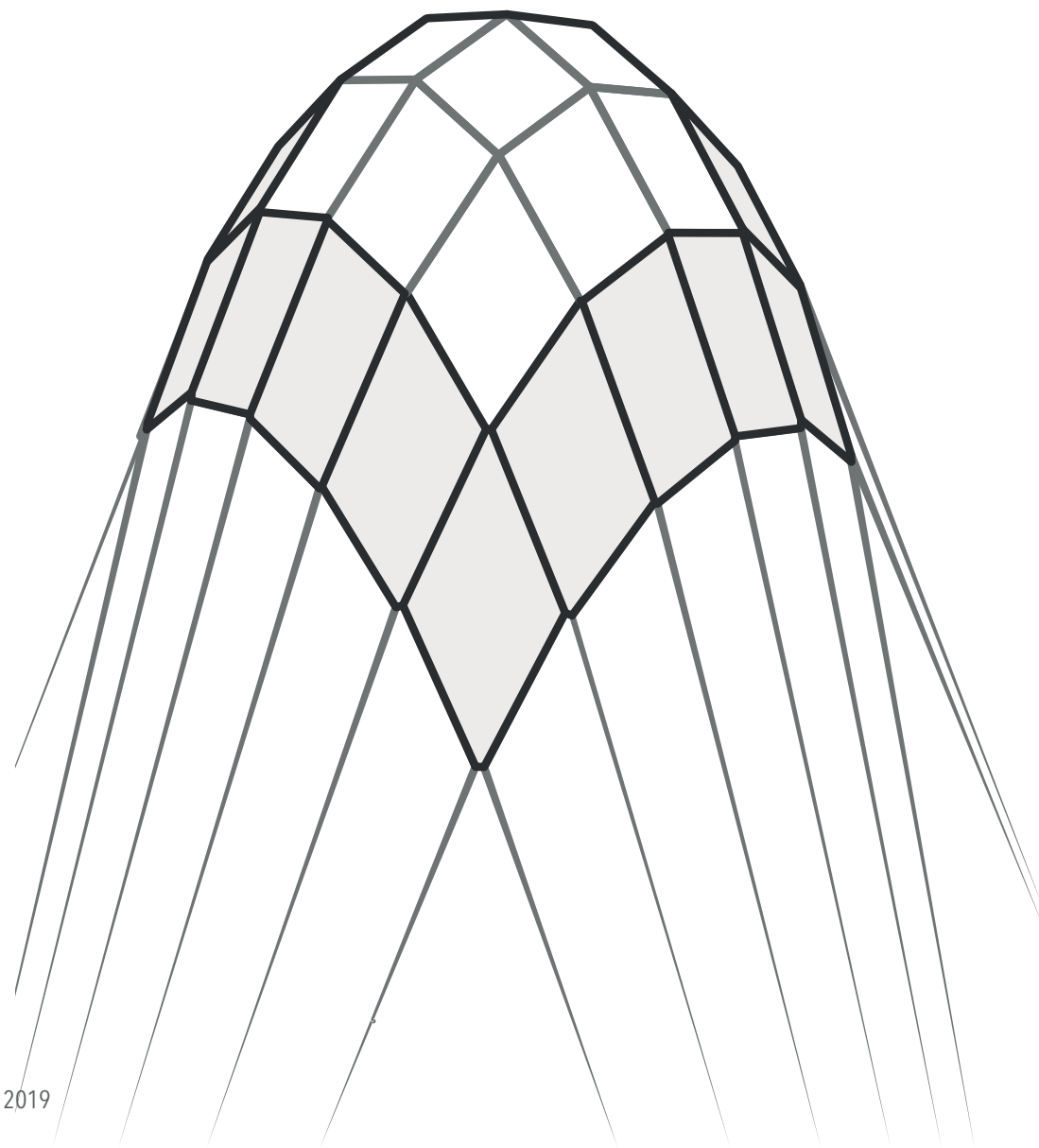
intersecting rings



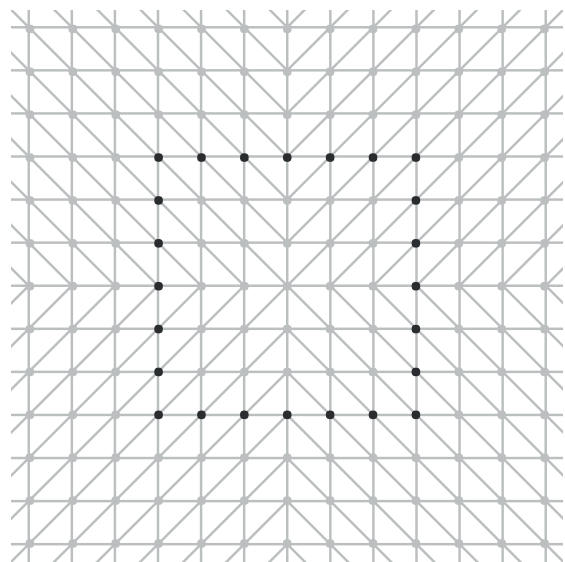
PARAMETRIC DESIGN TOOL



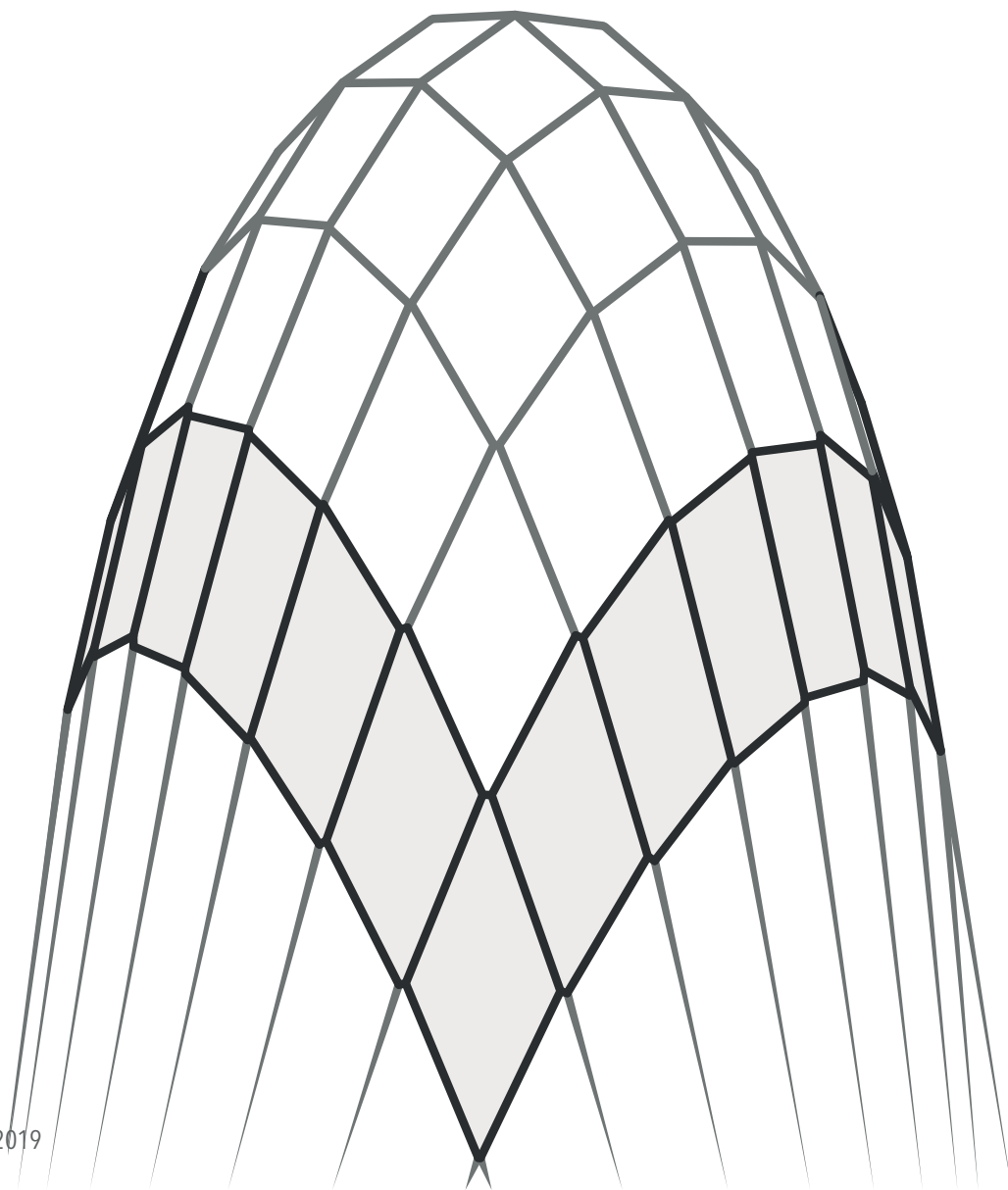
intersecting rings



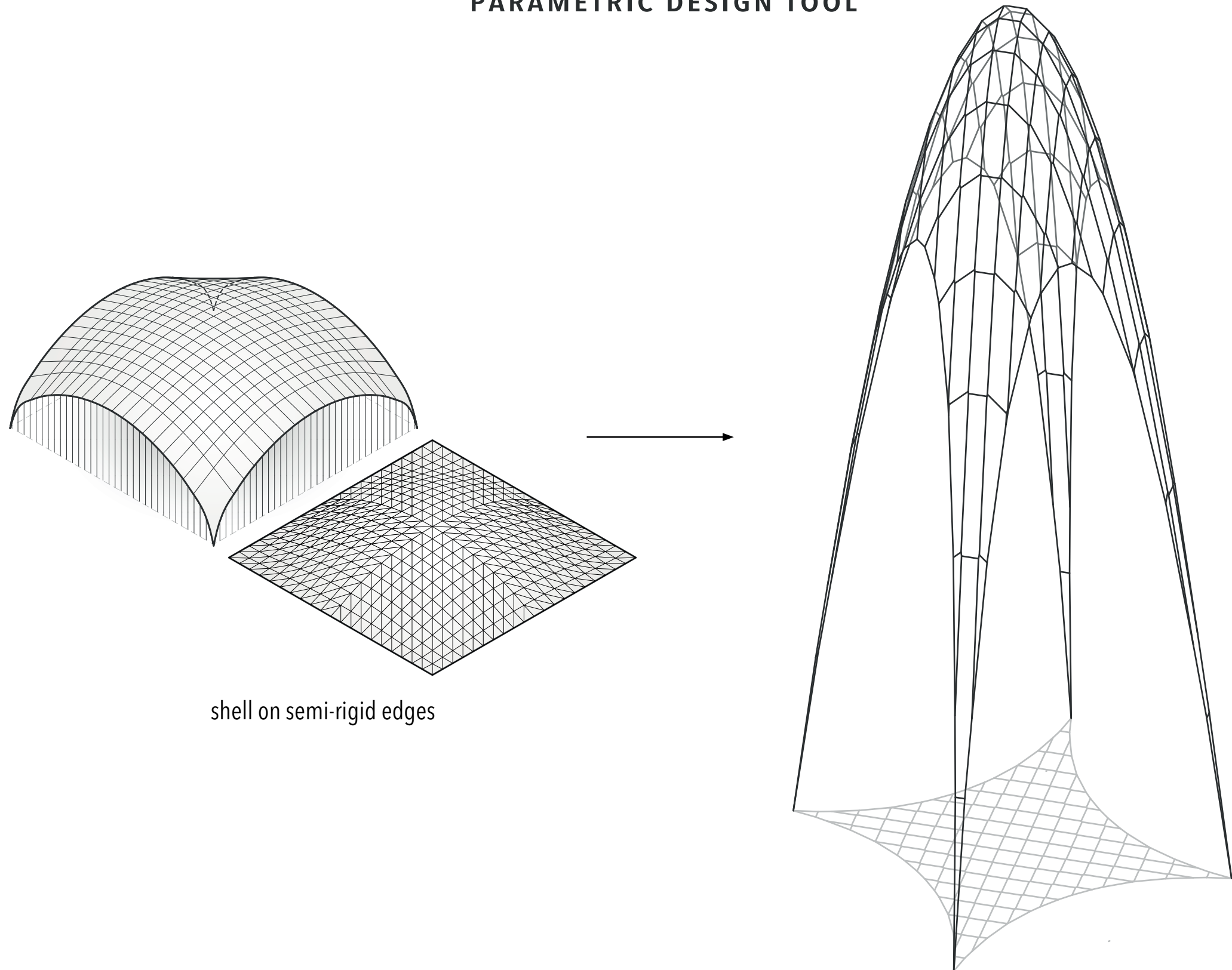
PARAMETRIC DESIGN TOOL



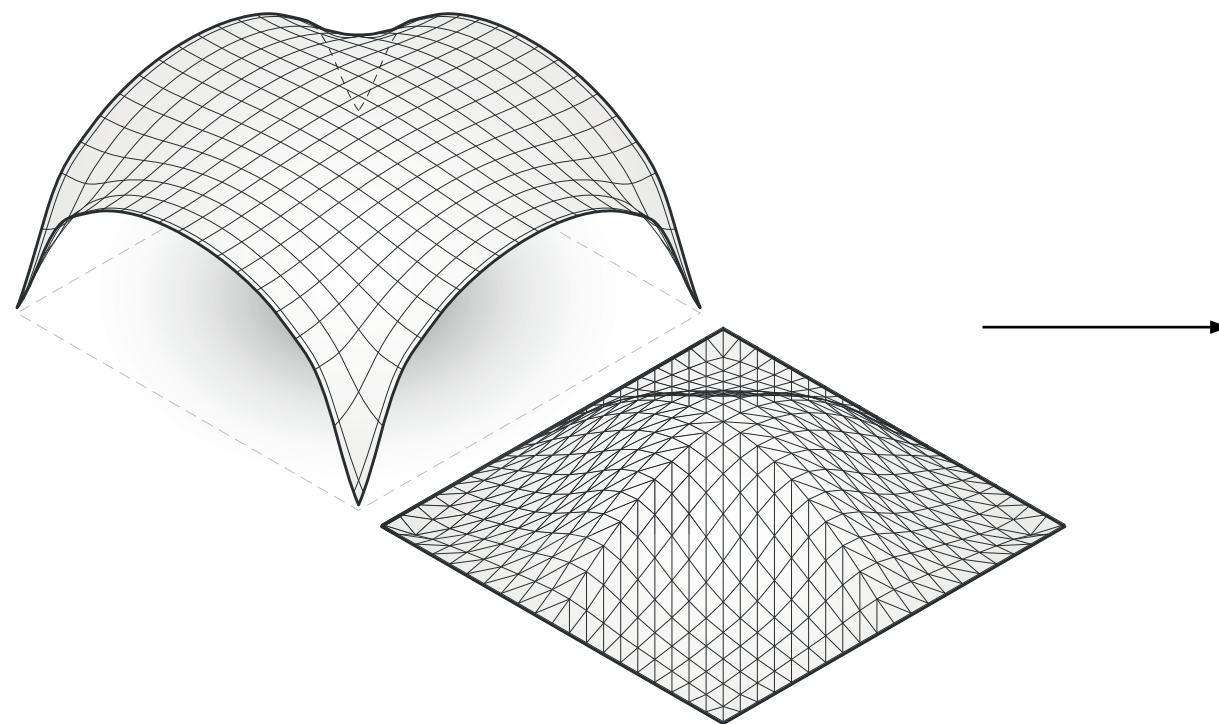
intersecting rings



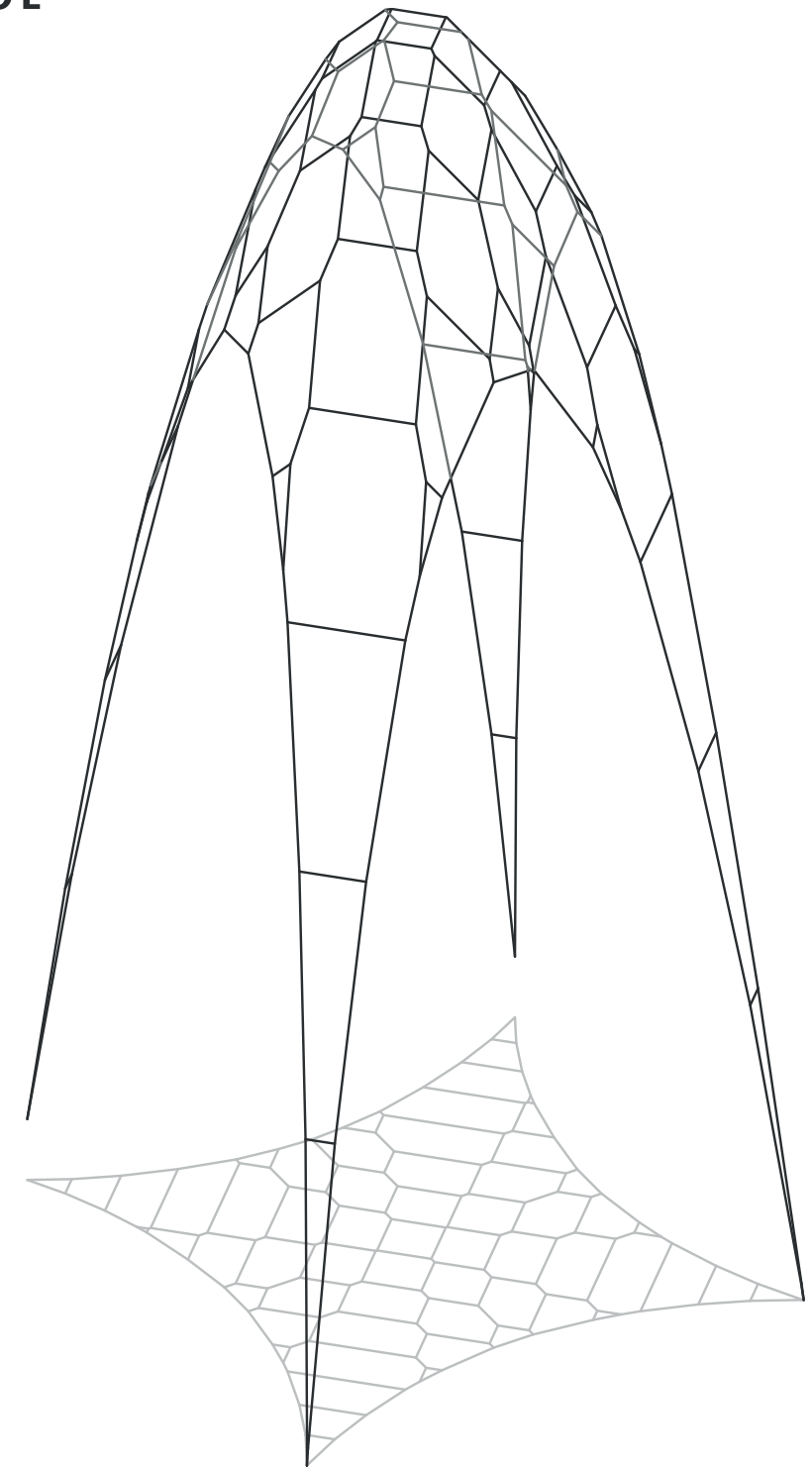
PARAMETRIC DESIGN TOOL



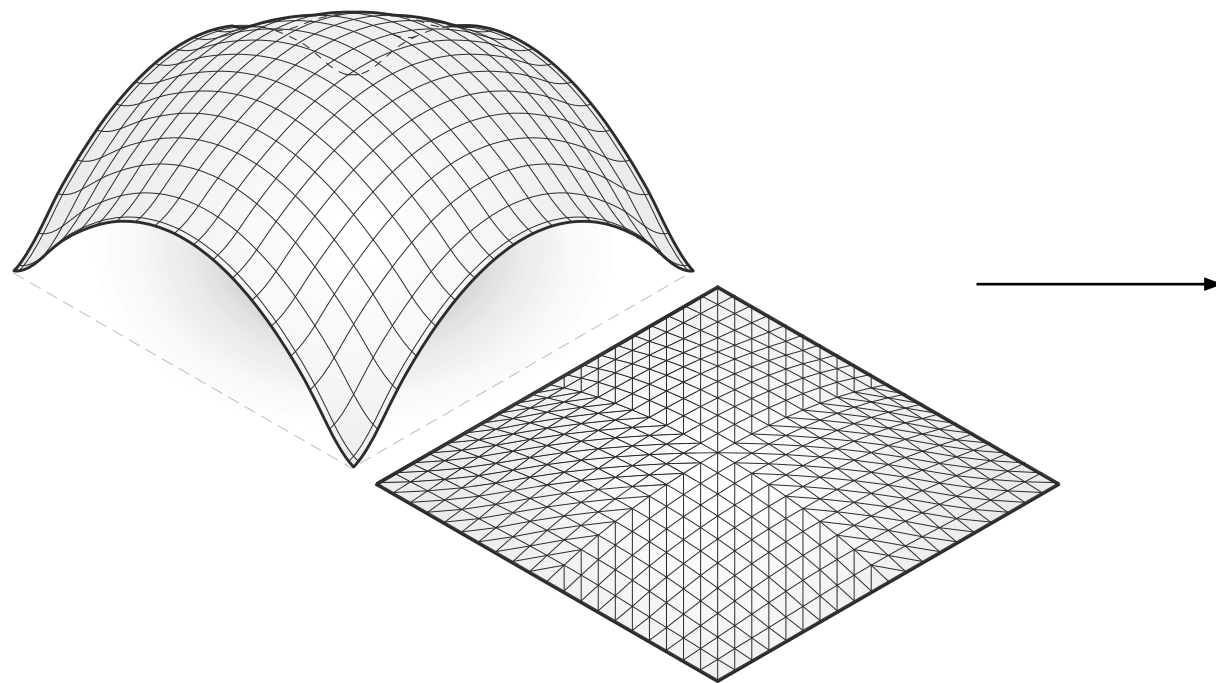
PARAMETRIC DESIGN TOOL



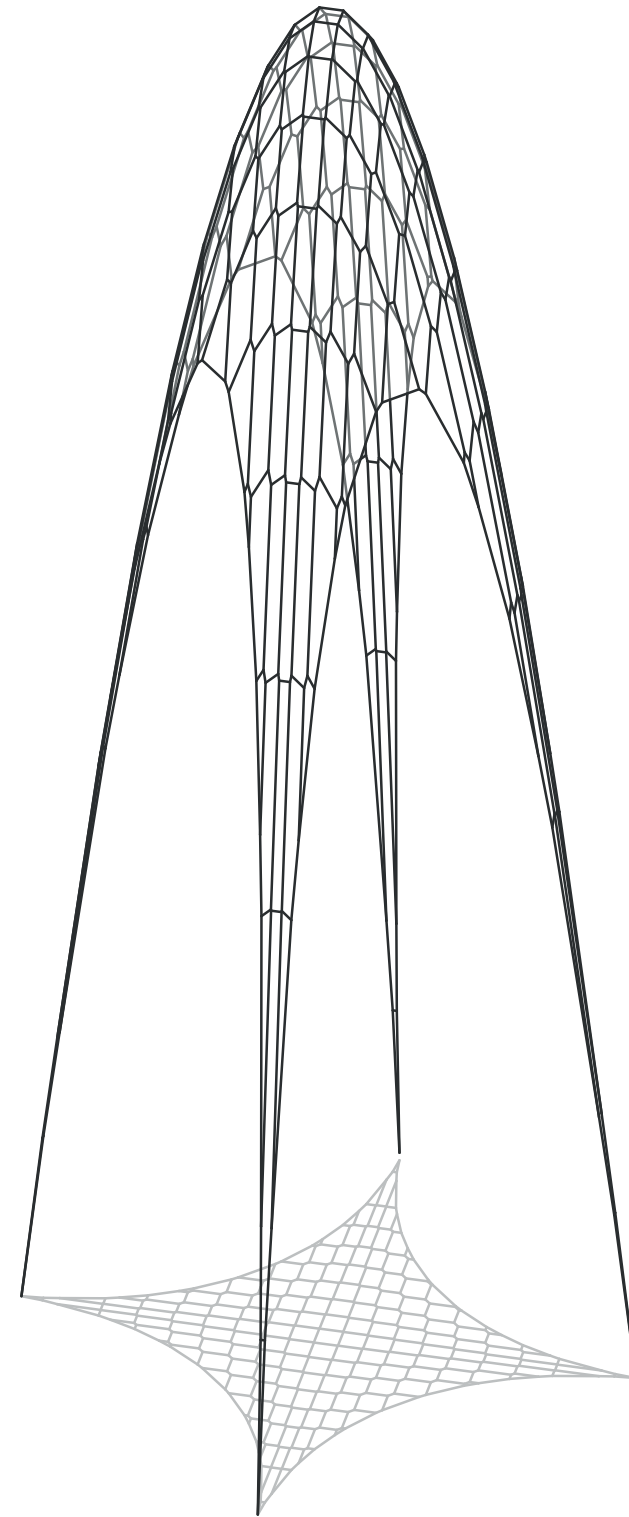
shell on four supports



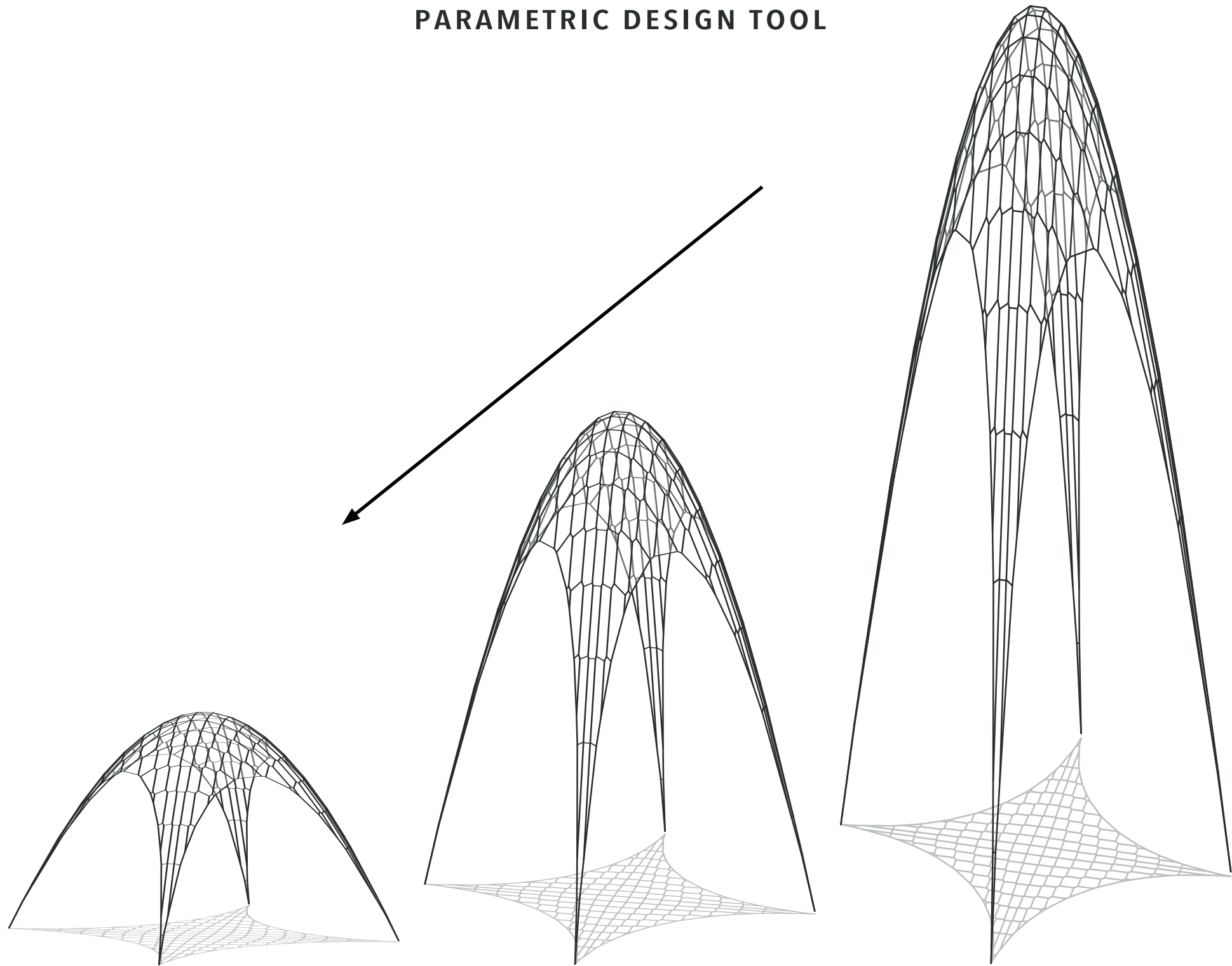
PARAMETRIC DESIGN TOOL



shell on four supports (from twistless plate)

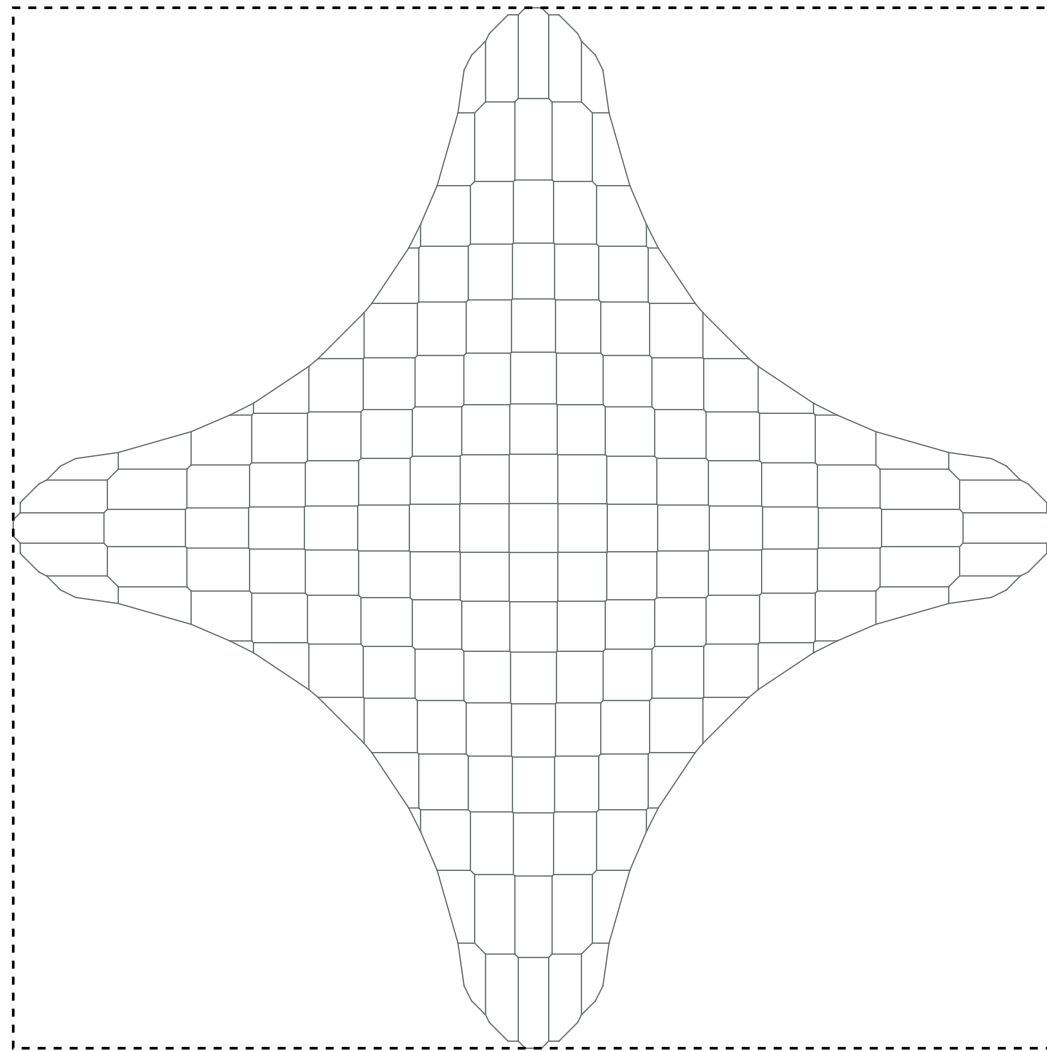


PARAMETRIC DESIGN TOOL



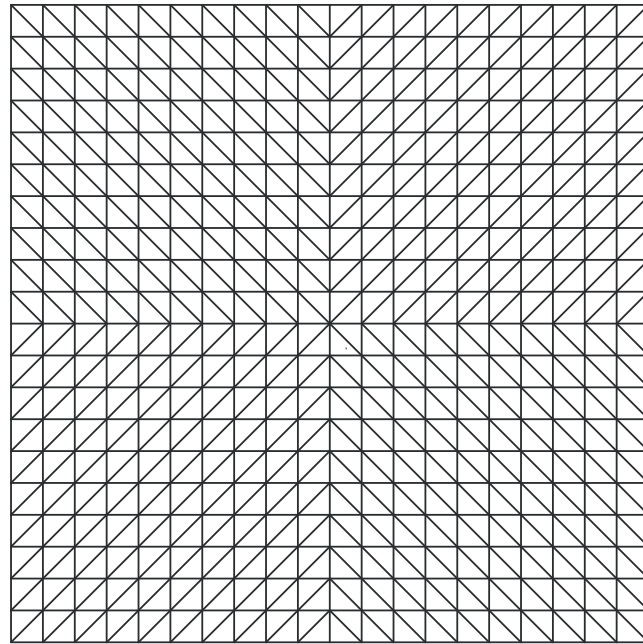
scaling

PARAMETRIC DESIGN TOOL

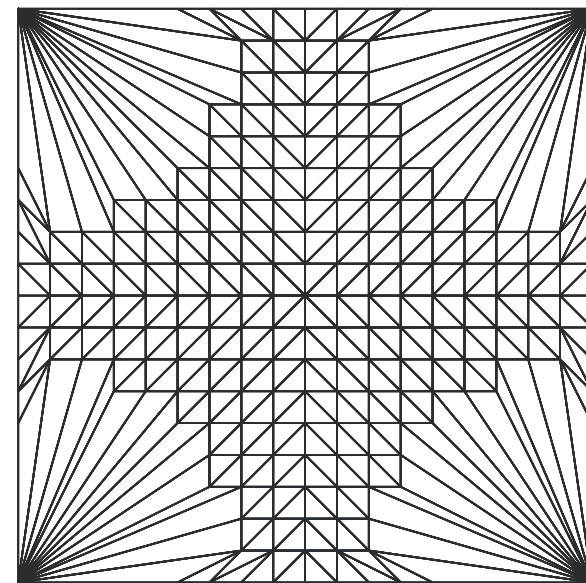


different footprint

PARAMETRIC DESIGN TOOL

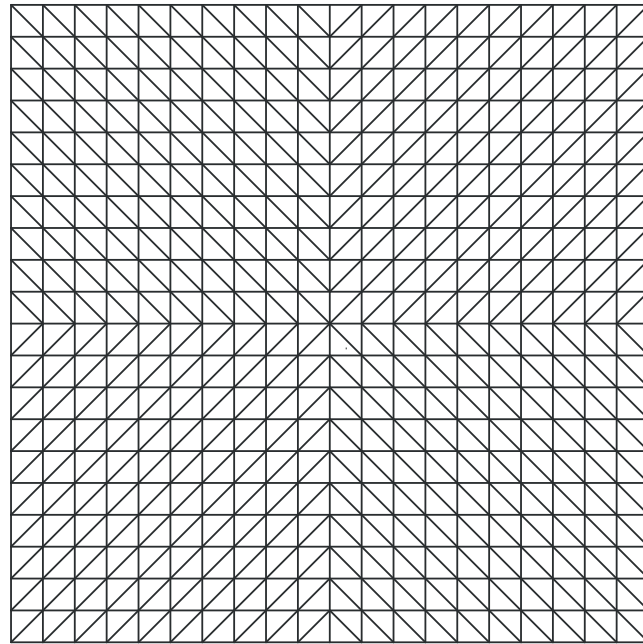


connectivity Airy stress mesh

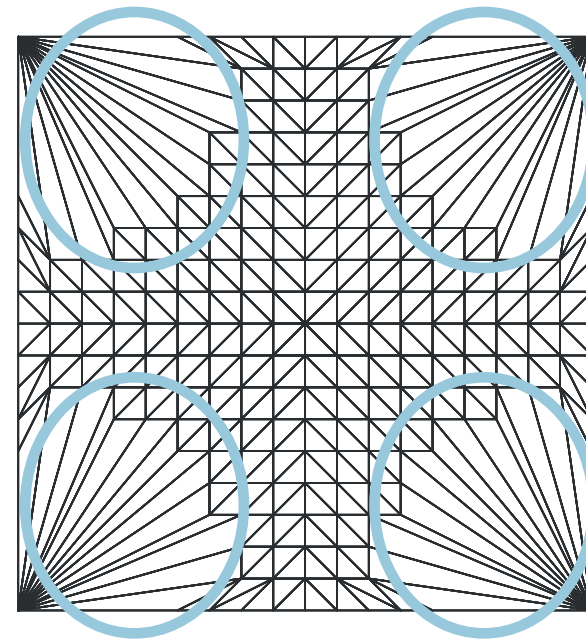


connectivity reciprocal figure

PARAMETRIC DESIGN TOOL

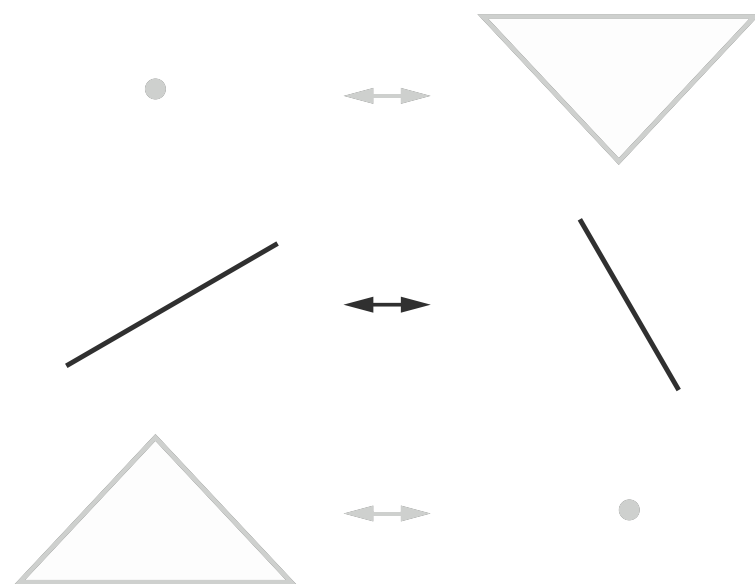


connectivity Airy stress mesh



connectivity reciprocal figure

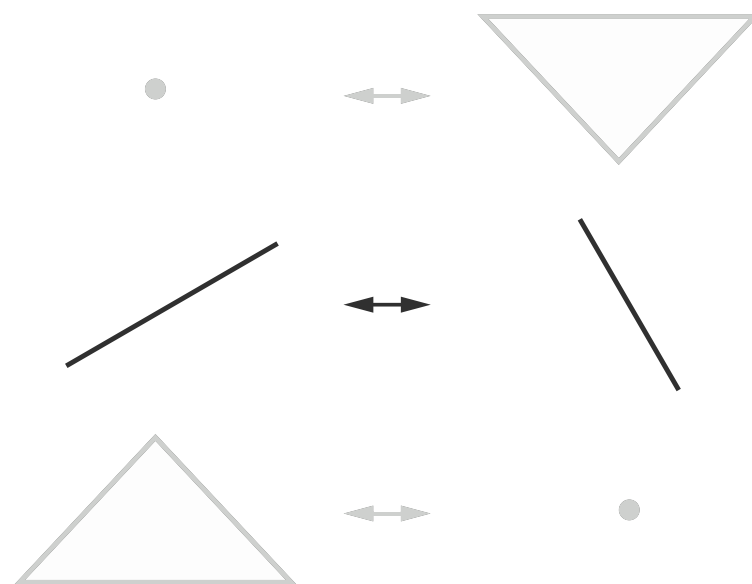
PARAMETRIC DESIGN TOOL: CONNECTIVITY



line Airy stress mesh

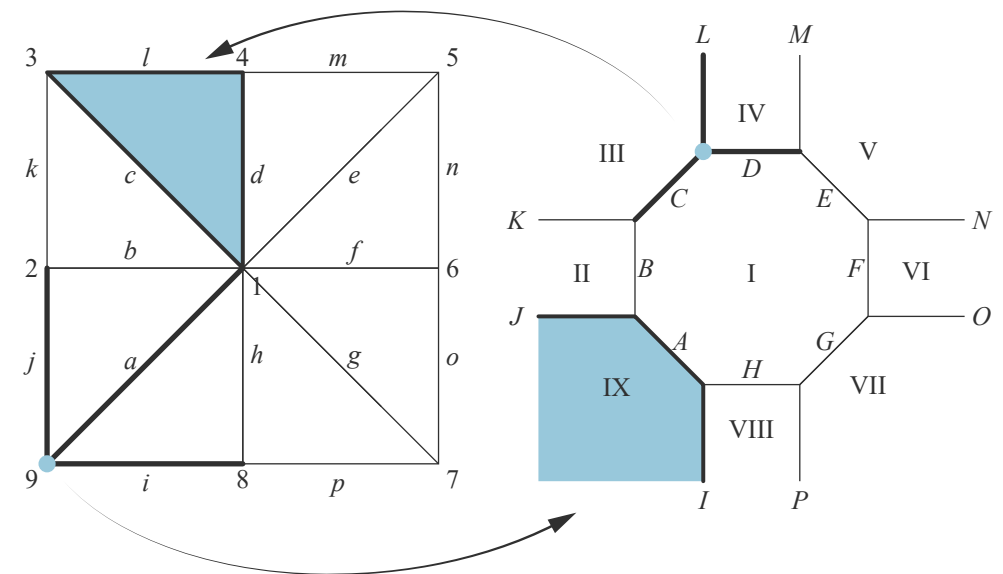
line thrust surface

PARAMETRIC DESIGN TOOL: CONNECTIVITY

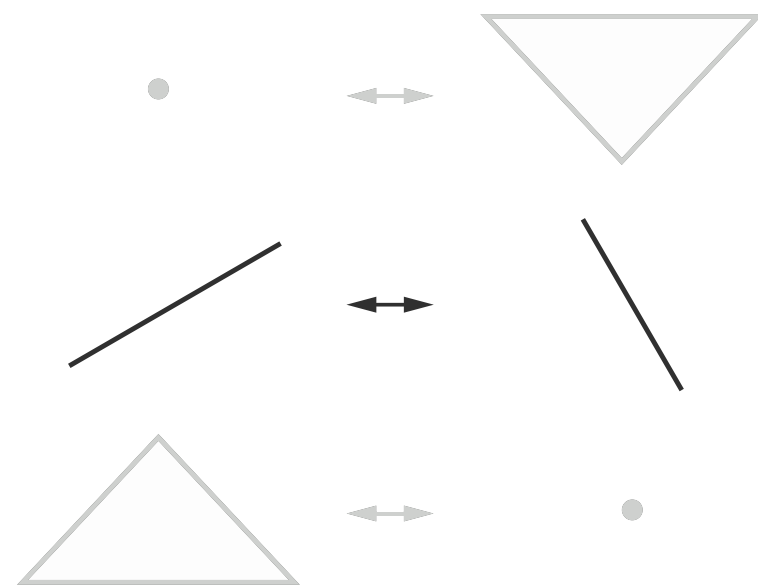


line Airy stress mesh

line thrust surface

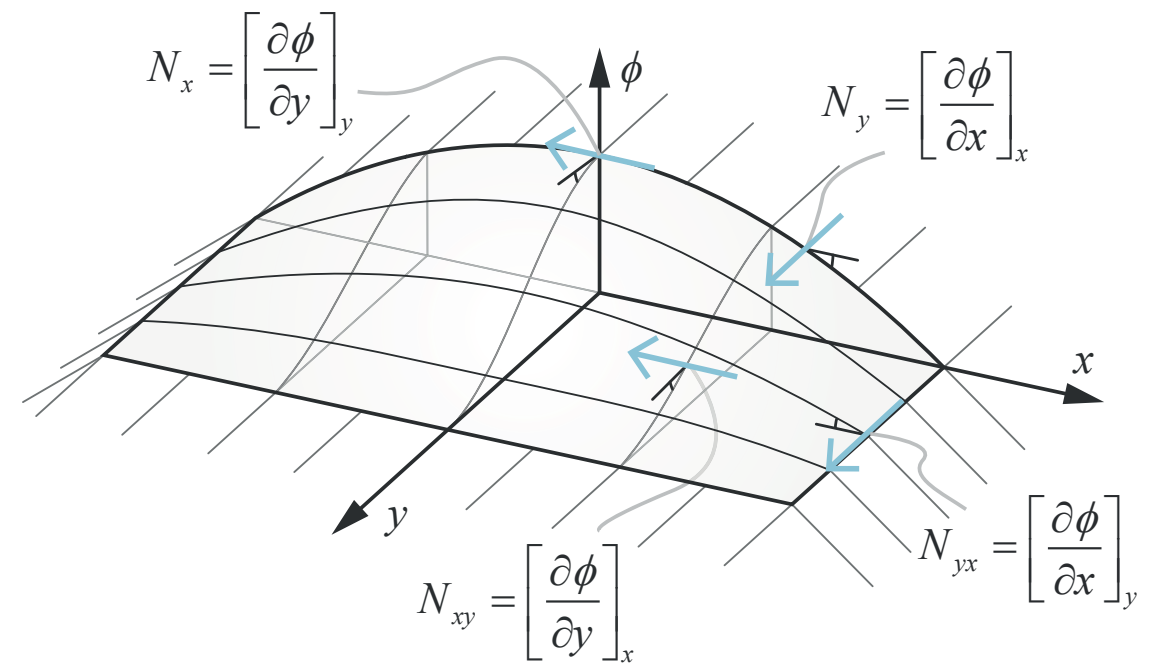


PARAMETRIC DESIGN TOOL: CONNECTIVITY

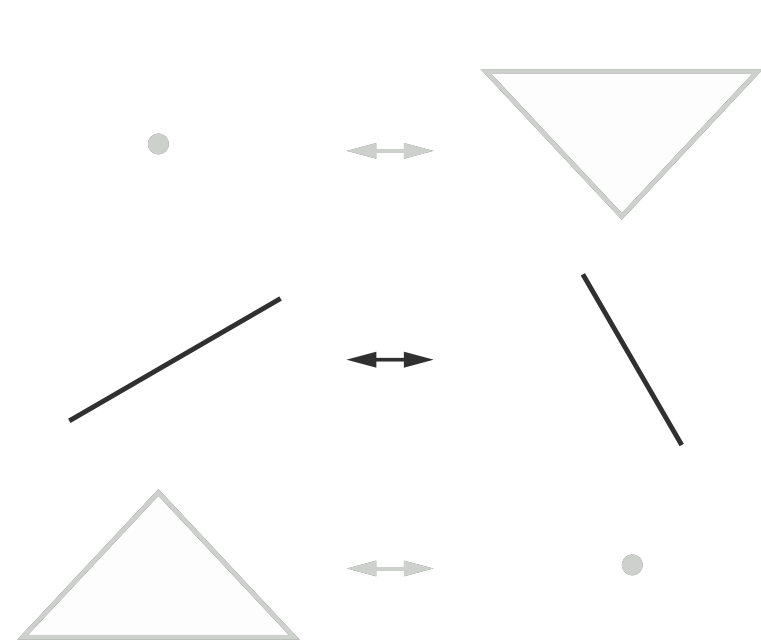


line Airy stress mesh

line thrust surface

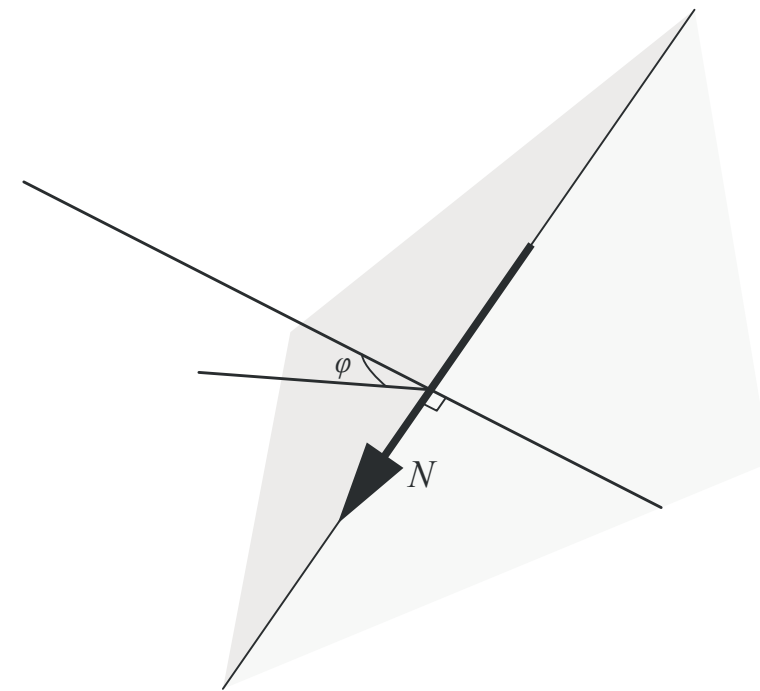


PARAMETRIC DESIGN TOOL: CONNECTIVITY



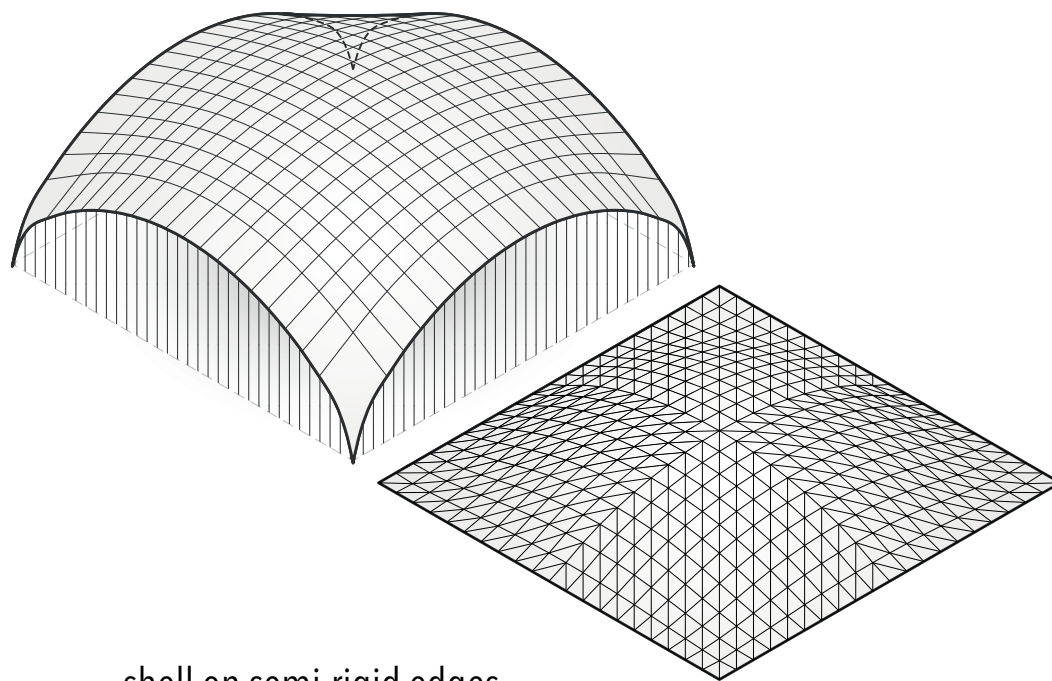
line Airy stress mesh

line thrust surface

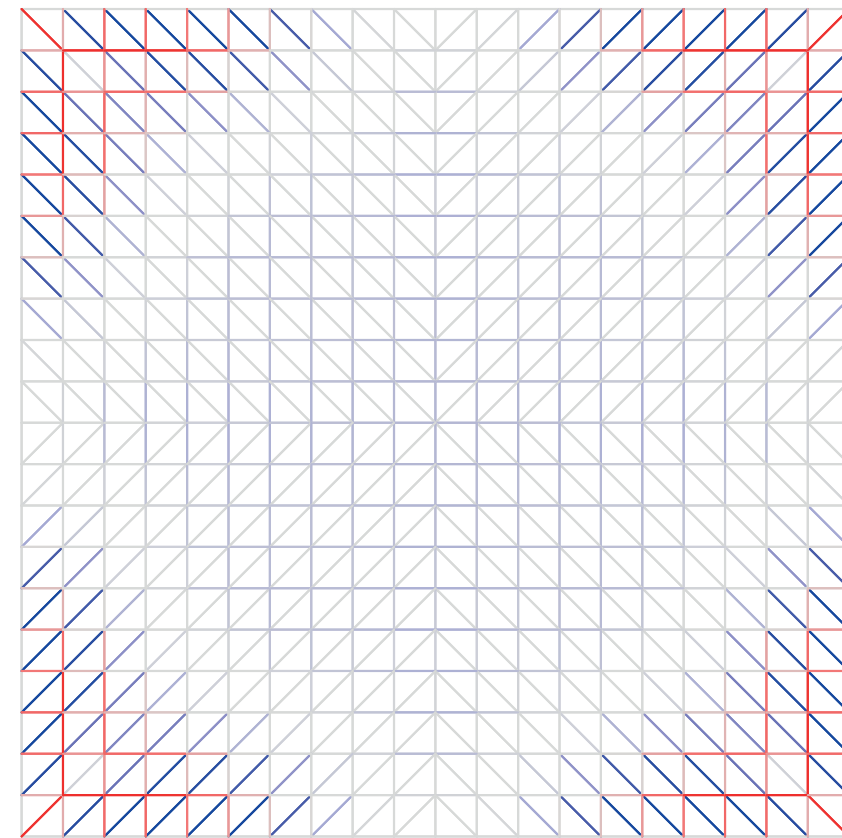


$$N_{BE;i} = \varphi_{AE;i}$$

PARAMETRIC DESIGN TOOL: CONNECTIVITY

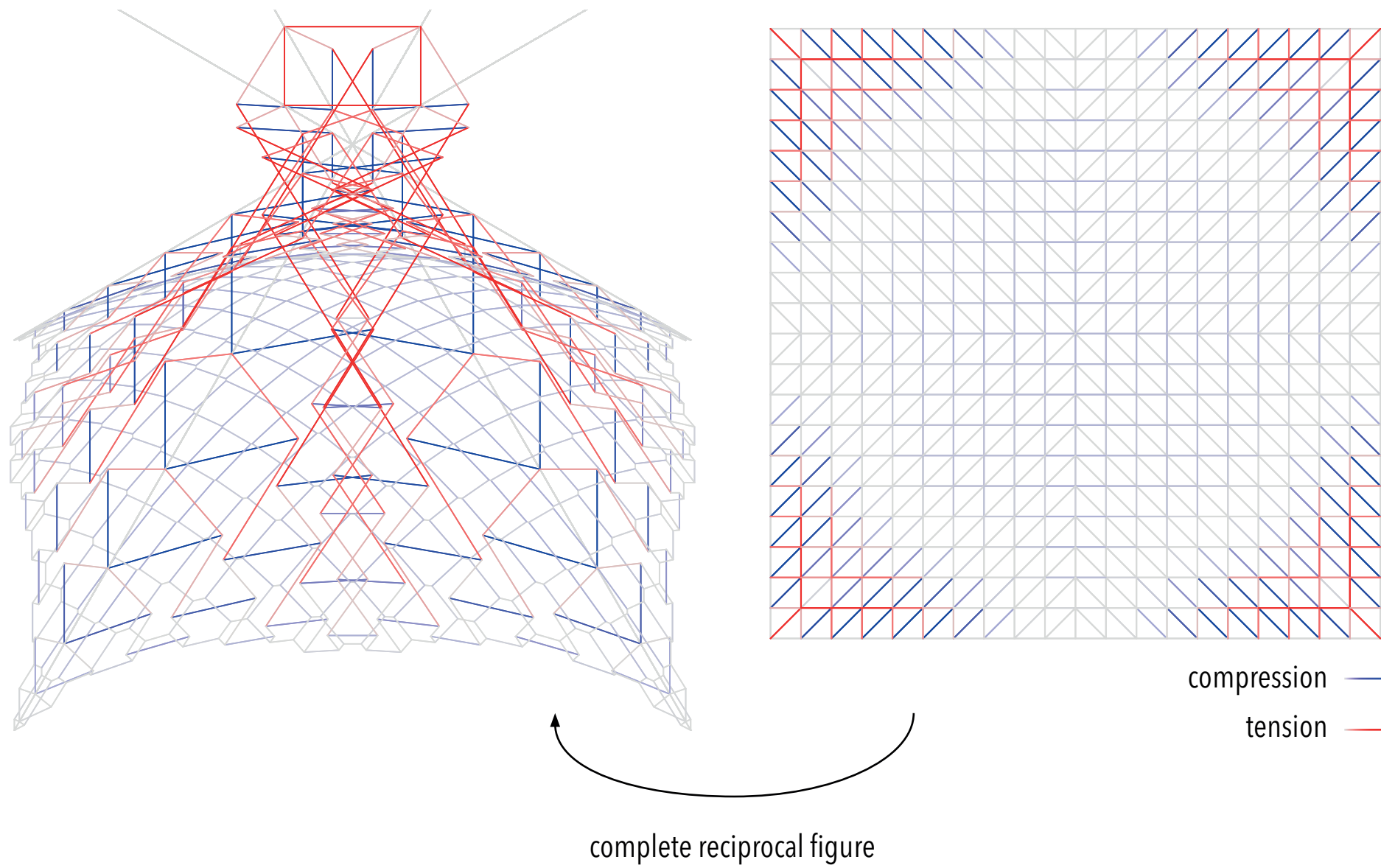


shell on semi-rigid edges
(subjected to distributed load)

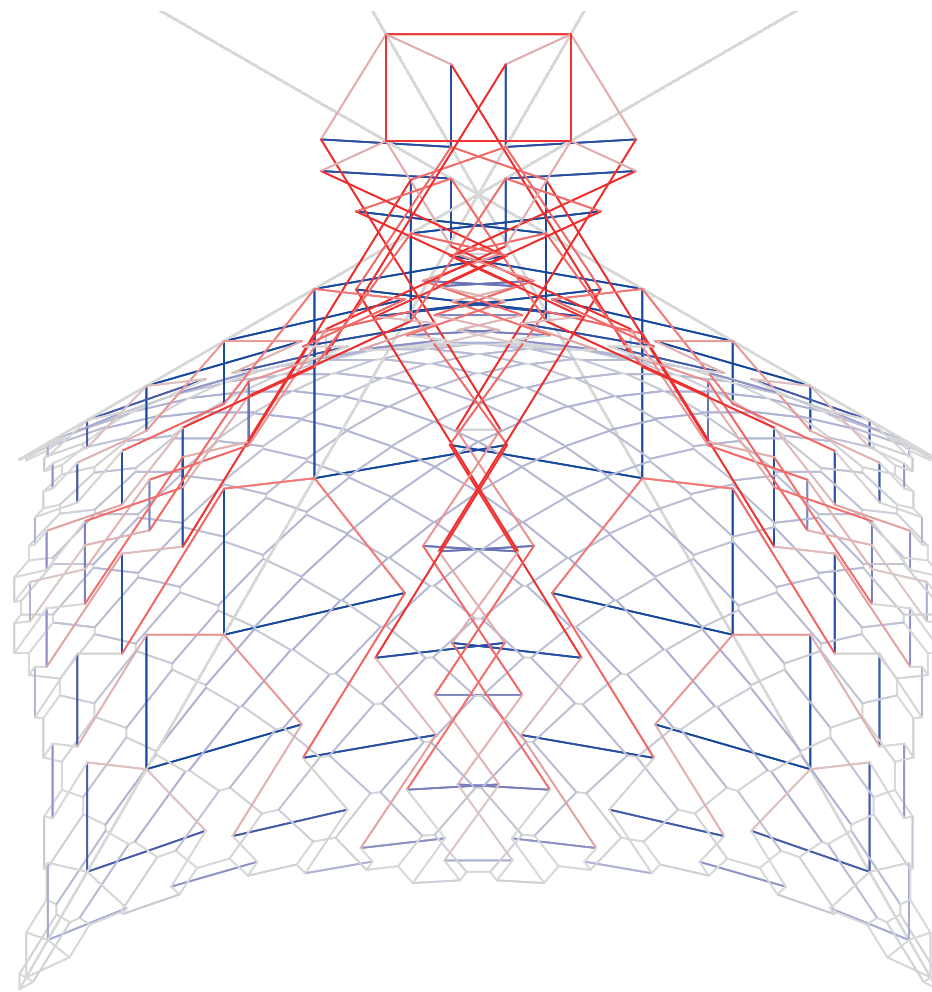


compression — blue —
tension — red —

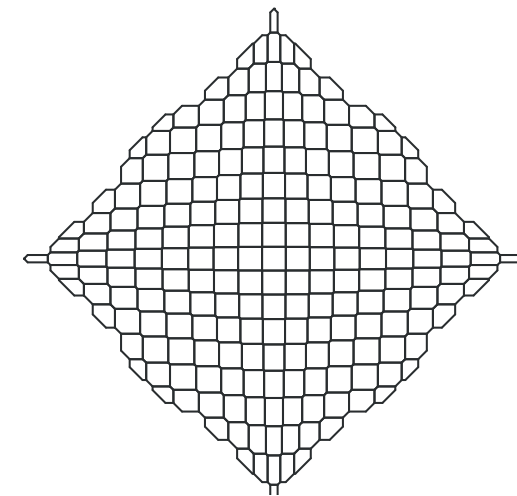
PARAMETRIC DESIGN TOOL: CONNECTIVITY



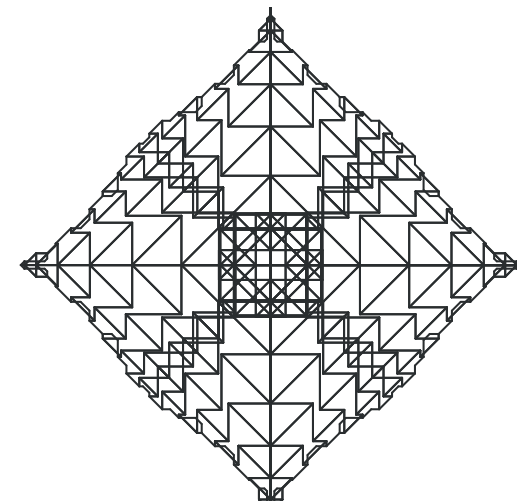
PARAMETRIC DESIGN TOOL: CONNECTIVITY



compression —
tension —

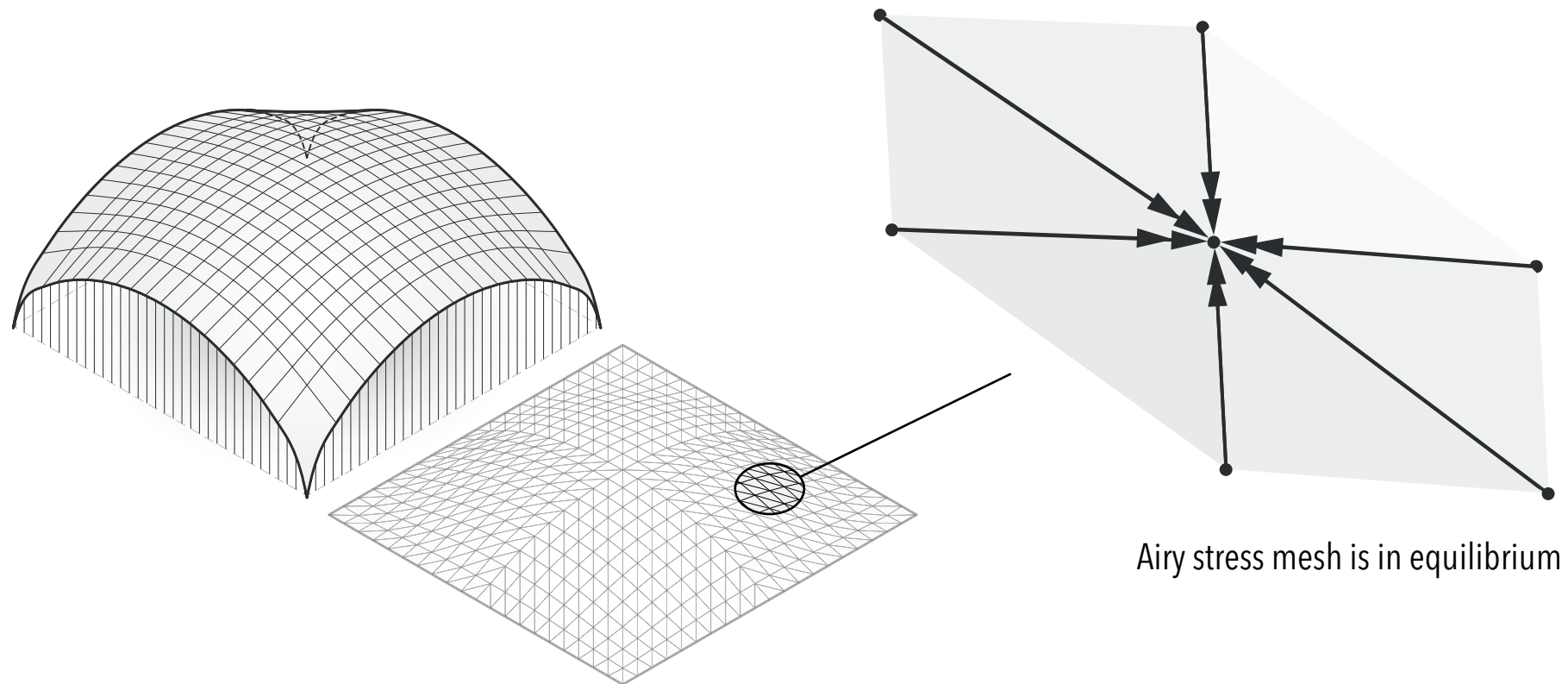


compression-only polygons

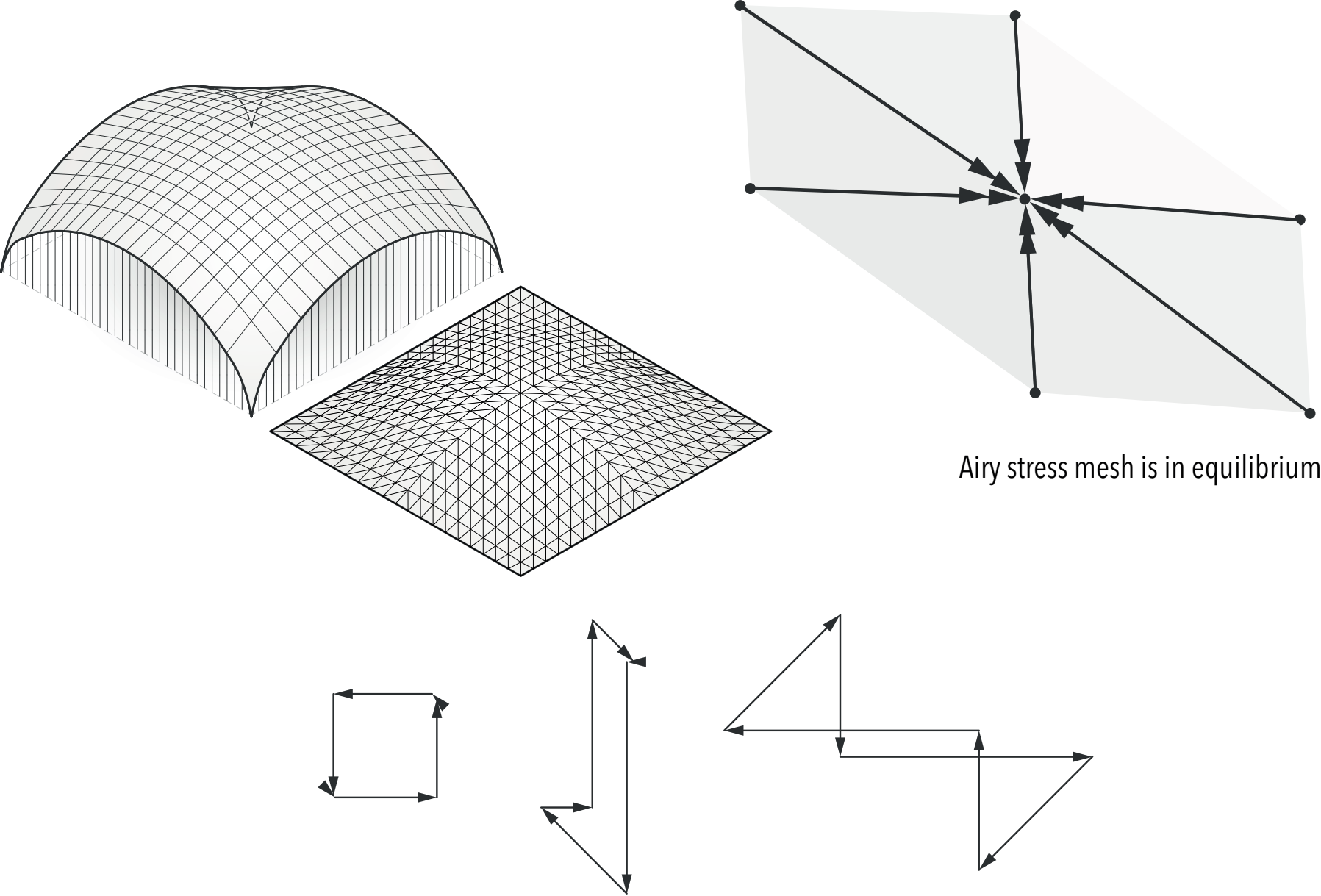


tension/compression polygons

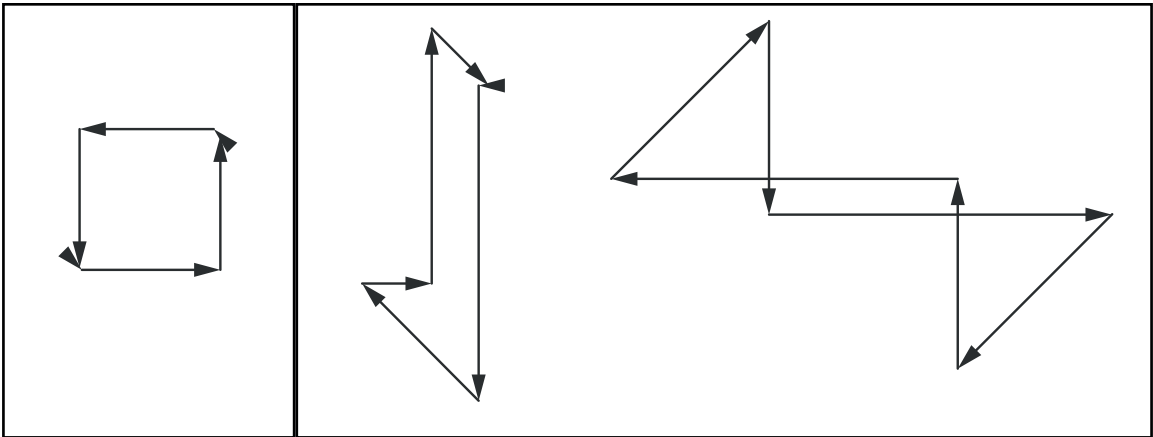
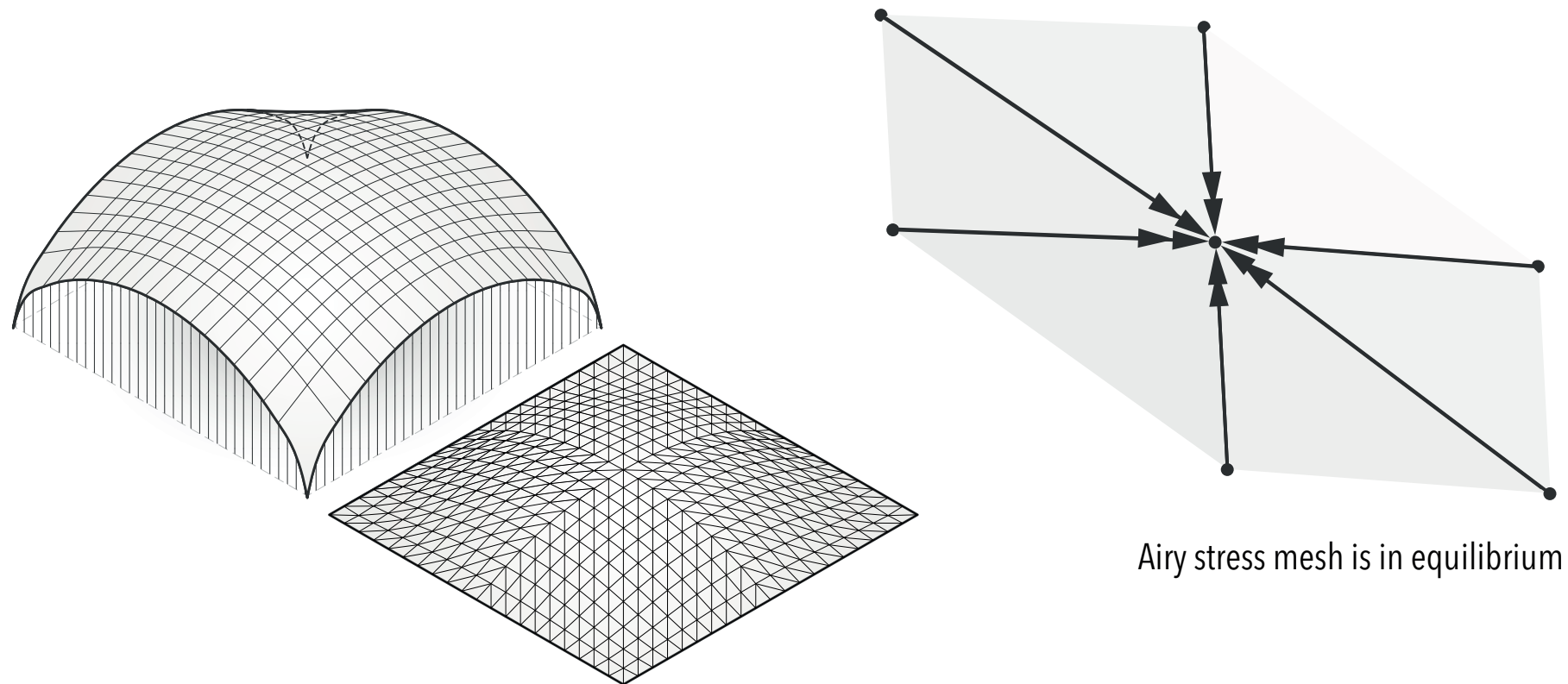
PARAMETRIC DESIGN TOOLS



PARAMETRIC DESIGN TOOLS



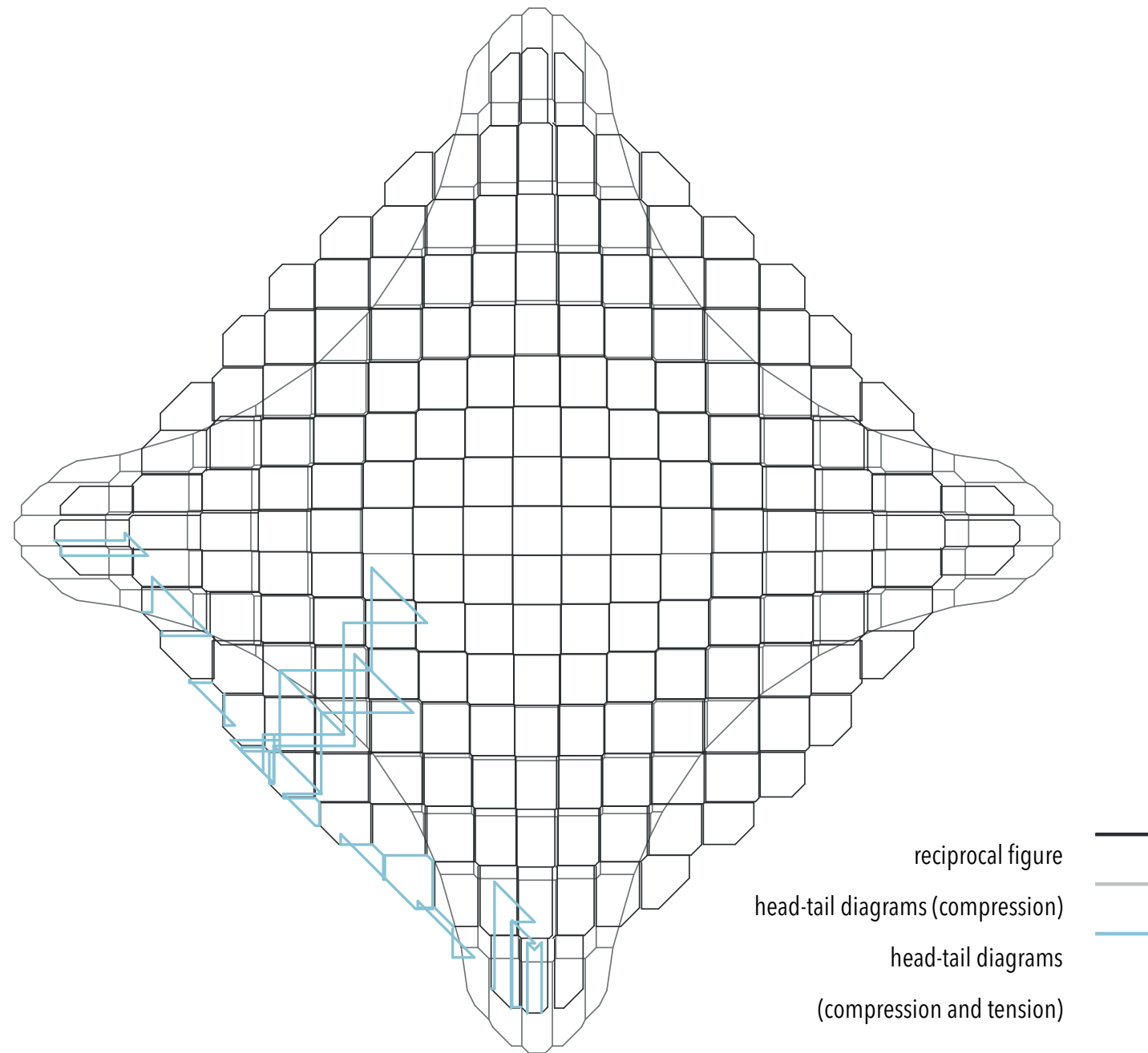
PARAMETRIC DESIGN TOOLS



compression only

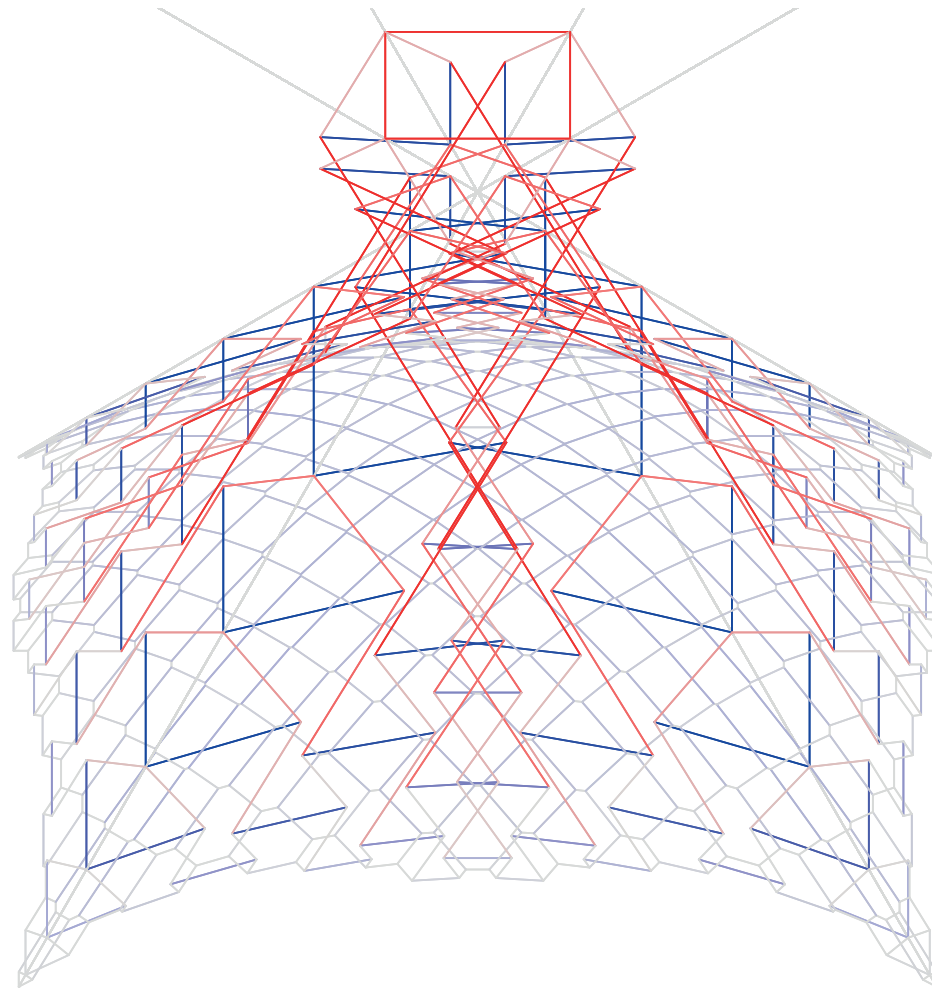
tension & compression

PARAMETRIC DESIGN TOOLS



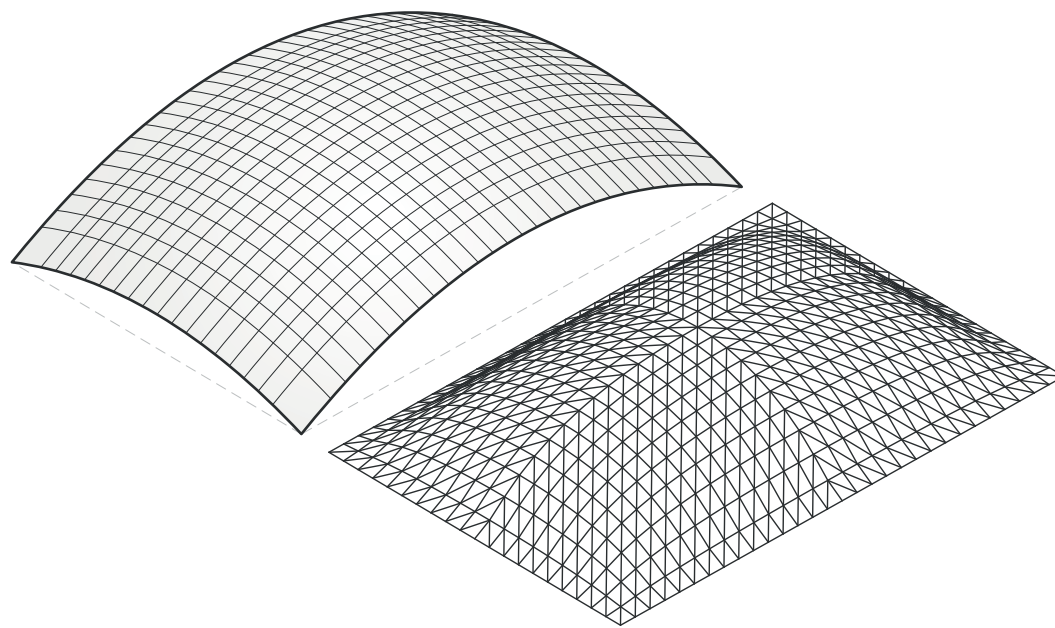
head-tail diagrams connected

PARAMETRIC DESIGN TOOLS



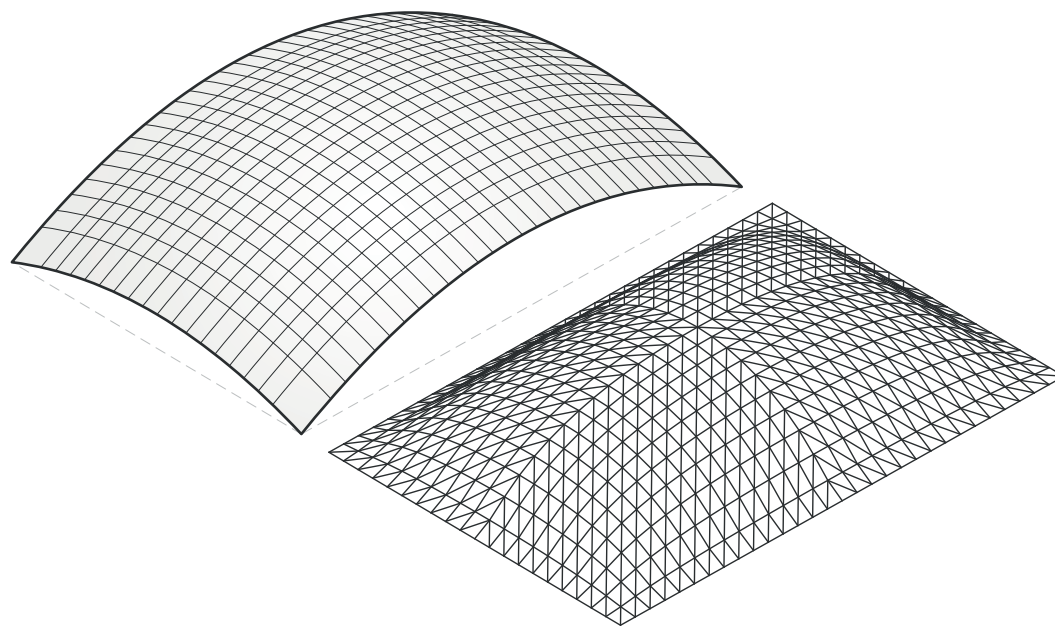
force polyhedron

PARAMETRIC DESIGN TOOL: CONNECTIVITY

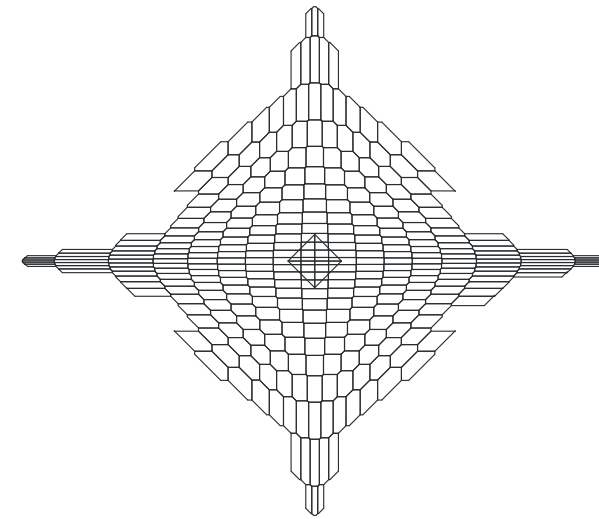


rectangular shell with free edges
(subjected to distributed load)

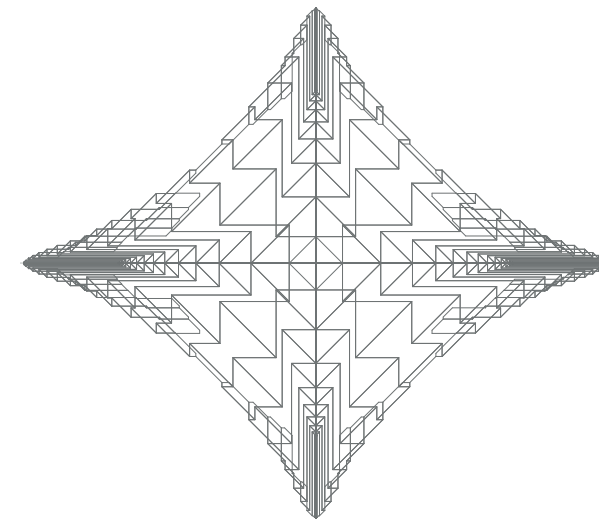
PARAMETRIC DESIGN TOOL: CONNECTIVITY



rectangular shell with free edges
(subjected to distributed load)

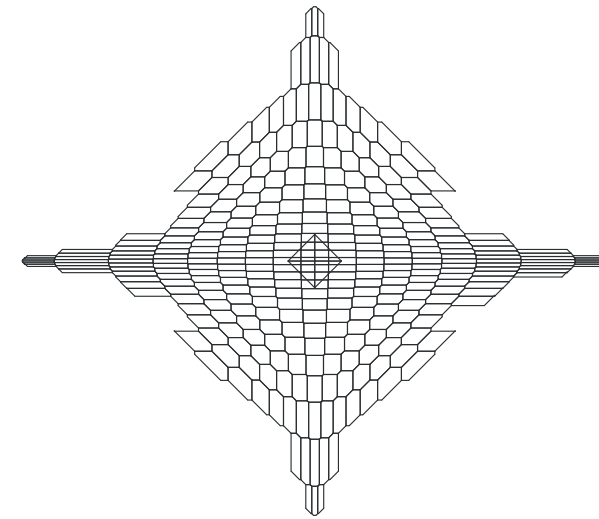
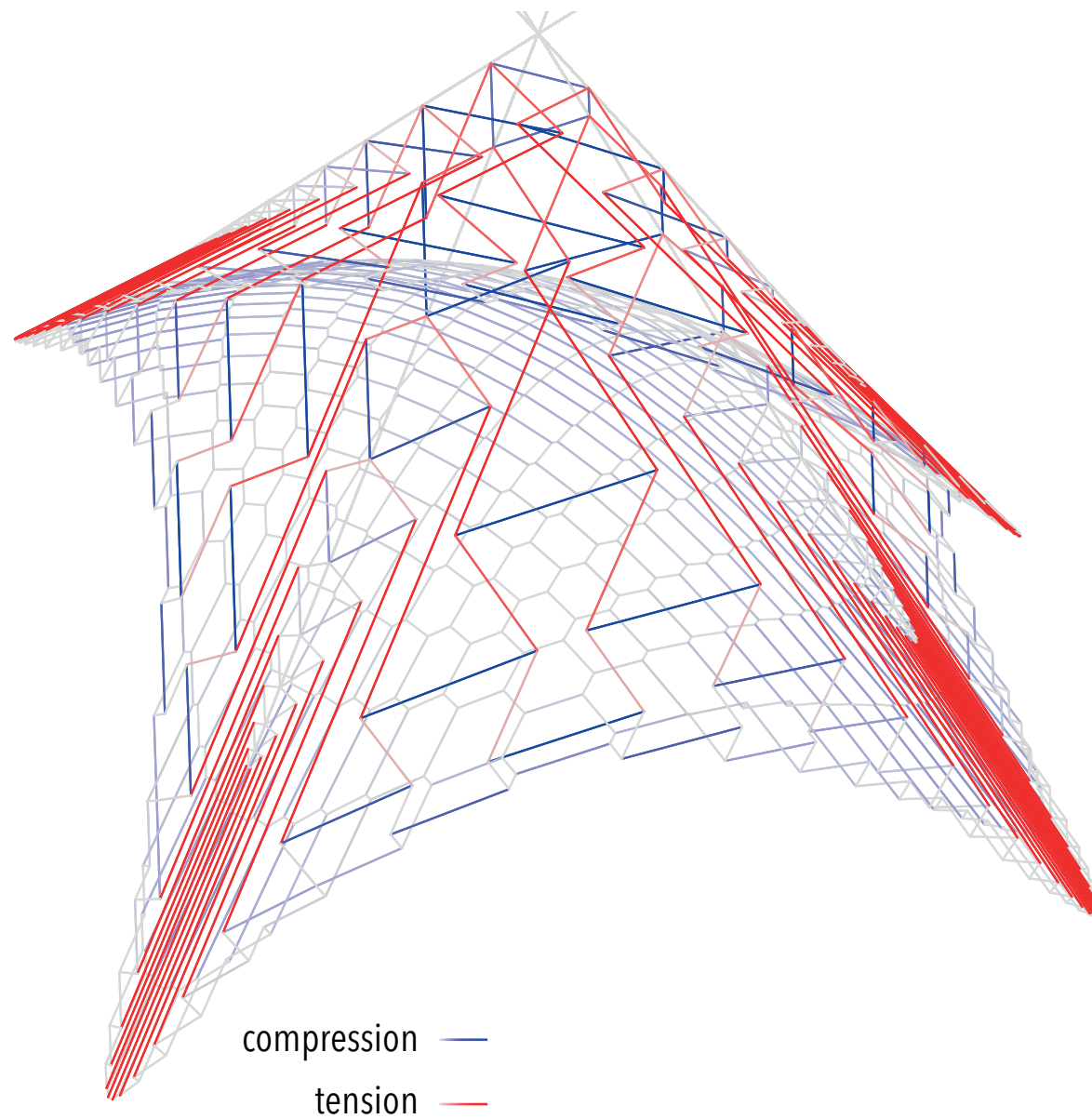


compression-only polygons

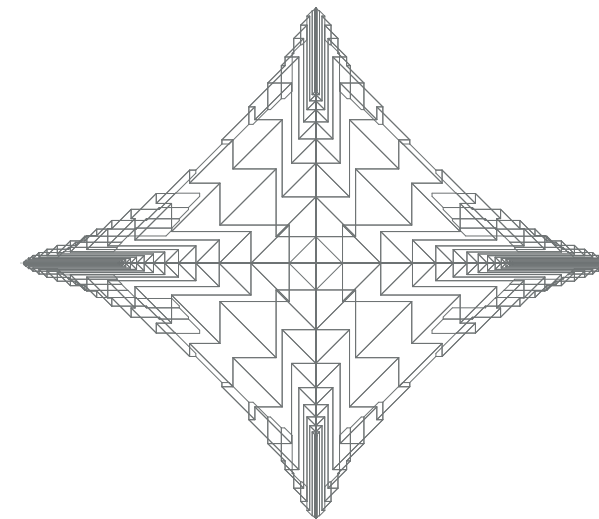


tension/compression polygons

PARAMETRIC DESIGN TOOL: CONNECTIVITY

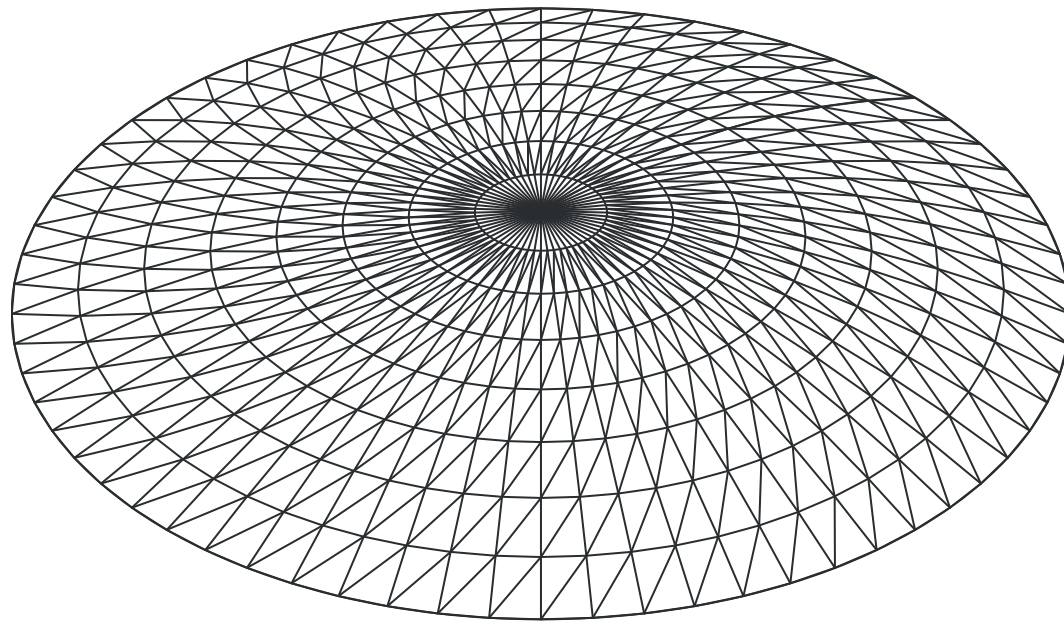


compression-only polygons

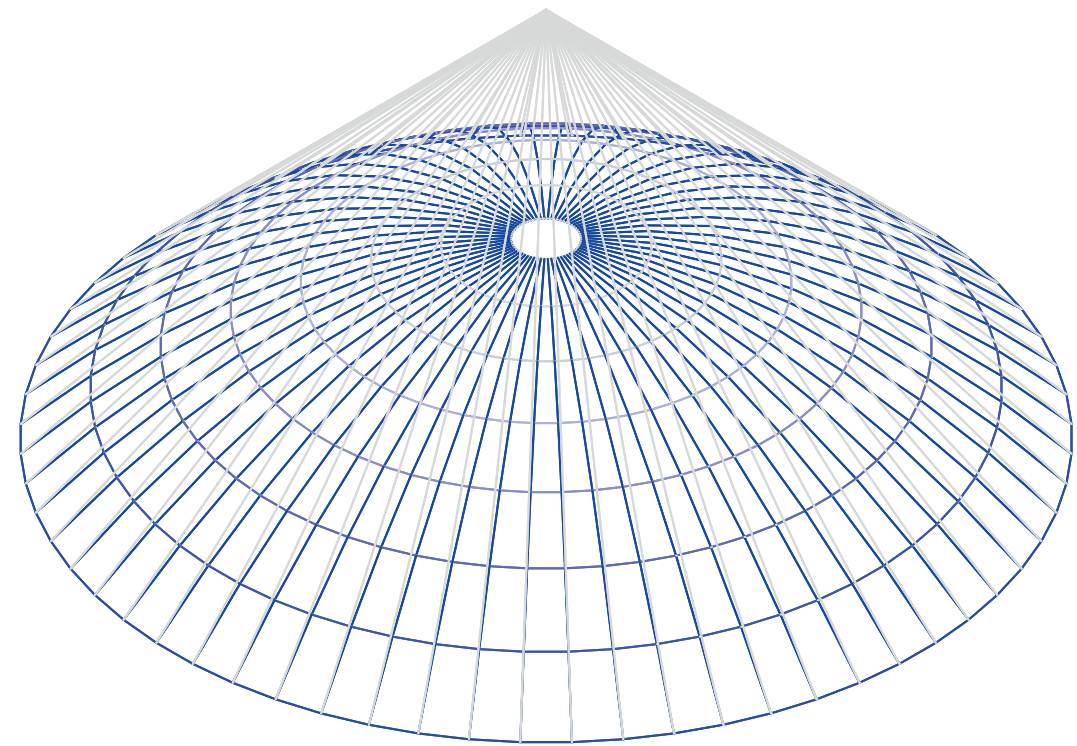


tension/compression polygons

PARAMETRIC DESIGN TOOL: CONNECTIVITY



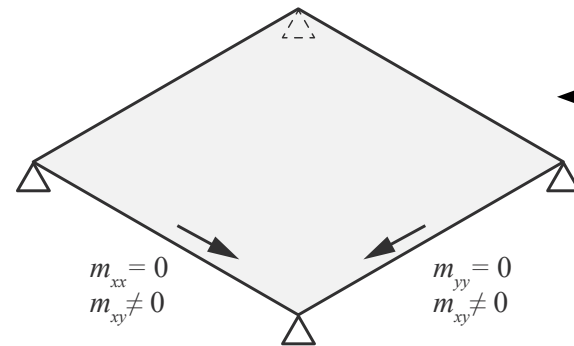
Airy stress function circular structure



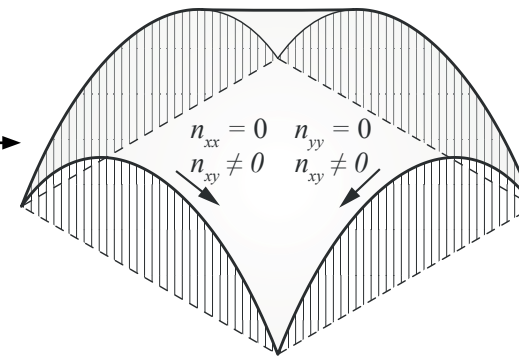
compression-only

'How can the relation between the **geometrical** and **mechanical** properties of shell structures be formulated and result in an intuitive and graphical design tool for (structural) designers?'

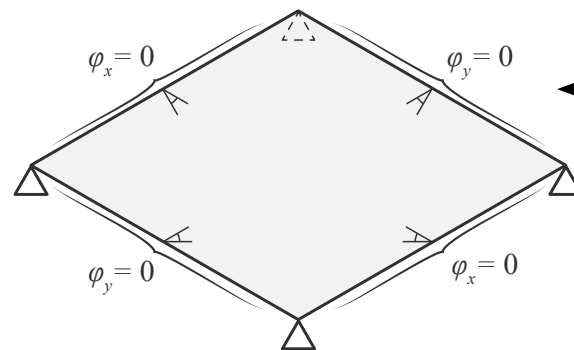
PLATE AND SHELL ANALOGIES



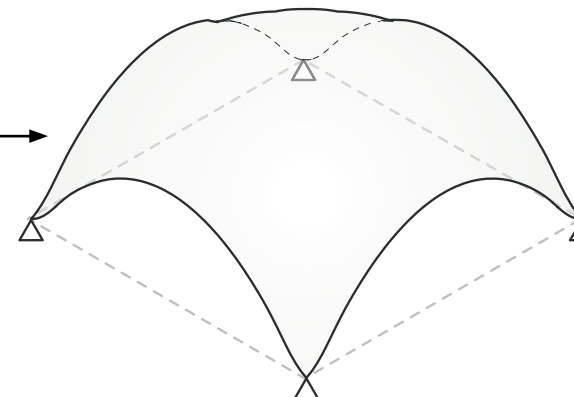
Simply-supported plate
(twistless)



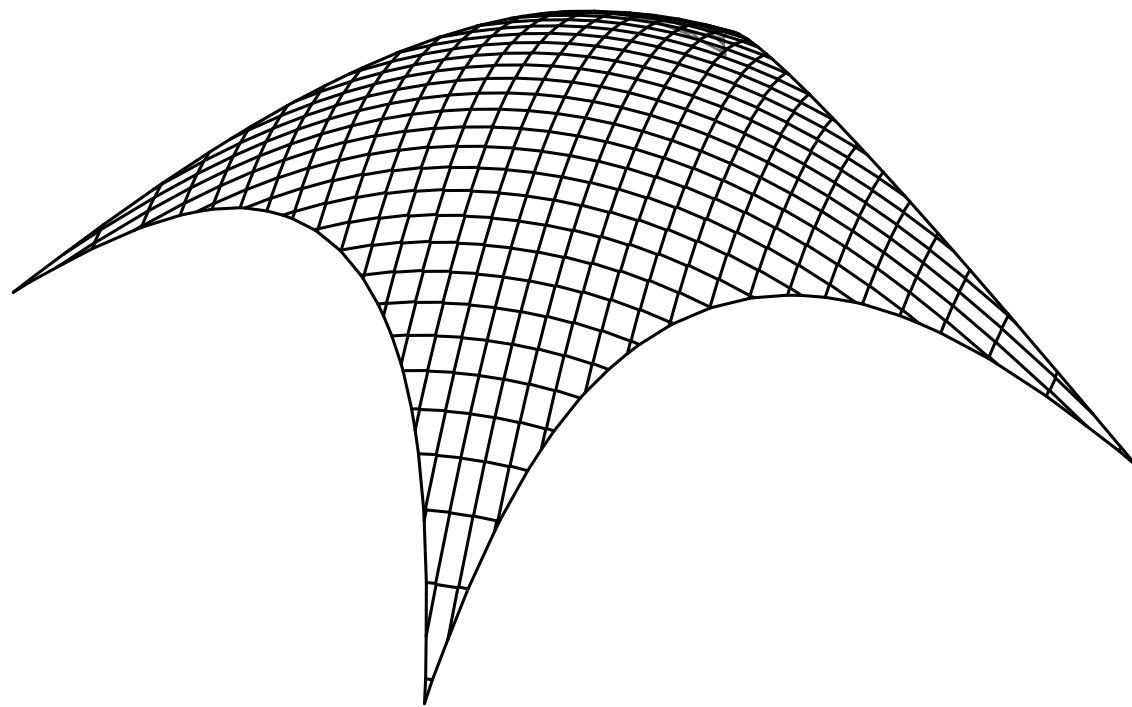
Shell on semi-rigid edge supports



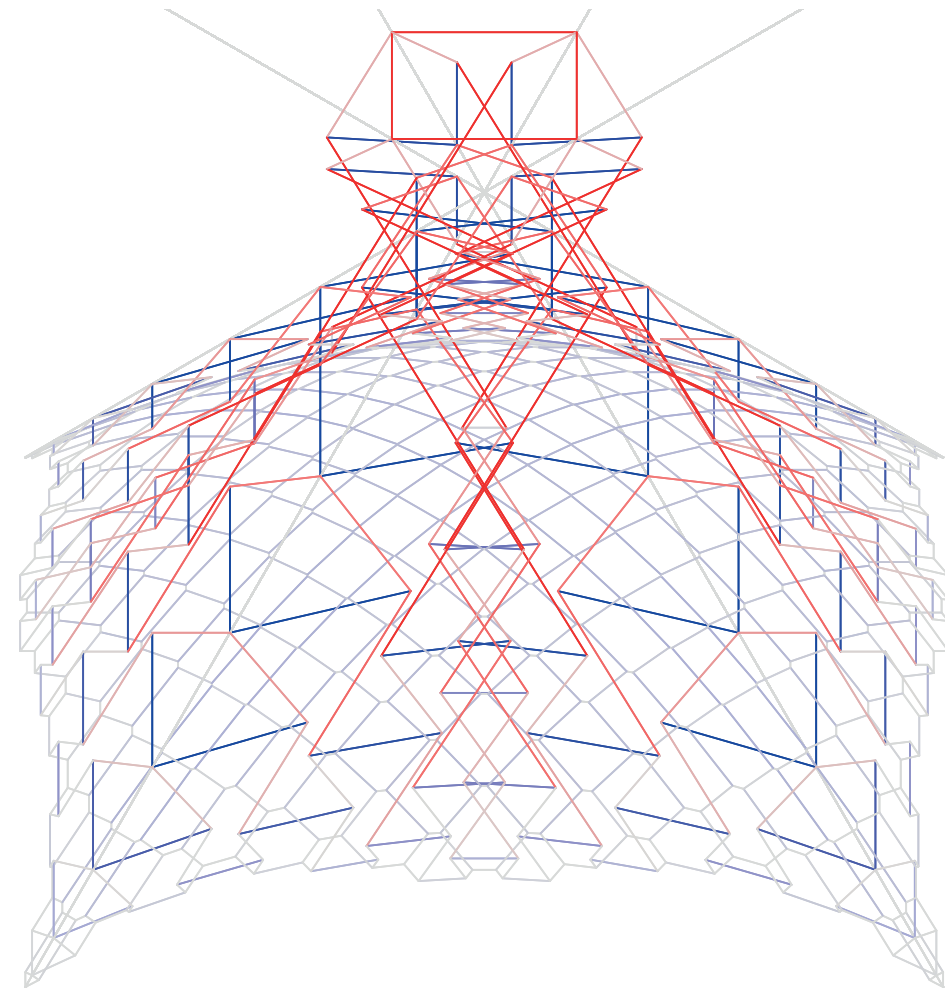
Simply-supported plate
(twistless & no rotation)



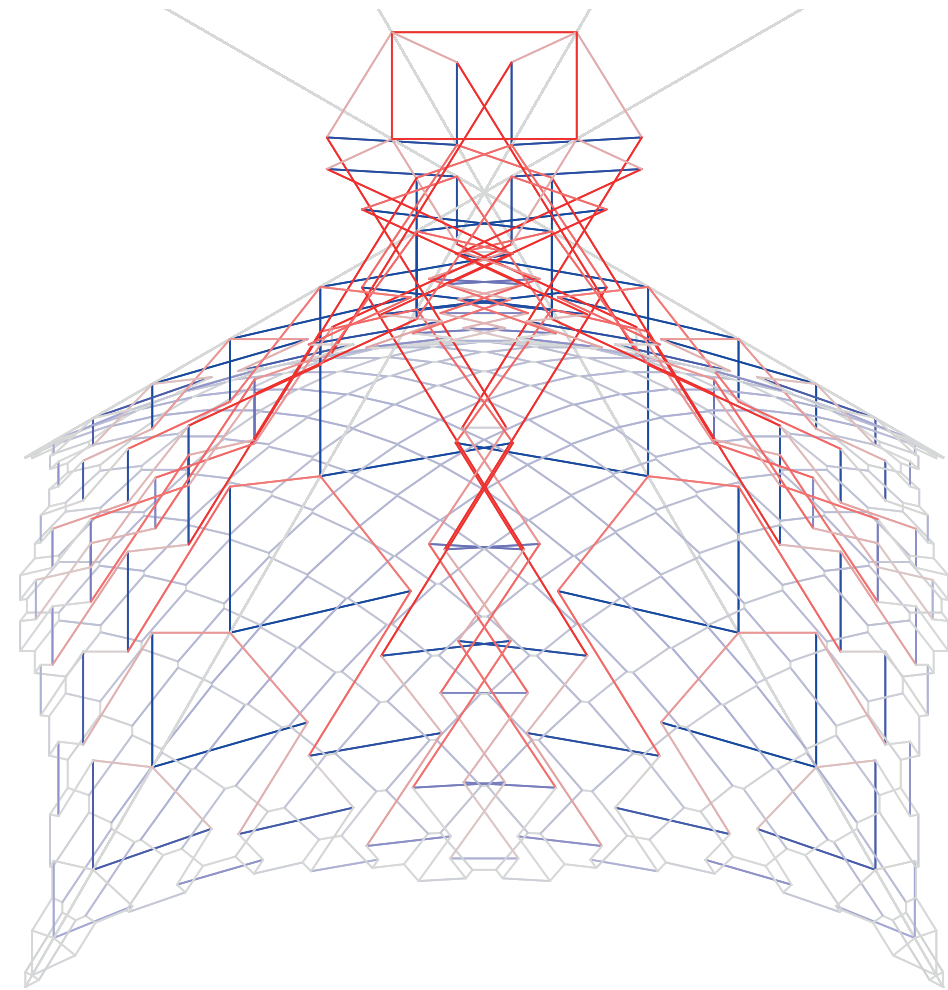
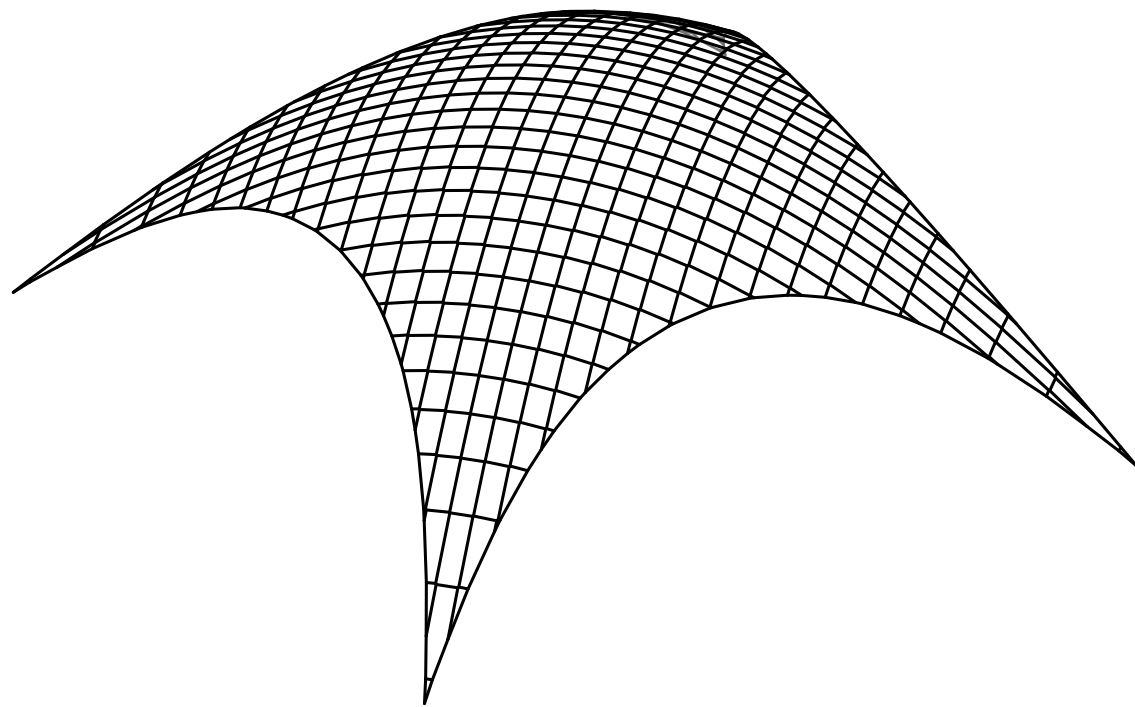
Simply-supported shell



form diagram



form diagram/force polygon



Thank you