

# Observation of Locally Negative Velocity of the Electromagnetic Field in Free Space

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Since the 1983 definition of the speed of light in vacuum as a fundamental constant with the exact value of 299 792 458 m/s the question has remained as to what apart from the wave front travels at that speed. It is commonly assumed that the entire electromagnetic waveform in free space does. Here it is demonstrated, both theoretically and experimentally, that the near- and intermediate-field dynamics of the vectorial electromagnetic field is much more complex than simple outwards propagation. In particular, it is shown that there exists a region close to the source, where, while the wave front travels outwards at the speed of light, the main body of the waveform appears to go inwards or back in time. The same effect may also lead to apparent superluminal results in free space.

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Around the year 1983 it had been decided to link the etalon of length to the speed of light in vacuum [1]. This has effectively put an end to the measurements of the “actual” velocity of light which is now defined to have an exact numerical value. However, there remains a seemingly simple question: what exactly travels at 299 792 458 m/s? It is more or less universally agreed that the wave front of the electromagnetic pulse in vacuum does. It is also often assumed that the electromagnetic waveform, which follows the wave front, also travels at that speed in free space. This assumption is essential for such technologies as radar ranging, travel-time tomography, and information transfer which all rely on an extremum or a slope of an impulse, or the envelope of a wave packet [2,3]. Yet, these practically useful waveform features do not always travel at the speed of light.

It is a well-known fact that dispersive media may drastically change the shape of the envelope, so that the group velocity significantly deviates from the velocity of light in the bulk medium [4–8]. Recently, it has been reported that “localized microwaves” travel at what appears to be a superluminal velocity in free space [9]. Although, the proposed theory and the accuracy of the measurements were subsequently disputed, there are no fundamental constraints prohibiting such behavior [10,11].

Here we look at the problem from a more general point of view, without specific reference to the Bessel-like  $X$  beams. It turns out that the apparent velocity of a characteristic feature (e.g., extremum or slope) of the electromagnetic waveform is not constant even in vacuum. This follows from the detailed analysis of the classical causal three-dimensional time-domain radiation formula. The near- and intermediate-field contributions both contain an additional relative time-delay with respect to the far-field

part. These relative time delays gradually disappear with the distance from the source. Thus the overall dynamics of the radiated field is a combination of two motions: one is the usual outward motion at the speed of light; the other is a relative inward motion. This leads, in particular, to the apparent superluminal results for the velocity of selected waveform features in the near- and intermediate-field zones. Here it is also predicted and experimentally demonstrated with microwaves that there exists a region close to the dipole source where the relative inward motion dominates and the main body of the waveform appears to travel back in time, which could be naively interpreted as an example of anticausal behavior.

According to the Maxwell equations, which govern the propagation of the electromagnetic field, the source of the field is the electric current density  $\mathbf{J}$ :

$$\begin{aligned} -\nabla \times \mathbf{H}(\mathbf{x}, t) + \varepsilon_0 \partial_t \mathbf{E}(\mathbf{x}, t) &= -\mathbf{J}(\mathbf{x}, t), \\ \nabla \times \mathbf{E}(\mathbf{x}, t) + \mu_0 \partial_t \mathbf{H}(\mathbf{x}, t) &= 0. \end{aligned} \quad (1)$$

Mathematically, causality enters here in the form of an assumption that the changes in the current *cause* the changes in the field. In particular, it is typically assumed that the field is zero everywhere before the current is switched on. Once the current density of the source is given as a function of space and time, and the surrounding medium is a free space, the Maxwell equations can be solved analytically, thus providing an explicit radiation formula for the fields everywhere outside the source at all times. In subscript notation to denote the Cartesian components of the electric field strength and the current density, using Einsteins summation convention, this important formula may be written as follows (see, e.g., [12]):

$$E_k(\mathbf{x}, t) = \int_{\mathbf{x}' \in D} \frac{1}{\epsilon_0 4\pi |\mathbf{x} - \mathbf{x}'|^3} (3\theta_k \theta_n - \delta_{kn}) \int_{-\infty}^{t_R} J_n(\mathbf{x}', t') dt' d\mathbf{x}' + \int_{\mathbf{x}' \in D} \frac{1}{c_0 \epsilon_0 4\pi |\mathbf{x} - \mathbf{x}'|^2} (3\theta_k \theta_n - \delta_{kn}) J_n(\mathbf{x}', t_R) d\mathbf{x}' + \int_{\mathbf{x}' \in D} \frac{1}{c_0^2 \epsilon_0 4\pi |\mathbf{x} - \mathbf{x}'|} (\theta_k \theta_n - \delta_{kn}) \partial_t J_n(\mathbf{x}', t_R) d\mathbf{x}'. \quad (2)$$

Where  $\theta_n = (x_n - x'_n)/|\mathbf{x} - \mathbf{x}'|$  represents a unit vector pointing from the location  $\mathbf{x}'$  inside the finite spatial domain  $D$  occupied by the source towards an observer situated at  $\mathbf{x}$ ,  $\delta_{kn}$  is the Kronecker delta, and  $t_R = t - |\mathbf{x} - \mathbf{x}'|/c_0$  is the so-called retarded time. It is this last simple expression for the retarded time, which one is tempted to apply to deduce the speed of light from the observed time-delay for a traveling electromagnetic impulse. Yet, the retarded time enters the three terms of the above radiation formula in three different ways. The terms correspond to the *near*-, *intermediate*-, and *far-field* zones, in accordance with the relative dominance of their spatial decay factors.

Sufficiently away from the source, the last term dominates. If it was the only term present, then the temporal dependence of the field would look like a somewhat distorted (differentiated) but otherwise just a shifted-in-time and reduced-in-magnitude copy of the current. Hence, having done measurements at two known locations, one further away from the source than the other, we can apply the retarded-time formula to some obvious feature of the waveform, say a particular extremum, and recover the speed of light. However, due to the presence of the near- and intermediate-field terms in (2) the field dynamics is far more complex than that.

To better understand the effect of the first two terms in (2) on the overall time-dependence consider a current in the form of a monocycle, i.e., a single period of a sinusoidal oscillation  $\sin(\omega_c t)$  of a given angular frequency  $\omega_c$ , starting at  $t = 0$  and ending at  $t = 2\pi/\omega_c$ . Elementary calculus shows that the first (leftmost in time) maximum of the current will happen at  $t_{\text{inter}} = \pi/(2\omega_c)$ . The time derivative of the current has its first maximum at  $t_{\text{far}} = 0$ , and the first maximum of the current's time integral is at  $t_{\text{near}} = \pi/\omega_c$ . Hence, for the relative positions of the first maxima we have the relation  $t_{\text{far}} < t_{\text{inter}} < t_{\text{near}}$ ; i.e., the first maxima of the near- and intermediate-field terms are retarded with respect to the first maximum of the far-field term. In the case of a monochromatic signal this is nothing else but the usual *local phase shift* between the current and the voltage in an ac circuit, which could be deduced already from the first of the Maxwell equations (1). It is difficult to prove but easy to convince yourself on a few simple examples that a similar relative shift of extrema will be observed with other, more general waveforms as well.

The total radiated field is a weighted sum of the three contributions, where the first one dominates very close to the source. Thus, the relative shift of an extremum with respect to its far-field shape will be a function of position.

We may expect, in particular, that the waveform extrema will be measurably retarded in the near-field zone, even in the immediate vicinity of the source. As we move further away from the source, the influence of the near- and intermediate-field terms on the overall time-dependence will rapidly diminish in inverse proportion to the cube and square of the distance. Hence, in addition to the causal time retardation, which shifts the waveform to the right along the time axis, the extrema will gradually loose their initial relative near- and intermediate-field retardations, i.e., move to the left along the time axis. It is difficult to tell which of the two tendencies will dominate in the overall time dependence. We resort here to simple numerical calculations which demonstrate, in particular, that there exists a region where the second (relative) motion dominates, and the main body of the waveform appears to be traveling back in time, namely, for progressively larger distances from the source we receive extrema of the waveform at progressively earlier times.

In the case of a point-dipole model the spatial integrals in the radiation formula are evaluated analytically [12,13]. The resulting formula is especially simple for the direction of observation orthogonal to the direction of the dipole moment (see Fig. 1), since along the measurement line the electric field vector is parallel to the current in the source. We take the excitation current to be a finite sinusoidal wave packet with 4 GHz central frequency and simulate the waveforms received at increasing distances from the source spanning the spatial interval from 10 to 100 mm, i.e., both the near- and the intermediate-field zones.

The simulated results are presented in Fig. 2, where the top part gives all the waveforms as a single image. The horizontal axis is time ( $c_0 t$  in metres), the vertical axis is the distance between the source and the receiver along the measurement line (also in meters), and the amplitude of the

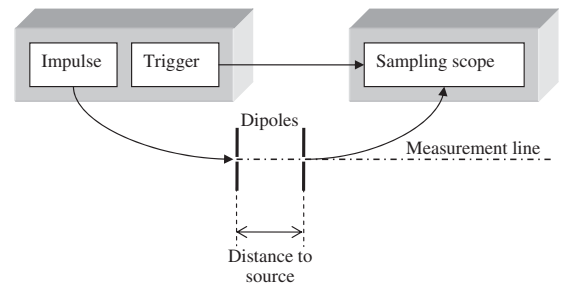


FIG. 1. The near-field measurement setup with two identical vertically polarized dipole antennas placed at a varying distance along the measurement line.

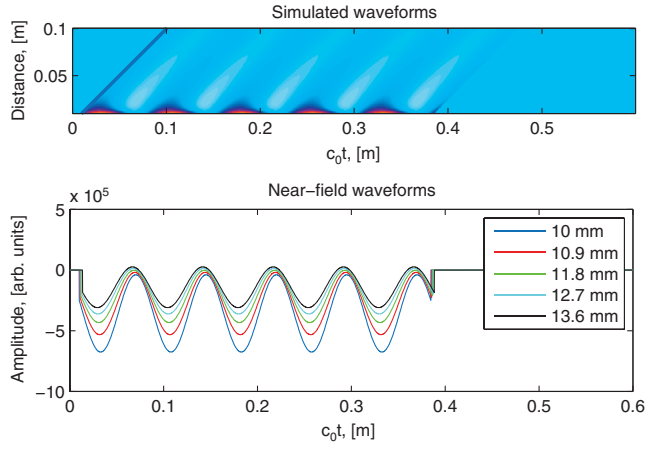


FIG. 2 (color). Simulation of the electric field strength in the near- and intermediate-field zones of the point-dipole antenna fed by a sinusoidal current. Top image: space-time wave packet dynamics (see text); lower part of this image corresponds to the near-field zone, where the leftward bending indicates negative velocity; middle and top part—positive velocity approaching light cone behavior. Bottom plot: the first five near-field waveforms received at increasing distances from the source; the outer edges shift rightwards—normal (light cone) behavior; the inner part shift leftwards—negative velocity.

electric field strength at the receiver as a function of time is encoded in color. The near-field zone corresponds to the bottom area of the top image.

A signal traveling at the speed of light should follow the so-called light cone. This light cone looks as a rising  $45^\circ$  line for convenience also shown in the left part of the top image. In such a color-coded image a waveform with a fixed shape traveling at the speed of light should look like a series of colored stripes all parallel to the light cone line with the color intensity decaying upwards. We see that the leftmost part of the wave packet—its wave front—does indeed follow the light cone. However, the main body (inner part) of the wave packet, at small distances (bottom of the image) makes an initial bend in time to the left and begins to follow the light cone only somewhat further from the source (middle and top of the image). To see what really happens, in the second plot of Fig. 2 we present the details of the first five near-field waveforms as a function of time, received at five progressively increasing distances from the source. These waveforms correspond to the bending in the bottom of the color-coded image. Now it is clearly visible that, while the edges of the wave packet shift to the right, the inner part shifts to the left, i.e., travels back in time. Paradoxically, an observer in the immediate vicinity of the source will receive the sequence of extrema somewhat later than another observer slightly further away.

The actual experiment was performed using a pair of unbalanced 31 mm long copper wire dipoles. The source antenna was fed by an approximately Gaussian impulse, with 71.4 ps FWHM duration (Picosecond Pulse Labs

impulse generator, model 3500). The internal trigger of the generator is used to trigger the Agilent Infiniium DCA-J 86100C mainframe with a 20 GHz sampling module (Agilent 86112A), which recorded and averaged the waveform received by the second dipole. Both antennas naturally deform the original Gaussian waveform, so that the received signal looks like a wave packet simulated above. The results are presented in Fig. 3 in the form identical to the simulations of Fig. 2.

The results are qualitatively similar to the previously discussed point-dipole simulation of Fig. 2 as we notice the presence of the leftward bend in the bottom area of the top image and the leftward shift of the main waveform body in the bottom plot of the first five waveforms. However, the radius of the negative velocity region of the actual dipole is approximately 8 mm, whereas for the point-dipole model it is almost twice as large (up to 15 mm). Also the relative back-in-time shift of the measured waveform is somewhat smaller. These discrepancies could be explained by several factors not included in the point-dipole model. For example, it does not take into account neither the mutual interaction between the source and the receiver nor the finite size of the real dipole antenna. Also the shape of the time-domain waveforms is different in the simulations and the experiment. Having said that, the phenomenon of the locally negative velocity of the electromagnetic field in free space is obviously confirmed.

What is the physical meaning of this effect? In particular, does it mean that the electromagnetic field and thus light can propagate with a negative velocity in vacuum? Obviously, if light would *propagate*, then it would do so at the speed of light. This is what happens when we consider the propagating (retarded) plane wave solution of the one-dimensional wave equation. On the other hand, wave radi-

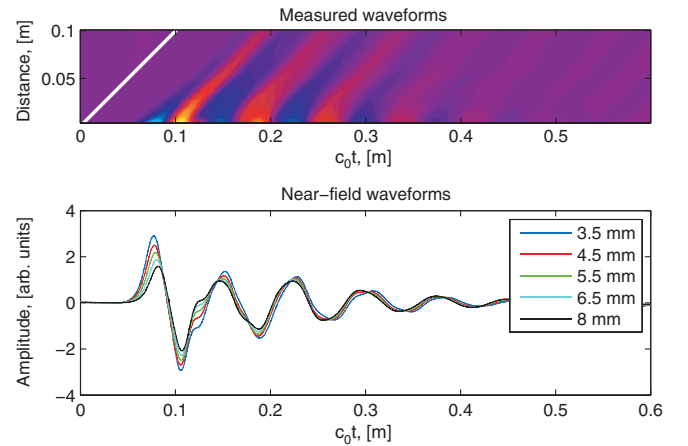


FIG. 3 (color). Experimental observation of the free-space negative waveform velocity. Top image: space-time waveform dynamics; the leftward bend in the lower part of the image indicates the negative velocity in the near-field. Bottom plot: details of the first five near-field waveforms with inner peaks traveling back in time.

ated by a *finite* 3D source is a superposition of infinitely many plane waves. So, is this effect due to superposition (interference)? Not entirely. One can easily obtain an explicit propagating solution looking almost exactly like the last (far-field) term of (2) for the source-driven three-dimensional *scalar* wave equation. Nevertheless, a similar effect was, in fact, predicted for the scalar case in [14], where the wave function of a free quantum particle with zero angular momentum is considered. There, however, the negative initial propagation of the average radial position for a ring-shaped wave packet is specific to the two-dimensional case, and is related to the violation of the Huygens principle in the space of even dimensions [15]. The present effect has more to do with the *vectorial* structure of the Maxwell equations (1) and the equivalent *vector* wave equation, which differs from the scalar one by the often omitted  $\nabla(\nabla\cdot)$ -term. Although the field is divergence-free in vacuum, it is not inside the source, since a time-dependent current may induce an instantaneous nonzero local charge distribution, which (as shown here) produces observable effects in the near- and intermediate-field zones. As a result, light radiated by a real three-dimensional source does not simply propagate, but rather *evolves* in space time. This evolution begins to look like propagation only sufficiently away from the source. Interestingly, these effects would not be observable if we could directly (and separately) measure the scalar and vector potentials, since the latter obey source-driven scalar wave equations. The near- and intermediate-field terms appear if potentials are linked by the relativistic Lorenz gauge, which is also something worth thinking about.

The counterintuitive nature of the negative velocity stems from our desire to interpret the complicated but perfectly causal evolution of the electromagnetic field as a simple propagation. In my view this also explains some of the recent apparently superluminal observations based on the time-delay and distance measurements [9]. Consider two observers (*A* and *B*) recording waveforms at different distances from the source. The influence of the near- and intermediate-field terms will be stronger for the nearest of the two observers (*A*) and weaker for the other (*B*). Hence, the extrema of the waveform will keep shifting backwards in time, although this shift is now relatively small compared to the overall forward motion of the wave packet. In any case, (*A*) will see the extrema shifted *more forward*, if compared to the same extrema as seen by (*B*). Hence, the time-delay between similar extrema at (*A*) and (*B*) would appear to be shorter than what is expected from the causal retardation alone. Calculations using the retarded-time formula will then produce an apparently superluminal velocity of propagation approaching the speed of light asymptotically from above as the distance from the source increases, in agreement to what was obtained in [9]. On the

practical side, the present results show that algorithms relying on the velocity of some particular feature of the electromagnetic waveform (e.g., radar ranging) may require a correction beyond the simple travel-time schemes, especially, if applied in the near- and intermediate-field zones.

We have considered here the classical picture of the electromagnetic field. Within this formalism it would be interesting to investigate the effects of the finite size and spatial and temporal coherence of the source on the extent of the negative velocity region. Further, it is well known that the quantum-optical formula for the dipole radiation is identical to its classical counterpart used here [16] and proven to contain the causal retardation up to all orders in the mutual interaction of the source and receiver atoms [17]. However, the role and physical meaning of the near- and intermediate-field terms is by far less understood in quantum optics.

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