

Maximum Energy Benefit of Compute-and-Forward for Multiple Unicast Sessions

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MAXIMUM ENERGY BENEFIT OF COMPUTE-AND-FORWARD FOR MULTIPLE UNICAST SESSIONS

by

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in partial fulfillment of the requirements for the degree of

Master of Science

in Electrical Engineering

at the Delft University of Technology,

to be defended publicly on Monday July 6, 2015 at 14:00 PM.

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This thesis is confidential and cannot be made public until July 6th, 2015.

PREFACE

My master thesis is done in the Cyber Security Group, Faculty of Electrical Engineering, Mathematics, and Computer Science (EEMCS), Delft University of Technology. Since I took my first step on the land of Netherlands and started my study in August 2013 in TU Delft, I have found life so different and amazing from that moment.

In these two years, I am receiving great help from professors, classmates and friends all the time. I feel extremely grateful to everyone who offered encouragements, suggestions, as well as support all through these years. Meanwhile on this occasion, I would like to express my sincere gratitude and heartfelt tribute to everyone who offered great help and guidance, or shared resources with me during this thesis work. Without the help from anyone of you, there is no possibility for me to complete the thesis program.

First of all, my wholehearted gratitude goes to my supervisor, Dr. ir. J. H. Weber, who is always supportive and helpful. During these nine months as well as all the lectures Dr. Jos gave, he made things clear and interesting. Even before I started my thesis work, he helped me with making clear plans, which offered guarantee to a good beginning. During the thesis work, he spent much time and effort on helping my project, giving constructive recommendations and reviewing my final thesis. He encouraged me when I came across problems with my thesis work and he always had patience with my mistakes.

Then I would like to give my gratitude to Dr. ir. G. J. M. Janssen, who is my coordinator during my two years study in TU Delft. He offered precious guidance about everything, especially for international students who were not familiar with the study and examination systems. He paid much attention to my academic development and oriented me towards my real interest.

I would like to give special thanks to Zhijie Ren. As my daily supervisor, he offered selfless help and encouragement to me. He gave useful criticism on every detail of my work, which helped me with deeper understanding of my thesis work. During these nine months he showed great patience and willingness to help whenever I had troubles. Without his help and inspiration, could I never get so far now.

I would like to thank all my friends sincerely for bring me so much fun in my two years life in Delft. They also gave me invaluable help, encouragement and warm accompany whenever I felt lonely and depressed. Because of their kindness could I get rid of homesick, find confidence and happiness in life.

I would like to thank my parents for their unconditional love, support and encouragement, even though we are eight thousand kilo miles away from each other. Thank you for giving me the whole freedom to chase my dreams. Last but not least, I would like to thank my boyfriend Li Zhen who is always there waiting for me. Your love completes me and makes me the luckiest girl in the world. Your full understanding and support healed me through difficult times, and gave me hope and joy during all these years.

Mengyao Liu
Delft, June 2015

ABSTRACT

In this thesis, we investigate the maximum energy benefit of compute-and-forward applying on general networks. The energy benefit is defined as the ratio of the minimum energy consumption in network when a symbol is communicated successfully for every session when the network is in traditional routing mode, and the minimum energy consumption in network when a symbol is communicated successfully for every session when the network is in compute-and-forward mode. The upper bound of the energy benefit is derived by proving the upper bound of the minimum energy consumption when applying traditional routing scheme, and the lower bound of the minimum energy consumption when applying compute-and-forward.

We give theorems and proofs about the energy benefit on general wireless networks and on some special wireless networks. Before that we give the model set-up for wireless networks. In general networks, we get the conclusion that for the benefit of energy consumption when applying compute-and-forward is upper bounded by the average distance of all sessions in the network. It is also upper bounded by the larger one of the maximum distance between each source node to the destination set, and the maximum distance between the source set and each destination node. For some special networks, we start by giving definitions of them, then we give upper bounds of the energy benefit on these special networks. We present the idea that applying compute-and-forward in a network does not make any benefit if the network is a single source network where the source node needs to transmit independent information to each destination node, or it is a single destination network where the destination node needs to receive independent information from each source node. In networks with non-collated source nodes and destination nodes, the energy benefit is upper bounded by $2K$, where K is the number of sessions in the network. Upper bounds of energy benefit on other special networks, e.g. line networks, 2D/3D rectangular lattice networks are studied in this thesis. We get the conclusion that in these special networks, the upper bounds of energy benefit are constants factors.

Keywords : wireless networks, compute-and-forward, energy benefit, upper bound

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1

INTRODUCTION

1.1. WIRELESS NETWORKS

These years, wireless networks are becoming so popular that it has captured great interests of electrical engineers. Compared to wired networks which connect devices to the Internet or other network using cables, a wireless network is a special kind of networks which uses wireless data connections for connecting network nodes. As 2G cell phone network first appeared in 1991 and 802.11 "WiFi" protocol was first released in 1997, wireless network technology has been developed quite fast [3].

Wireless networks have the following advantages of productivity, convenience, and cost over wired networks. Firstly, wireless network systems can provide wireless users with access to real-time information anywhere and every moment, while this mobility supports productivity and service opportunities which are not possible with wired networks. There are now thousands of universities, hotels and public places with public wireless connection. These free you from having to be at home or at work to access the Internet. Secondly, as for the installation, a wireless system can be installed fast and easily which eliminates the need to pull cables through walls and ceilings. The third advantage is the cost reduction: while the initial investment required for wireless networks can be higher than the cost of wired networks, overall installation expenses and life-cycle costs can be significantly lower. Long-term cost benefits are greatest in dynamic environments which require frequent moves and changes. The last but not least advantage is the scalability of wireless networks : a wireless network can be configured in a variety of topologies to meet the needs of specific applications and installations. Configurations are easily changed to realize roaming in peer-to-peer networks suitable for a small number of users, as well as in full infrastructure networks suitable for thousands of users.

When we refer to the application of wireless networks in real life, we think of cell phone networks naturally. As a cell phone offers a simple form of wireless networking, some of the same advantages that we enjoy while using a cell phone can be applied to a wireless network using computers, laptops and peripherals. Fig 1.1 shows how wireless devices, e.g., WiFi phones and notebooks, get connected to the Internet via a router.



Figure 1.1: An illustration figure for wireless networking. [1]

In Fig 1.1, wireless devices such as WiFi phones, wireless desktop and wireless storage get wireless access with the router which plays the role of relay, and the router is connected to the Internet via a cable. Then each wireless device could get access to the Internet via the router.

Another kind of wireless networks consists of a number of nodes or devices which communicate with each other over a wireless channel[2]. Some wireless networks have a wired backbone with only the last hop being wireless. Examples are cellular voice and data networks and mobile IP. In others, all links are wireless. One example of such networks is multi-hop radio networks or ad hoc networks, as is shown in Fig 1.2. Another possible example maybe collections of “smart homes” where computers, microwave ovens, door locks and other “information appliances” are interconnected by a wireless network.

In this thesis, we focus on wireless networks in which the links are all wireless ones. Such networks consist of a group of nodes which communicate with each other over a wireless channel without any centralized control. Nodes may cooperate in routing each others’ data packets.

Besides all the advantages, we cannot ignore big challenges of applying wireless networking. One of the well-known challenges is the throughput gain and the cost problem. Wired nodes are usually static, while wireless was built to support mobility and portability. The wired network design conflicts with the characteristics of the wireless medium. As a result, current wireless networks sometimes suffer low throughput, dead spots, and inadequate mobility support. Our medium access control, routing, and transport protocols are all imported from the wired domain, with minor tweaks. They are optimized to work over point-to-point links, for example a single predetermined path and a layered architecture. The cost of redesigning our network stack is non-negligible. But the wireless throughput is intrinsically limited, which warrants efforts to investigate the potential of new high-throughput architectures[4].

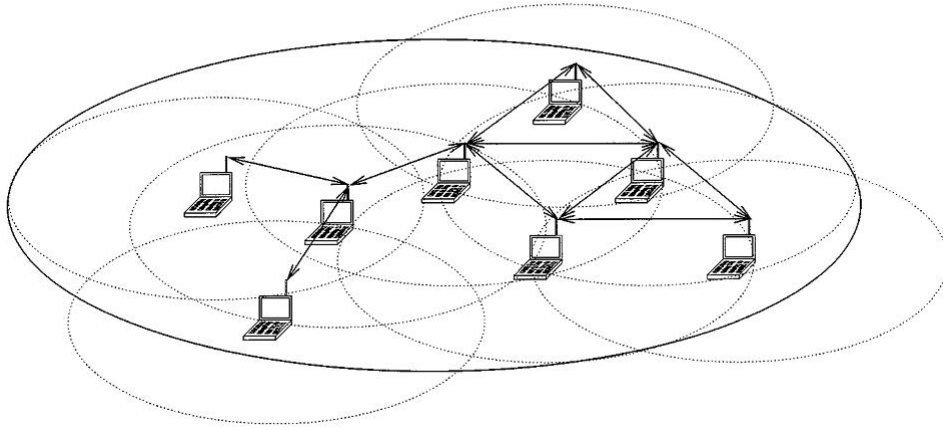


Figure 1.2: An example of wireless network with all wireless links: an ad hoc network. [2]

Another challenge is the energy limitation in wireless networks. In one hand, when using the traditional MAC and Point-to-Point based transmit protocols, the collisions are avoided by forbidding multiple devices transmitting simultaneously at the same frequency band, which does not actually deploy the broadcast nature of wireless transmissions. In other words, with most of the traditional communication protocols, most of the power of broadcasting is wasted since only a small portion of the power is useful for the decoding of the desired receiver. In another hand, the wireless communications are always related to wireless devices, such as mobile phones, laptops, etc., which are battery driven and power restricted.

The power of wireless devices are always limited, e.g., a wireless sensor network consists of many energy-autonomous micro sensors distributed throughout an area of interest. Each node monitors its local environment, locally processing and storing the collected data so that other nodes can use it. Network nodes share this information via a wireless link. Using data fusion, specific features of interest to the end user can be extracted from the information that several nodes collect, while a multi-hop communication scheme propagates this information to a base station node. Since these networks often are deployed in regions that are difficult to access, the nodes should not require maintenance. They must be energetically autonomous, using batteries that do not need to be replaced or recharged [5]. Thus, saving of energy consumption in wireless networks is a hot topic worth investigating for researchers.

Many challenges of wireless networks are caused by the broadcast nature of the wireless communications, due to the broadcast superimposed nature of electromagnetic (EM) waves, multiple sources transmitting simultaneously at the same frequency band might result in distorted signal at their destinations, which harms the decoding of the desired message. In traditional point of view, this situation is treated as collisions and is managed to be avoided by MAC protocols. However, as the progress in utilizing Network Coding and Compute-and-forward Scheme, which will be introduced later, the properties of broadcast and superposition are exploited as advantages in wireless networks.

1.2. NETWORK CODING

Network coding is a new and promising paradigm for wireless communication networks, as it allows intermediate nodes to mix messages received from multiple sources. Different from traditional hop-by-hop routing schemes, network coding fully uses the broadcast nature of wireless communication.

Network coding was firstly presented in 2000 in [6], which employed coding in the nodes, instead of regarding the information to be multicast as a “fluid” which can simply be routed or replicated. In the single-source problem, it gave characterization of the admissible coding rate region. It proved that the maximum flow capacity of a single multicast session can be achieved using network coding in an arbitrary network with directional links. Later in [7], advantages offered by network coding over traditional routing were proved, when network coding was used to realize mutual exchange of independent information between two nodes in a wireless network. A distributed scheme which obviated the need for synchronization and was robust to random packet loss and delay was also proposed in it.

To illustrate how different schemes perform in wireless networks, a simple three-node linear wireless network with the existence of local interference is presented. The model is also called two-way relay channel (TWRC). Efficient communication over a TWRC has attracted extensive research efforts, in light of the discovery of network coding. In a TWRC, two users exchange information via a relay without a direct link between the users. The model of TWRC is illustrated in Fig 1.3.



Figure 1.3: The Model of a two-way relay channel

In this model, N_1 (Node 1) and N_3 (Node 3) are communication nodes who want to exchange information, but they cannot exchange directly because of their being out of each other's transmission range. N_2 (Node 2) is the relay node between N_1 and N_3 . We assume the network is frame-based and define the time slot in this model as the time required for the transmission of one fixed-size frame. Each node is equipped with an omni-directional antenna, and the channel is half duplex, which means that transmission and reception at a node cannot happen in a same time slot.

Now we introduce the traditional routing scheme in comparison with network coding scheme. The goal for successful transmission is to realize mutual exchange of a frame in the model presented above. Traditional routing is the most basic and simplest method in wireless communications. In traditional routing, the source node transmits packets hop-by-hop through a predetermined single path in the unicast scenario or through a multicast tree to a group of receivers in the multicast scenario [8]. In traditional routing based networks, interference is usually avoided. Overlapping of signals from N_1 and N_3 to N_2 is prohibited in the same time slot. Fig 1.4 shows a possible transmission schedule when using traditional routing scheme. We define F_i as the frame initiated by N_i , and

F_i is a binary sequence which has fixed length. Firstly N_1 sends F_1 to N_2 , and then N_2 relays F_1 to N_3 . After that, N_3 sends F_3 in the reverse direction. The total number of needed time slots for the mutual exchange between N_1 and N_3 is then four.

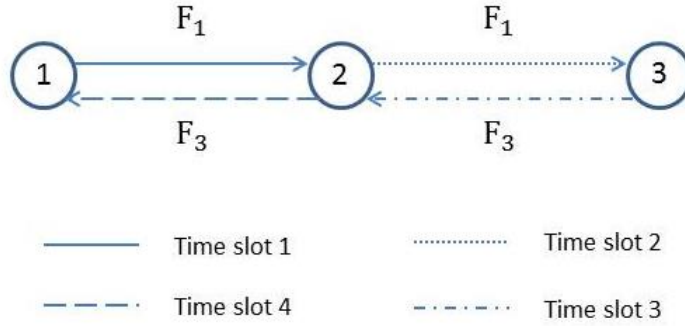


Figure 1.4: Traditional routing scheme

It is clear that the total number of needed time slots is four, and there are 4 transmissions and 4 receptions in total.

Instead of hop-by-hop routing, a network coding scheme exploits coding and broadcasting abilities of wireless nodes. Fig 1.5 illustrates the basic principle of network coding. First, N_1 sends F_1 to N_2 , after that N_3 sends frame F_3 to N_2 . Having received F_1 and F_3 , N_2 encodes frame F_2 as follows:

$$F_2 = F_1 \oplus F_3$$

where \oplus denotes bitwise exclusive OR operation being applied over the entire frames of F_1 and F_3 . Then N_2 broadcasts F_2 to both N_1 and N_3 . When N_1 receives F_2 , it extracts F_3 from F_2 using the local information F_1 :

$$F_1 \oplus F_2 = F_1 \oplus (F_1 \oplus F_3) = F_3$$

N_3 extracts F_1 in a similar way:

$$F_3 \oplus F_2 = F_3 \oplus (F_1 \oplus F_3) = F_1$$

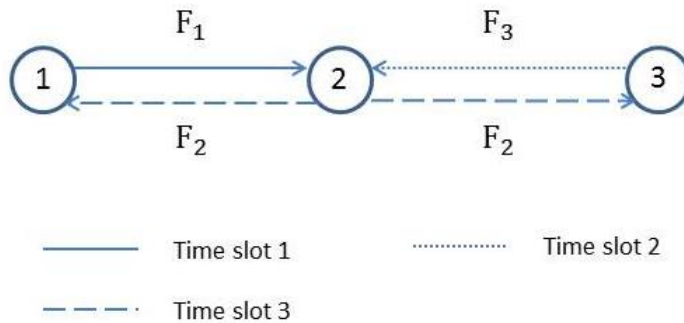


Figure 1.5: Network coding scheme

It is clear that the total number of needed time slots is three, instead of four compared to traditional routing. So the throughput improvement is 33% over the traditional routing scheme.

There are 3 transmissions and 4 receptions in total. Besides the throughput improvement, there is also an improvement in energy consumption since it reduces the number of transmissions by one when applying network coding. Hence, 25% of the transmissions energy has been saved for the system compared to traditional routing at the cost of the energy consumed for computation at N_2 , and N_1 and N_3 also need additional energy to extract useful message.

It has been proved in [6] that the information rate from the source to a set of nodes can reach the minimum of the individual max-flow bounds through coding. This benefit could be illustrated by the butterfly network example in Fig 1.6.

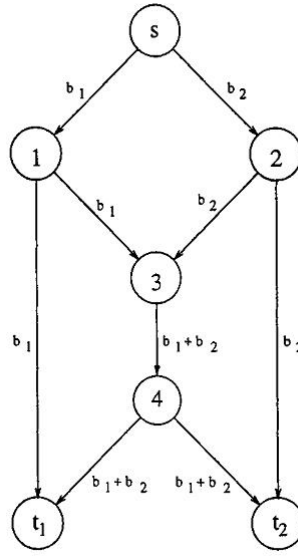


Figure 1.6: A butterfly network which illustrates the benefit using coding&broadcasting.

In this case, we want to multicast two bits b_1 , b_2 from the source S to both the nodes t_1 and t_2 . It shows a conjecture that by using linear network coding combined with wireless broadcasting, we can send b_1 and b_2 simultaneously. Information is coded at the node 3, and then the coded signal is broadcasted by it. At t_1 , b_2 can be recovered from b_1 and $b_1 \oplus b_2$, where \oplus denotes modulo 2 addition. Similarly, b_1 can be recovered at t_2 . This shows the advantage of network coding. In fact, without network coding, it is impossible to multicast two bits per unit time from the source to both the nodes t_1 and t_2 . Note that replication of data can be regarded as a special case of network coding.

1.3. PHYSICAL LAYER NETWORK CODING

Physical layer network coding (PLNC) is a new concept which was proposed in [9]. It is based on the fact that at physical layer of wireless networks, all data are transmitted through electromagnetic (EM) waves and when multiple EM waves come together within the same space, they add together.

The mixing of EM waves is a kind of network coding which is performed by nature. As it is clear that one of the main characteristics of wireless communication is its nature of broadcasting. Transmission of the EM signals from the sender is often received by more than one node, and at the same time, a receiver may be receiving EM signals transmitted by multiple nodes simultaneously. Additions of EM signals can be mapped to Galois field $GF(2)$, so that the interference becomes part of the arithmetic operation in network coding.

In [9], it shows how the concept of network coding can be applied at the physical layer to turn the broadcast property into a capacity-boosting advantage in wireless ad-hoc networks. It is also proved in [10] that by using PLNC, significantly higher throughput than both traditional routing and network coding is achieved, and it also presents a practical design that exploits analog network coding to increase network throughput.

As we already know, the basic idea of PLNC is to exploit the network coding operation that occurs naturally when EM waves are superimposed on one another. The main idea of PLNC is similar to that of network coding, but at the lower physical layer that deals with EM signal reception and modulation. Actually, received signals are mixed by the relay in wireless network without being decoded separately, which is in contrast to the case of network coding, where the relay decodes the packets when receiving them and then combines the decoded bits for transmission in sequence.

Now we use PLNC in TWRC model, and characteristics of both broadcasting and superposition are exploited. We assume that we use QPSK modulation in all nodes. We further assume symbol-level and carrier-phase synchronization, and power control is also used, so that F_1 and F_3 arrive at N_2 with the same phase and amplitude. As illustrated in Fig 1.7, N_1 sends F_1 to N_2 and at the same time N_3 sends frame F_3 to N_2 .

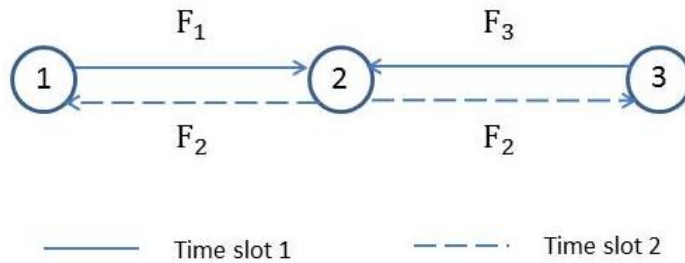


Figure 1.7: Physical layer network coding scheme

Assume the message sent by N_1 on time t is $s_1(t)$, and the message sent by N_3 on time t is $s_3(t)$. We define $r_2(t)$ as the message received by N_2 after time t . The combined bandpass signal received by N_2 during one symbol period is

$$\begin{aligned}
 r_2(t) &= s_1(t) + s_3(t) \\
 &= [a_1 \cos(\omega t) + b_1 \sin(\omega t)] + [a_3 \cos(\omega t) + b_3 \sin(\omega t)] \\
 &= (a_1 + a_3) \cos(\omega t) + (b_1 + b_3) \sin(\omega t)
 \end{aligned}$$

where a_1 and b_1 are the QPSK modulated information bits of N_1 ; a_3 and b_3 are the QPSK modulated information bits of N_3 , and ω is the carrier frequency. N_2 then receives two baseband signals, in-phase (I) and quadrature phase (Q), as follows:

$$I = a_1 + a_3;$$

$$Q = b_1 + b_3.$$

Note that N_2 is only a relay node and cannot extract the individual information transmitted by N_1 and N_3 . Then what we need is a modulation and demodulation mapping scheme, to obtain the summation of bits from N_1 and N_3 at the physical layer in Galois field $GF(2)$.

Recall that a QPSK data stream can be considered as two BPSK data streams: an in-phase stream and a quadrature-phase stream. Fig 1.8 illustrates the idea of the modulation and demodulation mapping for in-phase signals (I). In this figure, s_1 and s_3 are the in-phase data bits from N_1 and N_3 respectively, and a_1 and a_3 are the BPSK modulated bits of s_1 and s_3 respectively.

Modulation mapping at N_1 and N_3				Demodulation mapping at N_2		
				Input	Output	
Input		Output			Modulation mapping at N_2	
					Input	Output
s_1^I	s_3^I	a_1	a_3	$a_1 + a_3$	s_2^I	a_2
1	1	1	1	2	0	-1
1	0	1	-1	0	1	1
0	1	-1	1	0	1	1
0	0	-1	-1	-2	0	-1

Figure 1.8: Modulation and demodulation mapping for in-phase signals (I)

Similar to Fig 1.8, we can use the modulation and demodulation mapping in Fig 1.9 for quadrature-phase signals (Q).

Modulation mapping at N_1 and N_3				Demodulation mapping at N_2		
				Input	Output	
Input		Output			Modulation mapping at N_2	
					Input	Output
s_1^Q	s_3^Q	a_1	a_3	$a_1 + a_3$	s_2^Q	a_2
1	1	1	1	2	0	-1
1	0	1	-1	0	1	1
0	1	-1	1	0	1	1
0	0	-1	-1	-2	0	-1

Figure 1.9: Modulation and demodulation mapping for quadrature-phase signals (Q)

With reference to these two tables, N_2 obtains the information bits:

$$s_2^I = s_1^I + s_3^I$$

$$s_2^Q = s_1^Q + s_3^Q$$

Upon receiving $s_2(t)$, N_1 and N_3 can derive s_2^I and s_2^Q by ordinary QPSK demodulation. Physical layer network coding requires only two time slots for the exchange of two frames between two nodes (as opposed to three time slots in network coding scheme and four time slots in traditional routing scheme). So the throughput improvement is 100% over the traditional routing scheme.

There are 3 transmissions and 3 receptions in total. On the energy aspect, although the number of the transmissions of the system compared to network coding does not decrease, the number of the receptions when applying PLNC decreased by one. As a result, 25% of the receptions energy has been saved for the system compared to traditional routing and to the network coding as well. Note that the energy is also consumed for wireless devices to receive signals, e.g., to amplify the signal and decoding, etc. however they are not taken into account in this situation.

1.4. COMPUTE-AND-FORWARD

In recent years, compute-and-forward has become an important research topic in network information theory. Compute-and-forward, also known as Reliable Physical Layer Network Coding, is a technique which is proposed in 2011 for the application in wireless networks [11]. It presented the idea that by using judiciously chosen linear error-correcting codes, intermediate nodes in a wireless network can directly recover linear combinations of the packets from the observed noisy superpositions of transmitted signals. The main idea is that a relay node will decode a linear function of transmitted codewords in terms of the observed channel coefficients rather than ignoring the interference as noise. Instead of mapping the additions of EM signals to Galois field $GF(2)$, in compute-and-forward scheme, nested lattice codes are applied. Here we will not go into details about the process of compute-and-forward scheme, however, basic introductions about the channel model and nested lattice codes are given in the following subsections.

1.4.1. CHANNEL MODEL FOR COMPUTE-AND-FORWARD

The wireless medium in a wireless networks is of key importance for the propagation properties of signals. We explore compute-and-forward scheme by discussing the channel model of it. First of all, we assume the linear fading channel, which means that the EM signal undergoes a linear transformation between the transmitter and the receiver. Assume the communication is band-limited and flat fading, the received signal at any point in space can be expressed as the convolution of the transmitted signal with an impulse-response function that characterizes the signal propagation. We define the fading as h , which is often modeled as a Gaussian random variable, then the induced signal can be expressed as $hX[t]$, where $X[t]$ is the signal transmitted in time slot t . Note that it is usually assumed that the fading h is known exactly to the receiver.

Now we consider that N transmitters are active simultaneously. Assume each fading coefficient h_n is independent of each other, we could express the induced signal at any point in space as

$$\sum_{n=1}^N h_n X_n[t].$$

When we considering the noise distribution, we assume that it follows the Gaussian law and we describe it as $Z[t]$ in time slot t . Then the received signal at a particular receiver when N transmitters are simultaneously active in time slot t could be expressed as follows

$$Y[t] = \sum_{n=1}^N h_n X_n[t] + Z[t].$$

Till now the channel model for compute-and-forward scheme is defined. As it shows, the element of noise on the physical layer is undesirable for linear network coding. Moreover it will add up along stages of network, thus we need a suitable reliable coding method.

1.4.2. NESTED LATTICE CODES

A lattice Λ is a set of real-valued vectors such that for any two elements $\lambda_1, \lambda_2 \in \Lambda$ we have that

$$\lambda_1 + \lambda_2 \in \Lambda.$$

A simple example of lattice is the set of all integers \mathbb{Z} . If a lattice Λ is a subset of another lattice Λ_{FINE} , then Λ is said to be nested in Λ_{FINE} . Λ is usually referred to as the *coarse lattice*, and Λ_{FINE} as the *fine lattice*.

In [12], it presents the idea that upon utilizing the algebraic structure of lattice codes, i.e. the integer combination of lattice codewords is still a codeword as well, the intermediate relay node decodes and forwards an integer combination of original codewords. Receiving enough linear equations of the original transmitted codewords, the destination could decode the desired codewords successfully.

When we use the capacity-achieving nested lattice codes developed by Erez and Zamir[13] and lattice decoding, it has been demonstrated that lattice codes can achieve the additive white Gaussian noise (AWGN) channel capacity, by minimum-mean-square-error-scaled (MMSE-scaled) transformation of the AWGN into a modulo additive noise channel.

Fig 1.10 shows how the nested lattice looks like.

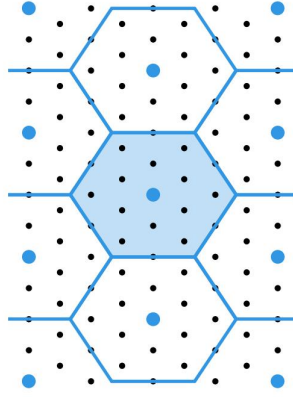


Figure 1.10: An example of nested lattices

The process to decode the modulo sum of the original message is illustrated in Fig 1.11. Each transmitter maps its finite-field message into an element of the nested lattice code and sends this vector on the channel, and all transmitters pick the same nested lattice code. Here, the channel coefficients are taken to be equal: $h_1 = h_2 = 1$. Therefore, the receiver observes a noisy sum of the transmitted vectors and determines the closest lattice point. After taking a modulo operation with respect to the coarse lattice, the receiver can invert the mapping and determine the modulo sum of the original messages. Note that the sum of the two vectors exceeds the boundary of the original nested lattice code. However, by decoding to the closest fine lattice point and then taking the modulo operation, the modulo sum of the codewords can be recovered.

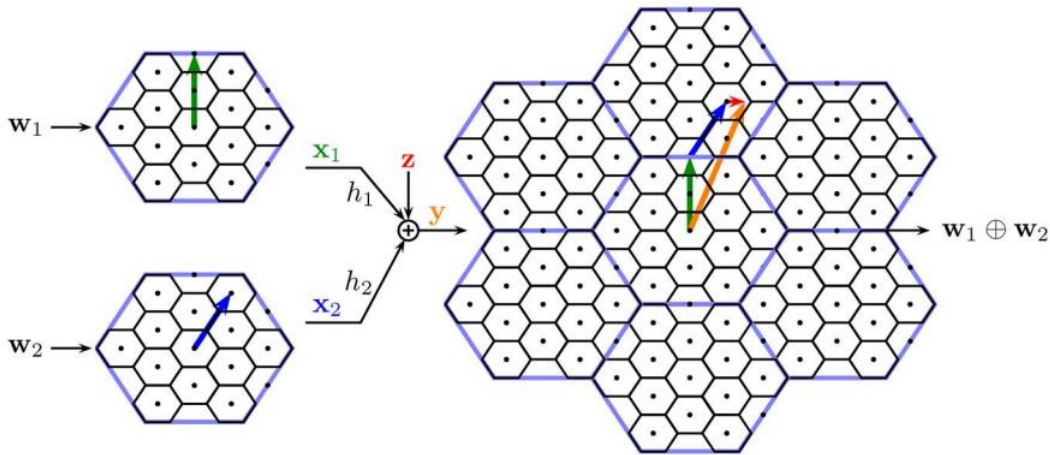


Figure 1.11: Recovery of the modulo sum of two original codewords in nested lattices

In [14], it proposed a scheme which used joint physical-layer network-layer code. It has been proved in [14] that in a symmetric TWRC over synchronized, average power constrained AWGN channels with a real input with SNR ratio, by using lattice codes and lattice decoding we can obtain a rate of

$$R_{lattice} = \frac{1}{2} \log\left(\frac{1}{2} + SNR\right)$$

bits per transmitter, which is essentially optimal at high SNR. This rate nearly matches the upper bound, except for a missing $1/2$ inside the logarithm, of the full capacity of the original power constrained AWGN channel,

$$R = \frac{1}{2} \log(1 + SNR)$$

The system model of the compute-and-forward scheme for a TWRC is depicted in Fig 1.12. The complete information exchange between the users is achieved in two time slots. In the first time slot, two source nodes N_1 and N_3 transmit simultaneously to the relay node N_2 . When N_2 receives the superimposed signal from the two source nodes, it computes the corresponding network codewords. In the second time slot, N_2 broadcasts the computed codewords back to N_1 and N_3 . After N_1 and N_3 receive the network codewords from the relay, both of them can retrieve each other's codeword by canceling its own codeword [15].

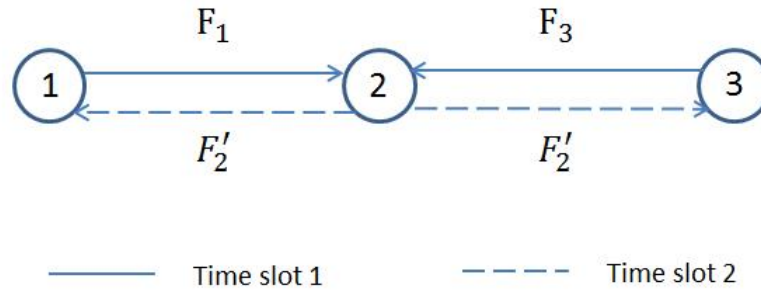


Figure 1.12: Compute-and-forward scheme

We then conclude from Fig 1.12 and the explanation above that the complete information exchange between the two users is accomplished in two time slots, which is the same as PLNC. So the throughput improvement is 100% over the traditional routing scheme when using compute-and-forward if the environment is noiseless, the same as PLNC.

There are 3 transmissions and 3 receptions in total, the same as PLNC. As a result, 25% of the receptions energy has been saved for the system compared to traditional routing and to the network coding as well, at the cost of the computation energy consumed at N_2 .

Note that the main difference between compute-and-forward scheme and PLNC is that when applying compute-and-forward, the noise of signals is removed by using the lattice codes. If we assume the capacity of each node is one symbol per time slot, when considering about the influence of noise, the capacity of each node may be smaller than one in PLNC scheme when decoding mistakes

are caused by noise, which means that more than one time slot are needed to decode the right symbol. However this capacity is always equal to one in compute-and-forward scheme because of its noise-removing property when decoding. Thus the rate compared to PLNC scheme is increased in such situations.

1.5. PROBLEM STATEMENT

More applications in wireless networks are emerging, e.g. ad hoc networks which could monitor environment in rural areas. There is a requirement for more resources with them. One of the most important limitations is the battery life of wireless devices. However nowadays battery technology is not developing so fast as the increasing demand from resource-consuming applications, thus there is a necessary that we should use the available energy more efficiently. More attentions are given to the subject of minimizing energy consumption in networks.

The benefits which are brought by compute-and-forward are of great interest for us. Actually, the throughput gain when using compute-and-forward in some specific network layouts has been already proved. By using compute-and-forward, the broadcast and superposition properties of wireless networks are turned into advantageous characteristics for achieving higher transmission rates and increasing the network throughput. Moreover, applying compute-and-forward in wireless networks makes a decrease in the number of transmissions and receptions, which brings great energy saving.

There are already many research works done in the subject of compute-and-forward. When a symbol is communicated successfully for every session, we call the ratio between the minimum energy consumption in the network when it is in traditional routing mode, and the minimum total energy consumption in the network when it is in compute-and-forward mode, as the energy benefit, and more precise definition of energy benefit and these two transmission modes will be given in Chapter 3. However, the maximum energy benefit of applying compute-and-forward on a general network with arbitrary placement for multiple unicast sessions is still an open problem. Till now, it is unclear whether the energy benefit a constant factor, or it is upper bounded by a factor which is related to the number of unicast sessions, just like what is already proved in [16] about the throughput gain of compute-and-forward. Moreover, if an upper bound is derived, it is also an interesting problem that whether there exist such a network with specific session placement, or a corresponding scheme, so that the energy benefit on such a network is upper bound approaching.

In this thesis, we focus on the energy benefit of compute-and-forward applying on a general network, and following problems will be studied:

- What is the maximum energy benefit of applying compute-and-forward on a general wireless network with arbitrary placement for multiple unicast sessions?
- What are the relations between the energy benefit and the properties of the network, e.g., number of unicast sessions?
- What is the maximum energy benefit of applying compute-and-forward on some special networks, e.g. line networks, rectangular networks?

1.6. STRUCTURE OF THE THESIS

The remainder of the thesis is organized as follows. In Chapter 2, state-of-the-art of both network coding and compute-and-forward are introduced. It also presents our contributions to the research work. In Section 2.1, we introduce the related results about the benefit of network coding. Recent results about gains applying compute-and-forward are introduced in Section 2.2. In Section 2.3, contributions that we made to the research is given.

In Chapter 3 we describe the network model, notations and different transmission modes used in this thesis. In Section 3.1, we define the wireless network with nodes, edges and sessions. We also introduce how the connectivity graph is built in an arbitrary network and what the traffic pattern is like. In Section 3.2, we define traditional routing mode and compute-and-forward mode which we will use in our networks. In Section 3.3, we focus on the energy consumptions and give definition of the energy benefit.

In Chapter 4, two bounds of energy benefit are presented and proved in general networks. In Section 4.1, a fundamental upper bound of energy benefit is derived which is related to the average distance. In Section 4.2, an advanced upper bound of the energy benefit is given by considering the distance from every source node to the set of destination nodes, and the distance from every destination node to the set of source nodes.

In Chapter 5 energy benefit in some special networks will be studied and discussed. In Section 5.1, energy benefit is derived in networks with non-collocated source nodes and destination nodes. In Section 5.2, we consider two situations where compute-and-forward scheme provides no benefit. In Section 5.3, we study the energy saving of compute-and-forward scheme in line networks. In Section 5.4 the energy benefit is studied in another special case of rectangular lattice networks, while both 2D and 3D cases will be studied.

In Chapter 6, we conclude this thesis by listing important outcomes in Section 6.1 and by answering questions which we have proposed in Section 1.5. In Section 6.2, we will show some possible influence that our results may have on real life. Suggestions for the following researchers who are interested in further investigation of this subject will be given in the last section.

2

STATE-OF-THE-ART AND OUR CONTRIBUTIONS

A lot of research has been done on the benefit of throughput and energy both for network coding and compute-and-forward. In Section 2.1, we introduce the related results about the benefit of network coding. Recent results about gains applying compute-and-forward are introduced in Section 2.2. In Section 2.3, contributions that we made to the research are introduced.

2.1. BENEFIT OF NETWORK CODING

Network coding is sometimes referred as *plain network coding* to distinguish from *physical layer network coding*. For wireless networks, network coding, combined with wireless broadcasting, has been proved to be beneficial in the performance on throughput efficiency. Before we focus on the throughput gain and energy saving of network coding, we introduce two different channel models in wireless networks with network coding.

In the network coding mode, there are two widely used channel models for successful reception of a transmission over one hop which are the *Protocol Model* and the *Physical Model* [2][17]. Under the Protocol model, a transmission is successfully received by a node if the source node is in the transmission range, and all other nodes in the network have distances from it greater than $(1 + \Delta)r$. Here, $r > 0$ is defined as the *transmission range* and $\Delta > 0$ is defined as the *interference parameter*. Under the Physical model a transmission is modeled as successful if the signal-to-interference-plus-noise ratio (SINR) is above a threshold.

2.1.1. BENEFIT OF NETWORK CODING IN TERMS OF THROUGHPUT

Multi-hop wireless networks have been intensively studied in recent years for both commercial and government applications. Such networks, static or mobile, have the potential to serve as either a self-contained network that provides communication without the presence of an established infrastructure, or as a bridge between end users and the high speed wired infrastructure. Two representative applications are wireless sensor networks and wireless mesh networks. Multi-hop wireless sensor networks can be deployed randomly in geographic regions to collect large volume of environment data and provide distributed query services. Wireless mesh networks can be potentially deployed in the streets of big cities, campuses, conference centers, etc. Hence, issues of the connectivity and capacity of such networks are of interest.

Even though these self-contained wireless mesh networks alone may not be enough to sustain all of the communication among users, they probably will be mandatory for supporting the last several hops to the end users, and thus serve as a glue between the end users at any corner and the wired infrastructure. For either case, one major concern with such wireless networks is scalability. Under a traditional communication model without network coding, it shows in [2] that as the total number of nodes increases, the many to many throughput decreases polynomially.

For the wired case, the benefit of network coding in terms of throughput and capacity is often quite limited. Specifically, for networks with bidirectional links that can be modeled as an arbitrary undirected graph, [18] shows that the throughput improvement is upper bounded by a factor of 2 for the single multicast case, and upper bounded by one, which means no benefit at all for the single unicast or broadcast case. Moreover it is conjectured that there is no throughput benefit of network coding for the multiple unicast case; this is called the *Li&Li* conjecture, which is still open with no counter-examples found yet.

In [19] they establish fundamental limitations to the benefit of network coding in terms of energy and throughput on two popular network scenarios: single multicast session and multiple unicast sessions. They prove the benefit of network coding in terms of throughput or energy saving is bounded by a constant factor for a single multicast session. In addition, they prove that network coding can increase the transport capacity of an arbitrary wireless network by at most a factor of π .

In [20], it studies the benefit of network coding and broadcasting on the many to many throughput of wireless networks under the framework proposed by Gupta and Kumar[2]. It shows that the benefit is upper bounded by a constant both for the protocol model and the physical model. Further, they develop bounds for these constants.

In an error-prone network, it proposes in [21] some network coding schemes which increase the bandwidth efficiency of reliable broadcast in a wireless network by reducing the number of broadcast transmissions from one sender to multiple receivers. The main idea is to allow the sender to combine and retransmit the lost packets in a certain way so that with one transmission, multiple receivers are able to recover their own lost packets. Both simulations and theoretical analysis confirm the advantages of the proposed network coding schemes over the automatic repeat-request (ARQ) ones.

In [22], it presents COPE, a new forwarding architecture which was inspired by the theory of network coding. It also proves that this new architecture substantially improves the throughput of stationary wireless mesh networks. They evaluate the design on a 20-node wireless network, and discuss the results of the first testbed deployment of wireless network coding. The results show that using COPE at the forwarding layer, without modifying routing and higher layers, increases network throughput. The gains vary from a few percent to several folds depending on the traffic pattern, congestion level, and transport protocol.

It has been proved in [6] that the information rate from the source to a set of nodes can reach the minimum of the individual max-flow bounds through coding. Moreover in [23] it is proved that by linear coding alone, the rate at which a message reaches each node can achieve the individual max-flow bound, which is somewhat stronger than the one in [6].

In [20], for random networks of any dimension under either the protocol or physical model that were introduced by Gupta and Kumar[2], it shows that the throughput benefit per node of network coding and broadcasting is upper bounded by a constant factor. The results indicate the difficulty in improving the scaling behavior of wireless networks without modification of the physical layer. In the multiple unicast scenario for the protocol model, the bounds on the constant is specifically conjectured to be 2. It is because network coding can only potentially improve the outgoing information rate from a node, while the incoming information rate is still constrained as previously.

2.1.2. BENEFIT OF NETWORK CODING IN TERMS OF ENERGY SAVING

There is also much work done in the aspect of energy saving in wireless networks when using network coding. In [24], a simple network coding strategy: *XOR* operations are used at each node in the hexagonal lattice with multiple unicast sessions, as shown in Fig 2.1. It is shown by simulations that this algorithm reduces the power consumption significantly for multiple unicasts on a wireless triangular grid, by reducing the number of transmissions and the corresponding power consumption.

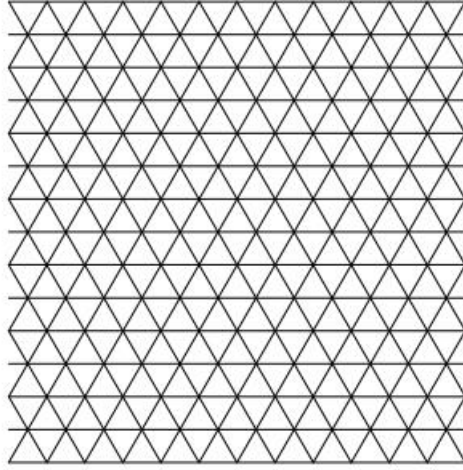


Figure 2.1: The nodes of the network lie on the vertices of a triangular lattice

In [25], it studies the energy savings that can be obtained by employing network coding instead of plain routing in wireless multiple unicast problems, and it provides lower bounds on the energy benefit of network coding for wireless multiple unicast. The energy benefit is defined as the ratio of the minimum energy consumption of routing solutions and the minimum energy consumption of network coding solutions, maximized over all node locations, multiple unicast sessions, and transmission ranges. It is proved that if coding and routing solutions are using the same transmission range, the benefit is at least $2d/\lfloor\sqrt{d}\rfloor$, where d represents that the network is a d -dimensional network. Moreover, it is shown that if the transmission range can be optimized for routing and coding individually, the benefit in 2-dimensional networks is at least 3.

As is mentioned in subsection 2.1.1, it is proved in [19] that the benefit of network coding in terms of throughput or energy saving is bounded by a constant factor for a single multicast session. Moreover in the situation of sensor networks where the sensors gather independent information for the sink, it is proved that network coding has no benefit in terms of energy.

In [26], it analyses the energy consumption of several network coding solutions to wireless multiple unicast problems. The energy gain over traditional routing when applying network coding is analyzed, with the energy of transmission and reception both taken into account. It has been demonstrated that under this model the benefit of using these coding solutions can be significantly different from results reported in the literature based on models that include only the energy emitted while transmitting. Moreover, it has been shown that by increasing the transmission power, it is possible to reduce the overall energy consumption in the network since more coding opportunities are created.

2.2. BENEFIT OF COMPUTE-AND-FORWARD

When compute-and-forward is applied in wireless networks, benefit over traditional routing is realized both in terms of throughput and energy saving. Wireless networks with different layouts have been studied on the throughput and energy benefit in some researches.

2.2.1. BENEFIT OF COMPUTE-AND-FORWARD IN TERMS OF THROUGHPUT

Benefit in terms of throughput when applying compute-and-forward is studied in some researches. In [29], it defines common rate as the long-term rate at which all sessions communicate. When compute-and-forward is applied in a wireless line networks with multiple bidirectional sessions, the common rate is improved by a factor close to 2 in most cases from the modulation results. Moreover, scheduling and coding schemes achieving these rates are presented.

In [27], the analysis has shown that the compute-and-forward scheme can significantly improve the throughput capacity. In [16], it defines throughput gain as the ratio of maximum achievable common rate in compute-and-forward mode and traditional mode respectively. It proves that the throughput gain for networks characterized by local interference and half-duplex constraints is upper bounded by $3K$, where K is the number of unicast sessions. Furthermore, a class of networks is also presented for which an improvement by a factor of $K/2$ is feasible by applying compute-and-forward. Hence it proved that the throughput gain of compute-and-forward is at most on the order of K for any network, and a gain in that order is indeed achievable for some networks.

In [28], it develops a multiple-antenna extension of the lattice-based compute-and-forward strategy for AWGN networks. It applies the framework to the multiple antenna TWRC and demonstrates improved performance, e.g. average rate, over traditional strategies.

In [30], it develops a coding scheme that enables relays to reliably recover equations of the original messages by exploiting the interference structure of the wireless channel. It shows that this framework can achieve end-to-end rates across an AWGN network that are not accessible with classical relaying strategies.

The problem of integer network coding coefficients design in a system level over a compute-and-forward multi-source multi-relay system is studied in [31]. By the proposed algorithms in it, the transmission rate of the multi-source multi-relay system is maximized. It also shows the effectiveness of the proposed algorithms over other strategies by simulation results.

2.2.2. BENEFIT OF COMPUTE-AND-FORWARD IN TERMS OF ENERGY SAVING

Compared to network coding, there are not so many researches been done on the problems about energy saving when applying compute-and-forward schemes. However there are some related works about computation coding, which is quite similar to the principle of compute-and-forward. It shows great potential on energy saving of compute-and-forward schemes. Over noisy multiple access channels, [32] introduces a technique called computation coding which allows many nodes to simultaneously and reliably compute their average at once within a neighborhood. It claims that if the size of the collaboration neighborhood is larger than a critical value that depends on the path loss exponent and the network size, interference can yield exponential benefits in the energy required to compute the average.

In [33], it gets the energy benefit of applying compute-and-forward on a wireless hexagonal lattice network with multiple unicast sessions with a specific session placement. Moreover, two compute-and-forward based transmission schemes are proposed, which allow the relays to exploit both the

broadcast and superposition properties of the wireless network. Note that the energy consumption of both transmission and reception of nodes are taken into account. Fig 2.2 shows the specific placements for nodes and sessions in an example of hexagonal lattice network model.

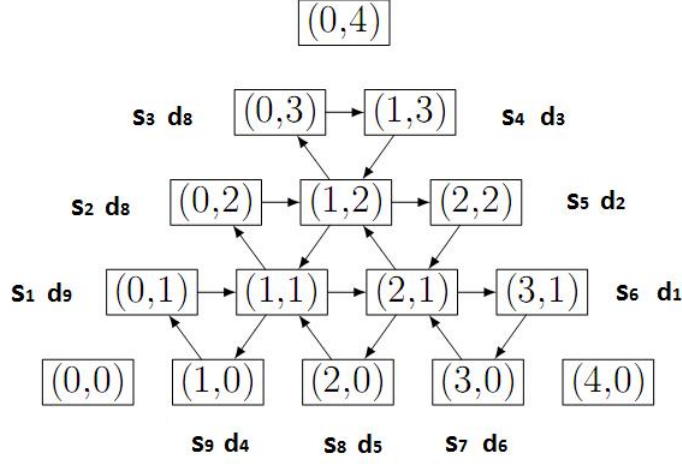


Figure 2.2: Placements for nodes and sessions in the hexagonal lattice network model

Note that in the model above, there are 9 sessions in total, and each node is defined by an index tuple which also indicates its position. s_i and d_i represent source and destination nodes for session i respectively. In this case, it claims that the energy consumption in the network can be saved by at least a factor of 1.5 using compute-and-forward compared to traditional routing.

2.3. OUR CONTRIBUTIONS

Compared to wired networks, in wireless channels energy efficiency represents an important constraint to consider, since wireless terminals use chargeable batteries and have to consume as little as possible power in order to be reliable enough. Network coding has been proved to be beneficial to a wireless network in the aspect of energy saving [19]. However, the maximum energy benefit of applying compute-and-forward on a general network with random session placement is still an open problem. It is still unclear whether the energy benefit can also be upper bounded by the number of unicast sessions, or it is upper bounded by a constant depending on the properties of the network. If an upper bound is derived, it is also an interesting problem that whether there exist such networks and schemes, so that the energy benefit on such networks is upper bound approaching.

Our contributions in this thesis are that we investigate the maximum energy benefit of compute-and-forward applying on general networks. The upper bound of the energy benefit is derived by proving the upper bound of the minimum energy consumption when applying a traditional routing scheme, and the lower bound of the minimum energy consumption when applying compute-and-forward.

We conclude that the benefit of energy consumption when applying compute-and-forward is upper bounded by the average distance of the network. It is also upper bounded by the maximum distance between each source node to the destination set, and the maximum distance between the source

set and each destination node. We also present the idea that when applying compute-and-forward in a network does not make any benefit in single source network and in single destination network. We give upper bounds on some special networks as well, e.g. networks with non-collated source nodes and destination nodes, line networks, 2D/3D rectangular lattice networks.

3

MODEL SET-UP

In this chapter, we describe the network model, notations and different modes used in this thesis. A communication network is a collection of links connecting communication devices (nodes). We consider arbitrary wireless networks constructed with a connected graph and multiple unicast sessions in our model, and we neglect the geometric properties of the wireless networks and only focus on the two key features of wireless networks: broadcast and superposition.

In Section 3.1, we define the wireless network with nodes, edges and sessions. We also introduce how the connectivity graph is built in an arbitrary network and what the traffic pattern is like. In Section 3.2, we define traditional routing mode and compute-and-forward mode which we will use in our networks. In Section 3.3, we give definitions of energy consumptions in traditional routing mode and compute-and-forward mode, and the definition of energy benefit respectively.

3.1. NETWORK MODEL AND TRAFFIC PATTERN

We consider an arbitrarily undirected and connected graph $G(V, E)$, with nodes $V = \{1, 2, \dots, N\}$ and edges $E = \{(u, v) \mid u, v \in V\}$. All configurations considered in this thesis are multiple unicast, i.e., there are several sessions in which a single source is communicating to a single destination. We assume that time is slotted and the capacity of each edge is one symbol per time slot. We define the symbol σ as a silent symbol, which represents no transmission and no reception. The transmitted symbol is from $GF(q) \cup \{\sigma\}$ where q is a prime power. Let $X_t(u)$ and $Y_t(u)$ represent the transmitted and received symbols by node u in time slot t , respectively. We assume half-duplex constraints, i.e. each node cannot both transmit and receive in the same time slot. Moreover, we define the transmit state as a node is only transmitting without receiving, and the receive state as a node is only receiving without transmitting. Moreover when node u and node v are called neighbors, it means that there is an edge between them.

We consider multiple unicast traffic in this thesis. The number of unicast sessions in the graph is K . In each session, information needs to be transmitted from a source to a destination, possibly via relays. We denote session i with $i \in \{1, 2, \dots, K\}$ by $S_i = (a_i, b_i)$, where $a_i, b_i \in V$ and $a_i \neq b_i$ are the source node and destination node respectively. We define the set of source nodes as $A = \{a_1, a_2, \dots, a_K\}$ and the set of destination nodes as $B = \{b_1, b_2, \dots, b_K\}$. The set of sessions is $S = \{S_1, S_2, \dots, S_K\}$.

Now define the function $d(i, j)$ as the minimum number of hops between two nodes i and j . For a certain session, we define the notation $d_i = d(a_i, b_i)$ as the minimum hop-count distance between

the source node a_i and the destination node b_i . Also, we define the distance of b_i from a set of nodes A as $d(A, b_i) = \min_{a_j \in A} \{d(a_j, b_i)\}$, and the distance of a_i from a set of nodes B as $d(a_i, B) = \min_{b_j \in B} \{d(a_i, b_j)\}$.

The communication network is defined by the undirected and connected graph $G(V, E)$ and the sessions S . Also we consider the network model without any noise. Thus the model of the network can be written as $N(G(V, E), S)$.

Example 1. We give a simple example of wireless networks in Fig 3.1, in which there are 3 sessions.

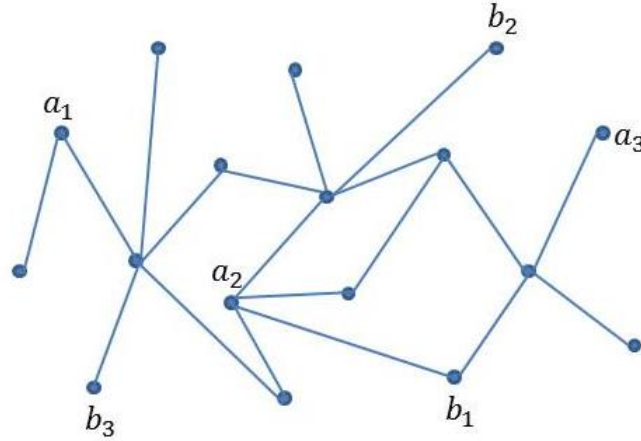


Figure 3.1: A wireless network model $N(G(V, E), S)$ with 3 sessions.

In the example network shown above, as we already defined, $S_1 = (a_1, b_1)$, $S_2 = (a_2, b_2)$, and $S_3 = (a_3, b_3)$. Minimum number of hops for these sessions are: $d_1 = 4$, $d_2 = 2$ and $d_3 = 6$. Moreover, $d(A, b_1) = 1$, $d(A, b_2) = 2$ and $d(A, b_3) = 2$; $d(a_1, B) = 2$, $d(a_2, B) = 1$, and $d(a_3, B) = 2$.

In order to study the improvement of compute-and-forward in such a network, we consider that the underlying physical and MAC layer can work in two different transmission modes. The two modes, which are traditional routing mode and compute-and-forward mode respectively, will be described in details in the following section.

3.2. TRANSMISSION MODES

We now define two transmission modes in the following two subsections, traditional routing mode and compute-and-forward mode respectively. In traditional routing mode, we assume that traditional routing schemes and MAC protocols are used, hence the broadcast and superposition properties are not exploited. In compute-and-forward mode, since by network coding, the transmitted message can be useful for multiple sessions and by compute-and-forward, it allows a node to directly retrieve a linear combination of all messages transmitted by its neighbors, we assume that both broadcast and superposition features can be exploited in this mode.

3.2.1. TRADITIONAL ROUTING MODE

In traditional routing mode, the network works based on a point-to-point communication scheme. Properties of broadcast and superposition are not exploited in traditional routing mode. It means that a source node u which is in the transmit state can transmit a symbol to only one neighbor v . A successful transmission of u and reception of v in time slot t can be realized only when other neighbors of v are silent in order to avoid collisions or interference, i.e.,

$$X_t(u') = \sigma, \forall u' \neq u : (u', v) \in E$$

3.2.2. COMPUTE-AND-FORWARD MODE

In compute-and-forward mode, both superposition and broadcast properties of wireless networks are exploited. Thus a node could efficiently and reliably recover a function of the messages from multiple senders, which means that a node could be receiving messages from several different neighbor nodes.

Note that broadcast channel means that a transmission will typically be received by several neighbors simultaneously, and superposition (or multi-access) means that simultaneous receptions from different nodes is feasible. Hence, for a node v which is in receive state in time slot t , the received symbol is

$$Y_t(v) = \sum_u X_t(u),$$

where $\sum_u X_t(u)$ is the summation of symbols from all non-silent neighbors u of v .

3.3. ENERGY CONSUMPTION MODEL

We will compare the energy consumption of traditional routing mode and compute-and-forward mode in this thesis. The energy consumption of these two transmission modes is comparable because they are essentially describing two different transmission schemes for the same model.

In [33], an energy consumption model is used that includes both the energy consumption for transmitting data and the energy consumption for receiving data. The energy consumed when receiving consists of, for instance, the energy consumed by supporting circuitry. This model is useful if the reception energy consumption cannot be neglected when compared to the transmission energy consumption. In this thesis, we study a similar energy consumption model as follows: in each time slot, a symbol from $GF(q)$ transmitted by node v can be successfully received by node u if u is a neighbor of v , and v transmits with energy consumption e_t and u receives with energy consumption e_r .

We give the definitions of P^{TR} and P^{CF} in network $N(G(V, E), S)$ as follows: P^{TR} is defined as the minimum total energy consumption in $N(G(V, E), S)$ when a symbol is communicated successfully for every session in S when the network is in traditional routing mode, and P^{CF} is defined as the minimum total energy consumption in $N(G(V, E), S)$ when a symbol is communicated successfully for every session in S when the network is in compute-and-forward mode. We only consider transmission energy consumption and reception energy consumption of nodes, and we neglect all the other energy consumptions during the process of communication, e.g., computation energy, synchronization energy, etc.

At last, we define the energy benefit I as the ratio of P^{TR} and P^{CF} ,

$$I = \frac{P^{TR}}{P^{CF}}.$$

4

ENERGY BENEFIT OF GENERAL NETWORKS

In this chapter, when applying compute-and-forward on general networks with multiple unicast sessions, an upper bound of energy benefit based on average distance of all sessions and another advanced upper bound which is also distance-based will be given and proved. In Section 4.1, the bound of energy benefit is determined by the average distance. In Section 4.2, another upper bound of the energy benefit is derived by giving a tighter lower bound of P^{CF} which is determined by the maximum distance from each source node to the set of destination nodes, and the maximum distance between each destination node from the set of source nodes.

4.1. FUNDAMENTAL DISTANCE-BASED BOUND OF ENERGY BENEFIT

In this section, the upper bound of the energy benefit in terms of average distance of all sessions is introduced. Before presenting Theorem 1, we give the lemma below which is about the energy consumption P^{TR} when traditional routing scheme is applied.

Lemma 1. *For network $N(G(V, E), S)$, the energy consumption when using traditional scheme is upper bounded by*

$$P^{TR} \leq (e_t + e_r) \sum_{i=1}^K d_i = (e_t + e_r) K \bar{d}$$

Proof. Now we propose a scheme which has an energy consumption of $(e_t + e_r) \sum_{i=1}^K d_i$. In this scheme, we simply let all sessions send their messages along their shortest paths respectively. For any session S_i , we define all nodes except a_i and b_i along the shortest path as relay nodes. For example, for session S_i , the message is sent by a_i and forwarded by all relay nodes along the shortest path, and finally received by b_i . We define the total energy consumption for this process as P_i , thus

$$P_i = d_i(e_t + e_r).$$

For all the K sessions, the energy consumption for the whole network is

$$(e_t + e_r) \sum_{i=1}^K d_i.$$

Since now we have proposed a scheme in traditional routing mode with energy consumption $(e_t + e_r) \sum_{i=1}^K d_i$, by the definition of P^{TR} , we have

$$P^{TR} \leq (e_t + e_r) \sum_{i=1}^K d_i.$$

As it is clear that

$$\sum_{i=1}^K d_i = K\bar{d},$$

So we come to the conclusion that

$$P^{TR} \leq (e_t + e_r) \sum_{i=1}^K d_i = (e_t + e_r) K\bar{d}.$$

□

Now we present Theorem 1 and give the proof of it as follows.

Theorem 1. *For network $N(G(V, E), S)$ with multiple unicast sessions, the energy benefit is upper bounded by*

$$I \leq \bar{d}$$

where \bar{d} is the average distance of all sessions.

Proof. The upper bound on the energy benefit of compute-and-forward can be derived by proving the upper bound of energy consumption when applying traditional routing scheme and the lower bound of energy consumption when applying compute-and-forward scheme. From lemma 1 we have the upper bound of energy consumption P^{TR} when applying traditional routing scheme

$$P^{TR} \leq K\bar{d}(e_t + e_r).$$

When applying compute-and-forward scheme and max-flow min-cut theory here, all source nodes need to transmit at least K times and all destination nodes need to transmit at least K times, thus we get that the lower bound of P^{CF}

$$P^{CF} \geq K(e_t + e_r).$$

Now we get an upper bound of the energy benefit when compute-and-forward scheme is applied as follows,

$$I = \frac{P^{TR}}{P^{CF}} \leq \frac{K\bar{d}(e_t + e_r)}{K(e_t + e_r)} = \bar{d}.$$

□

4.2. ADVANCED DISTANCE-BASED BOUND OF ENERGY BENEFIT

In this section, we derive an advanced upper bound of the energy benefit which is also distance-based. We present Theorem 2 and give the proof of it. This theorem provides a tighter upper bound of the energy benefit in a general network with multiple unicast sessions, and it is determined simply by considering the distance from every source node to the set of destination nodes, and the distance from every destination node to the set of source nodes. The proof of this theorem is similar to the proof of [19, Theorem 3].

Theorem 2. For a network $N(G(V, E), S)$ with K sessions, the energy benefit is upper bounded by

$$I \leq \frac{K\bar{d}}{\max[\sum_{j=1}^K d(a_j, B), \sum_{j=1}^K d(A, b_j)]}.$$

where a_j and b_j are source and destination nodes for session j , and A and B are the set of source nodes and set of destination nodes respectively.

Proof. Now in a network $N(G(V, E), S)$, we group the nodes of the network in terms of their distance from the subset B . We define $\tilde{H}_0 = B$ and $\tilde{H}_i = \{u \in V : d(u, B) \leq i\}$. Also we define $\tilde{G}_0 = \tilde{H}_0$ and $\tilde{G}_i = \tilde{H}_i \setminus \tilde{H}_{i-1}$. It is easy to show that the nodes of \tilde{G}_i are only connected to the nodes of \tilde{G}_{i-1} , \tilde{G}_i , \tilde{G}_{i+1} . Then for any $i \in \{0, 1, 2, \dots, \infty\}$ we define q_i as the number of source nodes which are not in \tilde{H}_i ,

$$q_i(B) = |\{a_j \mid d(a_j, B) > i\}|$$

Example 2. Here we show an example in Fig 4.1 to illustrate the way of grouping all nodes in terms of their distances from the set of destination nodes in a network $N(G(V, E), S)$ with 6 sessions.

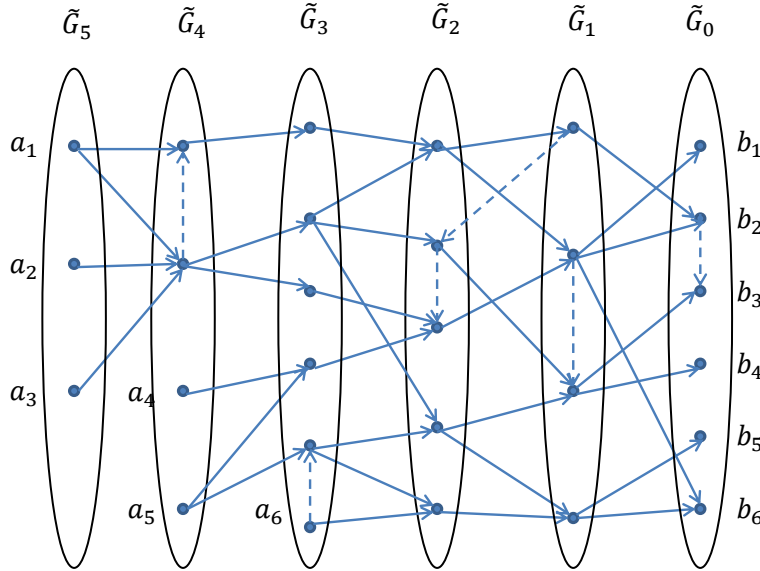


Figure 4.1: Grouping the nodes in terms of their distances from the set of destination nodes.

In this network, there are 6 sessions, and we have $q_0 = q_1 = q_2 = 6$, $q_3 = 5$, $q_4 = 3$ and $q_5 = 0$. Then $\sum_{i=0}^5 q_i = 6 \times 3 + 5 + 3 + 0 = 26$.

The cutset between $V \setminus \tilde{H}_i$ to \tilde{H}_i is the set of directed links from \tilde{G}_{i+1} to \tilde{G}_i . Then if successful transmission in each session is realized, the number of transmissions from $V \setminus \tilde{H}_i$ to \tilde{H}_i is at least $q_i(B)$.

Therefore, when a symbol is transmitted and received successfully in each session of a general network, the total expected number of transmissions in all session is at least $\sum_{i=0}^{\infty} q_i$, as well as the number of receptions. Then we have

$$\begin{aligned} P^{CF} &\geq (e_t + e_r) \sum_{i=0}^{\infty} q_i \\ &= (e_t + e_r) \sum_{i=0}^{\infty} |\{a_j \mid d(a_j, B) > i\}| \\ &= (e_t + e_r) \sum_{j=1}^K d(a_j, B) \end{aligned}$$

Last two steps are derived by simply counting and comparing, and it is easy to find that these two values below are equal,

$$\sum_{i=0}^{\infty} |\{a_j \mid d(a_j, B) > i\}| = \sum_{j=1}^K d(a_j, B).$$

Similarly, by grouping the nodes in terms of distance from the set of sources and use the same method we can get that

$$P^{CF} \geq (e_t + e_r) \sum_{j=1}^K d(A, b_j).$$

Then we come to the conclusion that the energy consumption when applying compute-and-forward is lower bounded by the larger one of these two lower bounds just presented above,

$$P^{CF} \geq (e_t + e_r) \max[\sum_{j=1}^K d(a_j, B), \sum_{j=1}^K d(A, b_j)].$$

From Lemma 1 we have the upper bound of energy consumption P^{TR} when applying traditional routing scheme

$$P^{TR} \leq (e_t + e_r) K \bar{d}$$

So the energy benefit is upper bounded by

$$\begin{aligned} I &\leq \frac{(e_t + e_r) K \bar{d}}{(e_t + e_r) \max[\sum_{j=1}^K d(a_j, B), \sum_{j=1}^K d(A, b_j)]} \\ &= \frac{K \bar{d}}{\max[\sum_{j=1}^K d(a_j, B), \sum_{j=1}^K d(A, b_j)]} \end{aligned}$$

□

Thus, by considering the maximum distance from each source node to the set of destination nodes, and the maximum distance from each destination node to the set of source nodes, together with the average distance of all sessions, can we get a tighter upper bound of energy benefit in the network.

5

ENERGY BENEFIT OF SPECIAL NETWORKS

In this chapter, energy benefit in some special networks will be studied and discussed. In Section 5.1, energy benefit is derived in networks with non-located source nodes and destination nodes. In this case, we present Theorem 3 and give the proof of it. In Section 5.2, we consider two situations where compute-and-forward scheme provides no benefit. We will present a theorem about these two cases and give the proof of it. In Section 5.3, we consider the energy saving of compute-and-forward scheme in special case of line network. In Section 5.4 the energy benefit is considered in another special case, rectangular lattice networks, and both 2D and 3D cases are studied here.

5.1. NETWORKS WITH NON-COLLOCATED SOURCE NODES AND DESTINATION NODES

In some special networks, every source node is different from each other, i.e., $a_i \neq a_j, \forall i \neq j$, and every destination node is also different from each other, i.e., $b_i \neq b_j, \forall i \neq j$. We also assume that the source node in one session cannot be the destination node in another session, i.e., $a_i \neq b_j, \forall i, j$. In this special case we will present Theorem 3 and the proof of it. Before that, we start by giving the definition of session-connected set M and two lemmas: Lemma 2 and Lemma 3 in order to prove another lower bound of the energy consumption P^{CF} when compute-and-forward is applied.

For a network $N(G(V, E), S)$, in which $a_i \neq a_j, b_i \neq b_j, \forall i \neq j$ and $a_i \neq b_j, \forall i, j$, we define session-connected set $M \subseteq V$ as follows: M is feasible for all sessions in S , which means that in M each session in S is connected, and for any graph $G(M', E')$ where $M' \subset M, E' \subset E$, the session placement S is not feasible, i.e. in graph $G(M', E')$, at least one pair of source and destination in S cannot be connected. Note that there are maybe more than one subsets M of V , however for network N , we define that subsets which have the least nodes among all M as smallest session-connected sets M_{\min} . Example 3 shows a smallest session-connected set M_{\min} in a network $N(G(V, E), S)$. Again, there are maybe more than one smallest session-connected sets M_{\min} , and they have the same number of nodes $|M_{\min}|$, i.e. for any $M' \in M$, it is always true that

$$|M_{\min}| \leq |M'|.$$

Example 3. Fig 5.1 shows a network and a possible smallest session-connected set M_{\min} .

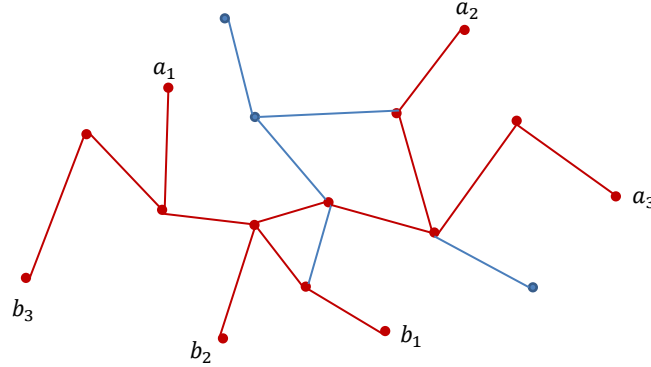


Figure 5.1: A possible smallest session-connected set M_{\min} for a network $N(G(V, E), S)$. All nodes with the color of red are elements in M_{\min} . Edges with the color of red show how all the sessions in the network can be connected.

There are 14 nodes in M_{\min} , thus we have $|M_{\min}| = 14$. Note that this is the only smallest session-connected set M_{\min} for this network. However for other networks there may be more than one smallest session-connected sets M_{\min} with the same number of nodes $|M_{\min}|$ in it.

Clearly we have the following property of session-connected sets.

Property 1. For any $N(G(M^*, E^*), S)$ which $|M^*| < |M_{\min}|$, $E^* \subseteq E$, S is not feasible, i.e., at least one pair of source and destination in S cannot be connected.

Before we present Theorem 3, we prove Lemma 2 and Lemma 3 in order to prove the lower bound of P_M^{CF} . We define P_M^{CF} as the minimum energy consumption when a symbol transmitted and received successfully for each session in a smallest session-connected set M_{\min} when applying compute-and-forward.

Lemma 2. In a smallest session-connected set M_{\min} , the minimum energy consumption when applying compute-and-forward scheme is lower bounded by

$$P_M^{CF} \geq (|M_{\min}| - K)(e_t + e_r).$$

Proof. In a smallest session-connected set M_{\min} , we consider the energy consumption P_M^{CF} when applying compute-and-forward as follows. Note that each source node in every session transmits at least once, and each destination node in every session receives at least once. The energy consumption for source nodes and destination nodes in M_{\min} is $Ke_t + Ke_r$.

As for a relay node, if it doesn't transmit or receive, it cannot relay any information, thus it could be removed from the set. According to the property of smallest session-connected sets, the set M_{\min}

will not be connected any more when this relay node is removed. Therefore we come to the conclusion that each relay node in M_{\min} needs to transmit at least once and receive at least once. The energy consumption for all the relay nodes in M_{\min} is $(|M_{\min}| - 2K)(e_t + e_r)$. Then we get the energy consumption P_M^{CF} when applying compute-and-forward

$$\begin{aligned} P_M^{CF} &\geq Ke_t + Ke_r + (|M_{\min}| - 2K)(e_t + e_r) \\ &= (|M_{\min}| - K)(e_t + e_r). \end{aligned}$$

□

Now based on Lemma 2, we present Lemma 3 and give the proof of it.

Lemma 3. *For the energy consumption when applying compute-and-forward scheme in network $N(G(V, E), S)$, in which $a_i \neq a_j, b_i \neq b_j, \forall i \neq j$ and $a_i \neq b_j, \forall i, j$, it holds that*

$$P^{CF} \geq (|M_{\min}| - K)(e_t + e_r).$$

Proof. This lemma could be proved by contradiction. We assume that there exists a transmission scheme, in which the energy consumption when applying compute-and-forward scheme is P' , and

$$P' < (|M_{\min}| - K)(e_t + e_r).$$

As to realize successful transmission and reception of one symbol in all sessions, each source node and each destination node need to transmit once and receive once, the energy consumption consumed by all the source nodes and destination nodes is $(Ke_t + Ke_r)$. Then we define P_r' as the energy consumption consumed by all the relay nodes in all sessions, so it comes that

$$\begin{aligned} P_r' &= P' - (Ke_t + Ke_r) \\ &< (|M_{\min}| - K)(e_t + e_r) - K(e_t + e_r) \\ &= (|M_{\min}| - 2K)(e_t + e_r) \end{aligned}$$

That is

$$P_r' < (|M_{\min}| - 2K)(e_t + e_r).$$

Since each relay node which is functioning in the network needs to transmit at least once and receive at least once, and we define the number of all the functioning relay nodes as N_r' . It is clear that

$$\begin{aligned} N_r' &= \frac{P_r'}{e_t + e_r} \\ &< \frac{(|M_{\min}| - 2K)(e_t + e_r)}{e_t + e_r} \\ &= |M_{\min}| - 2K. \end{aligned}$$

We define the total number of nodes in this special scheme as N' , then it is clear that

$$N' = N_r' + 2K < |M_{\min}|$$

Thus this scheme provides reliable transmission for S with less than $|M_{\min}|$ nodes, which contradicts with both Property 1 and Lemma 2. Thus we could say that the opposite assumption is true, that is

$$P^{CF} \geq (|M_{\min}| - K)(e_t + e_r).$$

□

Upon Lemma 3, we present Theorem 3 and the proof of it as follows.

Theorem 3. *For a non-located network $N(G(V, E), S)$ with K sessions, in which $a_i \neq a_j, b_i \neq b_j, \forall i \neq j$ and $a_i \neq b_j, \forall i, j$, the energy benefit is upper bounded by*

$$I < 2K.$$

Proof. First, we present the upper bound of the energy consumption P^{TR} when applying the traditional routing scheme in the network. Now we propose a scheme in traditional routing mode. For S_i , we find a shortest path in M_{\min} to transmit a symbol from a_i to b_i . By the definition of M_{\min} , this path can always be found. Assume this shortest path in M_{\min} has distance \tilde{d}_i . Then the energy consumption for one symbol transmitted from a_i to b_i is

$$\tilde{P}_i = (e_t + e_r) + (e_t + e_r)(\tilde{d}_i - 1),$$

where $(e_t + e_r)$ is the energy consumption by the source node and destination node in S_i , and $(e_t + e_r)(\tilde{d}_i - 1)$ is the energy consumption by all the relay nodes along this path. Note that $\tilde{d}_i \leq |M_{\min}| - 1$, we have

$$\tilde{P}_i \leq (e_t + e_r)(|M_{\min}| - 1).$$

Hence the energy consumption of this scheme is no larger than $K(e_t + e_r)(|M_{\min}| - 1)$, and by the definition of P_{TR} , we have

$$P_{TR} \leq (e_t + e_r)(|M_{\min}| - 1)$$

Secondly, in the compute-and-forward mode, we get the lower bound of the energy consumption P^{CF} from Lemma 3 which is

$$P^{CF} \geq (|M_{\min}| - K)(e_t + e_r).$$

By the definition of energy benefit, we now get the upper bound of I as follows

$$I = \frac{P^{TR}}{P^{CF}} \leq \frac{K(|M_{\min}| - 1)}{|M_{\min}| - K}$$

Note that M_{\min} is a nodes set consisted of all source nodes, all destination nodes and some other nodes. Thus we have the relationship between $|M_{\min}|$ and K as follows,

$$|M_{\min}| = 2K + x,$$

where x is the number of nodes which are neither source nodes nor destination nodes. Then the energy benefit becomes

$$\begin{aligned} I &\leq \frac{K(|M_{\min}| - 1)}{|M_{\min}| - K} = \frac{K(2K + x - 1)}{2K + x - K} \\ &= 2K - \frac{Kx + K}{K + x}. \end{aligned}$$

It is clear that x and K are both always non-negative integers, thus $\frac{Kx + K}{K + x}$ is always a positive number. Then we conclude that

$$I < 2K.$$

□

5.2. SINGLE SOURCE NETWORK AND SINGLE DESTINATION NETWORK

Before we present the theorem, we give two definitions about these two special cases where there is only one source or one destination.

Definition 1. In a single source network, we assume that all source nodes are the same one, i.e. $a_1 = a_2 = \dots = a_K = A$, and each of them sends independent information to its destination node, with the destination node set $B = \{b_1, b_2, \dots, b_K\}$, as is shown in Fig 5.2.

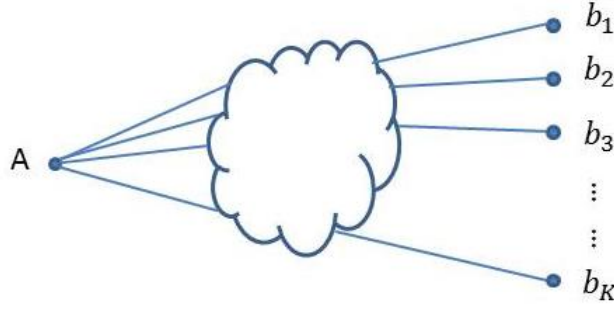


Figure 5.2: A network with one source

Definition 2. In a single destination network, we assume that all destination nodes are the same one, i.e. $b_1 = b_2 = \dots = b_K = B$, and each of them receives independent information from its source node, with the source node set $B = \{a_1, a_2, \dots, a_K\}$, as is shown in Fig 5.3.

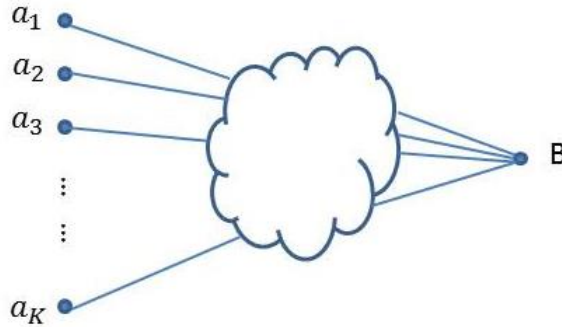


Figure 5.3: A network with only one destination.

For these two special cases, we present Theorem 4 and give the proof of it as follows.

Theorem 4. In a single source network or a single destination network, the energy benefit is upper bounded by

$$I \leq 1.$$

Proof. In a single source network, the information will be transmitted over the shortest path in each session in the network in traditional routing mode. It is clear that for S_i , the distance between source node and destination node is $d(a_i, b_i) = d(A, b_i)$, so the total number of transmissions in K sessions is $\sum_{i=1}^K [d(A, b_i)]$, as well as the number of total receptions. Thus the energy consumption P with this scheme is $(e_t + e_r) \sum_{i=1}^K [d(A, b_i)]$. Then we get the upper bound of P^{TR} ,

$$P^{TR} \leq (e_t + e_r) \sum_{i=1}^K [d(A, b_i)]$$

When it is in compute-and-forward mode, based on the result of Theorem 2 that

$$P^{CF} \geq (e_t + e_r) \max[\max_{\hat{B} \subseteq B} \sum_{i=1}^K d(a_i, \hat{B}), \max_{\hat{A} \subseteq A} \sum_{i=1}^K d(\hat{A}, b_i)],$$

we get the lower bound of P^{CF} ,

$$P^{CF} \geq (e_t + e_r) \sum_{i=1}^K [d(A, b_i)]$$

So the energy benefit is lower bounded by

$$I \leq 1.$$

In a single destination network, in traditional routing mode the information will be transmitted over the shortest path in each session in the network. It is clear that for S_i , the distance between source node and destination node is $d(a_i, b_i) = d(a_i, B)$, so the total number of transmissions in K sessions is $\sum_{i=1}^K [d(a_i, B)]$, as well as the number of total receptions. Thus the energy consumption P with this scheme is $(e_t + e_r) \sum_{i=1}^K [d(a_i, B)]$. Then we get the upper bound of P^{TR} ,

$$P^{TR} \leq (e_t + e_r) \sum_{i=1}^K [d(a_i, B)]$$

When it is in compute-and-forward mode, based on the result of Theorem 2 that

$$P^{CF} \geq (e_t + e_r) \max[\max_{\hat{B} \subseteq B} \sum_{i=1}^K d(a_i, \hat{B}), \max_{\hat{A} \subseteq A} \sum_{i=1}^K d(\hat{A}, b_i)],$$

we get the lower bound of P^{CF} ,

$$P^{CF} \geq (e_t + e_r) \sum_{i=1}^K [d(a_i, B)].$$

So the energy benefit is lower bounded by

$$I \leq 1.$$

So we come to the conclusion that a single source network or a single destination network, the energy benefit is always upper bounded by

$$I \leq 1.$$

□

We conclude from Theorem 4 that compute-and-forward in a single source network or a single destination network does not make any benefit at all. A good example of these two situations mentioned above is the base station when they do not play the role of relays in wireless communication

networks. A base station is a fixed communications location and is part of a network's wireless telephone system. Base stations use radio signals to connect mobile devices to the network, enabling people to send and receive calls, texts, emails, pictures, web, TV and downloads.

When more than one transmitting/receiving units, such as mobile phones, are transmitting their independent information to the base station, and when the base station is their final destination, it is regarded as the only destination in the network. In another situation when the base station is the source node and is transmitting independent information to more than one mobile phones, it could be regarded as the only source. Based on our conclusions, in both situations will there be no energy benefit using compute-and-forward schemes.

5.3. LINE NETWORK

The line network scenario has been widely studied to understand the benefits of network coding. Now we try to find possible energy benefits of compute-and-forward mode in line networks. We give definition of line networks model as follows.

Definition 3. In a line network $N(G(V, E), S)$, the nodes set is defined as $V = \{0, 1, 2, \dots, N\}$ and edges set is defined as $E = \{(0, 1), (1, 2), (2, 3), \dots, (N-1, N)\}$. The number of sessions is K . For each session S_i , we define d_i as the minimum number of hops between a_i and b_i . Moreover we define relay nodes as follows, if a node $\min(a_i, b_i) < v < \max(a_i, b_i)$, we say node v is the relay node of S_i .

In a line network $N(G(V, E), S)$, we define S_R as the sessions set in which all the sessions have the direction from left to right, i.e., $S_R = \{S_i : (a_i, b_i) \in S \mid b_i > a_i\}$. Similarly, we define S_L as the sessions set in which all the sessions have the direction from right to left, i.e., $S_L = \{S_i : (a_i, b_i) \in S \mid a_i > b_i\}$.

Example 4. An example of line network is shown in Fig 5.4.

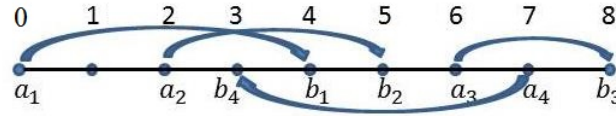


Figure 5.4: A line network example $N(G(V, E), S)$ with 4 sessions.

There are 4 sessions in the network, and $S_R = \{(0, 4), (2, 5), (6, 8)\}$, and $S_L = \{(7, 3)\}$.

We now present Theorem 5 in networks with the same features of Definition 3 and prove it as follows.

Theorem 5. In a line network $N(G(V, E), S)$, the energy benefit when using compute-and-forward is

$$I \leq 2.$$

Proof. We define P_R^{CF} as the total energy consumption in $N(G(V, E), S_R)$ when a symbol is communicated successfully for every session in S_R when the network is in compute-and-forward mode, and define P_L^{CF} as the total energy consumption in $N(G(V, E), S_L)$ when a symbol is communicated successfully for every session in S_L when the network is in compute-and-forward mode.

For node v , we now define n_v^R to be the number of sessions in which node v is the relay node, and meanwhile all these sessions are in S_R , where $1 \leq v \leq N$ and v is an integer, i.e., $n_v^R = |\{S_i \in S_R \mid a_i < v < b_i\}|$.

Similarly, we define n_v^L to be the number of sessions in which node v is the relay node, and meanwhile all these sessions are in S_L , where $1 \leq v \leq N$ and v is a positive integer, i.e., $n_v^L = |\{S_i \in S_L \mid b_i < v < a_i\}|$.

When applying compute-and-forward mode in the network, we start by considering the sessions which are in S_R . We focus on relay nodes in each session in S_R . Consider network $N(G(V, E), S)$ by max-flow min-cut theory, a node v has to transmit as well as receive at least n_v^R times to forward all the messages for sessions in S_R . Moreover, if it functions as source nodes for α_v^R sessions and functions as destination nodes for β_v^R sessions, it needs to transmit another α_v^R times and receive another β_v^R times. As it is clear that

$$\sum_{v=1}^N \alpha_v^R = \sum_{v=1}^N \beta_v^R = |S_R|$$

the minimum number of transmissions of all nodes in each session in S_R is

$$\sum_{v=1}^N n_v^R + \sum_{v=1}^N \alpha_v^R = \sum_{v=1}^N n_v^R + |S_R|,$$

so is the minimum number of receptions.

When applying compute-and-forward mode, we get the lower bound of energy consumption of all the sessions in S_R ,

$$P_R^{CF} \geq (e_t + e_r) \left(\sum_{v=1}^N n_v^R + |S_R| \right)$$

It can be easily calculated by counting that

$$\sum_{v=1}^N n_v^R + |S_R| = \sum_{S_i \in S_R} d_i.$$

So

$$P_R^{CF} \geq (e_t + e_r) \sum_{S_i \in S_R} d_i.$$

Similar derivation can be used when we only consider all the sessions in S_L . Then we get

$$P_L^{CF} \geq (e_t + e_r) \sum_{S_i \in S_L} d_i.$$

Of course adding sessions will not decrease the energy consumption, thus

$$P^{CF} \geq \max(P_L^{CF}, P_R^{CF}).$$

Obviously, it is always true that

$$\begin{aligned} [P^{CF} \geq \max(P_L^{CF}, P_R^{CF})] &\geq \frac{1}{2} (P_L^{CF} + P_R^{CF}) \\ &= \frac{1}{2} ((e_t + e_r) \sum_{S_i \in S_R} d_i + (e_t + e_r) \sum_{S_i \in S_L} d_i) \\ &= \frac{1}{2} (e_t + e_r) \sum_{S_i \in S} d_i. \end{aligned}$$

From Lemma 1 we have the upper bound of energy consumption P^{TR} when applying traditional routing scheme

$$P^{TR} \leq (e_t + e_r) \sum_{i=1}^K d_i$$

So we come to the conclusion that

$$\begin{aligned}
 I &= \frac{P^{TR}}{PCF} \\
 &\leq \frac{(e_t + e_r) \sum_{S_i \in S} d_i}{\frac{1}{2} [(e_t + e_r) \sum_{S_i \in S} d_i]} \\
 &= 2.
 \end{aligned}$$

□

This upper bound can be achieved when $N \rightarrow \infty$, and there are two sessions with sources and receivers at the endpoints of the network, which means that the line network is infinitely long so that energy consumption of two end nodes can be neglected. Achievable scheme applying compute-and-forward is given in [34, Lemma 13].

5.4. RECTANGULAR LATTICE NETWORK

Lattice networks are widely used in regular settings like grid computing, distributed control and wireless sensor networks. In practice, the wireless sensor network can be two dimensional (2D) plane or a three dimensional (3D) space. However, common devices used in sensor networks usually have limited power storage. So there is an urgent need for decreasing the energy consumptions in such networks. We now focus on the energy benefit when compute-and-forward mode is applied over traditional routing mode in 2D and 3D lattice networks respectively.

5.4.1. 2D RECTANGULAR LATTICE NETWORK

Now we give a definition of the model set-up in 2D rectangular lattice networks as follows.

Definition 4. In a 2D rectangular lattice network $N(G(V, E), S)$, we consider the lattice network with nearest neighbor connectivity. We define the nodes set as $V = \{v = (x, y) | 0 \leq x \leq M, 0 \leq y \leq N\}$, in which v is a node defined by an index tuple (x, y) and x, y, M, N are non-negative integers. The location of the node $v \in V$ in \mathbb{R}^2 is given by vG , where $G = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$. The number of sessions is also K in the 2D lattice network model, and we define session S_i by its source node and destination node as $S_i = (a_i, b_i)$ and $a_i = (x_{a_i}, y_{a_i}), b_i = (x_{b_i}, y_{b_i})$.

Then we define rows and columns based on the locations of nodes. If there are two nodes $v_1 = (x_1, y_1)$ and $v_2 = (x_2, y_2)$, we say that they are in the same row when $y_1 = y_2 = y$ and the row which they are in is called row y , $0 \leq y \leq N$, i.e., row $y = \{(x_i, y_i) | y_i = y\}$. Similarly, we say that two nodes $v_1 = (x_1, y_1)$ and $v_2 = (x_2, y_2)$ are in the same column when $x_1 = x_2 = x$, and the column which they are in is called column x , $0 \leq x \leq M$, i.e., column $x = \{(x_i, y_i) | x_i = x\}$.

In Fig 5.5, it shows the basic configurations of nodes, rows and columns in a 2D rectangular lattice network.

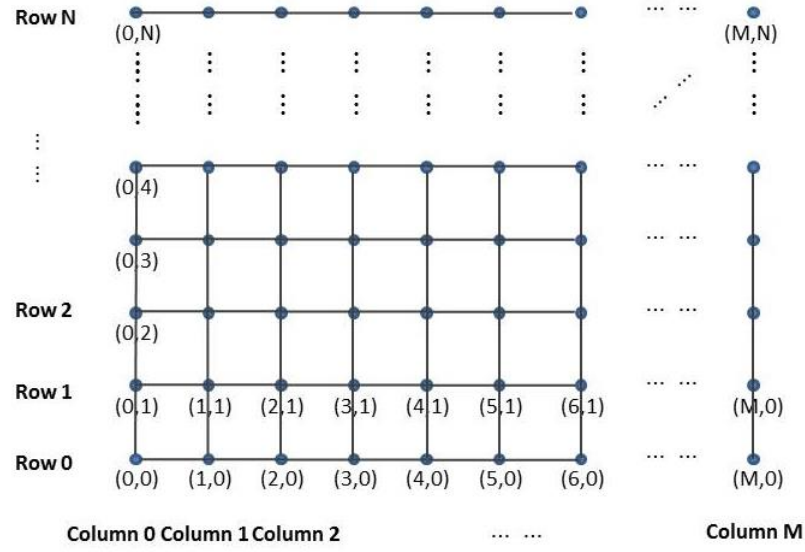


Figure 5.5: Basic configurations of nodes, rows and columns for a 2D rectangular lattice network.

Similar as what we did in line networks, in a 2D rectangular lattice network $N(G(V, E), S)$, we define S_R , S_L , S_U , and S_D as the sessions sets which have the directions of right, left, up and down respectively. S_R is defined as the sessions set, in which for a session $S_{R_i} = (a_i, b_i)$ where $a_i = (x_{a_i}, y_{a_i})$, $b_i = (x_{b_i}, y_{b_i})$, it is always true that $x_{a_i} < x_{b_i}$, i.e., $S_R = \{(x_{a_i}, y_{a_i}), (x_{b_i}, y_{b_i}) \mid x_{a_i} < x_{b_i}\}$. Then using similar principle to define S_L , S_U , and S_D , we can get $S_L = \{(x_{a_i}, y_{a_i}), (x_{b_i}, y_{b_i}) \mid x_{b_i} < x_{a_i}\}$, $S_U = \{(x_{a_i}, y_{a_i}), (x_{b_i}, y_{b_i}) \mid y_{a_i} < y_{b_i}\}$, and $S_D = \{(x_{a_i}, y_{a_i}), (x_{b_i}, y_{b_i}) \mid y_{b_i} < y_{a_i}\}$.

Example 5. Fig 5.6 shows an example of 2D rectangular lattice network.

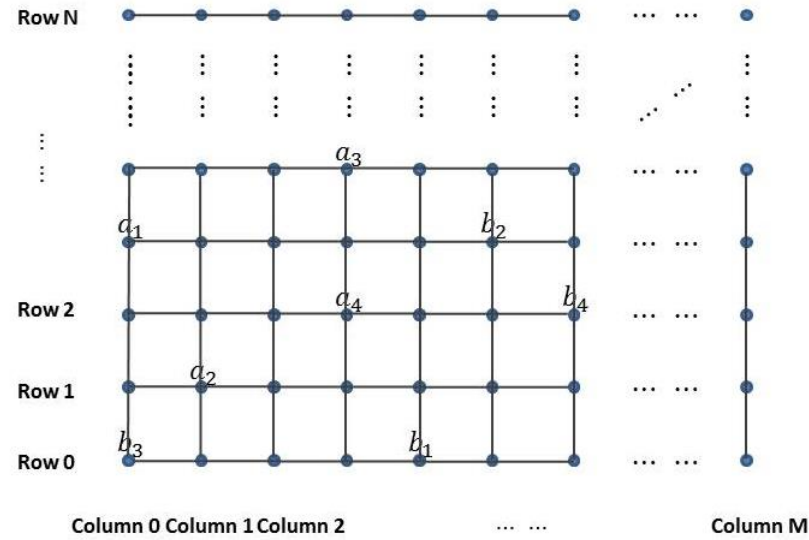


Figure 5.6: An example of 2D rectangular lattice network $N(G(V, E), S)$ with 4 sessions.

There are 4 sessions in the network, with $S_U : \{S_2\}$, $S_D : \{S_1, S_3\}$, $S_R : \{S_1, S_2, S_4\}$ and $S_L : \{S_3\}$.

Now we present Theorem 6 and give the proof of it.

Theorem 6. *In a 2D rectangular lattice network $N(G(V, E), S)$, the energy benefit when applying compute-and-forward mode is upper bounded by*

$$I \leq 4.$$

Proof. When only sessions in S_R are considered, we call the column in which the source node is as *source column* in S_R , and the column in which the destination node is as *destination column* in S_R . For each session in S_L , *source column* in S_L and *destination column* in S_L are defined in the same way. Then in S_U , we call the row in which the source node is as *source row* in S_U , and the row in which the destination node is as *destination row* in S_U . For each session in S_D , *source row* in S_D and *destination row* in S_D are defined in the same way.

Moreover, we define rows which are between *row* y_{a_i} and *row* y_{b_i} as relay rows for S_i , i.e. *row* y is a relay row for S_i if $\min(y_{a_i}, y_{b_i}) < y < \max(y_{a_i}, y_{b_i})$. Similar to the definition of relay rows, we define columns between *column* x_{a_i} and *column* x_{b_i} as relay columns for S_i , i.e. *column* x is a relay column for S_i if $\min(x_{a_i}, x_{b_i}) < x < \max(x_{a_i}, x_{b_i})$.

Firstly we consider all nodes in the network in terms of rows. We define P_U^{CF} as the total energy consumption in $N(G(V, E), S_U)$ when a symbol is communicated successfully for every session in S_U when the network is in compute-and-forward mode, and define P_D^{CF} as the total energy consumption in $N(G(V, E), S_D)$ when a symbol is communicated successfully for every session in S_D when the network is in compute-and-forward mode.

Now we focus only on relay rows in each session in S_U . We define n_r^U to be the number of sessions in which *row* r is the relay row in S_U , where $1 \leq r \leq N$ and r is a positive integer, i.e., $n_r^U = |\{S_i \in S_U \mid y_{a_i} < r < y_{b_i}\}|$. When applying compute-and-forward mode in the network, we start by considering the sessions which are in S_U . We focus on relay rows in each session in S_U . Consider network $N(G(V, E), S)$ by max-flow min-cut theory, *row* r has to transmit as well as receive at least n_r^U times to forward all the messages for sessions in S_U . Moreover, if it functions as *source row* in S_U for α_r^U sessions and functions as *destination row* in S_U for β_r^U sessions, it needs to transmit another α_r^U times and receive another β_r^U times. As it is clear that

$$\sum_{r=0}^N \alpha_r^U = \sum_{r=0}^N \beta_r^U = |S_U|$$

the minimum number of transmissions of all rows in each session in S_U is

$$\sum_{r=1}^N n_r^U + \sum_{r=0}^N \alpha_r^U = \sum_{r=0}^N n_r^U + |S_U|,$$

so is the minimum number of receptions. Thus we get the lower bound of P_U^{CF}

$$P_U^{CF} \geq (e_t + e_r) \left(\sum_{r=0}^N n_r^U + |S_U| \right).$$

Then we get the lower bound of P_D^{CF} as well in the same way. We define n_r^D to be the number of sessions in which *row* r is the relay row in S_D , where $1 \leq r \leq N$ and r is a positive integer, i.e., $n_r^D = |\{S_i \in S_D \mid y_{b_i} < r < y_{a_i}\}|$.

$$P_D^{CF} \geq (e_t + e_r) \left(\sum_{r=0}^N n_r^D + |S_D| \right).$$

Secondly we consider all nodes in the network in terms of columns. We define P_R^{CF} as the total energy consumption in $N(G(V, E), S_R)$ when a symbol is communicated successfully for every session in S_R when the network is in compute-and-forward mode, and define P_L^{CF} as the total energy consumption in $N(G(V, E), S_L)$ when a symbol is communicated successfully for every session in S_L when the network is in compute-and-forward mode. We now focus only on relay columns in each session in S_R . We define n_c^R to be the number of sessions in which *column* c is the relay column in S_R , where $1 \leq r \leq M$ and c is a positive integer, i.e., $n_c^R = |\{S_i \in S_R \mid x_{a_i} < c < x_{b_i}\}|$. When applying compute-and-forward mode in the network, we now consider the sessions which are in S_R . We focus on relay columns in each session in S_R . Consider network $N(G(V, E), S)$ by max-flow min-cut theory, *column* c has to transmit as well as receive at least n_c^R times to forward all the messages for sessions in S_R . Moreover, if it functions as *source columns* in S_R for α_c^R sessions and functions as *destination columns* in S_R for β_c^R sessions, it needs to transmit another α_c^R times and receive another β_c^R times. As it is clear that

$$\sum_{c=0}^M \alpha_c^R = \sum_{c=0}^M \beta_c^R = |S_R|$$

the minimum number of transmissions of all rows in each session in S_R is

$$\sum_{c=1}^M n_c^R + \sum_{c=0}^M \alpha_c^R = \sum_{c=0}^M n_c^R + |S_R|,$$

so is the minimum number of receptions. Then we get the lower bound of P_R^{CF} ,

$$P_R^{CF} \geq (e_t + e_r) \left(\sum_{c=0}^M n_c^R + |S_R| \right).$$

We can get the lower bound of P_L^{CF} in the same way. We define n_c^L to be the number of sessions in which *column* c is the relay column in S_L , where $1 \leq c \leq N$ and c is a positive integer, i.e., $n_c^L = |\{S_i \in S_L \mid x_{b_i} < c < x_{a_i}\}|$.

$$P_L^{CF} \geq (e_t + e_r) \left(\sum_{c=0}^M n_c^L + |S_L| \right).$$

Of course adding sessions will not decrease the energy consumption, thus

$$P^{CF} \geq \max(P_U^{CF}, P_D^{CF}, P_R^{CF}, P_L^{CF}).$$

Again, it can be easily calculated by counting that

$$\sum_{r=0}^N n_r^U + \sum_{r=0}^N n_r^D + \sum_{c=0}^M n_c^R + \sum_{c=0}^M n_c^L + |S_U| + |S_D| + |S_R| + |S_L| = \sum_{i=1}^K d_i$$

So

$$\begin{aligned} P^{CF} &\geq \frac{1}{4} (P_U^{CF} + P_D^{CF} + P_L^{CF} + P_R^{CF}) \\ &= \frac{1}{4} [(e_t + e_r) \left(\sum_{r=0}^N n_r^U + |S_U| \right) + (e_t + e_r) \left(\sum_{r=0}^N n_r^D + |S_D| \right) \\ &\quad + (e_t + e_r) \left(\sum_{c=0}^M n_c^R + |S_R| \right) + (e_t + e_r) \left(\sum_{c=0}^M n_c^L + |S_L| \right)] \\ &= \frac{1}{4} (e_t + e_r) \left(\sum_{r=0}^N n_r^U + \sum_{r=0}^N n_r^D + \sum_{c=0}^M n_c^R + \sum_{c=0}^M n_c^L \right. \\ &\quad \left. + |S_U| + |S_D| + |S_R| + |S_L| \right) \\ &= \frac{1}{4} (e_t + e_r) \sum_{i=1}^K d_i. \end{aligned}$$

From Lemma 1 we have the upper bound of energy consumption P^{TR} when applying traditional routing scheme

$$P^{TR} \leq (e_t + e_r) \sum_{i=1}^K d_i$$

So we come to the conclusion that

$$\begin{aligned} I &= \frac{P^{TR}}{P^{CF}} \\ &\leq \frac{(e_t + e_r) \sum_{i=1}^K d_i}{\frac{1}{4}(e_t + e_r) \sum_{i=1}^K d_i} \\ &= 4. \end{aligned}$$

□

5.4.2. 3D RECTANGULAR LATTICE NETWORK

Rather than 2D lattice networks, 3D wireless sensor networks reflect more accurate real life situations and 3D sensors have created a considerable degree of interests in both civil and military applications across geology, civil engineering, archeology, reverse engineering, cultural heritage, medicine, emergency medical care, criminal investigation, virtual reality, etc. Thus we now focus on the energy benefit when using compute-and-forward schemes over traditional routing in 3D lattice networks.

First, we give a definition of the model set-up in 3D rectangular lattice networks as follows.

Definition 5. In a 3D rectangular lattice network $N(G(V, E), S)$, we consider a homogeneous network model with identical nodes and with nearest neighbor connectivity in a 3D space. We define the nodes set as $V = \{v = (x, y, z) | 0 \leq x \leq M, 0 \leq y \leq N, 0 \leq z \leq H\}$, in which v is a node defined by an index tuple (x, y, z) and x, y, z, M, N, H are all non-negative integers. The location of the node $v \in V$ in \mathbb{R}^3 is given by vG , where $G = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$. We have K sessions in our lattice network, and we define session S_i as $S_i = (a_i, b_i)$ and $a_i = (x_{a_i}, y_{a_i}, z_{a_i})$, $b_i = (x_{b_i}, y_{b_i}, z_{b_i})$.

Then we define faces depending on the locations of nodes. If there are two nodes (x_1, y_1, z_1) and (x_2, y_2, z_2) , we say that they are in the same face when $x_1 = x_2 = x$ or $y_1 = y_2 = y$, or $z_1 = z_2 = z$ and the face which they are in is called face x , face y or face z respectively, with $0 \leq x \leq M$, $0 \leq y \leq N$, $0 \leq z \leq H$, i.e., face $x = \{(x_i, y_i, z_i) | x_i = x\}$.

Fig 5.7 shows the basic configurations of nodes and faces in a 3D rectangular lattice network.

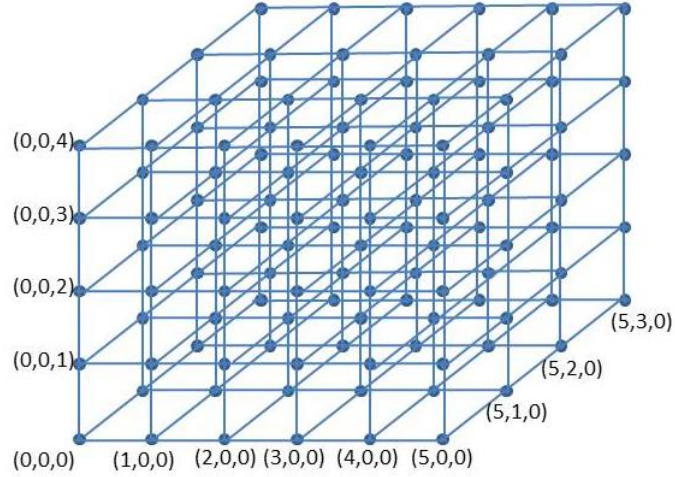


Figure 5.7: Basic configurations of nodes and faces for a 3D rectangular lattice network.

Similar as what we did in a 2D lattice network, in a 3D rectangular lattice network $N(G(V, E), S)$, we define S_R , S_L , S_U , S_D , S_F and S_B as the sessions sets which have the directions of right, left, up and down, front and back respectively. S_R is defined as the sessions set, in which for a session $S_i = (a_i, b_i)$ where $a_i = (x_{a_i}, y_{a_i}, z_{a_i})$, $b_i = (x_{b_i}, y_{b_i}, z_{b_i})$, it is always true that $x_{a_i} < x_{b_i}$, i.e., $S_R = \{(x_{a_i}, y_{a_i}, z_{a_i}), (x_{b_i}, y_{b_i}, z_{b_i}) \mid x_{a_i} < x_{b_i}\}$. Then using similar principle to define S_L , S_U , S_D , S_F and S_B , we can get that $S_L = \{(x_{a_i}, y_{a_i}, z_{a_i}), (x_{b_i}, y_{b_i}, z_{b_i}) \mid x_{b_i} < x_{a_i}\}$, $S_U = \{(x_{a_i}, y_{a_i}, z_{a_i}), (x_{b_i}, y_{b_i}, z_{b_i}) \mid z_{a_i} < z_{b_i}\}$, $S_D = \{(x_{a_i}, y_{a_i}, z_{a_i}), (x_{b_i}, y_{b_i}, z_{b_i}) \mid z_{b_i} < z_{a_i}\}$, $S_F = \{(x_{a_i}, y_{a_i}, z_{a_i}), (x_{b_i}, y_{b_i}, z_{b_i}) \mid y_{b_i} < y_{a_i}\}$, $S_B = \{(x_{a_i}, y_{a_i}, z_{a_i}), (x_{b_i}, y_{b_i}, z_{b_i}) \mid y_{a_i} < y_{b_i}\}$.

Now we present Theorem 7 and give the proof of it.

Theorem 7. In a 3D rectangular lattice network $N(G(V, E), S)$, the energy benefit when using compute-and-forward is upper bounded by

$$I \leq 6.$$

Proof. For $0 \leq i \leq K$, when only sessions in S_R are considered, we call the face x_{a_i} in which the source node $a_i = (x_{a_i}, y_{a_i}, z_{a_i})$ is as *source face in S_R* , and the face x_{b_i} in which the destination node $b_i = (x_{b_i}, y_{b_i}, z_{b_i})$ is as *destination face in S_R* . For each session in S_L , *source face in S_L* and *destination face in S_L* are defined in the same way. Then in S_U , we call the face z_{a_i} in which the source node $a_i = (x_{a_i}, y_{a_i}, z_{a_i})$ is as *source face in S_U* , and the face z_{b_i} in which the destination node $b_i = (x_{b_i}, y_{b_i}, z_{b_i})$ is as *destination face in S_U* . For each session in S_D , *source face in S_D* and *face in S_D* are defined in the same way. Then in S_F , we call the face y_{a_i} in which the source node $a_i = (x_{a_i}, y_{a_i}, z_{a_i})$ is as *source face in S_F* , and the face y_{b_i} in which the destination node $b_i = (x_{b_i}, y_{b_i}, z_{b_i})$ is as *destination face in S_F* . For each session in S_B , *source face in S_B* and *face in S_B* are defined in the same way.

Moreover, we define faces which are between face z_{a_i} and face z_{b_i} and at the same time are parallel to them as relay faces for S_i , i.e. face z is a relay face for $S_i = (a_i, b_i)$ if $\min(z_{a_i}, z_{b_i}) < z < \max(z_{a_i}, z_{b_i})$.

Firstly we define P_U^{CF} as the total energy consumption in $N(G(V, E), S_U)$ when a symbol is communicated successfully for every session in S_U when the network is in compute-and-forward mode, and define P_D^{CF} as the total energy consumption in $N(G(V, E), S_D)$ when a symbol is communicated

successfully for every session in S_D when the network is in compute-and-forward mode. We define n_f^U to be the number of sessions in which *face* f is the relay face in S_U , where $0 \leq f \leq H$ and f is a positive integer, i.e., $n_f^U = |\{S_i \in S_U \mid z_{a_i} < f < z_{b_i}\}|$. When applying compute-and-forward mode in the network, we focus on relay rows in each session in S_U . Consider network $N(G(V, E), S)$ by max-flow min-cut theory, a face f has to transmit as well as receive at least n_f^U times to forward all the messages for sessions in S_U . Moreover, if it functions as *source face* in S_U for α_f^U sessions and functions as *destination face* in S_U for β_f^U sessions, it needs to transmit another α_f^U times and receive another β_f^U times. As it is clear that

$$\sum_{f=0}^H \alpha_f^U = \sum_{f=0}^H \beta_f^U = |S_U|$$

the minimum number of transmissions of all rows in each session in S_U is

$$\sum_{f=0}^H n_f^U + \sum_{f=0}^H \alpha_f^U = \sum_{f=0}^H n_f^U + |S_U|,$$

so is the minimum number of receptions. Thus we get the lower bound of P_U^{CF}

$$P_U^{CF} \geq (e_t + e_r) \left(\sum_{f=0}^H n_f^U + |S_U| \right).$$

Now we can get P_D^{CF} in the same way. We define n_f^D to be the number of sessions in which *face* f is the relay face in S_D , where $0 \leq f \leq H$ and f is a positive integer, i.e., $n_f^D = |\{S_i \in S_D \mid z_{b_i} < f < z_{a_i}\}|$. The upper bound of P_D^{CF} is

$$P_D^{CF} \geq (e_t + e_r) \left(\sum_{f=0}^H n_f^D + |S_D| \right).$$

Secondly, we define P_R^{CF} as the total energy consumption in $N(G(V, E), S_R)$ when a symbol is communicated successfully for every session in S_R when the network is in compute-and-forward mode, and define P_L^{CF} as the total energy consumption in $N(G(V, E), S_L)$ when a symbol is communicated successfully for every session in S_L when the network is in compute-and-forward mode. We define n_f^R to be the number of sessions in which *face* f is the relay face in S_R , where $0 \leq f \leq M$ and f is a positive integer, i.e., $n_f^R = |\{S_i \in S_R \mid x_{a_i} < f < x_{b_i}\}|$, and n_f^L to be the number of sessions in which *face* f is the relay face in S_L , where $0 \leq f \leq M$ and f is a positive integer, i.e., $n_f^L = |\{S_i \in S_L \mid x_{b_i} < f < x_{a_i}\}|$. Based on the same reasons and derivation methods in getting P_U^{CF} and P_D^{CF} , we get the lower bounds of P_R^{CF} and P_L^{CF} ,

$$P_R^{CF} \geq (e_t + e_r) \left(\sum_{f=0}^M n_f^R + |S_R| \right),$$

$$P_L^{CF} \geq (e_t + e_r) \left(\sum_{f=0}^M n_f^L + |S_L| \right).$$

Thirdly, We define P_F^{CF} as the total energy consumption in $N(G(V, E), S_F)$ when a symbol is communicated successfully for every session in S_F when the network is in compute-and-forward mode, and define P_B^{CF} as the total energy consumption in $N(G(V, E), S_B)$ when a symbol is communicated successfully for every session in S_B when the network is in compute-and-forward mode. We then define

n_f^F to be the number of sessions in which *face f* is the relay face in S_F , where $0 \leq f \leq N$ and r is a positive integer, i.e., $n_f^F = |\{S_i \in S_f \mid y_{b_i} < f < y_{a_i}\}|$, and n_f^B to be the number of sessions in which *face f* is the relay face in S_B , where $0 \leq f \leq N$ and r is a positive integer, i.e., $n_f^B = |\{S_i \in S_B \mid y_{a_i} < f < y_{b_i}\}|$. Based on the same reasons and derivation methods in getting P_U^{CF} and P_D^{CF} , we get the lower bounds of P_F^{CF} and P_B^{CF} ,

$$P_F^{CF} \geq (e_t + e_r) \left(\sum_{f=0}^N n_f^R + |S_R| \right),$$

$$P_B^{CF} \geq (e_t + e_r) \left(\sum_{f=0}^N n_f^L + |S_L| \right).$$

Of course adding sessions will not decrease the energy consumption, thus

$$P^{CF} \geq \max(P_U^{CF}, P_D^{CF}, P_R^{CF}, P_L^{CF}, P_F^{CF}, P_B^{CF}).$$

Again, it can be easily calculated by counting that

$$\sum_{f=0}^H n_f^U + \sum_{f=0}^H n_f^D + \sum_{f=0}^M n_f^R + \sum_{f=0}^M n_f^L + \sum_{f=0}^N n_f^F + \sum_{f=0}^N n_f^B + |S_U| + |S_D| + |S_R| + |S_L| + |S_F| + |S_B| = \sum_{i=1}^K d_i$$

So

$$\begin{aligned} P^{CF} &\geq \frac{1}{6} (P_U^{CF} + P_D^{CF} + P_L^{CF} + P_R^{CF} + P_F^{CF} + P_B^{CF}) \\ &= \frac{1}{6} [(e_t + e_r) \left(\sum_{f=0}^H n_f^U + |S_U| \right) + (e_t + e_r) \left(\sum_{f=0}^H n_f^D + |S_D| \right) \\ &\quad + (e_t + e_r) \left(\sum_{f=0}^M n_f^R + |S_R| \right) + (e_t + e_r) \left(\sum_{f=0}^M n_f^L + |S_L| \right) \\ &\quad + (e_t + e_r) \left(\sum_{f=0}^N n_f^F + |S_F| \right) + (e_t + e_r) \left(\sum_{f=0}^N n_f^B + |S_B| \right)] \\ &= \frac{1}{6} (e_t + e_r) \left(\sum_{f=0}^H n_f^U + \sum_{f=0}^H n_f^D + \sum_{f=0}^M n_f^R + \sum_{f=0}^M n_f^L + \sum_{f=0}^N n_f^F + \sum_{f=0}^N n_f^B \right. \\ &\quad \left. + |S_U| + |S_D| + |S_R| + |S_L| + |S_F| + |S_B| \right) \\ &= \frac{1}{6} (e_t + e_r) \sum_{i=1}^K d_i. \end{aligned}$$

From Lemma 1 we have the upper bound of energy consumption P^{TR} when applying traditional routing scheme

$$P^{TR} \leq (e_t + e_r) \sum_{i=1}^K d_i$$

So we come to the conclusion that

$$\begin{aligned} I &= \frac{P^{TR}}{P^{CF}} \\ &\leq \frac{(e_t + e_r) \sum_{i=1}^K d_i}{\frac{1}{6} (e_t + e_r) \sum_{i=1}^K d_i} \\ &= 6. \end{aligned}$$

□

In the case of rectangular lattice networks, we now get the conclusion that the energy benefits when applying compute-and-forward in 2D/3D rectangular lattice networks are upper bounded by two constants respectively. However to the best of our knowledge, there are still no schemes in which these upper bounds can be achieved.

6

CONCLUSIONS AND SUGGESTIONS

In the very last chapter, we finally arrive at conclusions of our work, and we will also give recommendations for future researches on related subjects. In Section 6.1, we will conclude our work for each of the two different modes we have studied, by answering three questions which we have proposed in Section 1.5. In Section 6.2, we will show the possible influence that our results might make on real life. Suggestions for the following researchers who are interested in further investigation of this subject will be given in Section 6.3.

6.1. IMPORTANT OUTCOMES

As we defined in Session 3.3, for a network the energy benefit is the ratio of the minimum total energy consumption when a symbol is communicated successfully for every session when the network is in traditional routing mode, and the minimum total energy consumption when a symbol is communicated successfully for every session when the network is in compute-and-forward mode. Now we firstly review the three major problems that we proposed in Section 1.5:

- What is the maximum energy benefit of applying compute-and-forward on a general wireless network with arbitrary placement for multiple unicast sessions?
- What are the relations between the energy benefit and the properties of the network, e.g., number of unicast sessions?
- What is the maximum energy benefit of applying compute-and-forward on some special networks, e.g. line networks, rectangular networks?

In the following, we give conclusions of our work by answering these questions.

When applying compute-and-forward on a general wireless network with arbitrary placement for multiple unicast sessions, the energy benefit is upper bounded by the average distance \bar{d} over all sessions in the network. This conclusion is supported by Theorem 1 as follows,

$$I \leq \bar{d}.$$

A tighter upper bound of the energy benefit which is also distance-based is supported by Theorem 2 as follows,

$$I \leq \frac{K\bar{d}}{\max[\sum_{j=1}^K d(a_j, B), \sum_{j=1}^K d(A, b_j)]}.$$

where a_j and b_j are source and destination nodes for session j , and A, B are the set of source nodes and set of destination nodes respectively.

The maximum energy benefit of applying compute-and-forward on special networks are given by Theorem 3 to Theorem 7. Theorem 3 is for networks with non-located source nodes and destination nodes. In such networks, every source node is different from each other, every destination node is also different from each other, moreover one source node in one session cannot be the destination node in another session. The energy benefit in such networks is upper bounded by

$$I < 2K.$$

The upper bounds of energy benefit in single source or single destination networks, in line networks, in 2D rectangular lattice and 3D rectangular lattice network are presented in Table 6.1.

Table 6.1: Energy benefit (I) in some special networks.

Special Network	Single Source/Single Destination Network	Line Network	2D Rectangular Lattice	3D Rectangular Lattice
Energy Benefit (I)	≤ 1	≤ 2	≤ 4	≤ 6

6.2. INFLUENCE ON REAL LIFE

In this thesis, we derive upper bounds of the energy benefit for compute-and-forward based schemes over traditional routing schemes in aspect of energy saving in networks when both transmission energy and reception energy are considered. We studied the upper bound of the energy benefit for wireless networks with multiple unicast sessions.

However we prove that compute-and-forward has no energy gain in networks with only one source node sending independent information to several destination nodes, or networks with only one destination node receiving independent information from several source nodes. Examples of such scenarios include sensor networks, where the sensors gather independent sensing information for only one sink. Another example is the mesh network with an Internet gateway, where there is unidirectional and independent traffic from/towards the gateway. The same result has been proved in [19] for network coding.

Moreover, in some sensor networks in real life which have the same features as line networks or 2D/3D rectangular lattice networks, we make the conjecture that the energy benefit is upper bounded by constant factors in these cases, when not only the transmission energy and reception energy but also other kinds of energy consumption, e.g. computation energy in relay nodes, are taken into consideration.

6.3. SUGGESTIONS FOR FUTURE RESEARCH

It is our conclusion that the energy benefit is upper bounded by the average distance \bar{d} over all sessions in general networks, and is upper bounded by $2K$ in networks with non-collocated source and destination nodes. While it is still an open question worth investigating whether there exists a tighter upper bound based on the number of sessions K for general networks. We make a conjecture here that the upper bound of energy benefit for general networks is $O(K)$, which means that the upper bound has the same order as K .

For more special networks, whether the energy benefit is always upper bounded by a constant, as we already proved in the single source/single destination network, line network, 2D/3D rectangular lattice network, still needs to be answered. Another interesting question is that for specific networks, whether there are some schemes in which the upper bound of energy benefit that we derived can be approached.

As we know, our work on the upper bound of energy benefit proves the limitation of compute-and-forward scheme in wireless networks, however by using compute-and-forward the broadcast and superposition properties of wireless networks are turned into advantageous characteristics for achieving higher transmission rates and saving energy. Thus it is an interesting topic that what is the lower bound of the energy benefit when applying compute-and-forward in wireless networks, which will prove the great potential of compute-and-forward scheme.

As we know, more systems are currently being designed to determine to what extent the promised performance gains are attainable over the real wireless medium using compute-and-forward, but much of this exciting path still lays ahead.

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