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Dynamic Multilevel Methods

for Simulation of Multiphase Flow in
Heterogeneous Porous Media

Matteo Cusini

**DYNAMIC MULTILEVEL METHODS FOR SIMULATION
OF MULTIPHASE FLOW IN HETEROGENEOUS
POROUS MEDIA**

DYNAMIC MULTILEVEL METHODS FOR SIMULATION OF MULTIPHASE FLOW IN HETEROGENEOUS POROUS MEDIA

Dissertation

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by the authority of the Rector Magnificus prof. dr. ir. T. H. J. van der Hagen,
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by

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DARSim
Delft Advanced Reservoir Simulation



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*If people do not believe that mathematics is simple, it is only because they do not realize
how complicated life is.*

John von Neumann

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SYMBOLS

A	pressure matrix
$A_{\langle ij \rangle}$	Surface between cells i and j
B_g	gas formation volume factor
B_o	oil formation volume factor
c	compressibility
C	correction functions operator
CP	comulative production
D	diffusion-dispersion tensor
d_l	longitudinal dispersion coefficient
d_t	transversal dispersion coefficient
G	permutation matrix
g	gravitational acceleration
h	unit vector in vertical direction
i	cell index
I	identity matrix
J	Leverett's J function
J	Jacobian matrix
J_{nat}	Jacobian matrix for natural variable formulation
J_{oc}	Jacobian matrix for overall-comp. formulation
K	rock permeability tensor
k_c	k-value of component c
k_α	relative permeability
l	ADM resolution index
L	Second stage preconditioner
M	CPR decoupling operator
N_{ADM}	number of fine-scale cells
N_{fs}	number of ADM active cells
n	unit normal vector
n_c	number of components
n_{ph}	number of phases
P	static prolongation operator
$\hat{P}$	dynamic prolongation operator
P_{bub}	bubble point pressure
$P_{c_{\alpha,\beta}}$	capillary pressure between phases α and β
P_{crit}	critical pressure
P_{dew}	dew point pressure
P_{ST}	pressure at standard conditions
p	pressure

q	source/sink term
$\mathbf{R}$	static restriction operator
$\hat{\mathbf{R}}$	dynamic restriction operator
R_s	solution gas/oil ratio
r	residual
r_{nat}	residual of natural variables formulation
r_{oc}	residual vector of overall-comp. formulation
S_α	saturation
SR	solvent rate
T_{crit}	critical temperature
t	time
$\mathbf{u}$	Darcy velocity
$\mathbf{u}_\alpha$	velocity of phase α
$\mathbf{u}_t$	total velocity
$V_{(i)}$	volume of cell i
$\mathbf{x}$	vector of unknowns
$y_{c\alpha}$	mass fraction of component c in phase α
$\tilde{y}_s$	solvent volumetric concentration
z_c	overall mass fraction of component c
α	phase index
β	phase index
Γ	domain boundary
$\delta \mathbf{x}$	Newton's update of vector of unknowns $\mathbf{x}$
δp	pressure Newton's update
δS	saturation Newton's update
δz	overall-comp. Newton's update
ϵ_c	error in the solvent concentration
ϵ_{CP}	error in the cumulative production
ϵ_p	relative error in p
ϵ_S	relative error in S
ϵ_{SR}	error in the solvent rate
ϵ_x	relative error in variable x
ϵ	arbitrary small value
ϵ_r	refinement tolerance
ϵ_c	coarsening tolerance
ζ	DLGR resolution measure
θ	contact angle
λ	phase mobility
μ	viscosity
μ_α	viscosity of phase α
ν	iteration index
ρ	density
ρ_α	density of phase α
ρ_g	gas density
ρ_g^{STC}	gas density at stock tank conditions

ρ_l	liquid density
ρ_o	oil density
ρ_o^{STC}	oil density at stock tank conditions
ρ_v	vapour density
σ	surface tension
Φ	pressure basis function
ϕ	porosity
ψ	saturation basis function
Ω	simulation domain
$\tilde{\Omega}$	primal coarse domain
$\tilde{\Omega}$	dual coarse domain
ω	TL mixing parameter

PREFACE

Dear reader,

The dissertation you are about to read, or just leaf through, is the product of the research work I carried out at TU Delft as a PhD candidate, between February 2015 and December 2018, under the supervision of my promotors, Dr. Hadi Hajibeygi and prof. Cor van Kruijsdijk. I worked in the petroleum engineering section of the Geosciences and Engineering department on a Shell-funded project. The main goal was to develop a multilevel approach for simulation of multiphase flow in highly heterogeneous porous media. The scientific motivations of this work are carefully outlined in Chapter 1.

Most of the chapters of this manuscript have been taken, either partly or entirely, from the scientific articles that I have published throughout these years. As such, some repetitions or slightly inconsistent notations may be present, even though I have done my best to limit them as much as possible.

I would like to use the rest of this short preface to tell you something about my personal experience as a PhD candidate at TU Delft.

It is certainly a complicated story, rich in "sliding-doors" and too long to be told here, the one that explains how I ended up working in the field of applied geosciences and more specifically reservoir simulation. However, I can tell you that I only came to the decision of pursuing a PhD degree during the last few months of my master studies while I was an intern at Schlumberger in Abingdon (UK) working on my thesis project. It was at that time that I understood I enjoyed working at the boundaries between physics and computational science and how challenging and exciting research can be. Thus, when Hadi offered me, in July 2014, a PhD position I accepted it enthusiastically and a few months later I was sitting at my desk at TU Delft.

Some people say a PhD is just a continuation of your studies, a way to delay the start of a "real job" and extend for a few years the "easy student life" free from real life responsibilities. The experience I had could not be further from this description. Studying books and papers is certainly an important part of the life of a PhD candidate. However, the approach is very different from the one of a student as everything is functional to a specific goal which is defined by the scope of the project.

In my case, the main activities of the past four years have been reading and writing papers (a lot), developing algorithms, giving presentations, and writing code. Computer programming has taken a very consistent part of my time and it is an activity that I have found myself liking a lot more than I anticipated. Additionally, I have had to handle a number of responsibilities such as organising practical sessions for master courses, a few lectures and help with the supervision of students during their thesis projects. I have been, in other words, responsible, for the first time in my life, for transmitting my knowledge to someone else.

Along with the hard PhD work the last four years were also full of fun activities including nights out with colleagues, football games between PhDs of different faculties and trips around the world (either for pleasure or to conferences). One of the most valuable aspects of my time as PhD candidate is the number of remarkable people that I have met around the world. The scientific community is rich in brilliant individuals that have a lot more to share than just the knowledge in their fields. By looking back I see numerous occasions in which I had the chance to exchange views on a variety of topics with excellent minds. This, by itself, makes me feel that the choice I made over 4 years ago was the right one.

I now thank you for deciding to have a look at my thesis and hope you can find something useful in it!

Matteo Cusini
Delft, January 27, 2019

I

INTRODUCTION

1

INTRODUCTION

The utilisation of subsurface resources is of utmost importance for many applications such as energy production, water management and reduction of greenhouse gas emissions. In the sole year 2017 an increase of 2.2 % in the world energy demand was registered [1]. This, along with the scarcity of water resources in some geographical regions, justifies, by itself, the attention of the scientific community towards the study of the subsurface. In fact, most fresh water resources can be found in underground aquifers and the contribution of sources stored in geological formations (e.g., fossil fuels), to the overall energy production, is dominant compared to others.

In particular, fossil fuels still represent the major source of primary energy and their contribution is not expected to be significantly reduced in the near future [1]. Therefore, to meet the global demand, hydrocarbon extraction has evolved throughout the years and, nowadays, enhanced oil recovery (EOR) methods are drawing a lot of interest as they allow to increase the recovery factors reached with traditional techniques [2, 3]. Additionally, unconventional resources, e.g., shale gas [4], which require specific extraction techniques, have become more prominent. Consequently, optimal and safe management of hydrocarbon reservoirs has become more challenging compared to the past.

Another growing energy production strategy, directly related to the subsurface, is geothermal energy [5, 6] which has the advantage of not suffering from the variability in the production as other climate-dependent energy sources.

Besides their contribution to the energy sector, geological formations have a huge storage capacity for green house gases (i.e., CO_2), pollutants and energy vectors like hydrogen [7, 8]. For example, CO_2 sequestration [9, 10] has already seen some successful pilot projects [11–13] showing potential to be a viable option to control the anthropic impact on the environment.

All these geoen지니어ing applications require a thorough understanding of how fluids displace in the underground and how they interact, physically and chemically, with the rock hosting them. Hydrocarbon reservoirs, geothermal plants and subsurface storage facilities can only be operated efficiently and safely with the help of computational models. Numerical simulations allow, for example, to estimate the amount of hydrocar-

bons extracted by a reservoir, to choose the optimal extraction strategy [14], to predict the life-time of a geothermal field [15–18] or the amount of CO₂ that can be stored in a target formation [19]. Additionally, computer models can help to better understand the impact of these applications on the surrounding environment so that it is possible to avoid or mitigate phenomena such as induced seismicity and subsidence [20–23] that have direct impacts on local communities. As such it is crucial to develop efficient and accurate computer modelling tools (i.e., reservoir simulators) that allow to understand and predict the behaviour of these complex systems.

The next section presents the mathematical model that is used to represent multi-phase fluid flow in natural formations. Such equations, along with the proper boundary conditions, can be employed to describe the systems mentioned beforehand.

1.1. GOVERNING EQUATIONS

The governing equations describing flow and transport processes in natural formations vary depending on the physics considered and are here presented in increasing complexity, starting from single phase flow and concluding with multicomponent multi-phase flow where mass transfer between different phases is also taken into account.

1.1.1. SINGLE PHASE FLOW

Flow of a single phase fluid in a porous medium is described by the mass conservation equation, i.e.

$$\frac{\partial}{\partial t}(\phi\rho) + \nabla \cdot (\rho u) = \rho q \quad (1.1)$$

Here, t is the time, ϕ is the rock porosity, u is the fluid velocity and ρ is the fluid density. Finally, q is the sink/source term, representing, e.g., wells. The fluid velocity can be expressed by using Darcy's law

$$u = -\lambda \cdot (\nabla p - \rho g \nabla h), \quad (1.2)$$

where, $\lambda = \frac{K}{\mu}$ is the fluid mobility where μ is the fluid viscosity and K is the rock permeability tensor. Finally, g is the gravitational constant and h a vector pointing in the direction of the gravitational force.

1.1.2. IMMISCIBLE MULTIPHASE FLOW

Mass conservation for immiscible fluids of n_{ph} total phases flowing through a porous medium reads

$$\frac{\partial}{\partial t}(\phi\rho_\alpha S_\alpha) + \nabla \cdot (\rho_\alpha u_\alpha) = \rho_\alpha q_\alpha \quad \forall \alpha \in \{1, \dots, n_{ph}\}, \quad (1.3)$$

where S_α , and q_α are the phase saturation, mobility, and source terms respectively. Phase velocities, u_α can be expressed as

$$u_\alpha = -\lambda_\alpha \cdot (\nabla p_\alpha - \rho_\alpha g \nabla h). \quad (1.4)$$

The phase mobility λ_α includes fluid and rock properties, i.e., $\lambda_\alpha = K \frac{k_{r\alpha}}{\mu_\alpha}$, where $k_{r\alpha}$ is the phase relative permeability [24].

Phase pressures p_α are related by capillary pressure, P_c , i.e.

$$p_\alpha - p_\beta = (1 - \delta_{\alpha,\beta}) P_{c_{\alpha,\beta}}, \quad \forall \alpha, \beta \in 1, \dots, n_{ph}. \quad (1.5)$$

Here, $\delta_{\alpha,\beta}$ is the Kronecker delta, which is 1 if $\alpha = \beta$ and 0 otherwise. Capillary pressure is negative if α is the wetting phase and positive otherwise. In general, $P_{c_{\alpha,\beta}}$ is a non-linear function of the wetting-phase saturation. Additionally, the following constraint can be used to eliminate one saturation unknown, i.e.

$$\sum_{\alpha=1}^{n_{ph}} S_\alpha = 1. \quad (1.6)$$

1.1.3. MULTICOMPONENT MULTIPHASE FLOW

CONSERVATION EQUATIONS

Mass conservation equations for an isothermal system of n_c components partitioned in n_{ph} phases flowing through a porous medium read

$$\frac{\partial}{\partial t} \left(\phi \sum_{\alpha=1}^{n_{ph}} y_{c\alpha} \rho_\alpha S_\alpha \right) + \nabla \cdot \left(\sum_{\alpha=1}^{n_{ph}} y_{c\alpha} \rho_\alpha \mathbf{u}_\alpha \right) - \sum_{\alpha=1}^{n_{ph}} y_{c\alpha} q_\alpha = 0, \quad \forall c \in \{1, \dots, n_c\}. \quad (1.7)$$

Here, $y_{c\alpha}$ is the mole fraction of component c in phase α . Note that Eq. (1.7) can be expressed both in terms of moles or mass of each component.

THERMODYNAMIC EQUILIBRIUM EQUATIONS

In addition to the mass balance equation described above, thermodynamic equilibrium equations have to be satisfied whenever more than one phase is present, i.e.

$$f_{c,\alpha}(p, y_{c\alpha}) - f_{c,\beta}(p, y_{c\beta}) = 0, \quad \forall \alpha \neq \beta \in \{1, \dots, n_{ph}\} \wedge c \in \{1, \dots, n_c\} \quad (1.8)$$

$$z_c - \sum_{\alpha=1}^{n_{ph}} v_\alpha y_{c\alpha} = 0, \quad \forall c \in \{1, \dots, n_c\} \quad (1.9)$$

$$\sum_{c=1}^{n_c} y_{c\alpha} - 1 = 0, \quad \forall \alpha \in \{1, \dots, n_{ph}\} \quad (1.10)$$

$$\sum_{\alpha=1}^{n_{ph}} v_\alpha - 1 = 0. \quad (1.11)$$

Here, $f_{c,\alpha}$ is the fugacity of component c in phase α , and z_c is the overall mole fraction of component c , i.e.

$$z_c = \frac{\sum_{\alpha=1}^{n_{ph}} y_{c\alpha} \rho_\alpha S_\alpha}{\sum_{\alpha=1}^{n_{ph}} \rho_\alpha S_\alpha}, \quad (1.12)$$

and

$$v_\alpha = \frac{S_\alpha \rho_\alpha}{\sum_{\beta=1}^{n_{ph}} S_\beta \rho_\beta} \quad (1.13)$$

is the molar fraction of phase α . In this work, thermodynamic equilibrium equations are written in terms of k-values. For each component c the ratio between its mole fractions in two phases α and β is written as

$$k_{c,\alpha\beta} = \frac{y_{c\alpha}}{y_{c\beta}}. \quad (1.14)$$

Here, $k_{c,\alpha\beta}$ is the k-value of component c in phases α and β and it's a function of pressure and temperature. For all test cases studied in this work, phase α is the vapour (lighter) phase and phase β is the liquid (denser) phase.

Thermodynamic equilibrium equations as solved when the relevant phases are present. Thus, it is important to first determine the number of existing phases (stability check) [25]. For simplicity, only two phases are considered in this work. For a two phase isothermal system at pressure p , both the vapour and the liquid phase are existing if $P_{dew} < p < P_{bub}$. Here, P_{bub} and P_{dew} are the bubble and the dew point pressures. If $p > P_{bub}$, the system is entirely in liquid phase and

$$\sum_{c=1}^{n_c} z_c k_c - 1 < 0 \quad (1.15)$$

holds [26]. Similarly, if $p < P_{dew}$ the system is entirely in vapour phase and

$$\sum_{c=1}^{n_c} \frac{z_c}{k_c} - 1 < 0, \quad (1.16)$$

is verified. Equations (1.15) and (1.16) allow for determination of the number of existing phases.

COMPOSITIONAL FLUID PARAMETERS

Two compositional fluid parameters are considered in this thesis: (1) a fully compositional model where both components can exist in each phase, and (2) a black-oil fluid model where only the lighter component can exist in both hydrocarbon phases. K-values [25, 27] and properties of the fluids are defined appropriately for each compositional setting. More precisely, for fully compositional settings the experimental K-values are taken from the literature [25], and for the black-oil model, where two components of gas (g) and oil (o) are partitioned into two phases of vapour (v) and liquid (l), the K-values read

$$k_o = 0$$

$$k_g = \frac{\rho_g^{STC} R_s + \rho_o^{STC}}{\rho_g^{STC} R_s}. \quad (1.17)$$

Here, ρ_g^{STC} and ρ_o^{STC} are the gas and the oil density at stock tank conditions. R_s is the solution gas-oil ratio, which is the volumetric gas fraction dissolved in the liquid phase at standard conditions [24]. Also, phase densities read

$$\rho_l = \frac{\rho_o^{STC} + \rho_g^{STC} R_s}{B_o}$$

$$\rho_v = \frac{\rho_g^{STC}}{B_g}. \quad (1.18)$$

Here, B_o and B_g are the formation volume factors of the oil and gas respectively and are both functions of pressure. Remark that, for a black-oil system, there is no need to distinguish between mass and molar quantities since mass based k-values are provided. Also, it is worth to be mentioned that the phase distribution parameters y_{gv} , y_{gl} , y_{ov} , y_{ol} can be related to the black-oil notations [28] according to

$$\begin{bmatrix} y_{gv} & y_{gl} \\ y_{ov} & y_{ol} \end{bmatrix} = \begin{bmatrix} 1 & \frac{\rho_g^{STC} R_s}{\rho_g^{STC} R_s + \rho_o^{STC}} \\ 0 & \frac{\rho_o^{STC}}{\rho_g^{STC} R_s + \rho_o^{STC}} \end{bmatrix}. \quad (1.19)$$

An extra component, water (w), and an extra aqueous phase (a) may be considered in the black-oil model. However, the water component is considered to be the only component present in the aqueous phase (i.e., $y_{wa} = 1$) and not to appear in the two hydrocarbon phases (i.e., $y_{wl} = y_{wv} = 0$).

1.2. SOLUTION STRATEGIES

The non-linear system of equations presented in the previous section does not have a general analytical solution and it is usually solved numerically. A finite-volume discretization in space is by far the most common approach due to its simplicity and to the fact that it ensures mass conservation [24]. Usually, the convective term of the mass conservation equations is discretized with an upwind two-point flux approximation (TPFA), consistently with most commercial reservoir simulators. Both implicit and explicit time integration schemes have been proposed in the literature. Additionally, two main classes of solution strategies exist: a fully-coupled (or fully implicit) approach in which all equations are solved simultaneously and a sequential strategy in which the problem is split into a parabolic (or elliptic) part and a hyperbolic one. The fully-implicit and the sequential solution strategies are briefly reviewed in the next subsection

1.2.1. FULLY IMPLICIT (FIM) APPROACH

The fully implicit (FIM) approach [29] solves the coupled system of discretized equations for all unknowns simultaneously and implicitly. In general all equations are first discretized and written in residual form such that a set of discrete equations of the form

$$r(x) = 0 \quad (1.20)$$

is obtained. Here, r is the residual vector and x is the vector of unknowns. Usually, the residual is a non-linear function of the unknowns so a global linearization method is employed [24, 29]. Typically, the Newton-Raphson method is used leading to

$$r^{v+1} \approx r^v + \left. \frac{\partial r}{\partial x} \right|^v \delta x^{v+1} = 0, \quad (1.21)$$

where v is an iteration index and δx is the increment. Thus, at each iteration a sparse large linear system of equations, in the form,

$$\mathbf{J}^v \delta x^{v+1} = -r^v \quad (1.22)$$

is solved until convergence is achieved (i.e., $r^v < \epsilon$, where ϵ is an arbitrary small real number.). Here, $\mathbf{J}$ is the so-called Jacobian matrix and contains the derivatives of r with respect to the unknowns x .

For real field applications, Eq. (1.22) leads to a linear system that is too large to be solved directly so that an iterative strategy [30] has to be employed.

1.2.2. SEQUENTIALLY-COUPLED APPROACHES

In sequential simulation approaches, the system of equations is solved at each time step in two solution steps. First, the component balance equations (either with a volume- or a mass-based approach) are linearly combined to form the pressure equation. This pressure equation is solved first, where only pressure dependent terms are implicitly treated. Then, phase velocities u_α are computed using Eq. 1.4 and the total velocity is calculated as

$$\mathbf{u}_t = \sum_{\alpha=1}^{n_{ph}} \mathbf{u}_\alpha. \quad (1.23)$$

The second step includes solving mass balance equations using Eq. (1.6) to remove one of the phase saturation unknowns [24, 28, 31]. Note that all transport dependent terms are fixed when pressure equation is being solved, and all pressure dependent terms are fixed when saturation transport equations are being solved. Operating such a split in during the solution process allows to employ the most suitable solution method for each equation. In particular, the pressure equation is parabolic (elliptic for the incompressible case) whereas the transport one is hyperbolic. The simplest approach, IMPES [24], is to employ an implicit time integration scheme to solve the pressure equation and an explicit one to solve the transport. However, IMPES suffers from severe restrictions in the allowable time-step size [32] and it is usually preferable to employ an implicit time integration also for the transport equation. It is known that the stability of such an approach is limited to the cases where the coupling between the two equations, i.e., flow (pressure) and transport (saturation), are not strong. Therefore, applicability of this approach for cases with strong capillary and compositional effects can lead to solution instabilities [32].

For this reason the sequential fully-implicit (SFI) strategy was introduced. In SFI simulation iterations are added between the pressure and the transport equations until convergence is reached. This strategy was first developed for immiscible multiphase flow and then extended to multicomponent multiphase flow for both black-oil [33, 34] and general compositional models [35–37]. Fig. 1.1 shows a schematic representation of sequential simulation approaches.

1.3. CHALLENGES OF RESERVOIR SIMULATION

Simulation of multiphase flow in natural formations requires to deal with many difficulties deriving from the multi-scale (both in time and space) nature of the process. In fact, geological formations have very large length scales compared to those at which most physical and chemical interactions occur. Additionally, even at the so called Darcy scale, natural porous media have highly heterogeneous properties (e.g., permeability). To accurately capture the physics of interest and to honour the heterogeneous properties, very

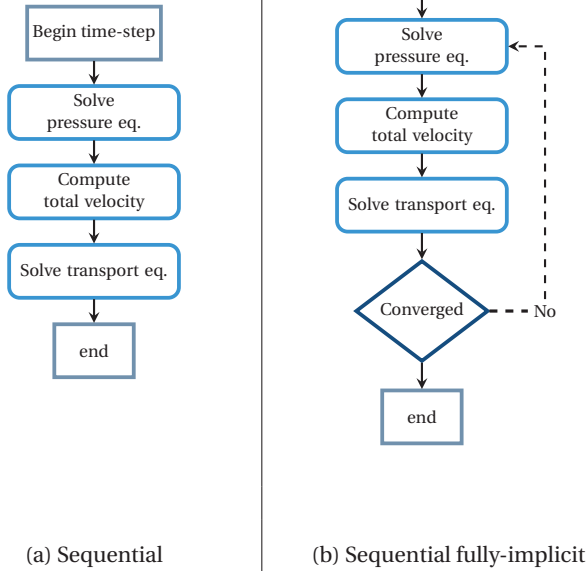


Figure 1.1: Schematic of sequentially-coupled (a) and sequential fully-implicit (b) simulation.

high resolution grids are required. However, the size of the domains and the necessity to run a large number of simulations to deal with the uncertainty of the parameters [38] make high resolution simulations impractical for field-scale applications.

Traditionally, upscaling techniques [39] have been used to reduce the computational cost by mapping rock and fluid properties to a coarser resolution, which makes computational simulation affordable. However, in presence of more complex physics, excessive upscaling may result in non-satisfactory results and, therefore, advanced algorithms and solvers have to be used to allow for higher resolution grids to be employed.

For example, due to the local nature of transport processes, a variety of dynamic local grid refinement (DLGR) techniques [40–47] have been proposed in the literature for both finite-element (FE) [48] and finite-volume (FV) [46, 49–53] schemes and for both sequential [51–54] and fully-coupled [50, 52] approaches. Note that if a method works for FIM simulations, it can be automatically employed for sequential approaches, as they are subsets of the generic FIM systems (with off-diagonal coupling terms being zero). In DLGR methods, the computational cost is reduced by adapting the grid resolution throughout the time-dependent simulation so that the high resolution grid is only employed at the advancing inter-phase front. However, such techniques are not easily employed in heterogeneous media, as mapping geological properties throughout different resolutions is challenging. The existing DLGR methods for heterogeneous media

[51, 52, 54] rely on pre-calculated upscaled (averaged) static quantities (e.g., permeability) at different levels. Therefore, the quality of their multilevel systems for multiphase simulations on highly heterogeneous domains may not be satisfactory. Once the grid resolution is determined, the linear systems need to be constructed. Classical DLGR methods provide a procedure for determination of non-neighbouring connections and adjustments of the data structure based on the dynamic number of grid cells present at each time step [52]. The overhead to restructure the connections and generate new interface quantities at each time-step (or iteration) might take a considerable amount of simulation costs. Consequently, it is important to develop an automatic (algebraic) procedure which constructs the system of discrete equations, at each time-step, for a dynamically generated multi-resolution grid.

Another class of advanced algorithms, trying to address the issue of combining different scales and speed up simulations, are multiscale methods [55, 56]. They were developed to efficiently solve the elliptic (or parabolic) pressure equation in sequentially coupled simulations. Such methods provide a systematic procedure to map across two grid resolutions (a fine and a coarse), with no dependency on upscaled quantities, the pressure equation and its solution. No general FIM multiscale procedure for flow and transport, which converges on an arbitrary adaptive multilevel resolution, exists in the literature. Developing an adaptive mesh simulation method which can be applied directly to heterogeneous problems, with no simplification of the underlying high-resolution heterogeneous data (e.g., permeability), is of great importance.

1.4. RESEARCH OBJECTIVES

Based on the challenges and the scientific needs introduced in the previous section, this work has two main objectives. The first one is to investigate how multiscale methods can be employed within FIM simulations. The second one is to develop a dynamic multilevel framework for fully-coupled simulation of multicomponent multiphase flow in heterogeneous porous media in which the whole system of equations and all unknowns are mapped between different resolutions. These objectives were achieved by setting the following intermediate steps:

- develop a method to employ multiscale methods within a FIM simulation strategy;
- develop a dynamic algebraic multilevel multiscale method suitable for FIM simulations in highly heterogeneous porous media;
- extend the method developed to more complex physics (i.e., compositional displacements).

1.5. THESIS OUTLINE

This dissertation consists of eight chapters, including this introduction, and one appendix. Firstly, in chapter 2, a brief literature review of multiscale methods is provided. Then, the multiscale finite-volume method is explained in detail, as many concepts are important for the main developments of this work. In chapter 3, multiscale methods are employed as a first-stage preconditioner in a two-stage preconditioning solution strat-

egy for FIM simulations. This represents a first attempt of employing multiscale methods for fully-coupled simulation.

Chapters 4 and 5 constitute the main body of this dissertation. In chapter 4 the algebraic dynamic multilevel method (ADM) is presented. This consists in the first dynamic multilevel multiscale strategy for FIM simulation of flow and transport in heterogeneous porous medium. Chapter 5, instead, contains the extension of ADM to more complex physics (i.e., compositional effects, capillarity and gravity) and 3D domains. In chapter 6 the ADM method is improved by employing an adaptive saturation interpolator that significantly increases its efficiency for some types of problems.

Chapter 7 shifts the focus from ADM to a specific physical phenomenon, incomplete mixing, for which a multi resolution simulation strategy is compared against a more traditional upscaling approach. This chapter shows the full potential of multilevel methods, like ADM, to accurately simulate complex physical processes.

Chapter 8 contains some general conclusions and a reflection about future perspectives of the work presented in this dissertation.

Finally, appendix A provides a brief description of the DARSim2 Matlab simulator which was developed throughout the PhD and served as platform in which to implement and test the methods and algorithms proposed.

2

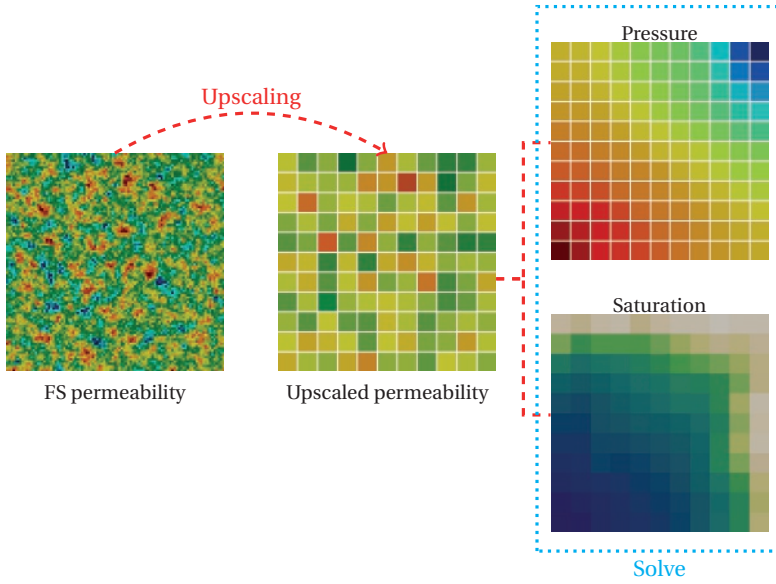
THE MULTISCALE FINITE-VOLUME (MsFV) METHOD

In this chapter, first, an overview of the main developments of multiscale methods for flow in porous media is provided. Then, the multiscale finite-volume method (MsFV) is described in details as it serves as a basis for most of the concepts introduced in the following chapters of this thesis.

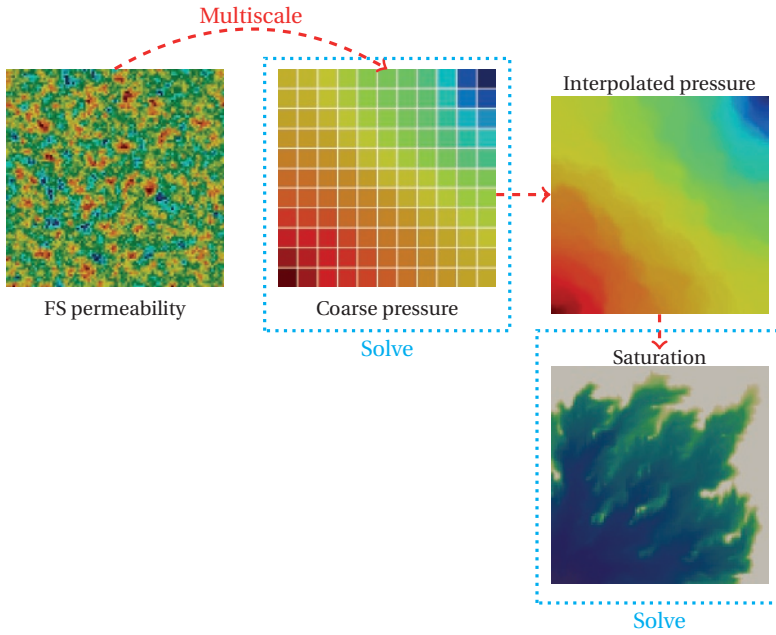
2.1. MULTISCALE METHODS FOR FLOW IN POROUS MEDIA

The Multiscale Finite Element (MsFE) [55, 57, 58] and Finite Volume (MsFV) [33, 56] methods were introduced to efficiently solve the elliptic (incompressible) or parabolic (compressible) pressure equation describing single-phase flow in porous media. These methods are used as an efficient solver for the pressure equation in simulations employing a sequential strategy, either IMPES [59] or sequential implicit [60], to treat the coupling between flow and transport. Multiscale methods reduce the computational cost by solving the pressure equation on a coarser grid resolution than that at which geological properties are defined. As such they represent an alternative to upscaling techniques [39] as methods to reduce the computational cost. However, contrary to upscaling, multiscale methods do not aim to define geological and fluid properties at different scales but only to construct an accurate and consistent mapping of the solution between two different resolutions as shown in Fig. 2.1. Multiscale methods construct a coarse-scale system for the pressure equation while preserving fine-scale information using local basis functions. Once the coarse pressure is found, the solution is prolonged (or interpolated) back to the original fine-scale resolution. Since such methods rely on a sequential split, transport equations are solved at the fine-scale resolution after reconstruction of a conservative velocity field [56]. Remark that, for field-size cases, the solution of the pressure system represents the real bottle neck of the simulation.

An algebraic description of multiscale methods has also been introduced, so that the coarse-scale system is formulated, starting from the fine-scale one, using prolon-



(a) Upscaling-based simulation



(b) Multiscale method simulation

Figure 2.1: Schematic representation of upscaling (a) and multiscale (b) simulations. Starting from a fine-scale geological map of the rock properties, upscaling defines geological properties on a coarse resolution on which governing equations are then solved. Multiscale methods, on the other hand, employ the fine-scale permeability field and then map the equations to a coarser solution on which a coarse-scale pressure equation is solved. Once a coarse pressure is found, it is interpolated back to the original resolution and used to solve the transport equations.

gation and restriction operators [61–65]. An iterative multiscale method has also been proposed to improve the quality of the interpolated pressure field and allow for convergence to the fine-scale solution [66]. This approach allows to employ multiscale methods as "physics-based" linear solvers and their performance has been thoroughly studied in the literature [65–67]. Recent advancements have shown the applicability of multiscale methods to fractured media [68–70], unstructured grids [71, 72], geomechanical deformations [73, 74], and even to real-field applications [75, 76]. The extensions also include enrichment strategies in presence of strong heterogeneity contrast that improve the multiscale solutions with additional basis functions (enrichment). These extensions were developed both within the FE and the FV framework [69, 77–79]. Multiscale methods, with a few exceptions [80, 81], have mainly been developed for the solution of the pressure (flow) equation for sequentially coupled simulations.

Recently, it has also been shown how to take advantage of multiscale methods within the context of inverse modelling using both analytical [82–84] and stochastic [85] gradient-based optimization algorithms. Additionally, it has also been suggested in the literature that multiscale methods may be a useful tool to bridge the differences between the scales of the collected data (e.g., seismic) and those of the geological models used, for example, in the oil and gas industry [86].

2.2. THE MSFV METHOD

To explain the multiscale finite volume method, the single phase incompressible flow equation is considered, i.e.

$$-\nabla \cdot (\boldsymbol{\lambda} \cdot \nabla p) = q \quad \text{on } \Omega \subseteq \mathbb{R}^d \quad (2.1)$$

where $d = \{1, 2, 3\}$ is the number of dimensions. The appropriate boundary conditions are also considered, i.e.

$$\begin{cases} -\boldsymbol{\lambda} \cdot \nabla p \cdot \mathbf{n} = \bar{u} & \text{on } \Gamma_u \\ p = \bar{p} & \text{on } \Gamma_p, \end{cases} \quad (2.2)$$

where, $\Gamma_u \cup \Gamma_p = \partial\Omega$ and $\mathbf{n}$ is the unit normal vector relative to $\partial\Omega$.

A fine-scale grid with N_f cells is imposed on the domain Ω and Eq. (2.1) is discretized with the finite-volume method, employing a two-point flux approximation (tpfa). The system of discrete equations is of the form

$$\mathbf{A}_f p_f = q_f. \quad (2.3)$$

Here, $\mathbf{A}_f$ is a $N_f \times N_f$ matrix and it is referred to as fine-scale pressure matrix. The MsFV method is designed to compute efficiently an approximate solution of Eq. (2.3) in presence of highly heterogeneous rock permeabilities.

The MsFV method superimposes to the fine-scale grid two overlapping coarse grids: (1) a primal coarse grid with N_c control volumes, $\tilde{\Omega}_k$ $k \in \{1, \dots, N_c\}$, indicated by the red lines in Fig. 2.2 (b); (2) a dual grid with N_d control volumes, $\tilde{\Omega}_j$ $j \in \{1, \dots, N_d\}$, shown in blue in Fig. 2.2 (c).

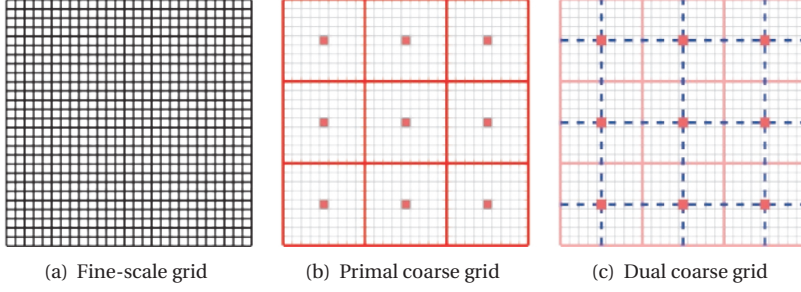


Figure 2.2: MsFV grids: the fine-scale (a), the primal coarse (b) and the dual coarse (c) grid are shown.

The fine-scale pressure, p_f , is approximated by

$$p_f \approx p' = \sum_{j=1}^{N_d} \left(\sum_{k=1}^{N_c} \Phi_j^k p^k \right). \quad (2.4)$$

Here, p^k is the pressure value relative to the coarse control volume k and Φ_j^k is the basis function relative to the primal node k with values in $\tilde{\Omega}_j$. Basis functions are obtained by solving, numerically, a localized flow problem with zero right-hand-side on each dual coarse domain $\tilde{\Omega}_j$. The reduced boundary condition is employed to localise the flow problem on each dual coarse domain, Ω_j , so that the following local problem is solved [65],

$$\begin{cases} -\nabla \cdot (\lambda \cdot \nabla \Phi_j^k) = 0 & \text{on } \tilde{\Omega}_j \\ -\nabla_{||} \cdot (\lambda \cdot \nabla \Phi_j^k)_{||} = 0 & \text{on } \partial \tilde{\Omega}_j \\ \Phi_j^k(x_i) = \delta_{ki} & \forall x_i \in \{1, \dots, N_c\}. \end{cases} \quad (2.5)$$

Here, δ_{ki} is the Kronecker delta and the subscript $||$ indicates that only the component parallel to the boundary $\partial \tilde{\Omega}_j$ is considered. This corresponds to solving a problem of reduced dimensions at the boundary of each dual coarse control volume. Thus, in 2D a 1D problem is solved as shown, for example, in Fig. 2.3. Note that, by construction, basis functions satisfy partition of unity, i.e.

$$\sum_{k=1}^{N_c} \sum_{j=1}^{N_d} \Phi_j^k = 1. \quad (2.6)$$

The MsFV pressure coarse system can be obtained by substituting Eq. 2.4 in Eq. 2.1 and by integrating over each primal coarse control volume $\tilde{\Omega}_k$, i.e.

$$-\int_{\tilde{\Omega}_k} \nabla \cdot \left(\lambda \cdot \nabla \sum_{j=1}^{N_d} \left(\sum_{k=1}^{N_c} \Phi_j^k p^k \right) \right) d\Omega = \int_{\tilde{\Omega}_k} q d\Omega \quad \forall k = \{1, \dots, N_c\} \quad (2.7)$$

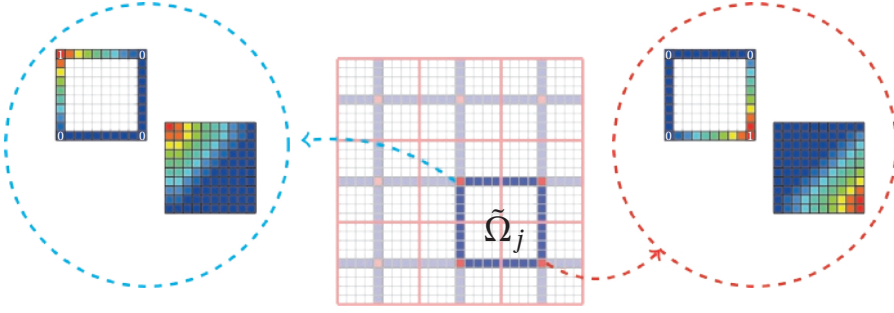


Figure 2.3: Representation of the procedure followed to compute two basis functions in a dual coarse domain $\tilde{\Omega}_j$.

which, by applying Gauss theorem, becomes

$$\sum_{k=1}^{N_c} p^k \sum_{j=1}^{N_d} \int_{\partial \tilde{\Omega}_k} \left(-\lambda \cdot \nabla \Phi_j^k \right) \cdot \mathbf{n}_k d\Gamma = \int_{\tilde{\Omega}_k} q d\Omega \quad \forall k = \{1, \dots, N_c\} \quad (2.8)$$

where $\mathbf{n}_k$ is the unit normal vector pointing out of $\partial \tilde{\Omega}_k$. Eq. (2.8) corresponds to a linear system of the form

$$\mathbf{A}_c p_c = q_c \quad (2.9)$$

where

$$\mathbf{A}_c(l, k) = \sum_{j=1}^{N_d} \int_{\partial \tilde{\Omega}_l} \left(-\lambda \cdot \nabla \Phi_j^k \right) \cdot \mathbf{n}_k d\Gamma \quad (2.10)$$

and

$$q_c(k) = \int_{\tilde{\Omega}_k} q d\Omega. \quad (2.11)$$

As such, the coarse scale pressure $p_c = [p_1, \dots, p_{N_c}]^T$ satisfies the mass balance equation for each coarse cell $\tilde{\Omega}_k$. Additionally, the coarse pressure matrix in Eq. (2.9) has a larger stencil compared to the fine-scale matrix, as diagonal fluxes are also considered.

The MsFV procedure to construct the coarse system can be expressed in terms of restriction and prolongation operators [61], i.e.

$$\underbrace{(\mathbf{R} \mathbf{A}_f \mathbf{P})}_{\mathbf{A}_c} p_c = \underbrace{\mathbf{R} q_f}_{q_c}. \quad (2.12)$$

Here, $\mathbf{R}$ is the restriction operator which is a $N_c \times N_f$ matrix with values in the k -th row being 1 at column j if cell j belongs to the coarse cell k , and 0 elsewhere, i.e.,

$$\mathbf{R}(k, j) = \begin{cases} 1 & \text{if cell } j \in \tilde{\Omega}^k, \\ 0 & \text{otherwise.} \end{cases} \quad (2.13)$$

The prolongation operator, $\mathbf{P}$, on the other hand, is a $N_f \times N_c$ matrix and each of its columns k contains the basis function, Φ_k , defined as

$$\Phi^k = \sum_{j=1}^{N_d} \Phi_j^k. \quad (2.14)$$

Alternatively, the multiscale coarse system can be constructed by employing a classical Galerkin-type operator, i.e., $\mathbf{R}_{FE} = \mathbf{P}^T$, resulting in the MsFE method [55]. This, ensures that the coarse matrix $\mathbf{A}_c$ is symmetric but does not guarantee mass conservation on the primal coarse grid. It has been shown that such an approach is more effective whenever the multiscale is used as an iterative linear solver [65].

2.3. ALGEBRAIC MULTISCALE SOLVER (AMS)

The procedure described in the previous section to construct the multiscale basis functions can be described with an algebraic formalism [65]. First, a wire-basket reordering [87] is applied to the fine-scale discrete system of Eq. (2.3), i.e.

$$\mathbf{A} = \mathbf{G} \mathbf{A}_f \mathbf{G}^T. \quad (2.15)$$

Here, $\mathbf{G}$ is an orthogonal permutation matrix that contains all the information about the dual coarse grid topology. Thus, the reordered linear system, for a 3D structured grid, is of the form

$$\underbrace{\begin{bmatrix} \mathbf{A}_{II} & \mathbf{A}_{IF} & & & \\ \mathbf{A}_{FI} & \mathbf{A}_{FF} & \mathbf{A}_{FE} & & \\ & \mathbf{A}_{EF} & \mathbf{A}_{EE} & \mathbf{A}_{EV} & \\ & & \mathbf{A}_{VE} & \mathbf{A}_{VV} & \end{bmatrix}}_{\mathbf{A}} \begin{bmatrix} p_I \\ p_F \\ p_E \\ p_V \end{bmatrix} = \begin{bmatrix} q_I \\ q_F \\ q_E \\ q_V \end{bmatrix}. \quad (2.16)$$

Here, the subscript I, F, E and V indicate interior, face, edge and vertex cells, as defined by the dual coarse grid partition (Fig. 2.4).

The localization assumption, described in the previous section, is equivalent to setting to zero the blocks, $\mathbf{A}_{FI}$ and $\mathbf{A}_{EF}$, corresponding to the connections between “face” and “interior” cells and between “face” and “edge” cells. Thus, assuming the coarse-scale system is of the form $\mathbf{A}_c p'_V = q_c$, the full multiscale stage reads

$$\begin{bmatrix} \mathbf{A}_{II} & \mathbf{A}_{IF} & & & \\ & \tilde{\mathbf{A}}_{FF} & \mathbf{A}_{FE} & & \\ & & \tilde{\mathbf{A}}_{EE} & \mathbf{A}_{EV} & \\ & & & \mathbf{A}_c & \end{bmatrix} \begin{bmatrix} p'_I \\ p'_F \\ p'_E \\ p'_V \end{bmatrix} = \begin{bmatrix} q_I \\ q_F \\ q_E \\ q_c \end{bmatrix}. \quad (2.17)$$

Here, $\tilde{\mathbf{A}}_{FF}$ and $\tilde{\mathbf{A}}_{EE}$ are the face and edge blocks that have been modified to account for the connections that have been deleted. The system of Eq. (2.17) is upper triangular and can be easily solved by employing a backward substitution algorithm. Consequently,

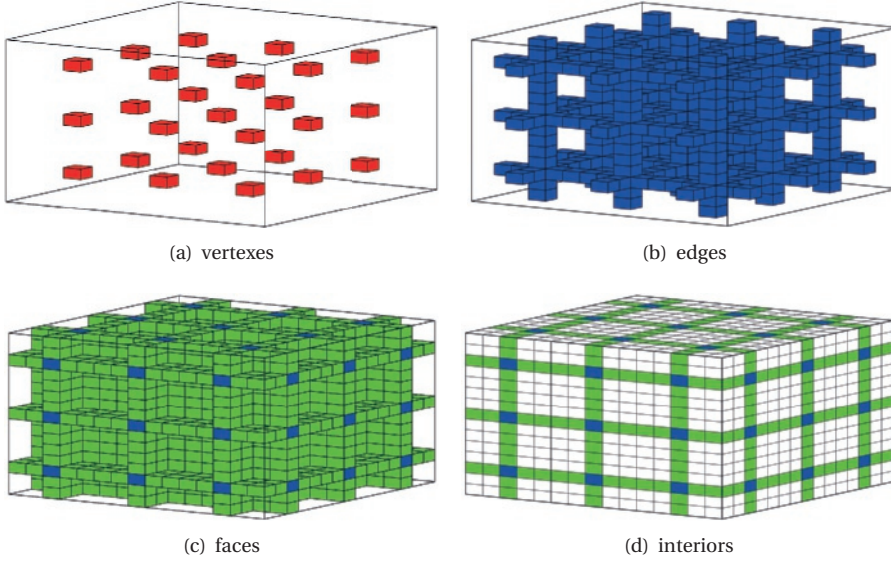


Figure 2.4: Dual grid partition for a 3D domain.

given the coarse solution p'_V the approximate fine-scale solution is found as

$$\begin{bmatrix} p'_I \\ p'_F \\ p'_E \\ p'_V \end{bmatrix} = \begin{bmatrix} -\mathbf{A}_{II}^{-1} \left(\mathbf{A}_{IF} \tilde{\mathbf{A}}_{FF}^{-1} (\mathbf{A}_{FE} \tilde{\mathbf{A}}_{EE}^{-1} \mathbf{A}_{EV}) \right) \\ \tilde{\mathbf{A}}_{FF}^{-1} (\mathbf{A}_{FE} \tilde{\mathbf{A}}_{EE}^{-1} \mathbf{A}_{EV}) \\ -\tilde{\mathbf{A}}_{EE}^{-1} \mathbf{A}_{EV} \\ \mathbf{I}_{VV} \end{bmatrix} p'_V + \mathbf{C} \begin{bmatrix} q_I \\ q_F \\ q_E \\ q_c \end{bmatrix}. \quad (2.18)$$

Thus, by definition, the multiscale prolongation operator, $\mathbf{P}$, is

$$\mathbf{P} = \mathbf{G}^T \begin{bmatrix} -\mathbf{A}_{II}^{-1} \left(\mathbf{A}_{IF} \tilde{\mathbf{A}}_{FF}^{-1} (\mathbf{A}_{FE} \tilde{\mathbf{A}}_{EE}^{-1} \mathbf{A}_{EV}) \right) \\ \tilde{\mathbf{A}}_{FF}^{-1} (\mathbf{A}_{FE} \tilde{\mathbf{A}}_{EE}^{-1} \mathbf{A}_{EV}) \\ -\tilde{\mathbf{A}}_{EE}^{-1} \mathbf{A}_{EV} \\ \mathbf{I}_{VV} \end{bmatrix}. \quad (2.19)$$

The matrix $\mathbf{C}$, instead, represents the correction functions operator which is not considered in this dissertation.

II

MSFV AS A PRECONDITIONER

3

CONSTRAINED PRESSURE RESIDUAL MULTISCALE (CPR-MS) SOLVER

As mentioned in chapter 2, all MsFV methods rely on a sequential (IMPES- or sequential implicit- type) strategy [89] to treat the flow-transport coupling. Unfortunately, when the coupling terms between flow and transport equations are strong, sequential strategies may not be efficient [32]. Strong coupling terms exist in many practical applications, for example in multiphase flows with strong capillary and compositional effects [90, 91]. For these cases, fully implicit (FIM) systems are generally more stable than sequential strategies [31, 92]. Of great importance in the multiscale community is to develop multiscale methods for fully implicit systems in order to further extend the boundaries of multiscale simulation strategies towards more challenging and realistic cases.

In this chapter, we devise the first multiscale (MS) method for fully implicit simulations of multiphase flow in highly heterogeneous porous media. Built on the fully implicit Jacobian matrix, a constrained pressure residual (CPR) preconditioning method [93, 94] is used to extract the pressure system. This pressure system is then coupled with a second-stage solver for the full residual correction [93, 94]. The multiscale solvers are then developed to solve the CPR-based pressure equations leading to a CPR-MS simulation strategy. More precisely, the global solver (for low-frequency errors) is constructed based on a multiscale system (either MsFV or MsFE), before and after which pre- and post- smoothing steps are applied on the fine-scale residual to eliminate high-frequency errors. For solution of the coarse-scale systems the effect of different solution strategies, including AMG [95], is studied.

The performance of CPR-MS for three-dimensional (3D) heterogeneous cases is presented in detail based on CPU times and the number of linear and nonlinear iterations

Parts of this chapter have been published in the Journal of Computational Physics, Vol. 299 (2015) [88].

for heterogeneous three-dimensional multiphase flows in presence of gravitational and capillarity effects.

This multistage multiscale strategy (i.e., CPR-MS) combines the advantages of multiscale methods with the stability of fully implicit systems for nonlinear coupled problems. As such, being integrated within the FIM iterative procedure, the CPR-MS captures the strong nonlinear coupling terms and highly heterogeneous coefficients efficiently. The CPR-MS method is an important advancement that put next-generation multiscale-based reservoir simulators into the context of strong nonlinear physics.

The chapter is structured as follows. In Section 3.1 the CPR-based FIM simulation strategy is briefly described. Then, the CPR-MS method is presented in Section 3.2, followed by Section 3.3, where numerical results are presented for 3D heterogeneous multiphase problems.

3

3.1. CPR-BASED FULLY IMPLICIT RESERVOIR SIMULATION

Here, the linear system arising from the FIM discretization of the equations describing multicomponent multiphase flow in porous media, Eqs. (1.7) - (1.11), is considered, i.e.,

$$\mathbf{J}\delta x = \begin{bmatrix} \mathbf{J}_{pp} & \mathbf{J}_{ps} \\ \mathbf{J}_{sp} & \mathbf{J}_{ss} \end{bmatrix} \begin{bmatrix} \delta x_p \\ \delta x_s \end{bmatrix} = -r = - \begin{bmatrix} r_p \\ r_s \end{bmatrix}, \quad (3.1)$$

Here, the block matrices $\mathbf{J}_{pp}$ and $\mathbf{J}_{ss}$ contain pressure and non-pressure (saturation S_α and component molar fractions $y_{c\alpha}$) coefficients, respectively. Also, $\mathbf{J}_{sp}$ represents the respective coupling coefficients. δx_p and δx_s are the primary (here, pressure) and secondary (here, S_α and $y_{c\alpha}$) variable increments, respectively.

In many practical scenarios, the linear solver step represents a dominant part of the overall computational time. It is a well-known fact that performance of iterative linear solvers strongly depends on robust and fast preconditioners, amenable for massive parallelization. Additionally, for realistic-scale problems, the memory requirement becomes also an important issue.

The CPR method [93, 96] was devised on the basis of targeting the parabolic part of the system (elliptic for incompressible flow) as a separate inner stage (predictor stage), combined with the FIM residual corrector stage. The combination of this predictor-corrector strategy lead to improved convergence properties compared with the case of iterating only on the FIM system.

The CPR pressure equation is extracted from the FIM Eq. (3.1). First a decoupling matrix $\mathbf{M}$ is multiplied to the system,

$$\underbrace{\mathbf{M}\mathbf{J}}_{\mathbf{J}^*} \delta x = - \underbrace{\mathbf{M}r}_{r^*}, \quad (3.2)$$

leading to the modified new system $\mathbf{J}^* \delta x = -r^*$, i.e.,

$$\begin{bmatrix} \mathbf{J}_{pp}^* & \mathbf{J}_{ps}^* \\ \mathbf{J}_{sp} & \mathbf{J}_{ss} \end{bmatrix} \begin{bmatrix} \delta x_p \\ \delta x_s \end{bmatrix} = - \begin{bmatrix} r_p^* \\ r_s \end{bmatrix}. \quad (3.3)$$

The choices of $\mathbf{M}$ can be made such that the off-diagonal $\mathbf{J}_{ps}^*$ becomes small, or, in the case of Schur complement, it becomes 0. Neglecting $\mathbf{J}_{ps}^*$ can be stated algebraically

as multiplying a matrix $\mathbf{C} = [\mathbf{I} \quad \mathbf{0}]^T$ to the system. The pressure system is then formed by solving for an approximate pressure increment $\delta x'_p$ as

$$\underbrace{(\mathbf{C}^T \mathbf{M} \mathbf{J} \mathbf{C})}_{\mathbf{A}_{cpr}} \delta x'_p = -\mathbf{C}^T \mathbf{r}^*, \quad (3.4)$$

which is called the CPR pressure system $\mathbf{A}_{cpr} = (\mathbf{C}^T \mathbf{M} \mathbf{J} \mathbf{C})$.

It is clear that the effectiveness of the CPR procedure depends, to a large extent, on the quality of the extracted pressure matrix $\mathbf{A}_{cpr}$, i.e., the choice of $\mathbf{M}$. Three of the most common options for $\mathbf{M}$ are outlined in the following subsections.

3.1.1. CPR: ABF REDUCTION APPROACH

In the Alternate-Block Factorization (ABF) procedure [95, 97], the decoupling operator is defined as

$$\mathbf{M}^{-1} = \begin{bmatrix} \mathbf{D}_{pp} & \mathbf{D}_{ps} \\ \mathbf{D}_{sp} & \mathbf{D}_{ss} \end{bmatrix} = \begin{bmatrix} \text{diag}(\mathbf{J}_{pp}) & \text{diag}(\mathbf{J}_{ps}) \\ \text{diag}(\mathbf{J}_{sp}) & \text{diag}(\mathbf{J}_{ss}) \end{bmatrix}. \quad (3.5)$$

Applying this operator to the linear system (3.1) leads to

$$\mathbf{M} \mathbf{J} = \begin{bmatrix} \Lambda^{-1} & \mathbf{0} \\ \mathbf{0} & \Lambda^{-1} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{D}_{ss} \mathbf{J}_{pp} - \mathbf{D}_{ps} \mathbf{J}_{sp} & \mathbf{D}_{ss} \mathbf{J}_{ps} - \mathbf{D}_{ss} \mathbf{J}_{ss} \\ \mathbf{D}_{pp} \mathbf{J}_{sp} - \mathbf{D}_{sp} \mathbf{J}_{pp} & \mathbf{D}_{pp} \mathbf{J}_{ps} - \mathbf{D}_{pp} \mathbf{J}_{ps} \end{bmatrix}, \quad (3.6)$$

with $\Lambda = \mathbf{D}_{pp} \mathbf{D}_{ss} - \mathbf{D}_{ps} \mathbf{D}_{sp}$. It can be verified that the main diagonal entries of $\mathbf{J}_{pp}^*$ and $\mathbf{J}_{ss}^*$ are all equal to one, and those of the off-diagonal blocks $\mathbf{J}_{ps}^*$, $\mathbf{J}_{sp}^*$ are all equal to zero [95]. Thus, the pressure equation to be solved is

$$\mathbf{J}_{pp}^* \delta x'_p = [\Lambda^{-1} (\mathbf{D}_{ss} \mathbf{J}_{pp} - \mathbf{D}_{ps} \mathbf{J}_{sp})] \delta x'_p = -\mathbf{r}_p^*. \quad (3.7)$$

3.1.2. CPR: QUASI-IMPES REDUCTION APPROACH

In the quasi-IMPES approach, the $\mathbf{M}$ matrix reads

$$\mathbf{M} = \begin{bmatrix} \mathbf{I} & -\text{diag}(\mathbf{J}_{ps}) \text{diag}^{-1}(\mathbf{J}_{ss}) \\ \mathbf{0} & \mathbf{I} \end{bmatrix}, \quad (3.8)$$

where $\text{diag}(\mathbf{J}_{ps})$ and $\text{diag}(\mathbf{J}_{ss})$ are the diagonal matrices consisting of the main diagonals of $\mathbf{J}_{ps}$ and $\mathbf{J}_{ss}$, respectively.

This choice would then lead to the following pressure equation

$$\mathbf{J}_{pp}^* \delta x'_p = [\mathbf{J}_{pp} - \text{diag}(\mathbf{J}_{ps}) \text{diag}^{-1}(\mathbf{J}_{ss}) \mathbf{J}_{sp}] \delta x'_p = -\mathbf{r}_p^*. \quad (3.9)$$

3.1.3. CPR: TRUE-IMPES REDUCTION APPROACH

In the true-IMPES approach, a simplified Schur complement is considered as the decoupling operator, i.e.,

$$\mathbf{M} = \begin{bmatrix} \mathbf{I} & -\text{colsum}(\mathbf{J}_{ps}) \text{colsum}(\mathbf{J}_{ss})^{-1} \\ \mathbf{0} & \mathbf{I} \end{bmatrix}, \quad (3.10)$$

where 'colsum' operator replaces $\mathbf{J}_{ps}$ and $\mathbf{J}_{ss}$ matrices by vectors whose elements are the sums of each column. Therefore, the off-diagonal sub-matrix reads

$$\mathbf{J}_{ps}^* = [\mathbf{J}_{ps} - \text{colsum}(\mathbf{J}_{ps})\text{colsum}(\mathbf{J}_{ss})^{-1}\mathbf{J}_{ss}], \quad (3.11)$$

which is nonzero, but will be eliminated in the pressure equation (by employing operator $\mathbf{C}$). Finally, the pressure equation reads

$$\mathbf{J}_{pp}^* \delta x_p' = [\mathbf{J}_{pp} - \text{colsum}(\mathbf{J}_{ps})\text{colsum}(\mathbf{J}_{ss})^{-1}\mathbf{J}_{sp}] \delta x_p' = -r_p^*. \quad (3.12)$$

Equation (3.4), using one of the three mentioned options for $\mathbf{M}$, forms the first stage of the iterative procedure at each time step. As mentioned before, properties of the pressure matrix $\mathbf{A}_{cpr}$ depend strongly on the choice made for the decoupling operator $\mathbf{M}$. The ABF approach generates a strongly nonsymmetric pressure matrix, compared to the original pressure block $\mathbf{J}_{pp}$, which would have negative impact on AMG methods [95]. IMPES-like CPR methods (quasi-IMPES and true-IMPES) would lead to more symmetric pressure equations (compared with ABF). In particular, true-IMPES would construct the closest to a symmetric system [98].

The linear system (3.1) is solved using Flexible GMRES (FGMRES) [99, 100] with a two-stage preconditioner. In the first preconditioner stage, typically one V-cycle of an AMG method is employed to obtain an approximate solution of Eq. (3.4). In the second preconditioner stage, a classical iterative solver, e.g., ILU(0) [30], is applied to the FIM system. The entire procedure for one time step is illustrated in Fig. 3.1 [94, 101].

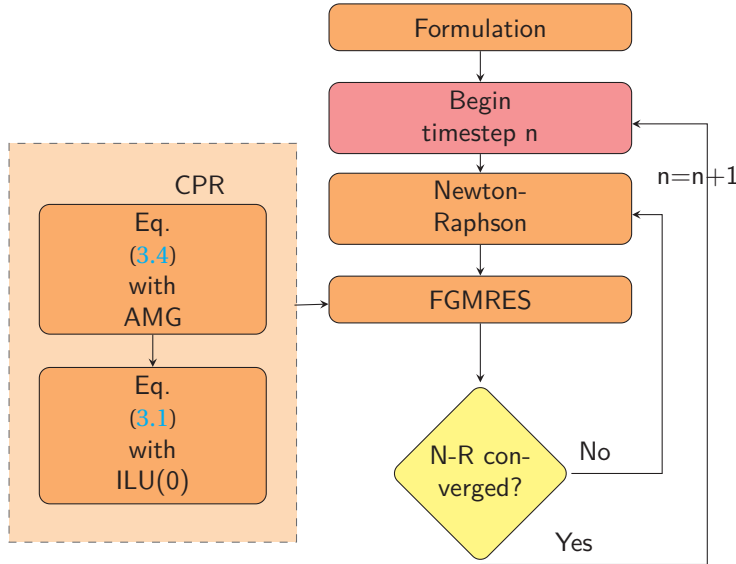


Figure 3.1: CPR-based FIM simulation framework: two stage CPR where AMG is used to solve the pressure equation and ILU(0) for the second-stage full residual correction.

3.2. CPR-MS SOLVER

The CPR-MS algorithm [102] employs a two-stage preconditioner, where an AMS method is used for solving the pressure system extracted by the CPR decoupling operator. Starting from the fully implicit Jacobian matrix $\mathbf{J}$, the solution strategy for one iteration loop is summarized in the following steps:

1. *Initialize:*
using FIM system solution x^{*v} , update FIM residual r^{*v}
2. *1st stage: pressure stage*
 - 2.1 *CPR stage:* extract $\mathbf{A}_{cpr}$ and r_p^v from the FIM system, as in Eq. (3.4).
 - 2.2 *Pre-smoothing stage:*
Perform n_{s_1} smoothing iterations on Eq. (3.4), update accumulated correction vector $\delta x_p^{v+s_1} = \sum_{i=1}^{s_1} \delta x_p^i$ and final residual $r_p^{v+s_1}$.
 - 2.3 *Employ multiscale stage:*
solve (or iterate on) $\underbrace{(\mathbf{R}\mathbf{A}_{cpr}\mathbf{P})}_{\mathbf{A}^c} \delta x_p^c = -\underbrace{\mathbf{R}r_p^{v+s_1}}_{r_p^c}$, and
update $\delta x_p^{v+s_1+1} = \delta x_p^{v+s_1} + \mathbf{P}\delta x_p^c$ and residual $r_p^{v+s_1+1}$.
 - 2.4 *Post-smoothing stage:*
Perform n_{s_2} smoothing iterations on Eq. (3.4), update $\delta x_p^{v+s_1+1+s_2}$ and residual $r_p^{v+s_1+1+s_2}$.
3. *2nd stage: FIM stage*
 - 3.1 *Update the FIM residual:* $r^{*v+1} = r^v + \mathbf{J}(\mathbf{C}\delta x_p^{v+s_1+1+s_2})$.
 - 3.2 *Employ the second stage preconditioner:* $\delta x^{*v+1} = -\mathbf{L}^{-1}r^{*v+1}$.
4. *Update the solution:* $x^{*v+1} = x^{*v} + \delta x^{*v+1} + \mathbf{C}\delta x_p^{v+s_1+1+s_2}$.

In this work, multiscale basis functions are computed, as described in Chapter 2, using the pressure equation obtained following an IMPES strategy. Although the IMPES pressure equation differs from that of CPR, it has been successfully employed as a preconditioner for FIM systems [103]. Thus, one may also use the pressure equation of the IMPES formulation for the basis function calculations. An other option would be to directly employ the CPR-based pressure equation to compute the basis functions. While both approaches are possible, convergence rates and numerical efficiencies may be affected by this choice. As such, a comparison study needs to be performed to obtain the optimum formulation for basis functions, similar to [67] for compressible problems, which is, however, beyond the scope of this work.

For optimum performance, MS restriction ($\mathbf{R}$) and prolongation ($\mathbf{P}$) operators (i.e., basis functions) should be adaptively updated based on the local change of parameters [60]. For the test cases of this chapter, however, they are computed at the beginning of each time step and kept constant throughout the Newton loop. In addition, the size and

sparcity of the coarse pressure matrix depend on the coarsening ratio and the choice of the restriction operator ($\mathbf{R}_{FV}$ and $\mathbf{R}_{FE}$). If the size of the coarse system is small enough, a direct solver can be used. Otherwise, an iterative method should be employed to achieve efficient coarse-scale solutions. Here, different iterative strategies involving AMG are considered.

Finally, Gauss Seidel (GS) and ILU(0) are considered as pre- and post smoothing steps on the fine-scale pressure system, i.e., Steps (2) and (4). Following [65, 67] no Correction Functions neither MsFV coarse-scale solver have been considered in this work.

Next, as a proof of concept, numerical results for three-dimensional heterogeneous problems are presented.

3.3. NUMERICAL RESULTS

Two different heterogeneous three-dimensional models with multiphase flow under gravitational force and injection/production wells are considered: (1) the SPE 9 model (2) the Brill model. The coupling parameters are set such that a sequential strategy encountered difficulties in convergence when capillary effect was considered.

Results and performance were compared with the current version of a modern reservoir simulator which uses CPR preconditioning with TRUE-IMPES decoupling, 1 V-cycle of AMG for solving the CPR pressure system and ILU(0) as a second-stage FIM preconditioner. This reference solution strategy is referred to as CPR-AMG. The coarse grids in AMG solver are constructed by the parallel maximally independent set (PMIS) coarsening scheme with the Gauss-Seidel smoothing process. The coarse level is solved by FGMRES preconditioned by ILU(0) and maximum number of levels is limited to 50 by default settings.

The CPR-MS employs the same decoupling technique and the same second-stage preconditioner, thus, the only difference between CPR-AMG and CPR-MS is the solution strategy for the pressure system. For this reason, all solutions obtained with the CPR-MS and CPR-AMG solvers have been verified to perfectly match each other, as shown, e.g., in Fig. 3.2.

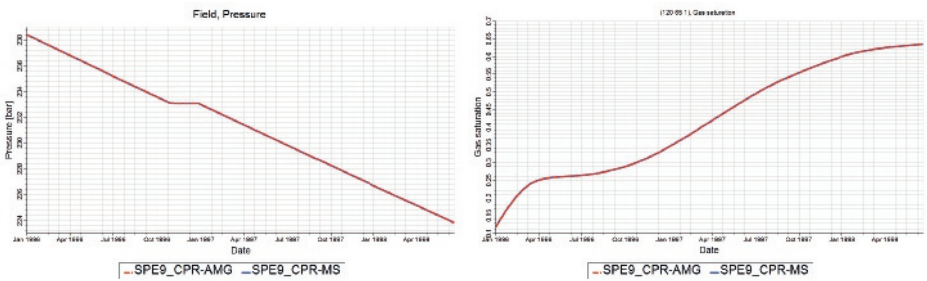


Figure 3.2: Field pressure (averaged pressure over the domain) and gas saturation in grid cell (120,65,1) for CPR-AMG and CPR-MS, showing that the two solutions match perfectly.

All results presented were obtained in serial runs using a desktop PC with 20GB RAM with Intel Xeon X5677 3.47GHz CPU.

3.3.1. BLACK OIL SPE9 MODEL

A modified version of the SPE9 model presented in [104] is considered. The original SPE9 grid is refined and a 3-phase (oil, water and gas) black oil case is considered for two scenarios: with and without capillary pressure. The following equilibrium settings are used. Top and bottom of the reservoir are located at 2735.14 m and 3231.52 m, respectively. Datum depth is set at 3032.76 m, and datum pressure is at 3500 psi. The gas-oil contact is located at 3032.76 m, while oil-water contact is located at 3032.76 m. As a result, a two-phase flow is observed. A water injection well and fifteen hydrocarbon productions wells are considered [104]. The fine grid contains $144 \times 150 \times 30$, i.e., 648000, cells in total, which is a refinement of the original SPE9 model by factor of $6 \times 6 \times 2$ for each cell. Permeability and porosity were copied from coarse cells into fine cells to maintain original geology at the fine scale. The porosity distribution and the permeability field are shown in Fig. 3.3. With the coarsening ratio of $3 \times 5 \times 5$, the multiscale grid contains $48 \times 30 \times 6$ grid cells, 8640 in total.

This coarsening ratio has been selected as it provided efficient simulations (in terms of CPU) for this model. Note that basis functions are computed using a direct solver, there is no adaptivity applied in their computations, and AMG solver is used to solve the multiscale coarse system. Increasing the coarsening ratio resulted in more computationally expensive basis functions, which without adaptivity would not offset the benefit in convergence properties and thus lead to increased overall CPU times. Efficient implementation of the CPR-MS including adaptivity based on a rigorous error estimate analysis is a subject of our ongoing research.

The initial conditions of the reservoir are shown in Fig. 3.4. The FE-based restriction operator was considered, with basis functions which were updated at the beginning of each time-step. Also, the simulation was run for the duration of 900 days.

SIMULATIONS WITHOUT CAPILLARY PRESSURE

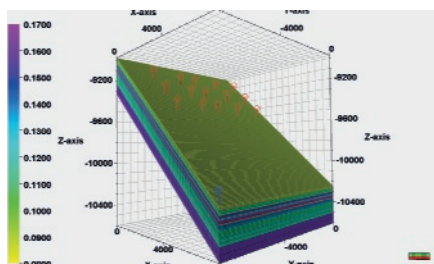
Table 3.1 summarizes the settings of the three CPR-MS runs. Two different solvers were used on the coarse pressure system.

Table 3.1: SPE9 model with capillary pressure: settings. Differences can be seen in the smoothers and in the solver used on the coarse scale pressure system.

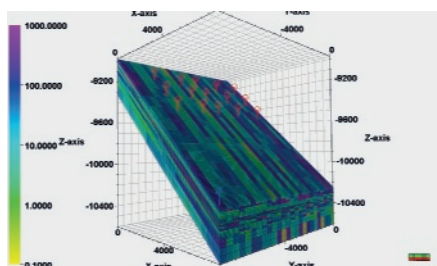
Runs	Pre-smoothing	Post-smoothing	Coarse Solver
Run 1	GS	GS	AMG
Run 2	GS	GS	GMRES-AMG
Run 3	None	GS	GMRES-AMG

Saturations and pressure maps at the end of simulations are shown in Fig. 3.5. Table 3.2 presents the performance of each simulation (number of iterations and CPU time) for the three CPR-MS runs and the CPR-AMG (as the reference) solver.

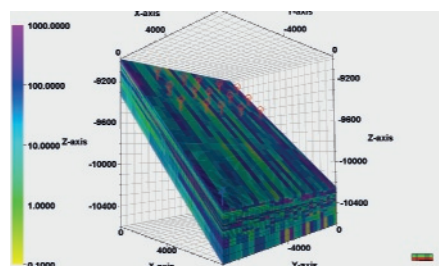
Note that the number of nonlinear iterations for all 4 runs is the same, meaning that CPR-MS treats the coupling terms as efficiently as CPR-AMG.



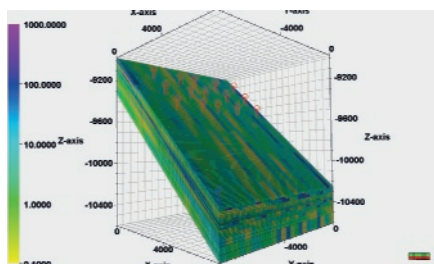
(a) Porosity



(b) X-Permeability (mD)



(c) Y-Permeability (mD)



(d) Z-Permeability (mD)

Figure 3.3: SPE9 model: porosity and permeability fields. Also shown are the water injection (red) and the hydrocarbon production wells (red).

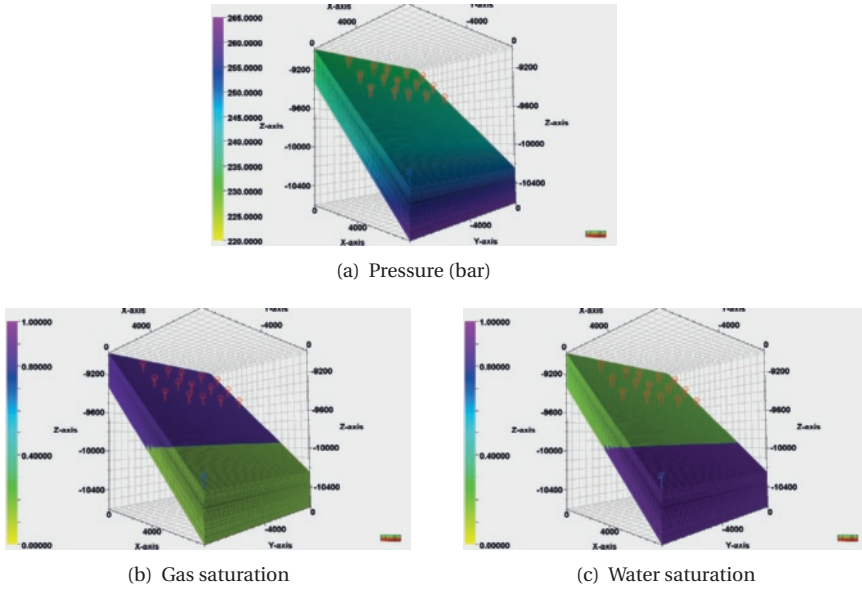


Figure 3.4: SPE9 model: pressure, gas and water saturations at the beginning of the simulation. Oil saturation was set to zero for this data set.

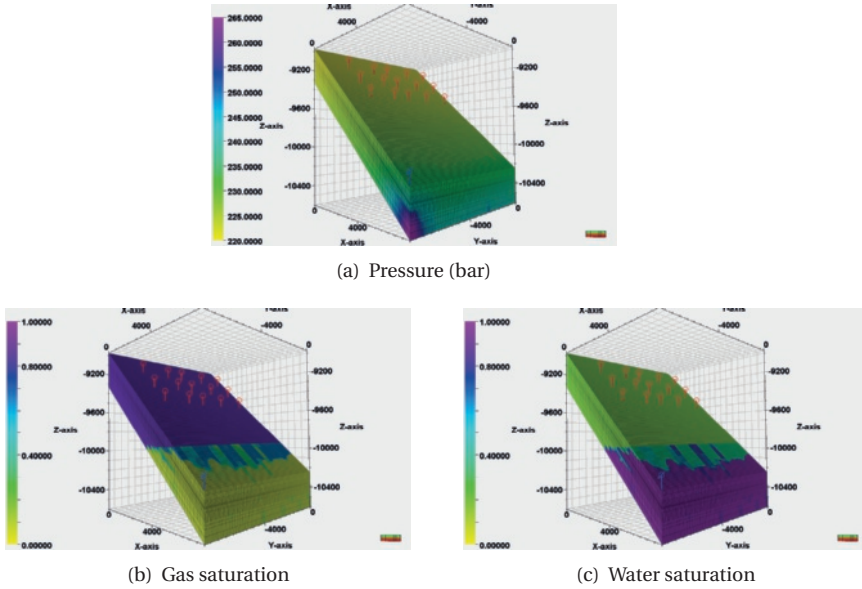


Figure 3.5: SPE9 model: pressure, gas and water saturations at the end of the simulation (900th day).

Table 3.2: SPE9 without capillary pressure: overall performance of the CPR-MS and CPR-AMG.

Runs	Timesteps	Iteration Count		CPU time (sec.)	
		Nonlinear	Linear	Linear solver	Total
Run 1	64	369	2872	1615	2699
Run 2	64	369	2845	1119	2203
Run 3	64	369	3175	1969	3075
CPR-AMG	64	369	2187	1571	2537

Figure 3.6 focuses on the linear system CPU time, split into setup and solution phases. The results illustrate the efficiency of the CPR-MS research code developed for this project. Clearly, an adaptive strategy for calculation of the basis functions, and more importantly, implementation of an efficient parallel multiscale algorithm would significantly improve the setup time for CPR-MS. The Run 2, which was found as the optimum, involves both pre- and post-smoothing steps with GMRES preconditioned by AMG as the coarse-scale solver.



Figure 3.6: SPE9 without capillary pressure: LS CPU time (s) breakdown.

A breakdown of the CPU time of the CPR-MS pressure solver is presented in Fig. 3.7. The setup stage of CPR-MS will further improve if an adaptive strategy for basis functions update is employed. Here, the focus is not to find an optimum CPR-MS strategy, but to show fully implicit simulations where AMS is used -for the first time- as the pressure preconditioner. Note that the most time-consuming stages, shown in this figure,

are computation of basis functions (dark green bars) and construction of coarse-scale system, (**RAP**) (i.e., matrix-matrix multiplications).

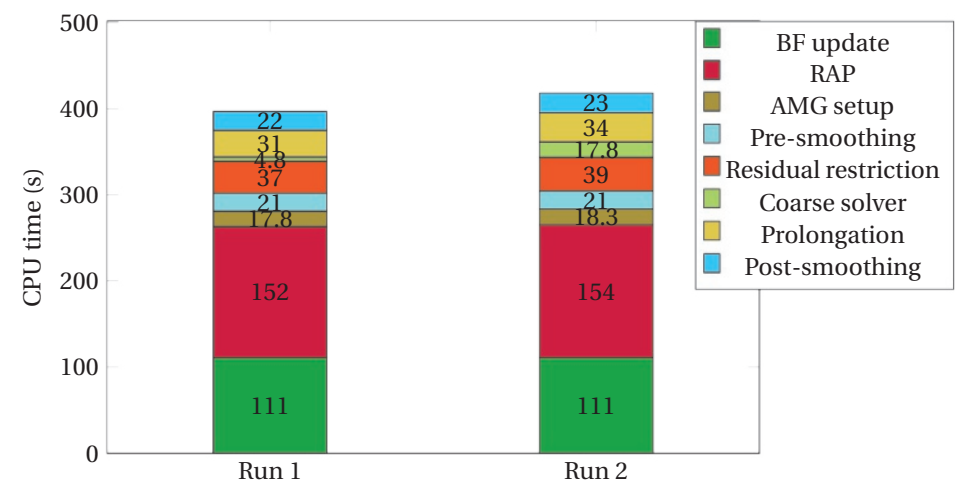


Figure 3.7: SPE9 black oil without capillary pressure: pressure solver detailed CPU time breakdown.

The weight of the coarse pressure system solver in the overall CPU time is very small. Comparing Run 2 and 3 for the coarse-system solvers (light green bars in Fig. 3.7) and the overall performance (i.e., Table 3.2), the additional costs due to GMRES preconditioned by AMG is offset by the reduction of the overall linear iterations and therefore the overall CPU time.

SIMULATIONS WITH CAPILLARY PRESSURE

Keeping the same parameters as in the previous case, the capillary pressure curve (See Fig. 3.8) is also considered. It is clear that the number of required time steps is increased for all FIM simulators (CPR-MS and CPR-AMG) compared with the previous test case without capillary effects.

Table 3.3 shows the settings of the different runs that were done with capillary pressure.

Table 3.3: SPE9 model with capillary pressure: settings. Differences can be seen in the smoothers and in the solver used on the coarse scale pressure system.

Runs	Presmoothing	Post-smoothing	Coarse Solver
Run 1	GS	GS	AMG
Run 2	GS	GS	GMRES-AMG
Run 3	2xGS	2xGS	GMRES-AMG

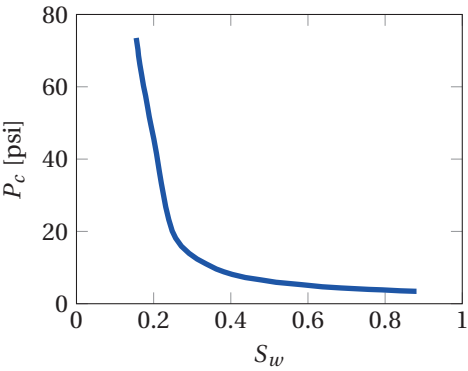


Figure 3.8: Capillary pressure curve considered for the case SPE9.

Table 3.4: SPE9 with capillary pressure: the table shows the total numbers of nonlinear and of linear iterations and the total and the linear solver CPU time of each run.

Runs	Timesteps	Iteration Count		CPU time (sec.)	
		Nonlinear	Linear	Linear solver	Total
Run 1	75	284	1888	759	1672
Run 2	75	284	1867	1202	2130
Run 3	75	281	1627	1105	2036
CPR-AMG	75	281	1381	1093	1863

Table 3.4 shows that the CPR-MS and CPR-AMG perform almost the same in terms of the number of time steps and the nonlinear iterations. Note that the performance of CPR-MS, as for the previous case, is influenced by the choice of the coarse pressure solver and the number of smoothing iterations.

3.3.2. BRILL MODEL

The Brill model is a 3-phase (water, gas and oil) black oil case that was run both with and without considering capillary pressure. The fine grid contains $180 \times 135 \times 27$, 656100 in total, cells. There exist 7243 non-neighboring connections in this model. The following equilibrium data has been considered. Top and bottom of the reservoir are located at 2128.57 m and 2490.73 m. Datum depth is located at 2143.05 m, and datum pressure is set at 4500 psi. The gas-oil contact is located at 2143.05 m, while oil-water contact is located at 2438.4 m. As a result, a two-phase flow is observed. The fluid model description can be found in Tables 3.10 and 3.9 (see Appendix), where the Stone three-phase relative permeability for oil phase is used [105].

The porosity distribution and the permeability field are shown in Fig. 3.9. There exist 8 water injection and 43 hydrocarbon production wells.

The simulation was run for 2520 days. The initial conditions of the reservoir are shown in Fig. 3.10. The coarsening factors for the three directions were $5 \times 5 \times 3$, which resulted in a coarse grid of $36 \times 27 \times 9$, total of 8748, cells.

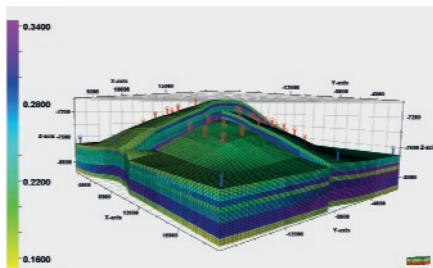
SIMULATIONS WITHOUT CAPILLARY PRESSURE

The settings of the three runs chosen for this test case are shown in Table 3.5. The results obtained with Run 1 and Run 2 suggested that an extra iteration for each smoothing step could help limiting the number of linear iterations, which was then considered for Run 3.

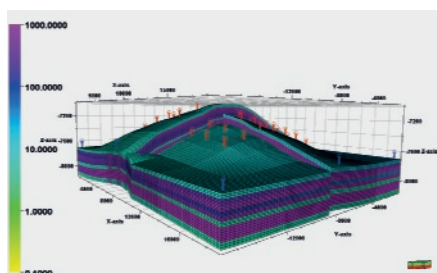
Table 3.5: Brill model without capillary pressure: settings. Differences can be seen in the smoothers and in the solver used on the coarse scale pressure system.

Runs	Presmoothing	Post-smoothing	Coarse Solver
Run 1	GS	GS	AMG
Run 2	GS	GS	GMRES-AMG
Run 3	2xGS	2xGS	GMRES-AMG

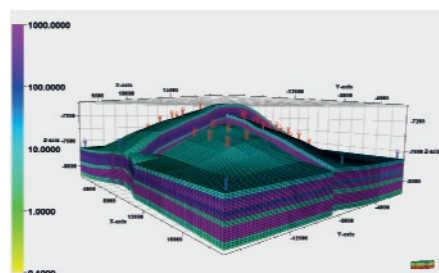
Results for the three runs are shown in Table 3.6. The best performance is obtained by introducing an extra pre- and post-smoothing iteration as this allows to reduce the number of linear iterations. The improved quality of the pressure solution also affects the number of nonlinear iterations. In fact, the difference in the number of nonlinear iteration between Run 1 and the reference one is much higher than the one observed with the previous model. This results in a higher CPU time for the overall simulation. A promising finding is that simulation times for all three runs are comparable with that of CPR-AMG.



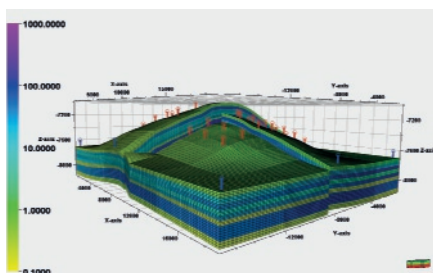
(a) Porosity



(b) X-Permeability (mD)



(c) Y-Permeability (mD)



(d) Z-Permeability (mD)

Figure 3.9: Brill model: porosity and permeability fields. The injection wells (red) and the production wells (red) are represented.

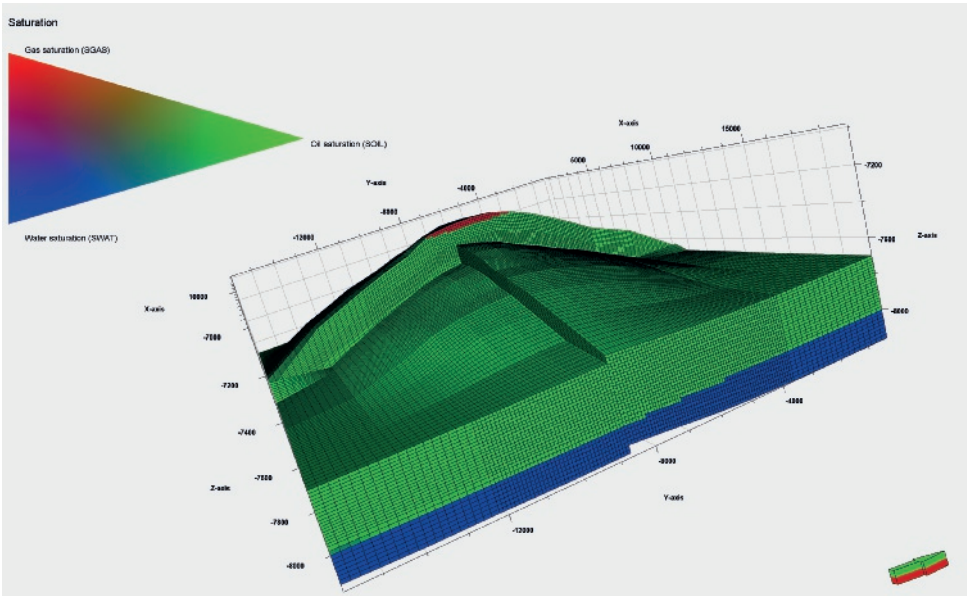


Figure 3.10: Brill model: ternary graph of gas (red), oil (green) and water (red) saturations showing the conditions of the reservoir at the beginning of the simulation.

Table 3.6: Brill model without capillary pressure: the table shows the total numbers of nonlinear and of linear iterations and the total and the linear solver CPU time of each run.

Runs	Timesteps	Iteration Count		CPU time (sec.)	
		Nonlinear	Linear	Linear solver	Total
Run 1	498	3329	22201	11376	28859
Run 2	483	3122	21211	10686	27035
Run 3	484	3091	18178	10158	26270
CPR-AMG	478	2981	9764	8967	22951

SIMULATIONS WITH CAPILLARY PRESSURE

The capillary pressure curve as shown in Fig. 3.11 was then added to the Brill model. As for the previous model, this was done to test the CPR-MS solver on a test case with a stronger coupling between the variables.

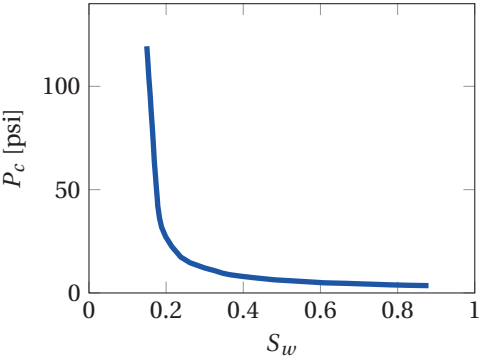


Figure 3.11: Capillary pressure curve considered for the case Brill.

Table 3.7 summarizes the settings of the 3 runs that are considered for the Brill model with capillary pressure. Once again, different number of iterations of the smoothers and different solution strategies on the coarse scale pressure system were used.

Table 3.7: Brill model with capillary pressure: settings. Differences can be seen in the smoothers and in the solver used on the coarse scale pressure system.

Runs	Presmoothing	Post-smoothing	Coarse Solver
Run 1	GS	GS	AMG
Run 2	GS	GS	GMRES-AMG
Run 3	2xGS	2xGS	GMRES-AMG

The results for this test case are shown in Table 3.8. The performance of the CPR-MS and CPR-AMG for this challenging test case is quite comparable, though the CPR-MS CPU time is about 11.5% more than that of CPR-AMG for the best scenario which is Run 1. Note that the number of iterations may be reduced with adaptive update of the basis functions and space-time adaptive employment of iterations within the AMS framework [106, 107] for FIM systems, which is the topic of our ongoing research. As observed for the previous test cases, the number of linear iterations for CPR-MS is typically more than CPR-AMG which is consistent with the literature [65, 67].

Table 3.8: Brill model with capillary pressure: the table shows the total numbers of nonlinear and of linear iterations and the total and the linear solver CPU time of each run.

Runs	Timesteps	Iteration Count		CPU time (sec.)	
		Nonlinear	Linear	Linear solver	Total
Run 1	340	1596	11863	5739	14213
Run 2	341	1608	12234	5863	14348
Run 3	341	1610	10679	5745	14454
CPR-AMG	342	1610	5749	5045	12750

APPENDIX

The phase properties used of the simulations are presented here. Thermodynamics

Table 3.9: Phase properties used for simulations presented in this chapter.

S_w dependent			S_o dependent			S_g dependent	
S_w	k_{rw}	p_{cow} [psi]	S_o	k_{row}	k_{rog}	S_g	k_{rog}
0.15	0.0	120	0.12	0.00	0.0	0.00	0.0
0.16	0.0	91.47231262	0.29	0.0016	0.08333	0.04	0.0
0.18	0.0	39.28921117	0.40	0.0162	0.16667	0.10	0.02
0.20	0.0001	26.5228644	0.54	0.0950	0.25	0.20	0.1
0.35	0.0003	9.130558739	0.60	0.1714	0.33333	0.30	0.24
0.36	0.0103	8.794231502	0.62	0.4350	0.41667	0.40	0.34
0.37	0.0222	8.485097318	0.63	0.5680	0.5	0.50	0.42
0.38	0.0359	8.199865113	0.64	0.7005	0.58333	0.60	0.5
0.40	0.0695	7.690440048	0.65	0.8302	0.66667	0.70	0.813
0.46	0.1049	6.517609798	0.80	0.9788	0.75	0.85	1.0
0.60	0.2104	4.893140943	0.82	0.9916	0.83333		
0.71	0.3187	4.135515361	0.84	0.9995	0.91667		
0.88	0.4900	3.372596478	0.85	1.0	1.0		

equilibrium data of Table 3.10 has been used in order to obtain the simulations of this chapter. Note that B_j stands for formation volume factor of j , the ratio of volumes at reservoir condition to stock tank condition [24].

Table 3.10: Phase thermodynamics equilibrium properties of oil and gas used for simulations of this chapter. For water, $C_w = 10^{-6}$ [1/psi], $\mu_w = 0.96$ [cp], and $B_w = 1.0034$ are used.

Oil PVT				Gas PVT		
R_s	p_{bub} [psi]	B_o	μ_o [cp]	p [psi]	B_g	μ_g [cp]
0.0	14.7	1.062	1.04	14.696	166.6	1.0
0.371	1114.7	1.295	1.037	1114.7	3.197	1.0
0.636	3314.7	1.435	1.035	3314.7	1.614	1.0
0.93	5514.7	1.565	1.03	5514.7	1.080	1.0
1.27	8814.7	1.695	1.02	8814.7	0.811	1.0
	15014.7	1.579	1.04	15014.7	0.386	1.0

III

THE ALGEBRAIC DYNAMIC MULTILEVEL METHOD

4

THE ALGEBRAIC DYNAMIC MULTILEVEL (ADM) METHOD

The Algebraic Dynamic Multilevel (ADM) method is presented in this chapter. ADM extends the formulation, methodology, and applicability of grid refinement strategies for FIM simulations of multiphase flow in homogeneous and heterogeneous subsurface geological formations. It also develops the first multilevel multiscale procedure for FIM simulations. Starting from the FIM linear system, the ADM hierarchical nested grid resolution is determined on the basis of an error estimate criterion. Once the grid resolution is determined, the FIM system is described on this dynamic multilevel grid by employing sequences of prolongation and restriction operators. More precisely, $\mathbf{R}_j^i$ restricts the system at level i to a system at level j and $\mathbf{P}_j^i$ prolongs (interpolates) the solution from level i to level j .

ADM allows for several options for prolongation operator, namely: (1) constant, (2) bilinear and (3) multiscale-based local basis functions [55, 56, 64, 109, 110]. The first two options can be also seen as a basis function with constant and bilinear solutions, respectively. As such, the basis functions of level i are entries of prolongation operator $\mathbf{P}_j^i$, and form partition of unity. As for the restriction operator, ADM employs a finite-volume-based operator, due to the importance of a conservative map between the scales. Such a conservative map allows for stable and robust multiphase coupled flow-transport simulations. This algebraic formulation allows for direct applicability of ADM for both homogeneous and heterogeneous problems. Note that ADM allows for different prolongation for pressure (flow solution) and saturation (transport solution), which is also crucial for treating different quantities of different natures (i.e., pressure being global, while saturation being local).

The performance of ADM is investigated in detail for homogeneous and heterogeneous test cases, including the challenging SPE10 comparative test cases [111]. It is shown that only a small fraction of the fine-scale grid cells are enough to provide ac-

Parts of this chapter have been published in the Journal of Computational Physics, **Vol. 314** (2016) [108].

curate (compared with reference fine-scale) solutions. ADM can be seen as a significant step forward in the application of DLGR methods, in the sense that it is algebraic, allows for systematic map across different scales, and applicable to heterogeneous test cases without any upscaling of fine-scale high resolution quantities. Also, it is the first multi-scale method which allows for obtaining FIM solutions on an arbitrary multilevel grid. Thus, it casts a promising framework for next-generation FIM simulators for multiphase flow in large-scale natural subsurface formations.

The chapter is organised as follows. In Section 4.1 the governing equations describing multiphase flow in porous media are presented along with the FIM simulations strategy. Then, in Section 4.2 the ADM method along with its procedure of dynamic grid generation, construction of prolongation and restriction operators, and simulation strategy are described. Numerical simulation results are presented for homogeneous and heterogeneous test cases in Section 4.3.

4

4.1. FINE-SCALE FORMULATION

The mass conservation equations for a system of N_{ph} incompressible and immiscible phases, flowing in a porous media are considered, i.e.

$$\frac{\partial}{\partial t}(\phi S_\alpha) - \nabla \cdot (\lambda_\alpha \cdot \nabla p) = q_\alpha \quad \forall \alpha \in \{1, \dots, n_{ph}\}. \quad (4.1)$$

Equations (4.1) along with the constraint of Eq. (1.6) form a well-posed system of equations for $(n_{ph} + 1)$ unknowns. The constraint can be used to eliminate the $S_{n_{ph}}$, so that the system of n_{ph} equations (4.1) is solved for n_{ph} unknowns. FIM discretization of these equations is obtained if all the pressure- and saturation-dependent terms are calculated implicitly at new time step $(n + 1)$, and the residual r_α^{n+1} is set to be zero, i.e.,

$$r_\alpha^{n+1} = \frac{\phi}{\Delta t} (S_\alpha^{n+1} - S_\alpha^n) - \nabla \cdot (\lambda_\alpha^{n+1} \cdot \nabla p^{n+1}) - q_\alpha^{n+1} = 0 \quad \forall \alpha \in \{1, \dots, n_{ph}\}. \quad (4.2)$$

As the residual is a non-linear function of the primary unknowns $(p, S_1, \dots, S_{N_{ph}-1})$, the system is solved using the Newton-Raphson method [29], i.e.,

$$r^{n+1} \approx r^{v+1} = r^v + \left. \frac{\partial r}{\partial p} \right|^v \delta p^{v+1} + \sum_{i=1}^{N_{ph}-1} \left. \frac{\partial r}{\partial S_i} \right|^v \delta S_i^{v+1}, \quad (4.3)$$

where v and $(v + 1)$ denote the previous and new iteration steps, respectively. Then, the linear system $\mathbf{J}^v \delta x^{v+1} = -r^v$ is obtained and solved iteratively until convergence is reached. Here, δx is the vector of unknowns and $\mathbf{J}$ is the Jacobian (derivatives) matrix.

For two-phase system of oil (o) and water (w), taking (p, S_w) as the primary unknowns, this linear system reads

$$\underbrace{\begin{bmatrix} \mathbf{J}_{op}^v & \mathbf{J}_{os}^v \\ \mathbf{J}_{wp}^v & \mathbf{J}_{ws}^v \end{bmatrix}}_{\mathbf{J}^v} \underbrace{\begin{bmatrix} \delta p^{v+1} \\ \delta S_w^{v+1} \end{bmatrix}}_{\delta x^{v+1}} = - \underbrace{\begin{bmatrix} r_w^v \\ r_o^v \end{bmatrix}}_{r^v}. \quad (4.4)$$

Here, each row belongs to each phase α and contains two block entries, $\mathbf{J}_{\alpha p}$ and $\mathbf{J}_{\alpha s}$, representing the pressure and the saturation dependencies corresponding to the linearized residual equation, i.e., Eq. (4.2), of the phase α .

As mentioned before, several processes require high resolution grids over large length scales. As such, FIM simulations, leading to Eq. (4.4), on fine-scale static grid become computationally too expensive for natural formations.

Next, the ADM method for FIM simulations is presented. ADM casts a new algebraic framework for advanced reservoir simulators, applicable for both homogeneous and heterogeneous media.

4.2. ADM METHOD

Equation (4.4), described on fine-scale grid resolution, is solved on a dynamically defined, adaptive grid resolution by the ADM method. A domain with $N_f = N_{fx} \times N_{fy}$ grid cells is considered (in 3D N_{fz} is also considered). The resolution of the fine grid is assumed to be good enough to capture the complex physics involved in the process, which is beyond the resolution of state-of-the-art simulators for realistic size domains. ADM employs a hierarchy of n_l coarse grids, where l refers to the level index (higher values denote coarser resolutions). At each level l , the corresponding coarse grid consists of $N_l = N_{lx} \times N_{ly}$ control volumes. The coarsening ratio at each level γ^l is defined as

$$\gamma^l = (\gamma_x^l, \gamma_y^l) = \left(\frac{N_{fx}}{N_{lx}}, \frac{N_{fy}}{N_{ly}} \right). \quad (4.5)$$

Also, Π^l is defined as a set that includes all the grid cells belonging to the level l .

ADM procedure is built on constructing an approximate fine-scale solution by interpolating the solution at any level l , δx_l , through a sequence of prolongation operators $\mathbf{P}$, i.e.,

$$\delta x_f \approx \delta x'_f = \mathbf{P}_f^1 \dots \mathbf{P}_{l-1}^l \delta x_l, \quad (4.6)$$

It is clear that the quality of the approximated solution, $\delta x'_f$, depends highly on the quality of the interpolation operators. The prolongation operator $\mathbf{P}_i^{i+1}$ interpolates the solution at level $(l+1)$ to the higher resolution (finer) level l .

On the basis of Eq. (4.6), the FIM system of Eqs. (4.4) can be mapped onto the level l as

$$\underbrace{\mathbf{R}_l^{l-1} \dots \mathbf{R}_1^f \mathbf{J} \mathbf{P}_f^1 \dots \mathbf{P}_{l-1}^l}_{\mathbf{J}_l} \delta x_l = - \underbrace{\mathbf{R}_l^{l-1} \dots \mathbf{R}_1^f}_{r_l} r_f, \quad (4.7)$$

where $\mathbf{R}_{l+1}^l$ is the restriction operator, mapping the solution from higher resolution (finer) scale l to coarser scale $(l+1)$.

The operators $\mathbf{R}$ and $\mathbf{P}$ are block-diagonal matrices of the form

$$\mathbf{R}_l^{l-1} = \begin{bmatrix} (\mathbf{R}_p)_l^{l-1} & \\ & (\mathbf{R}_s)_l^{l-1} \end{bmatrix}_{N_l \times N_{l-1}} \quad (4.8)$$

and

$$\mathbf{P}_{l-1}^l = \begin{bmatrix} (\mathbf{P}_p)_{l-1}^l & \\ & (\mathbf{P}_s)_{l-1}^l \end{bmatrix}_{N_{l-1} \times N_l}, \quad (4.9)$$

respectively. As to preserve the conservative solution at each level, a finite-volume (FV) restriction operator is considered for both pressure $(\mathbf{R}_p)_l^{l-1}$ and saturation $(\mathbf{R}_s)_l^{l-1}$ blocks.

Thus $(\mathbf{R}_p)_l^{l-1} = (\mathbf{R}_s)_l^{l-1}$ holds. The entry (i, j) of the restriction operator $(\mathbf{R}_p)_l^{l-1}$, e.g., reads

$$(\mathbf{R}_p)_l^{l-1}(i, j) = \begin{cases} 1 & \text{if cell } j \text{ is inside cell } i, \\ 0 & \text{otherwise.} \end{cases} \quad (4.10)$$

As the saturation (local, hyperbolic) and pressure (global, elliptic or parabolic) are quantities of different natures, one may consider different prolongation operators for them. More precisely, $(\mathbf{P}_p)_l^{l-1}$ is in general different than $(\mathbf{P}_s)_l^{l-1}$. In this work, three different options for $(\mathbf{P}_p)_l^{l-1}$ are considered, namely, constant, bilinear, and local basis function [55, 56, 60, 60, 65, 81, 112] approaches. In general, constant and bilinear functions can also be considered as simplified basis functions, which are mathematically described without consideration of the underlined heterogeneity. On the other hand, multiscale basis functions, which are local solutions of the pressure equations capture the finer-scale transmissibility heterogeneity. As such, in this general form, $(\mathbf{P}_p)_l^{l-1}$ consists of sparse vectors -basis functions- Φ^k in each column k . Basis functions belonging to a coarse level l are constructed by applying the Algebraic Multiscale procedure, described in Chapter 2, to the pressure system of level $l - 1$. Note that the local-solution-based basis functions, as in the multiscale community, need to be updated infrequently only where and when the local transmissibility changes beyond a threshold value [106]. Note that all options for prolongation operators form partition of unity by construction.

For saturation, preserving mass across scales, constant interpolation is considered; this means that the saturation prolongation operator between the two levels l and $l - 1$ reads

$$(\mathbf{P}_s)_l^{l-1} = (\mathbf{R}_l^{l-1})^T, \quad (4.11)$$

where super-index T denotes the transpose operator. The incorporation of more accurate interpolation strategies for saturation, as the one discussed in [81], will be discussed in Chapter 6.

After the general descriptions of the prolongation and restriction operators, ADM method can be presented in detail. Starting at time step n , one has to select the adaptive multilevel grid resolution through the computational domain. One would naturally need to accurately capture the location of fronts (steep gradients), close to boundary conditions and wells (source terms), and near complex geological features (fractures, faults, etc.). Such an adaptive multi-level grid would contain $N_{l_{max}}^{ADM}$ (solution) grid cells, consisting of all grids at all levels (fine to a user-defined maximum level l_{max}). Figure 4.1 illustrates an example of such a solution grid. In general, for each level l , a grid of N_l^{ADM} blocks, composed of nodes belonging to all grids of level l and finer (up to fine-scale resolution) can be constructed.

The ADM coarse system is then constructed on the basis of Eq. (4.4) by introducing a sequence of restriction and prolongation operators, i.e.,

$$\underbrace{\hat{\mathbf{R}}_l^{l-1} \dots \hat{\mathbf{R}}_1^f \mathbf{J} \hat{\mathbf{P}}_f^1 \dots \hat{\mathbf{P}}_{l-1}^l}_{\hat{\mathbf{J}}_l} \delta x_l^{ADM} = \underbrace{\hat{\mathbf{R}}_l^{l-1} \dots \hat{\mathbf{R}}_1^f}_{\hat{\mathbf{r}}_l} r_f, \quad (4.12)$$

where δx_l^{ADM} is the solution on the l -level ADM solution grid and the fine-scale solu-

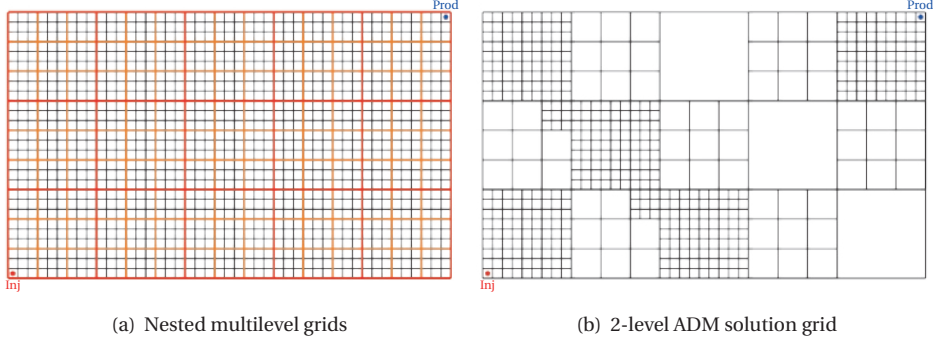


Figure 4.1: 2-level ADM method: (a) fine and the Nested multilevel grids and (b) the ADM solution grid $N_{l_{max}=2}^{ADM}$.

tion is approximated by

$$\delta x_f \approx \delta x'_f = \hat{\mathbf{P}}_f^1 \dots \hat{\mathbf{P}}_{l-1}^l \delta x_l^{ADM}. \quad (4.13)$$

Finally, ADM approximates the fine-scale reference solution as

$$\delta x_f \approx \delta x'_f = \hat{\mathbf{P}}^l (\hat{\mathbf{R}}_l \mathbf{J} \hat{\mathbf{P}}^l)^{-1} \hat{\mathbf{R}}_l r_f, \quad (4.14)$$

where,

$$\hat{\mathbf{P}}^l = \hat{\mathbf{P}}_f^1 \dots \hat{\mathbf{P}}_{l-1}^l \quad (4.15)$$

and

$$\hat{\mathbf{R}}_l = \hat{\mathbf{R}}_l^{l-1} \dots \hat{\mathbf{R}}_1^f. \quad (4.16)$$

The ADM restriction and prolongation operators, e.g., $\hat{\mathbf{R}}_l^{l-1}$ and $\hat{\mathbf{P}}_{l-1}^l$ read

$$\hat{\mathbf{R}}_l^{l-1} = \begin{bmatrix} (\hat{\mathbf{R}}_p)_l^{l-1} & \\ & (\hat{\mathbf{R}}_s)_l^{l-1} \end{bmatrix} \quad (4.17)$$

and

$$\hat{\mathbf{P}}_{l-1}^l = \begin{bmatrix} (\hat{\mathbf{P}}_p)_{l-1}^l & \\ & (\hat{\mathbf{P}}_s)_{l-1}^l \end{bmatrix}. \quad (4.18)$$

The set Ω^l contains all ADM grid cells belonging to level l and finer levels (up to and including those at fine scale). Moreover, the set Γ^l is defined such that it involves those cells which are at level l only, i.e.,

$$\Gamma^l = \Omega^l \cap \Pi^l. \quad (4.19)$$

For ADM restriction sub-blocks $(\hat{\mathbf{R}}_p)_l^{l-1} = (\hat{\mathbf{R}}_s)_l^{l-1}$ holds. In addition, their entries, e.g. for the pressure sub-block read

$$(\hat{\mathbf{R}}_p)_l^{l-1}(i, j) = \begin{cases} (\mathbf{R}_p^{l-1})_l^{l-1}(i, j) & \text{if cell } i \in \Gamma^l \text{ and cell } j \in \Gamma^{l-1}, \\ \delta_{ij} & \text{otherwise.} \end{cases} \quad (4.20)$$

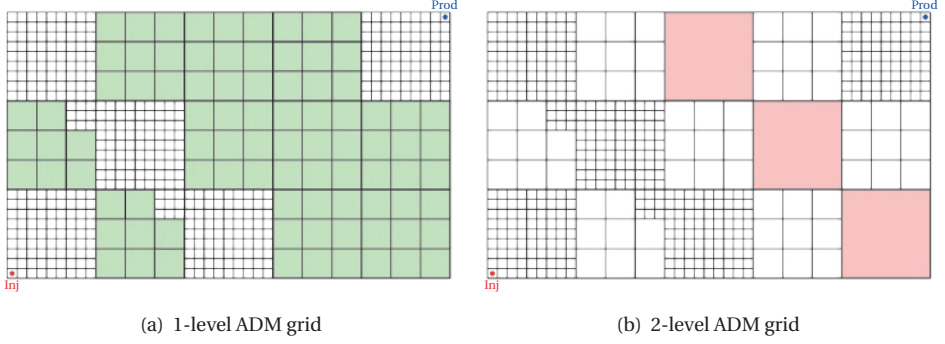


Figure 4.2: Example of 1-level (a) and 2-level (b) ADM solution grids. The green blocks are the nodes belonging to the set Γ^1 whereas the red ones belong to the set Γ^2 . The ADM restriction operator $\hat{R}_2^1$ maps the grid shown in (a) to the one shown in (b).

Here, δ_{ij} is the Dirac delta function. Figure 4.2 shows examples of 1-level and 2-level ADM solution grids with highlighted cells belonging to set Γ^1 and Γ^2 , respectively.

In addition to the ADM restriction operator, $\hat{\mathbf{R}}_l^{l-1}$, the ADM prolongation operator $\hat{\mathbf{P}}_{l-1}^l$ needs to be defined. In order to preserve the partition of unity through interpolating between multi-level ADM solution grid, one has to carefully follow the next procedure. The set $\hat{\Pi}^l$ is defined as the set of all nodes in Ω^l that are located at the centres of a coarse node belonging to level l . Then, given the ADM grid at level l , $(\hat{\mathbf{P}}_p)_l^l$ reads

$$(\hat{\mathbf{P}}_p)_l^l(i, j) = \begin{cases} (\mathbf{P}_p)_l^l(i, j) & \text{if cell } i \in \Gamma^{l-1} \text{ and cell } j \in \hat{\Pi}^l, \\ \delta_{ij} & \text{otherwise.} \end{cases} \quad (4.21)$$

Since a constant interpolation is used for saturation, the ADM interpolation saturation operator reads

$$(\hat{\mathbf{P}}_s)_l^l = ((\hat{\mathbf{R}}_s)_l^{l-1})^T. \quad (4.22)$$

Figure 4.3 illustrates the sequences of prolongation from level 2 to fine-scale (level 0). Note that the centres of each block at level l , i.e., $\hat{\Pi}^l$, are also demonstrated in this figure.

4.2.1. DYNAMIC MULTILEVEL MULTISCALE BASIS FUNCTIONS

As mentioned in the previous subsection, multiscale basis functions can be used as pressure interpolator for heterogeneous domains and they are computed following the Algebraic Multiscale Solver (AMS) [65] procedure described in Chapter 2 which, in this chapter, is extended to multiple levels.

Even though the fine-scale pressure equation is a TPFA-based system, starting from coarse level 1, all the coarse-scale systems develop MPFA-based connectivities, due to the local basis functions. Here, the MPFA-based pressure matrix $\tilde{\mathbf{A}}_p^{l-1}$, which stands as fine-scale system for level $l-1$, is constructed by the multiscale finite-element procedure

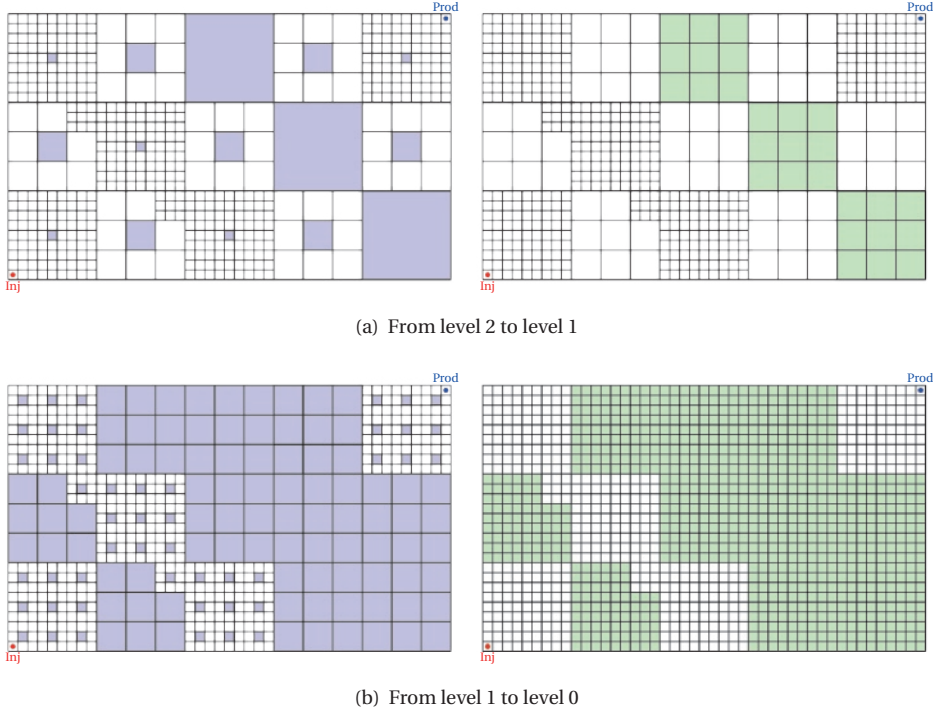


Figure 4.3: ADM multilevel interpolation: the nodes used to interpolate are shown in blue on the left whereas those for which an approximated solution is found are shown in green on the right.

as

$$\tilde{\mathbf{A}}_p^{l-1} = ((\mathbf{P}_p)_{l-2}^{l-1})^T \mathbf{A}_p^{l-2} (\mathbf{P}_p)_{l-2}^{l-1}. \quad (4.23)$$

Then, $\tilde{\mathbf{A}}_p^{l-1}$ is reduced to a TPFA-based system, i.e.,

$$\mathbf{A}_p^{l-1} = tpfa(\tilde{\mathbf{A}}_p^{l-1}), \quad (4.24)$$

where the $tpfa(\bullet)$ extracts the TPFA components of the matrix (heptadiagonal matrix for 3D Cartesian grids), by considering the transmissibility between cells i and j equal to entry $\mathbf{A}_{ij}$ of the original matrix. The algebraic multiscale procedure for the calculation of basis functions is now performed on this TPFA system at level $l-1$, in order to obtain the basis functions at level l , i.e., $(\mathbf{P}_p)_{l-1}^l$. Fig. 4.4 shows, e.g., some multilevel multiscale basis functions for a 2D heterogeneous reservoir. To avoid non-physical peaks in the interpolated pressure solution, the permeability contrast is limited to a few order of magnitudes for the computation of basis functions. Alternative solutions are well documented in the literature [77, 79, 113, 114] but were not considered in this work.

Multiscale basis functions map the pressure solution from a coarse resolution l to a finer resolution $l-1$. However, basis functions are defined on the static grid resolutions

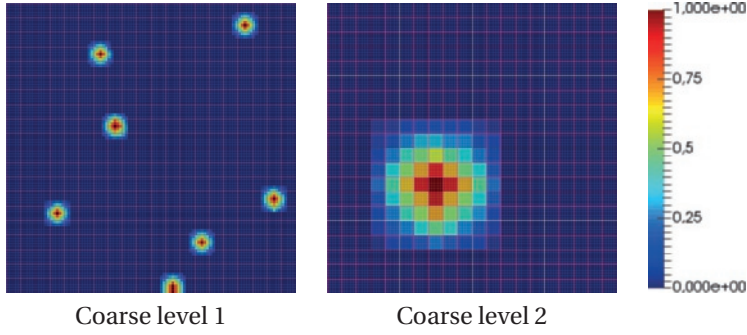


Figure 4.4: Multilevel multiscale basis functions for a 2D heterogeneous reservoir are shown. The fine-scale and the coarse grids are also visible. The coarsening factors in the x and y directions are equal to 5.

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initially defined. As such, they have to be adapted to adjust to the dynamically defined ADM grid resolution; basis functions have to be set to zero in those cells that are already at the fine-scale resolution in the ADM grid, as no interpolation is needed for those cells as mentioned in the previous subsection. Figure 4.5 illustrates, e.g., how basis functions are modified to accommodate the topology of the ADM grid for a 2D domain.

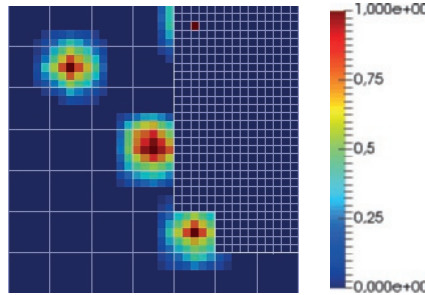


Figure 4.5: ADM multiscale basis functions. The figure shows some basis functions that were modified to adapt to the ADM grid resolution.

The ADM grid resolution selection and the overall simulation strategy will be presented in the following subsections.

4.2.2. SELECTION OF THE ADM GRID RESOLUTION

The ADM efficiency, similar to other mesh refinement methods, depends highly on the criterion used to choose the multilevel grid. In other words, one needs to apply fine-scale resolution grids only at the location of high importance, e.g. sharp fronts, while allowing for coarser grids everywhere else. Moreover, close to wells (i.e., source terms) one needs to keep the fine-scale resolution in order to capture the complex flow scenario accurately. The location of fronts and other dynamic features need to be automatically detected through out the simulation on the basis of an error estimate criterion. A study of such error criteria for multiphase flow in homogeneous media is presented in the lit-

erature [50]. Note that though only some selective criteria based on phase saturation distribution will be investigated in this chapter, ADM allows for any user-defined criterion.

The ADM multilevel grid procedure is as following. A block Π_l^l belonging to the coarse level l is included in the ADM solution grid if the maximum saturation difference between the following sets of cells is below a user-defined threshold value. The set of cells consists of its four neighbours (six in 3D) at level l and the two fine-scale cells inside Π_l^l where the saturation is maximum and minimum, respectively. Figure 4.6 illustrates an overview of this procedure.

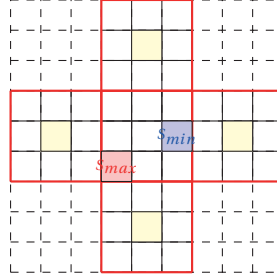


Figure 4.6: ADM coarsening criterion: shown in solid red is the coarse grid level l , with the cell in the middle of the figure being Π_l^l , surrounded by the shown four neighboring blocks. Also, fine-scale grid is shown in black. To involve Π_l^l in the ADM solution grid, the maximum difference between the saturation values of the highlighted cells S_{max} (filled in red), S_{min} (filled in blue), and the four neighboring cells of the same level l (filled in yellow) needs to be lower than a user-defined value. Note that S_{max} and S_{min} are the fine-scale cells inside Π_l^l with maximum and minimum saturation values, respectively.

To maintain numerical accuracy, the transition of grid resolution is limited to 1 among the neighboring cells. For example, grid cells around a cell at level l can be either at $(l+1)$, l , or $(l-1)$ resolution.

4.2.3. SIMULATION STRATEGY

The ADM method is implemented within a FIM simulation framework. The linearized system of equation (4.4) is obtained at fine scale, to be iteratively solved until convergence to non-linear equation (4.2) is reached. ADM applies sequences of prolongation and restriction to the fine-scale FIM system, and converges on the adaptive multilevel grid, where fine-scale grids are present only where and when needed. As such, ADM provides a good quality (but not exactly the same) solution to the FIM system of fine-scale resolution. In order to enhance the non-linear convergence of both fine-scale reference and ADM simulations, the FIM system at the first time-step is preconditioned by a pressure-block solver (providing a good initial pressure solution). Furthermore, the time-step size is set based on a user-defined CFL value. However, during the FIM simulation, the time-step size is halved whenever the non-linear loop does not converge within a maximum of 12 iterations.

A schematic overview of the ADM solution strategy is provided in Fig. 4.7. At the beginning of each time-step the ADM solution grid is selected and the sequence of operators $\hat{\mathbf{R}}_l$, Eq. (4.16), and $\hat{\mathbf{P}}^l$, Eq. (4.15), are constructed. Then, the ADM system is

constructed and solved, and its solution is prolonged to the fine scale. The process is iterated until convergence is reached on the ADM grid.

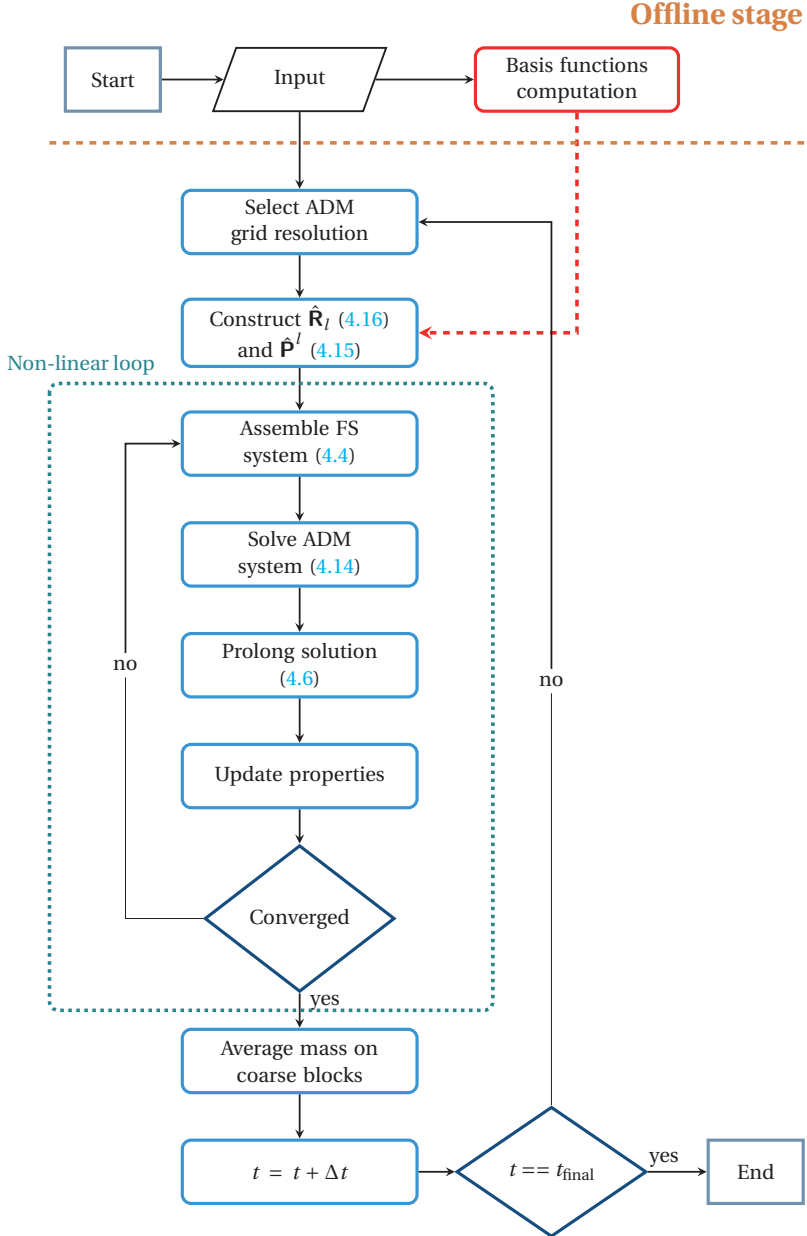


Figure 4.7: ADM method: schematic overview of the FIM ADM solution strategy.

Next, numerical results for different homogeneous and heterogeneous test cases are pre-

sented.

4.3. NUMERICAL RESULTS

The performance of ADM is investigated for four test cases. All case have the same geometrical dimensions of 2160×540 [m²] on which the fine-scale reference simulator imposes a 216×54 grid. In addition, all cases are subject to no-flow boundary conditions at all boundaries. Table 4.1 summarizes the characteristic of the permeability fields and the location of the wells.

Table 4.1: Overview of the test cases presented.

Test case	Fine Grid	Permeability	Injector	Producer
1	216×54	Homogeneous	Bottom left	Top right
2	216×54	Patchy	Bottom left	Top right
3	216×54	Channelized	Top left	Bottom right
4	216×54	Hormo + barriers	vert. + horiz.	vert + horiz.

4

Test case 1 is a homogeneous reservoir whereas test cases 2 and 3 are extracted from the highly heterogeneous SPE10 test case [111] top and bottom layers, respectively. For these three cases, an injector and a producer are consider, having fixed pressures of 100 and 0 [kPa], respectively [115]. The last Test case, Test case 4, is a homogeneous reservoir that entails high-contrasts low-permeable regions with long-correlation lengths, which adds to the complexity of the flow pattern once combined with more complex configuration of the wells. For all four cases, quadratic relationship is used for the phase relative mobility function, i.e., $k_{r\alpha} = S_\alpha^2$. In addition, phase viscosities read $\mu_w = 10^{-5}$ and $\mu_o = 1.5 \cdot 10^{-5}$ [Pa . s]. ADM performance is investigated in terms of number of non-linear iterations, number of grid blocks and the accuracy of its solution, compared against the fine-scale fully resolved solutions. The accuracy of the solution is evaluated during the simulation at 10 fixed simulation times t as

$$\epsilon_p(t) = \frac{\|p(t) - p_{ref}(t)\|_2}{\|p_{ref}(t)\|_2} \quad (4.25)$$

for pressure, and

$$\epsilon_S(t) = \frac{\|S(t) - S_{ref}(t)\|_2}{N_f} \quad (4.26)$$

for saturation. $p_{ref}(t)$ and $S_{ref}(t)$ are the pressure and saturation reference solutions at simulation time t . An average error over these 10 simulation times is then computed as

$$\epsilon_p = \text{mean}(\epsilon_p(t)) \quad (4.27)$$

and

$$\epsilon_S = \text{mean}(\epsilon_S(t)). \quad (4.28)$$

For all cases, different pressure interpolators (constant, bilinear, and for SPE10-top layer MS-basis functions) are considered. Also, the ADM solution quality and efficiency is investigated for different threshold values of saturation change. All numerical results are obtained using a MATLAB code. It is also important to emphasize that all linear systems are solved with MATLAB direct solver. A study of the scalability of ADM is presented in section 4.4, where the increase of ADM active nodes and its stability (non-linear iteration counts) are presented for domains of different sizes.

4.3.1. TEST CASE 1

The first test case is a water flooding experiment, for the period of 3000 days, in a homogeneous reservoir with the permeability value of 10^{-12} [m²]. ADM simulations are run both with constant and bilinear prolongation operators for pressure and with constant interpolators for saturation.

Figure 4.8 shows a comparison of the ADM solution and the reference one after 1500 simulation days. Note that fine-scale cells are used only around the two wells and the moving water front. It is important to also note that using bilinear functions as pressure interpolator improves significantly the pressure solution. This also results in a more accurate saturation distribution, which can be seen comparing the position of the water front. Note that more accurate pressure interpolator would allow for globally more accurate pressure solutions, and through the coupling terms, it would provide more accurate saturation solutions. Also, note that the bilinear functions are accurate for the homogeneous domains.

Figure 4.9 shows history of active nodes employed by ADM during the simulation. The maximum number of nodes is registered close to the water breakthrough. From then on, the number of active grid blocks starts decreasing again, because coarse blocks can be used behind the front where the saturation gradient is moderate.

Figure 4.10 (a) shows the number of non-linear iterations used by the ADM method compared against the non-linear iterations used in the standard FIM fine-scale simulation. Note that the two methods (i.e., ADM and fine-scale reference) provide solutions on different grid resolutions. As such, they converged to different solutions even though they use the same tolerances. Therefore, the non-linear iteration count does not provide an exact comparison of the non-linear stability between the two methods.

Figure 4.10 (b) shows that the average number of grid cells used in a 2-level ADM simulation increases if more restrictive (smaller) threshold values are used. As shown in Fig. 4.11 (a) and (b) smaller threshold values, and consequently more active grid cells, reduces the error norms significantly. Note that at the beginning of the simulation, coarse blocks are employed in most part of the reservoir, irrespectively of the threshold value. This is due to the local nature of the saturation front. Also, since bilinear function are quite accurate interpolators for homogeneous problems, the relative error in the pressure (red curve in Fig. 4.11 (a)) is not as sensitive to the threshold value chosen for the coarsening criterion as in the case with constant interpolator.

4.3.2. TEST CASE 2

Natural logarithm of the permeability for Test case 2 is presented in Fig. 4.12, which is extracted from SPE10 top layer [111]. The simulation time is 3000 days and it is run with

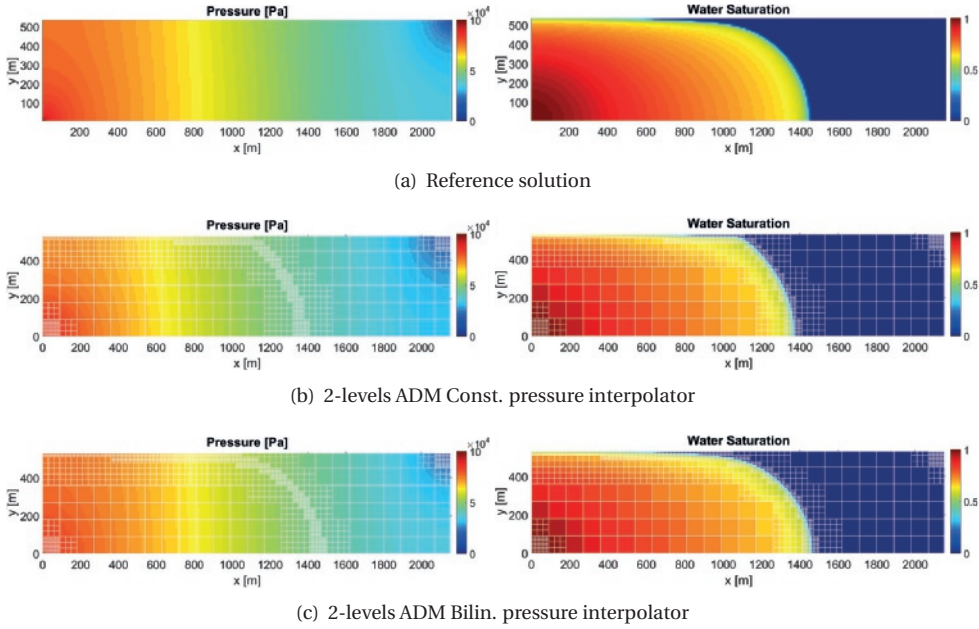


Figure 4.8: Test case 1 - Pressure and water saturation at 1500 days after injection. As for the prolongation operators, the ADM method employs constant (b) and bilinear (c) functions for pressure and constant function for saturation. The threshold value for the front tracking criterion is 0.2, corresponding to an average 10.2% and 11.7% grid cells, compared with the reference solution (a), for constant and bilinear interpolators, respectively.

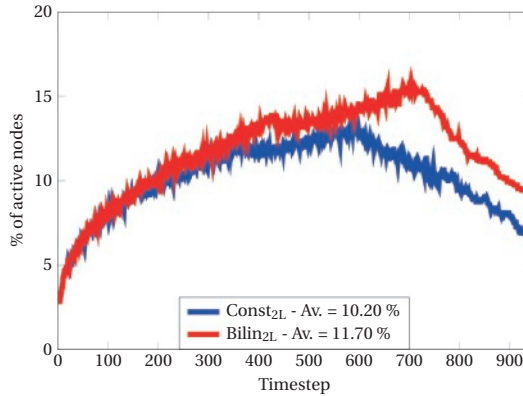


Figure 4.9: Test case 1 - history of the number of active nodes employed by ADM during simulation. The threshold value for coarsening criterion is 0.2. Also, average ADM active grid cells are mentioned in the graph.

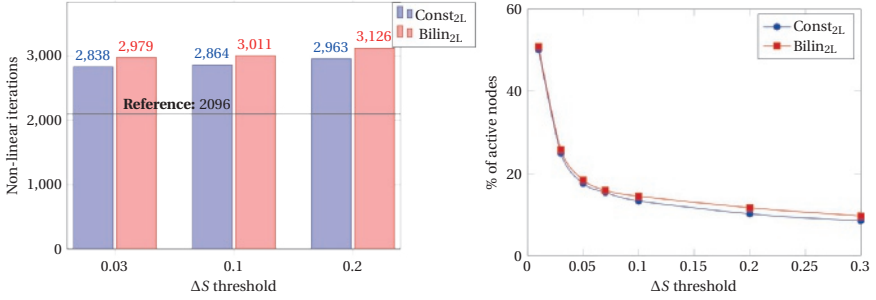


Figure 4.10: Test case 1 - (left) count of ADM non-linear iterations with different threshold values. Also shown in solid black line is the one for the reference solution. Note that the same non-linear convergence threshold values are used for both ADM and fine-scale systems, while they converge on different grid resolutions. (right) ADM active cells as a function of the coarsening threshold value.

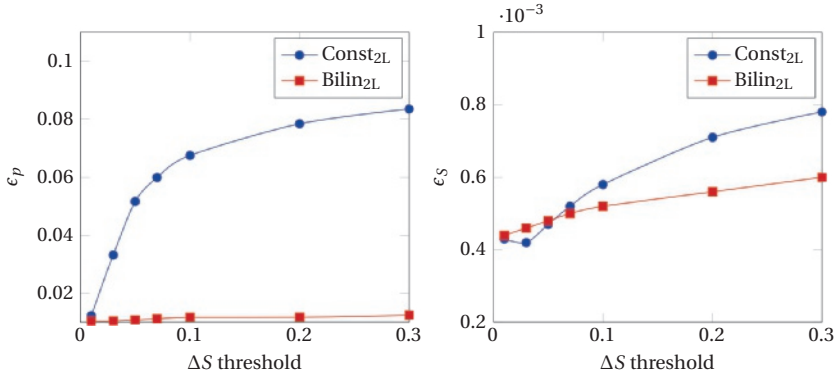


Figure 4.11: Test case 1 - pressure (left) and saturation (right) relative errors as a function of the coarsening threshold value.

a time-step size corresponding to an initial CFL number equal to 5. For ADM prolonga-

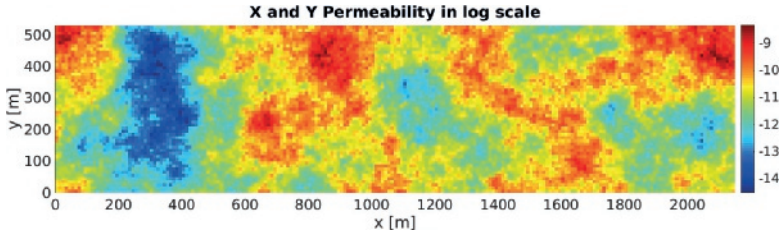


Figure 4.12: Test case 2 - Logarithm in base 10 of the permeability.

tion operator, in addition to the constant and bilinear functions, here, multiscale basis functions (local solutions of pressure equation) is also considered.

Figures 4.13 and 4.14 show the reference and 2-level ADM solutions at 750 and 1500 days after injection. Note that coarse blocks are employed behind the front, at heterogeneous field. All pressure interpolators reproduce the fine-scale solution quite accurately. However, bilinear and multiscale basis functions lead to more accurate solutions, compared to the constant interpolator.

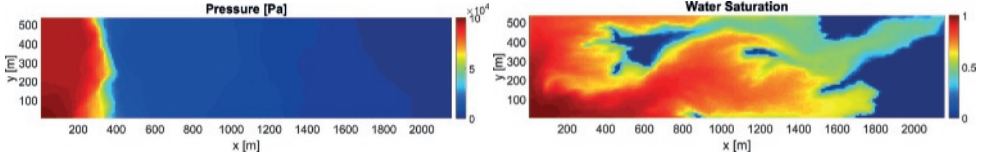
Figure 4.15 shows history of the active blocks employed by ADM. Note that the grid resolution, and consequently the total employed grid cells, are dynamically selected based on the dynamic evolution of the saturation map.

Figure 4.16 (a) shows the count of non-linear iterations employed by the ADM simulation for three different values of the coarsening criterion threshold. The average number of ADM grid cells is shown in Fig. 4.16 (b). As shown in Fig. 4.17 (a) and (b) using more active nodes improves the solution quality significantly. Note also that bilinear and multiscale-based interpolators outperform the constant interpolator. The pressure and saturation solutions are slightly (and not significantly) more accurate when multiscale basis functions are employed (compared with the bilinear ones). Note that, in our implementation, the multiscale basis functions are not updated during the multiphase simulation. In addition to this, the sub-regions where strong non-linear dynamic happens are treated at fine-scale in both (i.e., multiscale and bilinear) cases.

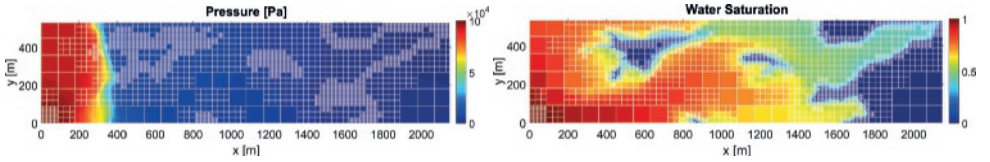
4.3.3. TEST CASE 3

Permeability of the SPE10 [111] bottom layer is considered for the last Test case, as shown in Fig. 4.18.

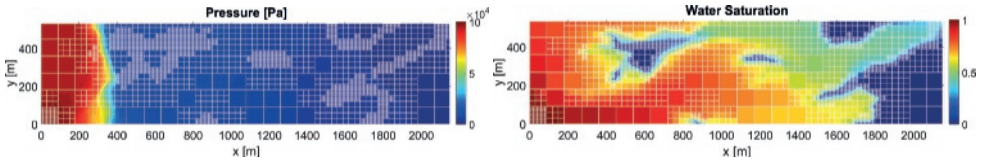
Figures 4.19 and 4.20 present a comparison between the ADM and the reference simulations at 3 and 15 days after injection, respectively. Note that, due to the high permeability contrast, more active grid cells are used by ADM, compared to the previous Test cases. In fact, the highly permeable channels get flooded first, creating a sharp saturation front between high and low permeability zones. As for the previous heterogeneous Test case, along with constant and bilinear functions, multiscale basis functions are also considered as pressure interpolators. However, due to the high permeability contrast, to maintain monotonicity of multiscale solutions, the basis functions are computed with a modified permeability field that limits the maximum permeability contrast to two orders of magnitude [113, 116]. Note that even though the solution obtained with constant



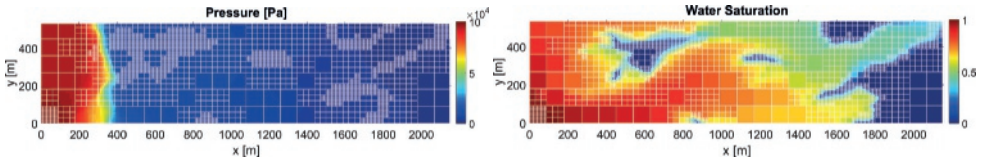
(a) Reference solution



(b) ADM const. interp.



(c) ADM bilin. interp.



(d) ADM MS interp.

Figure 4.13: Test case 2 - pressure and water saturation at 750 days after injection. The ADM method uses constant (b) and bilinear (c) and multiscale (d) interpolation for pressure and constant interpolation for saturation. The threshold value for the front tracking criterion is 0.2, corresponding to an average 18% of the reference fine-scale grid cells being employed by the ADM.

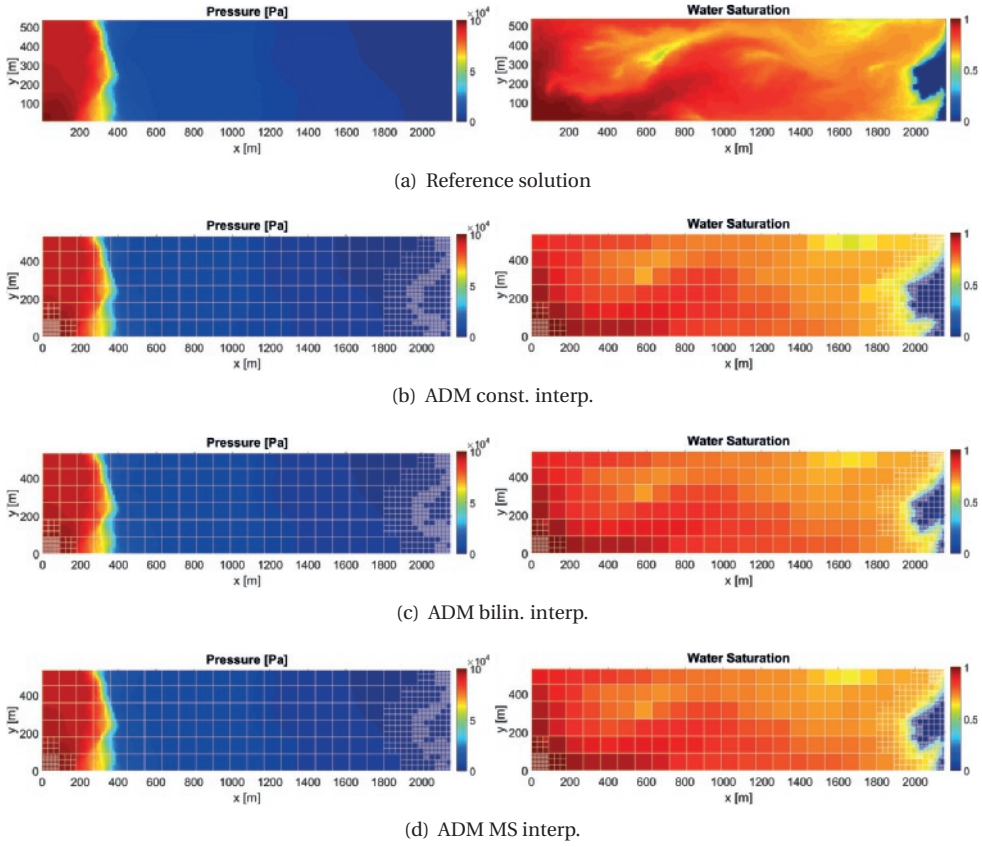


Figure 4.14: Test case 2 - pressure and water saturation at 1500 days after injection. The threshold value for the front tracking criterion is 0.2, corresponding to an average 18% of the reference fine-scale grid cells being employed by the ADM.

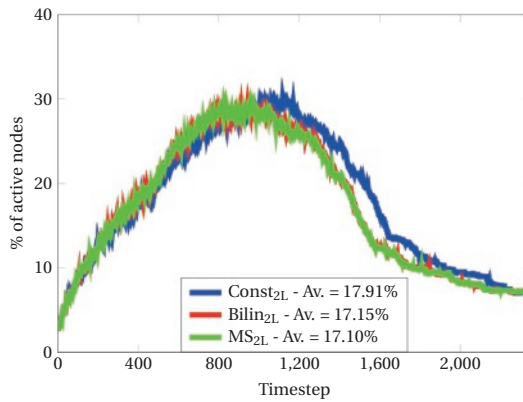


Figure 4.15: Test case 2 - history of the number of active nodes employed by ADM during simulation. The threshold value for coarsening criterion is 0.2. Also, average ADM active grid cells are mentioned in the graph.

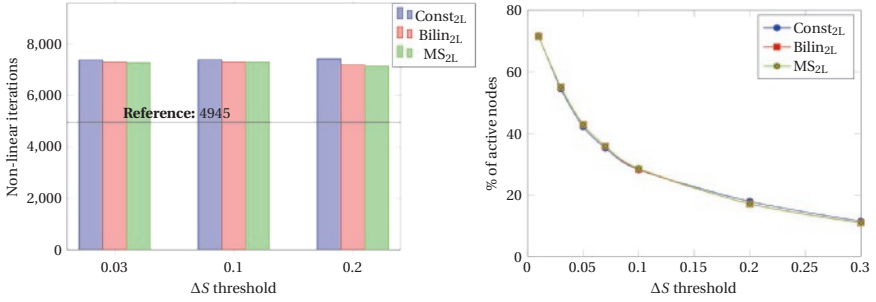


Figure 4.16: Test case 2 - (left) count of ADM non-linear iterations with different threshold values. Also shown in solid black line is the one for the reference solution. Note that the same non-linear convergence threshold values are used for both ADM and fine-scale systems, while they converge on different grid resolutions. (right) ADM active cells as a function of the coarsening threshold value.

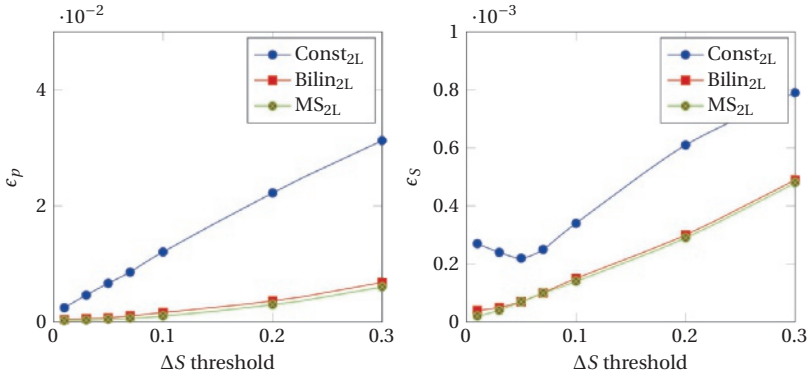


Figure 4.17: Test case 3 - pressure (left) and saturation (right) relative errors as a function of the coarsening threshold value.

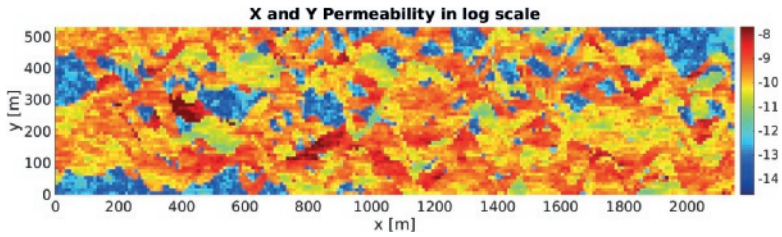


Figure 4.18: Test case 3 - Logarithm in base 10 of the permeability field.

pressure interpolators is fairly accurate, bilinear and multiscale basis functions still lead to improved solutions.

The number of nodes used by the ADM method for a coarsening criterion threshold of 0.2 is shown in Figure 4.21. Remark that at the beginning of the simulation the number of active cells is 5 – 10% of the fine-scale ones.

Figure 4.22 (a) and (b) show, respectively, the total number of non-linear iterations and the number of active blocks in a ADM simulation for different values of the coarsening criterion threshold. Note that the curve in Figure 4.22 (b) is less steep than for the other Test cases. This is due to the highly channelized nature of the permeability field that generates high saturation gradients throughout the whole domain.

Figure 4.23 shows the pressure and saturation relative errors as functions of the coarsening criterion threshold. As in the previous test cases, bilinear and multiscale basis functions outperform the constant interpolators. Note that multiscale basis functions - for this challenging case- provide the best pressure solution. The benefit is more evident for greater values of the coarsening threshold, as more coarse blocks are employed. Also, for the channelized and high-contrast nature of the permeability field, the errors are less sensitive to the type of the interpolator. This is due to the fact that, compared with the previous Test cases, more fraction of the cells remain at the fine-scale resolution.

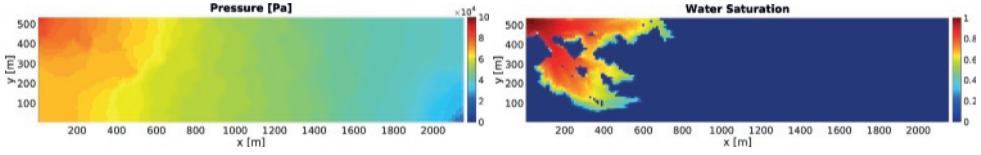
4.3.4. TEST CASE 4

To test the ADM for even more challenging scenario, a homogeneous reservoir that entails low-permeable long-coherent structures with six complex wells (two injectors and four produces) is considered. The permeability value of the low-perm features is 10^{-15} [m²], as illustrated in Fig. 4.24. The geometry and the location of the wells, along with the presence of the low-permeable areas, ensure a complex flow path. Table 4.2 provides the location of all wells.

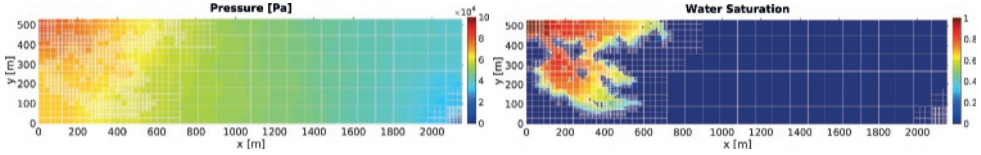
Table 4.2: Test case 4 - location of the wells, where $(x_b, y_b) - (x_e, y_e)$ denote the beginning ($_b$) and end ($_e$) coordinates, for linear perforating well segments. Note that wells "Inj 1" and "Prod 1" consist of two linear segments.

Well	Coordinates: $(x_b, y_b) - (x_e, y_e)$
Inj 1	(70, 20)-(70,46) and (71,18)-(75,15)
Inj 2	(143,50)-(145,50)
Prod 1	(200,40)-(200,44) and (197,40)-(199,40)
Prod 2	(184,30)-(184,33)
Prod 3	(100,3)-(101,3)
Prod 4	(5,3)

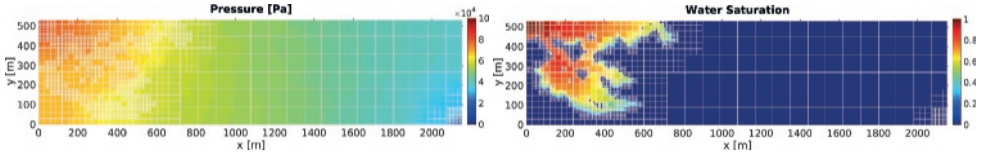
Figure 4.25 shows ADM solution using constant and bilinear interpolators along with those obtained by a fine-scale reference simulator, at the end of the simulation (200 days). Note that, similarly as in Test case 3, refinement is automatically preserved at the critical location of the high contrasts.



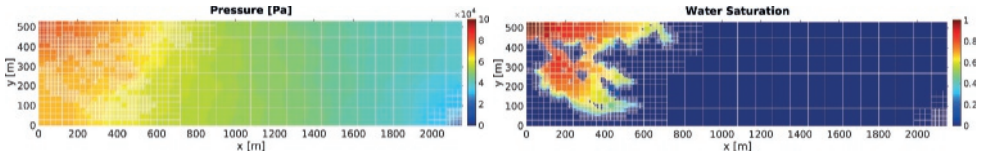
(a) Reference solution



(b) ADM const. interp.



(c) ADM bilin. interp.



(d) ADM MS interp.

Figure 4.19: Test case 3 - pressure and water saturation at 3 days after injection. The ADM method uses a coarsening criterion threshold of 0.2.

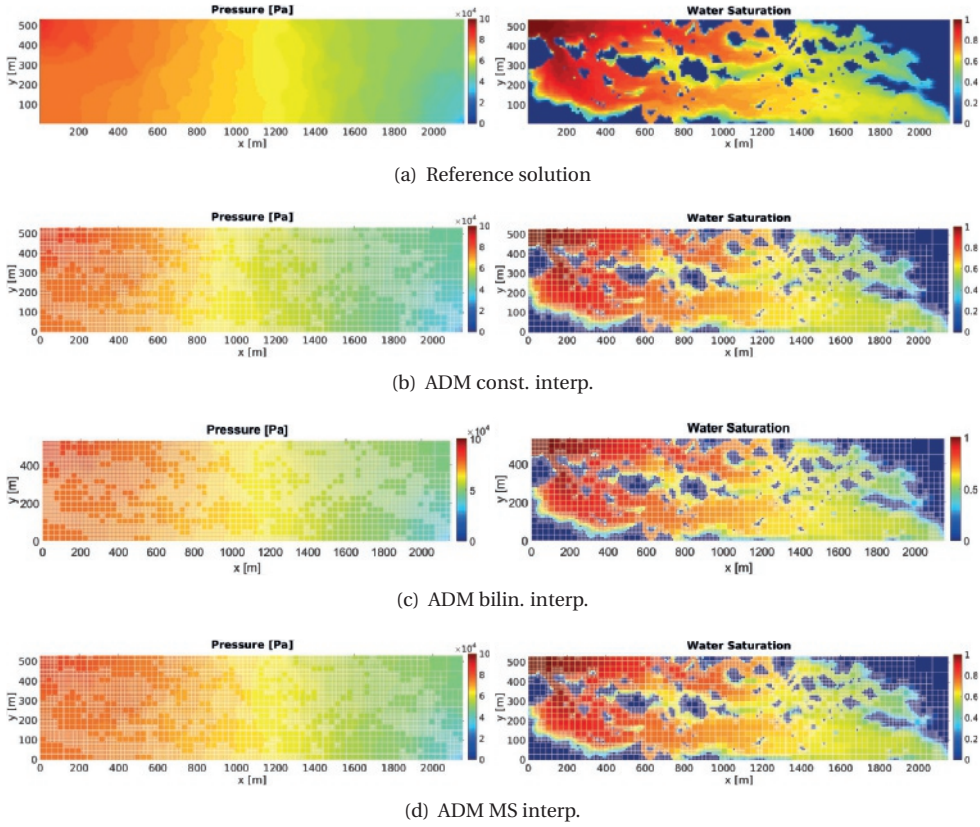


Figure 4.20: Test case 3 - pressure and water saturation at 15 days after injection. The ADM method uses a coarsening criterion threshold of 0.2, corresponding to an average 38% of the number of fine-scale blocks.

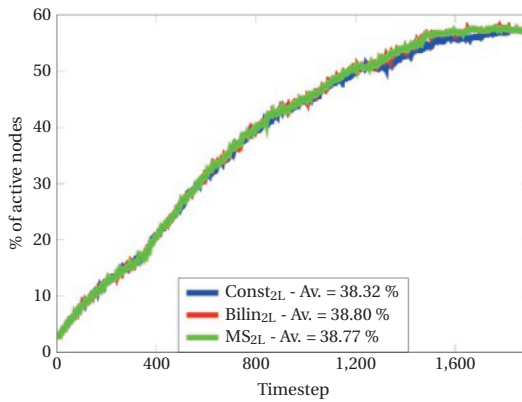


Figure 4.21: Test case 3 - history of the number of active nodes employed by ADM during simulation. The threshold value for coarsening criterion is 0.2. Also, average ADM active grid cells are mentioned in the graph.

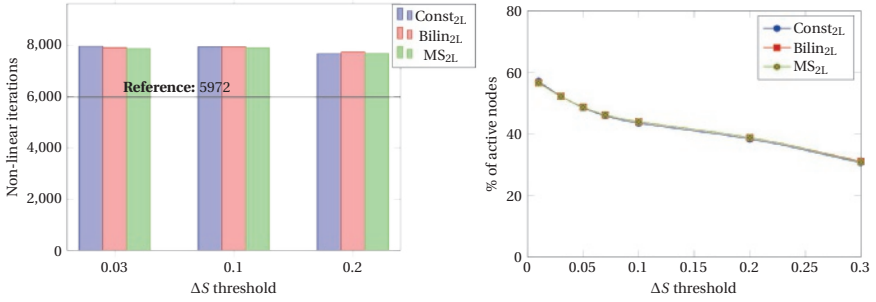


Figure 4.22: Test case 3 - (left) count of ADM non-linear iterations with different threshold values. Also shown in solid black line is the one for the reference solution. Note that the same non-linear convergence threshold values are used for both ADM and fine-scale systems, while they converge on different grid resolutions. (right) ADM active cells as a function of the coarsening threshold value.

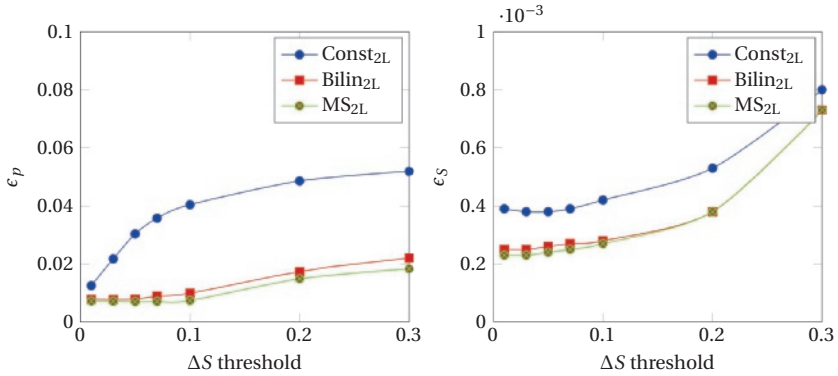


Figure 4.23: Test case 3 - pressure (left) and saturation (right) relative errors as a function of the coarsening threshold value.

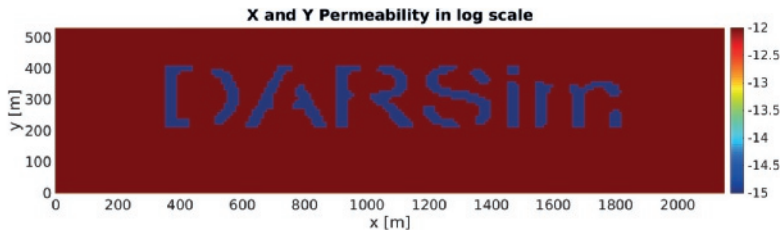


Figure 4.24: Test case 4 - Logarithm in base 10 of the permeability field.

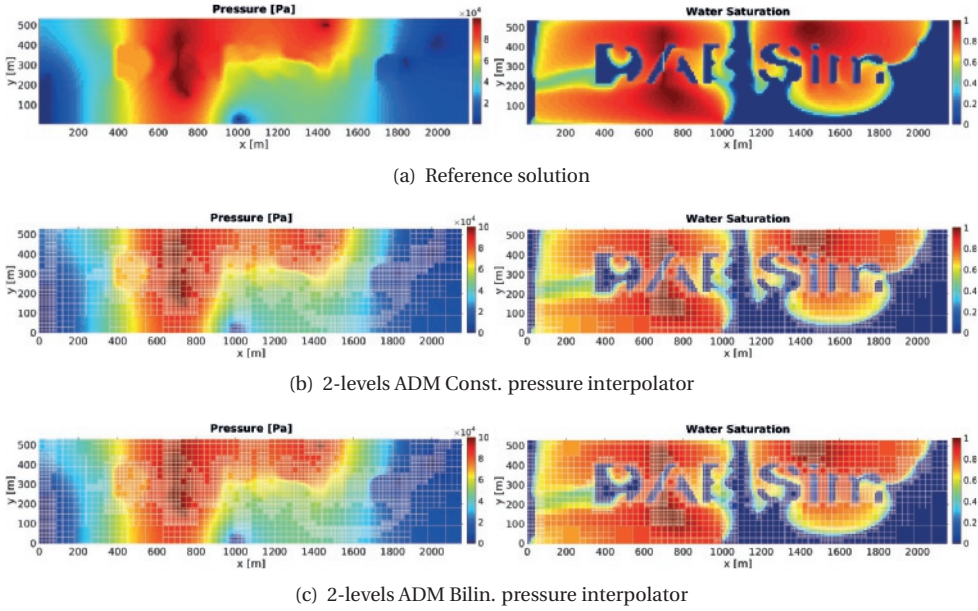


Figure 4.25: Test case 4 - Pressure and water saturation at 200 days after injection. The ADM method uses constant (b) and bilinear (c) interpolation for pressure and constant interpolation for saturation. The threshold value for the coarsening criterion is 0.2, corresponding to about 40 % of fine-scale grid cells.

Figure 4.26 shows the history of the active grid cells employed by ADM during the simulation, corresponding to the results of Fig. 4.25.

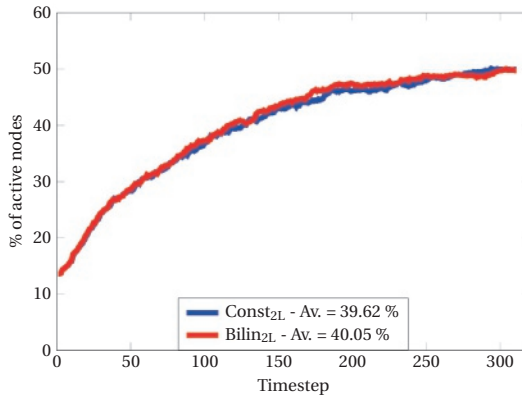


Figure 4.26: Test case 4 - history of the number of active nodes employed by ADM during simulation. The threshold value for the coarsening criterion is 0.2. Also, average ADM active grid cells are mentioned in the graph.

Figure 4.27(a) presents the count of non-linear iterations. In addition, Fig. 4.27(b) shows the average number of cells used in the ADM simulations as a function of the

threshold value chosen for the coarsening criterion. Note that, compared to the homogeneous problem (Test case 1), the average number of blocks has increased. This is due to the refinement required around the low permeable barriers, and the presence of a greater number of wells.

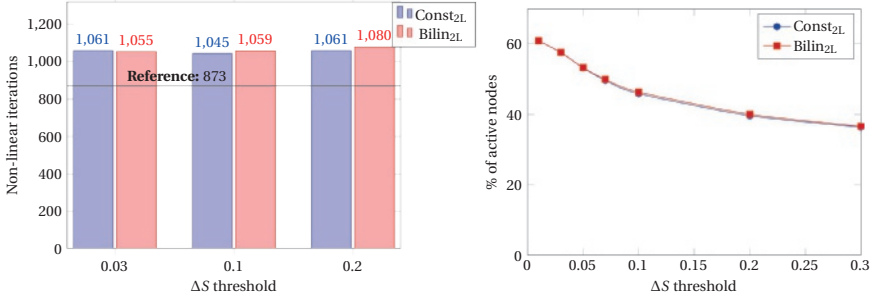


Figure 4.27: Test case 4 - (left) count of ADM non-linear iterations with different threshold values. Also shown in solid black line is the one for the reference solution. Note that the same non-linear convergence threshold values are used for both ADM and fine-scale systems, while they converge on different grid resolutions. (right) ADM active cells as a function of the coarsening threshold value.

Figure 4.28 shows the pressure and the saturation relative errors as functions of the threshold value chosen for the coarsening criterion. Using bilinear functions improves both the pressure and the saturation solutions, compared with constant interpolator. As expected, decreasing the threshold value for the coarsening criterion reduces the pressure and saturation relative errors. It is clear that using bilinear interpolation allows for lower sensitivity to the threshold value, and, consequently, the number of active fine grid cells.

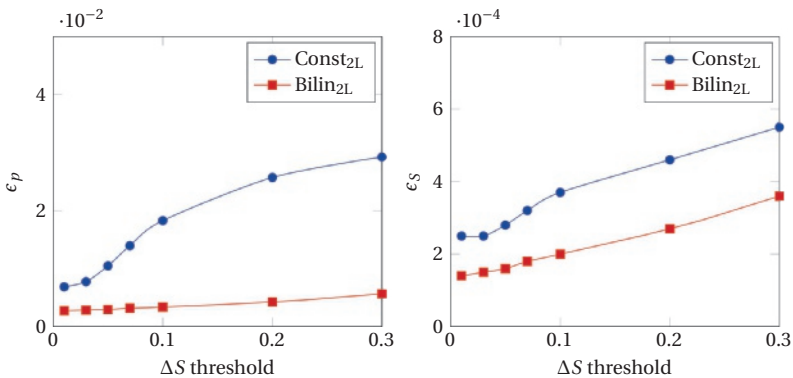


Figure 4.28: Test case 4 - pressure (left) and saturation (right) relative errors as a function of the coarsening threshold value.

4.4. COMPUTATIONAL COST AND SCALABILITY

The computational complexity of the ADM algorithm can be linked to three main components: (1) adaptive computation of interpolators (only if basis functions are used), (2) adaptive dynamic construction of multilevel restriction and prolongation operators, and (3) solution of the ADM linear system. Note that a systematic study of the ADM performance would require an efficient implementation of the method in a compilable language (e.g., C++) along with using advanced iterative solvers, such as Algebraic Multi-Grid (AMG) [95], for ADM linear system. A detailed assessment of the performance of the ADM method is, consequently, out of the scope of this manuscript. However, the current MATLAB research-based implementation of ADM allows one to study the scalability of the ADM system as the domain size becomes bigger. This study is presented in this section.

Here, three homogeneous domains with different sizes of 108×27 , 216×54 and 432×108 are considered. Figure 4.29 shows the average number of non-linear iterations per time-step employed by the ADM method. Remark that the convergence of the Newton's method on the ADM grid is not influenced by the problem size.

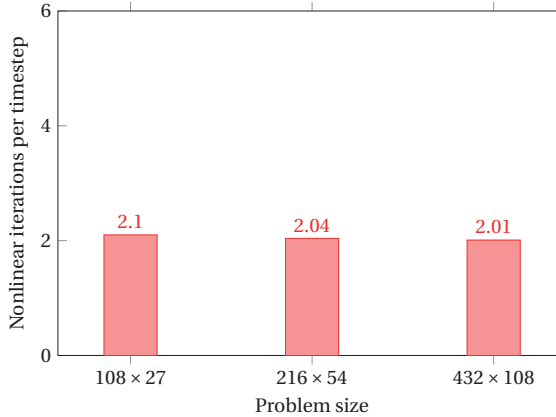


Figure 4.29: Average number of non-linear iterations per time-step employed by the ADM algorithm for different problem sizes.

Figure 4.30 shows, instead, the average number of active grid cells employed by ADM for the three different problem sizes. Due to the localised nature of the transport problem, the number of active nodes does not increase linearly with the number of fine-scale unknowns. This is a significant achievement made possible by the ADM method, as a promising approach for next-generation simulators. It is also important to note that the significantly smaller linear systems solved by the ADM are denser than the original big fine-scale reference system. Along with the size of the ADM system, the sparsity of the ADM and fine-scale systems is presented in Fig. 4.31.

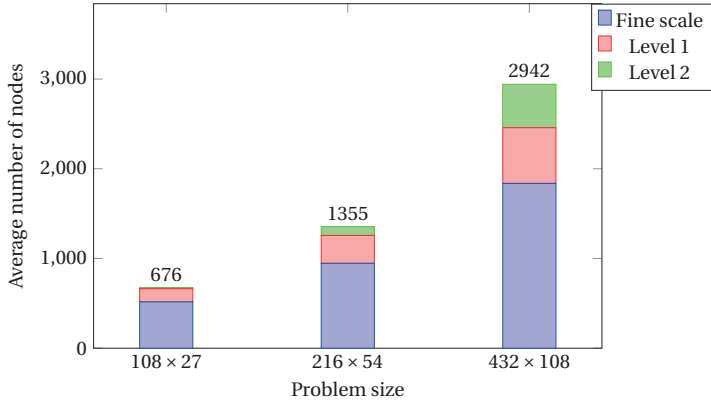


Figure 4.30: Average number of cells employed by the ADM algorithm for different problem sizes. The total count of cells used on average by ADM is shown on top of the bars.

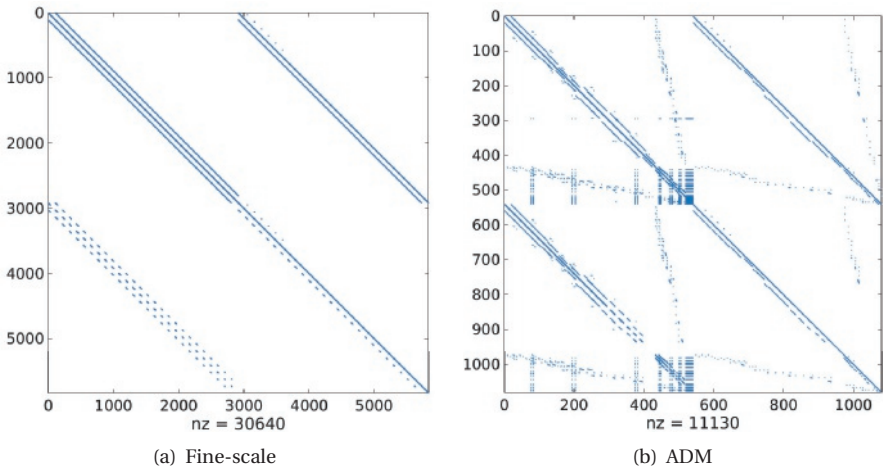


Figure 4.31: Examples of the sparsity pattern of the Jacobian matrix for fine-scale (a) and ADM (b) simulations. The fine-scale system has $34 \cdot 10^6$ elements, of which 30640 are non-zeros. The ADM Jacobian has instead $1.1 \cdot 10^6$ elements, of which 11130 are non-zeros. This example shows how the ADM system, although being significantly smaller, leads to denser systems.

5

ADM METHOD FOR COMPOSITIONAL DISPLACEMENTS

In the previous chapter, ADM was developed for two-phase immiscible displacement in 2D heterogeneous reservoirs [88]. However, multilevel methods play a more important role when simulating processes involving complex physics [48, 50, 53]. In this chapter, ADM is extended to compositional simulations. Gravitational and heterogeneous capillary forces are also taken into account, adding significantly to the non-linear complexity of the multiphase flow. Two different fine-scale compositional formulations, differing for the choice of primary variables, are considered: the so-called natural variables formulation [118] and the overall-composition formulation. In both cases the set of primary unknowns is mapped to the dynamically defined grid resolution from the fine-scale grid. While multiscale basis functions are employed as pressure interpolators, piece-wise constant functions are used for the remaining variables (i.e., saturations or overall-compositions). The grid resolution is chosen based on the difference of the mass fraction of each component between two neighbouring cells. Additional consideration is taken when natural variable formulation is used because of the variable substitution that has to be performed for single-phase cells.

ADM is applied successfully to challenging homogeneous and heterogeneous 2D and 3D compositional test cases. Systematic studies of the performance of ADM are conducted in terms of error with respect to the reference fine-scale solution, number of non-linear iterations and number of the active grid-blocks. Sensitivity to the fine-scale formulation (i.e., choice of primary unknowns) and to the problem size is also investigated. Numerical experiments show that ADM provides accurate results by employing only a fraction of the fine-scale grid cells. It provides a promising framework for field-scale simulation of multiphase flow in 3D natural formations with non-linear physics.

This chapter is organised as follows. In Section 5.1 the two fine-scale formulations are presented along with the FIM solution strategy. Then, in Section 5.2, the ADM method

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along with its procedure of dynamic grid generation, construction of prolongation and restriction operators, and simulation strategy are described. Numerical simulation results are presented for homogeneous and heterogeneous test cases in Section 5.3.

5.1. FINE-SCALE FORMULATIONS AND SOLUTION STRATEGY

In this chapter the full set of equations describing multicomponent multiphase flow in porous media is considered. It is shown in the literature that it is possible to solve for as few as n_c primary variables using n_c equations [31]. Usually, mass conservation equations are included in the set of primary equations. The rest of the variables (secondary) can be subsequently updated using the remaining equations. Different formulations are present in the literature that differ for the choice of the primary unknowns. Two formulations are considered in this work as representative of the two main alternatives, natural and molar (mass) variables. The compositional ADM method, presented in the next section, is developed for these two formulations. A detailed analysis of the existing compositional fine-scale formulations can be found in [119].

5

5.1.1. NATURAL VARIABLES FORMULATION

The natural variable formulation as presented in [118] employs n_{ph} phase pressures, n_{ph} saturations and $n_{ph}n_c$ phase compositions as the set of base unknowns. Capillary relations, Eq. (1.6) and Eq. (1.10) are used to reduce the number of unknowns. The discrete residual form of Eq. (1.7) for component $c \in \{1, \dots, n_c\}$ in cell $\langle i \rangle$ reads

$$r_{c,\langle i \rangle}^{n+1} = \frac{V_{\langle i \rangle} \phi_{\langle i \rangle}}{\Delta t} \sum_{\alpha=1}^{n_{ph}} \left(x_{c\alpha,\langle i \rangle}^{n+1} \rho_{\alpha,\langle i \rangle}^{n+1} S_{\alpha,\langle i \rangle}^{n+1} - x_{c\alpha,\langle i \rangle}^n \rho_{\alpha,\langle i \rangle}^n S_{\alpha,\langle i \rangle}^n \right) + \sum_j^{N_n} A_{\langle ij \rangle} \sum_{\alpha=1}^{n_{ph}} y_{c\alpha} \rho_{\alpha} \mathbf{u}_{\alpha} \Big|_{\langle ij \rangle}^{n+1} \cdot \mathbf{n} - \sum_{\alpha=1}^{n_{ph}} x_{c\alpha,\langle i \rangle} q_{\alpha,\langle i \rangle} = 0, \quad (5.1)$$

and the discrete form of Eq. (1.8) for component c distributed among phases α and β reads

$$e_c^{n+1} = k_{c,\alpha,\beta}^{n+1} y_{c\beta}^{n+1} - y_{c\alpha}^{n+1} = 0. \quad (5.2)$$

Here, $V_{\langle i \rangle}$ is the volume of cell i , $A_{\langle ij \rangle}$ is the surface area between cells i and j , N_n indicates the number of neighbours of cell i and $\mathbf{n}$ is the unit vector normal to its boundary. The Newton-Raphson method is used to linearize and iteratively solve these equations, i.e.

$$r_{c,\langle i \rangle}^{v+1} \approx r_{c,\langle i \rangle}^v + \frac{\partial r_{c,\langle i \rangle}^v}{\partial p_1} \Big|_v \delta p_1^{v+1} + \sum_{\alpha=1}^{n_{ph}-1} \frac{\partial r_{c,\langle i \rangle}^v}{\partial S_{\alpha}} \Big|_v \delta S_{\alpha}^{v+1} + \sum_{m=1}^{n_c-1} \sum_{\alpha=1}^{n_{ph}} \frac{\partial r_{c,\langle i \rangle}^v}{\partial y_{m\alpha}} \Big|_v \delta y_{m\alpha}^{v+1} = 0. \quad (5.3)$$

$$e_{c,\alpha,\beta}^{v+1} \approx e_{c,\alpha,\beta}^v + \left. \frac{\partial e_{c,\alpha,\beta}^v}{\partial p_1} \right|^\nu \delta p_1^{v+1} + \sum_{\alpha=1}^{n_{ph}-1} \left. \frac{\partial e_{c,\alpha,\beta}^v}{\partial S_\alpha} \right|^\nu \delta S_\alpha^{v+1} + \sum_{m=1}^{n_c-1} \sum_{\alpha=1}^{n_{ph}} \left. \frac{\partial e_{c,\alpha,\beta}^v}{\partial y_{m\alpha}} \right|^\nu \delta y_{m\alpha}^{v+1} = 0. \quad (5.4)$$

Here, v and $v+1$ indicate the iteration steps. Finally, the linear system

$$\mathbf{J}_{nat}^v \delta x_{nat}^{v+1} = -r_{nat}^v, \quad (5.5)$$

is solved, where $\mathbf{J}$ is the Jacobian matrix and δx^{v+1} is the Newton's update. The process is repeated until convergence is reached and $\delta x^{n+1} \approx \delta x^{v+1}$. For example, in presence of 2 components (1, 2) and 2 phases (v, l) Eq. (5.5) reads

$$\underbrace{\begin{bmatrix} \frac{\partial r_1}{\partial p_v} & \frac{\partial r_1}{\partial S_v} & \frac{\partial r_1}{\partial y_{1v}} & \frac{\partial r_1}{\partial y_{2l}} \\ \frac{\partial r_2}{\partial p_v} & \frac{\partial r_2}{\partial S_v} & \frac{\partial r_2}{\partial y_{1v}} & \frac{\partial r_2}{\partial y_{2l}} \\ \frac{\partial e_{1,v,l}}{\partial p_v} & \frac{\partial e_{1,v,l}}{\partial S_v} & \frac{\partial e_{1,v,l}}{\partial y_{1v}} & \frac{\partial e_{1,v,l}}{\partial y_{2l}} \\ \frac{\partial e_{2,v,l}}{\partial p_v} & \frac{\partial e_{2,v,l}}{\partial S_v} & \frac{\partial e_{2,v,l}}{\partial y_{1v}} & \frac{\partial e_{2,v,l}}{\partial y_{2l}} \end{bmatrix}}_{\mathbf{J}_{nat}^v} \underbrace{\begin{bmatrix} \delta p_v \\ \delta S_v \\ \delta y_{1v} \\ \delta y_{2l} \end{bmatrix}}_{\delta x_{nat}^{v+1}} = - \underbrace{\begin{bmatrix} r_1 \\ r_2 \\ e_{1,v,l} \\ e_{2,v,l} \end{bmatrix}}_{r_{nat}^v}. \quad (5.6)$$

Also, note that the additional complexity related to the natural variable formulation is that a variable substitution has to be performed for single phase cells, where no thermodynamic equilibrium equations need to be solved.

5.1.2. OVERALL COMPOSITIONS FORMULATION

The overall compositions formulation [28, 119] uses one phase pressure and the n_c total mole (or mass) fractions (z_i) as primary variables. The constraint $\sum_{i=1}^{n_c} z_i = 1$ is used to eliminate one unknown. This formulation is based on the hypothesis that thermodynamic equilibrium holds at each Newton's iteration. The residuals of the n_c conservation equations are linearised with respect to the n_c primary variables (1 phase pressure and $n_c - 1$ total fractions), i.e.,

$$r_{c,\langle i \rangle}^{v+1} \approx r_{c,\langle i \rangle}^v + \left. \frac{\partial r_{c,\langle i \rangle}^v}{\partial p_1} \right|^\nu \delta p_1^{v+1} + \sum_{m=1}^{n_c-1} \left. \frac{\partial r_{c,\langle i \rangle}^v}{\partial z_m} \right|^\nu \delta z_m^{v+1} = 0. \quad (5.7)$$

Here, v and $v+1$ are the current and the following iteration step. Remark that derivatives of the phase compositions ($x_{i\alpha}$) with respect to the total component mole fractions are necessary to have a consistent fully-implicit discretization scheme [24]. These derivatives are computed from the thermodynamic equilibrium equations. The linear system

$$\mathbf{J}_{oc}^v \delta x_{oc}^{v+1} = -r_{oc}^v \quad (5.8)$$

is solved and the primary variables are updated. In presence of 2 components (1, 2) and 2 phases (v, l) the linear system to be solved is of the form

$$\underbrace{\begin{bmatrix} \frac{\partial r_1}{\partial p_v} & \frac{\partial r_1}{\partial z_1} \\ \frac{\partial r_2}{\partial p_v} & \frac{\partial r_2}{\partial z_1} \end{bmatrix}}_{\mathbf{J}_{oc}^v} \underbrace{\begin{bmatrix} \delta p_v \\ \delta z_1 \end{bmatrix}}_{\delta x_{oc}^{v+1}} = - \underbrace{\begin{bmatrix} r_1 \\ r_2 \end{bmatrix}}_{r_{oc}^v}. \quad (5.9)$$

Once the primary variables have been updated the number of phases existing in each cell is determined using Eqs. ((1.15))-(1.16). Nothing is done for single-phase cells whereas flash calculations are performed for two-phase cells. The entire process is repeated until non-linear convergence is reached.

5.1.3. WELLS MODELLING

Wells are modelled according to the Peaceman well model [115, 120]. Each well can consist of many completions, each of them perforating a single cell. The incoming flux of phase α from a well to a perforated cell is given by

$$q_{\alpha, \langle i \rangle} = \text{WI} K_{\langle i \rangle} \rho_{\alpha}^{up} \frac{k_{r, \alpha}^{up}}{\mu_{\alpha}} (p_{well} - p_{\langle i \rangle}). \quad (5.10)$$

Here, WI is the well index and it contains all geometrical information. The flux $q_{\alpha, \langle i \rangle}$ is positive for injectors and negative for producers. According to the phase-based upwind scheme, fluid properties ($\rho_{\alpha}, k_{r, \alpha}, \mu_{\alpha}$) are evaluated at the pressure of the perforated cells for producer wells. On the other hand, properties of the injected fluid (computed at the injection pressure) are used for injectors.

5.1.4. NON-LINEAR CONVERGENCE AND TIME-STEPPING

The Newton-Raphson procedure is considered to be converged when both the infinite norm of the residual and the Newton's updates (δx) of all variables are below a user-defined tolerance. A simple adaptive time-stepping strategy is employed to limit the number of wasted Newton's iterations. A maximum and a minimum time-step size are defined by the user along with an initial one. Three other values are defined by the user to control the time-stepping strategy, a , b and It_{max} such that $a < b < It_{max}$. If, for a given time-step t , non-linear convergence is reached with less than a Newton's iterations, time-step $t + 1$ will be twice the size of time-step t . If, instead, more than b iterations are necessary the following time-step will have half the size of time-step t . If none of these conditions is the case the time-step size remains the same. If the number of Newton's iterations exceeds It_{max} the time-step size is divided by two and solved again. A maximum of 10 time-step chops is allowed.

The given strategy cast the fine-scale compositional simulation, which becomes too expensive to be solved for real-field applications. Next, the Dynamic Multilevel Method for compositional displacements (ADM) is presented, to resolve this limitation.

5.2. COMPOSITIONAL ADM

ADM reduces the computational cost associated with the solution of the linear systems of Eq. (5.5) and Eq. (5.8) by algebraically mapping them to a dynamically defined grid so that fine-scale resolution is employed only in a small fraction of the domain. The ADM grid is constructed combining grid blocks belonging to a set of n_l grids of nested topology (e.g., Fig. 5.1). Each grid is formed by $N_x^l \times N_y^l \times N_z^l$ grid-blocks and it corresponds to a resolution $l \in \{0, \dots, n_l\}$. The ratios $\text{cf}_x = \frac{N_x^{l-1}}{N_x^l}$, $\text{cf}_y = \frac{N_y^{l-1}}{N_y^l}$ and $\text{cf}_z = \frac{N_z^{l-1}}{N_z^l}$ are called coarsening factors. Here, $l = 0$ indicates the fine-scale grid, which is assumed to have

the proper resolution to capture the physics of interest. At each time-step, the grid resolution is determined based on the contrast of specific fluid properties defined by the user, and on the presence of particular features, e.g wells.

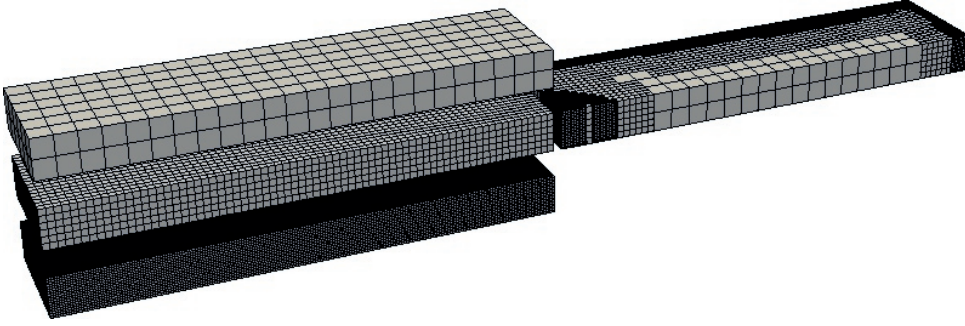


Figure 5.1: An example of the ADM grids for a 3D reservoir is shown. Three static grids ($l = \{0, 1, 2\}$) are considered. On the left, a possible dynamic grid is shown combining grid-cells of the three resolutions.

Let us call N_l the number of grid-blocks of a ADM grid where the coarsest grid-block is at resolution l . At each Newton's iteration, ADM approximates the fine-scale solution, x_0 , as

$$x_0^v \approx \prod_1^{n_l} \hat{\mathbf{P}}_l^{l-1} x_l^v. \quad (5.11)$$

Here, $\hat{\mathbf{P}}_l^{l-1}$ is a block diagonal matrix referred to as prolongation operator. Each block is a $N_{l-1} \times N_l$ matrix and its columns contain the basis functions used to interpolate the corresponding primary variables. Sequences of restriction operators, $\hat{\mathbf{R}}_l^{l-1}$, are employed along with the prolongation operators to map the fine-scale linear system to the ADM grid resolution, i.e.

$$\left(\prod_1^{n_l} \hat{\mathbf{R}}_l^{l-1} \mathbf{J}_0^v \prod_1^{n_l} \hat{\mathbf{P}}_l^{l-1} \right) \delta x_l^{v+1} = - \prod_1^{n_l} \hat{\mathbf{R}}_l^{l-1} r_0^v. \quad (5.12)$$

Here, $\hat{\mathbf{R}}_l^{l-1}$ is a block diagonal matrix of the form

$$(\hat{\mathbf{R}})_l^{l-1} = \begin{bmatrix} (\hat{\mathbf{R}})_l^{l-1} & & \\ & \ddots & \\ & & (\hat{\mathbf{R}})_l^{l-1} \end{bmatrix} \quad (5.13)$$

where each block $(\hat{\mathbf{R}})_l^{l-1}$ is a $N_l \times N_{l-1}$ matrix such that the entry in row i and column j is

$$\hat{\mathbf{R}}(i, j) = \begin{cases} 1 & \text{if cell } j \text{ belongs to cell } i \\ 0 & \text{otherwise.} \end{cases} \quad (5.14)$$

The choice of employing a finite-volume restriction operator ensures mass conservation to be preserved at all resolutions. Thus, the pressure field is conservative on the ADM grid resolution on which the transport equations are solved.

The following subsections illustrate how ADM was adapted to the two fine-scale formulations described in the previous section.

5.2.1. ADM FOR NATURAL VARIABLES FORMULATION

For simplicity, but without any loss of generality, let us consider a 2-component (1, 2), 2-phase (v, l) system. In the natural variables formulation mass conservation and thermodynamic equilibrium equations are solved simultaneously. Consequently, all equations have to be mapped to the ADM grid resolution. Thus, the ADM prolongation operator in Eq. (5.12) is a block matrix of the form

$$\hat{\mathbf{P}}_{l-1}^l = \begin{bmatrix} (\hat{\mathbf{P}}_p)_{l-1}^l & & & \\ & (\hat{\mathbf{P}}_{S_v})_{l-1}^l & & \\ & & (\hat{\mathbf{P}}_{y_{1v}})_{l-1}^l & \\ & & & (\hat{\mathbf{P}}_{y_{2l}})_{l-1}^l \end{bmatrix}. \quad (5.15)$$

Note, that all variables have to be prolonged. ADM allows for different interpolation operators for each variable. Here, piece-wise constant, bilinear and multiscale basis functions are considered as pressure interpolators whereas piece-wise constant functions are used to interpolate the remaining variables. Remark, that since different equations are solved for cells in different phase states, it is important to avoid clustering grid-cells in which different number of phases are present.

The solution procedure of one time-step, using the ADM method with the natural variable formulation is presented in Algorithm 1. Remark that convergence is achieved on the dynamically defined grid resolution and that different tolerances can be employed for mass balance and thermodynamic equilibrium equations.

Algorithm 1: ADM with natural variables formulation

```

Data: Solution at time-step  $n$ 
Result: Solution at time-step  $n + 1$ 
1 Selection of the grid resolution;
2 Construction of ADM operators;
3 initialization;
4 while Converged = false do
5   Define phase state /* number of phases in each cell */
6   Build fine-scale system /* Eq. (5.5) */
7   Solve Coarse System /* Eq. (5.12) */
8   Prolong solution /* Eq. (5.11) */
9   Update fluid properties /* Compute  $\rho, \mu$  */
10  Compute residuals ( $r^{v+1}, E^{v+1}$ );
    /* Check convergence on the ADM grid */
11  if  $\|r_{ADM}^{v+1}\|_\infty < \varepsilon_1$  &  $\|e_{ADM}^{v+1}\|_\infty < \varepsilon_2$  &  $\|\delta x^{v+1}\|_\infty < \varepsilon_3$  then
12    | Converged := true;
13  end
14 end

```

5.2.2. ADM FOR OVERALL COMPOSITIONS FORMULATION

In the overall-composition formulation, conservation equations and thermodynamic equilibrium equations are solved in two separate steps. ADM maps the linear system of Eq. (5.8) describing the first set of equations to the dynamically defined grid resolution. Thus, the prolongation operator is of the form

$$\hat{\mathbf{P}}_{l-1}^l = \begin{bmatrix} (\hat{\mathbf{P}}_p)^l_{l-1} & \\ & (\hat{\mathbf{P}}_{z_1})^l_{l-1} \end{bmatrix}. \quad (5.16)$$

Once the coarse system has been solved and the solution (p and z_c) has been prolonged to fine-scale resolution, thermodynamic equilibrium equations are solved on the fine-scale resolution. Note that the local nature of this second sets of equations makes this step suitable for efficient distributed computations. The main steps of the solution of one time-step are shown in Algorithm 2.

Algorithm 2: ADM with overall composition formulation

Data: Solution at time-step n
Result: Solution at time-step $n + 1$

- 1 Selection of the grid resolution;
- 2 Construction of ADM operators;
- 3 initialization;
- 4 **while** *Converged* = *false* **do**
 - 5 Build fine-scale system /* Eq. (5.8) */
 - 6 Solve Coarse System /* Eq. (5.12) */
 - 7 Prolong solution /* Eq. (5.11) */
 - 8 Perform flash calculations /* Compute all $x_{i\alpha}$ */
 - 9 Update fluid properties /* Compute ρ, μ */
 - 10 Compute mass balance residual (r^{v+1});
/* Check convergence on the ADM grid */
 - 11 **if** $\|r_{ADM}\|_\infty < \varepsilon_1$ & $\|\delta x\|_\infty < \varepsilon_2$ **then**
 - 12 *Converged* := *true*;
 - 13 **end**
- 14 **end**

5

5.3. NUMERICAL RESULTS

The described fine-scale and ADM solution strategies were implemented in the Matlab research code DARSim2 (Appendix A). Numerical studies are presented in this section. Five test cases are presented to study the performance of ADM, i.e.

- **Test case 1: influence of the pressure interpolator.** The benefit of using multiscale basis functions as pressure interpolators, rather than piece-wise constant functions, is analysed in this 3D compositional test case.

- **Test case 2: influence of the problem size.** Three similar models of different sizes are considered. This is done to show how ADM is more efficient, in terms of number of grid-cells employed throughout the simulation, for larger problems.
- **Test case 3: robustness of ADM.** Different permeability heterogeneities are considered and the performance of ADM are detailly analysed in terms of number of non-linear iterations, number of grid-cells used and accuracy of the solution.
- **Test case 4: capillarity heterogeneities.** The ability of ADM to deal with heterogeneous capillary curves is assessed in this test-case.
- **Test case 5: gas injection in a 3D heterogeneous reservoir.** A gas injection process in a 3D heterogeneous reservoir is considered. Both gravitational and compositional effects are included.

5.3.1. TEST CASE 1: INFLUENCE OF THE PRESSURE INTERPOLATION

A $108\text{m} \times 27\text{m} \times 18\text{m}$ homogeneous reservoir with uniform porosity $\phi = 0.2$ is considered. A fine-scale computational grid of $108 \times 27 \times 18$ cells is imposed on the domain. Two components (Comp₁, Comp₂) partitioning into two phases (vapour and liquid) are present. The properties of the components are summarized in Table 5.1. Note that the pressure-dependent K-values are obtained by employing Wilson's equation.

Table 5.1: Test case 1 - Fluid properties.

	Comp ₁	Comp ₂	Vapour	Liquid
ρ [$\frac{\text{kmol}}{\text{m}^3}$]	-	-	1.378	4.829
μ [Pa s]	-	-	$0.018 \cdot 10^{-3}$	$0.33 \cdot 10^{-3}$
c [$\frac{1}{\text{Pa}}$]	-	-	0	0
P_{crit} [bar]	73.8	25	-	-
T_{crit} [K]	304	568	-	-
ω	0.239	0.398	-	-

The model is run both with fine-scale and ADM simulators. ADM employs two coarse levels with coarsening factors of 3 in each direction. Piecewise constant and multiscale basis functions are considered as ADM pressure interpolators; while constant interpolators are used for the transport equation unknowns. Figure 5.2 compares the pressure and the saturation solution after 1.5 days of injection obtained with the reference fine-scale and ADM simulators, employing the two different pressure interpolators. Note that the solutions obtained using multiscale basis functions are significantly more accurate than those obtained with piecewise constant basis functions. In particular, ADM with constant pressure interpolation overestimates the saturation front location. Figure 5.3 shows a comparison of the vapour and the liquid cumulative production curves. Note,

that ADM with constant pressure interpolation estimates the gas breakthrough to happen earlier than in the reference solution. Also, there is a mismatch in the liquid production curve in the early times. On the other hand, the production curves obtained with ADM employing multiscale basis functions match perfectly the fine-scale ones.

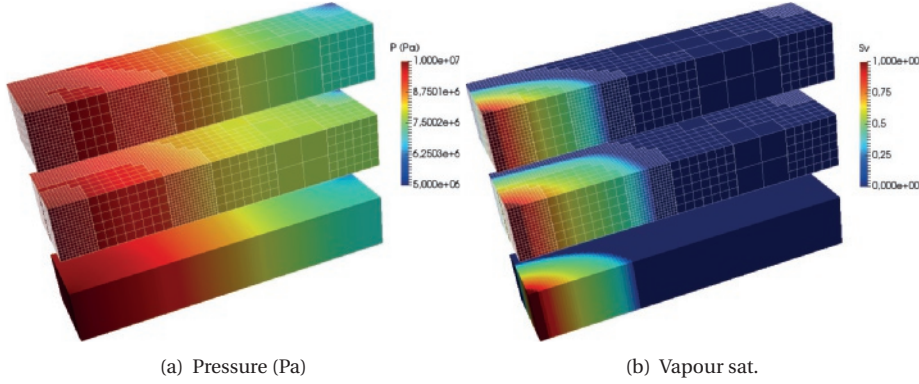


Figure 5.2: Test case 1 - Comparison of the pressure and the saturation solutions obtained with the fine-scale simulator (bottom) and with ADM employing piece-wise constant (middle) and multiscale (top) basis functions.

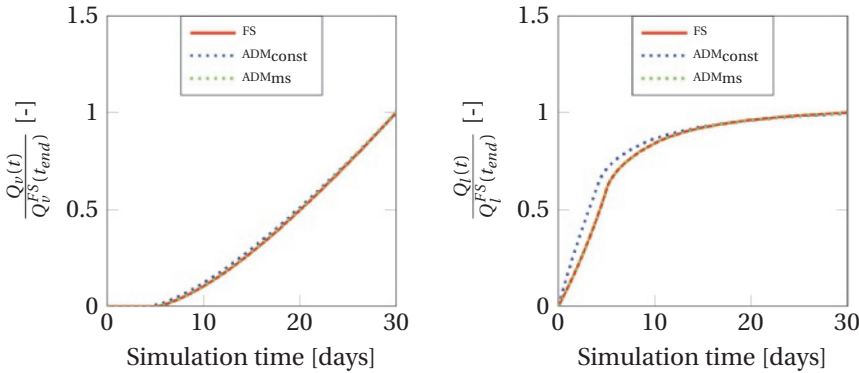


Figure 5.3: Test case 1 - Vapour (left) and liquid (right) cumulative production curves.

5.3.2. TEST CASE 2: SCALABILITY TEST IN 3D

Three 3D homogeneous reservoirs of sizes $108\text{ m} \times 27\text{ m} \times 18\text{ m}$, $216\text{ m} \times 54\text{ m} \times 18\text{ m}$ and $432\text{ m} \times 108\text{ m} \times 18\text{ m}$ are considered (see Fig. 5.4). A cartesian mesh with $\Delta x = \Delta y = \Delta z = 1\text{ m}$ is imposed on each reservoir. The fluid properties are the same of those used in the test case 1. Both gravitational and capillary forces are neglected. A pressure-constrained injection well ($P_{\text{inj}} = 100\text{ bar}$) is placed in the left corner and a pressure-constrained producer is placed in the top right corner ($P_{\text{prod}} = 50\text{ bar}$). Both wells are perforating all 18

cells in the vertical direction. The characteristic velocity is defined as $\bar{U} = K \frac{P_{\text{inj}} - P_{\text{prod}}}{d}$, where d is the distance between the wells. Permeability of each reservoir is chosen so that $\bar{U}$ is the same in the three models. Simulations are run until 2 pore volumes of gas are injected. The maximum time-step size is set to 30 days. ADM employs two coarse levels with coarsening factors of 3 in each direction, with multiscale basis functions as pressure interpolators.

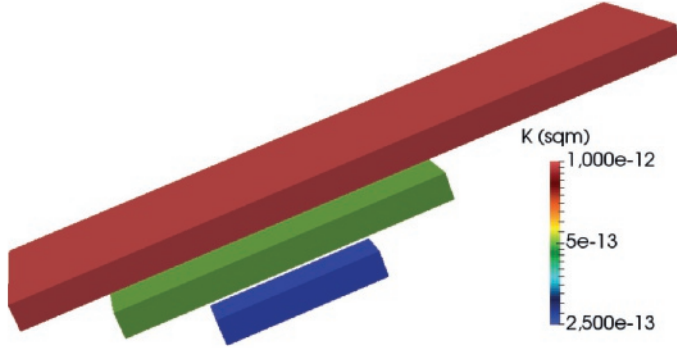


Figure 5.4: Test case 2 - Permeability of the three homogeneous reservoirs considered.

Figure 5.5 shows the evolution of the number of ADM grid-blocks (expressed as a percentage of the number of fine-scale grid-cells) throughout the time-dependent simulation. Remark that, as the size of the problem grows, ADM becomes more effective in reducing the number of grid-cells with respect to the fine-scale simulation.

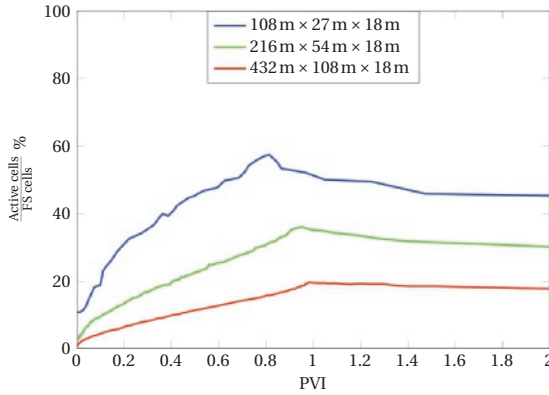


Figure 5.5: Test case 2 - History of the number of active nodes employed by ADM during simulation. The threshold value for the coarsening criterion is 0.07.

5.3.3. TEST CASE 3: GAS INJECTION IN A 2D

A $500\text{ m} \times 500\text{ m}$ reservoir is considered on which a 99×99 grid is imposed. The reservoir thickness is 100m and its porosity is uniform and equal to 0.2. Five sets of log-

normally distributed permeability field are chosen with different layering orientations. These fields are created using sequential Gaussian simulations with spherical variogram and dimensionless correlation lengths 0.5 and 0.02 [121]. Each set is formed by 20 statistically identical realisations and one of them is shown in Fig. 5.6.

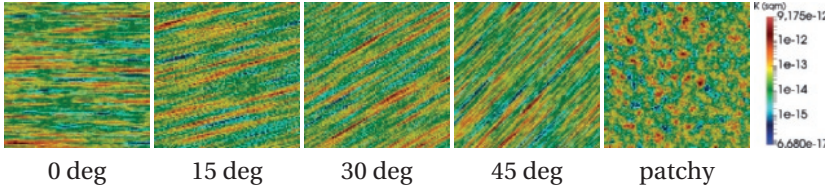


Figure 5.6: Test case 3 - One realisation of each of the 5 sets of heterogeneous permeability fields considered is shown.

A black oil fluid model is used with properties as summarised in Table 5.2. Phase viscosities are constant whereas densities are computed using Eq. (1.18). Quadratic relative permeability curves are considered with end points equal to 1 and with an irreducible oil saturation of 0.1 [24]. Capillary pressure is neglected in this test case.

Table 5.2: Test case 3 - Fluid properties. Here, $P^* = 250$ bar and $P_{st} = 1$ bar is the pressure at standard conditions. The reservoir is considered to be isothermal and no temperature dependencies of the fluid properties are considered.

	Gas properties	Oil properties
$\rho^{ST} [\frac{\text{kg}}{\text{m}^3}]$	1	800
$\mu [\text{Pa s}]$	10^{-4}	10^{-3}
$B [-]$	$\frac{P_{st}}{2P}$	1
$R_s [\frac{\text{m}^3}{\text{m}^3}]$	-	$100 \frac{P}{P^*} + 0.2$

The initial pressure and composition of the reservoir are set to be $P_{init} = 150$ bar and $z_g = 0.1$, respectively, resulting in an initial vapour saturation of $S_{v_{init}} = 0.08728$. Two pressure-constrained wells are present: a producer in the bottom left corner (cell (1, 1)) and a producer in the top right one (cell (99, 99)) with $P_{inj} = 200$ bar and $P_{prod} = 100$ bar. Simulations are run for a total of 10000 days and the minimum and maximum time-step sizes are 0.1 and 30 days, respectively. The non-linear convergence tolerance is set to 10^{-5} and the maximum number of non-linear iterations is 15. All linear systems are solved exactly using Matlab's direct solver (back-slash). ADM employs two coarse levels with coarsening factor in each direction equal to 3. Three different values are considered for the coarsening criterion thresholds, i.e., $dz_1 = 0.05$, $dz_2 = 0.07$, and $dz_3 = 0.1$. This is done to analyse the sensitivity of ADM to the number of grid-cells used. Here, multilevel multiscale basis functions are used as pressure interpolator. The permeability contrast is limited to 2 order of magnitudes when computing basis functions.

ADM results are in good agreement with the fine-scale ones for all simulations. Figure 5.7 shows, e.g., the saturation map after 5000 days of injection for one realisation of each permeability field, for both fine-scale and ADM with two different coarsening threshold values. It is clear that ADM employs fine-scale resolution only in a small portion of the domain. The maximum CFL number (averaged over 20 realisations) for each set of permeability fields is presented in Table 5.3. Here, the CFL number is computed as

$$\text{CFL} = \frac{N_{\text{cells}}}{\max_1} \left(\frac{\Delta t}{\phi \Delta V} \max_{c=1}^{n_c} \frac{\sum_{\alpha=1}^{n_{ph}} x_{c,\alpha} \rho_{\alpha} F_{\alpha}}{\sum_{\alpha=1}^{n_{ph}} x_{c,\alpha} \rho_{\alpha} S_{\alpha}} \right), \quad (5.17)$$

where, F_{α} is the total influx of phase α .

Table 5.3: Test case 3 - Maximum CFL number (averaged over 20 realisations) of fine-scale runs for both compositional formulations.

	0 deg	15 deg	30 deg	45 deg	patchy
Natural var.	115	132	170	209	147
Overall-comp	115	132	170	209	147

To quantify the error of the ADM results for pressure and saturation,

$$\epsilon_p = \text{mean}_{t=1}^{n_r} \left(\frac{\|p(t) - p_{fs}(t)\|_2}{P_{\text{inj}} - P_{\text{prod}}} \right) \quad (5.18)$$

and

$$\epsilon_s = \text{mean}_{t=1}^{n_r} \left(\text{mean} |S(t) - S_{fs}(t)| \right). \quad (5.19)$$

are used, respectively. Here, the subscript fs indicates the fine-scale results, and $t = 1, \dots, n_r$ with $n_r = 10$ are the fixed report times at which solutions are saved. The error is averaged over the 20 realisations for each set of permeabilities. The averaged pressure and saturation errors are shown in Fig. 5.8. Remarkably, no significant difference between employing ADM with the two formulations is observed. Also, as expected, the errors decrease as the coarsening criterion threshold is reduced. Table 5.4 summarizes the average and the maximum number of ADM grid-blocks, normalized by the number of fine-scale cells, for each permeability set. Note that the minimum number of ADM grid-blocks is the same for all runs. In fact, the refinement around the wells and some geometrical constraints determine the minimum number of grid-cells employed by ADM. Finally, Fig. 5.9 shows the average number of non-linear iterations for the 5 sets of permeability fields. From this figure, ADM and fine-scale simulators employ comparable non-linear iterations.

5.3.4. TEST CASE 4: CAPILLARITY HETEROGENEITIES

A $500\text{m} \times 500\text{m}$ heterogeneous reservoir is considered, on which a 99×99 Cartesian grid is imposed. The same black-oil fluid model presented in the previous test case is employed. Five pressure-constrained wells are present, placed in a 5-spot pattern shown in

Table 5.4: Test case 3 - average, maximum and minimum number (averaged over the 20 realisations) of grid-blocks employed by ADM (normalised –in percentage– by the number of grid cells used by the fine-scale simulations).

		Nat. var.			Over. comp.		
		Av.	Min	Max	Av.	Min	Max
0 deg	dz1	56.5 %	5.3 %	68.7 %	56.0 %	5.3 %	68.0 %
	dz2	20.2 %	5.3 %	24.4 %	19.9 %	5.3 %	23.9 %
	dz3	7.2 %	5.3 %	7.9 %	7.1 %	5.3 %	7.8 %
15 deg	dz1	62.5 %	5.3 %	76.0 %	62.0 %	5.3 %	74.6 %
	dz2	27.1 %	5.3 %	34.2 %	27.0 %	5.3 %	33.8 %
	dz3	8.0 %	5.3 %	9.1 %	8.0 %	5.3 %	9.1 %
30 deg	dz1	70.7 %	5.3 %	85.8 %	70.3 %	5.3 %	85.1 %
	dz2	29.7 %	5.3 %	39.5 %	29.5 %	5.3 %	39.0 %
	dz3	7.0 %	5.3 %	7.9 %	7.0 %	5.3 %	7.9 %
45 deg	dz1	75.4 %	5.3 %	92.3 %	75.3 %	5.3 %	91.9 %
	dz2	30.9 %	5.3 %	42.8 %	30.8 %	5.3 %	42.7 %
	dz3	6.7 %	5.3 %	7.3 %	6.7 %	5.3 %	7.6 %
patchy	dz1	77.7 %	5.3 %	92.9 %	77.5 %	5.3 %	92.3 %
	dz2	27.0 %	5.3 %	34.7 %	26.7 %	5.3 %	34.3 %
	dz3	6.6 %	5.3 %	7.1 %	6.6 %	5.3 %	7.3 %

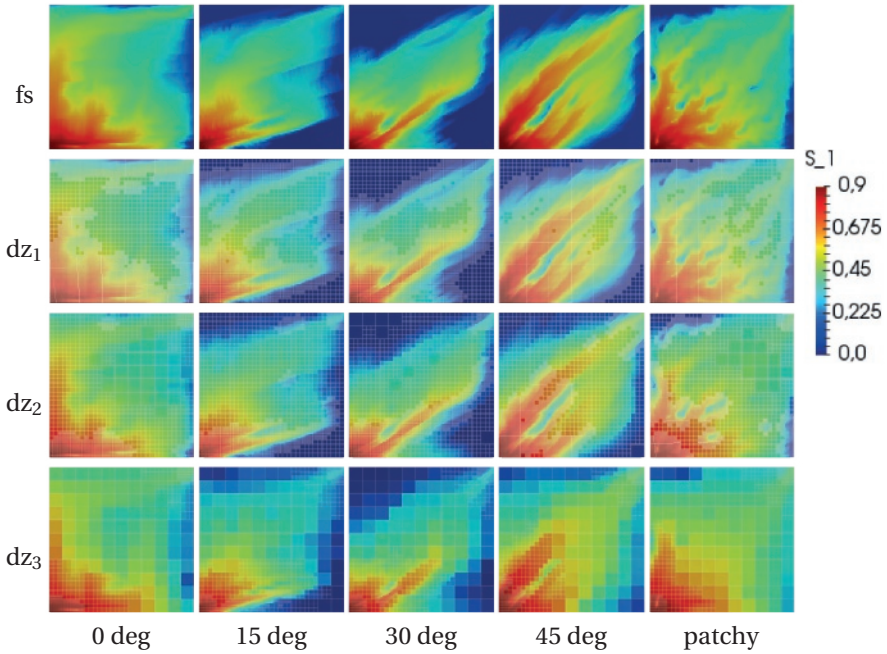


Figure 5.7: Test case 3 - Comparison of the saturation map, for one realisation of each set of permeability fields, after 5000 days of injection. Fine-scale (top row) and ADM solutions are shown. Three different coarsening thresholds are employed for the ADM simulations, i.e., $dz_1 = 0.05$, $dz_2 = 0.07$ and $dz_3 = 0.1$. The dynamic grid is also shown.

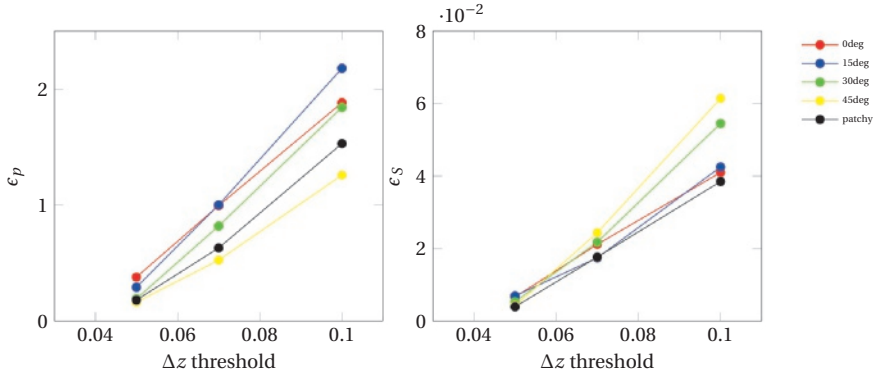
Fig. 5.10. The location and the constraint of each well is presented in Tab. 5.5. As for the previous test case, ADM employs two coarse levels with coarsening ratio of 3 in each direction and multiscale basis functions as pressure interpolators.

Table 5.5: Test case 4 - Coordinates and wells BHP constraints.

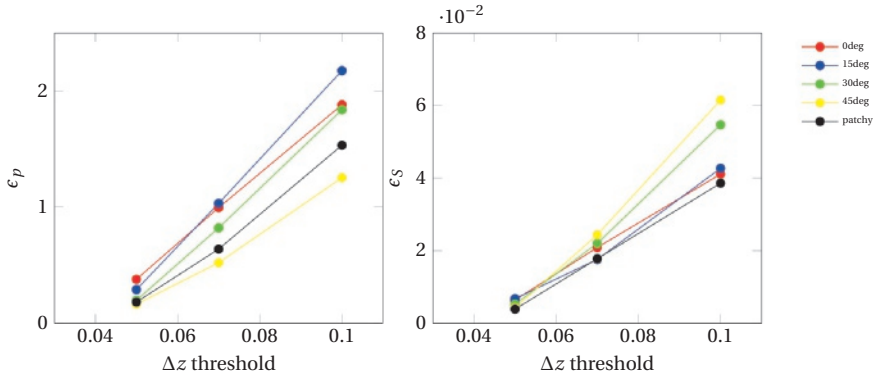
Well	x	y	Pressure [bar]
Injector 1	50	50	150
Producer 1	1	1	120
Producer 2	99	1	120
Producer 3	99	99	100
Producer 4	1	99	100

The model is first solved neglecting capillary pressure and then, the following capillary pressure curve is considered [122]:

$$P_c = \sigma \cos(\theta) \sqrt{\frac{\phi}{K}} J(S_l) \quad (5.20)$$

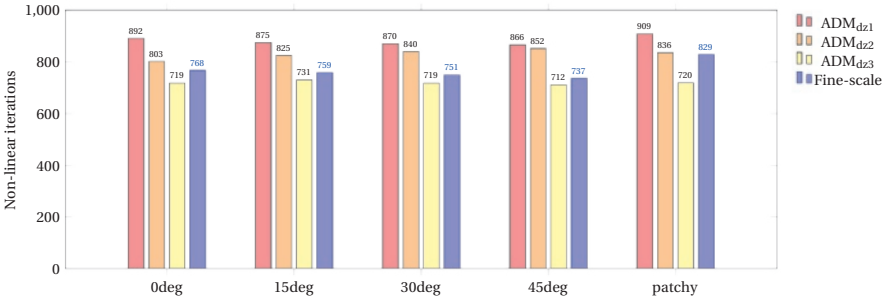


(a) Natural variables

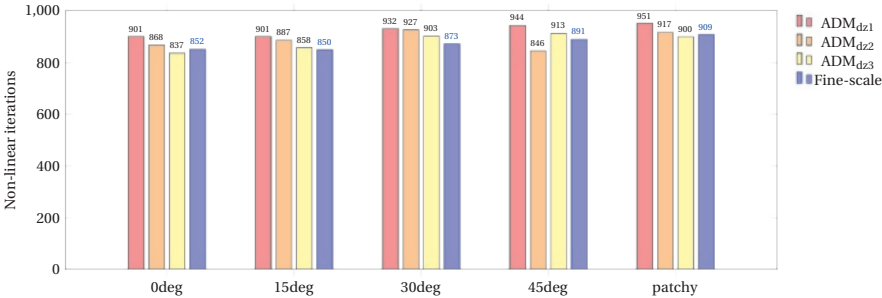


(b) Overall-compositions

Figure 5.8: Test case 3 - pressure and saturation relative errors (averaged over the 20 realisations) as a function of the coarsening threshold value for both natural variables (a) and overall-composition (b) formulations.



(a) Natural variables formulation



(b) Overall-compositions formulation

Figure 5.9: Test case 3 - Comparison of the number of non-linear iterations (averaged over the 20 realisations) employed in ADM and fine-scale simulations both using natural variables (a) and overall-composition (b) formulation.

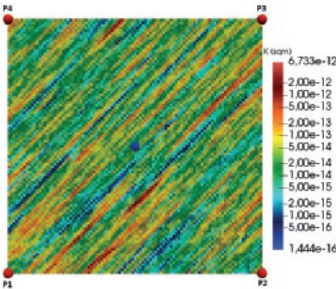


Figure 5.10: Test case 4 - base 10 logarithm of the permeability field.

where

$$J(S_l) = 0.1 \left(\frac{1 - S_{l_{irr}}}{S_l - S_{l_{irr}}} \right)^{-0.5}. \quad (5.21)$$

Here, S_l is the saturation of the liquid phase, considered to be wetting, $\sigma = 4.361 \cdot 10^{-2} \text{ Pa} \cdot \text{m}$ is the surface tension and $\theta = 0$ is the contact angle.

Figure 5.11 compares the reference (fine-scale) and the ADM saturation maps after 5000 days of injection both with and without considering capillary pressure for three different ADM threshold values of $dz_1 = 0.07$, $dz_2 = 0.12$ and $dz_3 = 0.25$. The ADM grid is also shown in the figure. The presence of capillary heterogeneity (permeability and saturation dependent) forces ADM to employ fine-scale resolution in a much larger portion of the domain compared to the scenario neglecting capillary effects. Even for this small test case, where the saturation front and capillary effects reach a significant part of the entire computational domain, as shown in Fig. 5.12, increasing the coarsening threshold value decreases the number of ADM grid cells.

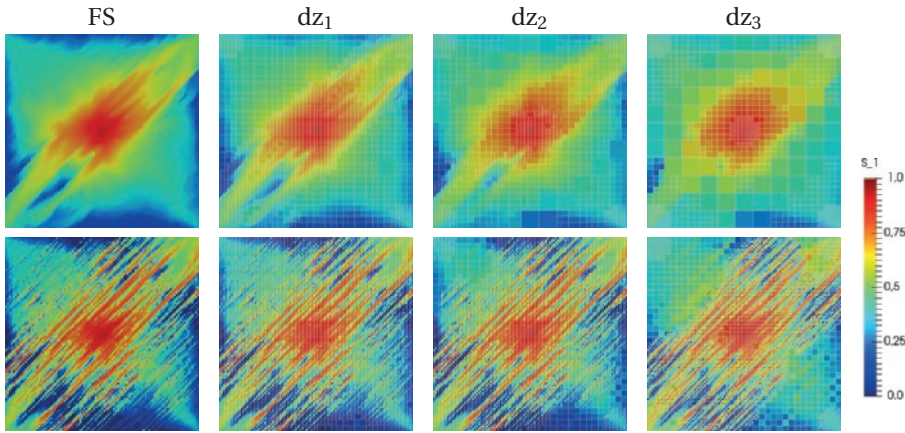


Figure 5.11: Test case 4 - Fine-scale and ADM saturation maps both with (bottom) and without (top) capillary pressure. ADM solution is shown employing two difference threshold for the coarsening criterion ($dz_1 = 0.07$, $dz_2 = 0.12$ and $dz_3 = 0.25$).

5.3.5. TEST CASE 5: GAS INJECTION IN A 3D

A 3D heterogeneous reservoir of $432\text{m} \times 432\text{m} \times 90\text{m}$ is considered, discretised with a $54 \times 54 \times 54$ fine-scale grid. The permeability field, shown in Fig. 5.13 (a), was taken from one of the realisations of the Egg model [123] that was refined along the z-axis. The same fluid properties employed for the previous two test cases are used. One injection and one production well are placed in the two opposite corners, both perforating the whole reservoir thickness. Both wells are pressure-constrained and the injection and production bottom-hole pressures are 200 and 100 bars. The initial gas mass fraction is $z_g = 0.1$ and the initial pressure is shown in Fig. 5.13 (b). Capillary effects are neglected in this test case whereas gravitational acceleration acts along the z-axis. Simulations are run up to 100 days from the beginning of the gas injection process. ADM employs two

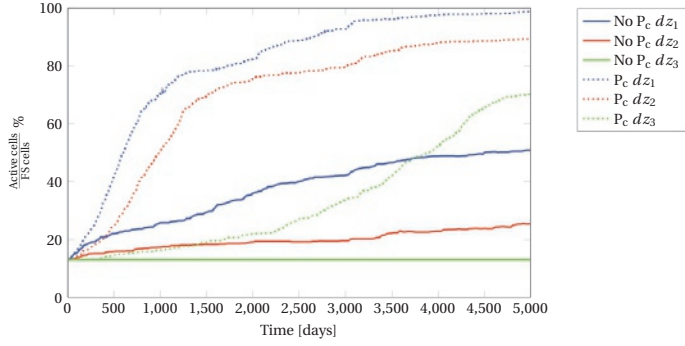


Figure 5.12: Test case 4 - history of the number of active nodes employed by ADM during the simulation.

coarse levels with coarsening factors $cf_x = cf_y = cf_z = 3$.

Figure 5.14 shows a comparison of the production curves obtained with ADM and with the reference fine-scale simulations. Remark that ADM results are in good agreement with fine-scale ones. For this test case, in average, ADM employs about 45 % of the number of fine-scale grid-blocks. Note that about 10 % of refinement does not depend on the dynamic displacement process, but imposed to keep the fine-scale resolution close to the wells. The history of the number of grid-blocks, expressed in percentage of the fine-scale cells, employed by ADM is shown in Fig. 5.15.

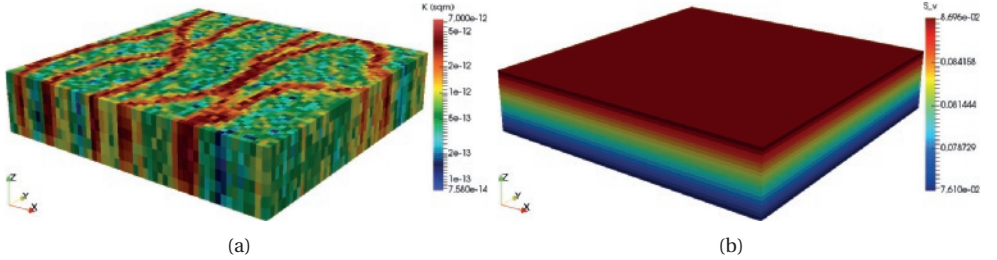


Figure 5.13: Test case 5 - base 10 logarithm of the permeability (a) and initial vapour saturation (b).

Figure 5.16 shows the ADM saturation map with part of the fine-scale grid at simulation times of 25 and 100 days. Note that the lighter vapour phase migrates to the top of the reservoir since the early time-step, and the ADM grid resolution automatically adjusts to follow the gravitational segregation process.

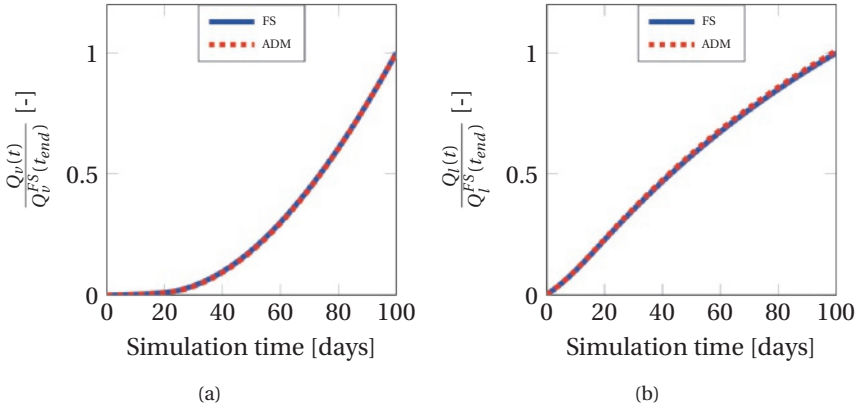


Figure 5.14: Test case 5 - Comparison of the cumulative production curves obtained with ADM and fine-scale simulations.

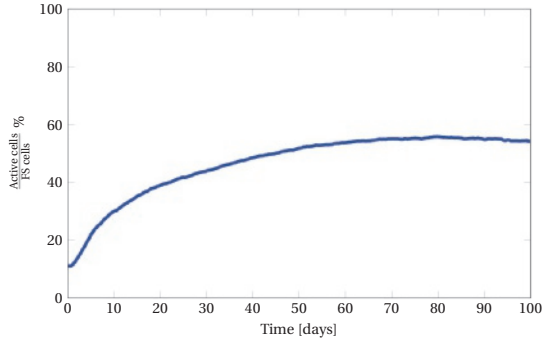


Figure 5.15: Test case 5 - history of the number of active nodes employed by ADM during the simulation.

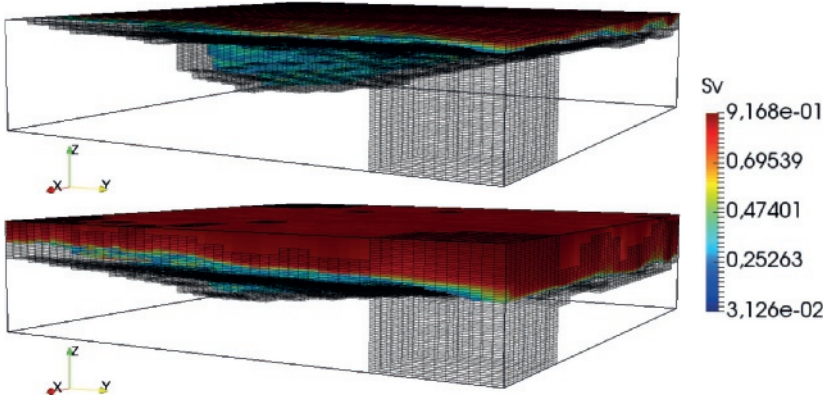


Figure 5.16: Test case 5 - ADM vapour saturation map at simulation time of 25 days (top) and 100 days (bottom). Cells with saturation values smaller than 0.25 were made invisible to show the saturation front inside the domain. The fine-scale grid-cells employed by ADM are also shown.

6

ADM WITH AN ADAPTIVE SATURATION INTERPOLATOR

The Algebraic Dynamic Multilevel (ADM) method presented in chapters 4 and 5 was introduced to allow to employ a dynamically defined grid resolution to highly heterogeneous domains. The system of equations is discretized on a high-resolution grid (referred to as fine-scale) and then mapped to coarser grid resolutions in those regions where a high-resolution is not required. The appropriate grid resolution was chosen based on the contrast of relevant fluid properties and the presence of specific features (e.g., wells). Mapping the unknowns across different resolutions is performed through sequences of restriction and prolongation (interpolation) operators. While the choice of finite-volume restriction operators [56, 65] is dictated by the need of honouring mass conservation at all scales, different options can be considered for the interpolation strategy. In the previous chapters it was shown that multiscale basis functions are an effective choice as pressure interpolators as they contain information of the underlying heterogeneities. All other variables (i.e., saturations and mass/mole fractions), instead, were always interpolated using piece-wise constant functions.

Here, an adaptive multilevel prolongation operator for saturation unknowns is introduced. Based on the saturation change over simulation time, the sub-regions in which the saturation values slowly change are first detected. Then, following [81], the ratio between fine and coarse saturation changes over time is assumed to be constant for these sub-regions. The value of this constant ratio is found dynamically throughout the time-dependent simulation [81]. The grid is coarsened wherever the time derivative (in discrete form, the change over a time step) of the ratio between fine and coarse scale saturations is below a user-defined tolerance. The ratio between coarse and fine-scale saturation is then stored and employed as the saturation basis-functions for subsequent time-steps. Such an approach allows for a better reconstruction of the saturation distribution behind the fast-moving fronts, i.e., where the saturation changes are slow. Several numerical examples are presented to investigate the advantages of including this

Parts of this chapter have been published in the Proceedings of ECMOR XVI [124].

enhanced saturation prolongation for ADM, compared to the originally proposed piece-wise constant interpolation.

6.1. ADM WITH AN ADAPTIVE SATURATION INTERPOLATOR

The linear system arising from the solution through a Newton-Raphson method of the system of discretized governing equations describing incompressible two-phase flow in porous media is considered, i.e.

$$\underbrace{\begin{bmatrix} \mathbf{J}_{wp}^v & \mathbf{J}_{ws}^v \\ \mathbf{J}_{nw_p}^v & \mathbf{J}_{nw_s}^v \end{bmatrix}}_{\mathbf{J}_0^v} \underbrace{\begin{bmatrix} \delta p^{v+1} \\ \delta S^{v+1} \end{bmatrix}}_{\delta x_0^{v+1}} = - \underbrace{\begin{bmatrix} r_{w0}^v \\ r_{nw0}^v \end{bmatrix}}_{r_0^v}. \quad (6.1)$$

Here, the subindex w and nw indicate the wetting and non-wetting phases.

As shown in the previous chapters, the ADM method was developed to reduce the computational cost associated with the solution of Eq. (6.1) without having to define upscaled quantities. ADM employs a hierarchy of nested grid at different resolutions, defined at the beginning of the simulation. At each time-step, the proper grid resolution is defined for each area of the domain. Thus, the fine-scale Jacobian system of Eq. (6.1) is mapped to the dynamically defined grid resolution by the means of sequences of restriction ($\hat{\mathbf{R}}$) and prolongation ($\hat{\mathbf{P}}$) operators. Considering $l_{\max}$ resolution levels, at each Newton's iteration the following restricted system has to be solved

$$\prod_{l=1}^{l_{\max}} \hat{\mathbf{R}}_{l-1}^l \mathbf{J}_0^v \prod_{l=1}^{l_{\max}} \hat{\mathbf{P}}_l^{l-1} \delta x_{l_{\max}}^{v+1} = - \prod_{l=1}^{l_{\max}} \hat{\mathbf{R}}_{l-1}^l r_0^v. \quad (6.2)$$

Once the system of Eq. (6.2) is solved, the solution is prolonged (interpolated) to the original fine-scale resolution, i.e.

$$\delta x_0^{v+1} \approx \prod_{l=1}^{l_{\max}} \hat{\mathbf{P}}_l^{l-1} \delta x_{l_{\max}}^{v+1}. \quad (6.3)$$

The restriction and prolongation operators are block matrices of the form

$$\hat{\mathbf{R}}_{l-1}^l = \begin{bmatrix} \hat{\mathbf{R}}_{l-1}^l & \\ & \hat{\mathbf{R}}_{l-1}^l \end{bmatrix}, \quad \hat{\mathbf{P}}_l^{l-1} = \begin{bmatrix} (\hat{\mathbf{P}}_l^{l-1})_p & \\ & (\hat{\mathbf{P}}_l^{l-1})_s \end{bmatrix}. \quad (6.4)$$

Here, all blocks of the restriction operator are identical and correspond to a finite-volume restriction operator [56, 65] so that mass conservation is ensured. On the other hand, the columns of the blocks $(\hat{\mathbf{P}}_l^{l-1})_p$ and $(\hat{\mathbf{P}}_l^{l-1})_s$ contain the basis functions employed to interpolate pressure and saturation unknowns, respectively. Multiscale basis functions [55, 56, 65] are employed as pressure interpolator as it was shown that they provide an accurate pressure approximation for heterogeneous problems. In all previous works piece-wise constant basis functions were used for saturation unknowns.

Let us consider, for the sake of simplicity, a case where only a coarse and a fine resolution level are present. These can represent any adjacent pair of resolution levels $I = l$

(coarser) and $J = (l - 1)$ (finer). The discrete mass balance equation at iteration v for a cell i belonging to the finer scale (J) reads

$$\frac{\phi V_i}{\Delta t} \left((\rho_i S_i)^v - (\rho_i S_i)^n \right) + \sum_{j=1}^{n_{nb}} F_{ij}^v = \rho_i^v q_i^v \quad \forall i \in \Omega^J, \quad (6.5)$$

where V_i is the volume of cell i , F_{ij} is the discrete flux exchanged between cells i and its neighbours j and n_{nb} is the number of cells neighbouring cell i . Let us consider a coarse block h , containing cell i , belonging to the coarse level I . The coarse scale saturation can be defined as

$$S_h^v = \frac{\sum_{i=1}^{cf} V_i \rho_i^v S_i^v}{V_h \rho_h^v}, \quad (6.6)$$

where

$$V_h \rho_h^v = \sum_{i=1}^{cf} \rho_i^v V_i. \quad (6.7)$$

Here, cf is the coarsening factor and indicates the number of fine cells i contained in a coarse block h . Thus, by applying the FV restriction operator to Eq. (6.5) the coarse system of equations reads

$$\frac{\phi}{\Delta t} \left(\sum_{i=1}^{cf} V_i (\rho_i S_i)^v - \sum_{i=1}^{cf} V_i (\rho_i S_i)^n \right) + \sum_{i=1}^{cf} \left(\sum_{j=1}^{n_{nb}} F_{ij} \right)^v = \sum_{i=1}^{cf} \rho_i^v q_i^v. \quad (6.8)$$

By substituting Eq. (6.6) the following coarse scale mass-balance equation is obtained,

$$\frac{\phi V_h}{\Delta t} \left((\rho_h S_h)^v - (\rho_h S_h)^n \right) + \sum_{i=1}^{cf} \left(\sum_{j=1}^{n_{nb}} F_{ij} \right)^v = \sum_{i=1}^{cf} \rho_i^v q_i^v \quad \forall h \in \Omega^I. \quad (6.9)$$

Let us define ψ_i^n as the ratio between the fine and coarse saturation of a fine-scale cell i contained in coarse cell h , i.e.

$$\psi_i^n = \frac{\Delta S_i^n}{\Delta S_h^n} \quad (6.10)$$

where

$$\Delta S_i^n = S_i^n - S_i^{n-1} \quad \text{and} \quad \Delta S_h^n = S_h^n - S_h^{n-1}. \quad (6.11)$$

The fine-scale saturation update at iteration $v + 1$ is approximated as

$$\delta S_i^{v+1} \approx \psi_i^n \delta S_h^{v+1} \quad \text{if } i \text{ is in cell } h. \quad (6.12)$$

Remark, that the originally proposed ADM method [108] would consider $\psi = 1$ and assign the average saturation to each coarse block.

6.2. COARSENING CRITERION

Equation (6.12) states that the fine-scale saturation change is approximated by a constant fraction of the coarse-scale saturation. Consequently, such an approximation will be accurate whenever the ratio ψ_i defined in Eq. (6.10) is actually close to be constant, i.e.

$$\frac{\partial \psi_i}{\partial t} \approx 0. \quad (6.13)$$

Thus, it is natural to choose to coarsen the grid whenever such condition is verified. At a given time-step n , a cell j belonging to level l , with $l > 0$, is refined whenever the following condition is met:

$$\frac{|\psi_i^n - \psi_i^{n-1}|_\infty}{\Delta t} > \varepsilon_{r_1} \quad \text{where cell } i \text{ is contained in } j. \quad (6.14)$$

Additionally, in order to detect an incoming saturation front, cell j is also refined whenever the following conditions occur

$$\left(\Delta S_j^{n-1} < \varepsilon_{r_2} \wedge \Delta S_j^n > \varepsilon_{r_2} \right) \quad (6.15)$$

or

$$\max(\Delta S_i^n) \cdot \min(\Delta S_i^n) < 0 \quad \text{where cell } i \text{ is contained in } j. \quad (6.16)$$

Three numerical experiments comparing the original ADM method and the proposed improvements are presented in the following section.

6.3. NUMERICAL EXAMPLES

In this section numerical examples are presented as proof of concept. Results obtained with the proposed adaptive saturation interpolation strategy and coarsening criterion are compared against those obtained with the piece-wise constant saturation basis functions and a coarsening criterion based on the saturation difference between neighbouring cells as described in Chapter 4.

In all the presented test cases, two immiscible incompressible phases, water (w) and oil (o), are considered. The phase viscosities and densities are $\mu_w = 10^{-3}$ Pa s, $\rho_w = 1000 \frac{\text{kg}}{\text{m}^3}$ for the water and $\mu_o = 1.5 \cdot 10^{-3}$ Pa s, $\rho_o = 800 \frac{\text{kg}}{\text{m}^3}$, for the oil. The initial water saturation is uniform and equal to 0.1, quadratic relative permeability functions are considered and capillary forces are neglected.

The ADM settings employed for all test cases are shown in Table 6.1. The following

Table 6.1: ADM settings for all runs.

	Sat. basis functions	Coarsening criterion	Tolerances
Run 1	piece-wise constant	ΔS	$\varepsilon_{r_1} = 0.05$
Run 2	adaptive	$\frac{\partial \psi}{\partial t}$	$\varepsilon_{r_1} = \varepsilon_{r_2} = 10^{-3}$

error measure is employed for the variable x (i.e., pressure and saturation),

$$\epsilon_x = \frac{\|x - x_{fs}\|_2}{\|x_{fs}\|_2} \quad (6.17)$$

Here, the subscript fs indicates a fine-scale simulation which is used as a reference. The average error is computed by averaging over all time-steps.

6.3.1. TEST CASE 1: 1D HOMOGENEOUS RESERVOIR

A $99\text{ m} \times 10\text{ m} \times 10\text{ m}$ homogeneous reservoir with permeability $K = 1\text{e-}15\text{ m}^2$ and porosity $\phi = 0.2$ is considered on which a $297 \times 1 \times 1$ grid is imposed. A rate constrained injection well injects pure water in the first cell at $0.001 \frac{\text{pv}}{\text{day}}$ whereas a pressure constrained producer perforates cell 99 with a bottom hole pressure of 100 bar. The initial water saturation is uniform and equal to 0.1 and quadratic relative permeabilities are considered. The simulation is run until 2 pore volumes are injected in the reservoir.

Figure 6.1 shows a comparison of the saturation profile at three different time-steps obtained by employing ADM with piece-wise constant saturation basis functions and the adaptive prolongation operator presented in the previous section. Note that when employing piece-wise constant interpolation the coarse scale saturation is assigned to all fine-cells belonging to a coarse cell whereas, the adaptive saturation interpolator allows to preserve most fine-scale details of the saturation distribution.

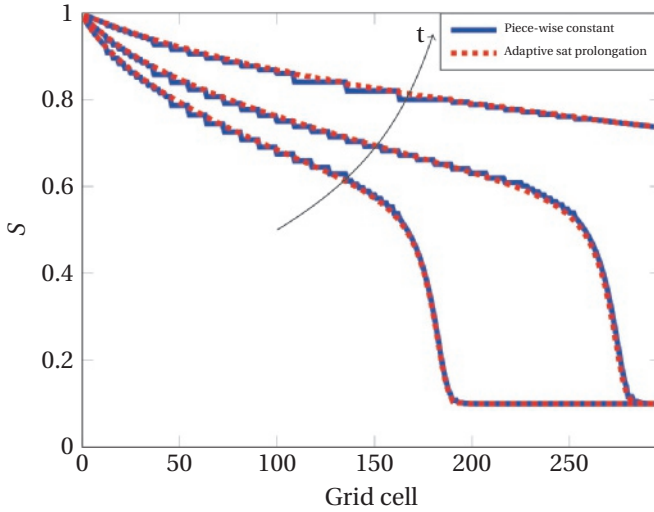


Figure 6.1: Test case 1 - saturation distribution at three different time-steps obtained using a 3-level ADM method employing piece-wise constant saturation interpolation (blue curves) and the adaptive saturation prolongation (red dotted curves).

Figure 6.2 shows the adaptive grid employed by the two ADM simulations. In both cases ADM employs 3 coarse levels (4 levels in total). Note that, since the coarsening criteria are also different the grid resolutions also differ.

The evolution of the number of grid cells employed in each ADM simulation is shown in the left graph of Fig. 6.3. The right graph of Fig. 6.3, instead, illustrates the average pressure and saturation errors. The errors of the two ADM simulations are similar although employing the adaptive saturation prolongation operator reduces the saturation error. This is related to the higher quality of the interpolation of the saturation field behind the front and probably to the different choice of ADM grid resolution.

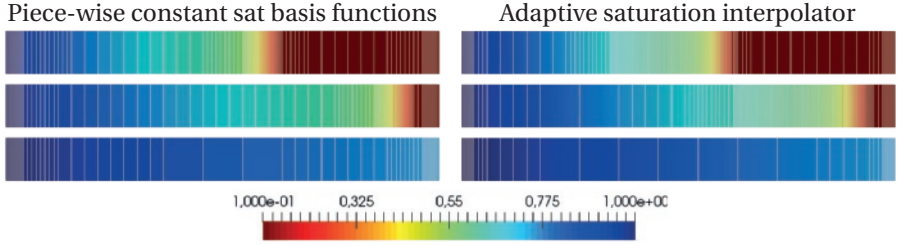


Figure 6.2: Test case 1 - saturation map and ADM grid at the three different time-steps for which the solution is shown in Fig. 1.

6

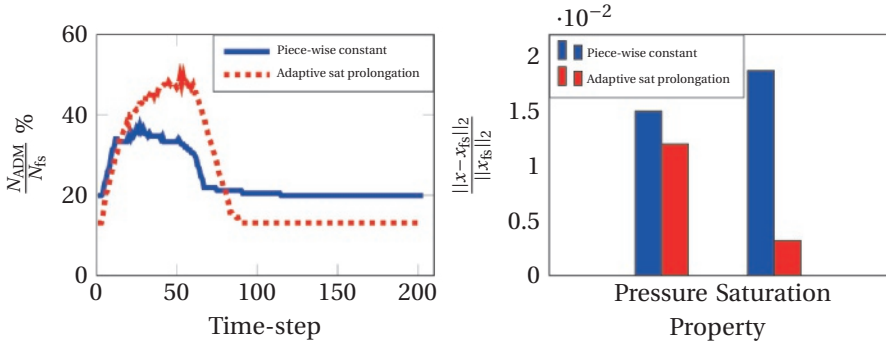


Figure 6.3: Test case 1 - number of grid-cells employed in ADM simulations expressed as a percentage of the fine-cells grids (left) and average pressure and saturation errors (right) for ADM employing piece-wise constant saturation basis functions (blue) and the adaptive saturation interpolation (red).

6.3.2. TEST CASE 2: 2D HOMOGENEOUS RESERVOIR

A $99\text{m} \times 99\text{m} \times 10\text{m}$ homogeneous reservoir with permeability $K = 10^{-15} \text{ m}^2$ and porosity $\phi = 0.2$ is considered. A $99 \times 99 \times 1$ cartesian grid is imposed on the domain. Two wells are present in cells (1,1) and (99,99); the first one is injecting water a constant rate of $0.001 \frac{\text{pv}}{\text{day}}$ whereas the one has a constrained BHP = 100 bar. The simulation is run until 2 pore volumes are injected in the reservoir. Simulations are run with the ADM method employing 2 levels of coarsening with both the adaptive saturation prolongation and piece-wise constant saturation basis functions.

Figure 6.4 shows a comparison of the saturation map and the ADM grid resolution for the two different settings of the ADM method. Remark the differences in the grids

employed in the two ADM simulations. At early times (column 1) the saturation gradient based criterion (top row) concentrates most fine-scale grid-cells around the front and seems to have a more efficient distribution of the fine-cells grids. However, at later times (columns 2 and 3) regions where stationary or quasi-stationary saturation gradients are present are kept refined whereas the newly proposed criterion allows for coarsening in these regions and the fine-scale details of the saturation distribution are recovered by the saturation interpolation strategy.

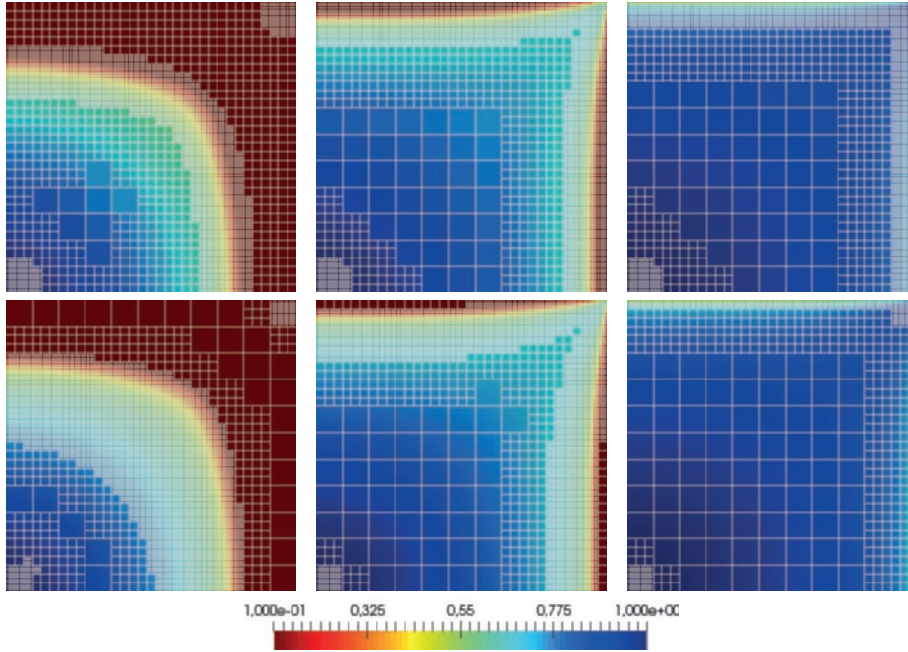


Figure 6.4: Test case 2 - saturation map and ADM grid at three different time-steps (columns) for ADM simulations employing piece-wise constant basis functions (first row) and the adaptive saturation prolongation operator (second row).

The evolution of the number of grid cells employed in each ADM simulation is shown in the left graph of Fig. 6.5. The right graph of Fig. 6.5, instead, illustrates the average pressure and saturation errors, which, as for the previous test cases, are reduced by employing the adaptive saturation interpolation.

6.3.3. TEST CASE 3: 2D HOMOGENEOUS RESERVOIR WITH BARRIERS

Here, two low permeability barriers are added to the reservoir presented in the previous test case as shown in Fig. 6.6. Boundary conditions and wells are the same as those of the previous test case.

Figure 6.7 shows the saturation map and the ADM solution grid at three different time-steps for simulations run employing piece-wise constant (top-row) and adaptive (bottom row) saturation basis functions. Remark how the newly proposed coarsening criterion and the adaptive saturation interpolator allow for much more aggressive coars-

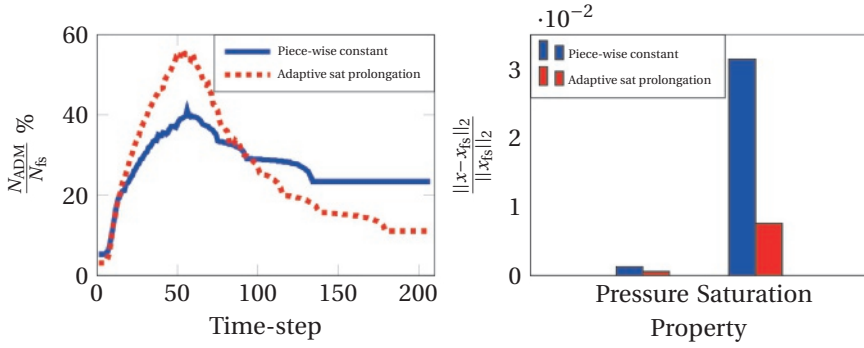


Figure 6.5: Test case 2 - number of grid-cells employed in ADM simulations expressed as a percentage of the fine-scale grid cells (left) and average pressure and saturation errors (right) for ADM employing piece-wise constant saturation basis functions (blue) and the adaptive saturation interpolation (red).

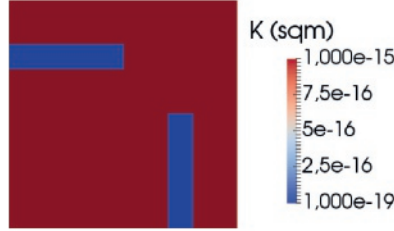


Figure 6.6: Permeability field for test case 3.

ening at the interface between high and low permeability regions. Such differences are highlighted in yellow in Fig. 6.7. The newly proposed criterion identifies regions where no dynamics are present despite the existence of a high saturation gradients. Here, the fine-scale saturation distribution can be effectively reconstructed by employing the adaptive saturation interpolation strategy.

The evolution of the number of grid cells employed in each ADM simulation is shown in the left graph of Fig. 6.5. Remark the considerable difference in the number of grid-cells employed by the newly proposed ADM method compared to the original one. The average pressure and saturation errors are instead shown in the right graph of Fig. 6.5. In spite of the much smaller number of cells employed with the adaptive saturation prolongation compared to the original ADM method, pressure and saturation errors are only slightly higher and still in the same order of magnitude.

6.3.4. TEST CASE 4: SPE10 BOTTOM LAYER

The permeability field of the bottom layer of the SPE10 model [111] is considered in this test case. The domain size is $216m \times 54m \times 10m$ and a $2016 \times 54 \times 1$ fine-scale cartesian grid is imposed on it. No-flow boundary conditions are applied to all domain boundaries and a water injection and a production well are present in cells (1, 54) and (216, 1). Water is injected at a constant rate of $0.001 \frac{pv}{day}$ and simulations are run until injection of 2 pv

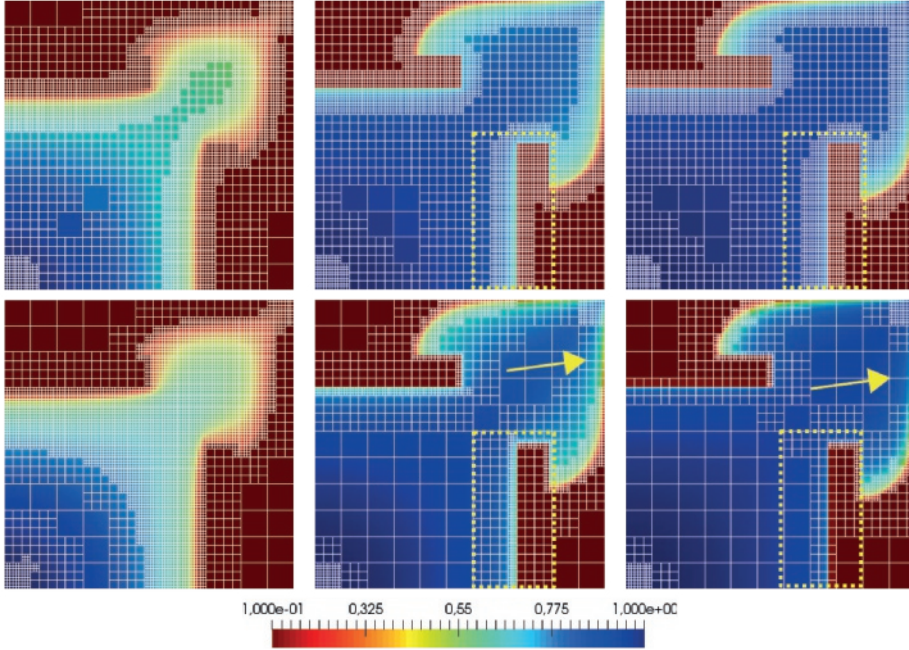


Figure 6.7: Test case 3 - saturation map and ADM grid at three different time-steps (columns) for ADM simulations employing piece-wise constant basis functions (first row) and the adaptive saturation prolongation operator (second row).

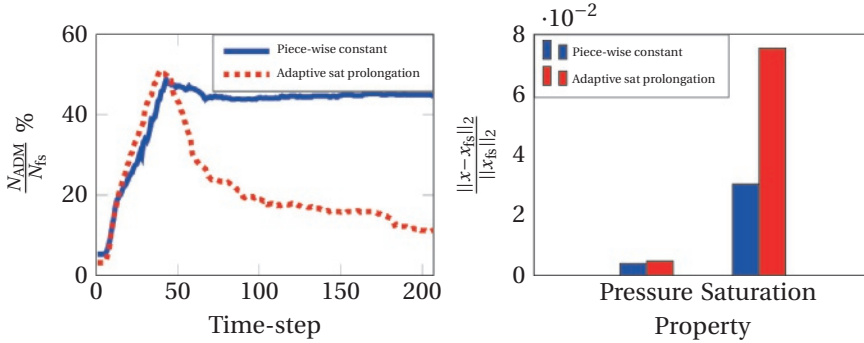


Figure 6.8: Test case 3 - number of grid-cells employed in ADM simulations expressed as a percentage of the fine-scale grid cells (left) and average pressure and saturation errors (right) for ADM employing piece-wise constant saturation basis functions (blue) and the adaptive saturation interpolation (red).

has been reached (i.e., 2000 days).

Fig. 6.9 shows the saturation map and the ADM grid resolution at an early and a late time-step for both ADM simulations. Remark that the simulations using the adaptive saturation interpolator allow for a much coarser grid at the boundary between the high permeability channels and the low permeable regions. In fact, whenever stationary satu-

ration fronts are detected the grid is coarsened and the saturation interpolation strategy described in the previous subsection is used. On the other hand, the original ADM strategy employs a large number of fine-scale cells to resolve all the details of the saturation distribution.

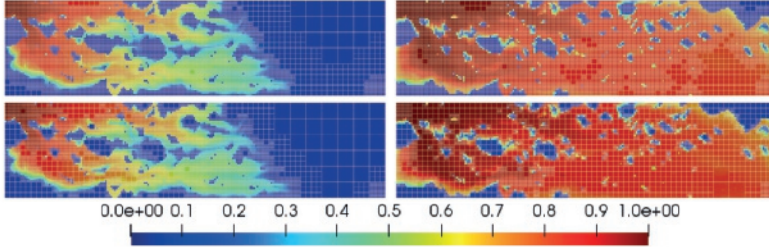


Figure 6.9: Test case 4 - saturation map and ADM grid at two different time-steps (columns) for ADM simulations employing piece-wise constant basis functions (first row) and the adaptive saturation prolongation operator (second row).

Figure 6.10, instead, shows the evolution of the number of nodes employed by ADM simulations along with the average pressure and saturation errors. Remark that the adaptive interpolation strategy for saturation along with its coarsening criterion allows for much more efficient ADM simulations, especially once some stationary saturation fronts have developed.

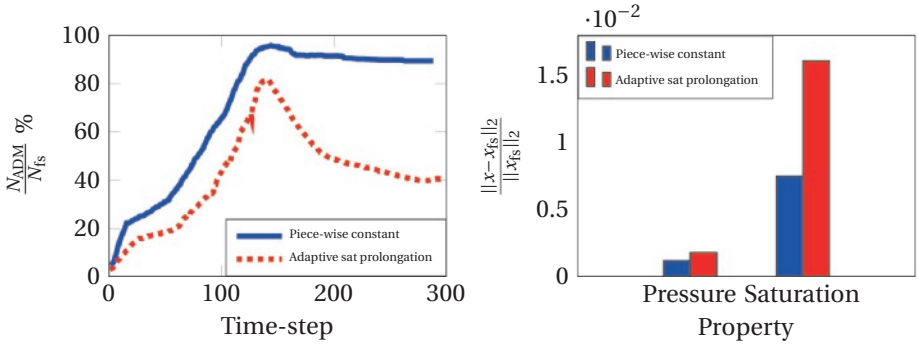


Figure 6.10: Test case 4 - number of grid-cells employed in ADM simulations expressed as a percentage of the fine-scale grid cells (left) and average pressure and saturation errors (right) for ADM employing piece-wise constant saturation basis functions (blue) and the adaptive saturation interpolation (red).

IV

MULTILEVEL METHODS VS UPSCALING

7

THE CASE OF INCOMPLETE MIXING

The miscible and immiscible displacements in subsurface porous media can develop instabilities at the interfaces of the fluids with different properties [126]. Moreover, reservoir rock heterogeneity plays a dominate effect on how fluids are displaced through the porous reservoir rock. Depending on its values, variance and spatial correlation, the permeability distribution can either accentuate or suppress the viscous fingering phenomenon [2, 127–132]. For timely decisions with the available computational capacities, the state-of-the-art reservoir simulations are typically performed on far lower grid resolutions than the scale at which the incomplete mixing, i.e., fingering and unstable interfaces, occur. Apart from introducing additional numerical diffusion, these low-resolution grid cells may suppress small-scale physical phenomena that contribute to unstable displacements, leading to inaccurate simulations [53]. There has been a significant effort to account for these sub-grid features through effective upscaled fluid properties [133–135]. Among them, Todd and Longstaff’s (TL) model [134] is widely used in the petroleum industry.

Similar to alternative upscaling-based models, a major disadvantage of the TL approach is that the effective fluid properties are described by an undefined scalar mixing parameter, which must be obtained from either high resolution simulation or experimental data. The use of full-field high resolution simulations, in order to find these tuning (effective) parameters, is impractical. Moreover, the tuned parameters obtained for a given simulation settings may not be valid if the simulation inputs are changed. As such, the applicability of these approaches to properly address incomplete mixing processes is questionable. Of particular interest is to investigate the applicability of the TL approach, and, if it is found impractical, to develop an alternative simulation approach applicable for field-scale studies.

This challenge motivates the development of Dynamic Local Grid Refinement (DLGR) techniques [41, 44, 49, 50, 52, 136–138], and of Adaptive Mesh Refinement (AMR) methods [42, 46, 48, 51, 54, 139] in which coarser grid resolutions are employed when and

Parts of this chapter have been published in the Computational Geosciences journal (2018) [125].

where the solution (e.g., saturation and concentration) variations are low. On the other hand, the fine-scale grid is employed where sharp gradients exist.

In this chapter, the accuracy and robustness of DLGR and the widely-used TL model is investigated for the simulation of incomplete mixing. The sensitivity of DLGR simulations to different error estimate strategies in order to refine or coarsen the grid is also assessed. Both DLGR and TL are implemented in a commercial-grade simulator for quality benchmarking purposes. As the studied fluid flow involves unstable displacements, the variations of the fine-scale reference solution based on different perturbations are used to define "acceptable solution" range rather than a specific fine-scale solution map.

Numerical experiments show that DLGR is capable of simulating incomplete mixing phenomena with only a fraction of the fine-scale grid blocks. Important is that although TL upscaling approach was employed after tuning parameters based on the fine-scale reference solution, its results are sensitive to changes of the simulation inputs (e.g., grid resolution and mobility ratio). The DLGR strategy, on the other hand, is consistent when simulation parameters are changed. Based on the results presented here, a DLGR approach is recommended for the efficient and reliable simulation of complex fluid displacements as in the incomplete mixing process.

The chapter is organised as follows. The fine-scale governing equations are presented in Section 7.1. Then, the DLGR simulation strategy is briefly described in Section 7.2, with details of several error criteria approaches that have been considered in this work. The formulation employed by the upscaled TL model is described in Section 7.3. To investigate the accuracy and consistency of DLGR and TL model, numerical test cases are presented in Section 7.4 for both homogeneous (7.4.1) and heterogeneous (7.4.2) media.

7

7.1. FINE-SCALE EQUATIONS AND SOLUTION STRATEGY

The equations describing a single-phase incompressible system of two miscible components (oil and a solvent) flowing in a porous medium [140] are

$$\nabla \cdot \mathbf{u} = q_t \quad (7.1)$$

and

$$\phi \frac{\partial y_s}{\partial t} + \nabla \cdot (y_s \mathbf{u}) - \nabla \cdot (\phi \mathbf{D} \cdot \nabla c) = q_s, \quad (7.2)$$

where ϕ is the porosity of the porous medium, t is the time, $\tilde{y}_s$ is the volume fraction (or concentration) of the solvent and q_t and q_c denote, respectively, the total and tracer source term with [1/sec] dimension (only through injection and production wells). Note that since the source term is only induced by the wells, the maximum concentration value cannot exceed 100% void volume of a grid block [24]. Also note that at $\tilde{y}_s = 0$ the convective term and concentration source in Eq. (7.2) become zero, and at $\tilde{y}_s = 1$, $q_s = q_t$. Thus Eq. (7.1) enforces the net outgoing fluxes in Eq. (7.2) to balance the tracer source term. Furthermore, the Darcy velocity $\mathbf{u}$ in the absence of gravitational effects reads

$$\mathbf{u} = - \frac{K}{\mu_{mix}} \cdot \nabla p, \quad (7.3)$$

where μ_{mix} is the viscosity of the mixture, $\underline{\underline{K}}$ is the permeability tensor and p is the pressure. Equation (7.3) is substituted in Eq. (7.1) to obtain a pressure equation, i.e.

$$-\nabla \cdot \left(\frac{\underline{\underline{K}}}{\mu_{mix}} \cdot \nabla p \right) = q_t. \quad (7.4)$$

Here, μ_{mix} , is calculated based on a quarter-power mixing rule [133], i.e.,

$$\mu_{mix}(\tilde{y}_s) = \left(\frac{\tilde{y}_s}{\mu_s^{\frac{1}{4}}} + \frac{(1 - \tilde{y}_s)}{\mu_o^{\frac{1}{4}}} \right)^{-4}. \quad (7.5)$$

Finally, $\mathbf{D}$ in Eq. 7.2 is the dispersion tensor representing molecular diffusion and mechanical dispersion [141]. In a 2D domain the dispersion tensor reads

$$\mathbf{D} = \begin{bmatrix} D_{xx} & D_{xy} \\ D_{xy} & D_{yy} \end{bmatrix} \quad (7.6)$$

where

$$D_{xx} = d_m + d_l \frac{u_x^2}{|u|} + d_t \frac{u_y^2}{|u|}, \quad (7.7)$$

$$D_{yy} = d_m + d_l \frac{u_y^2}{|u|} + d_t \frac{u_x^2}{|u|}, \quad (7.8)$$

and

$$D_{xy} = D_{yx} = (d_l - d_t) \frac{u_x u_y}{|u|}. \quad (7.9)$$

Here u_x and u_y are the x and y component of the Darcy velocity u . Additionally, d_m is the molecular diffusion constant [$\frac{m^2}{s}$], and d_l , and d_t are the longitudinal and transverse dispersion constants [m], respectively.

Thus, the longitudinal (Pe_l) and transverse (Pe_t) Péclet numbers are defined as

$$Pe_l = \frac{q_{inj} L_x}{A D_{xx}} \quad (7.10)$$

and

$$Pe_t = \frac{q_{inj} L_y}{A D_{yy}}, \quad (7.11)$$

where L_x and L_y are the dimensions of the domain along the x and y axis respectively and A is the cross section. Finally, q_{inj} is the volumetric flux of the injected solvent.

An unstable front (i.e., viscous fingering) is generated throughout the miscible displacement process whenever the injected solvent reduces the viscosity of the fluid phY-ortsos1986ase (thus $\mu_s < \mu_o$) as shown by the linear stability analysis [131, 142, 143].

The system of governing equations (i.e., Eqs. (7.2)-(7.4)) is discretised with a finite-volume method and solved using a sequential approach. First, Eq. (7.4) is solved to obtain the pressure field that is used to compute velocity by using Eq. (7.3). Then, Eq. (7.2) is solved using an operator splitting technique. The linear advection term is solved using an explicit time discretization whereas the dispersion term is solved implicitly due to the stability considerations. A second-order (two-point upstream) scheme along with high resolution time step sizes are employed to limit the effect of discretisation error that may interfere with the physical dispersion.

7.2. DLGR METHOD

A nested DLGR technique with implicit evaluation of the grid resolution is employed [50] to solve Eqs. (7.2) - (7.4). At the beginning of each time-step n , the solution grid of time-step $n - 1$ is locally coarsened or refined based on a pair of user-defined refinement and coarsening criteria that aim at employing high grid resolution only at the advancing front location. Once the solution grid has been chosen, the non-linear system of equations is solved. If the converged solution violates the refinement criteria, the grid is updated and a new solution is found. This process is referred to as repeat time-step and it is shown in Fig 7.1. Previous studies have shown that the overhead inherent to the inclusion of the additional solution step is minimal [50]. Here, the maximum resolution jump between two neighbouring cells is forced to be at most equal to one level of grid refinement to limit the discretisation error [41, 144].

A variety of front-tracking criteria has been presented in the literature both involving fluid properties [41, 136, 138, 145–147] and mass or volume fluxes [44, 138, 148]. Here, given a variable x (e.g., saturation, concentration and viscosity) the following quantities can be considered bluet to determine the appropriate grid resolutions:

- Difference: Δx
- Normalised gradient: $\frac{\nabla x}{\|\nabla x\|_\infty}$
- Normalised third order spatial derivative: $\frac{\nabla^3 x}{\|\nabla^3 x\|_\infty}$
- Normalised time derivative of the gradient: $\frac{\frac{\partial}{\partial t} \nabla x}{\|\frac{\partial}{\partial t} \nabla x\|_\infty}$

For any given measure (m), two threshold values, ε_r and ε_c , are defined by the user to determine, respectively, when to refine (if $m > \varepsilon_r$) and coarsen (if $m < \varepsilon_c$) the grid. Note that the threshold values for refinement and coarsening can be set independently as long as $\varepsilon_c \leq \varepsilon_r$. In this work concentration, mixture mobility and mixture viscosity have been considered as properties to locate the advancing solvent front.

Whenever a grid-cell is refined, piece-wise constant function are employed to interpolate all variables (i.e., pressure and concentration) to the finer resolution. On the other hand, when coarsening is performed, the average values are assigned to the coarse blocks.

In heterogeneous domains the finest level (or maximum) corresponds to the resolution of the given permeability field. Coarser permeabilities are obtained with a local flow based upscaling procedure (Fig. 7.2) [39]. Here, the focus of the studies is on comparing TL and the DLGR approach.

7.3. TL MODEL EQUATIONS AND SOLUTION STRATEGY

The TL model [134] was originally introduced to represent mixing phenomena when a solvent is added to the three phase black oil displacement [24]. Its main advantage was that it allowed for using the same black oil simulator (instead of a compositional one) after adjusting fluid properties based on upscaled mixing parameters. Furthermore, the TL model aims to represent the incompletely-mixed fluid displacement at a much coarser

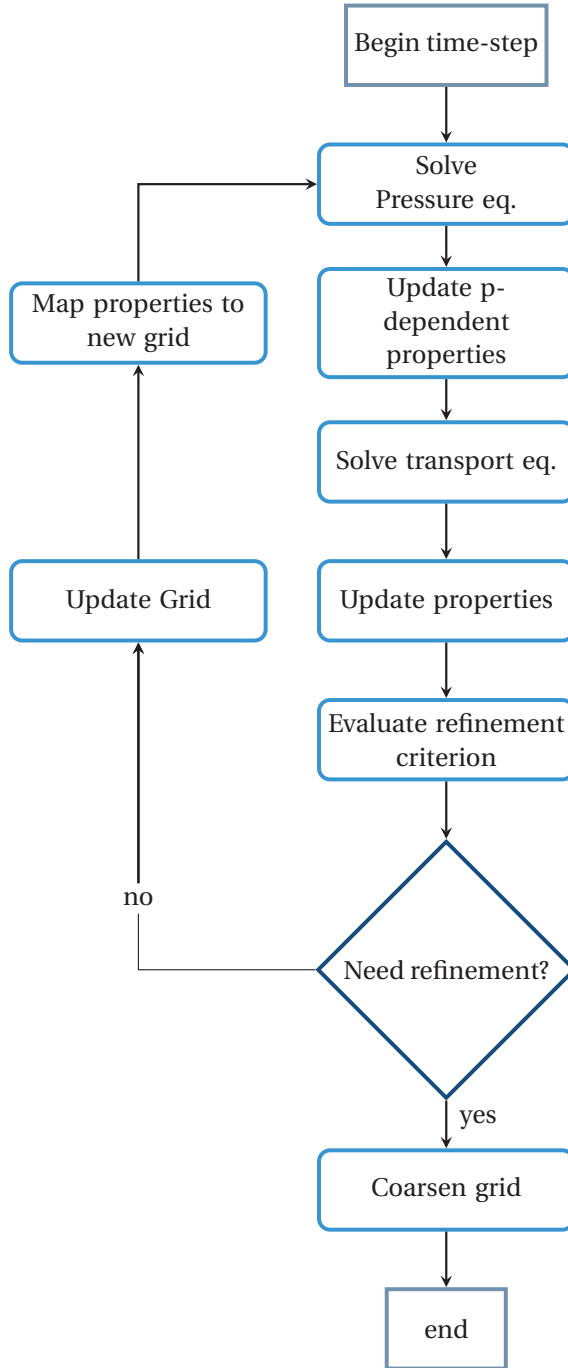


Figure 7.1: Schematic description of the solution of one time-step in DLGR simulations.

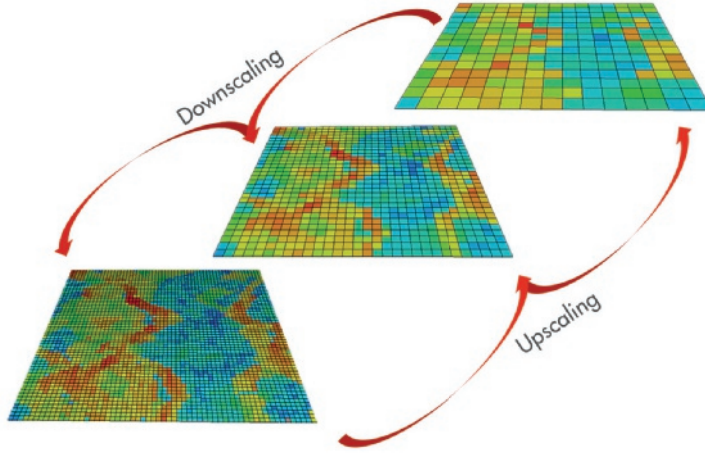


Figure 7.2: Multi-level grid permeability obtained by local flow-based upscaling approach.

grid resolution than the mixing (fingering) resolution. Consequently, the effect of mixing is captured by introducing the effective mixture properties, which are obtained, typically, from expensive fine-scale simulations. The equations of the TL-based approach for miscible oil (o) and solvent (s) flow with no gravitational effects, read

$$\phi \frac{\partial S_\alpha}{\partial t} + \nabla \cdot \left(- \frac{\mathbf{K} k_{r,\alpha}}{\mu_{\alpha,\text{eff}}} \cdot \nabla p \right) = q_\alpha, \quad \alpha = o, s. \quad (7.12)$$

Although represented by two different phase indices, the two components (oil and solvent) are assumed miscible with no interfacial forces and linear relative permeabilities [134]. Additionally, $\mu_{\alpha,\text{eff}}$ is the effective viscosity of phase α computed as

$$\mu_{\alpha,\text{eff}} = \mu_\alpha^{1-\omega} \mu_{\text{mix}}^\omega. \quad (7.13)$$

The mixture viscosity μ_{mix} is calculated employing Eq. (7.5) where $\tilde{y}_s = S_s$. Here, ω is the TL mixing parameter and its value determines the mixture effective properties. In particular, $\omega = 1$ corresponds to the case of complete mixing and $\omega = 0$ corresponds to the immiscible one. Todd and Longstaff reported that the use of $\omega = \frac{2}{3}$ gave satisfactory results for the simulation of miscible displacements with an oil/solvent viscosity ratio of 86 [134]. However, it should be noted that the value of the empirical mixing parameter, ω , highly depends on changes in properties such as grid block size, permeability, geometry and fluid properties and, therefore, cannot be easily generalized. For example, Fig. 7.3 shows two possible concentration distribution scenarios within a simulation grid-block. The left figure shows a stable dispersed front and the right one shows an unstable displacement with both frontal dispersion and viscous fingering. The ratio between the size of the grid-block and the thickness of the mixing zone determines the value of ω that should be used. If the grid-block size is small compared to the dispersed zone the solvent and oil fluid parameters can be seen as completely mixed and $\omega = 1$ can be considered. On the other hand, if frontal dispersion is negligible compared to the grid block size, pure components properties can be employed thus $\omega = 0$ [134].

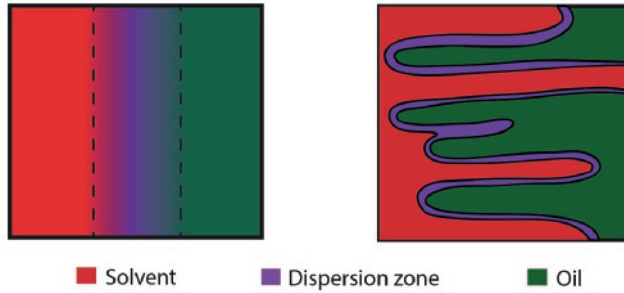


Figure 7.3: Frontal dispersion for a stable displacement (left) and unstable displacement (right).

The constraint $S_o + S_s = 1$ can be used to eliminate one unknown so that Eq. (7.12) leads to a well-posed the system of equation for two unknowns (i.e., p and S_s). Here, Eq. 7.12 is solved employing the finite-volume method on structured Cartesian grids and an implicit time discretization. A two-point flux approximation is used to evaluate interface fluxes along with a first-order upwind for all fluid properties.

7.4. NUMERICAL RESULTS

7.4.1. CASE 1: HOMOGENEOUS POROUS MEDIA

A $100\text{m} \times 100\text{m}$ reservoir, with a permeability of 400 mD and uniform porosity equal to 0.3, is considered. No flow boundary conditions are considered at all boundaries and an injection well perforates all cells at the left boundaries whereas a producer is located in all cells at the right boundary. The solvent is injected at the fixed rate of $1 \frac{\text{ft}}{\text{day}}$ (in the left boundary). At the right boundary, i.e., production cells, the fixed bottom-hole pressure of 350 bar is imposed. The viscosities of the single components are $\mu_s = 1 \text{ cP}$, for the solvent, and $\mu_o = 100 \text{ cP}$, for the oil. The reservoir is initially saturated with oil and simulations are run until breakthrough occurs with time-step sizes corresponding to a CFL number equal to 1. Instabilities are generated by randomly perturbing the saturation distribution in the cells perforated by the injection well (see the Appendix). Three different physical scenarios are analysed: a zero, a medium and a high dispersion case, resulting in different longitudinal and transverse Péclet numbers as presented in Tab. 7.1.

Table 7.1: Case 1: Péclet numbers for the three physical scenarios (zero, medium and high dispersion).

	Pe_l	Pe_t
Case 1a	∞	∞
Case 1b	333	10.000
Case 1c	333	2000

CONVERGENCE OF FINE-SCALE SIMULATIONS

The solvent injection process, for all three physical scenarios, is solved with different fine-scale grid resolutions. Starting from a 25×25 grid, 2×2 refinement is applied up to obtaining a 400×400 grid. Figure 7.4 shows the grid sensitivity of the solvent concentration profile at 0.2 pore volume injected (PVI) upon grid refinement. The left column of Fig. 7.4 corresponds to the solutions obtained without any physical dispersion, so that discretization errors are the only cause of smearing out the solution. Note that, since discretization error reduces with grid refinement, the solution does not converge. The second and third column show reducing instabilities at the front, caused by the increased physical dispersion. As a result of the physical dispersion, small perturbations are smeared out, allowing only large fingers to grow. It can be observed in Fig. 7.4 that the two highest resolution simulations show a converging trend, which indicates that the physical dispersion dominates numerical diffusion.

ACCURACY OF THE DLGR METHOD

In this subsection the accuracy of the DLGR method and its sensitivity to the choice of the refinement and coarsening criteria is evaluated for Cases 1a, 1b and 1c. The high resolution (400×400 grid) simulations discussed in the previous subsection are employed as the reference. In order to mimic the physical uncertainty, a range of possible results is obtained by generating ten statistically equivalent initial tracer distribution at the injector. The standard deviations of the oil production and solvent rates are presented in Table 7.2. This allows for quantification of the underlying uncertainty, so to allow for an acceptable solution range for DLGR and TL simulations.

Table 7.2: Case 1: uncertainty of the unstable fluid flow for the three different dispersion cases

	oil prod.	solvent rate
Case 1a	1.0%	11.0%
Case 1b	1.2%	17.1%
Case 1c	1.1%	11.4%

The error of DLGR simulations is quantified using the following error indicators:

- Relative error of the cumulative oil production (CP):

$$\epsilon_{CP} = \frac{|CP_{dlgr} - CP_{fs}|}{CP_{fs}}; \quad (7.14)$$

- Relative error of solvent rate (SR) at the producer well

$$\epsilon_{SR} = \frac{|SR_{dlgr} - SR_{fs}|}{SR_{fs}}; \quad (7.15)$$

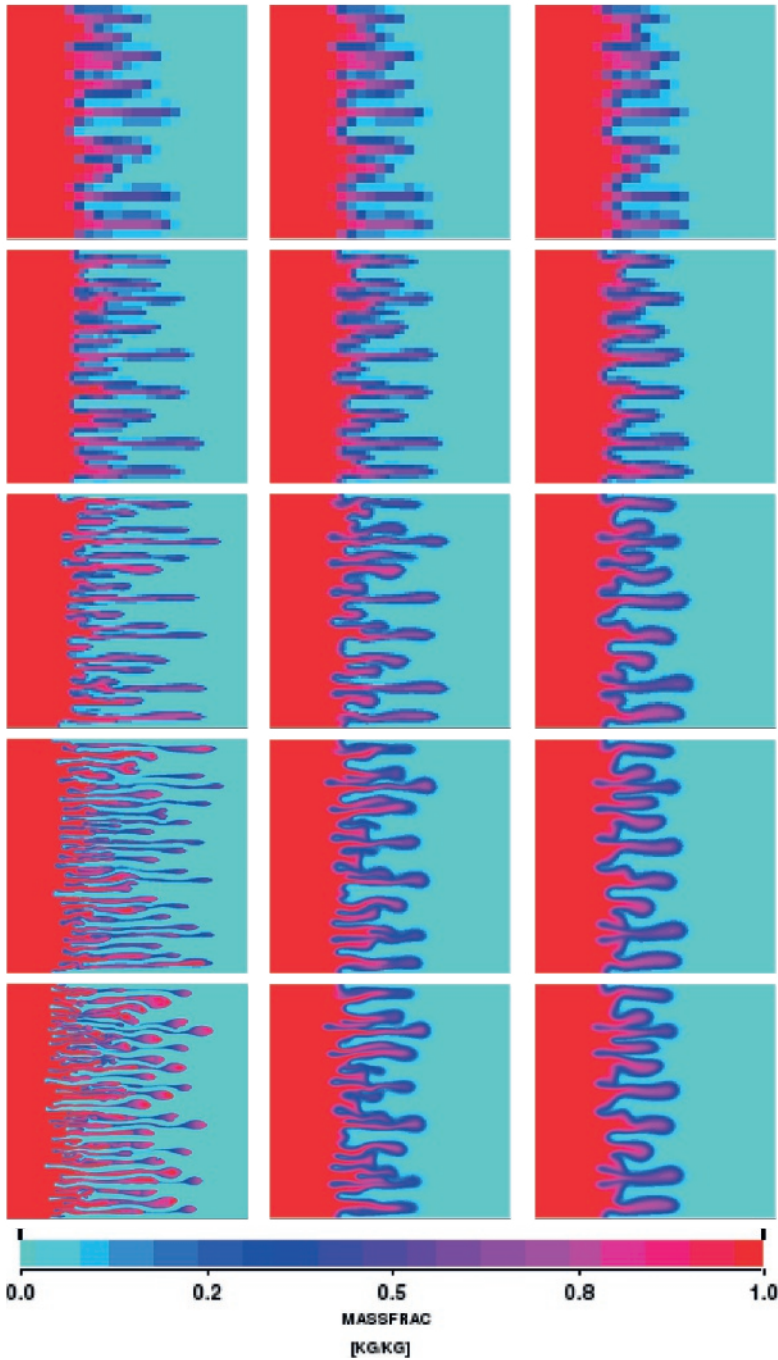


Figure 7.4: Case 1: solvent concentration obtained with simulations on different grid resolutions for the case with zero (left), medium (middle) and high (right) dispersion coefficients. The grid resolutions are 25×25 (row 1), 50×50 (row 2), 100×100 (row 3), 200×200 (row 4) and 400×400 (row 5).

- Second norm of the concentration difference:

$$\epsilon_c = \frac{1}{\text{NOE}} \sqrt{\sum_{i=1}^{\text{NOE}} ((\tilde{y}_s)_{\text{dlgr}}(i) - (\tilde{y}_s)_{\text{fs}}(i))^2} \quad (7.16)$$

Here, the subscript fs stands for fine-scale (reference) and NOE is the number of grid blocks with a solvent concentration $(\tilde{y}_s)$ higher than 0.01.

For the DLGR simulation, the grid resolution has been selected based on the values of the following quantities:

- | | |
|---|---|
| 1. $\zeta_1^c = \frac{\nabla \tilde{y}_s}{\ \nabla \tilde{y}_s\ _\infty}$ | 5. $\zeta_1^\lambda = \frac{\nabla \lambda_s}{\ \nabla \lambda_s\ _\infty}$ |
| 2. $\zeta_2^c = \Delta \tilde{y}_s$ | 6. $\zeta_2^\lambda = \Delta \lambda_s$ |
| 3. $\zeta_3^c = \frac{\frac{\partial}{\partial t} \nabla \tilde{y}_s}{\ \frac{\partial}{\partial t} \nabla \tilde{y}_s\ _\infty}$ | 7. $\zeta_1^\mu = \frac{\nabla \mu_{mix}}{\ \nabla \mu_{mix}\ _\infty}$ |
| 4. $\zeta_4^c = \frac{\nabla^3 \tilde{y}_s}{\ \nabla^3 \tilde{y}_s\ _\infty}$ | 8. $\zeta_2^\mu = \Delta \mu_{mix}$ |

Here, $\lambda_s = \frac{\tilde{y}_s}{\mu_{mix}}$ is the mobility of the solvent. For each criterion a total of 7 simulations for each scenario (1a, 1b, 1c) are performed, each using the tolerance shown in Table 7.3.

DLGR simulations	Tolerances ($\epsilon_r = \epsilon_c$)
1	0.005
2	0.01
3	0.05
4	0.1
5	0.15
6	0.20
7	0.25

Table 7.3: Case 1: tolerances employed for DLGR refinement and coarsening criteria.

Employing the value of the time derivative of the concentration gradient (ζ_3^c) as an indicator of the front location has shown promising results in the literature [50]. However, in the case of viscous fingering, this will lead to very early coarsening of the rear part of the fingers which is not subject to large changes in time. This affects the development of the instabilities. Therefore, the criterion $\zeta_3^c > \epsilon_r$ is used for refinement whereas $\zeta_1^c < \epsilon_c$ is used to determine where to coarsen.

Figure 7.5 shows the DLGR errors for different refinement criteria as functions of the average number of the employed grid-blocks (indicated as a percentage of the number of cells of the fine-scale grid). The red line indicates the allowable mean deviation from the reference solution, due to the variations within the fine scale results for different perturbations. The average number of repeated time-steps (for the implicit dynamic grid strategy) is also shown in Fig. 7.5. Note that DLGR performance is similar for difference- and gradient-based operators, but different between the quantities of μ , c and λ .

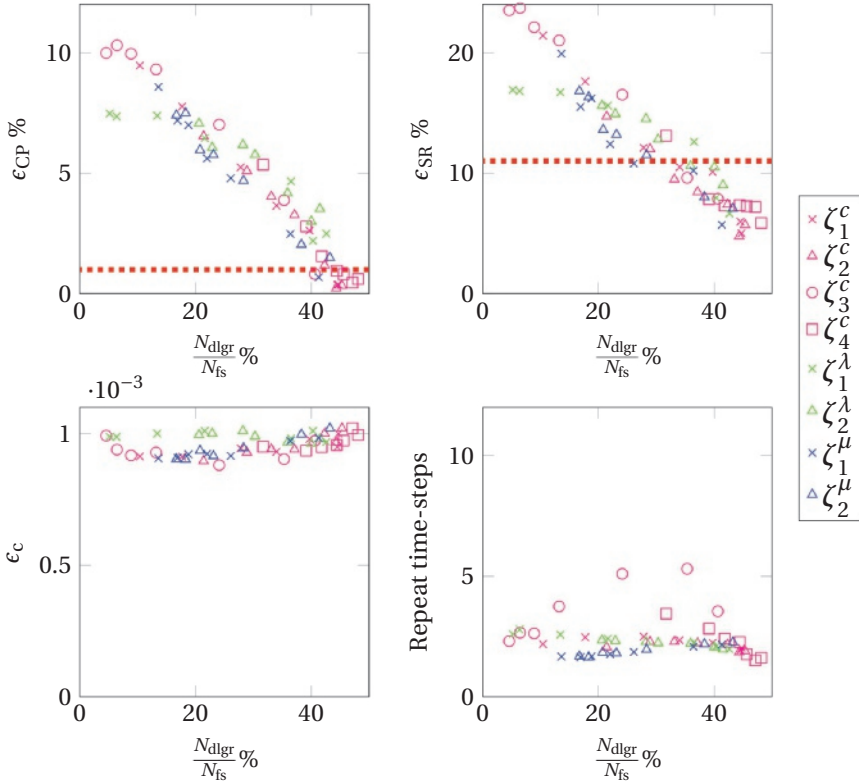


Figure 7.5: Case 1a: DLGR solution errors for different refinement criteria as functions of the average active grid cells. Shown in different colours are: concentration (magenta), viscosity (blue), and mobility (green).

In fact, viscosity based criteria (shown in blue) perform slightly better than the mobility (in green) and the concentration (in magenta) ones. A possible explanation can be deduced when analysing the normalized gradients illustrated in Fig. 7.6.

As described earlier by Eq. (7.5), viscosity is a strong function of the concentration. Figure 7.6 illustrates that the gradient of viscosity criterion tends to refine the grid relatively towards the tip of the front, compared with the gradients of mobility and concentration strategy. As such, the front is captured more accurately, when a viscosity gradient (or difference) is being used.

Figure 7.7 illustrates this concept by showing the concentration solution and the grid

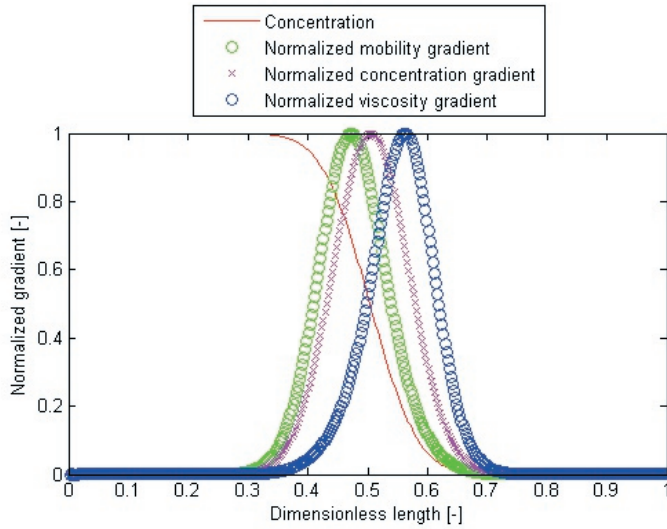


Figure 7.6: Normalized gradients of phase viscosity (blue), concentration (magenta) and phase mobility (green) against dimensionless length. The corresponding concentration gradient is shown in red.

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at 0.2 PVI for the normalized viscosity and mobility gradient criteria using the tolerance of 0.05. It is observed that as mentioned above, the viscosity criteria refine the grid more towards the front tips where the resolution is actually needed. It is also clear from Fig. 7.5 that the viscosity-based approach requires the minimum re-adjustment of the implicit dynamic grid strategy. It should be also stressed that these results obtained for the phase viscosity and mobility criteria are based on the quarter-power mixing formulation, as mentioned in Section 7.1. Therefore, if other functions are used, one has to first study which criteria tends to employ the fine grids towards the tip of the front.

Due to the unstable nature of the front, different grid resolutions impose different perturbations, which in the absence of the physical dispersion can cause different fingering patterns. As such, the concentration error ζ_c , shown in Fig. 7.5, does not show any specific trend when more refined grids are being employed.

Both the space-time derivative and the third order spatial derivative of concentration were able to perform within the allowable range of cumulative oil production and solvent rate of 1% and 11% respectively. Moreover, the mean repeat time steps needed per simulation time step strongly increases for both error estimators with the tolerance value is relaxed (as high as 5.3 repeat time steps per simulation time step shown in Fig. 7.5).

Figure 7.8 shows the accuracy evaluation for the medium and high dispersion cases, respectively. The acceptable error ranges are shown by the red lines, which indicate the uncertainty within the fine-scale solution.

For the medium dispersion case, based on the cumulative oil production, acceptable results are obtained by employing 15% of the fine-scale grid cells. For the high disper-

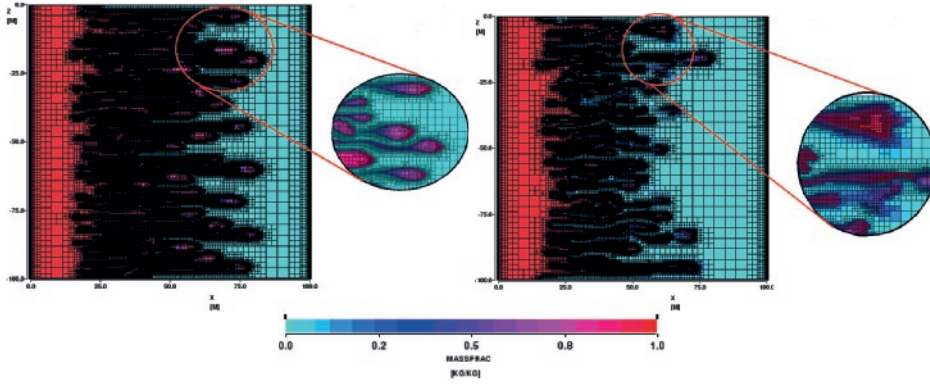


Figure 7.7: Case 1a: DLGR concentration solution at 0.4 PVI, with ζ_1^μ (left) and ζ_1^λ (right) criteria of 0.05 (both).

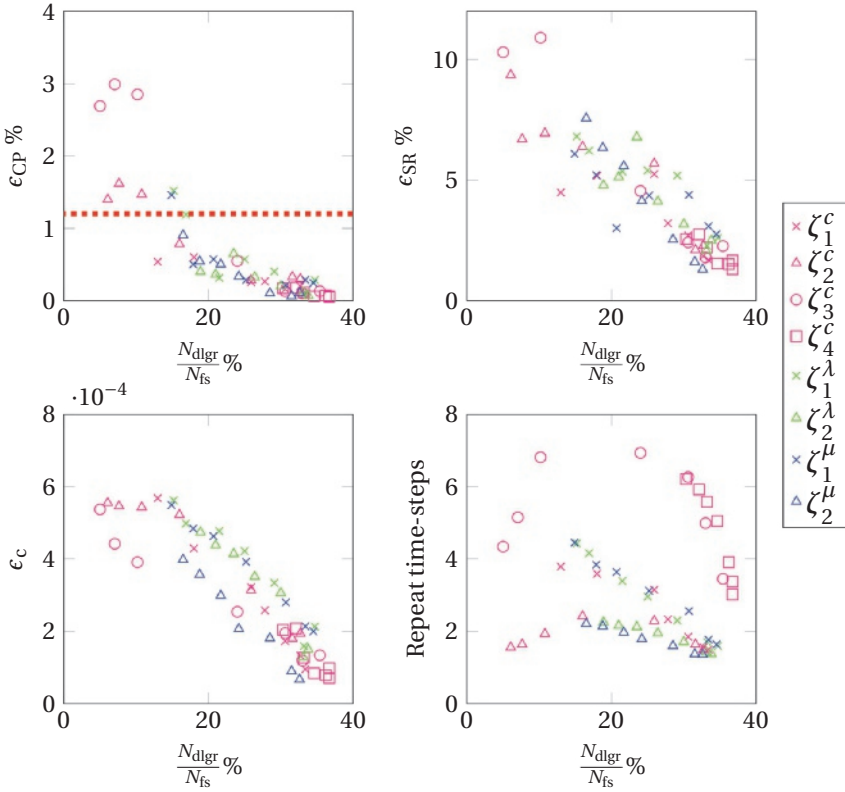


Figure 7.8: Case 1b: DLGR solution errors for different refinement criteria as functions of the average active grid cells. Shown in different colours are: concentration (magenta), viscosity (blue), and mobility (green).

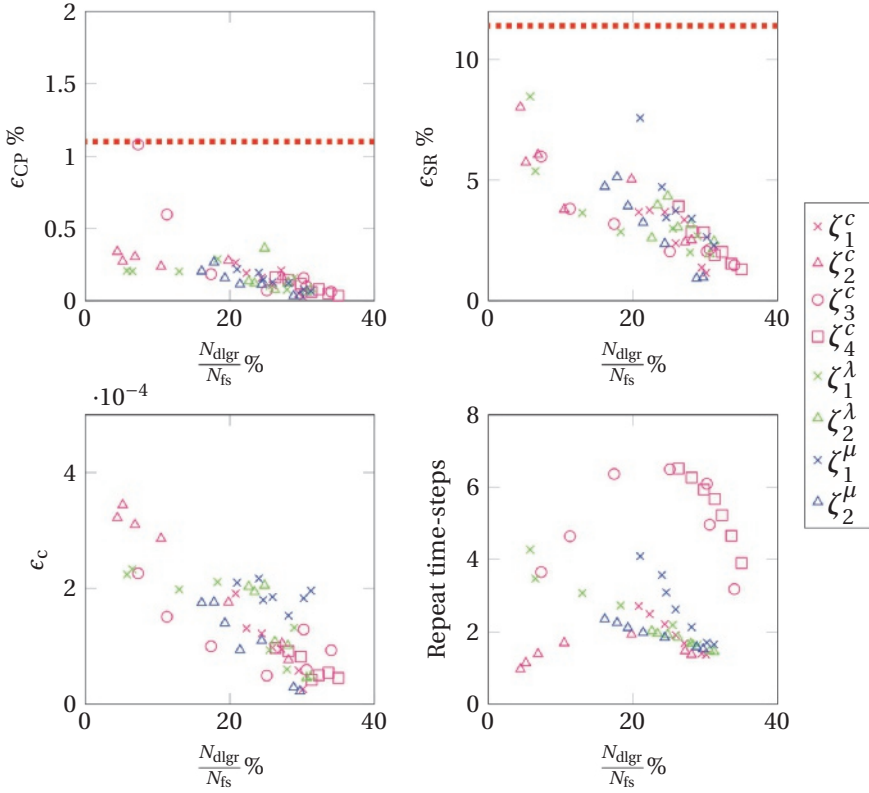


Figure 7.9: Case 1c: DLGR solution errors for different refinement criteria as functions of the average active grid cells. Shown in different colours are: concentration (magenta), viscosity (blue), and mobility (green).

sion case, all the studied values fall below the allowable error line, thus only a few percent active grid cells are enough. The concentration error (mean l_2 – norm) and the number of repeat time steps, illustrated in Fig. 7.8, show that DLGR is more accurate when using difference, i.e., Δ , based criteria. This can be explained by the different asymptotic behaviour, i.e.

$$\lim_{\Delta x \rightarrow 0} \frac{\Delta f}{\Delta x} \rightarrow \frac{\partial f}{\partial x} \quad (7.17)$$

$$\lim_{\Delta x \rightarrow 0} \Delta f \rightarrow 0. \quad (7.18)$$

Here, f is an arbitrary smooth function of location x and Δx is the discretization step. Additionally, $\Delta f = f(x + \Delta x) - f(x)$. When Δx tends towards zero, the ratio $\frac{\Delta f}{\Delta x}$ converges to the first derivative of f with respect to x (i.e., $\frac{\partial f}{\partial x}$) whereas Δf tends to zero. This means that the gradient criteria requires an enforced maximum refinement level in order to limit the amount of refinements because, the true derivative, $\frac{\partial f}{\partial x}$, may be larger than the refinement tolerance (so the refinement never meets the tolerance value). On the other hand, the gradients can be lowered by simply coarsening the grid, which then leads to

smearing out the fronts. This is illustrated in Fig. 7.10 showing the solution for concentration difference (top) and gradient (bottom) criterion at 0.6 PVI using the normalized tolerance of 0.2. Figure 7.11 shows the amount of grid blocks used in each refinement level during the simulation time for both the concentration difference and gradient criteria, confirming that more intermediate refinement levels are used with the difference criterion. The distinct difference between gradient-based and difference-based criteria in the mean repeat time steps used during the simulation, as shown in Figs. 7.8 - 7.9, emphasizes the added benefit of the difference-based strategy to refine the grid.

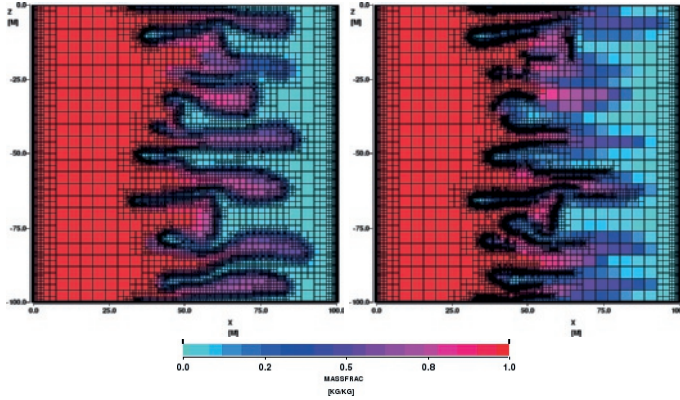


Figure 7.10: Case 1a: DLGR concentration solution at 0.6 PVI, with ζ_2^C (top) and ζ_1^C (bottom) criteria of 0.2 (both).

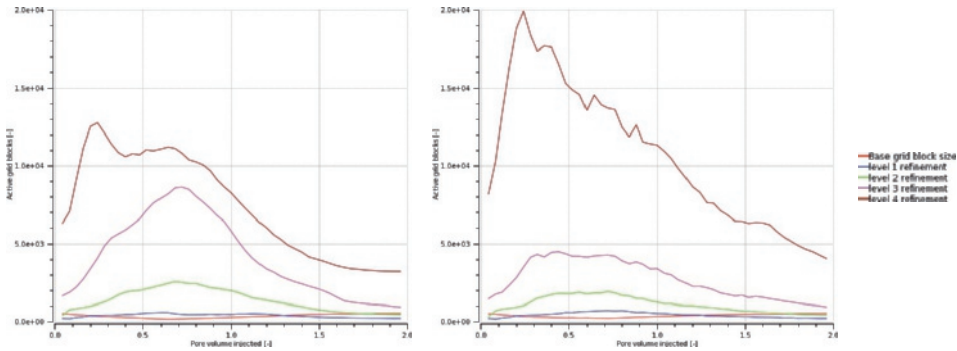


Figure 7.11: Case 1b: active grid blocks for each refinement level against PVI for DLGR simulations employing difference (left) and gradient based criteria (right).

It is observed that the error criteria using the space-time derivative of the spatial gradient of the solvent concentration performed well for simulations including dispersion. This is reflected in the error in production data and the concentration solution, as can be seen in Fig. 7.9. However, as the tolerances are loosened, the error in cumulative production strongly increases, suggesting that the space-time derivative criteria are smaller than the tolerances and thus no local grid refinements are introduced.

UPSCALED TL MODEL SIMULATIONS

In this subsection the sensitivity of the upscaled TL model to the grid-block size and the parameter ω is investigated for Case 1b (medium dispersion). Figure 7.12 shows a surface plot of the error made by the TL model in the cumulative oil production as a function of the grid block size and the mixing parameter. At three different locations on the surface plot the actual difference in cumulative production is visualized in order to relate the error.

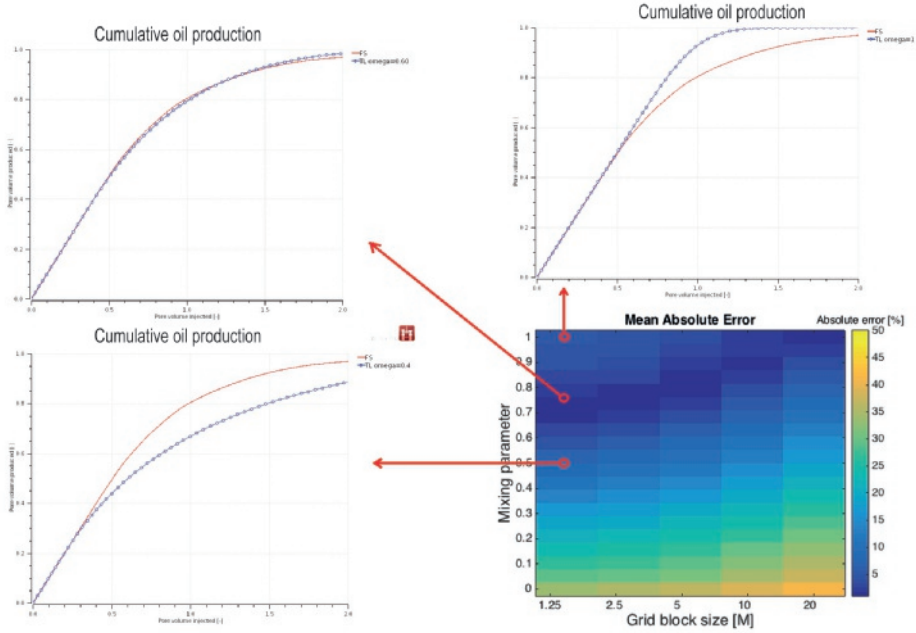


Figure 7.12: Case 1b: effect of grid block size and mixing parameter on the error [%] made by the TL model compared to a fine scale simulation in cumulative oil production for medium dispersion. The fine-scale reference solution is obtained on a 400×400 grid.

From Fig. 7.12 it can be observed that the range of accurate TL results (i.e. results within the 'allowable deviation' of 1.2%) obtained for a given grid resolution is narrow.

TL MODEL VS. DLGR METHOD: SENSITIVITY TO DOMAIN SIZE

Here, four homogeneous reservoirs, of equal heights (25 m) and lengths of 200, 100, 50 and 25 m, are considered. The same injection rate (1 ft/day) and fluid properties are used in all reservoirs and no dispersion is present. The TL mixing parameter is fit to match the fine scale simulation of the largest reservoir. The cumulative oil production curves obtained with fine-scale, DLGR and TL simulations for all 4 reservoirs are shown in Fig. 7.13. In each figure the high resolution simulation curves are depicted in red, DLGR in green and TL effective model in blue. The top left (a) corresponds to a reservoir length of 200 m. DLGR uses the concentration difference across grid block interfaces as error criterion, using a threshold value of 0.05. On average this resulted in a mean

active grid block count of 31% of the fine scale simulation grid block count. It is clearly observed that DLGR is capable of capturing the physics independent of the scale of the system. It is also observed that the number of grid blocks used by DLGR reduces when increasing the size of the domain. On the other hand, the TL model is not able to scale with the physical solution. The mixing parameter used shows an underprediction of local mixing, resulting in incrementally earlier solvent break-through for the downscaled reservoir sizes.

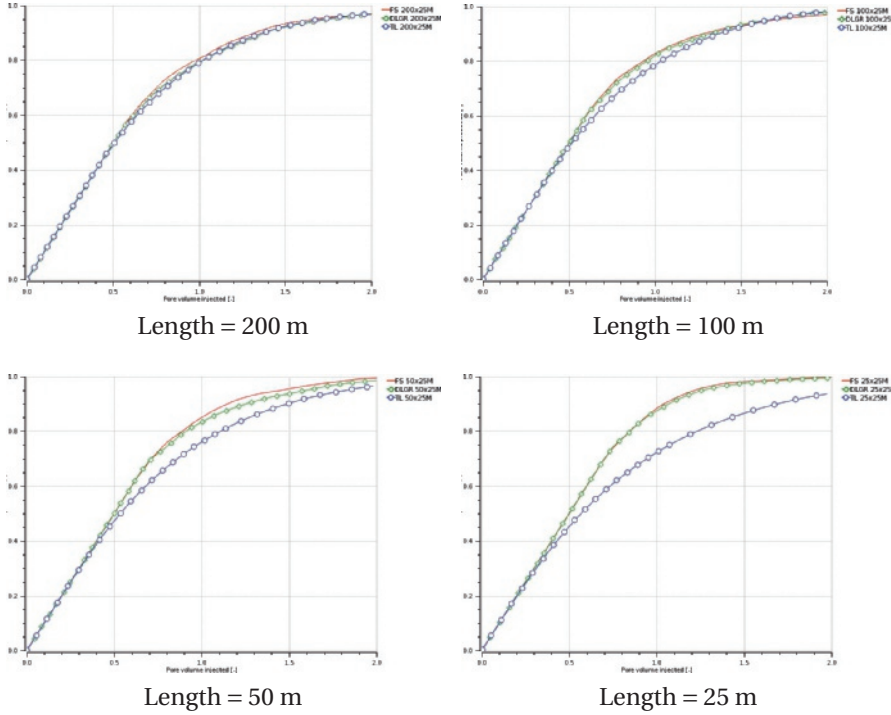


Figure 7.13: Cumulative oil production curves obtained with fine-scale simulations (red), DLGR (green) and TL (blue) for a homogeneous reservoir with a length of 200 (row 1), 100 (row 2), 50 (row 3) and 25 m (row 4). The mean active grid block used by DLGR for the simulations of the 4 reservoirs are 28, 29, 33 and 34%.

TL MODEL VS. DLGR METHOD: SENSITIVITY TO MOBILITY CONTRAST

The same $100\text{ m} \times 100\text{ m}$ reservoir previously described is considered along with the three different physical scenarios (Cases 1a, 1b, 1c). The injection process is simulated with fine-scale (400×400), DLGR and TL model (40×40), for three different mobility ratios $M = \frac{\mu_o}{\mu_s}$: $M=10$, $M=100$ and $M=500$. DLGR simulations use the concentration difference error criterion and the mixing parameter of TL is fit to match the fine-scale simulation cumulative oil production curve for $M=10$.

Figures 7.14 to 7.16 show the cumulative oil production curves obtained with the three simulation strategies for all scenarios. For all three cases, DLGR provides accurate cumulative oil production curves irrespective of the mobility ratio. Note that the

mean active grid blocks employed by DLGR tend to increase with the mobility ratio. On the other hand, the TL model only provides accurate solutions for the mobility ratio for which the parameter is tuned (here, $M = 10$). Figure 7.17 illustrates the matched (optimum) mixing parameter values against the mobility ratio for the three different scenarios. Clearly a linear relation exists on a semi-log scale, allowing for potential rescaling of the mixing parameter in order to fit the data consistently. Literature reported a relation of the mixing parameter as a function of the mobility ratio [43, 149, 150]. This was obtained by equating the fractional flow formulation derived from TL effective (upscaling) model with the fractional flow obtained from Koval's effective (upscaling) model [133], assuming an average solvent concentration at the front. This relationship, depicted with yellow markers, shows a good fit with the zero dispersion case, however, is incapable of predicting an optimal mixing parameter for cases with added dispersion.

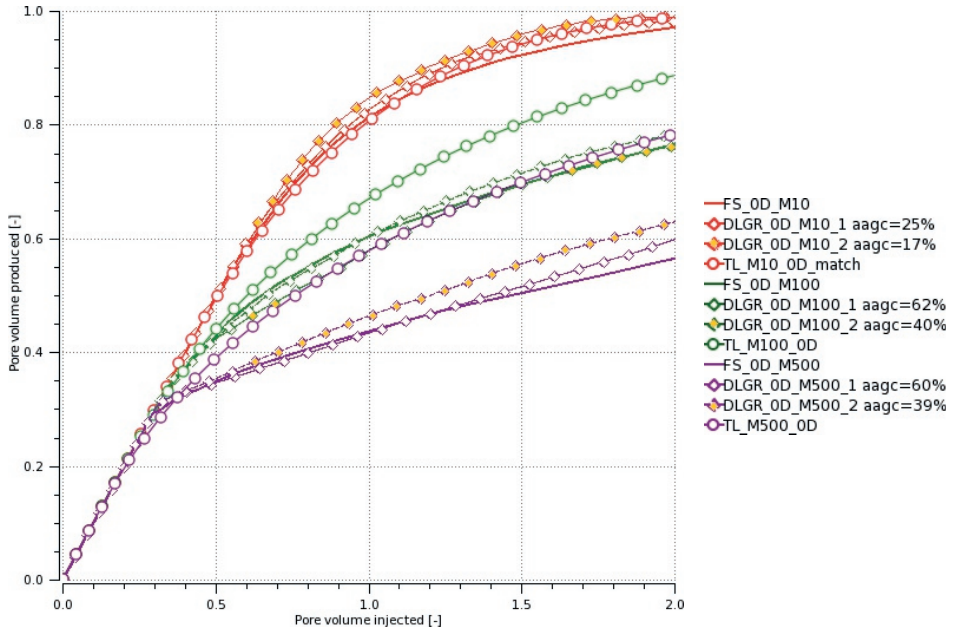


Figure 7.14: Case 1a: cumulative oil production curves for fine-scale, DLGR and TL. The different colours indicate the different mobility ratios; $M=10$ (red), $M=100$ (green) and $M=500$ (magenta). Two different results for DLGR are shown, obtained with different average active grid block counts (aagc) over simulation time.

7.4.2. CASE 2: PERMEABILITY WITH SMALL CORRELATION LENGTH

A $100\text{ m} \times 20\text{ m}$ 2D heterogeneous reservoir is considered with no flow boundary conditions at all boundaries. A rate-constrained injection and a production well perforate all the cells at the left and right boundaries, respectively. Solvent is injected with a constant rate of 1 ft/day. The solvent and oil viscosities are 1 and 10 cP respectively. Three permeability fields with an average value of 90 mD, a variation corresponding to a Dykstra-Parson (V_{dp}) coefficient of 0.63 and a correlation length, $\lambda_D=0.01$ are used, one of which

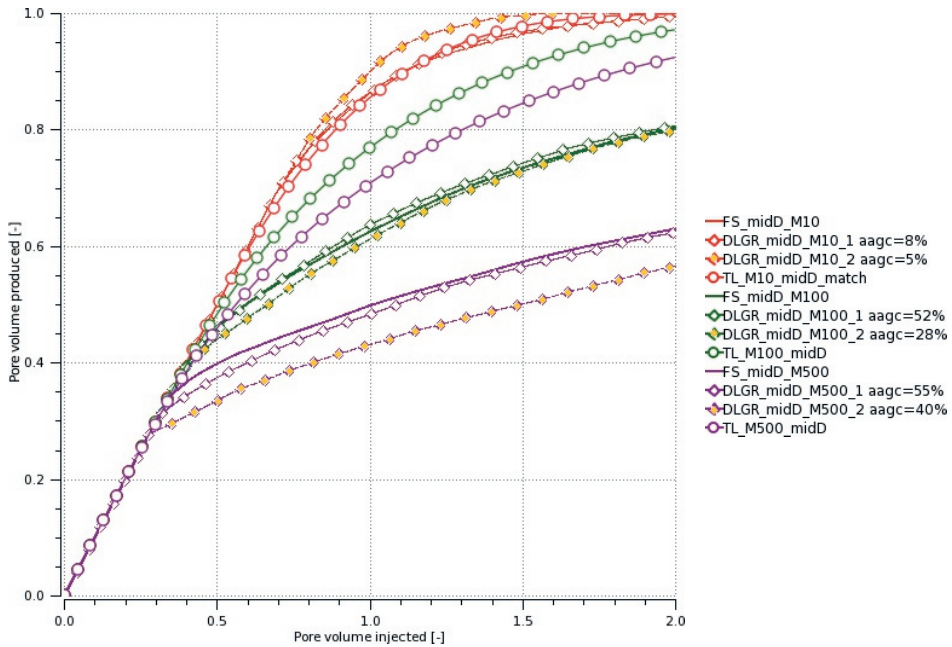


Figure 7.15: Case 1b: cumulative oil production curves for fine-scale, DLGR and TL. The different colours indicate the different mobility ratios; $M=10$ (red), $M=100$ (green) and $M=500$ (magenta). Two different results for DLGR are shown, obtained with different average active grid block counts (aagc) over simulation time.

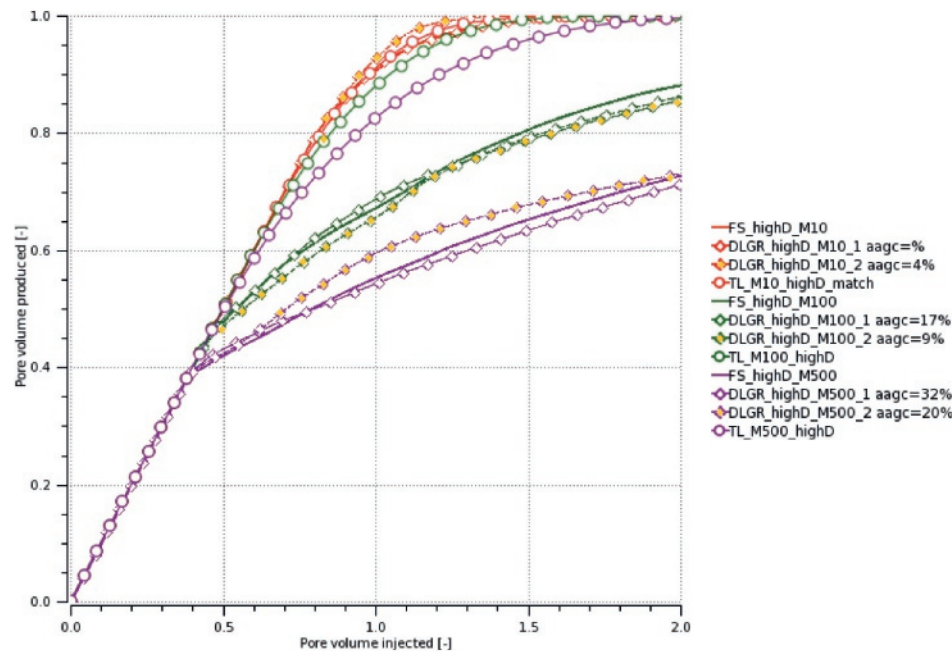


Figure 7.16: Case 1c: cumulative oil production curves for fine-scale, DLGR and TL. The different colours indicate the different mobility ratios; $M=10$ (red), $M=100$ (green) and $M=500$ (magenta). Two different results for DLGR are shown, obtained with different average active grid block counts (aagc) over simulation time.

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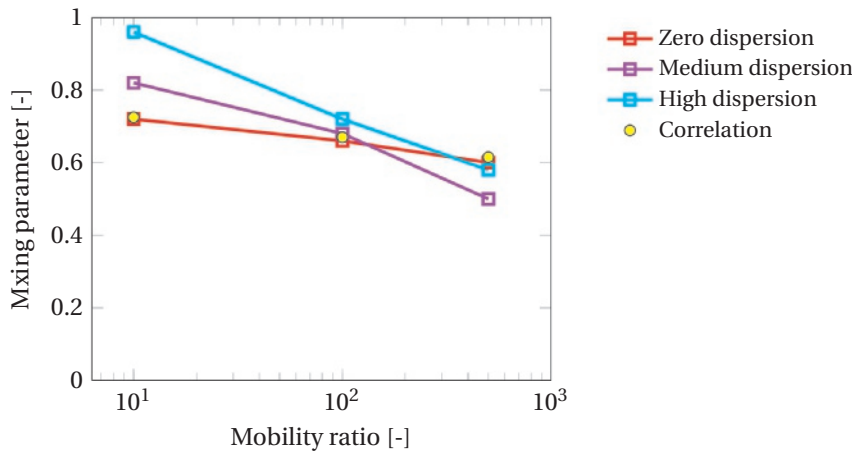


Figure 7.17: Case 1: mixing parameter match against the logarithm of mobility for zero, medium and high dispersed cases. The results from the mixing parameter relation is shown in yellow, showing good results for zero dispersion cases.

is shown in Fig. 7.18. Table 7.4 illustrates all simulation parameters. Note that, in DLGR

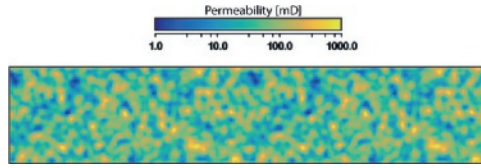


Figure 7.18: Case 2: permeability realization with dimensionless correlation length $\lambda=0.01$ and $V_{dp}=0.63$.

simulations, the highest refinement level (level 4) corresponds to the fine scale permeability map. Permeability fields of coarser resolutions are obtained with a flow-based upscaling procedure.

Table 7.4: Case 2: simulation parameters.

	Fine-scale	DLGR	TL
Base Grid	400×80	25×5	40×8
Refinement	-	2×2	-
Ref. levels	-	4	-
Time-step size	1 day	1 day	10 days

The concentration solutions for all three simulations techniques as well as the grid generated by DLGR, for one permeability realisation, at 0.4 PVI are shown in Fig. 7.19. DLGR employs a concentration difference tolerance of 0.05 as refinement criterion. Note that DLGR is capable of capturing all small scale flow features using, on average, 35% of the grid blocks compared to the fine scale simulation (see Fig. 7.19).

The calibration of the TL model mixing parameter for heterogeneous systems obtained by equating the fractional flow formulation of the Koval and TL model is the only method described in the literature [149]. However, this leads to the estimation of an erroneous negative value (-0.043) for the mixing parameter. By tuning the mixing parameter to fit the data of the high resolution reference simulation an optimal mixing parameter of 0.6 is found. The concentration profile obtained with the empirical TL model is also shown in Fig. 7.19.

Figures 7.20 and 7.21 show the cumulative oil production and the oil rate. In Fig. 7.20 DLGR employs a refinement criterion tolerance of 0.05, resulting in an average grid-block count of 37% of the fine-scale grid-blocks. Figure 7.21, instead, shows results with DLGR using a tolerance value of 0.15, resulting in using only in 19% of the fine-scale grid-blocks on average. It can be observed that the TL model shows low sensitivity to the variation in production and local solvent concentration distributions compared to the high resolution and DLGR simulations.

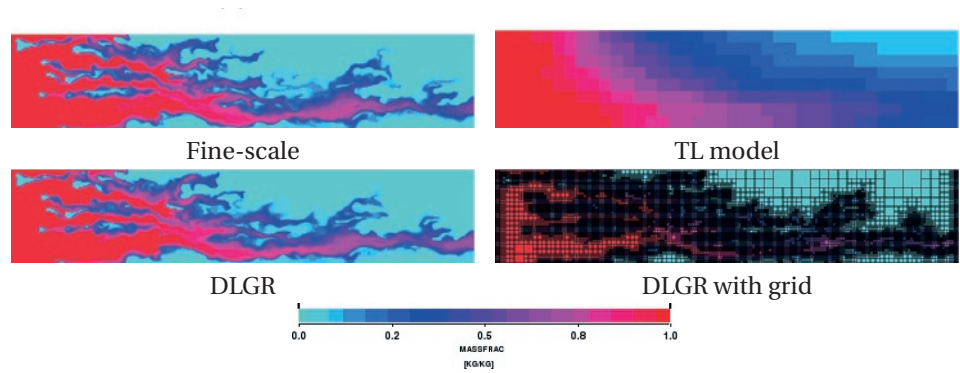


Figure 7.19: Case 2: solvent concentration map at 0.4 PVI for fine-scale, TL model and DLGR. The mixing parameter for TL was equal to 0.6 and DLGR employs, on average, 35% of the fine-scale grid cells.

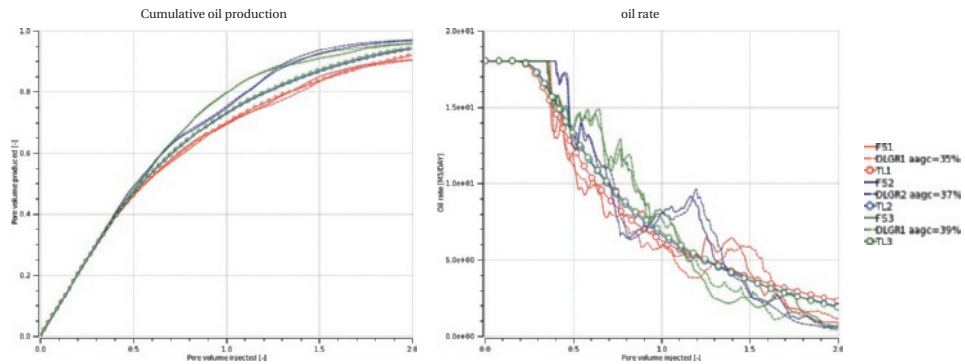


Figure 7.20: Case 2: comparison of the cumulative oil production and oil rate obtained with fine-scale (solid line) simulation, with a 400×80 grid, DLGR (dashed line) and TL (solid line with white circles), with a 40×8 grid and $\omega=0.6$. DLGR employs a refinement criterion tolerance of 0.05. The different colours indicate the different permeability realisations.

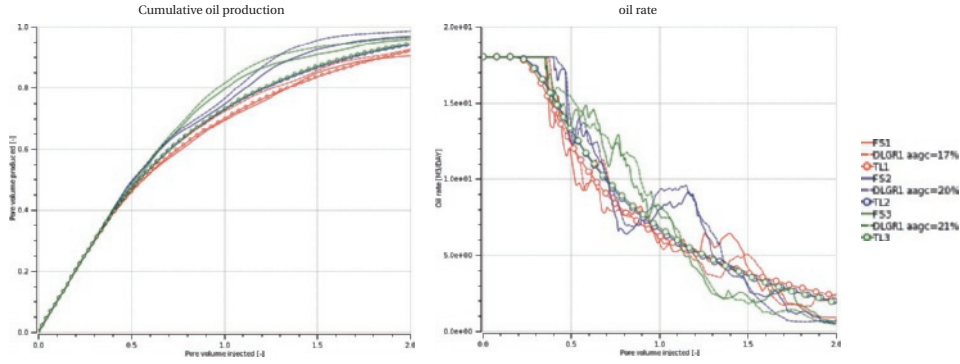


Figure 7.21: Case 2: comparison of the cumulative oil production and oil rate obtained with fine-scale (solid line) simulation, with a 400×80 grid, DLGR (dashed line) and TL (solid line with white circles), with a 40×8 grid and $\omega=0.6$. DLGR employs a refinement criterion tolerance of 0.15. The different colours indicate the different permeability realisations.

7.4.3. CASE 3: PERMEABILITY WITH LONG CORRELATION LENGTH

The same reservoir as the one of the previous case is considered along with the same fluid properties and the same boundary conditions. Here, a permeability field with a higher correlation length is employed as shown in Fig. 7.22. As for the previous case, the fine scale permeability map is upscaled (locally) for TL and the coarser grid resolutions used by DLGR.

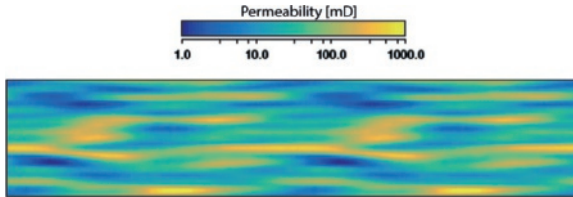


Figure 7.22: Case 3: permeability realization with dimensionless correlation length $\lambda=0.1$ and $V_{dp}=0.63$.

Figure 7.23 shows the solvent concentration map at 0.4 PVI for fine-scale, DLGR and TL simulations. Here, DLGR employs, on average, 29% of the fine-scale grid cells. The TL model is run using the optimal mixing parameter $\omega = 0.375$. Note that the long correlation length reduces the local mixing as shown by the optimal value of the TL mixing parameter. As for the previous case, DLGR is capable of capturing the physics of fluid flow altered by the underlying permeability field. However, due to the increased correlation length of permeability, the frontal surface area is higher than in the previously described case resulting in a higher amount of refinements during simulation.

Figures 7.24 and 7.25 show the cumulative oil production and the oil rate. In Fig. 7.24 DLGR uses a refinement criterion tolerance of 0.05 and in Fig. 7.25 a refinement criterion tolerance equal to 0.15. Similarly to the previous case, DLGR results are in good agreement with the fine-scale reference one for all permeability realisations whereas the TL ones show a considerable error for some of them.

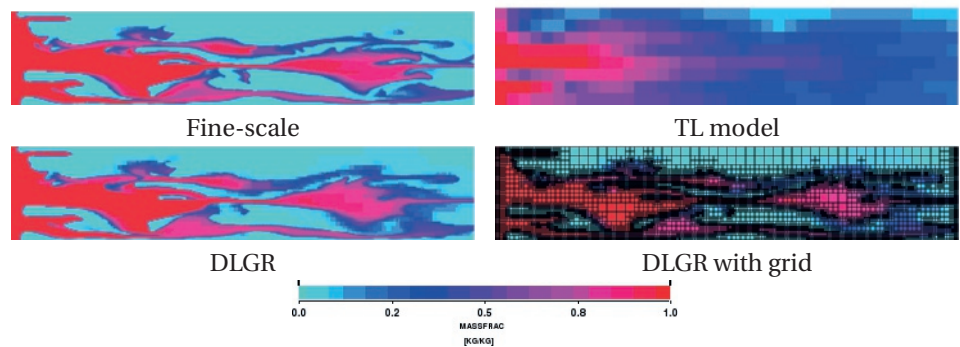


Figure 7.23: Case 3: solvent concentration map at 0.4 PVI for fine-scale, TL model and DLGR. The mixing parameter for TL was equal to 0.375 and DLGR employs, on average, 29% of the fine-scale grid cells.

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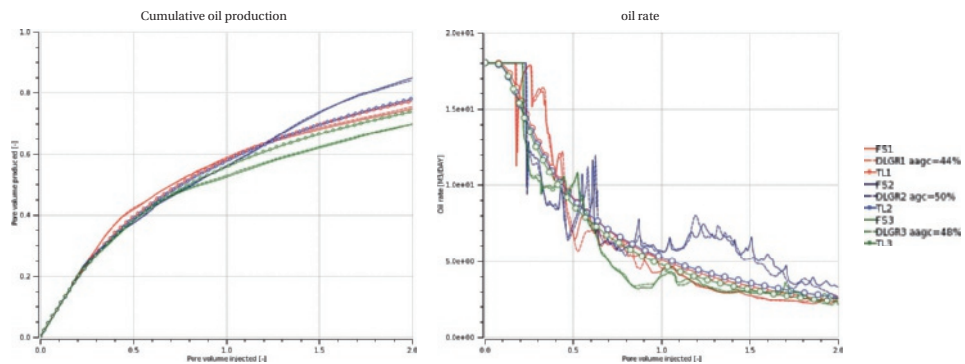


Figure 7.24: Case 3: comparison of the cumulative oil production and oil rate for fine-scale (solid line) simulation, with a 400×80 grid, DLGR (dashed line) and TL (solid line with white circles), with a 40×8 grid and $\omega=0.6$. DLGR employs a refinement criterion tolerance of 0.05. The different colours indicate the different permeability realisations.

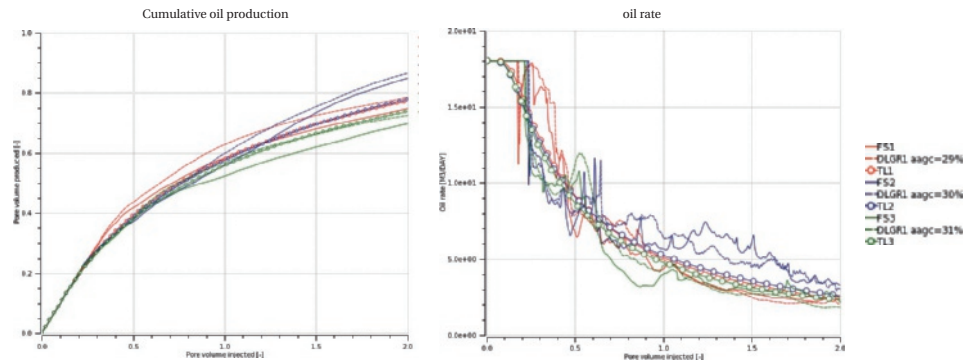


Figure 7.25: Case 3: comparison of the cumulative oil production and oil rate for fine-scale (solid line) simulation, with a 400×80 grid, DLGR (dashed line) and TL (solid line with white circles), with a 40×8 grid and $\omega=0.6$. DLGR employs a refinement criterion tolerance of 0.15. The different colours indicate the different permeability realisations.

APPENDIX: CONCENTRATION PERTURBATIONS

The solvent (randomly generated) perturbations (for the initial concentration values) imposed in the first column of the domain are shown in Table 7.5. To perturb the initial concentration values at the finer grid resolution N_y , these values are applied per patches of $N_y/25$ cells.

Table 7.5: Perturbations imposed in the first column at the injection boundary in order to initiate the unstable displacement.

Cell	Perturbation	Cell	Perturbation	Cell	Perturbation
(1,1)	0.100	(1,2)	0.800	(1,3)	0.500
(1,4)	0.200	(1,5)	0.900	(1,6)	0.400
(1,7)	0.700	(1,8)	0.965	(1,9)	0.958
(1,10)	0.993	(1,11)	0.547	(1,12)	0.188
(1,13)	0.279	(1,14)	0.547	(1,15)	0.097
(1,16)	0.308	(1,17)	0.632	(1,18)	0.221
(1,19)	0.914	(1,20)	0.969	(1,21)	0.127
(1,22)	0.840	(1,23)	0.900	(1,24)	0.820
(1,25)	0.140				

V

CONCLUSIONS

8

CONCLUDING REMARKS AND FUTURE PERSPECTIVES

8.1. CONCLUSIONS

In this section, some conclusions are drawn about the contents presented in each part of this dissertation.

8.1.1. PART II

In chapter 3, the first attempt of employing multiscale methods within fully implicit simulation of multiphase multicomponent flow in heterogeneous porous media is presented. Here, multiscale is employed as a linear solver within a two-stage preconditioning strategy. The CPR decoupling operator is used to extract the pressure system to which the multiscale method is applied. Following the steps of CPR preconditioning, after having solved the pressure system, a second stage preconditioner (i.e., ILU(0)) is used as the second stage solver for FIM system. Multiscale restriction and prolongation operators are obtained by solving basis functions using the IMPES formulation.

The results for the test cases presented, which are challenging for a sequential simulator, show promising results for the application of multiscale methods as a pressure linear solver for FIM simulations. Compared with AMG solvers (nowadays the industry standard), multiscale methods benefit from aggressive coarsening ratios with accurate prolongation operators on the basis of locally computed basis functions. Further studies are required to exploit full adaptivity of the multiscale methods along with the conservative property of MsFV into the FIM system.

Although multiscale methods have proven to be efficient linear solvers, in the author's opinion, the potential of such methods extends beyond being very efficient solvers. In fact, their physics-based nature makes them suitable methods to build smarter non-linear solution approaches as proposed in the following chapters.

8.1.2. PART III

In chapter 4, a novel algebraic dynamic multilevel (ADM) method for fully implicit simulation of multiphase flow in porous media is developed. The FIM system is described on an adaptive multilevel grid through sequences of prolongation and restriction operators. As such, the size of the FIM system changes during the simulation dynamically, with refined grid resolutions being used only where and when needed.

While constant prolongation operators are employed for saturation, different prolongation operators for pressure are studied. These options are constant, bilinear, and multiscale-based interpolators. All of them are leading to prolongation operators with partition of unity. Several test cases ranging from homogeneous to highly heterogeneous channelized fields are considered, in order to investigate the applicability of the new method.

The numerical results, compared with those obtained from a fine-scale fully resolved simulator, show that the ADM method leads to accurate and efficient FIM simulations by employing only a fraction of the total fine-scale grid cells. Results of the scalability study of the method, based on the count of active grid cells, show that ADM is a promising approach for real-field applications. In addition, its algebraic description allows for a convenient integration of such an advanced method, as a blackbox, within existing reservoir simulators. It is important to note that, though it is presented for FIM systems, ADM method can be readily used for sequential approaches.

In chapter 5, the algebraic dynamic multilevel method (ADM) is extended to 3D geometries with compositional simulations with capillary and gravitational effects. This extension is successfully developed both for the natural variables and for the overall-compositions formulations, with proper considerations of the primary unknowns based on the phase distribution. The algorithm, thanks to its operator based nature, can be integrated into an existing simulator irrespectively of the formulation used. The performance of the method is investigated for a wide range of test cases with strong non-linearity and heterogeneity. These numerical results show that ADM provides, even in presence of complex fluid physics, satisfactory solutions employing a fraction of the fine-scale grid cells. It is shown that ADM tends to be more effective when applied to larger test cases. As such, it is a promising approach for large-scale studies.

In chapter 6, an adaptive saturation prolongation operator for the ADM method is presented along with an alternative grid selection criterion. The adaptive saturation interpolator enables to capture the details of the rarefaction saturation wave even though the region behind the front is coarsened. Additionally, the newly proposed interpolation strategy and coarsening criterion allow for a more aggressive coarsening in regions with high fluid-property gradients that do not evolve with time. Such situations may occur in presence of large permeability contrast. The fine-scale details of the saturation distribution are preserved by employing adaptive saturation basis functions.

Numerical examples are shown to illustrate the differences between the original ADM method and the improved one proposed in this work. Employing the adaptive saturation interpolator has evident advantages compared to employing piece-wise constant basis functions, especially in presence of high permeability contrast that generate stagnant zones. The proposed improvements to the original ADM method seem a promising way of reducing the number of grid cells employed without excessive loss of accuracy.

In general, ADM combines the advantages of multiscale methods with those of dynamic local grid refinement techniques, and provides an algebraic multilevel multiscale framework for FIM simulations, the first of its kind.

8.1.3. PART IV

In chapter 7 the advantages of employing a multilevel approach compared to a more traditional upscaling technique are analysed for the specific case of miscible displacements. In particular, the accuracy and sensitivity of a DLGR method are compared against the TL upscaling approach for incomplete mixing processes.

Numerical results show that DLGR has a great potential to overcome the limitations of effective models like those of the TL one both for homogeneous and heterogeneous media. In fact, it is shown that, for a homogeneous domain, the TL model is very sensitive to modifications of the simulation parameters (both size of the domain and mobility contrast). Consequently, TL simulations require adequate tuning of the mixing parameter ω every time the simulation input changes. In many cases the correct value of the TL mixing parameter can only be found by running high-resolution simulations and an a priori estimate cannot be found. On the other hand, DLGR provides accurate solutions for simulations of incomplete mixing processes by employing only a fraction of the grid cells used for fine-scale fully resolved simulations.

Accuracy and robustness of the described DLGR strategy and the TL model for heterogeneous media with different correlation lengths are also assessed. DLGR allows for local preservation of the underlying geological features, clearly indicating the potential of DLGR to solve for incomplete mixing in heterogeneous media. This is illustrated by the fact that the production results of DLGR show the variations similarly observed in the high resolution simulations that are used as a reference. Furthermore, it was shown that the error of DLGR simulations can be controlled by regulating the tolerances used for the refinement criteria.

As such, dynamic multilevel strategies represent a promising approach to obtain more accurate results and overcome the limitations of upscaled models for simulations of incomplete mixing processes. Note that efficient implementation of the DLGR method is an important factor for its successful implementation in field-scale commercial simulators. Additionally, the use of upscaled properties (i.e., permeabilities) in coarse grid-blocks may negatively affect the accuracy of the dynamic multilevel simulations, specially in presence of more complex physics. Some of these issues have been addressed by the algebraic multiscale-based ADM method. As such, the DLGR approach employed in chapter 7 could be implemented in a commercial-grade simulator with an ADM-based implementation strategy, where constant interpolators are used for both pressure and concentration across different scales.

8.2. FUTURE PERSPECTIVES

Since 2015, year in which the study about CPR-MS [88] was published, there has been a great effort, by different scientific groups to improve even further the performance of multiscale methods as a fully converged iterative solver for the elliptic pressure equation [151] and extend it to more realistic scenarios [69, 75, 76]. Therefore, the author

thinks that this is a fairly mature research topic. On the other hand, in chapters 4 - 6 a dynamic multilevel framework for simulation of multicomponent multiphase flow was introduced. This is a significant step forward and sheds new light on both multiscale methods and DLGR techniques. However, there exist further scope for research and the remaining of this chapter discuss some possible topics for future research activities.

CPU BENCHMARKING AND PARALLEL IMPLEMENTATION

No CPU study has been presented in this work to prove the efficiency of the newly developed ADM method compared to more traditional DLGR techniques. In fact, the ADM method has only been implemented in the Matlab DARSim2 simulator developed by the author of this work. The implementation of ADM in a compiled language, (e.g., C++) that would allow to thoroughly study its performance, should be the object of future work. Such an implementation would also allow to study the best strategy to have a parallel implementation of the algorithm and assess its scalability.

DEVisING APPROPRIATE LINEAR SOLVERS FOR ADM

In this work, the ADM coarse system of chapters 4 - 6 is solved, exactly, employing a direct solver. However, for field-scale applications, the solution of the ADM coarse system may require the use of an iterative solver. The properties of the ADM Jacobian can be quite different from those of the fine-scale one and, as a consequence, ad hoc preconditioners may have to be devised.

UNSTRUCTURED GRIDS

In this work ADM has only been employed on structured cartesian grids, mainly for ease of implementation. Real field applications, however, may not allow for such grid regularity as natural formations have much more complex geometries. A significant step forward would be to devise an ADM strategy for unstructured grids.

FRACTURED MEDIA

The focus of this thesis are highly heterogeneous porous media. Another aspect that is crucial to build predictive models for many real applications is to include the effect of fractures. The extension of ADM to fractured media has also been addressed by the author of this dissertation and it has shown promising results [152]. However, fractures add, greatly, to the complexity of the system and further studies are required. In particular, the hydrodynamical properties of the fractures are strictly related to the geomechanical configuration. Therefore, it is important to extend the ADM method to problems involving coupled flow and geomechanics.

ADAPTIVITY IN THE TIME DISCRETIZATION

Most efforts in the literature, including this work, have focused on reducing the computational cost of simulations by mapping efficiently and accurately equations (or properties) from a high resolution grid to a coarse one. However, the physical phenomena involved in multicomponent multiphase flow simulations do not only involve a variety of length scales but also different time scales as both fast (e.g., chemical reactions, flow in fractures) and slow (e.g., Darcy flow, diffusion) processes take place.

A

DARSIM2 MATLAB SIMULATOR

The ADM method presented in chapters 4, 5 and 6 was implemented and tested in a simulator that was developed throughout the course of the PhD. In this chapter the DARSim2 simulator is presented and validated by comparison against well-established simulators.

DARSim2 is an object oriented Matlab simulator that can be easily used as a basis for testing new methods and accommodates various formulations, coupling strategies and fluid models. Figure A.1 shows the class diagram.

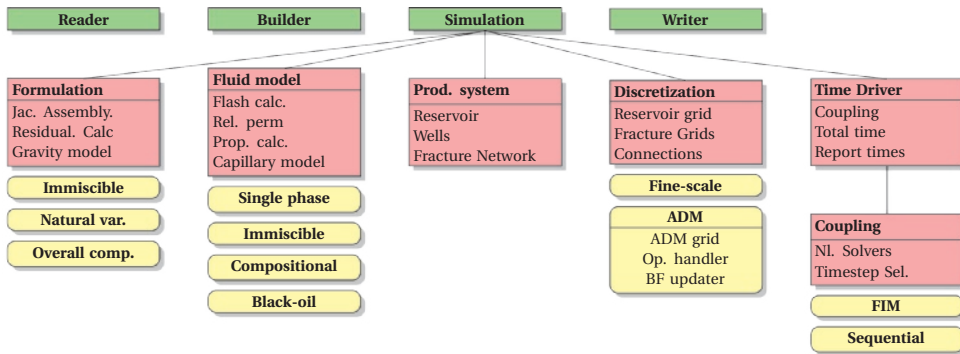


Figure A.1: DARSim2 reservoir simulator class diagram. The green boxes are the 4 main objects constituting the simulator. The most important classes forming the simulation are shown in the red boxes. The existing specializations of those classes are shown in yellow.

Remark, that the code supports different fluid models and was already designed to include fractures in the computational domain. Also, since the aim of chapter 5 was extending ADM to compositional displacements, two compositional formulations were

Parts of this chapter have been published in the Journal of Computational Physics, **Vol. 354** (2018) [117].

implemented: (1) natural-variable based formulation; (2) overall-composition formulation.

A.1. VALIDATION OF DARSIM2

In this section some numerical experiments are conducted to validate the results of the DARSim2 simulator. In particular, two well established reservoir simulators are used as a reference; (1) the Matlab Reservoir Simulation Toolbox (MRST) [153–155]; (2) the AD-GPRS simulator [28, 31, 92, 119].

A.1.1. IMMISCIBLE DISPLACEMENT

The fine-scale implementation of DARSim2 for immiscible displacements is validated against the Matlab Reservoir Simulation Toolbox (MRST). A 1D homogeneous reservoir with a permeability of 1 D and porosity of 20 % is considered. The domain has a length of 1000 m and is discretized with a 100 equidistant cells. Two incompressible immiscible phases are present. The phase densities are $\rho_1 = 1000 \frac{\text{kg}}{\text{m}^3}$ and $\rho_2 = 700 \frac{\text{kg}}{\text{m}^3}$ whereas their viscosities are $\mu_1 = 10^{-2} \text{ Pa s}$ and $\mu_2 = 10^{-3} \text{ Pa s}$. An injection and a producer well are present at the left and right ends of the reservoir, respectively. The initial saturation of phase 1 in the reservoir is $S_1 = 0.1$. Phase 1 is injected at a constant pressure of $p_{\text{inj}} = 10 \text{ bar}$ and produced at a pressure of $p_{\text{prod}} = 1 \text{ bar}$.

Figure A.2 shows a comparison of the results obtained with DARSim2 and MRST at four different time-steps. Note that the results obtained with the two simulators are in very good agreement.

A.1.2. COMPOSITIONAL DISPLACEMENTS

The fine-scale implementation of the compositional formulations in DARSim2 is validated against the General-Purpose Research Simulator (AD-GPRS). A 1D homogeneous black-oil reservoir with a permeability of 1 mD and porosity $\phi = 0.2$ is considered. The reservoir length is 99 m and it is discretized with 99 equidistant cells. The initial pressure of the reservoir is 10 bar and the initial gas mass fraction $z_g = 0.03973$ leading to an initial vapour saturation $S_v = 0.027$. The gas and oil densities at standard conditions are $\rho_g^{STC} = 100 \frac{\text{kg}}{\text{m}^3}$ and $\rho_o^{STC} = 800 \frac{\text{kg}}{\text{m}^3}$, respectively; and the formation volume factors are $B_g = 0.25$ and $B_o = 1$. The solution gas-oil ratio is considered to have a linear dependency on pressure, i.e., $R_s = 0.2p^* + 0.2$ where $p^* = \frac{p}{p_{ref}}$ and $p_{ref} = 100 \text{ bar}$. The vapour and liquid viscosities are respectively $\mu_v = 10^{-4} \text{ Pa s}$ and $\mu_l = 10^{-3} \text{ Pa s}$ and are considered to be constant [28]. A pressure-constrained gas injector well with $p_{\text{inj}} = 100 \text{ bar}$ is located at cell 1, while a producer well with $p_{\text{prod}} = 10 \text{ bar}$ is placed in the last cell 99.

Figure A.3 shows a comparison of the solution obtained with DARSim2 and AD-GPRS at 5 different simulation times. Remark that results of the two simulators are in perfect agreement.

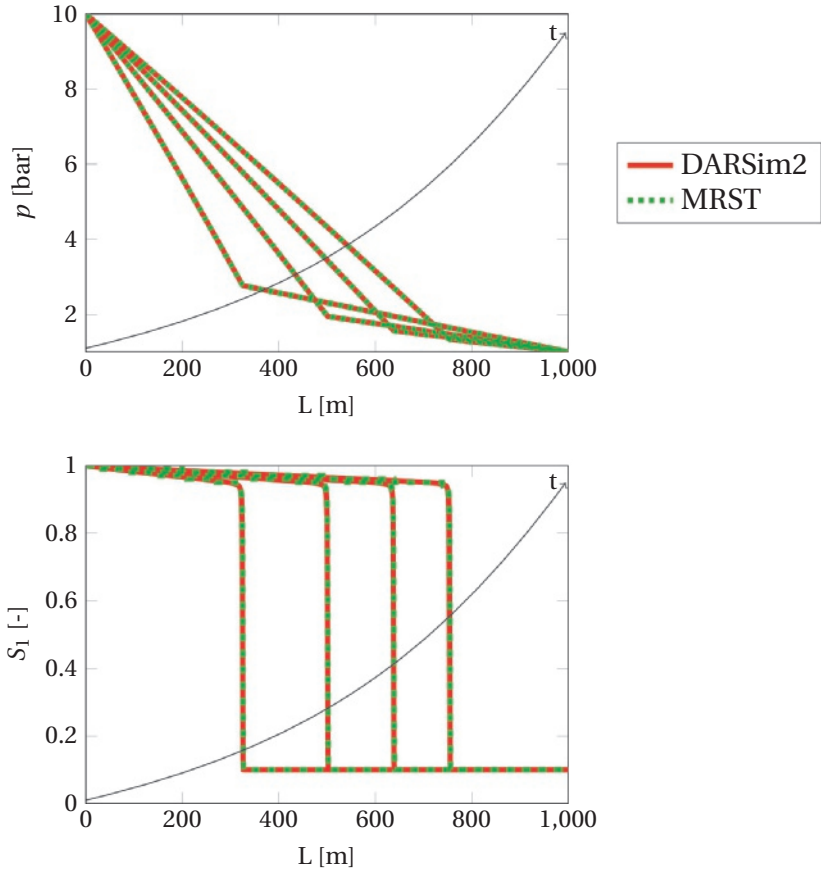


Figure A.2: Pressure, and saturation as functions of the position obtained with MRST and DARSim2 simulator. The solution is shown at 4 different simulation times: 2000, 4000, 6000 and 8000 days after injection has started.

A

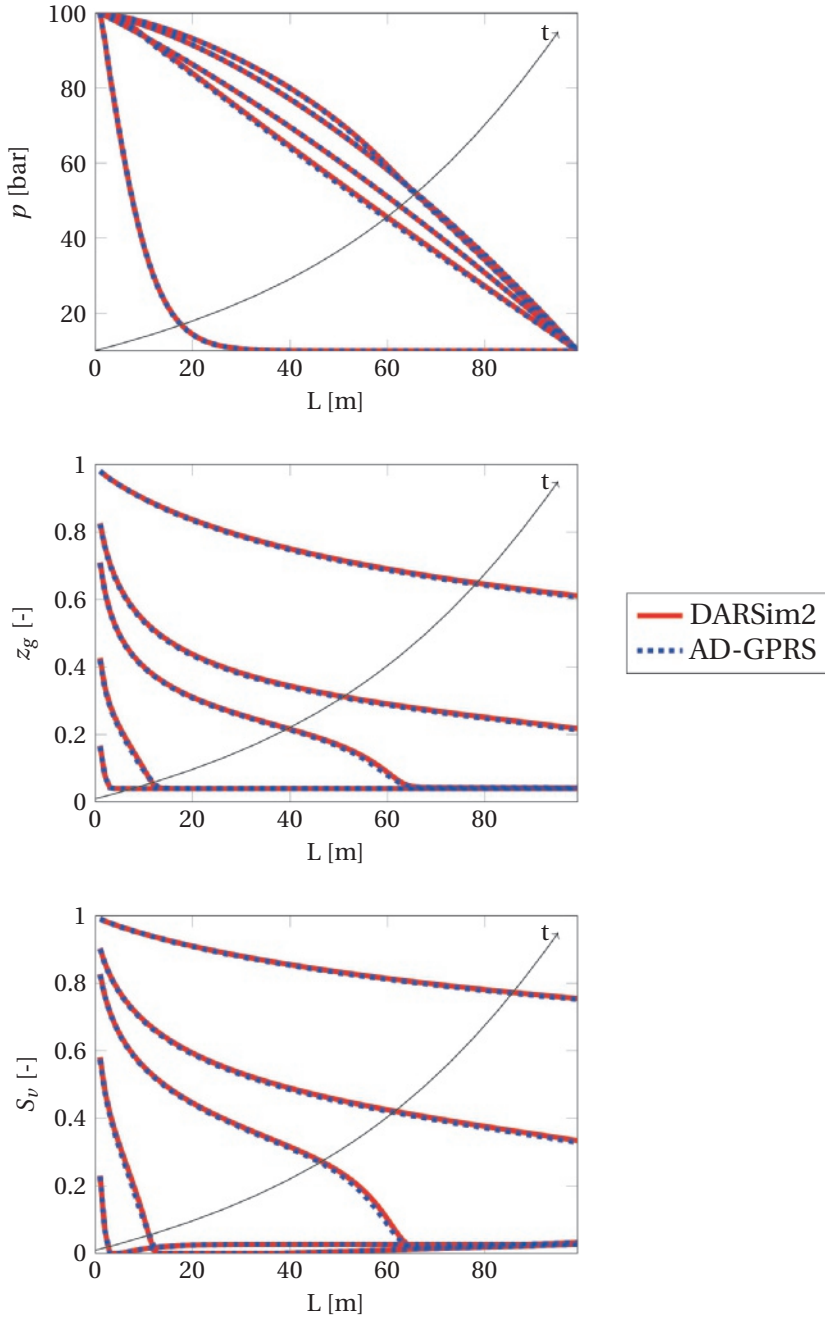


Figure A.3: Pressure, total gas mass fraction and vapour saturation as functions of the position obtained with AD-GPRS and the FS DARSim2 simulator. The solution is shown at 5 different simulation times: 0.5, 5, 50, 1000 and 5000 days after injection has started.

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LIST OF PUBLICATIONS

PEER-REVIEWED PUBLICATIONS

7. **M. Cusini, Fanxiang Xu, H. Hajibeygi**, *A local time-stepping strategy for fully-implicit simulation of multiphase flow in porous media*, in preparation.
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5. **M. Cusini, R. Gielisse, H. Groot, C. van Kruijsdijk, H. Hajibeygi**, *Incomplete Mixing in Porous Media: Todd-Longstaff upscaling approach versus a dynamic local grid refinement method*, [Computat. Geosciences](#) (2018).
4. **M. HosseiniMehr, M. Cusini, C. Vuik, H. Hajibeygi**, *Algebraic dynamic multilevel method for embedded discrete fracture model (F-ADM)*, [J. Comput. Phys.](#) **373**, 324-345 (2018).
3. **M. Cusini, B. Fryer, C. van Kruijsdijk, H. Hajibeygi**, *Algebraic dynamic multilevel method for compositional flow in heterogeneous porous media*, [J. Comput. Phys.](#) **354**, 593-612 (2018).
2. **M. Cusini, C. van Kruijsdijk, H. Hajibeygi**, *Algebraic dynamic multilevel (ADM) method for fully implicit simulations of multiphase flow in porous media*, [J. Comput. Phys.](#) **314**, 60-79 (2016).
1. **M. Cusini, A. Lukyanov, J. Natvig, H. Hajibeygi**, *Constrained pressure residual multiscale (CPR-MS) method for fully implicit simulation of multiphase flow in porous media*, [J. Comput. Phys.](#) **299**, 472-486 (2015).

CONFERENCE PROCEEDINGS

5. **M. HosseiniMehr, R. Arbarim, M. Cusini, C. Vuik, H. Hajibeygi**, *Algebraic Dynamic Multilevel method for single-phase flow in heterogeneous geothermal reservoirs*, [Proceedings of ECMOR XVI - 16th European Conference on the Mathematics of Oil Recovery, Barcelona 3-6 Sept. 2018](#).
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2. **M. Cusini, C. van Kruijsdijk, H. Hajibeygi**, *Algebraic Dynamic Multilevel (ADM) Method for Immiscible Multiphase Flow in Heterogeneous Porous Media with Capillarity*, [Proceedings of ECMOR XV - 15th European Conference on the Mathematics of Oil Recovery, Amsterdam 29th Aug. - 1st Sept. 2016](#).
1. **M. Cusini, A. Lukyanov, J. Natvig, H. Hajibeygi**, *A Constrained Pressure Residual Multiscale (CPR-MS) Compositional Solver*, [Proceedings of ECMOR XIV - 14th European Conference on the Mathematics of Oil Recovery, Catania 8-11 Sept. 2014](#).

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Matteo Cusini was born on March 25th, 1989, in Milan, Italy. He earned a bachelor's degree in Energy Engineering in 2012 at Politecnico di Milano. Having attended the Ecole Centrale de Paris between 2010-2012, he then obtained, in December 2014, a double master's degree from the two universities. During that time, the author also took part in the Unitech International programme, completing two internships in Romania, the UK and the Netherlands. In February 2015, Matteo started a PhD at the faculty of Civil Engineering and Geosciences at the TU Delft, under

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Besides his passion for Maths and Physics, the author enjoys outdoor activities and sporting, having as main hobbies hiking, cycling and, since recently, surfing. He also has practised skiing, running, soccer and American football at a competitive level for several years.

A passionate traveller, Matteo, despite his young age, has already visited a large portion of the world, and intends to continue doing so.

