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RESEARCH PAPER

Towards intentional aesthetics within topology optimization by applying the principle of unity-in-variety

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Abstract

Topology optimization is increasingly applied to design consumer products, for which aesthetics plays an important role to consumer acceptance. In industrial design, it is known that preferences or taste judgement obey certain rules or principles. These principles are not directly quantifiable, but can qualitatively predict and explain aesthetic responses. In this paper, we empirically evaluate whether or not these design principles are effective for increasing the appealingness of topology optimized shapes. Our starting point is an overarching principle known as *Unity-in-Variety*. Variety stimulates our interests, whilst unity helps us make sense of a design in its entirety. According to this principle, aesthetic appreciation is maximized when a balance in unity and variety is attained. Since designs from topology optimization often exhibit remarkable complexity and variety, we hypothesize that increasing unity is the key to reach a balance and thus to elevate aesthetic appreciation in topology optimization. In our experimental setup, designs from topology optimization were manually post-processed, with the intention to increase unity, by following the "principles of perceptual grouping", known as *Gestalt principles*. Our user study shows that in 11 out of the 12 pairs of topology optimized designs and their modified counterparts, the modified designs are perceived by the majority as visually more appealing, confirming our hypothesis. These findings provide a good basis for improving the aesthetic pleasure of topology optimized designs, either manually or ultimately by integrating them in the topology optimization formulation. It is expected that this eventually will contribute to a wider acceptance of topology optimization for consumer product design.

Keywords Topology optimization · Design aesthetics · Unity-in-variety · Gestalt principles

1 Introduction

Topology optimization is a computational approach for designing structures. It takes structural design requirements as input, and automatically generates the best performing structure, based on modelling of the physical system and mathematical optimization. It has been widely used for designing lightweight structures in industries such as aerospace and automotive. In recent years, along with advances in additive manufacturing for producing complex structures, topology optimization is increasingly applied to design consumer products as well, for which aesthetics is critical to consumer acceptance.

Structures designed by topology optimization typically have an organic appearance, a quality that many people find novel in industrial products. This quality is also considered as visually attractive. For example, the "bone chair" (Fig. 1), designed by Dutch designer Joris Laarman using topology optimization, became part of the permanent collection of multiple design and modern art museums. Appealing designs like this one have promoted a lot of expectation of topology optimization beyond its engineering value, i.e., to create aesthetic products.

Whilst structures designed using topology optimization may look appealing, this is not always the case; the visual attractiveness is largely coincidental. Topology optimized designs are determined by a number of factors. These include structural design requirements (e.g., mechanical boundary conditions, material properties, manufacturability constraints) as well as parameters in topology optimization

Fig. 1 "Bone chair" designed using topology optimization. Image courtesy of Joris Laarman (www.jorislaarman.com)

algorithms (e.g., material interpolation schemes, design update strategies, design initialization). The visual quality amongst different designs varies significantly.

Wouldn't it be wonderful to steer topology optimization towards designs that are functionally optimal and visually appealing? Many researchers and practitioners in topology optimization have raised this question or similar ones. Appealingness is a challenge to mathematical modelling, in stark contrast to functional performance. *"Beauty is in the eye of the beholder".* A design that one person finds beautiful may not appeal to another.

Despite the subjective nature of aesthetic appreciation, it has been demonstrated that preferences or taste judgments obey certain rules or principles (Hekkert 2006). These principles are not directly quantifiable, but can qualitatively predict and explain aesthetic responses of the majority. A correct application of these principles is thus expected to enhance products' aesthetic value. In the conventional design process where the product shape is thought out and tailored by human minds and hands, industrial designers are instructed to actively apply and reflect on these principles.

In this paper, we examine the design principles, that have been guiding manual design processes, and empirically evaluate whether or not they are also effective for increasing the appealingness of topology optimized shapes. Our starting point is an overarching principle known as *Unity-in-Variety* (Hekkert 2006; Post et al. 2016). Variety stimulates our interests, whilst unity helps us make sense of a design in its entirety. According to this principle, aesthetic appreciation is maximized when a balance in unity and variety is attained (Post et al. 2016). Compared to the manually sketched designs that surround us in our daily lives, designs from topology optimization exhibit much more complexity and variety. We thus hypothesize that increasing unity is the

key to reach a balance and thus to elevate aesthetic appreciation of topology optimized designs. In our experimental setup, designs from topology optimization were manually post-processed, with the intention to increase unity. The manual post-processing follows the *Gestalt principles* of visual perception ("Gestalt" is German for "unified whole") (Wagemans et al. 2012). The original and altered designs are then assessed in a user study.

Aesthetics has been a recognized aspect in topology optimization research since the early days of this field. Bendsøe and Rodrigues (1991) indicated the utilization of the ingenuity of mechanical designers to interpret topology optimization results, considering aesthetics as well as ease of production. Beghini et al. (2014) advocated the role of topology optimization in connecting architecture and engineering by creating optimized geometric patterns. Dapogny et al. (2017) presented a set of geometric constraints for structural optimization in architectural design, with a focus on similarity to a prescribed reference design or pattern. Aesthetics has been a (secondary) source of motivation of novel topology optimization approaches for controlling geometric features such as member sizes (Guest 2009), symmetry (Rozvany 2011), and pattern gradation (Stromberg et al. 2011). These features are well aligned with the known design principles, from which one can even identify further features that have not yet been studied in the topology optimization literature. To the best of the authors' knowledge, a comprehensive study of design principles and an empirical validation are missing in topology optimization. Our scientific contribution is an experimental validation of the hypothesis that increasing unity is the key for elevating aesthetics in topology optimization.

Besides in optimization-driven design, aesthetics has also received attention in other generative design approaches such as parametric modelling and shape grammars (Wu et al. 2019). Orsborn et al. (2009) proposed to quantify customer preferences as functions of selected shape parameters that were calibrated by a consumer study. It was validated on exploring automobile silhouettes. Lugo et al. (2016) investigated relationship between aesthetic preference and quantified Gestalt principles in automobile wheel rims. Mata et al. (2019) presented a method to integrate aesthetic design rules into shape grammars and parametric models. It was demonstrated by a tool for creating vases. The number of design parameters in these validations is small (i.e., ranging from a few to a few dozens), and an analogical transfer of these findings to topology optimization may not be feasible.

The remainder of this paper is organized as follows. In Sect. 2 we review the design principles. After explaining the setup of the user study in Sect. 3, we present and discuss results in Sect. 4. The paper is concluded in Sect. 5 with some ideas for future work.

2 Unity-in-variety

To start with, let us clarify what we mean by aesthetics in this paper. This clarification is important, since, for example, an industrial designer and an experienced structural engineer may evaluate aesthetics from different perspectives and come to different or even opposite opinions. Aesthetics, referring to sensory perception, is most often used to describe (visual) arts, which are mostly created to gratify our senses. In contrast to visual arts, industrial products are designed for very specific functions, e.g., a chair for sitting on, and a beam for carrying mechanical loads. Our visual evaluation of products thus does not stay on the sensory perception level, but also involves various cognition processes. For instance, when viewing a design, we often instantly bring in our knowledge and experience to predict its functional performance. The perceptual and cognitive interpretation processes are highly related and not clearly separable. They together contribute to an overall assessment of design aesthetics.

In this paper we focus on the perceptual level of aesthetics. On the perceptual level, it's human nature to find order in chaos. Consequently, aesthetic pleasure is often high when a design creates a balance between unity and variety, i.e., unity-in-variety (Hekkert 2006). Unity-in-variety is an age-old principle, and the acknowledgement of this can be traced back to the Greeks.

- Variety is defined as the amount of differences or variations that exist between different parts within a design.
- Unity means the degree of coherence that can be perceived.

An example contrasting variety and unity is shown in Fig. 2. From the layout on the right hand side, a clear alignment amongst the different parts can be perceived.

Unity increases aesthetic pleasure since it allows easy and efficient perceptual processing. From an evolutionary psychology perspective, unity is beneficial for our survival, whilst variety stimulates a sense of accomplishment

Variety   Unity

Fig. 2 Illustration of variety and of unity. (Author/Copyright holder: Teo Yu Siang and Interaction Design Foundation. Copyright terms and licence: CC BY-NC-SA 3.0)

(Hekkert 2006). This principle has been empirically validated on multiple studies, including product design (Post et al. 2016) and website layout (Post et al. 2017). The results showed that both unity and variety, whilst suppressing each other's effect, positively influence aesthetic perception. It was argued that when an optimum balance between unity and variety is achieved, aesthetic appreciation is maximized (Post et al. 2016).

In the conventional, manual design process, unity is achieved by applying principles of perceptual grouping: Gestalt principles. Industrial designers, as well as graphic designers and user-interface designers, consciously apply Gestalt principles during the design process. These principles are based on the tendency of human beings to perceptually group and bring order to things to create a meaningful whole. The idea of Gestalt principles dates back to the psychologist Max Wertheimer in the beginning of the 20th century, who discovered stimulus factors that influence the perception of grouping different elements (Wagemans et al. 2012). The most common Gestalt principles are described below and visualized in Fig. 3.

- Closure. In seeking a single, recognizable pattern, people can fill in the missing parts of a design or image to create a whole.
- Continuity. People group elements together if they are interpreted as continuing in line or form.
- Similarity. Different parts of a design may be (dis)similar in colour, material, size, and orientation. It is human nature to group similar elements together.
- Parallelism. People tend to group together elements that are parallel to each other.
- Proximity. Elements that are closer to each other are seen as a group opposed to elements that are distant from each other.
- Symmetry. When a product is perceived as symmetrical, it is easier for the brain to process it and it creates balance within the product.

As topology optimization algorithms already create a great deal of variety within a design, we hypothesize that increasing unity within a design would help reach a balance in unity and variety, and thus increase the aesthetics. For topology optimized designs, unity could potentially be enhanced by following Gestalt principles. In this research, three principles, i.e., similarity, continuity, and closure, are investigated. Similarity is specified to "uniform thickness". Closure is specified to the balance between the areas of solid and void regions as well as the balance amongst the void sizes. Continuity concerns the orientation of beam-like substructures across the joint where they meet. These three were chosen because of their general applicability. Uniform thickness for example, could be applied throughout the entire
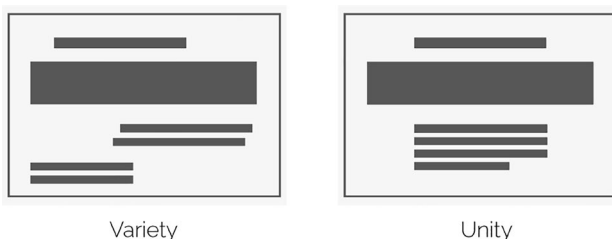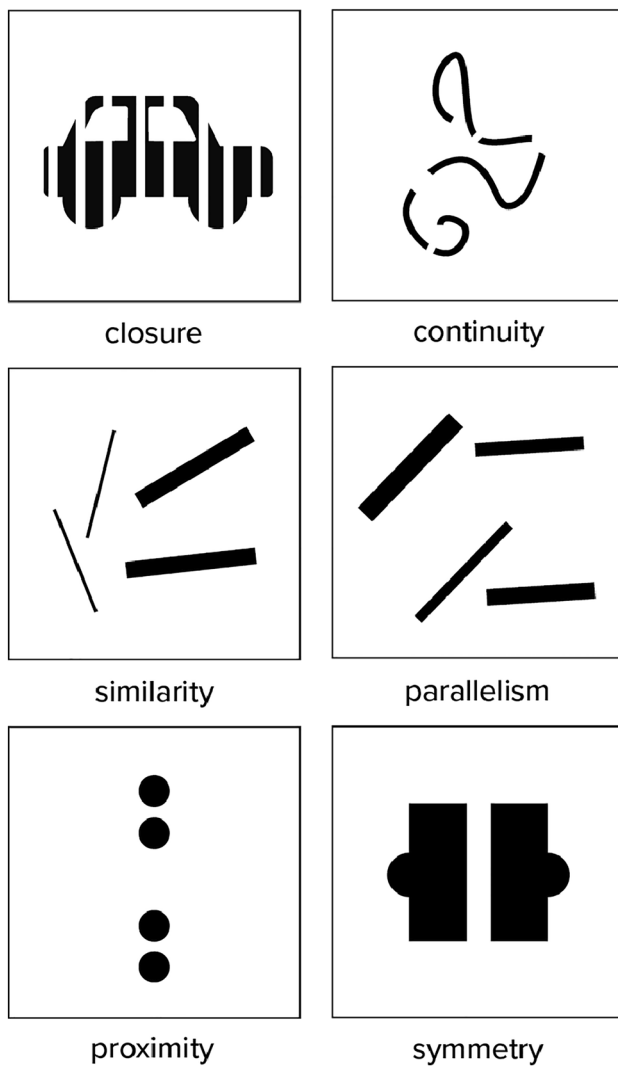
Fig. 3 Illustration of six common Gestalt principles

design. Symmetry in a design, on the other hand, is highly dependent on the problem set-up. For this reason, symmetry was not included in this research.

3 User study

To investigate whether or not Gestalt principles are effective in elevating the aesthetic appreciation of topology optimized designs, a user test was executed, in the form of an online survey. In the survey, participants were asked to rate 12 pairs of 2D shapes that were created based on topology optimization.

3.1 Stimuli

Two categories of shapes are included in the user study as stimuli: chairs and cantilever beams. Chairs are a representative of consumer products that people are familiar with. Topology optimized chairs are popular in the design field, e.g., the bone chair (Fig. 1). Chairs have also been used as an example to demonstrate novel topology optimization algorithms. In designing consumer products such as chairs, aesthetics is typically given more attention than their mechanical performance. Cantilever beams represent mechanical components where aesthetics is often less of interest than their mechanical performance.

The stimuli include 6 pairs of chairs and 6 pairs of beams. In each pair, one was created using the TopOpt app (version 5.3.4p1) (Aage et al. 2013). It was then reshaped to produce the second one, by carefully applying Gestalt principles to enhance unity. The shape modification was performed using Adobe Illustrator (version CC 2018). Figure 4 shows one of the original chairs and one of the original beams.
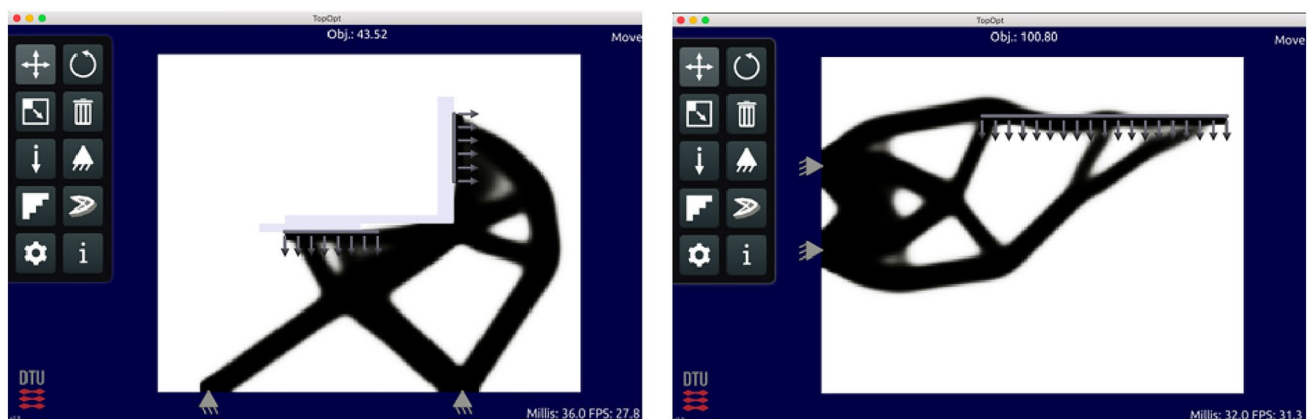

Fig. 4 A 2D chair and a cantilever beam, created using the TopOpt App

To design the 2D chairs, two fixations were placed at the bottom of the design domain, initiating the chair legs. The loads include distributed forces on the seat and backrest, mimicking a person sitting on the chair. The locations of the fixations and forces as well as the material percentage were varied to create different designs. The cantilever beams have two fixed joints to a wall on the left, and have either single forces or a distributed load pointing downwards, all consisting of unit forces. Whilst creating these designs, it was kept in mind that each shape needed to be clearly recognizable as a chair or a cantilever beam.

In total, twelve chairs and twelve beams were initially created using the TopOpt app. The chairs and beams were randomly divided over the three Gestalt principles: uniform thickness, closure and continuity. Four chairs and four beams were placed for each principle. From these, two chairs and two beams for each principle were selected for manual modification, and the rest were disregarded. The selection was based on an assessment of whether or not the principle is most applicable, i.e., applying that principle results in a significant change in shape. For example, for the uniform thickness principle, we selected designs that have large variations in thickness. This selection gave a total of six chairs and six beams to include in the user test, 2 per Gestalt principle. The next step was to modify the chairs and beams (in Adobe Illustrator CC 2018) following the respective Gestalt principle to change the aesthetics. To this end, we first traced the contour of each design, and then edited the control points to change the design. Figure 5 highlights some of the modifications that were made to increase unity.

Figures 6 and 7 show the six pairs of chairs and six pairs of beams, respectively. The order of the pairs in each category is randomized. Also randomized is the order of the original and altered design in each pair. To clarify the usage of the products for the participants, the fixations are illustrated as a blue line. The load on the chair is portrayed by a person sitting on it. This person also makes the proportions and use of the chair more clear. The loads on the beams are indicated with red arrows.

3.2 Participants

A total of 37 subjects completed the survey. All participants are current students or recent graduates of Delft University of Technology. 21 subjects are students in the master programme Design for Interaction and have prior knowledge of the Gestalt principles and the principle of Unity-in-Variety. The other 16 participants are bachelor students who have different academic backgrounds ranging from Industrial design engineering and Mechanical engineering to Aerospace engineering and Nano biology. They had no or limited prior knowledge of these aesthetic principles. Eight of the 16 had followed a 2-hour introductory lecture on topology optimization. It was deliberately chosen to invite participants with and without prior knowledge of aesthetic principles, in order to minimize survey bias. It was expected that participants without prior knowledge may respond to the survey differently to those who could recognize the aesthetic elements in the test.

3.3 Procedure

The user study was conducted via an online survey, consisting 54 questions in total. Participants were instructed that the survey would take approximately 25 minutes. They however progressed with the survey at their own pace, with no restriction on the minimum nor maximum time per question or for the entire survey.

In the survey, the two shapes in each pair were placed side by side, enabling a direct comparison. The following five questions were asked for each pair of the chairs.

Fig. 5 The original design, on the left of each pair, is altered by applying Gestalt principles. From top to bottom, continuity, similarity (i.e., uniform thickness) and closure (i.e., balance between solid and void)
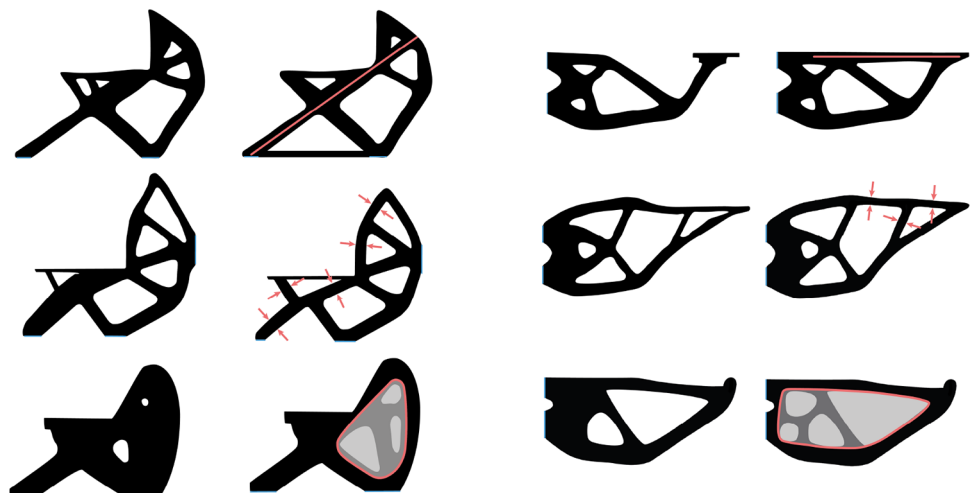
Fig. 6 Left: The six pairs of chairs in the user study*. Right: The comparative rating on aesthetics and comfort on a 7-point scale. The rating has a scale from −3 to 3. To the left side of the scale, i.e., −3, indicates a strong preference of the design on the left hand side of the pair – It can be an original or altered one, depending on their order in that pair. Similarly, to the right side of the scale, i.e., 3, indicates a strong preference of the design on the right hand side of the pair. A scale of 0 indicates a neutral opinion. * The illustrations of sitting poses are reproduced from Dimensions.com (https://www.dimensions.com/collection/people-sitting)
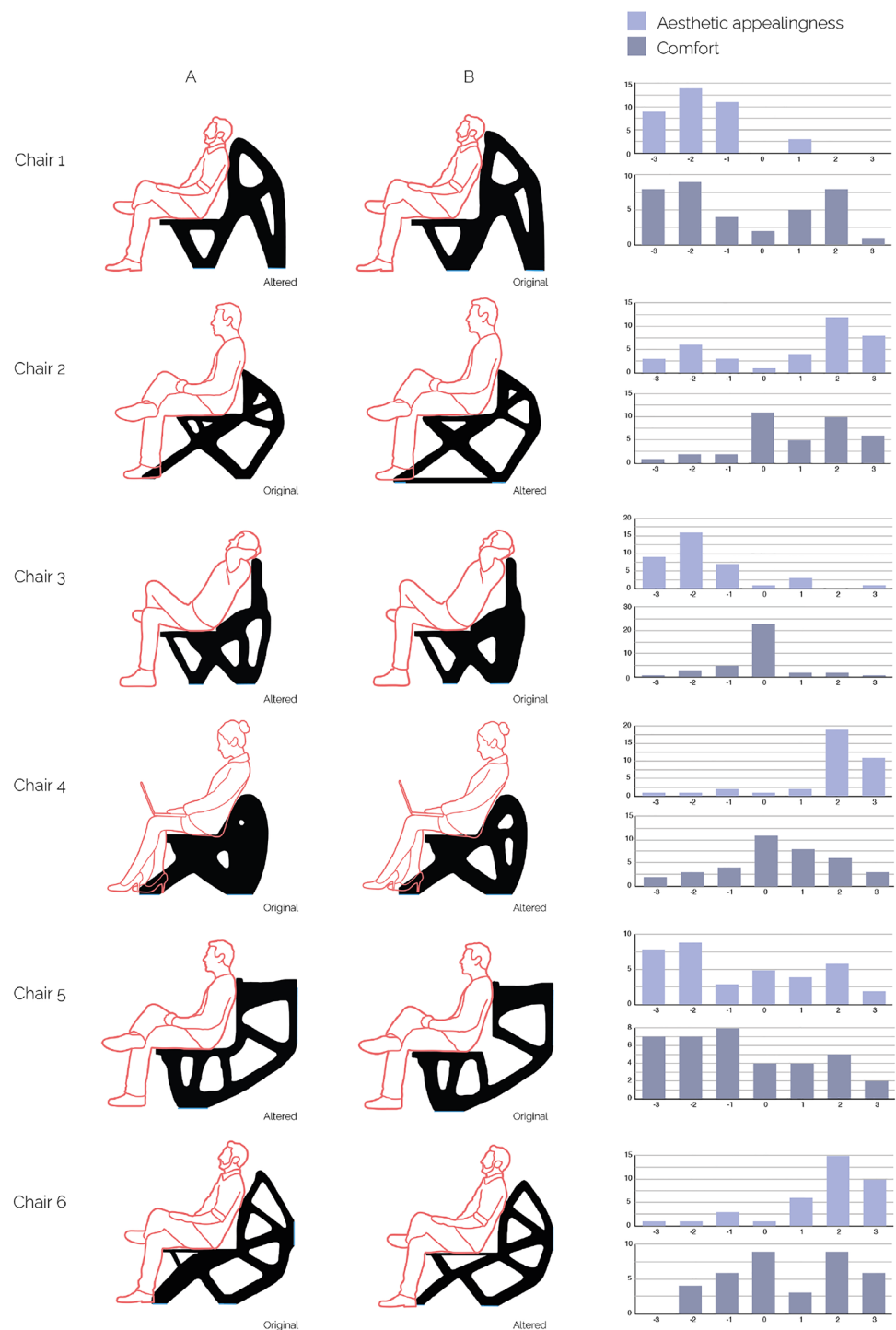

Q1 *"Which chair do you find the most beautiful?"*
Q2 *"Please explain your rating —-"*
Q3 *"Which chair do you find the most comfortable?"*
Q4 *"Which chair is the most unified?"*
Q5 *"Which chair is the most varied?"*

Q1 is the main question of interest. Q2 is included to understand the participants' rating, from their own perspective.

The purpose of Q3 is to remind participants to exclude comfort from their rating on aesthetics; A pilot study before this survey, with only Q1, showed that participants often took comfort into consideration whilst they rated aesthetics. With Q4 and Q5, we intend to gain insights on the relation between the aesthetic appreciation and degrees of unity and variety.

Fig. 7 Left: The six pairs of beams in the user study. Right: The comparative rating on aesthetics on a 7-point scale
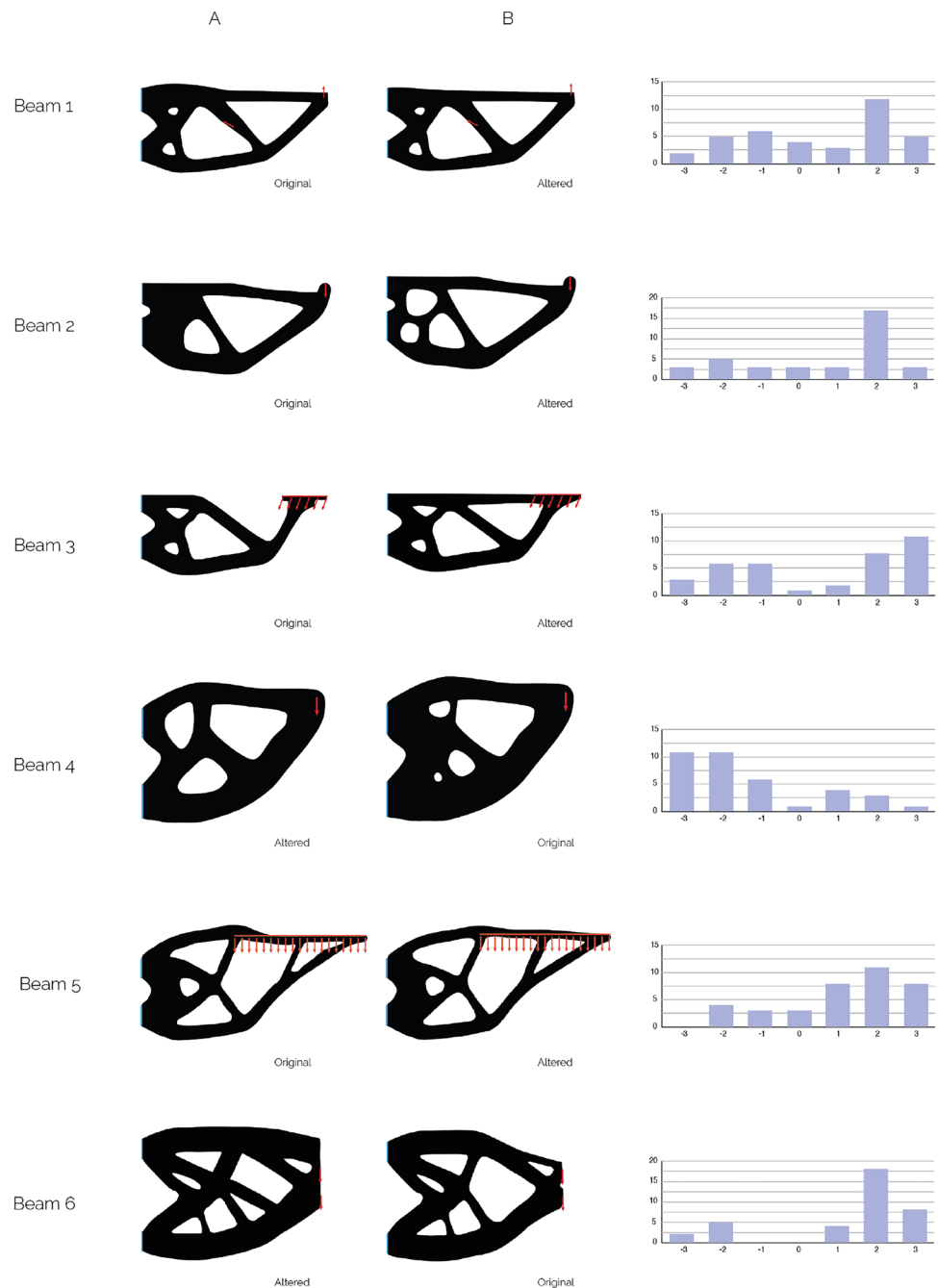
Q1 and Q3 use a 7-point scale, whilst Q4 and Q5 use a 5-point scale. This difference in scales is commonly used to remind participants of the change of questions. In a 7-point scale, 1 (resp. 7) indicates a strong preference towards the shape on the left (resp. right) hand side of the pair, whilst 4 indicates a neutral opinion. Likewise for a 5-point scale. Later, for a more intuitive interpretation of the results, the scale for aesthetic preference is transformed from 1 till 7 to -3 till 3. Likewise, for unity and variety the 5-point scale is transformed to -2 to 2.

The questions are split into 2 parts. The first part includes Q1 and Q2 (for chairs, also Q3), and the second part covers Q4 and Q5. The questions regarding unity and variety are asked after the rating on aesthetics, for all pairs, has been completed. This prevents an explicit evaluation of "unity" and "variety" from affecting the rating on aesthetics, especially by participants with prior knowledge of design principles. In each part, the chairs appear before the beams.

Fig. 8 Aesthetic preference results. In each pair of designs, an icon is placed on the left or right, indicating the side of the altered design. This follows the order in Fig. 6 and Fig. 7. In 11 out of the 12 pairs, i.e., except the last pair, the aesthetic preference is towards the altered design
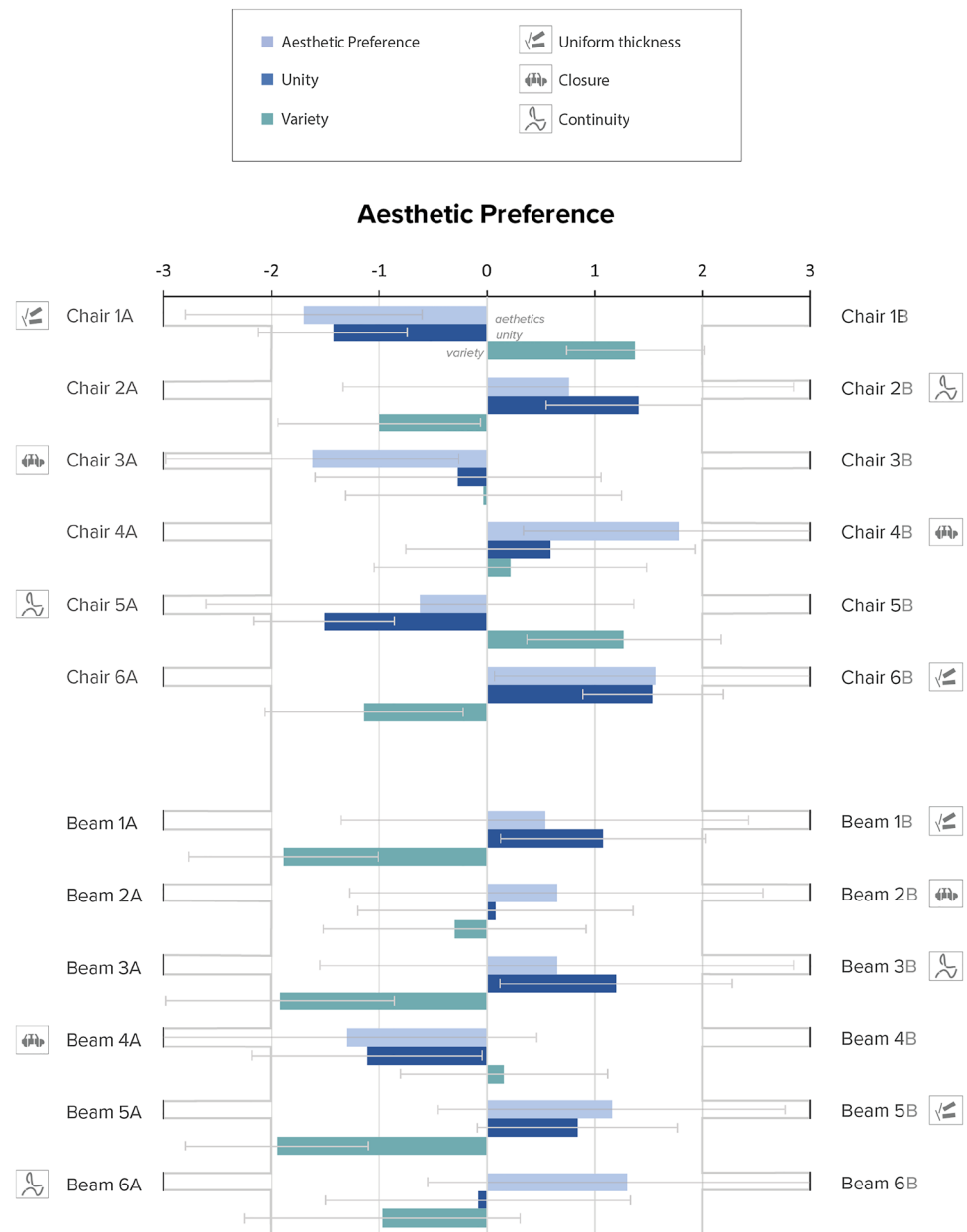
4 Results and discussion

The results of the user study are summarized in Fig. 8. All 12 pairs of shapes are included, in the order of their appearance in the survey. Each pair is compared in terms of aesthetic preference, unity and variety. An icon is placed next to the modified one in each pair, indicating which of the three Gestalt principles (i.e., uniform thickness, closure and continuity) was mainly used for modifying the original design from topology optimization. The altered design is perceived visually more pleasing in 11 out of the 12 pairs. Only in the last pair (Beam 6), the original design is preferred.

The aesthetic preference is concluded from the data visualized by the bar graph. For each pair, the first row in the bar graph shows the average rating of aesthetic preference. A negative value means a preference of the design on the left hand side over the one on the right, whilst a positive value means that the right one is preferred. Similarly, the second and third rows in the graph for each pair show which one is perceived more unified and more varied, respectively. In addition to the average, the standard deviation is also included in the bar graph. The rating of comfort on the chairs is not included in this graph, since it was introduced as a means to separate comfort from aesthetic appealingness. The rating of comfort can be found in Fig. 6.

4.1 Discussion

Amongst the twelve pairs, except the last one (Beam 6), the altered designs are perceived more appealing than the original topology optimized designs. In all these cases, the altered design is also seen as more unified than the original design. This positive correlation validates our hypothesis that increasing unity of topology optimized designs is expected to elevate the aesthetic appreciation. In nine out of the eleven pairs, the increase in unity is accompanied with a decrease in variety. The two exceptions are Chair 3 and 4, where both unity and variety have been increased in the altered designs. In these two pairs the magnitudes of unity and variety are smaller, indicating that the contrast in unity and variety are less pronounced in these two pairs than in the other nine. Nevertheless, these two exceptions show that unity and variety, whilst being opposites literally, can coexist in a design. This is in line with the principle of unity-in-variety, which states that a design that maximizes both unity and variety will be perceived more pleasing. However, the majority, i.e., nine pairs out of the eleven, suggests that the aesthetic preference is positively correlated with unity and negatively correlated with variety. This is not too surprising since the altered designs, whilst being less varied than the original designs, are more varied than the everyday products that people are used to.

For Beam 6, the original design on the right hand side is voted as the aesthetically preferred one. The altered design is perceived slightly more unified, but much more varied than the original design. This agrees with the observation from the other eleven pairs that there is an overall negative correlation between the aesthetic preference and degree of variety. The topology of this design is more complex than that in the others. This complexity may have played a role that makes the manually introduced unifying feature less visible or even distracting.

Standard deviation The graph shows a relatively large standard deviation. This can be partially explained by the nature of the survey questions – each question asks for comparing a pair of designs. In such a comparative rating scheme, the outcome is typically split into two clusters towards the two ends. This results in a large standard deviation. We note that large standard deviations are not uncommon in aesthetics research, e.g., (Berghman and Hekkert 2017). The large standard deviation reflects the subjective nature of aesthetic perception.

Chairs vs beams In four out of the six chairs (Chair 1, 3, 4, and 6), the magnitude of the aesthetic preference is about 1.5. The average of magnitudes in all six chairs is 1.3, larger than that in the beams (0.5). Perhaps this is due to the fact that beams are abstract items, whilst chairs are familiar items and thus tend to stimulate a stronger opinion from the participants.

For chairs, comfort is an important attribute. We intended to minimize the influence of comfort on the rating of aesthetics by additionally asking for a rating of comfort. The comments we received from 10 participants suggest that comfort was not fully excluded from the rating of aesthetics. They explained their rating for aesthetics by saying: "*I prefer chair B, because it looks more comfortable and fitting to the person*". In the survey, the question to judge the chairs on comfort was asked below the question on attractiveness, whilst on the same page. Thus participants might have already included the degree of comfort in their answer on visual attractiveness. In hindsight, it may be a good idea to ask comfort before aesthetic.

Participants More than half of the participants have a background in design related fields. These participants already know Gestalt principles and unity-in-variety. This was a concern that could potentially bias their responses. Upon analysis of the results, we observed variations in the magnitude of the average of aesthetic ratings from participants with and without a design background. However, the sign of the average of aesthetic ratings was the same between the two groups, for all 12 pairs, meaning that both groups had the same aesthetic preferences. This is an interesting finding reinforcing the premise that Gestalt principles are generally applicable. Another interesting finding is that the aesthetic ratings from the group with a design background show a larger standard deviation than the ratings from the other group, whilst the magnitude from the former group is not always higher. This is a bit unexpected, since the education background of this group is less diverse. A speculation is that designers are more used to expressing distinctive preferences, whilst engineers seem more conservative; We noticed frequent 3 and -3 in the ratings from design students, and also noticed for instance a participant with an engineering background gave neutral opinions to many questions.

It shall be restated that all participants are bachelor/master students or recent graduates. A portion of them know the concept of topology optimization, but none of them is considered to be an expert in topology optimization or structural mechanics. Thus, the rating might be different for experienced topology optimization researchers. Domain experts often evaluate aesthetics not only on the perceptual level, but instantly also on the cognitive level: Is this a structurally good design? In our current work, we chose to focus on the perceptual level, mimicking the behaviour of general consumers.

Interplay between different Gestalt principles In creating the altered designs we applied three Gestalt principles. A question we asked ourselves was, which of the three is most effective for elevating appealingness. This remains a difficult question. Each altered design was created with one of the three Gestalt principles in mind. However, a change

in one aspect often affects the other two. For instance, in Chair 3, an intended increase in closure is accompanied with less variation in thickness. Participants perceive the design as a whole, and it is not exactly clear which change is the dominant factor in stimulating their perception. Related to this, in Beam 6, material was added to make the shape more continuous. This results in more variations in the thickness in the beam. The higher variation might partly explain why the altered design is not the aesthetically preferred one.

Implication for topology optimization Modifying the shapes in a post-process as we did for the user study is expected to compromise structural performance. As a logical next step, the aesthetic principles shall be incorporated in the optimization loop. Amongst the three principles we have tested, the uniform thickness has been investigated (e.g., Guest 2009), often motivated by structural robustness and ease of production. Closure relates to the balance between the areas of solid and void regions, as well as the balance between the void sizes. The former can be possibly regularized by the local volume constraint (Wu et al. 2018), whilst the latter is related to the length scale of the void. Continuity, however, is less recognized in the topology optimization literature. It is more a global property, and we consider it an interesting avenue to explore in future research.

5 Conclusion

In this paper we have presented a user study that is conducted to investigate whether unity-in-variety is applicable to improve aesthetics in topology optimization. The results have validated our hypothesis that increasing unity is key for regulating the aesthetics of topology optimized designs. The findings provide a basis for improving aesthetic pleasure, either manually or ultimately by integrating them in the topology optimization formulation. It is expected that this eventually will contribute to a wider acceptance of topology optimization for consumer product design.

We consider this study as the start of a comprehensive understanding of the aesthetic implications of topology optimization. There are many interesting questions remain to be answered. For example, extending from 2D to 3D, the visibility of structures in 3D is expected to affect aesthetic appreciation. Furthermore, the rising use of multi-scale structures may evoke different aesthetic experience. Last but not the least, learning-based approaches may prove valuable for evaluating aesthetics.

Declarations

References

Aage N, Nobel-Jørgensen M, Andreasen CS, Sigmund O (2013) Interactive topology optimization on hand-held devices. Struct Multidisc Optim 47(1):1–6. https://doi.org/10.1007/s00158-012-0827-z

Beghini LL, Beghini A, Katz N, Baker WF, Paulino GH (2014) Connecting architecture and engineering through structural topology optimization. Eng Struct 59:716–726. https://doi.org/10.1016/j.engstruct.2013.10.032

Bendsøe MP, Rodrigues HC (1991) Integrated topology and boundary shape optimization of 2-d solids. Comput Method Appl Mech Eng 87(1):15–34. https://doi.org/10.1016/0045-7825(91)90144-U

Berghman M, Hekkert P (2017) Towards a unified model of aesthetic pleasure in design. New Ideas in Psychology 47:136–144. https://doi.org/10.1016/j.newideapsych.2017.03.004

Dapogny C, Faure A, Michailidis G, Allaire G, Couvelas A, Estevez R (2017) Geometric constraints for shape and topology optimization in architectural design. Comput Mech 59(6):933–965. https://doi.org/10.1007/s00466-017-1383-6

Guest JK (2009) Imposing maximum length scale in topology optimization. Struct Multidisc Optim 37(5):463–473. https://doi.org/10.1007/s00158-008-0250-7

Hekkert P (2006) Design aesthetics: principles of pleasure in design. Psychol Sci 48(2):157

Lugo JE, Schmiedeler JP, Batill SM, Carlson L (2016) Relationship between product aesthetic subject preference and quantified gestalt principles in automobile wheel rims. J Mechanical Design 138(5):051101. https://doi.org/10.1115/1.4032775

Mata MP, Ahmed-Kristensen S, Shea K (2019) Implementation of design rules for perception into a tool for three-dimensional shape generation using a shape grammar and a parametric model. J Mech Design 10(1115/1):4040169

Orsborn S, Cagan J, Boatwright P (2009) Quantifying aesthetic form preference in a utility function. J Mechanical Design 131(6):061001. https://doi.org/10.1115/1.3116260

Post R, Blijlevens J, Hekkert P (2016) To preserve unity while almost allowing for chaos Testing the aesthetic principle of

unity-in-variety in product design. Acta Psychologica 163:142–152. https://doi.org/10.1016/j.actpsy.2015.11.013

Post R, Nguyen T, Hekkert P (2017) Unity in variety in website aesthetics: a systematic inquiry. Int J Human-Comput Stud 103:48–62. https://doi.org/10.1016/j.ijhcs.2017.02.003

Rozvany GI (2011) On symmetry and non-uniqueness in exact topology optimization. Struct Multidisc Optim 43(3):297–317. https://doi.org/10.1007/s00158-010-0564-0

Stromberg LL, Beghini A, Baker WF, Paulino GH (2011) Application of layout and topology optimization using pattern gradation for the conceptual design of buildings. Struct Multidisc Optim 43(2):165–180. https://doi.org/10.1007/s00158-010-0563-1

Wagemans J, Elder JH, Kubovy M, Palmer SE, Peterson MA, Singh M, von der Heydt R (2012) A century of gestalt psychology in visual perception: I. perceptual grouping and figure-ground organization. Psychol Bulletin 138(6):1172. https://doi.org/10.1037/a0029333

Wu J, Aage N, Westermann R, Sigmund O (2018) Infill optimization for additive manufacturing - approaching bone-like porous structures. IEEE Transact Visual Comput Graph 24(2):1127–1140. https://doi.org/10.1109/TVCG.2017.2655523

Wu J, Qian X, Wang MY (2019) Advances in generative design. Computer-Aided Design 116:102733. https://doi.org/10.1016/j.cad.2019.102733