# On the use of FTLE to educe 3D coherent structures in the left ventricular flow

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## ABSTRACT

Although the use of Finite Time Lyapunov Exponents (FTLE) to infer Lagrangian Coherent Structures (LCS) is acknowledged in literature, few studies have been devoted to the analysis of LCS derived from experimental three-dimensional velocity fields, especially in case of pulsatile flows. In this work we investigate the application of FTLE for the identification and tracking of LCS developing in a left ventricle laboratory model, where 3D velocity fields are reconstructed from two dimensional image analysis measurements on two sets of orthogonal planes. Obtained FTLE fields show how diastolic LCS progressively stretches and moves following a diagonal propagation direction, and redirects the flow towards the aortic orifice prior to its opening, helping an efficient ejection during the systole.

## INTRODUCTION

The computation of Finite Time Lyapunov Exponents (FTLE, [1, 2]) allows the identification and tracking of Lagrangian Coherent Structures (LCS) of the flow. With respect to the traditional instantaneous analyses techniques (vorticity, $\lambda_2$-criterion, etc.) this methodology has the advantage to reveal the vortical structures boundaries and encode the fundamental Lagrangian description into each single field, conveying key information about the underlying mechanism of fluid transport [3]. This powerful technique was proved to be successful to identify LCS in different contexts, such as geophysical and biological flows, but it has been seldom applied in literature to three-dimensional experimental fields [4].

Recently, researchers have applied FTLE computation to magnetic resonance (MR) velocity fields for the characterization of vortical structures in the left ventricle [5, 6]. These studied are motivated from the awareness that the formation and evolution of the vortical structures inside the left ventricle (LV) during the diastolic inflow deeply affect the physiologic and pathophysiologic cardiovascular conditions. In fact, previous studies performed on experimental and numerical models, as well as clinical observation have demonstrated that, in physological conditions, ventricle filling is optimized for blood transport, and the flow structure formed by the jets entering through the mitral valve during the diastole appears to be favorable to ejection through the aortic valve during the systole (see [7] for a review). However, a deep comprehension of the complex LV flow dynamics, which is scientifically relevant and has important implications in the medical field, is yet to be achieved.

From this perspective, an experimental laboratory investigation on assessing which kind of information can be extracted from FTLE analysis, and more specifically on the characterization of the LCS is worthwhile, not only from a scientific point of view but also for the interpretation of the results of the MR clinical studies, which are inherently conditioned by the difficulty to obtain data in repeatable and controlled conditions during the time consuming scan acquisitions needed for this purpose.

Espa and coauthors [8] recently showed how important information can be inferred from the analysis of 2D FTLE obtained from 2D velocity fields measured on a LV laboratory model. However, due to the three-dimensionality of the flow and the asymmetrical propagation of the diastolic jet, the identification and detailed characterization of the LCS in the LV requires the computation of 3D FTLE fields.

In order to meet this target, we here analyze LCS obtained from the FTLE computation on 3D velocity fields, measured in a laboratory LV ventricle model, and reproducing the main flow dynamics features observed in the human heart in controlled and repeatable conditions.

## DEFINITIONS AND METHODS

Investigations were performed in a laboratory model shown in Figure 1. The left ventricle is simulated by means of a transparent sack made of silicone rubber, housed in a rectangular tank (A) with transparent walls, allowing for the optical access. The ventricle sack is mounted on a circular plate, 56 mm in diameter, having one inlet and one outlet (simulating aortic and mitral orifice, respectively) that are connected to a constant head reservoir and provided with two one-way valves.

The motion of a piston, driven by a linear motor (C), produces the ventricular volume change, $\Delta V(t)$. Its derivative, Q(t), represents the flow rate through the mitral orifice during the diastole, and through the aortic one during the systole. The dynamic flow similarity (i.e. matching the ratio of inertial to viscous effects) is assured by matching Reynolds (Re) and Womersley (Wo) numbers:

$$\mathrm{Re} = \frac{UD}{\upsilon} \tag{1}$$

$$Wo = \sqrt{\frac{D^2}{T\upsilon}} \tag{2}$$

between the real case and the model. Here, D is the maximum diameter of the ventricle, U the peak velocity through the mitral orifice, T the period of the cardiac cycle, ν the kinematic viscosity of the working fluid. The geometrical scale is 1:1. Working parameters used during the runs of the experiments are listed in Table 1, and have been chosen in order to obtain non-dimensional parameters within the physiological range and reproduce the observed left ventricular fluid dynamics observed in vivo.

Table 1. Experimental parameters.

| Stroke Volume [ml] | T [s] | U [m/s] | Re [-] | Wo [-] |
|---|---|---|---|---|
| 6 | 6 | 0.145 | 8322 | 22 |

While most of the previous studies performed using similar apparatus, provided two dimensional velocity measurements on one [9, 10, 11, 12] or two planes [8], in the present work three dimensional velocity fields were reconstructed from two dimensional image analysis measurements on two sets of orthogonal planes. Specifically, for each set, 50 cycles of the cardiac flow on 24 parallel planes spaced 2.5 mm have been recorded (Figure 2). During each acquisition, the measurement plane is illuminated by a 12 W, infrared laser, while the working fluid inside the ventricle is seeded with neutrally buoyant particles, about of 30 μm in diameter. A high-speed digital camera (250 Hz, 1280×1024 pixel) is triggered by the motor to capture the time evolution of the phenomenon at known instants of the cycle. Images are then analyzed using a Feature Tracking algorithm [10], and velocity vectors are subsequently interpolated over a regular three-dimensional grid. Hence, the mean cycle is obtained by phase averaging.

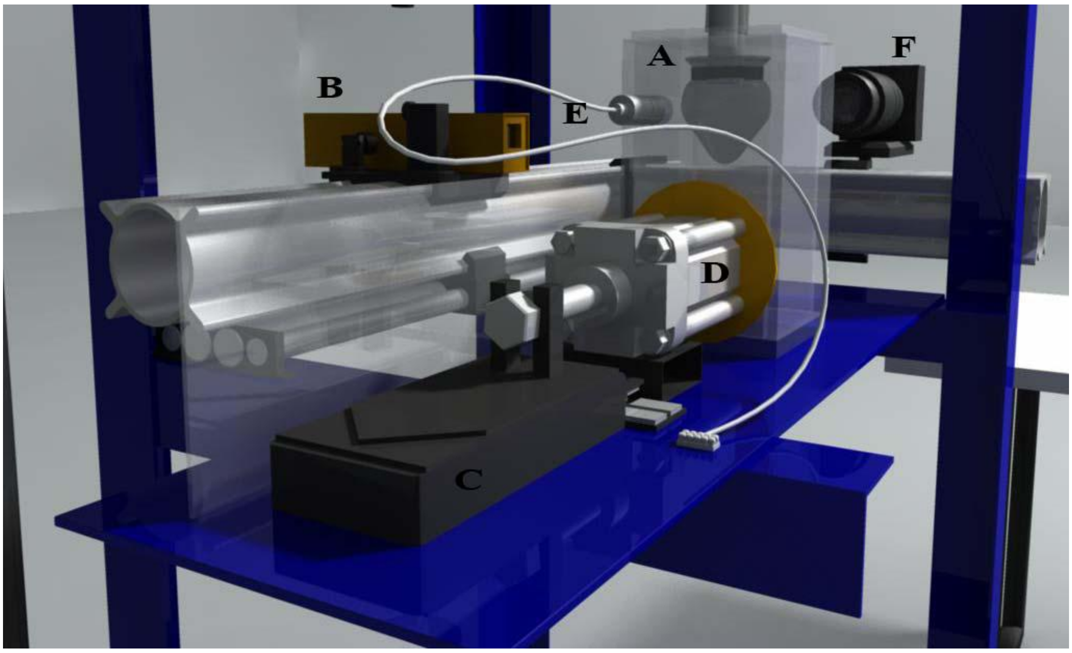


**Figure 1** Experimental set-up. A: Ventricle chamber; B: Laser; C: Motor; D: Piston; E: pressure transducer; F: fast camera.

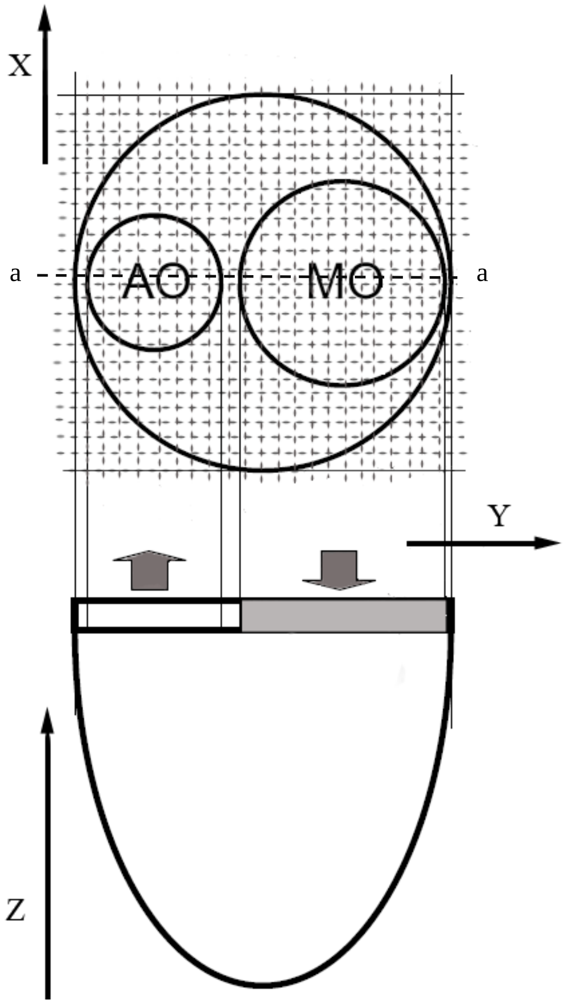


**Figure 2** Position of all the measurement planes (dashed lines). AO: aortic orifice. MO: mitral orifice.

## THEORETICAL REMARKS

Finite Time Lyapunov Exponents (FTLE) are a measure of the maximum linearized rate of growth of the distance among initially adjacent particles advected by the flow over a finite time interval.
FTLE computation is based on the definition of the flow:

$$\Phi_t^{t+T^*} : \boldsymbol{x}(t) \longrightarrow \boldsymbol{x}(t+T^*) \tag{3}$$

that maps a material point x(t) at time t to its position at time t + T* along its trajectory. After a linearization, the amount of stretching about a trajectory can be defined in terms of the matrix:

$$\Delta = \left( \frac{d\Phi_t^{t+T^*}(\boldsymbol{x})}{d\boldsymbol{x}} \right)^2 \tag{4}$$

where $\frac{d\Phi_t^{t+T^*}(\boldsymbol{x})}{d\boldsymbol{x}}$ is the Cauchy-Green deformation tensor and $\Delta$ represents its finite time representation. Since the maximum stretching occurs when the initial separation is aligned with the maximum eigenvalue of $\Delta$, the FTLE is defined as:

$$\sigma(\boldsymbol{x}, t, T^*) = \frac{1}{|T^*|} \ln \sqrt{\lambda_{\max}} \tag{5}$$

where $\lambda_{max}$ is the maximum eigenvalue of $\Delta$, and $\sqrt{\lambda_{max}}$ corresponds to the maximum stretching factor.

Trajectories can be integrated forward or backward in time: when a positive T* is considered, the FTLE measure separation forward in time, thus identifying repelling structures, on the contrary, if a negative T* is considered, FTLE measure separation backward in time, thus highlighting attracting structures.
As shown by Shadden and coauthors [2], even though the locally maximizing FTLE surfaces (or ridges) tend to become sharper with increasing integration time T*, the structure of the FTLE fields is quite persistent with varying T*. However, the choice of an appropriate value for T* has to be related to the characteristic flow time scales.
Even though Haller [13] has recently pointed out that this definition may sometime not be rigorous, LCS are generally identified as the locally maximizing surfaces, or ridges, of the FTLE field. Given a generic scalar function $f \subset \Re^N$, its ridges are characterized as a set of points where $f$ is maximized in p < N independent directions, thus forming a set of dimension $n = N-p$. According to the height ridge definition [14], ridges are computed by means of the eigenvectors of the Hessian matrix $H = \nabla^2 f$ associated with the $p$ smallest eigenvalues $\lambda_1<\lambda_2<...<\lambda_p$, provided that these eigenvalues are negative. Spurious structures can be then filtered out based on the value of $f$ and of the crease strength $|\lambda_p|$.
However, in order to minimize artifacts and holes, the extraction of ridge surfaces requires advanced visualization techniques, and LCS straightforward identification by means of a visual inspection and an appropriate thresholding of FTLE fields is generally used.

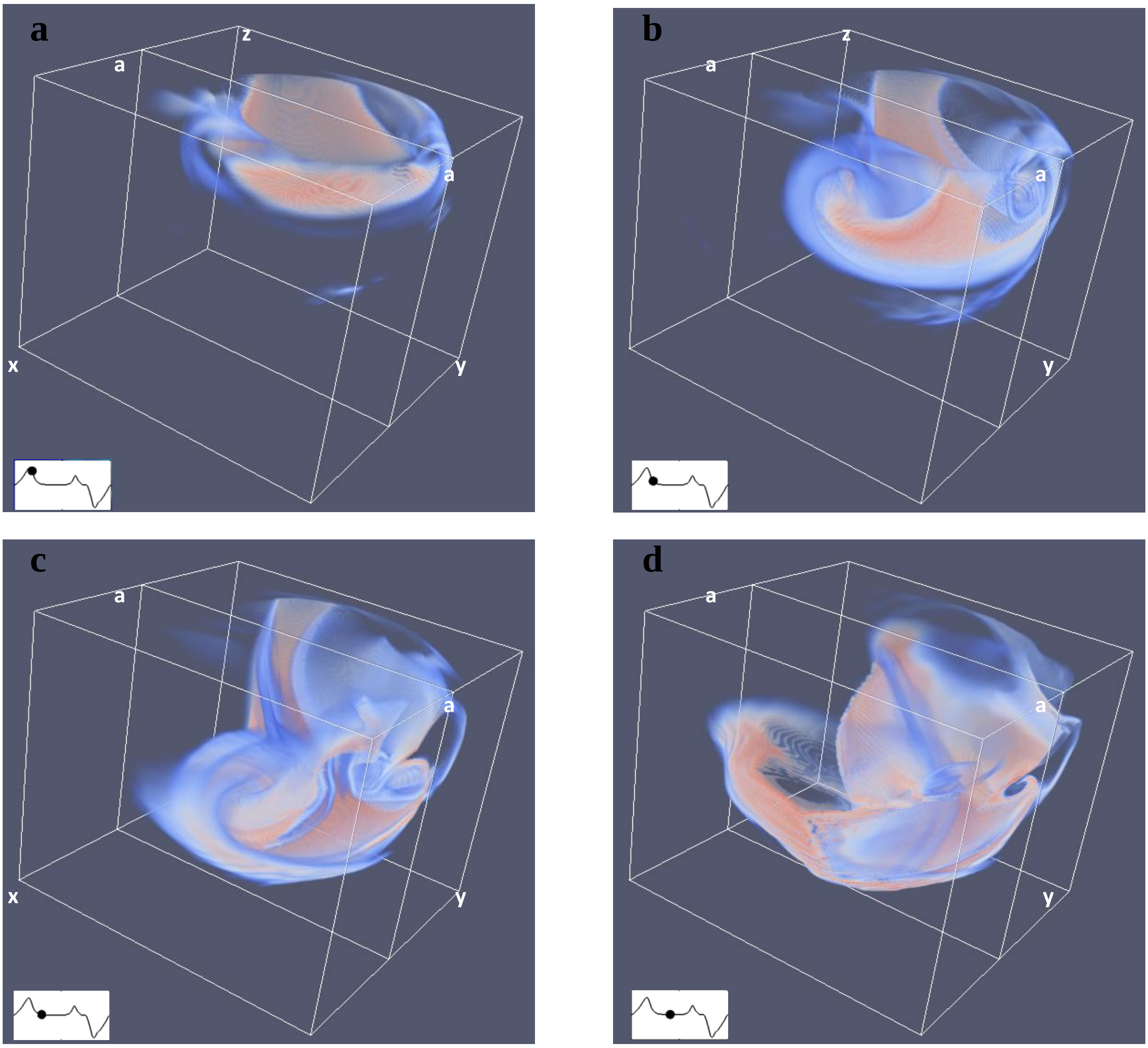


**Figure 3** Backward FTLE isosurfaces. Initial times for FTLE computation are (a) t/T = 0.18, (b) t/T = 0.23, (c) t/T = 0.29, (d) t/T = 0.4. The symmetry plane a-a (Figure1) is displayed.

## RESULTS

In the present work, FTLE computation was performed on the averaged experimental phase velocity fields by means of a public domain code, NEWMAN [15]. We adopted an integration time, T*, equal to twice the advection time scale, i.e. the ratio between the end-diastolic ventricle diameter and the peak mitral velocity (D/U ~ 0.40 s). FTLE were computed on a grid with finer space resolution than original velocity fields to better capture the FTLE ridges [4].
It has to be considered that, since the 2D velocity measurements could not be taken on planes almost tangent to the ventricle wall, 3D data were reconstructed on a grid which does not contain the whole ventricle. Hence, structures highlighted from 3D FTLE are partially cut next to the ventricle boundaries.
Figure 3 shows backward FTLE isosurfaces for some characteristic instants of the cardiac cycle. The small panel in the bottom-left corner of each plot shows the diagram of the flow rate as a function of the non-dimensional time (solid line), while a solid circle represents the initial time used for corresponding FTLE computation. We recall here that the first peak in the cardiac cycle, corresponding to the dilation of the ventricle, is called E-wave, while the second peak, called A-wave, is due to the contraction of the left atrium.
Figure 3a shows the upstream front of the vortex ring originated from the mitral orifice during the E-wave, and the leading edges of the jet still connected to the orifice. Although the structure is still in its first stage, its asymmetry with respect to the diametral a-a plane is already apparent and the surface is stretched along a diagonal direction. This situation is clearer in Figure 3b: here the surface has grown and is still connected to the mitral orifice, but is more elongated with respect to previous panel, and the rolling of the vortex at both sides of the surface display an anisotropic propagation. In fact, Figure 3c shows how the jet posterior lobe tends to be slower than the anterior one due to the interaction with the ventricle wall. After the impingement on the ventricle wall (Figure 3d), the remains of the anterior lobe fold onto the ventricle wall, creating a recirculation path that facilitates subsequent flow ejection [17], while there is still a channel connected to the mitral orifice driving the flow.
Figure 4 displays forward FTLE fields computed at the same instants of Figure 3. During E-wave (Figure 4a), one can identify its downstream front, corresponding to a mushroom-shaped surface, not yet fully entered into the LV. In Figure 4b, apart from the evolution of the structure and the rolling at the sides of the entering jet, there is another surface (partially depicted also in Figure 4a) that is directed towards the point of impingement of the jet on the ventricle wall. Figure 4c and 4d show how the downstream front of the LCS moves towards the aoric orifice well before the systole occurs, thus preparing the flow for optimal ejection.
Apart from the classical FTLE thresholding, we also applied the algorithm developed by Schultz and coauthors [16], which is a voxel based ridge surface extraction method that includes a set of topological principles to improve both correctness and performance. As an example, Figure 5 displays the superposition of the backward and forward LCS obtained by means of the Schult's algorithm and computed at the end of the E-wave. Displayed crease surfaces closely resemble those obtained by thresholding the corresponding FTLE fields. The feasibility to obtain LCS by means of an unsupervised extraction algorithm strengthen the possibility to use this methodology to characterize LV dynamics using rigorous criteria.
The joint analysis of backward and forward FTLE is a powerful tool for the localization of regions enclosing eddies [2], the superposition of attracting and repelling LCS clearly identify the elliptical region enclosing the diastolic vortex ring (Figure 5), and allows to following its deformation and evolution during the cardiac cycle (here not shown). Moreover, the study of the lobes formed by the intersection of attracting and repelling LCS can be used to identify fluid that will be entrained or detrained from the LCS.

## CONCLUSIONS

Proper formation and evolution of the Lagrangian Coherent Structures in the left ventricle has been acknowledged to play fundamental role for the overall functionality of the heart, being responsible for efficient mixing and blood recirculation. The Finite Time Lyapunov Exponent approach, which is a powerful technique to identify and characterize LCS in non-stationary flows, is becoming popular in clinical research studies. In-vivo results on the use of FTLE for LCS characterization are very promising, however cannot be performed in controlled and repeatable conditions.
In this perspective, the left ventricular flow was experimentally investigated in a laboratory model, where three dimensional velocity fields were reconstructed from two dimensional image analysis measurements on two sets of orthogonal planes. The analysis showed how FTLE fields help to elucidate the complex dynamics associated with the intraventricular flow, unveiling the asymmetric and anisotropic LCS. Future work will be devoted to determine, in both physiological and pathological conditions, characteristic and synthetic LCS parameters that might provide a term of reference for future clinical investigations.

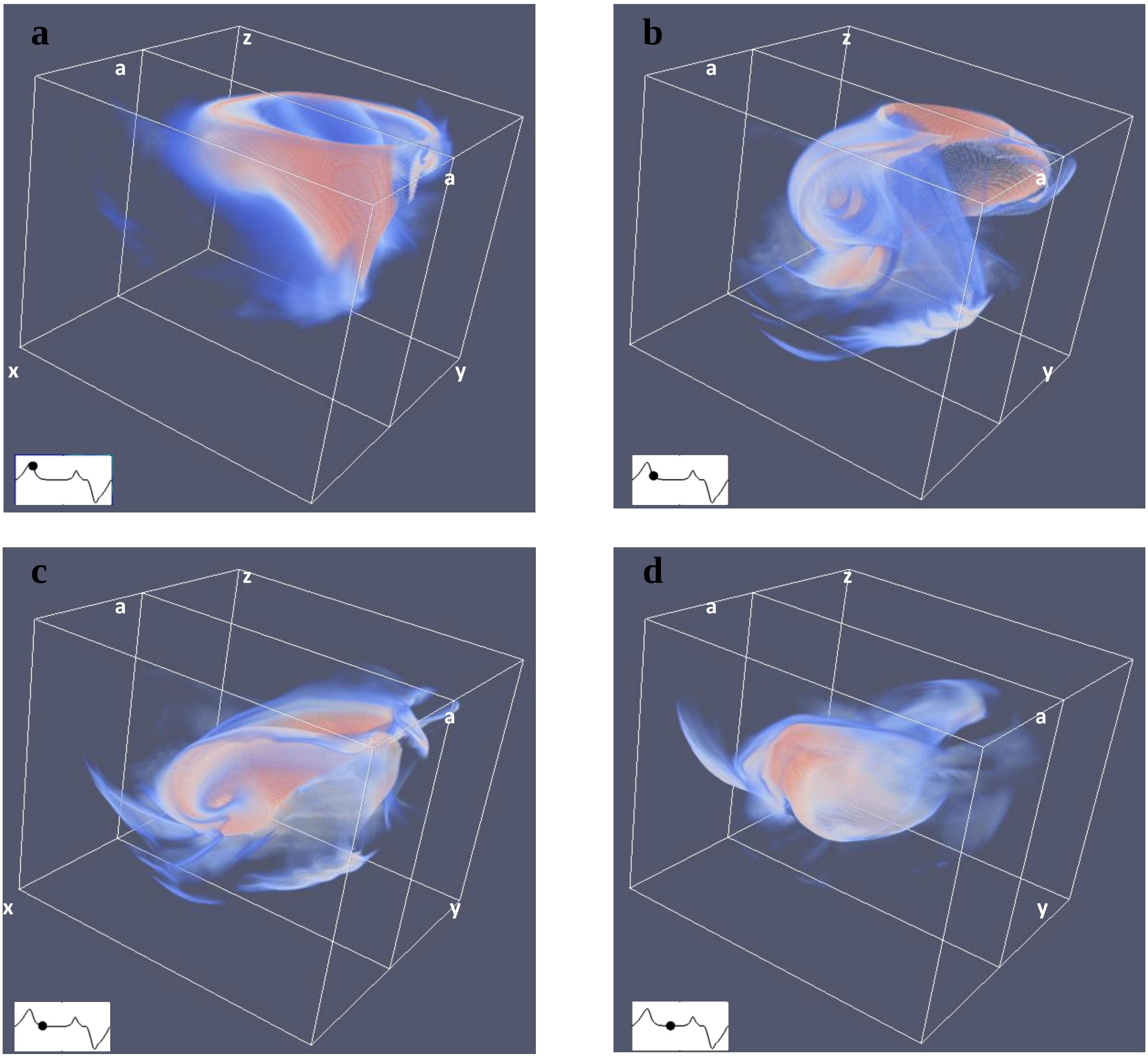


**Figure 4** Forward FTLE isosurfaces. Initial times for FTLE computation are (a) t/T = 0.18, (b) t/T = 0.23, (c) t/T = 0.29, (d) t/T = 0.4. The symmetry plane a-a (Figure1) is displayed.

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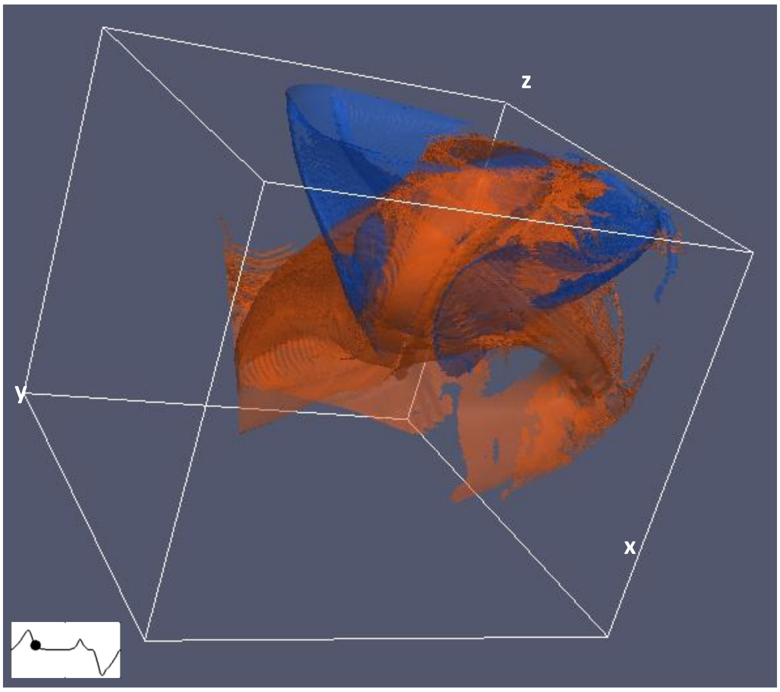


**Figure 5** Superposition of attracting and repelling LCS (respectively blue and orange surfaces) computed at t/T = 0.23.