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Data-driven Expressions for the Control of Network Systems with Asynchronous Experiments

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Abstract— **This paper proposes a direct data-driven approach to address decentralized control problems in network systems, i.e., systems formed by the interconnection of multiple subsystems, or agents. Differently from previous work, in this paper we assume that coordination among agents is limited in the data collection phase. Specifically, while we allow for multiple experiments to be performed on the network, these can be asynchronous (meaning that we do not require that all agents take part to each experiment). We focus this study on an open-loop optimal control problem, and propose a strategy to reconstruct the missing experimental data, i.e., data from the agents not participating to a given experiment. Importantly, our data-reconstruction strategy does not compromise the performance or numerical reliability of the approach, as we give conditions under which the missing data can be exactly reconstructed. We complement our findings with numerical simulations, showcasing the effectiveness of our approach in decentralized control scenarios.**

I. Introduction

Over the last few decades, the controls community has extensively focused on the study of network systems [1]. Modeling interactions among agents through networks has proven useful in various scientific and engineering domains, including robotics [2], [3], transportation [4], [5], social science [6], and energy markets [7]. However, the inherent heterogeneity and large-scale nature of these systems, coupled with the continuous evolution in complexity of human-made network systems, pose a significant challenge in accurately identifying a model for their underlying dynamics. In the end, employing inaccurate models can jeopardize the analysis and control design phases, leading to unreliable controllers.

At the same time, thanks to the recent improvements in sensor accuracy, storage efficiency and processing power, large amounts of data can be collected even for these complex systems. This abundance in data availability motivated the development of the numerous recent data-based techniques for controller design. Within this data-driven framework, a recent trend involves the so-called *direct* data-driven approach to controller design: at the core of this approach is to use data to directly design a desired control action, without explicitly identifying a model of the system [8]–[10]. Numerous interesting results in this direction have already been discussed in the literature. However, these often rely on limiting assumptions, e.g., they implicitly require coordination among agents in the data collection phase. Requiring constant coordination among all agents can be especially restrictive when dealing with network systems, since these cannot always rely on a centralized coordinating entity when collecting experimental data. In this work we propose a strategy to extend the data-driven control literature by collecting experiments where agents collect and store data locally and experiments are asynchronous.

Related work. Over the past few years, the field of direct data-driven control has evolved rapidly, and we refer the reader to the seminal papers [8], [9], [11] and to the recent review papers [10], [12] for a broader overview of the topic. Within this framework, the intersection between data-driven control and network systems has been studied, for example, in [13]–[16]. What all these works have in common, however, is the implicit assumption that each agent within the network takes part to all experiments concurrently. This assumption translates in the necessity of a central planner who is able to coordinate the data collection phase among all agents. To the best of our knowledge this is the first work that aims at lifting this assumption by allowing for asynchronous experiments.

Paper contribution. This paper introduces a novel framework for designing optimal controllers for network systems in the presence of data collected from asynchronous experiments. Unlike traditional scenarios, the assumption of data collection from asynchronous experiments challenges the classical notion of persistency of excitation in the data. To address this, we extend the concept of persistently exciting data to accommodate asynchronous experiments. We demonstrate that, under certain assumptions on the collected data, it is possible to precisely reconstruct data from idle agents not participating in a specific experiment. The combination of recorded and reconstructed datasets, as illustrated in this work, forms a comprehensive basis for solving the considered quadratically optimal control problem.

Paper organization. The paper is organized as follows. In Section II we formulate our problem and review existing data-driven results. Section III contains the main contributions of this work. In Section IV numerical results are shown.

Notation. We let $\mathbb{R}$ and $\mathbb{N}$ denote the set of real and natural numbers, respectively. Given a matrix $A \in \mathbb{R}^{n \times m}$, $\mathrm{Rank}(A)$, $\mathrm{Basis}(A)$, $\mathrm{Ker}(A)$, and $A^\top$ denote the rank, a basis of the column space, the kernel, and the transpose of A. We let A_{ij} refer to the element from the i-th row and j-th column of A.

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For any non-square matrix A, we denote its Moore-Penrose pseudoinverse as $A^\dagger$. For vectors $x_1 \in \mathbb{R}_1^n$, $x_2 \in \mathbb{R}_2^n$, we let $\mathrm{vec}(x_1, x_2) = \begin{bmatrix} x_1^\top & x_2^\top \end{bmatrix}^\top \in \mathbb{R}^{n_1+n_2}$.

II. Problem setup and preliminary results

Consider a linear time-invariant system

$$x_{t+1} = Ax_t + Bu_t, \tag{1}$$

where $x_t \in \mathbb{R}^n$ and $u_t \in \mathbb{R}^m$ are the state and input at time $t \in \mathbb{N}$, respectively. In this paper we discuss how to design optimal controllers when the system model in (1) is unknown but when data from the system is available. With respect to previous work on data-driven control, in this paper we relax some assumptions of the data collection phase and, in particular, we allow for experiments to be asynchronous and distributed among multiple agents in network. We leave the details of our assumptions on the asynchronous experiments to Section III. In the remainder of this section, instead, we detail the control problem under investigation, together with the setup for the distributed data experiments. In particular, we frame our analysis within the bounds of a simple but insightful problem, i.e., the minimum energy control problem. Of course, while the solution we propose in this paper is particular for the problem at hand, the framework is broad and can be extended to more complex and general problems. Consider the problem of computing the control input $\mathbf{u}_T^* = \mathrm{vec}(u_0^*, \dots, u_{T-1}^*)$ that, given a desired final state $\hat{x}_T$, solves

$$\begin{aligned} \underset{\mathbf{u}_T}{\text{minimize}} \quad & \|\mathbf{u}_T\|_2^2 \\ \text{subject to} \quad & x_{t+1} = Ax_t + Bu_t, \\ & x_0 = 0, \\ & x_T = \hat{x}_T. \end{aligned} \tag{2}$$

While the solution to (2) is well known in the traditional case of known A and B [17], in this paper we find a solution to (2) by relying only on a series of asynchronous experiments. First, we recall how (2) can be solved when data is both synchronous and available at a centralized location. Then, we focus on strategies to lift some of the assumptions on the data, i.e., by (i) assuming that data is distributed among multiple agents in the network and (ii) that the experiments are performed asynchronously.

Let $N \in \mathbb{N}$ be the cardinality of the available experiments, each of length $T \in \mathbb{N}$. In particular, for experiment $j \in \{1, \dots, N\}$, let $\mathbf{u}_T^j = \mathrm{vec}(u_0^j, \dots, u_{T-1}^j)$ be the input supplied to (1) during that experiment, and let x_T^j be the associated final state. In this paper we consider episodic experiments where the network state is reset to zero before running a new trial: $x_0^j = 0$, for $j \in \{1, \dots, N\}$. The data recorded throughout the experiments is collected in matrices

$$U = \begin{bmatrix} \mathbf{u}_T^1 & \dots & \mathbf{u}_T^N \end{bmatrix} \in \mathbb{R}^{mT \times N}, \tag{3a}$$

$$X = \begin{bmatrix} x_T^1 & \dots & x_T^N \end{bmatrix} \in \mathbb{R}^{n \times N}. \tag{3b}$$

Then, the following holds.

Lemma 2.1: ***(Data-driven solution to*** (2) ***[11])*** If the matrix U in (3a) is full row rank then, for any final state $\hat{x}_T$, the input minimizing (2) can be computed as

$$\mathbf{u}_T^* = (XU^\dagger)^\dagger \hat{x}_T. \tag{4}$$

□

The centralized data-driven controller (4) requires the full knowledge of both data matrices X and U, which is a limiting assumption for network systems. Instead, we shall assume that data is collected and stored locally by a set of M agents, each assigned to a subset of the states of (1). Further, we allow for agents to exchange information through a communication graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ in order to solve the control objective. Formally, (1) can be rewritten as

$$x_{i,t+1} = A_{ii}x_{i,t} + \sum_{\substack{j=1 \\ j\neq i}}^{M} A_{ij}x_{j,t} + \sum_{j=1}^{M} B_{ij}u_{j,t}, \tag{5}$$

where $x_{i,t} \in \mathbb{R}^{n_i}$ and $u_{i,t} \in \mathbb{R}^{m_i}$ are the state and input assigned to agent $i \in \{1, \dots, M\}$. For simplicity of notation we assume that $x_t = \mathrm{vec}(x_{1,t}, \dots, x_{M,t})$, which is always satisfied after a reordering of the elements of x_t (and a similar transformation over A and B). Clearly, $\sum_{i=1}^M n_i = n$ and $\sum_{i=1}^M m_i = m$. In this setting, each agent is able to collect data referred to its assigned states $x_{i,t}$ and inputs $u_{i,t}$ only, and has control authority over $u_{i,t}$. Then, the data matrices in (3) can be partitioned among the M agents as

$$U_i = \begin{bmatrix} u_{i,1}^1 & & u_{i,1}^N \\ \vdots & \dots & \vdots \\ u_{i,T-1}^1 & & u_{i,T-1}^N \end{bmatrix}, \tag{6a}$$

$$X_i = \begin{bmatrix} x_{i,T}^1 & \cdots & x_{i,T}^N \end{bmatrix}, \tag{6b}$$

and, in the following, we shall refer to the input sequence of agent i during experiment j as

$$\mathbf{u}_{i,T}^j = \mathrm{vec}(u_{i,1}^j, \dots, u_{i,T-1}^j).$$

III. Learning minimum energy controls with missing data and asynchronous experiments

A. Asynchronous experiments and data-missing matrices

The experimental setup described in Section II assumes that all agents collect experimental data in a synchronous fashion. Namely, every agent i is aware that an experiment is taking place, and consequently it records its input sequence $\mathbf{u}_{i,T}$ and final state $x_{i,T}$. This assumption is restrictive in distributed scenarios, as it requires an implicit coordination among agents. In this paper, we relax this assumption and allow agents to perform experiments asynchronously.

We design asynchronous experiments by allowing for a subset of all agents to take part to an experiment, while the other agents remain idle. Specifically, if agent i does not take part to the j-th experiment, then i does not apply an input sequence, and $\mathbf{u}_{i,T}^j = 0$. Although i is not taking part to experiment j, generally $x_i^j(t) \neq 0$ as a result of the activity of the other agents in the network. Therefore, when an agent

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i does not take part to an experiment j, its associated final state remains unknown, which we refer to as $x_{i,T} = *$.

Definition 1: ***(Data-missing matrix)*** We call $P \in \mathbb{R}^{r\times c}$ a *data-missing matrix* if it contains at least an unknown element, i.e., there exist i and j such that $P_{ij} = *$. $\square$

In the asynchronous scenario considered in this paper, the data matrix X (as well as the local data matrices X_i) are, in general, data-missing matrices. Hence, known results, such as Lemma 2.1, cannot be employed to compute minimum energy controls. In the upcoming sections we discuss a strategy to compute minimum energy controls when X is a data-missing matrix. We remark that these results are general and are not restricted to the asynchronous experiments scenario. For instance, other applications of these results include scenarios in which elements of X are missing, e.g., as a result of packet losses or sensor malfunctions, see also [18].

B. Centralized minimum energy with data-missing matrices

For each agent $i \in \{1, \ldots, M\}$ in (1), let $S_i \subseteq \{1, \ldots, N\}$ be the set of experiments to which i takes part to, and let $\bar{S}_i$ be its complement (the set of experiments in which i is idle). For the matrices U and X in (3) and a set $S \subseteq \{1, \ldots, N\}$, we let $U\{S\}$ (resp. $X\{S\}$) be the matrix formed by the columns of U (resp. X) corresponding to the experiments indexed by the elements of S. Finally, let Q denote the set of experiments to which all M agents take part to, i.e.,

$$Q = \{j \ : \ j \in S_i \ \text{ for all } i \in \{1, \ldots, M\}\}. \qquad (7)$$

Notice that $(U\{Q\}, X\{Q\})$ is the dataset comprised of the data collected during the experiments in which all agents participated to (i.e., the synchronous experiments). It is easy to show that if $U\{Q\}$ is full-row rank, then the minimum energy control can be found through Lemma 2.1 as

$$\mathbf{u}_T^* = \left(X\{Q\}U\{Q\}^\dagger\right)^\dagger \hat{x}_T. \qquad (8)$$

The above is the trivial case in which asynchronous experiments can be discarded and the remaining experiments are sufficiently informative to compute the optimal solution. In the following, instead, we will focus on the case in which $U\{Q\}$ is not full-row rank and Lemma 2.1 cannot be used.

We begin by proving a preliminary result, which will be used to prove the main theorem of this work.

Lemma 3.1: ***(Rank relationship between partitions of*** U ***and*** $\mathrm{Ker}(U)$*)* For $N \geq mT$, let $U = \begin{bmatrix} U_1 & U_2 \end{bmatrix} \in \mathbb{R}^{mT\times N}$ be full-row rank, and let $K = \begin{bmatrix} K_1 \\ K_2 \end{bmatrix} = \mathrm{Basis}\left(\mathrm{Ker}(U)\right)$. Then, U_1 is full-row rank if and only if K_2 is full-row rank. $\square$

Proof:

(i) [U_1 full-row rank $\Rightarrow$ K_2 full-row rank]: Suppose by contradiction that K_2 is not full-row rank. This implies that there exists a vector $v \neq 0$ such that $v \notin \mathrm{Im}(K_2)$. However, since by hypothesis U_1 is full-row rank then there exists a vector w that solves $U_1 w + U_2 v = 0$. Define

$$z := \begin{bmatrix} w \\ v \end{bmatrix}$$

and note that $z \notin \mathrm{Im}(K)$. (Otherwise, there would exist a vector α such that $z = K\alpha$ and this would imply that $v = K_2\alpha$ contradicting the fact that $v \notin \mathrm{Im}(K_2)$). Thus, z jointly satisfies $Uz = 0$ and $z \notin \mathrm{Im}(K)$. This implies that K is not a basis of the null space of U, a contradiction.

(ii) [K_2 full-row rank $\Rightarrow$ U_1 full-row rank]: Suppose by contradiction that U_1 is not full-row rank. This implies that there exists a vector $v \neq 0$ such that $v^\top U_1 = 0$. Recalling that $UK = 0$, we obtain

$$0 = v^\top UK = v^\top \begin{bmatrix} U_1 & U_2 \end{bmatrix} \begin{bmatrix} K_1 \\ K_2 \end{bmatrix} = v^\top U_2 K_2.$$

This implies that $v^\top U_2 = 0$ since K_2 is full-row rank by hypothesis. This, along with $v^\top U_1 = 0$ implies that $v^\top U = 0$, which contradicts the assumption that U is full-row rank. ■

Through the above lemma we have shown that, for a partitioned matrix, the rank of a partition is closely related to the rank of the null-space of the complementary partition. This conclusion is instrumental to derive the following theorem.

Theorem 3.2: ***(Reconstructing missing elements of*** X*)* Let the matrices U and X be as in (3), with U full-row rank and X data-missing (see Definition 1). For each agent i, with $i \in \{1, \ldots, M\}$, let X_i be as in (6b) and let S_i be the set of experiments to which i takes part to. Further, let

$$\bar{X}_i = X_i\{\bar{S}_i\}, \qquad (9)$$

and

$$K_i = (K^\top\{\bar{S}_i\})^\top, \qquad (10)$$

where $K = \mathrm{Basis}(\mathrm{Ker}(U)$. Then, if $U\{S_i\}$ is full-row rank, the missing dataset $\bar{X}_i$ can be reconstructed as

$$\bar{X}_i = b_i K_i^\dagger, \qquad (11)$$

where

$$b_i = \begin{bmatrix} b_i^1 & \ldots & b_i^{N-mT} \end{bmatrix}, \qquad (12)$$

and $b_i^j = -\sum_{q\in S_i} x_{i,T}^q K_{qj}, \forall j \in \{1, \ldots, N - mT\}$. $\square$

Proof: The dynamics of the network can be described introducing the controllability matrix $C_T = \begin{bmatrix} B & AB & \cdots & A^{T-1}B \end{bmatrix}$ as follows

$$x_T = C_T \mathbf{u}_T, \qquad (13)$$

and equivalently

$$X = C_T U. \qquad (14)$$

From (14) notice that $\alpha \in \mathrm{Im}(K) \implies \alpha \in \mathrm{Im}(K_X))$ for $\alpha \in \mathbb{R}^N$, where K_X denotes a matrix whose columns form a basis of the kernel of X and $\mathrm{Im}(\cdot)$ represents the image of a given matrix. Then it holds that $XK = 0$ and, consequently,

$$X_i K = 0, \qquad (15)$$

which can simply be rewritten as

$$\sum_{q=1}^{N} x_{i,T}^q K_{qj} = 0, \forall j \in \{1, \ldots, N - mT\}. \qquad (16)$$

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Let S_i and $\bar{S}_i$ be the sets of indices defined as in the previous section, and notice that $\bar{S}_i$ contains the indices of the unknown elements of X_i we aim to reconstruct. Then, we can split (16) into

$$\sum_{q\in S_i} x^q_{i,T} K_{qj} + \sum_{q\in \bar{S}_i} x^q_{i,T} K_{qj} = 0, \forall j \in \{1,\dots,N-mT\}. \quad (17)$$

By introducing $\bar{x}_{i,T}, K_i$ and b_i as defined in (9), (10), (12) respectively, (17) can be reformulated as

$$\bar{X}_i K_i = b_i. \quad (18)$$

The equation (18) demonstrates that the reconstruction of each row i of the data-matrix X consists in solving a simple linear system. Therefore, if K_i is full row rank, then the closed-form solution to (18) is

$$\bar{X}_i = b_i K_i^{\dagger}. \quad (19)$$

From Lemma 3.1 we notice that K_i is full row rank if and only if $U\{S_i\}$ is full row rank, which is true by hypothesis, and this ends the proof of the theorem. Notice that, through this latter step, results of the theorem are reformulated in terms of experimental conditions, in line with the overall data-driven approach existing in literature. ■

Remark 1: ***(Asynchronous experimental setup)*** We remark that in our approach we overcome the requirement of a centralized planner coordinating the experiments execution by allowing autonomous agents to perform experiments asynchronously and collect experimental measurements locally. However, the reconstruction and controller design phases still call for some degree of coordination, as they rely on global information K_i, which is built from the data matrix U (sharing of data matrix X, instead, is not required). This is a limitation of our approach since a central planner is required to compute the control laws (see, e.g., [13], [19] for fully decentralized approaches with synchronous experiments.) □

Remark 2: ***(Redundancy in the experiments)*** In order to reconstruct the data-missing matrix X some redundancy in the experiments is needed. If $U\{S_i\}$ is full row rank, then necessarily $N - |\bar{S}_i| \geq mT$, which implies $N \geq mT$. □

Remark 3: ***(Pairwise coordination among agent)*** A consequence of Theorem 3.2 is that any agent i must take part to at least mT experiments, to ensure that $U\{S_i\}$ is full-row rank. Of these, some must involve two or more agents in order for the dataset to be suitable for our control goal. In fact, if agent i takes part to $|S_i|$ experiments without overlapping with any other agent j, that is $S_i \cap S_j = \emptyset, \forall j \neq i$, then $\mathrm{Rank}(U\{S_i\})$ is at most $m_i T < mT$. However, under specific assumptions, pairwise coordination among agents can be sufficient to satisfy the requirements of Theorem 3.2. For instance, if each agent i takes part to $m_i T$ linearly independent experiments by itself, plus at least a set S_{ij} of experiments together with another agent j, repeated for all agents $j \neq i$, and such that $|S_{ij}| = m_j T$, and $\mathbf{u}^k_{j,T}, \forall k \in S_{ij}$, are linearly independent, then $U\{S_i\}$ is full row rank. The sufficiency of this condition can be proved by noticing that

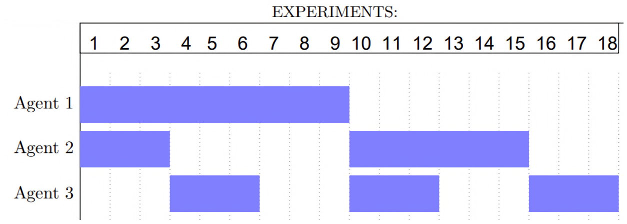

Fig. 1: Experiment schedule discussed in Remark 3 ($M = 3$, $T = 3$, fully connected network, $m_i = 1$, for each i). Each agent performs $mT = 9$ experiments. Specifically, it takes part to $m_i T = 3$ experiments by itself, plus to $m_j T = 3$ experiments pairwise overlapping with agent $j, \forall j \neq i$.

matrix U, after a reordering of its rows, can be reconducted to an upper triangular blocks matrix, with non singular blocks on the diagonal. This is illustrated in Fig. 1. □

In this section, we presented a strategy to compute a minimum energy control when the dataset X is distributed among multiple agents in a network and where some of its elements are missing due to the asynchronous nature of the experiments. The proposed strategy involves a preliminary centralized step to share data matrix U through which missing experimental data is reconstructed (cf. Remark 1). By leveraging this new information, the data-driven minimum energy controller can then be computed. Crucially, the data is collected without a central authority coordinating the data-acquisition phase. This means that, in our framework, experimental data can be collected by every agent in the network, at their best convenience. Sharing U can then be achieved via distributed approaches, e.g., see [20]. Moreover, the strategy that we propose to reconstruct data-matrix X is not restrictive, as it can be applied to the data-driven computation of any control law relying on data-matrices X and U. In the next section, numerical results are shown.

IV. Numerical results

In this section we verify the results of the previous section by means of numerical examples. First, consider a low-dimension LTI system with $M = 2$ agents, with $n = 2$, $m = 2$, i.e., each agent's input and output are scalars. Suppose that $N = 5$ asynchronous experiments of length $T = 2$ are performed and that we have access to the following, partially unknown, data-matrices:

$$U = \begin{bmatrix} 0.5377 & 0.3188 & 3.5784 & 0 & -0.1241 \\ 1.8339 & -1.3077 & 2.7694 & -0.0631 & 0 \\ -2.2588 & -0.4336 & -1.3499 & 0 & 1.4090 \\ 0.8622 & 0.3426 & 3.0349 & -0.2050 & 0 \end{bmatrix} \quad (20a)$$

$$X = \begin{bmatrix} -0.8590 & 0.2278 & 5.2634 & * & 1.2849 \\ 0.4372 & -1.3987 & 4.4545 & -0.2680 & * \end{bmatrix}. \quad (20b)$$

Agent $i = 1$ does not take part to experiment $j = 4$, thus $S_1 = \{1,2,3,5\}$ and $\bar{S}_1 = \{4\}$ (i.e., $\mathbf{u}^4_{1,T} = 0$ and $x^4_{1,T} = *$). Agent $i = 2$ does not take part to $j = 5$, as $\mathbf{u}^5_{2,T} = 0$ and $x^5_{2,T} = *$, hence $S_2 = \{1,2,3,4\}$ and $\bar{S}_2 = \{5\}$. Both

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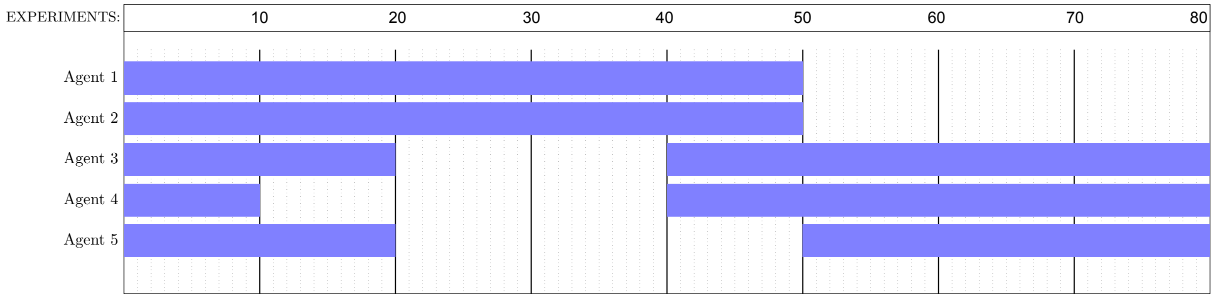

Fig. 2: Experiment schedule of each agent. N=80 asynchronous experiments are performed. All the agents take part only to experiments $j \in \{1, \ldots, 10\}$. However, each agent takes part to a sufficient number of experiments.

the agents take part to experiments $j \in Q = \{1, 2, 3\}$. Since $U\{Q\}$ is not full-row rank, this example does not reduce to the trivial case in which $(U\{Q\}, X\{Q\})$ can be directly used to compute the optimal controller through (4). Thus, columns of data-matrices containing unknown elements can not be simply discarded and a reconstruction step is needed. As required by Theorem 3.2, we firstly compute the matrix K, from which K_1 and K_2 can be easily extracted by selecting rows of K indexed by elements contained in $\bar{S}_1$ and $\bar{S}_2$ respectively. Specifically:

$$K = \begin{bmatrix} 0.2596 \\ 0.2308 \\ -0.0441 \\ 0.8243 \\ 0.4449 \end{bmatrix} \tag{21a}$$

$$K_1 = K\left\{\bar{S}_1\right\} = \begin{bmatrix}0.8243\end{bmatrix} \tag{21b}$$

$$K_2 = K\left\{\bar{S}_2\right\} = \begin{bmatrix}0.4449\end{bmatrix}. \tag{21c}$$

By relying on the sub-matrix $U\{S_1\}$, the local state of agent $i = 1$ during the 4th experiment, i.e, $x^4_{1,T}$, can be reconstructed by applying (11). Specifically, it holds that

$$x^4_{1,T} = b_1 K_1^{\dagger} = -0.2052, \tag{22}$$

where $b_1 = -0.1691$ is computed as in (12). By relying on $U\{S_2\}$, instead, $x^5_{2,T}$ is reconstructed as follows

$$b_2 = 0.6267, \tag{23a}$$

$$x^5_{2,T} = 1.4086. \tag{23b}$$

The reconstructed values (22) and (23b) can be used to make the data-matrix X fully known. The aim of this example is to better understand the key concept of Theorem (3.2), which lies in the fact that unknown elements of X_i can be reconstructed if and only if agent i took part to an informative enough subset of experiments $\{S_i\}$.

We now discuss a second example to show how the overall performance of our approach improves as the number of reconstructed elements of X grows. The idea is that by increasing the number of fully-known columns of X, and in turn increasing the cardinality of Q, the minimum energy control inputs can be computed basing on a larger dataset. This leads to an improvement of numerical reliability and accuracy of our approach. Specifically, we want to identify a data-driven open-loop minimum energy control problem over a finite time horizon $T = 10$, given a randomly chosen final state $\hat{x}_T$, by relying on a data-missing dataset. We consider a network composed of 5 agents and generated by the Erdös–Rényi model, with edge probability $p = \log n/n + \epsilon, \epsilon = 0.5$. The network's state and input dimensions are $n = 20$ and $m = 10$ respectively, therefore each agent i has control authority over an input of dimension $m_i = 1$ and measures its assigned local state having dimension $n_i = 4$. We assume that $N = 80$ asynchronous experiments are performed, according to the schedule shown in Fig. 2. In particular, all the agents take part only to experiments $j \in \{1, \ldots, 10\}$. Therefore matrix X is data-missing, as the columns indexed by $j \in Q = \{1, \ldots, 10\}$ only are fully known, and a reconstruction step is needed in order to compute the data-driven control input. We iteratively compute the data-driven control input (4) relying on $(U\{Q(k)\}, X\{Q(k)\})$, where $Q\{k\}$ is a set whose elements index columns of X which are fully-known at step k. This means that we start by considering the synchronous dataset $(U\{Q\}, X\{Q\})$ and then we gradually consider further experiments as the corresponding columns of X are reconstructed. The first aspect we aim at investigating regards the minimum number of columns needed to be reconstructed in order to properly apply (4). Fig. 3(a) shows that, if at least 40 columns of X have been reconstructed, then $U\{Q(k)\}$ is full row rank and the available dataset $(U\{Q(k)\}, X\{Q(k)\})$ is informative enough to compute the data-driven control input. In particular, the norm of the data-driven control inputs [11] reaches its minimum value when at least 40 experiments have been reconstructed, that is when the data-matrix $U\{Q(k)\}$ is full row rank (see Fig. 3(b)). Moreover, if a sufficiently large number of experiments has been reconstructed, by applying to the network the data-driven control strategy, the reached final state is almost as close to $\hat{x}_T$ as the one computed via the model-based approach (see Fig. 3(c)). This example shows that the reconstruction step does not affect the data-driven performances presented in [11].

V. Conclusions and Future Work

In this study, we investigate a direct data-driven approach to tackle decentralized control problems in networked systems. We focus on open-loop optimal control scenarios, assuming that agents conduct experiments asynchronously

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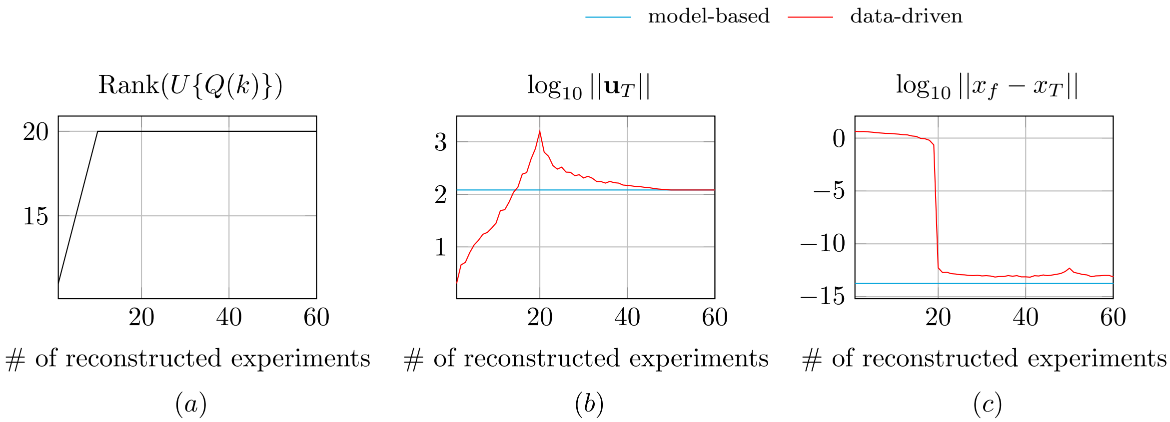

Fig. 3: This figure shows how data from asynchronous experiments can be used to build an informative dataset for the design of optimal controls. The data-driven control input (4) is iteratively computed relying on $(U\{Q(k)\}, X\{Q(k)\})$, where in $Q\{k\}$ are the columns of X which are fully-known at step k. If at least 40 columns of X have been reconstructed, then (i) $U\{Q(k)\}$ is full row rank and the available dataset $(U\{Q(k)\}, X\{Q(k)\})$ is informative enough to compute the data-driven control input (see panel (a)), and (ii) the norm of the data-driven control input, computed as in (4), reaches its minimum value, and the final state is almost as close to $\hat{x_T}$ as the one computed via the model-based approach, as shown in panel (b) and (c). If the number of reconstructed columns is not sufficiently large, the data-driven technique performs worse than the optimal case, both in terms of control input energy (see panel (b)) and of error in the final state (see panel (c)). Specifically, when less than 10 experiments have been reconstructed, the reached final state is significantly far from the target value $\hat{x}_T$, allowing the norm of the data-driven control input to be smaller than the model-based one.

and store data locally. This leads to potentially incomplete datasets, requiring a reconstruction step to fill in missing information before computing a data-driven minimum energy controller. Notably, when some conditions on the persistency of excitation of the asynchronous dataset are met, the reconstruction phase we propose involves solving a system of linear equations, leading to an exact reconstruction of the missing data. While our approach aligns with the decentralized nature of network systems in the experiment collection phase, it still relies on some coordination among agents in the data reconstruction and controller design phases. Further limitations include restrictions on the experiment's initial condition, as well as the employment of noisy data and the extension to nonlinear or time-varying systems, which we leave as key topics for future research.

References

[1] F. Bullo, *Lectures on Network Systems*. Kindle Direct Publishing, 1.4 ed., 2020. With contributions by J. Cortes, F. Dörfler, and S. Martinez.

[2] V. Kumar, N. Leonard, and A. S. Morse, *Cooperative Control: A Post-Workshop Volume, 2003 Block Island Workshop on Cooperative Control*, vol. 309. Springer Science & Business Media, 2004.

[3] J. Shamma, *Cooperative control of distributed multi-agent systems*. John Wiley & Sons, 2008.

[4] S. Coogan, E. Kim, G. Gomes, M. Arcak, and P. Varaiya, "Offset optimization in signalized traffic networks via semidefinite relaxation," *Transp. Research Pt. B: Methodological*, vol. 100, pp. 82–92, 2017.

[5] G. Como, E. Lovisari, and K. Savla, "Convexity and robustness of dynamic traffic assignment and freeway network control," *Transp. Research Pt. B: Methodological*, vol. 91, pp. 446–465, 2016.

[6] M. O. Jackson, *Social and Economic Networks*. Princeton University Press, 2008.

[7] F. Dörfler, M. Chertkov, and F. Bullo, "Synchronization in complex oscillator networks and smart grids," *Proc. Natl. Acad. Sci. U.S.A.*, vol. 110, no. 6, pp. 2005–2010, 2013.

[8] J. Coulson, J. Lygeros, and F. Dörfler, "Data-enabled predictive control: In the shallows of the DeePC," in *European Control Conference*, (Naples, Italy), pp. 307–312, 2019.

[9] C. De Persis and P. Tesi, "Formulas for data-driven control: Stabilization, optimality and robustness," *IEEE Transactions on Automatic Control*, vol. 65, no. 3, pp. 909–924, 2020.

[10] F. Celi, G. Baggio, and F. Pasqualetti, "Closed-form and robust formulas for data-driven LQ control," *Annual Reviews in Control*, vol. 56, 2023.

[11] G. Baggio, V. Katewa, and F. Pasqualetti, "Data-driven minimum-energy controls for linear systems," *IEEE Control Systems Letters*, vol. 3, no. 3, pp. 589–594, 2019.

[12] I. Markovsky and F. Dörfler, "Behavioral systems theory in data-driven analysis, signal processing, and control," *Annual Reviews in Control*, vol. 52, pp. 42–64, 2021.

[13] A. Allibhoy and J. Cortés, "Data-based receding horizon control of linear network systems," *IEEE Control Systems Letters*, vol. 5, no. 4, pp. 1207–1212, 2020.

[14] F. Celi, G. Baggio, and F. Pasqualetti, "Distributed data-driven control of network systems," *IEEE Open Journal of Control Systems*, vol. 2, pp. 93–107, 2023.

[15] J. Jiao, H. J. van Waarde, H. L. Trentelman, M. K. Camlibel, and S. Hirche, "Data-driven output synchronization of heterogeneous leader-follower multi-agent systems," in *IEEE Conf. on Decision and Control*, (Austin, TX), pp. 466–471, Dec. 2021.

[16] A. Nazerian, K. Bhatta, and F. Sorrentino, "Exact decomposition of optimal control problems via simultaneous block diagonalization of matrices," *IEEE Open Journal of Control Systems*, vol. 2, pp. 24–35, 2022.

[17] T. Kailath, *Linear Systems*. Prentice-Hall, 1980.

[18] H. J. van Waarde, C. De Persis, M. K. Camlibel, and P. Tesi, "Willems' fundamental lemma for state-space systems and its extension to multiple datasets," *IEEE Control Systems Letters*, vol. 4, no. 3, pp. 602–607, 2020.

[19] F. Celi, G. Baggio, and F. Pasqualetti, "Distributed learning of optimal controls for linear systems," in *IEEE Conf. on Decision and Control*, (Austin, TX), pp. 5764–5769, Dec. 2021.

[20] S. Boyd, N. Parikh, E. Chu, B. Peleato, and J. Eckstein, "Distributed optimization and statistical learning via the alternating direction method of multipliers," *Foundations and Trends® in Machine Learning*, vol. 3, no. 1, pp. 1–122, 2011.

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