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A study on wave energy converter arrays using data-driven polynomial chaos expansion.

Avni Jain, Jian Tan, Vaibhav Raghavan, and George Lavidas

Abstract—Wave energy converter (WEC) arrays should be designed to ensure consistent and optimal power production over long operational periods. This requires an understanding of stochastic wave variability, interactive effects among devices and their mutual dependence. In this work, a computationally efficient surrogate modelling framework was developed using data-driven polynomial chaos expansion (PCE) to analyze the performance of WEC arrays under realistic sea state conditions spanning 30 years. For this purpose, using Latin hypercube sampling scheme on a joint probability distribution derived from the ECHOWAVE hindcast dataset, resulting 10^6 combinations of significant wave height (Hs), wave period (Tp), and WEC radius (R) for two array configurations—interacting and non-interacting cases were evaluated. The surrogate model was set up to evaluate the performance of WEC arrays by means of global sensitivity analysis using Sobol indices. The results conclude that the interactive effects significantly alter the contribution of design parameters (like geometry and spatial configurations) to power output, emphasizing the inadequacy of single-device analysis for array optimization. The findings highlight the importance of tailored WEC design within arrays and offer a robust approach for long-term performance prediction and optimization of wave energy farms.

Keywords—Wave energy converter arrays, polynomial chaos expansion, surrogate modeling, probabilistic wave modeling, renewable energy optimization.

I. INTRODUCTION

WITH an increasing demand for renewable energy sources, wave energy shows great promise due to its highest potential, availability, and predictability [1]. However, unlike the wind energy, wave energy is lacking behind in commercial deployments. According to many researchers [2]–[4], the reasons can be categorized mainly due to large number of WEC designs , uncertainities involved in prediction of the sea states which often rely on limited data, and the lack of knowledge of the interactive behaviour of the WECs in an array for larger number of stohastic wave conditions.

Designing WEC arrays for optimal performance demands understanding of the parameters that contribute to constructive/destructive or no interaction of the waves with WECs in an array. In current literature, researchers have tried to achieve either optimizing a WEC/ array design for particular sea states [5]–[7] or study a particular design of WEC for probabilistic variability, however, it is crucial to study both together. One of the critical challenges in literature is the inadequate consideration of probabilistic variability in ocean wave conditions when analyzing WEC array performance. Ocean waves exhibit inherent randomness and variability, significantly affecting the consistency and reliability of energy production over extended periods. Traditionally, deterministic approaches have predominantly been used, resulting in limited insights into long-term WEC array behaviour under realistic conditions. Consequently, there exists a clear research gap in applying probabilistic methods that adequately reflect realistic conditions over the operational period of these devices.

Another crucial limitation is the computational complexity involved in accurately simulating WEC arrays, particularly when exploring multiple interacting parameters such as wave height, wave period, WEC dimensions and array configurations. Detailed numerical simulations, though accurate, often demand extremely high computational resources, limiting their practicality for extensive sensitivity and probabilistic analyses. Therefore, there is a need for faster models [8], [9] that can run a high number of simulations in a reasonable duration of time. The computational time and accuracy of the frequency domain model used and the surrogate model developed have been compared in [10].

To address these research gaps, this study develops a computationally efficient surrogate model based on Polynomial Chaos Expansion (PCE) to study the interactive effects of WEC arrays under realistic, probabilistically defined wave conditions spanning a period of 30 years. By systematically varying key input parameters including metocean conditions, and the radius of individual WECs, this study evaluates the power production both at the individual converter level and for the array as a whole. Utilizing joint distributions and sensitivity analyses, specifically through first-order and total Sobol indices, this study quantifies the influence of each input parameter on power generation, and interactions of these

parameters with each other. The outcome of this comprehensive analysis provides a new approach for selecting appropriate dimensions of WECs in an array. The findings can enable developers to make informed decisions, ensuring optimal and consistent power output from wave energy arrays over extended operational lifetimes.

II. MODELS

The surrogate model (SM) is generated using the steps shown in Fig. 1 to compute the power capture of individual WECs and the total power output of various array configurations for varying wave parameters and radius. A Latin hypercube sampling technique is employed to generate 200 samples of wave parameters and radius of WECs. The generated dataset is used to construct a surrogate model using data-driven PCE, allowing us to approximate the power response of the array for a wide range of wave conditions. By running the surrogate model for all significant wave height (H_s), peak period (T_p) and radius (R) combinations, we evaluate a total of 10^6 cases, enabling a comprehensive probabilistic analysis. Fig. 1 shows the methodology to create a surrogate model using polynomial chaos expansion to evaluate the performance of an array of a configuration of five wave energy converters (WECs). In the following section, each step of Fig. 1 will be discussed in detail.

A. Probabilistic input

The probabilistic input parameters studied in based on wave parameters (H_s and T_p) and radius (R) of each of the WEC. The probabilistic input for parameters H_s and T_p used in this work has been adopted from ECHOWAVE 30-years hindcast dataset. The ECHOWAVE 30-years hindcast provides spatial (~ 2.3 km) and temporal (1 hr) resolution of wave fields and spectral data within the European coastal shelf. One of the main characteristics of this dataset is the use of the TUD-165 parameterisation and wind intensities correction proposed by [11]. The use of TUD-165, together with the selected forcing fields, helped to reduce the overall wave heights' biases in the North-East Atlantic. Adjustments that led to the proposed parametrisation were extensively verified (and then validated) with measurements from the ESA Sea State CCI V3 altimeter product [12]. The resolution and accuracy of ECHOWAVE deems it an excellent tool for a detailed estimation of the energy flux within areas of interest for the development of wave energy projects (typically in depths below 200m). Fig. 2 shows the steps involved in the sampling of wave parameter data for the surrogates. In the first step, 30 years of scatter data for North sea was transformed into contour showing the number of occurrences associated with different H_s and T_p combination. This contour was used to fit a surface to the joint probability distribution of H_s and T_p using a gaussian copula which was then used to sample 200 sets of input variables for the simulations

to train the surrogates.

For the radius of the WEC, a uniform distribution of R ranging from 2m - 12m is assumed. As shown in Fig. 3, 200 samples of radius were chosen to train the surrogate model.

B. Frequency domain model

The frequency-domain (FD) model was initially developed for a single wave energy converter and was extended to an array of five WECs for this work as shown in Fig. 4. The dynamic response of the WEC system is derived based on the linear FD model for each sea state. The FD model for array calculations considers the heave motion for the point absorber type WECs. The PTO damping is optimized for wave period and the radius of the WECs. Nemoh [13] was used for hydrodynamic calculations. The geometric details of the WEC are discussed below.

Geometric details: Fig. 4b shows the geometry of the WEC used in this study and Fig. 4a shows the relative distance (20R) of each WEC in the array studied in this work. The WEC used in this work is a floating heaving point absorber (constrained in all directions other than heave), which is illustrated in Fig. 4b. The geometry of the floating buoy is considered as a sphere of variable radius. The mass of the buoy is assumed to be the same as that of the displaced water by the buoy. The details regarding the WEC can be found in [14].

Cases studied: In this work, two cases have been studied to analyse the influence of constructive/ destructive interference of waves, experienced by each WEC in the array. Case1 is the configuration shown in Fig. 4, where each WEC is placed at a distance of 20R in a staggered configuration. Case2 is the scenario where each WEC is placed 2 km far (it was found to be sufficiently far to avoid interactive effects) away from each other.

C. Model Response

The power produced by each WEC in the array is computed in addition to the total power produced by the WEC array configuration under study for each of the combinations of H_s, T_p and R.

D. Polynomial Chaos Expansion

The PCE model in this work is built using UQlab, a framework developed at ETH Zurich [15], [16]. This framework provides a high-level implementation of Uncertainty Quantification analysis. A hyperbolic (q-norm) polynomial degree truncation scheme [17] with $q = 0.75$ was chosen in the search of optimal basis. The other details regarding PCE parameters can be found in [18].
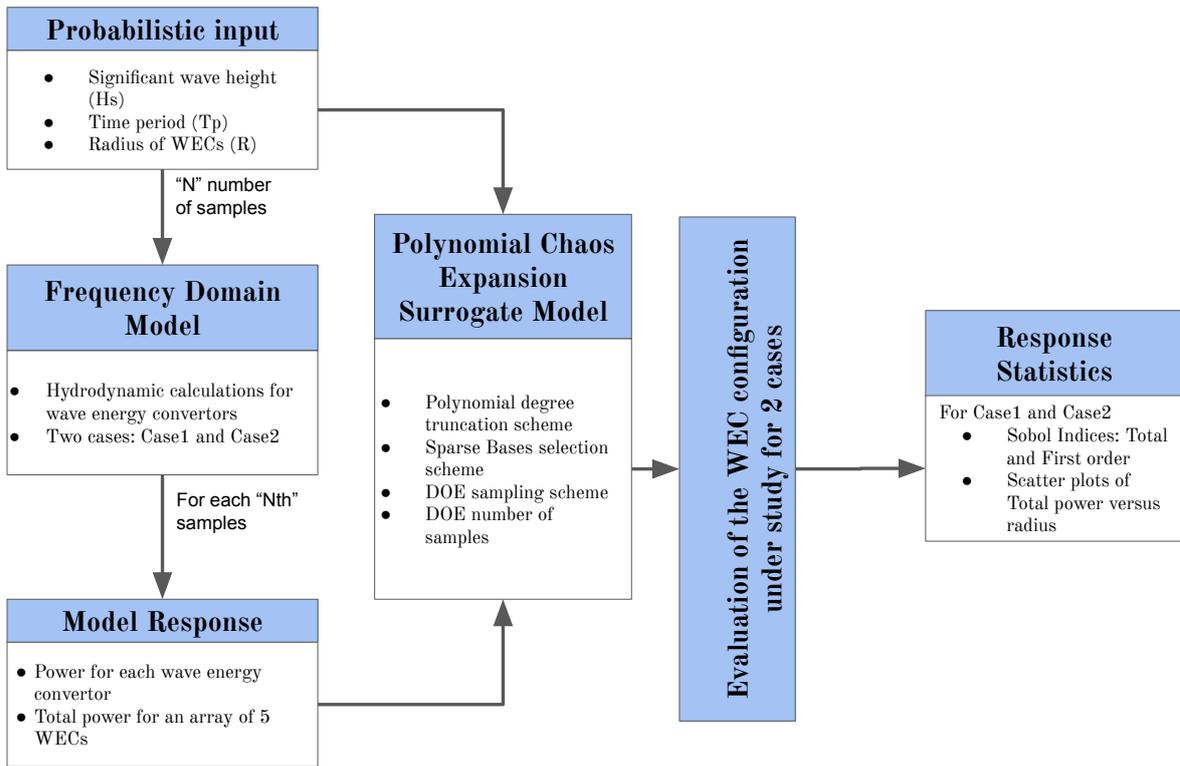
Fig. 1. Flowchart showing steps for creating a surrogate model for evaluation of wave energy converter arrays.

E. Evaluation of the WEC array for the cases under study

Case1 (WEC in a staggered array configuration as shown in Fig. 4) and Case2 (individual WEC behaviour) are studied and compared using Sobol indices and scatter plots to analyse the influence of input parameters (H_s, T_p and R) on the total power produced in both the cases.

Sobol indices are sensitivity measures used in global sensitivity analysis to quantify the contribution of each input variable (or combinations of variables) to the output variance of a model. They help understand how uncertainties in input parameters affect the variability of the model output.

- **First-Order Sobol Indices (S_i):** The first-order Sobol index indicates the direct contribution of a single input variable to the output variance. The first-order Sobol index quantifies the effect of varying a single input variable X_i on the output variance while keeping all other variables fixed. It is mathematically expressed as:

$$S_i = \frac{\mathrm{Var}[E(Y \mid X_i)]}{\mathrm{Var}(Y)} \qquad (1)$$

where, Y is the model output, X_i is the input variable of interest, $\mathrm{Var}(Y)$ is the total variance of the output, $E(Y \mid X_i)$ is the expected value of Y given X_i.

- **Total Sobol Indices (S_{Ti}):** The total Sobol index captures all effects that include X_i, whether they are first-order effects or interactions with other variables. The total Sobol index accounts for both

the main effect of a variable and all interaction effects involving that variable. It is expressed as:

$$S_{Ti} = 1 - \frac{\mathrm{Var}[E(Y \mid X_{\sim i})]}{\mathrm{Var}(Y)} \qquad (2)$$

where, $X_{\sim i}$ represents all input variables except X_i.

- **Relation Between First-Order and Total Sobol Indices**: The relationship between the first-order and total Sobol indices is given by:

$$S_i \leq S_{Ti} \qquad (3)$$

Equality ($S_i = S_{Ti}$) holds if and only if the variable does not interact with any other variables. If interactions exist, the difference ($S_{Ti} - S_i$) represents the contribution of interaction effects involving the variable X_i.

In the end, the scatter plots of the relationship between the total power output of the array under study and the radius of the WECs is studied for Case1 and Case2.

III. RESULTS

Fig. 5 shows the first-order Sobol indices for each WEC in an array for Case1 in a pie chart demonstrating contributions from each of the parameters (H_s, T_p and R) under study. The major observations are that the behaviour is the same along the lines of symmetry as expected. WEC 1 and WEC3 show similar behaviour and WEC4 and WEC5 are identical too. However, a clear difference can be seen in the response and parametric contributions in the first column (WEC1,2,3) and the second column (WEC4,5) of the WEC array
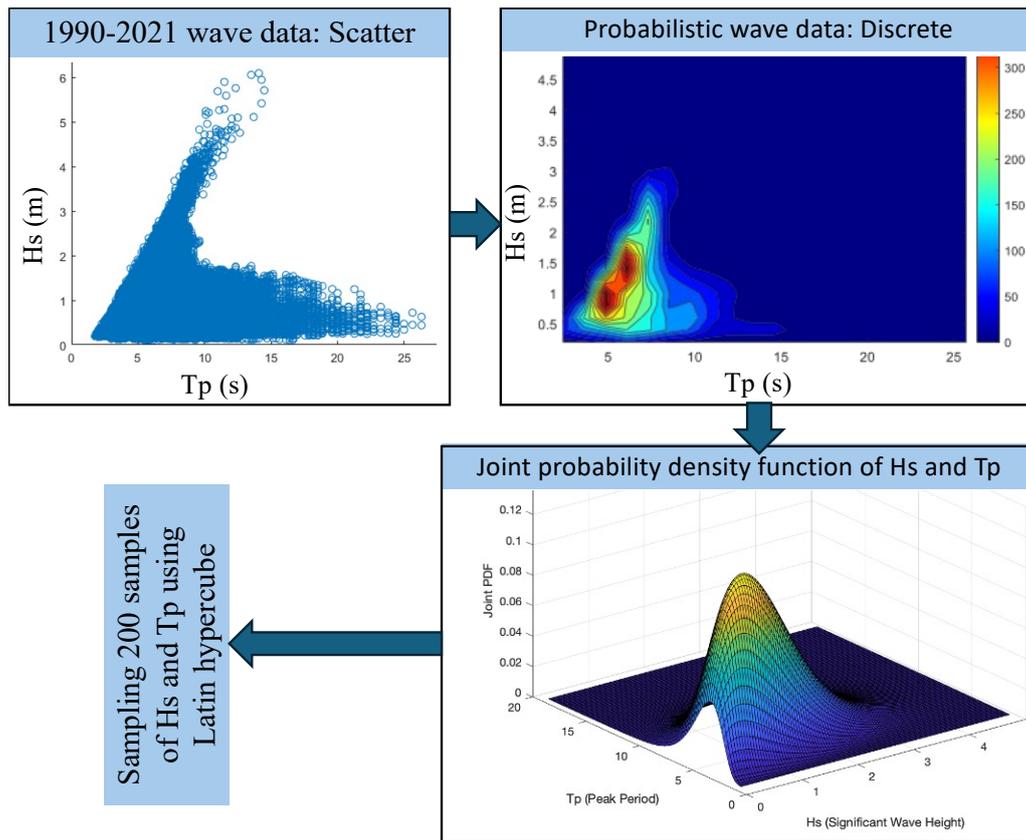
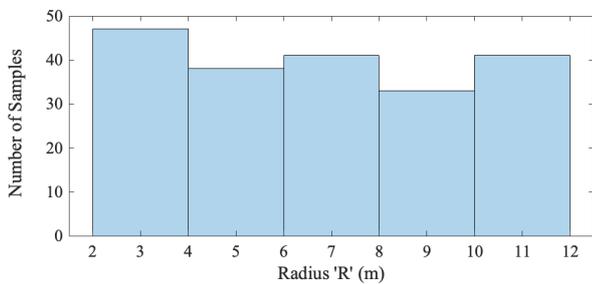Fig. 2. Sampling process used to create data points from ECHOWAVE dataset.

Fig. 3. Histogram of 200 samples of radius used for the design of experiments for SM.

Fig. 4. Geometric details of (a) WEC array and (b) point absorber type WEC.

configuration due to interactive effects (both due to interference of waves and interactive effect between the parameters). The interactive effects between the parameters will be discussed later in terms of the total Sobol index. The following can be concluded in terms of first order Sobol indices:

- H_S has the highest contribution to the power produced by each WEC as expected as it depends on $(H_s)^2$.
- For the first column (WEC1,2,3), the radius of the WEC was more influential for WEC2 than for WEC1 and WEC3.
- Comparing the first row of WECs with the second, R was more influential (relative to T_p) for the first row compared to the second row of WECs.
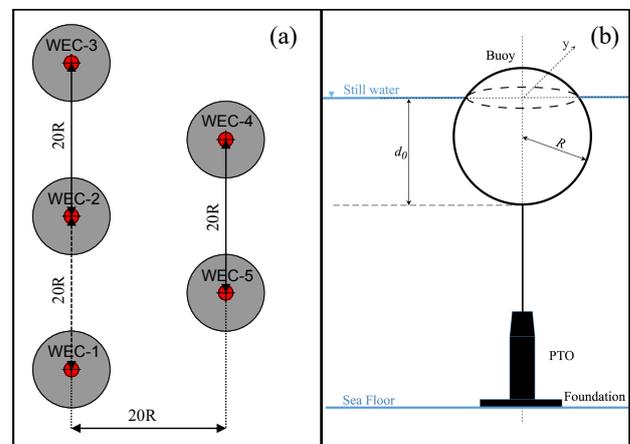
In the end, it can be concluded that the relative contributions of the parameters vary significantly based on the location of the WECs in an array. It might be beneficial if the radius of WEC2 is designed differently than the surrounding WECs.

Fig. 6 shows the comparison of the Total Sobol index discussed above and the first-order Sobol indices for the parametric (H_s, T_p and R) influence in Case 1. It is to be noted that the first-order Sobol indices do not take into account the cross-contribution of the parameters unlike the Total Sobol indices.
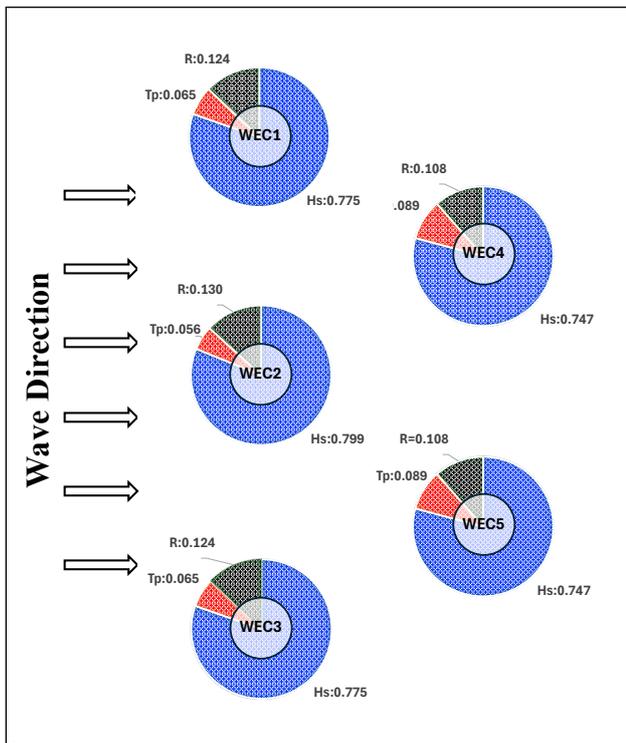
Fig. 5. First order Sobol indices for Case1 showing contributions of H_s, T_p and R.
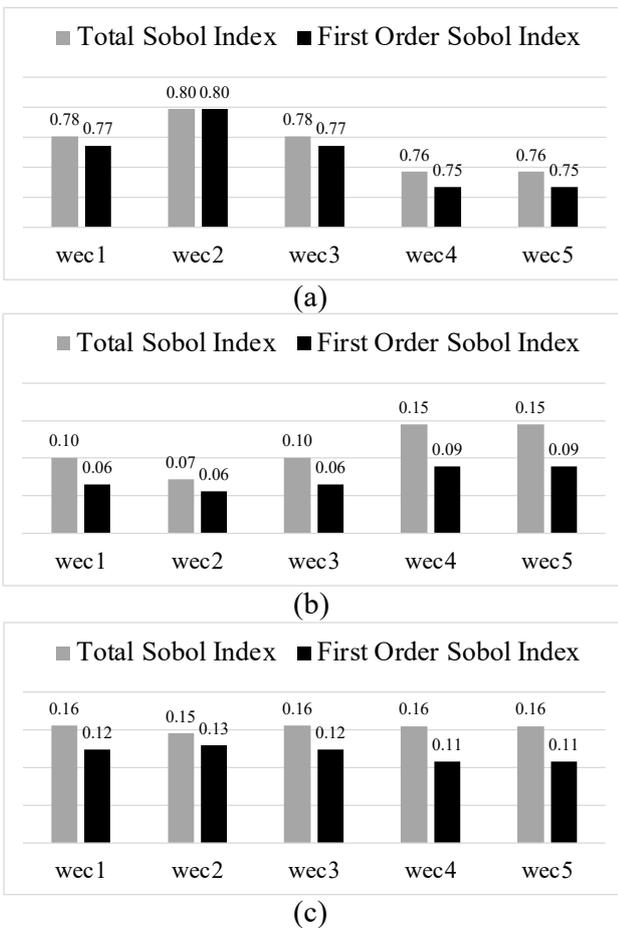

Fig. 6. Total Sobol indices for Case1 showing contributions of (a) H_s, (b) T_p and (c) R.

The contribution of H_s to the power produced by each WEC is almost the same when comparing the first-order and total Sobol indices, implying that H_s does not have much cross contribution with the other parameters (T_p and R). A significant interaction between T_p and R is observed in terms of Sobol indices. This can be explained in terms of natural frequencies (related to R) of the WECs interacting with the T_p of the incident waves. It is important to highlight that this interaction between R and T_p can also be attributed to the fact that power take off (PTO) damping has been optimised in the model for the wave period and the radius of the WECs. This interaction was significantly stronger for the second row of the WECs (WEC4,5), where the wave interference is expected to be stronger. This leads to the conclusion that the design of each WEC must be optimized based on the strength of the interactive effects between the most influential parameters. Therefore, the same design for all the WECs in a configuration is not optimum.

Fig. 7 shows the first-order Sobol index (a) and the total Sobol indices (b) for Case2 (WECs not at interactive distances). It can be clearly seen that the power produced by each WEC is the same as expected due to no interactive effects. The contribution of H_s remains similar to Case1. However, compared to Case1, the contribution of R is much higher than T_p (almost negligible) in Case2. This leads to the conclusion that the design methodology, and parametric sensitivity of an individual WEC are inherently different from that of an array. Consequently, the design choices must not be based on the response of a single device. Moreover, unlike Case1, the difference between the first-order and total Sobol indices is almost negligible for Case2, implying no interaction between the parameters under study contributing to the total power generation of a WEC.

Fig. 8 shows the scatter plots for total power for an array of 5 WECs and radius (R) of the WECs for Case1 (a) and Case2 (b). The color in the plots shows the corresponding T_p value. For Case1 the power for the array increases with increasing radius and the higher values of power are associated to T_p values around 5s. Power output starts to decrease after around $R = 10$m. This implies that the radius of WECs interacts with the wave parameters (H_s and T_p) leading to either constructive or destructive interference phenomena. Increasing the radius of the WECs beyond certain values might not lead to higher power generation, but rather be detrimental for the array considering given long-term metocean conditions. For Case2, where WECs are placed far enough to behave as individual WECs, the correlation between the total power and R is significantly different, as expected. The power output shows an increasing trend with increasing radius and the higher values of power are associated to higher values of T_p (specially for radius in the range of 6-10m), unlike in Case1. It is important to highlight that Case1 for all values of R produces always a
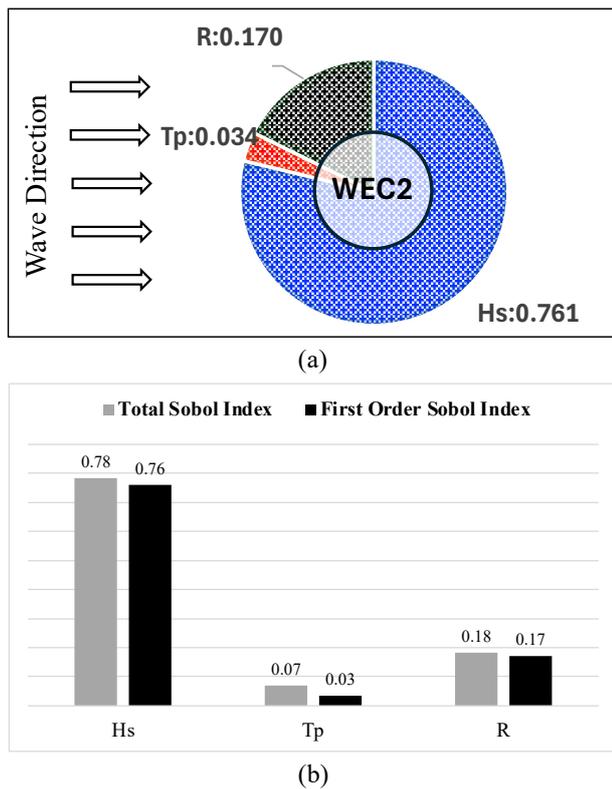
Fig. 7. (a) First order Sobol indices and (b) Total Sobol indices for Case2 showing contributions of H_s, T_p and R.

Fig. 8. Scatter plots for (a) Case1 and (b) Case2 showing correlation between total power (5 WECs) and R. The colorbar shows the values of T_p.

higher power compared to Case2, implying that it's not favourable to choose a configuration where the devices are kept very far in order to avoid interactive effects. In summary, the choice of WEC radius for optimum power production is highly influenced by the configuration of the WEC arrays. The design choices adopted for a single WEC are not extendable to any array configuration.

IV. CONCLUSIONS AND FUTURE DIRECTIONS

In this paper, a surrogate model has been developed using data-driven polynomial chaos expansion for an array of 5 wave energy converters. The results have been studied in terms of Sobol indices and scatter plots for two cases: a staggered configuration (at a distance of 20 times the radius) and the same configuration with an extremely large (2km) distance between the WECs to simulate no interaction conditions.

The correlation between wave parameters (H_s and T_p), radius of WECs (R), position of WECs and the power output was studied in detail for a high-fidelity probabilistic wave input data. It was found that the behavior of WECs for both Cases was significantly different in terms of the contribution (individual and combined) of the input parameters to the power output as well as the interaction of the WECs with each other. The discussions presented in this paper leads to questions for future investigations regarding the optimisation of WEC configuration for varying geometries and probabilistic wave input data
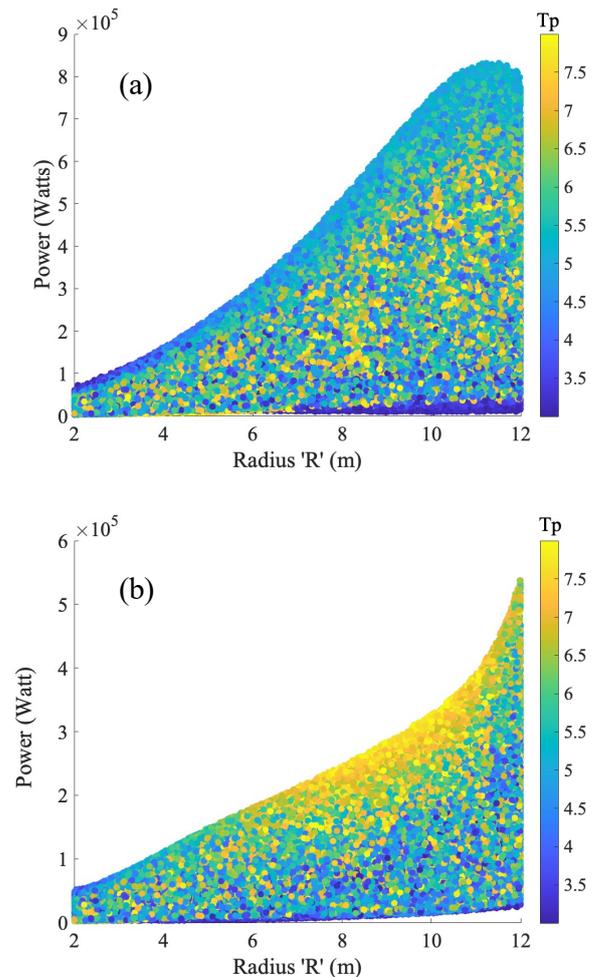
simultaneously. In addition to this, presenting a robust framework for surrogate modelling that can be used for long-term performance studies of wave energy converter arrays.

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