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A model to study the hydraulic performance of controlled irrigation canals

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A model to study the hydraulic performance of controlled irrigation canals

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Abstract

The function of an irrigation canal system is to convey and distribute irrigation water throughout an irrigation scheme. In many irrigation systems the water distribution performance is below expectation, which implies an unreliable and unequal supply. One of the causes underlying a poor water distribution performance is the unsteady flow phenomena. As a result the daily operation of the system is troublesome.

If the system would reacted instantaneously on changes in gate settings, it would be much more simple to determine the proper timing and magnitude of gate adjustments. In reality, the water flow does not react instantaneously, but shows a retarded and diffusive response. The aim of the study was to develop a tool to analyze the unsteady flow in irrigation systems in order to improve the daily operation of these systems.

The mathematical description of the gradually varied one dimensional unsteady flow phenomenon of open canal systems (De Saint Venant Equations) was already known more than a hundred years ago. Three methods of solution of these equations have been reviewed: analytical solutions of simplified equations, integration by characteristics, and numerical methods of solution. In this study the numerical method has been selected as the most appropriate one due to its flexibility and ability to handle complex canal systems.

A computer program which solves the unsteady flow equations numerically is called a hydrodynamic flow model. A review of existing hydrodynamic flow models, available in the Netherlands, demonstrated a number of shortcomings, which were mainly related to the computation flow through structures and control of structures. From the best available existing hydrodynamic flow model, a new model has been developed to simulate and evaluate the hydraulic performance of controlled irrigation canal systems for unsteady flow conditions. The main adaptations comprise the incorporation of typical irrigation structures with variable flow conditions, and the possibility of simulating real time controlled regulators. Moreover, to allow for a fast and diagnostic interpretation of the simulation results, operation performance indicators were defined and incorporated in the model.

Various case studies have been carried out in order to demonstrate the applications and limitations of the developed model. All case studies are dealing with the daily operation of irrigation systems. Several (manual, automatic, self-regulating and non-selfregulating) irrigation canal control systems have been investigated on their operational performance for unsteady flow conditions.

In conclusion, it can be stated that the unsteady flow phenomenon should be taken into account in the daily operation of different types of irrigation canals in order

to improve the water distribution performance. The developed model proved to be a convenient and powerful tool to determine the timing and magnitude of operational instructions that are required to achieve this improved distribution performance. The use of the developed model is not restricted to the daily operation only. It is advised to use the model already in the design or modernization phase of irrigation systems in order to select the most appropriate canal control system. With this model the effects of alternative control systems and operation strategies can be quantified in advance. By doing so, problems in the operation phase can be reduced.

Preface

The research work of this thesis was carried out at the Delft University of Technology, Faculty of Civil Engineering, Department of Sanitary Engineering & Watermanagement, Division of Irrigation. Although this thesis is not a composition of a set of papers, most of the research work has been published before. This holds especially for chapter four "Model applications".

I would like to thank all those who supported and assisted me during this study. In the first place I wish to express my gratitude to professor Robert Brouwer for his willingness to act as a promotor. He found a perfect balance between giving me sincere guidance and granting me sufficient freedom for own ideas and interpretations.

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Finally I want to acknowledge the financial support of the "Lamminga Fonds" and the "Nederlandse Organisatie voor Wetenschappelijk Onderzoek" (NWO) which facilitated me to visit scientific conferences all over the world.

Curriculum Vitae

The author was born in Geleen, the Netherlands, on 18 November 1963. He studied at the Delft University of Technology, Faculty of Civil Engineering from 1982 to 1987. He graduated in the department of hydraulic engineering with a Master of Science thesis entitled "Rehabilitation of secondary unit S1AK". During his research he worked for 6 months in Bangladesh at the Ganges Kobadak Rehabilitation Project for DHV-Consulting Engineers. After graduation he was employed as Assistant Researcher at the Sanitary Engineering and Watermanagement department at Delft University of Technology, Faculty of Civil Engineering.

In addition to his research work he gave lectures for Master of science students from the Delft University of Technology and the International Institute for Hydraulic and Environmental Engineering (IHE) in the field of irrigation, drainage and computational hydraulics. During his research period he participated in the development of a new lecture series "Control of Watermanagement Systems". Many of these Master of Science students participated in his research project for their Msc theses.

He was project leader at the University for the preparation of a hydraulic computer model used in a modernization project in Jordan executed in cooperation with HASKONING Royal Consulting Engineers and Architects. He worked as Consultant with IIMI-Pakistan to assess the applicability of canal operation simulation models for an irrigation system in the Punjab, Pakistan. The author is a member of the ASCE Task Committee on Irrigation Canal System Hydraulic Modeling and of the KIVI Working Group on Automation in watermanagement.

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1 Introduction

1.1 Irrigation water distribution performance

Many irrigation projects appear to perform below expectation. Due to increasing water demands, accurate control of the water flow in any irrigation system is becoming more and more important, but in practice control of water flow in many irrigation systems is still rather poor. To solve the problem of poor water distribution, first the problem itself should be clearly defined and thereafter its causes investigated.

A poor water distribution performance implies an inefficient, unreliable and/or inequitable water distribution, which may occur in the main canal system as well as in the minor canal system. What is meant with efficient, reliable and equitable water distribution? An efficient water distribution means that the designed conveyance and distribution efficiencies are met. Reliability means the extent to which the farmers' expectations of future water deliveries can be met satisfactorily; they can be measured by comparing the actual water deliveries with the allocated planned ones. An equitable distribution reflects a fair water distribution: some offtakes do not receive relatively more water than others.

Two possible causes of inadequate water distribution can be distinguished: an improper water scheduling and an inappropriate operational realisation of the intended water distribution. Both aspects, in turn, can be caused by a combination of social, institutional, economical and technical problems. Most studies of water distribution deal with the aspect of an improper water scheduling. A better scheduling of the available water might indeed lead to higher irrigation efficiencies. However, it seldom improves the reliability and equitability of supply. An unreliable and unequitable supply are usually caused by an inappropriate realisation of the scheduled supplies, which in turn can originate from design, maintenance and/or operation shortcomings. To find the causes of an inappropriate realisation of the scheduled supplies, the shortcomings before-mentioned will be briefly discussed.

Design shortcomings, apart from those which lead to physical collapse may lead to maintenance or operational problems. The absence of measuring devices, for example, is a design shortcoming that will cause operational problems. Lack of maintenance leads to a poor physical condition of the system infrastructure, which in turn makes a proper operation impossible. Operational errors can be the result of inability of the operators and/or of unwillingness of the operators (corruption, laziness). Inability can be caused by absence or poor condition of regulators and measuring devices, but can also be a result of not knowing how to operate the regulators correctly in order to realize the scheduled deliveries.

This study deals with this latter aspect: not knowing how to operate the regulators. Control of regulators in large canal systems can be compared with steering a large vessel to its right place in a harbour. Like large vessels, the system reacts retardedly to control actions and without experience and/or training, it is nearly impossible to perform the job successfully. Control of irrigation systems is also difficult, because of the long response times of the canal system and the non-uniform flow in the open canal systems. Furthermore, various types of regulators with different operational characteristics can be found in an irrigation scheme. To operate them successfully the operator has to be aware of their characteristics.

Summarising, it can be stated that one of the causes of poor performance of water distribution in irrigation systems is the inability to control the regulators in such a way that the actual supplies match the intended supplies. To improve the operation the real life non-uniform and unsteady flow phenomena rather than the simplified uniform steady flow phenomena should be taken into account. The non-uniform unsteady flow phenomenon in controlled irrigation canals is rather complex and can only be calculated by using a model. This model can be a mathematical, a scale, or a numerical one. Due to the progressive development of one-dimensional unsteady flow models and computer resources, a numerical model is most suitable to study the non-uniform unsteady flow phenomena in controlled irrigation canals.

1.2 Role of a flow model

The user of a flow model should be well aware of the limitations of his flow model (it only calculates the hydraulic behaviour of the canal system), but at the same time identify the impact of hydraulics to implicitly related aspects. Any model, by definition, only covers a limited number of all aspects of physical reality. This implies that the use of a model must be restricted to those aspects which are covered by the model. At the other hand, one should not forget that the aspects covered by the model might be (implicitly) related to others. In an irrigation system social, institutional, economical and technical aspects are involved. All these aspects are interrelated. This implies, for example, that institutional problems can be caused by technical shortcomings and, hence, they can sometimes be solved by paying attention to the technical aspects.

To use the model effectively, one should not only simulate the hydraulic behaviour of the system, but also interpret the results. In order to interpret the flow model's output, water levels and discharges varying in time and place, performance indicators are indispensable. But, as soon as performance indicators are introduced, the question is raised: "what is a good performance?". To answer this question the objectives of irrigation have to be reviewed, for a system will perform well if it meets its objectives.

The objective of irrigation is to guarantee or to increase the agricultural production in a certain area, in order to improve the quality of life. These questions remain: whose, and on what terms ? Since there are many participants involved in an irrigation enterprise, and it cannot be assumed that they all share the same view as to what constitutes a good performance, a viewpoint should be defined (Abernethy 1989). Hence, it is impossible to specify *the* objective of irrigation, and consequently, *the* optimal water distribution does not exist. For, the optimal water distribution from an economic point of view might not be optimal from a social viewpoint.

Instead of looking to the objectives of irrigation another more narrow approach is followed: the hydraulic performance of the operation of the system is examined rather than the performance of the system itself. This implies that, firstly, the deliveries to the water users (on farm, tertiary unit or secondary unit level) have to be defined in a water supply schedule. This schedule should be the result of optimization of all different performance requirements of the irrigation system. Once a water supply schedule, in which all hydrological, agronomical, social and economical factors have been incorporated, has been established the supply system should deliver the irrigation water according to this intended schedule. Thus, this study is not dealing with the determination of the optimal water delivery schedule but with the realisation of a given delivery schedule.

A perfect operational performance is achieved when the actual supplies match the intended supplies. This implies that the intended supply must have been agreed upon before the operation performance can be assessed. The objectives of irrigation are encapsulated in the scheduled deliveries.

Good performance indicators should reflect to which extent actual supplies match intended supplies. Although some operation performance parameters do exist (Makin 1986), (IIMI 1987) and (Francis & Elawad 1989), new operation performance parameters have been developed mainly because the existing parameters appeared not suitable for the evaluation during unsteady flow conditions.

Once a model that is able to calculate the real life non-uniform unsteady flow phenomena in controlled canals and to compute operation performance indicators has been developed it can be used to improve the reliability, equality, and efficiency of the water distribution in every phase of the life-cycle of irrigations projects.

■ **Design phase**

"Irrigation district canal systems are typically designed for constant, steady state peak flow rates. Canals are rarely installed with sufficient attention given to the operational criteria for unsteady flow rate conditions which actually exist most of the time" (Burt 1989). As far as the dimensioning of the system for peak flows is concerned, the assumption of steady flow is generally valid. However, for the operation of the system, the unsteady flow phenomena should be taken into account, as the unsteady flow may seriously affect the water distribution. Unsteady flow can be caused by a variable inflow and by operation of regulators in the system. The traditional design criteria for dimensioning the system should therefore be extended with operation performance criteria of the designed system. In the design phase it should already be investigated whether or with which efforts it is possible to realize the intended water distribution. This is especially important as in the design phase one is most flexible to implement changes in the system. The operational performance of the designed system for non-uniform unsteady flow conditions can only be investigated with a (computer) model in which the design can be modelled and tested on its operational performance for variable conditions.

For the design of automatic real time control systems, such as local and regional downstream control, a hydrodynamic flow model is indispensable for the assessment of the control algorithm and its gain parameters and for the determination of the required canal bank elevation.

■ **Operation phase**

In the operation phase, the problem of matching the intended supply with the actual supply is most apparent. In a real life environment operators usually do not have the possibility to experiment with alternative operation strategies. A computer model, on-line or off-line, will provide the operators the opportunity to test various operation strategies. This will lead to more precise and more reliable operational instructions. Furthermore, the operators will gain more confidence in their actions and more understanding of the system's behaviour. In the same way, the model can be used as a training tool for (unexperienced) operators.

■ **Modernization phase**

In the modernization or rehabilitation phase, a poor water distribution is often the initiator of modernizing the system. The real art of modernizing, then, is to tune the required infrastructural and operational modifications in such a way that the causes rather than the symptoms of the poor water distribution are cured. A computer model in which the existing condition of the irrigation infrastructure and the proposed modifications can be easily analyzed and evaluated, would be a useful tool to assist in the development of a sound rehabilitation design. In the model various modernization alternatives and their operational behaviour for real life non-uniform unsteady flow conditions can be studied.

In each of the phases of the life-cycle of an irrigation system, a flow model in which the irrigation system can be modelled and the water distribution simulated and evaluated will be useful. However, the requirements of the model are dependent of its usage. If a model is used as a training tool, the emphasis will be put on the user interface and the presentation of the model results to give the trainee a quick insight in the consequences of the operational manipulations he has entered in the model. If, at the other hand, a model is used for the design of an irrigation system, the emphasis will be put on all types of modelling features such as easily modelling of the irrigation system, easily removing from or adding regulators to the system and a quick definition of alternative control systems for the regulators.

1.3 Scope of the study

1.3.1 Problem description

Although many one dimensional hydrodynamic flow models do exist, none of them have been tailored for irrigation practices or, in general, for controlled watermanagement systems. A comparative study of existing hydrodynamic flow models available in the Netherlands (Schuurmans 1988), demonstrated, that, in the existing flow models, the hydraulic structures and the control of these structures in particular are in an early unsophisticated stage of development. Recently, SAMWAT (SAMWAT 1989) has finalised an extensive inventory of existing computer models available in the Netherlands. It is this very inventory which underlines the findings of the author's comparative study. The "Task Group on Real Time Control of Urban Drainage Systems" also comes up with the conclusions that "*Models for the state of the system ... have been developed in a great number for static, non-controllable systems. However, hardly any model has been described allowing to simulate automatic regulators and external control input during the simulated process*". (Schilling 1987). The same conclusion was drawn two years later: "*.. a key to efficient research on canal automation is the existence of an easy-to-use, accurate, and flexible unsteady flow canal hydraulic simulation program. This research project did not find such a program*". (Burt, 1989)
In 1988, the Irrigation and Drainage Division of the American Society of Civil Engineers, has formed a Task Committee on " Irrigation Canal System Hydraulic Modelling". One of the first activities of this Task Committee has been to evaluate existing flow models on their applicability for irrigation practices. This very action indicates that the existing hydrodynamic flow models were not developed with irrigation practices in mind, and, partly because of the fact that there were no suitable models available, there is hardly any experience with these hydrodynamic models in irrigation engineering.

What makes an irrigation flow model different from the widely spread, river hydraulic flow models? First, it should be noted that there are many similarities between river flow models and irrigation flow models. As a result, river flow models can be used for the development of flow models for irrigation. The main deficiencies of river flow models are listed;

- In most river flow models, a limited amount of structures can be modelled, whereas a wide variety of structures are found in irrigation systems.
- The flow conditions and the transition between flow conditions e.g. from free to submerged and vice versa, can usually not be handled satisfactorily by river flow models.
- Open and closed loop control of regulators is usually not possible at all or is very limited in river flow models.
- Operation performance indicators are lacking. The lack of operation performance parameters, both in flow models and in literature, again indicates that flow models are presently hardly used in irrigation engineering at all.

1.3.2 Objective of the study

To investigate the impact of unsteady flow on the hydraulic performance of irrigation canals, and to examine possible design and/or operational measures to improve the performance, a flow model is needed. Given the present need for a flow model in irrigation practices and the present none existence of irrigation flow models, the objective of the study has been formulated as follows.

The development of a user-friendly computer model which can calculate the non-uniform unsteady flow phenomenon in controlled irrigation systems and compute operation performance indicators. The potentials and limitations of the model so developed has to be demonstrated with a number of applications.

To meet the objective of the study the following steps have to be undertaken.

At first, an analytical expression for the response time of an open canal will be derived. If the response time is considerable, and, as a result, the impact of unsteady flow is significant, an appropriate tool is needed to investigate and to quantify its impacts.

This tool will have to be new model as existing river flow models showed some shortcomings, which were mainly related to the computation of flow through structures and to the operation of structures. Yet, extensive use can be made of existing flow models.

The applicability of the new flow model will be demonstrated by a number of case studies. In these studies the impact of unsteady flow on the hydraulic performance will be quantified to allow for rational based decision between design and/or operation alternatives of irrigation schemes.

1.4 Structure of the report

The mathematical description of non-uniform gradually varied unsteady flow in open canals and the methods of solution of the resulting equations will be discussed in chapter 2. The complete derivation of the De Saint Venant Equations without the simplifications often found will be presented. Approximated methods of solution of the De Saint Venant Equations are reviewed, as no analytical solutions of these equations are known. By reviewing all types of solution techniques, insight is gained in the shortcomings of each method and the physical process of non-uniform unsteady flow.

A first method of solution is presented by using simplified De Saint Venant Equations for which analytical methods of solution are known. Two analytical methods of solution of simplified De Saint Venant Equations, which formerly could not be found in literature and which have been published in the international journal on Irrigation and Drainage Systems (Schuurmans 1990), are presented. The first solution can be used to calculate the response time of a canal reach bounded downstream by a structure after a change in inflow discharge. The second solution can be used to calculate the response time of an (infinitely long) open canal as a function of time and place, after a change in inflow discharge. With the aid of these analytical solutions the need of unsteady flow computations is demonstrated as it can be proven that the response time of open canals can be considerable and ,therefore, may seriously affect the water distribution. In addition, insight in the unsteady behaviour of an open canal system is provided. The application of both analytical formulae appears to be practically restricted to simple uniform canal systems. For more complex canal networks, equipped with numerous structures, the analytical solutions are inadequate.

A second method of solution reviewed is the method of characteristics which is a well known (graphically and numerically) solution technique. The characteristic relationships of the De Saint Venant Equations are derived and the graphical solution technique and the physical interpretation of the characteristics are explained.

A third method of solution deals with the numerical integration of the De Saint Venant Equations. The principals of computational hydraulics will be treated. Furthermore, the numerical equations which are found by using the well known Preissmann scheme will be discussed. In the next chapter, dealing with the developed computer model, more attention will be paid to the applied numerical method of solution.

Chapter three presents a description of the developed modelling package, called MODIS. MODIS is an acronym for "*Mo*delling *D*rainage and *I*rrigation *S*ystems" and has been developed out of the existing river modelling package "Rubicon". A comparative study of existing hydrodynamic flow models, available in the

Netherlands, showed that Rubicon was the most suited one for further development. The numerical methods applied in MODIS and the changes which have been made will be described in detail, as this constitutes the heart of the model. Furthermore, the features of MODIS models developed for this study, such as the hydraulic structure library, the closed loop controllers and the operation performance indicators, are discussed.

In chapter four, applications of the MODIS model will be presented. The applications comprise cases of both theoretical and practical nature and were partly published earlier in the form of articles and papers. The theoretical application deals with the stability and performance of regional downstream controlled irrigation canals. For a proper design of computer controlled canal systems, numerical simulations to assess the best control algorithm and its parameters are indispensable. The practically oriented case studies are dealing with the selection of an appropriate control system for a new irrigation system in Iran and for an existing irrigation system in Jordan. For both systems the effects of alternative control systems and operation strategies have been quantified. The applications clearly demonstrate the potentials and limitations of the MODIS model.

The conclusions and recommendations of the study are presented in chapter five. Here, the future role of a flow model in irrigation engineering is presented, and the impact of unsteady flow will be reviewed.

2 Hydrodynamics of unsteady flow

The theory of motion of fluids is part of continuum mechanics. In the engineering applications the theory appears under the name "hydraulics", a word composed of the Greek words for water and tube. A better denomination for the theory is "hydrodynamics" presently used in the etymologically correct meaning (Schoemaker).

2.1 Introduction

The scientific study of unsteady flow in open canals started some two centuries ago in order to dimension canals for irrigation and drinking water and to study water levels in rivers and estuaria. Ever since numerous publications on the subject have been written. A complete overview of the literature till 1975 is given by (Yevjevich, 1975).

The first publications dealt with the mathematical formulation of unsteady flow. It was Jean-Claude Barré De Saint Venant who published in 1871 the two partial differential equations for gradually varied unsteady flow, now called the De Saint Venant Equations. However, it was Belanger (1789-1874) who had derived these equations some fifty years earlier (Rouse,1957). In 1872 Joseph Boussinesq derived the equations in a mathematically correct way. Analytical solutions of the De Saint Venant Equations have still not been found.

To study the unsteady flow phenomena solutions of the De Saint Venant Equations are to be found. The reliability of the solutions is determined in the first place by the limitations underlying the De Saint Venant Equations. To investigate the limitations it is necessary to review the derivation of the equations. Three methods are available to find approximated solutions of the De Saint Venant Equations: (1) the method of simplified equations, (2) the method of characteristics, and (3) the method of numerical integration.

The first method of solution of the De Saint Venant Equations is to find analytical solutions of simplified equations. Two analytical formulae are discussed to determine the response time of an open canal system after a change in flow rate. The first

method of canal routing is a simple method and is used to determine the filling time of short canal reaches. The prediction error increases for longer canal reaches, as a result of neglecting the non-uniform flow conditions. The second method to determine the response time after a change in flow rate, the diffusion approximation, is a more accurate method and valid for long canal reaches.
The analytical solutions found in literature are valid for the propagation of floods waves, but not for the propagation of step variation in discharge. Furthermore, the assumption of a rectangular canal profile is usually made, which is normally not valid for irrigation canals.

The second method of solution is the classical method of characteristics which is a suitable method to understand the physical process of unsteady flow. Besides, the method has many similarities with the numerical integration method and ,as such, it is an instructive way to explain the numerical integration method that will be discussed later.

The third method of solution of the De Saint Venant Equations is the method of numerical integration. Due to the present availability of cheap and reliable computers the numerical method is at present the most popular solution method. In this chapter the principals of computational hydraulics are discussed, whereas in the next chapter a more detailed description of the computer model developed will be presented.

Despite the present availability of computers the need for analytical approximation formulae still exist. These analytical formulas can be used for a preliminary study of the unsteady flow phenomenon, and for checking the outcome of a computer model on possible modelling or for other kinds of errors.

2.2 The De Saint Venant Equations

The assumptions underlying the De Saint Venant Equations can be found by reviewing the derivation of the equations. In Appendix A this derivation, based on the law of mass and momentum conservation, is presented. The main simplifications underlying the De Saint Venant Equations are listed:

1) The velocities perpendicular to the direction of flow are negligible (and thus the accelerations also) as compared to the velocity in the direction of flow. This implies that the slope of the water level perpendicular to the direction of flow is assumed to be horizontal.
2) A negligible curvature of the water surface implies a hydrostatic pressure distribution perpendicular to the flow direction (or bed slope).
3) Friction losses in unsteady flow are not significantly different from those in steady flow and therefore the same resistance formulae may be applied.
4) Density of the fluid is assumed to be constant.

5) The derivatives of the coefficient of Boussinesq ß, expressing the non-uniformity of the velocity distribution across the wetted area, are assumed to be negligible small.

The De Saint Venant Equations expressed in Q and A read :

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = q \qquad (2.1)$$

$$\frac{\partial Q}{\partial t} + 2\beta \frac{Q}{A}\frac{\partial Q}{\partial x} + c^2(1-\beta Fr^2)\frac{\partial A}{\partial x} = W \qquad (2.2)$$

Where:

$$c = \sqrt{g \cos(s) \frac{A}{T}} \qquad (2.3)$$

$$Fr = \frac{v}{c} = \frac{Q}{A\,c} \qquad (2.4)$$

$$W = g A \left[\sin(s) - \frac{Q\,|\,Q\,|}{k^2 R^{4/3} A^2} \right] \qquad (2.5)$$

$$\beta \equiv \frac{\int_0^A u^2 dA}{v^2 A} \qquad (2.6)$$

Where,

A	= cross sectional area	m^2
c	= critical celerity or celerity of an infinitesimal disturbance	m/s
Fr	= Froude number (=v/c)	-
g	= gravitational acceleration	m/s^2
k	= Strickler resistance coefficient (= 1/n)	$m^{1/3}/s$
n	= Manning's resistance coefficient	$s/m^{1/3}$
Q	= discharge	m^3/s
q	= lateral inflow per unit length	m^2/s
R	= hydraulic radius	m
s	= canal bed slope	-
T	= width of canal at water level	m
u	= fluid velocity	m/s
v	= mean fluid velocity	m/s
W	= gravity force minus resistance	m^3/s^2
ß	= Boussinesq coefficient	-

Eq. (2.1) represents conservation of volume which equals continuity of mass when the density of water is assumed to be constant. Eq. (2.2) represents conservation of momentum, which can be interpreted as dynamic equilibrium of forces whereby the acceleration is incorporated. The two equations are non-linear partial differential equations of hyperbolic type (Sommerfield, 1949) with variable coefficients for which only approximated solutions exist. In the following, three methods of solution of the (simplified) De Saint Venant Equations will be discussed, starting with two analytical methods of solution. The first and most simple one is the method of canal routing.

2.3 Canal routing

2.3.1 Introduction

The most simple approximation formula for the system response time after a change in inflow discharge is obtained when a canal reach is simplified as a storage basin, whereby the non-uniform flow phenomenon is neglected. The method is known as the method of canal routing. In literature (Yevjevich 1975) the method of canal routing has been extensively discussed, but not for determining the response time of a canal reach bounded by an outflow structure. This situation often occurs in irrigation canals consisting of a series of canal pools. An analytical formula for the determination of the response time has been derived (Schuurmans 1990a).

The method of canal routing uses the equation of continuity only and it omits the equation of motion. However, the equation of motion can implicitly be taken into account for the determination of the boundary conditions. The equation of continuity states that the net inflow in a canal reach is equal to the change in storage volume of that canal reach. In formula this equity reads:

$$\frac{\delta S}{\delta t} = Q_{inflow} - Q_{outflow} \tag{2.7}$$

in which S is the storage volume, t is time, Q is the inflow and outflow rate. Two problems and their solutions will be discussed.

2.3.2 Problem I

Given a canal reach which has to be filled. The volume required to fill the canal reach is equal to S. If the net inflow rate (Q_{inflow} - $Q_{outflow}$) is constant the time T to fill the reach simply reads:

$$T = \frac{S}{Q_{inflow} - Q_{outflow}} \tag{2.8}$$

Where T is the filling time (s), S is the storage volume to be filled (m^3), and Q_{inflow} - $Q_{outflow}$ is the net inflow rate (m^3/s).

2.3.3 Problem II

A canal reach is bounded downstream by a check structure. The inflow rate is known and fixed and the outflow rate depends on the downstream water level of the reach. If the inflow rate is changed then the water level in the reach will change and consequently the outflow rate will be affected. When the outflow rate equals the inflow rate, a new equilibrium will be reached. To determine the time before the system has reached the new steady state, the following series of assumptions are made.

a) The new inflow rate (Q_{in}) is assumed to be constant after the change has been made.

b) The difference of storage volume S between the previous and the new steady state equals:

$$S = \int_{h_0}^{h_1} A\, dh = A_s\, \Delta h \tag{2.9}$$

Where A is storage surface area (m^2), A_s is the mean storage surface area (m^2), h is the water level (m), Δh is the total variation in water level near the outflow structure (m). In case of a wedge storage, the mean storage surface area can be taken equal to:

$$A_s = \frac{S}{\Delta h} \tag{2.10}$$

Both the storage volume S and the variation in water level Δh can be determined by calculating the depth for the new steady state using Strickler resistance formula.

c) The stage discharge curve of the check structure at the downstream end of the canal reach reads:

$$Q_{outflow} = K\, h^u \tag{2.11}$$

in which $Q_{outflow}$ is the outflow discharge (m^3/s), K is a discharge coefficient of the outflow structure (m^{3-u}/s), h is the water depth with reference to the sill level of the structure (m), and u is an exponent (e.g. u equals 0.5 for underflow and 1.5 for overflow structures). To obtain an analytical solution of Eq. (2.7), the stage discharge curve of the check structure at the downstream end of the canal reach has to be linearized:

$$Q_{outflow} = K\, h^u \approx K h_m^u + K u h_m^{u-1}\, dh + (\textit{neglected rest terms of second order})$$

in which h_m is a mean water depth (m) and equal to $\frac{1}{2}\{h_0 + h_1\}$, h_0 is the old water depth (m), h1 is the new water depth (m), and dh is the variation of the water level (m) with respect to h_m.

Substitution of the formulated expressions for S, Eq.(2.9), and the linearized expression of $Q_{outflow}$, Eq.(2.12), into the general equation of canal routing, Eq.(2.7), yields the following differential equation:

$$\frac{dS}{dt} = A_s\frac{dh}{dt} = Q_{inflow} - K h_m^u - K u h_m^{u-1}(h-h_m) \tag{2.13}$$

Rearranging the terms yields,

$$A_s\frac{dh}{dt} = \left(Q_{inflow} - K h_m^u + K u h_m^{u-1} h_m\right) - \left(K u h_m^{u-1}\right) h \tag{2.14}$$

A solution of this linear differential equation for the initial condition $h = h_0$ on $t = 0$, reads:

$$h(t) = h_1 - (h_1 - h_0)\, e^{-\alpha t} \tag{2.15}$$

Where,

$$\alpha = \frac{u K h_m^u}{h_m A_s} \tag{2.16}$$

The correctness of the solution can be demonstrated by substituting the solution found in the differential equation. The new water level h_1 can be calculated with Strickler resistance formula, stating that at the new steady state the inflow rate must equal the outflow rate. In formula:

$$h_1 = \left[\frac{Q_{inflow}}{K}\right]^{1/u} \tag{2.17}$$

Now that the variation of the water level in time is known, the lapse of time before η% of the total variation in water level has been achieved can be found by rewriting Eq.(2.15) and yields:

$$T(\eta\%) = \frac{h_m A_s}{u K h_m^u} \ln\left[\frac{100\%}{100\% - \eta\%}\right] \tag{2.18}$$

2.3.4 Example calculation I

The following example calculation has been made, to illustrate the use of the presented solution of the method of canal routing. An irrigation canal of 1000 m length with a weir at the downstream end, is supplied by a night reservoir. In the morning, irrigation water is released from the reservoir into the canal with a discharge of 0.5 m^3/s, and in the evening irrigation water supply is stopped. During the night, the water level in the irrigation canal will become horizontal, whereby the water level in the canal becomes equal to the crest level of the weir.

The irrigation canal has a Manning roughness coefficient n of 0.028 s $m^{-1/3}$, a bottom width of 1 m, a side slope m of 1.5 and a bed slope s of 2.10^{-4}. The crest level of the weir z is situated 0.50 m above the canal bed. The stage discharge curve of the weir reads:

$$Q_{outflow} = K h^{u} \tag{2.19}$$

with K equal to 4.5 $m^{3/2}/s$ and u equals 3/2. Using the Manning resistance formula (Chow 1959),

$$Q = \frac{1}{n} A R^{2/3} s^{1/2} \tag{2.20}$$

the equilibrium water depth in the canal is calculated, and equals 0.78 m, for a discharge of 0.5 m^3/s.

To determine the filling time of the canal reach, it is assumed that first the storage wedge S_1 is filled whereby the water level becomes perpendicular to the bed slope and whereby the outflow discharge is still zero. Thereafter the volume S_2 is filled, whereby the outflow rate is gradually increased. (Fig. 2.1).

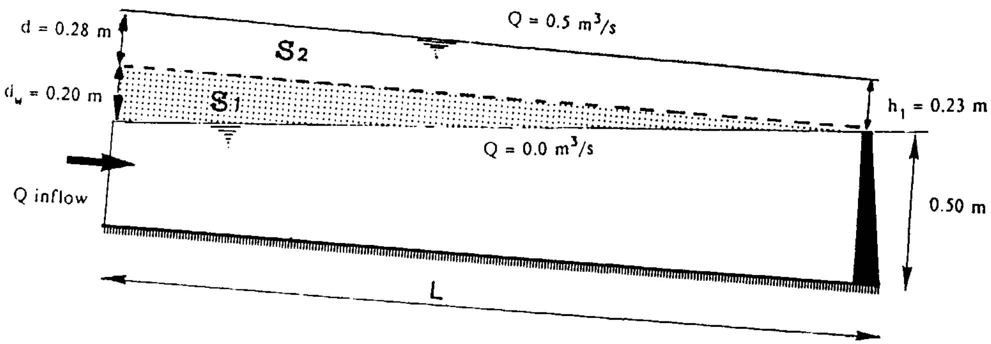


Fig. 2.1 Filling process of a canal reach bounded by an outflow structure

■ **Filling time of S_1**

The volume of the storage wedge S_1 can be approximated by:

$$S_1 = \frac{1}{2} B_m L\, d_w \tag{2.21}$$

in which B_m is the mean storage surface width and d_w is the upstream depth of the storage wedge and equal to:

$$d_w = sL = 2 \bullet 10^{-4} \bullet 1000 = 0.20m \tag{2.22}$$

The mean storage width B_m can be approximated by:

$$B_m = \frac{1}{2}\left[B_{bottom} + 2 sideslope\ z + B_{bottom} + 2\ sideslope\ (z - \frac{1}{2}d_w\right] = 2.35\ m \tag{2.23}$$

Hence the volume of the storage wedge S_1 can be approximated by:

$$S_1 = \frac{1}{2} B_m L\ d_w = \frac{1}{2} 2.35 \bullet 1000 \bullet 0.20 = 235\ m^3 \tag{2.24}$$

The filling time of this storage wedge can be computed with Eq.(2.8), and yields:

$$T_{filling} = \frac{S_1}{Q} = 470\ s \approx 8\ \text{min} \tag{2.25}$$

■ **Filling time of S_2**

To calculate the filling time of the storage volume S_2, whereby the outflow discharge gradually increases until it equals the inflow discharge, Eq. 2.18 can be applied. The reference water level is taken at the crest level of the weir for calculation convenience. The water level at the new steady state just upstream of the weir, h_1, can be computed by Eq.(2.17):

$$h_1 = (\frac{Q}{K})^{\frac{1}{u}} = (\frac{0.5}{4.5})^{\frac{2}{3}} = 0.23m \tag{2.26}$$

So, the mean water level near the weir h_m, becomes

$$h_m = \frac{1}{2}(h_1 - h_0) = \frac{1}{2} \bullet 0.23 = 0.115\ m \tag{2.27}$$

The mean storage surface area A_s of storage volume S_2, is computed, by neglecting the influence of the backwater curve upstream of the weir. The new water level in the canal will have an equilibrium depth of 0.78 m above bottom level. The raise of the water level (d) along the canal becomes equal to:

$$d = 0.78 - 0.50 = 0.28m \tag{2.28}$$

The mean canal surface width B_m for S_2 is equal to:

$$B_m = B_{bottom} + 2\ sideslope\ (z + \frac{1}{2}d) = 2.92\ m \tag{2.29}$$

hence, the storage volume S_2 becomes:

$$S_2 = B_m L d = 2.92 \bullet 1000 \bullet 0.28 = 818 \ m^3 \tag{2.30}$$

Reminding Eq. (2.10), the mean storage surface area becomes:

$$A_s = \frac{S_2}{\Delta h} = \frac{818}{0.23} = 3555 \ m^2 \tag{2.31}$$

Now that all variables have been determined, the time before 90% of the new water level in the canal reach has been achieved can be computed using Eq.(2.18) and yields,

$$T_{90\%} = \frac{h_m A_s}{u K h_m^u} \ln\left[\frac{100\%}{100\%-90\%}\right] = \frac{0.115 \bullet 3555}{1.5 \bullet 4.5 \bullet 0.115^{1.5}} \ln(10) = 3576 \ s = 60 \ \text{min} \tag{2.32}$$

The overall time before 90% of the change has been achieved is found by adding both response times and yields:

$$T_{90\%} = 8 + 60 = 68 \ \text{min} \tag{2.33}$$

2.3.5 Applicability of the method of canal routing

To evaluate the applicability of the method of canal routing a comparative calculation was made with MODIS hydrodynamic flow model. A response time of 60 minutes was found, which is about 10% less than the result of the manual calculation. The accuracy of the method of canal routing depends to a great extent on how the filling process is simplified, and on the accuracy of the computation of the storage volumes. For, in physical reality the water levels are no straight lines. The non-uniformity of the flow will play an increasingly important role for increasing canal lengths, and consequently the accuracy of the method of canal routing will reduce for longer canals. However, the accuracy can be improved by applying the method to series of canal reaches.

For long canals not bounded by a structure, the non-uniformity of the flow cannot be neglected anymore. In that case, an analytical expression for the response time can by found by using a simplified equation of motion rather than neglecting the equation of motion. The equation thus obtained is called a diffusion equation for which analytical solutions do exist.

2.4 Diffusion approximation

2.4.1 The diffusion equation

The most accurate form of the diffusion equation is found by neglecting the local and partly the convective acceleration terms, (Appendix A & B), of the equation of motion of the De Saint Venant Equations with respect to a moving reference system (Schoemaker 1989). The diffusion equations found in literature are less accurate because a rectangular profile is usually assumed, and because the local and convective acceleration terms are totally neglected. The assumption of (partly) neglecting the acceleration terms is valid as long as the Froude number of the flow remains small, which is usually the case in irrigation canals.

The diffusion equation reads, Appendix B (Schoemaker 1989) :

$$D \frac{\partial^2 Q}{\partial x^2} = \varphi \frac{\partial Q}{\partial x} + \frac{\partial Q}{\partial t} \qquad (2.34)$$

Where, Q is the discharge (m^3/s), D is the diffusion coefficient (m^2/s), φ is the diffusion celerity (m/s), and the x (m) and t (s) refer to distance and time respectively. The diffusion coefficient D reads,

$$D = \frac{Q}{2\ T\ tg(s)}\ \lambda \qquad (2.35)$$

and the diffusion celerity is defined by,

$$\varphi = \frac{Q}{A}\ \kappa \qquad (2.36)$$

Where, A is the area of cross section, T is the top flow width of the canal, Q is the flow rate, and s is canal bed slope. The coefficients λ and κ are defined by,

$$\lambda = 1 - \left[\frac{2r}{3a} Fr\right]^2 \ (\approx\ 1.0) \qquad (2.37)$$

$$\kappa = 1 + \frac{2r}{3a}\ (\approx\ 1.5) \qquad (2.38)$$

Where, Fr is the Froude number, and the coefficients (a) and (r) are called hydraulic exponents of Bakhmeteff, and follows after approximation of the trapezoidal profile by a parabolic canal profile[1] (Schoemaker 1976, 1979). The hydraulic exponents are defined as (Appendix C):

$$a = \frac{\ln(\frac{A_{max}}{A_{min}})}{\ln(\frac{H_{max}}{H_{min}})} \qquad (2.39)$$

$$r = \frac{\ln(\frac{R_{max}}{R_{min}})}{\ln(\frac{H_{max}}{H_{min}})} \qquad (2.40)$$

Whereby the subscripts max and min refer to the maximum and minimum value respectively, H is the mean (hydraulic) water depth (= A/T), and R is the hydraulic radius.

The diffusion equation can be read in many forms by applying various reference systems. A dimensionless notation will be used. The advantage of a dimensionless notation is that existing mathematical solutions of the diffusion equation can be used directly. This implies that standard tables can be used. When the coordinates x and t are replaced by dimensionless coordinates ξ and τ. The relationship between these coordinates reads:

$$d\xi = \frac{\varphi}{D}\,dx \approx \xi = \frac{\varphi}{D}\,x \qquad (2.41)$$

$$d\tau = \frac{\varphi^2}{D}\,dt \approx \tau = \frac{\varphi^2}{D}\,t \qquad (2.42)$$

The dimensionless notation of the diffusion equation reads:

$$\frac{\partial^2 Q}{\partial \xi^2} = \frac{\partial Q}{\partial \xi} + \frac{\partial Q}{\partial \tau}, \qquad (2.43)$$

[1] The use of power functions for the sharp approximation of complex relationships between variables is old. The best known examples are the many forms of the factor C of the flow resistance of Chezy. A much wider use of power functions has been introduced by Boris Bakhmeteff in his "Hydraulics of Open Channels" (1932) and his method of approximation has been followed here. The characterizing geometrical variables of the cross-section in this way is an extension of Tolkmitt's parabola formula (1898).

2.4.2 Solutions of the diffusion equation

In literature, analytical solutions of the diffusion equation have been derived for a wide variety of initial and boundary condition (Crank 1975), (Abramowitz & Stegun 1972). However, the solution for an instantaneous variation in intake discharge could be deduced from general solutions given in literature and will be presented here (Schoemaker 1990).

▪ Unit impulse function

A well known solution of the diffusion equation, for a sudden increase in discharge during an infinitesimal period of time (impulse function), is the normal or Gaussian probability function. The correctness of the solution follows after substitution into the diffusion equation, Eq.(2.43). This solution reads,

$$Q = Q_0 + \Delta Q \; Z(\xi,\tau) \; , \qquad (2.44)$$

Where, Q_o is the original discharge and ΔQ is the variation in discharge. The function Z is the Gaussian probability function and the function variables ξ and τ, are dimensionless distance and time coordinates. The function Z is defined by:

$$Z(\xi,\tau) = \frac{1}{\sqrt{2\pi}} \frac{1}{\sqrt{2\tau}} \, e^{\left(-1/2 \frac{(\xi-\tau)^2}{2\tau}\right)} \qquad (2.45)$$

▪ Unit step function

The diffusion equation is for constant (D) and (φ) a linear equation. This implies that elementary solutions can be added to describe a composite solution. The solution for an instantaneous variation in discharge (unit step function) is composed of endless series of elementary solutions of the impulse function. This is mathematically obtained by integrating the normal probability function $Z(\xi,\tau)$ with respect to time. In appendix B, this integration has been carried out (Schoemaker). The result is not found in literature and yields,

$$Q = Q_0 + \Delta Q \int_0^{\tau} Z(\xi,\tau)\, d\tau = Q_0 + \Delta Q \; I(\xi,\tau) \qquad (2.46)$$

The function I is the integrated Z function, and has been tabulated in Table 2.1 for $I(\xi,\tau) = 0.90$.

2.4.3 Example calculation II

To illustrate the use of the presented solution of the diffusion equation, an example calculation has been made.

Given an irrigation canal with a trapezoidal profile and a bottom width of 15 m, a side slope of 1:2, a bed slope of 1.10^{-4}, and a Manning roughness coefficient (n) of 0.025.

The initial intake discharge Q_0 equals 15 m^3/s and is instantaneously increased to 18 m^3/s.

Table 2.1 Values of ξ and τ for $I(\xi,\tau) = 0.90$

ξ	τ	ξ	τ
0.00	0.00	5.50	9.60
0.50	1.21	6.00	10.33
1.00	2.37	6.50	11.05
1.50	3.39	7.00	11.75
2.00	4.32	7.50	12.45
2.50	5.14	8.00	13.13
3.00	5.90	8.50	13.82
3.50	6.59	9.00	14.50
4.00	7.34	9.50	15.15
4.50	8.12	10.00	15.82
5.00	8.87	10.50	16.48

To calculate the lapse of time before 90% of the variation in flow rate has reached a point located 10 km downstream, Eq.(2.46) is used. The 90% change is reached when the function $I(\xi,\tau)$ is equal to 0.90. The dimensionless distance coordinate ξ can be computed for a distance of 10 km, and the corresponding dimensionless time coefficient τ can then be read from Table 2.1. The relationship between the dimensionless coordinates and the real life coordinates is completely determined by the diffusion coefficient D and the diffusion celerity φ. D and φ in turn, are determined by the geometrical and flow parameters of the canal. Therefore, the geometrical and flow parameters of the canal have to be calculated first. They can easily be found using Manning resistance formula,

$$Q = \frac{1}{n} A R^{2/3} s^{1/2} \qquad (2.47)$$

The result are depicted in Table 2.2.

Table 2.2 Values of flow and geometrical variables

	h (m)	A (m^2)	T (m)	H (m)	R (m)	Q (m^3/s)
minimum	1.67	30.63	21.68	1.41	1.36	15.06
maximum	1.85	34.60	22.40	1.54	1.49	18.02
90%-values	1.83	34.15	22.32	1.53	1.47	17.66

Using the 90%-values, the Froude number is found to be equal to:

$$Fr = \frac{Q}{A\,c} = \frac{Q}{A\sqrt{g\frac{A}{T}}} = \frac{17.66}{34.15\sqrt{9.81\frac{34.15}{22.32}}} = 0.13 \qquad (2.48)$$

The hydraulic exponents "a" and "r" are calculated by Eq. (2.39) & (2.40),

$$a = \frac{\ln(\frac{A_{max}}{A_{min}})}{\ln(\frac{H_{max}}{H_{min}})} = 1.37 \qquad (2.49)$$

$$r = \frac{\ln(\frac{R_{max}}{R_{min}})}{\ln(\frac{H_{max}}{H_{min}})} = 0.97 \qquad (2.50)$$

Now, the coefficients λ and κ values can be computed by Eq. (2.37) & (2.38),

$$\lambda = 1 - \left(\frac{2r}{3a}Fr\right)^2 = 0.996 \qquad (2.51)$$

$$\kappa = 1 + \frac{2r}{3a} = 1.47 \qquad (2.52)$$

The diffusion coefficient D and the diffusion celerity φ thus become, Eq. (2.35) & (2.36),

$$D = \frac{Q}{2\,B\,tg(s)}\lambda = \frac{17.66}{2\;22.32\;tg(1.10^{-4})}0.966 = 3940\ m^2/s \qquad (2.53)$$

$$\varphi = \frac{Q}{A}\kappa = \frac{17.66}{34.15}\,1.47 = 0.76\ m/s \qquad (2.54)$$

Now the values of the diffusion parameters are known, and the relationship between the real life and dimensionless coordinates is fixed. A real life distance of 10,000 m is equivalent with a dimensionless distance of:

$$\xi = \frac{\varphi}{D}x = \frac{0.76}{3940}\,10.000 = 1.93 \qquad (2.55)$$

From Table 2.1, a corresponding τ-value of 4.18 is computed by linear interpolation. This is the time to reach 90% of the variation in discharge. A dimensionless time of 4.18 is equivalent with a real life time of:

$$t = \frac{D}{\varphi^2}\tau = \frac{3940}{0.76^2}\,4.18 = 28{,}513\ s = 7{:}55\ hours \qquad (2.56)$$

2.4.4 Applicability of the diffusion approximation

To evaluate the applicability of the diffusion approximation a comparative calculation was made using MODIS hydrodynamic flow model as a reference computation. Two canals were examined: one with a flat bed slope (10 cm/km), and the other one with a normal bed slope (20 cm/km). The Manning resistance coefficient of both canals was taken at 0.025 s $m^{-1/3}$, the side slope was 1:2, and the bottom width of the flat canal was 15 m, and the other one had a bottom width of 10 m. Both canals had an original discharge of 15 m^3/s, which was increased to 18 m^3/s, in a period of 5 minutes. It was assumed that the flow in the canals is not affected by the conditions downstream. The diffusion coefficients of both canals are presented in Table 2.3.

Table 2.3 Diffusion coefficients of the canals with flat and normal bed slopes

	λ	κ	D (m^2/s)	φ (m/s)
Flat canal	0.966	1.472	3940	0.76
Steep canal	0.993	1.434	2524	1.00

The results of both the analytical solution and the computer simulations are presented in Fig. 2.2.

- **Discussion**

On the basis of the comparative computations the following conclusions can be drawn. (1) The diffusion approximation gives an accurate prediction of the advancing time of the discharge with a maximum deviation of 10 minutes or about 3%. The error is mainly a result of the applied constant coefficients D and φ. (2) The error increases for increasing (dimensionless) canal lengths. (3) If one compares the discharges of both computations at a certain moment in time, then the difference in discharge at that time level will be far less than 3%. (4) The advancing time of the discharge is not equal to the advancing time of the water levels. One should realize that the steady uniform relationship between discharge and water depth, is not applicable for non-uniform and/or unsteady flow.

2.4.5 Factors affecting the system response time

On basis of the results of both analytical formulae, the factors influencing the response time can be found. Basically, the system response time can be viewed upon as the time that is required to fill or to empty a canal reach. What makes things complicated is that the outflow discharge is not constant but variable during the process.

When the canal reach is bounded by a structure, the rate of outflow discharge is determined by the stage discharge relationship of that structure. If an orifice type of

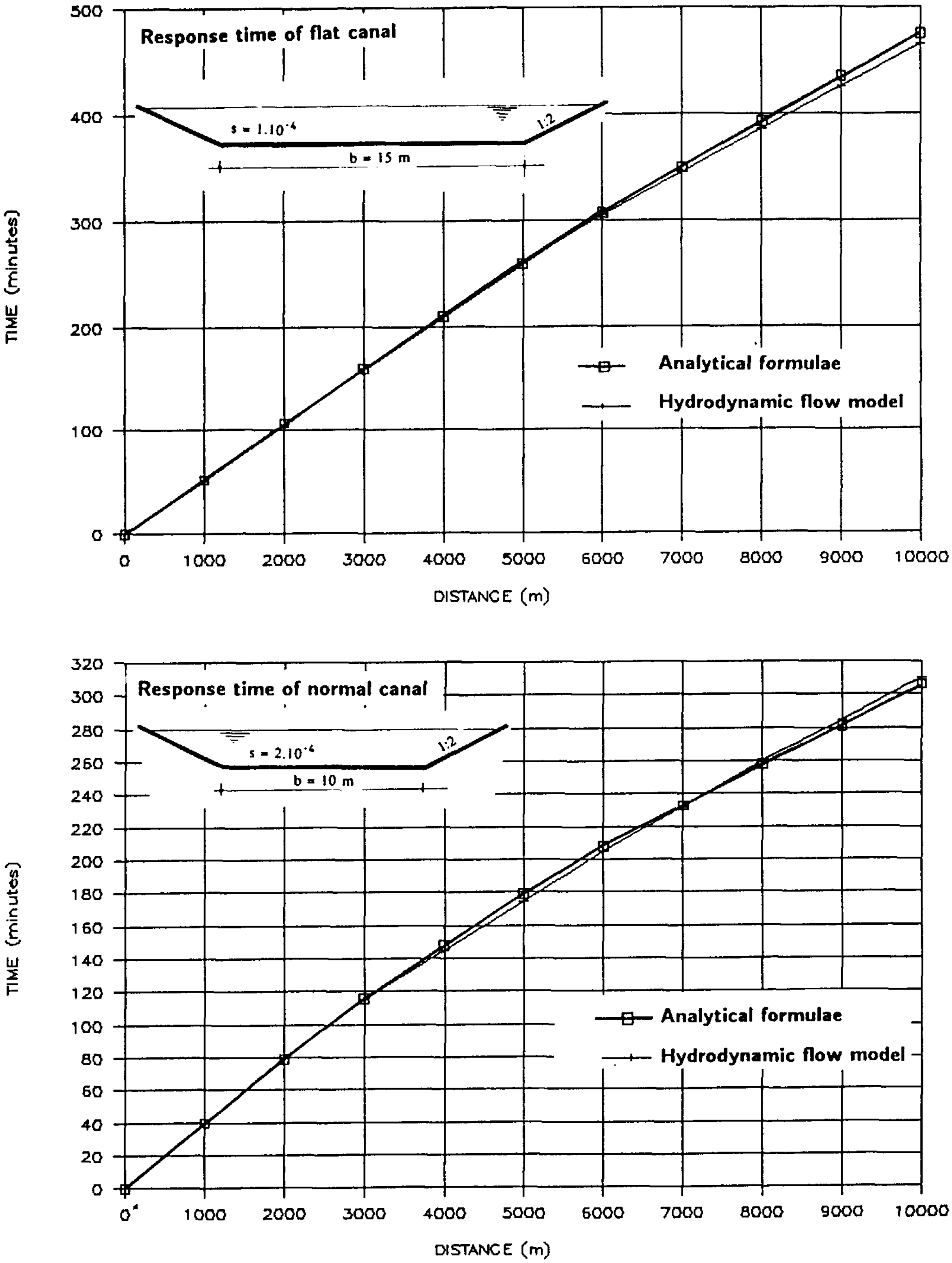


Fig. 2.2 Propagation of a variation in discharge according to the diffusion equation and a hydrodynamic flow model

outflow structure is used, the variation of water level will generally be higher than for a weir type of outflow structure. Consequently, the response time will be longer if an orifice is used rather than a weir. This result can also be read from Eq. (2.18): a higher u-value yields a shorter response time.

If the canal is not bounded by a check structure, the non-uniformity of the flow will have to be taken into account in order to determine the variation of the outflow rate at an imaginary downstream boundary. In order to do so, the momentum equation has to be incorporated. This approach leads to the diffusion equation. Basically, the response time is still determined by the additional volume that has to be filled or emptied in order to reach a new steady state. The magnitude of this volume is affected by the top width of the canal, the length of the canal and the variation of water depth. The variation of the water depth in turn, is determined by the resistance factor, the canal profile and the bed slope of the canal. All these factors are incorporated in the diffusion coefficient D and the diffusion celerity φ.

The system response time is hardly affected by the magnitude of variation in flow rate at all, because the relationship between additional inflow discharge and additional storage volume is almost linear for relative small changes. In other words, the magnitude of variation in flow rate is compensated by the required storage volume to reach a new steady state.

Misleading formulae are often used in practice and they often underestimate the response time. There is no physical relationship between the propagation celerity of large disturbances such as a variation in discharge and the flow velocity in the canal. The critical celerity or the celerity of infinitesimal disturbances ($\approx \sqrt{gh}$) is also not related to the propagation celerity of disturbances in discharges in canals with friction. (Apart from bores which, however, hardly ever occur in irrigation canals).

2.5 Method of characteristics

2.5.1 Introduction

In the previous sections two analytical formulae were discussed for the computation of the system response time after a variation in discharge. Another method to find solutions of the De Saint Venant Equations is to convert the partial differential equations into a set of ordinary differential equations. This method is called the method of characteristics.

According to Abbott (Abbott 1975) the method of characteristics may be described as, *"a technique whereby the problem of solving two partial differential equations can be replaced by the problem of four ordinary differential equations"*. This description is true for the De Saint Venant Equations, but, in general, every set of partial differential equations can be transformed into ordinary difference equations, by using the method of characteristics. The set of differential equations thus obtained do not have to include any simplifications, although in practice, simplifications are often introduced, e.g. rectangular cross-sections without resistance. The characteristics presented here (hardly) do not include any simplification.

To solve the ordinary differential equations, graphical or numerical methods of solutions will have to be adapted. Up to now no exact solution has been found: yet it is the most accurate method of solution. Another advantage of using characteristics is that they are physically interpretable and, hence, they provide insight in the unsteady flow phenomena. Furthermore, the numerical and graphical solution method of the ordinary differential equations have many similarities with the numerical integration method of the De Saint Venant Equations, to be discussed later on. The method of characteristics was extensively discussed in (Abbott 1979) and (Cunge 1980).

2.5.2 Characteristic equations

The De Saint Venant Equations, Eq. (2.1) & (2.2), consist of two partial differential equations with two dependent variables (e.g. A and Q), which vary both in time and place. So, four partial differentials can be distinguished: A_t, A_x, Q_t and Q_x, where the subscripts t and x refer to the partial derivatives with respect to time and place. To obtain a unique solution for the partial differentials, two additional equations are needed. The additional equations are found by stating that the dependent variables Q and A are varying with the independent variables x and t only. The resulting set of equations thus obtained read:

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = q \tag{2.57}$$

$$\frac{\partial Q}{\partial t} + 2\beta \frac{Q}{A}\frac{\partial Q}{\partial x} + c^2(1-\beta Fr^2)\frac{\partial A}{\partial x} = W \tag{2.58}$$

$$\frac{\partial A}{\partial x}dx + \frac{\partial A}{\partial t}dt = DA \tag{2.59}$$

$$\frac{\partial Q}{\partial x}dx + \frac{\partial Q}{\partial t}dt = DQ \tag{2.60}$$

Where D represents the total differential. For the meaning of the other symbols reference is made to paragraph 2.2.

In order to eliminate the partial differentials, we are looking for directions in which the partial differentials are not defined. These directions are called the primary characteristics or characteristics. They can be pictured as lines in the x-t field, along which the partial differentials, A_x, A_t, Q_x and Q_t, do not exist. Instead, ordinary differentials which replace the original partial differentials are valid. The ordinary differentials are called the secondary characteristics or characteristic relationships. In other words, a new, natural, reference system is formulated, which is called the characteristics. On this new reference system the original partial differential equations are converted into ordinary differential equations; the secondary characteristics.

The mathematical derivation of the characteristics, out of the set of four partial differential equations, is given in Appendix D. The characteristics and the characteristic relationships, expressed in Q and A, read:

$$\frac{dx}{dt} = c\,(Fr \pm 1) \tag{2.61}$$

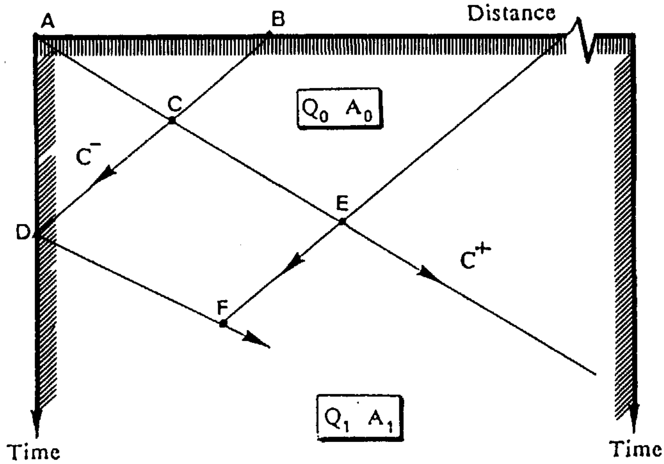
$$dQ = c\,(Fr \mp 1)\,dA + W\,dt \tag{2.62}$$

In the presented characteristic relations it has been assumed that the lateral inflow is zero and that the velocity distribution across the wetted area does not substantially affect the wave propagation so that $\beta = 1$. These are the only and not necessarily simplifications. For the rest they are equivalent to the original De Saint Venant Equations. When the resistance term W is equal to zero, the secondary characteristics are referred to as the Riemann invariants. To solve the characteristic equations, graphical or numerical methods of solution can be applied. In the following the graphical method of solution will be explained, as this method gives the best insight in the physical process and frames the basis of computational hydraulics.

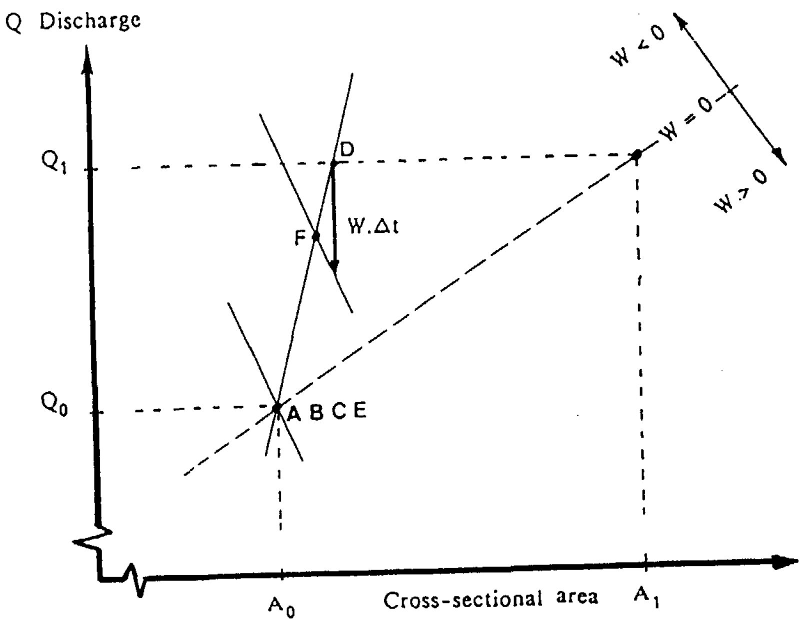
2.5.3 Computations by means of characteristics

Graphically the characteristics, Eq.(2.61), can be represented as lines in the x-t field. When the flow is subcritical, the Froude number is smaller than one, both a positive and a negative characteristic exist. Along these lines the secondary characteristics, Eq.(2.62), are valid. The secondary characteristics specify a relationship between Q and A.

In Fig. 2.3 the characteristics are shown. The time axis in the x-t field is directed downwards, so that negative and positive characteristics have the same direction as the corresponding secondary characteristics in the Q-A field. Unfortunately, both the characteristics and the secondary characteristics are curves and not straight lines, because the slopes of the lines are varying with Q and A. In a numerical and graphical computation straight lines are normally assumed. This assumption leads to small errors as long as the variation in Q and A is small, which is generally the case when short space steps Δx and time steps Δt are used. Beware that this simplification is not inherent to the characteristic equations, but only to the numerical or graphical solution.



x-t field



Q-A diagram

Fig. 2.3 The method of characteristics

■ **Condition in point C**

To determine the condition (Q,A) in a point C located in the x-t field of Fig. 2.3, two characteristics crossing point C have to be determined. Along these characteristics, corresponding (secondary) characteristic relationships are valid. When the positive characteristic through AC is called C+ and the negative characteristic through BC is called C-, and the conditions in point A and B are known, then the condition in point C can be graphically determined by crossing the corresponding secondary characteristics of C+ and C-. In the Q-A diagram the secondary characteristics through point A and B are shown. It can be seen that the condition in point C is equal to the condition in A and B as the initial conditions of both points are equal. A and B have the same discharge and cross-sectional area.

■ **Condition in point D**

To determine the condition in a point D, located at the left boundary of the x-t field, the same procedure as described above is followed. The condition in point D is determined by the characteristic through CD and the boundary condition. In this case the boundary condition is a discharge Q_1. In Fig. 2.3 this boundary condition is represented as a line with a constant Q in the Q-A diagram. The condition in point D is found by crossing the secondary negative characteristic of point C with the boundary condition in point D.

■ **Condition in point E**

The condition in a point E (Fig. 2.3) is determined by applying the same procedure as for point C.

■ **Condition in point F**

The condition in point F is somewhat more difficult to determine, as the resistance has to be taken into account. In the x-t diagram it is read that the condition in F is determined by the characteristics through D (the positive characteristic) and E (the negative characteristic). The condition in point F is then found by crossing the corresponding secondary characteristics in the Q-A field. However, in point D the resistance W is less than zero, Eq.(2.5). It can be read from Eq.(2.62), that $W.\Delta t$ must be added to the secondary characteristic. A negative W graphically means a lowering of the secondary characteristic.

From the example computation it becomes clear that for subcritical flow conditions, the condition in every points can be determined when an initial condition and two boundary conditions are given (one at both sides). The initial condition specifies the Q and A at every location at the initial time level. The boundary conditions specify Q, A or any Q-A relationship along the x-boundaries of the x-t field for any moment in time.

2.5.4 Physical interpretation of the characteristics.

The primary characteristics represent the celerity of an infinitesimal disturbance. This is the highest celerity with which a disturbance can propagate through water. This implies, for example, that information cannot be transferred to upstream locations when the flow velocity is greater than or equal to the critical celerity. Therefore, the flow condition for super critical flow is determined by the upstream boundary condition only, as the downstream boundary can not influence the flow upstream.

Mathematically, a non existing of the partial derivatives indicates a discontinuity. Physically a discontinuity in A and Q is manifested as a hydraulic jump or bore. It will be shown that the celerity of a bore with infinitesimal head equals the characteristic velocity of the undisturbed water.
The celerity of a bore has been computed in Appendix E, and reads

$$u = v_2 \pm c_0\,\mathrm{f}(a,n) \tag{2.63}$$

Where u is the celerity of a bore, v_2 is the downstream flow velocity, c_0 is the characteristic velocity of the undisturbed flow and the function f(a,n) reads,

$$\mathrm{f}(a,n) = \sqrt{\frac{n_1^a}{n_2^a}\,\frac{a}{a+1}\,\frac{n_1^{a+1}-n_2^{a+1}}{n_1^a-n_2^a}} \tag{2.64}$$

The coefficient a is the hydraulics exponent of Bakhmeteff and follows after approximation of the trapezoidal profile by a parabolic canal profile (Appendix C). The coefficients n_1 and n_2 represent the ratio of the actual mean water depth and a reference mean water depth upstream and downstream of the bore. The value of the function f{a,n} is less than one but tends to one if n_1 equals n_2. The celerity of the bore then equals the characteristic celerity:

$$\lim_{n_1 \to n_2} u = v_2 \pm c_0\,\mathrm{f}(a,n) = v \pm c \tag{2.65}$$

Summarized, it is stated that the characteristics celerity can be considered as the celerity with which information is passed on to a neighbouring location. Using the method of characteristics can be interpreted as using a natural reference system, on which the original partial differential equations are replaced by ordinary difference equations which can be more easily analyzed. The graphical computation, carried out with the characteristics, can also be done numerically. In that case, one speaks of numerical integration by means of characteristics.

Integration by means of characteristics is the most accurate method, but not the most convenient one. Inconvenience is caused by two reasons. The first one is that interpolation in time and place has to be carried out in order to obtain results on a fixed time and space grid. Furthermore, the time step is bounded by the characteristic celerity. In practice larger time might be preferred although the accuracy will become less.

If a larger time step is applied, the computation will become unstable, which implies that the numerical solution does not converge to the real solution. Numerical instability can be avoided by applying unconditionally numerical integration schemes, the so-called implicit schemes.

2.6 Numerical integration methods

2.6.1 Introduction

Besides integration, by means of characteristics, other integration methods can be used to solve the De Saint Venant Equations. The finite difference method replaces the partial differential equations by finite difference equations. This method is less accurate than the method of characteristics, but more convenient in use. As a result, the finite difference methods are most widely applied in hydrodynamic flow models

In this section the basic principles of computational hydraulics will be briefly discussed. The term "computational hydraulics" was introduced some 20 years ago at the 13th Congress of the International Association for Hydraulic Research, Kyoto, to describe *"a hydraulics that was formulated in such a way as to suit it specifically to the ways of working of digital computers"* (Abbott 1990). The user of a hydrodynamic model needs some basic understanding of computational hydraulics in order to make a reliable interpretation of the results and to select the best value of some computational parameters which affect the computation speed, memory allocation and accuracy of the computation. To gain some understanding of computational hydraulics, the terms consistency and convergency have to be explained.

2.6.2 Consistency and convergency

In a numerical computation only difference equations can be used, whereas the De Saint Venant Equations are continuous differential equations. Cantor proved in 1873 that there is a non-equivalence between both methods of representation. Because of the non-equivalence between both equations, the numerical solution can only be an approximated solution. In a numerical computation two conditions should be met: (1) the differential equations and difference equation should be consistent with each other, and (2) the solution of the difference equation should converge to the real solution.

- **Consistency**

If one substitutes the analytical solution of the differential equation in the difference equation, a rest term will appear. This rest term is called the truncation error and is a direct result of the difference between the differential and difference equation. The magnitude of the truncation error can be examined by expansion of the differential equation in Taylor series. The difference equation is called consistent with the differential equation, when the truncation error tends to zero as the step size of the independent variables, Δx and Δt, tends to zero. Difference equations with a high order truncation error are more accurate as the truncation error is smaller.

■ **Convergency**
If one considers the difference equation only, it is found that the solution of the difference equation does not always converge to the real solution, but it can oscillate around the real solution with increasing magnitude. The solution is then said to be unstable, and this phenomenon is denoted as numerical instability. In this study, stability is defined as non- amplifying oscillations in time. It can be proved (Richtmeyer 1967) that Lax's Equivalence theorem is valid: *"Given a properly posed initial-value problem and a finite-difference approximation to it that satisfies the consistency condition, stability is the necessary and sufficient condition for convergence"* (Lax 1954). In other words, stability ensures convergency, if certain conditions are met.

Consistency and convergency are easily mixed up. Richtmyer expressed the difference between consistency and convergency in the form of a paradox, as follows (Richtmeyer 1967): *"If the mesh is refined, but in such a way as to violate the stability condition, the exact solution of the differential equation comes closer and closer to satisfy the difference equations, but the exact solutions of the difference equations departs, in general, more and more from the true solution of the initial value problem"*.

Summarized, it can be stated that only difference equations can be used in a numerical computation, whereas the original equations to be solved are differential equations. To guarantee a realistic solution of the numerical computation, two vital requirements are to be met: the difference equation should be consistent with the differential equation, and the solution of the difference equation should converge to the real solution. The latter condition is satisfied when a stable solution is found.

2.6.3 Finite difference methods

Among all the methods available for computing function value at a new time level, only the linear multi-step methods or finite difference methods shall be considered, as they are most commonly used in simulation programs. It will be demonstrated that the numerical scheme underlying the developed flow model, the Preissmann scheme (Preissmann 1961), reaches the optimal finite difference method.

A finite difference method is a numerical method which computes new function values based on old values and first order derivatives (Otten 1982, pp 49):

$$y^{n+1} = \sum_{i=0}^{i=p} a_i\, y^{n-i} + \sum_{i=-1}^{i=p} \Delta t\, b_i \left[\frac{dy}{dt}\right]^{n-i} \tag{2.66}$$

Where

y^{n+1}	=	new function value at time level n+1
y^n	=	old function value at time level n
a,b,p	=	coefficients
i	=	variable
Δt	=	time step

The forward euler ($p=0$, $a_0=1$, $b_0=1$),

$$y^{n+1} = y^n + \Delta t \left[\frac{dy}{dt}\right]^{n} \tag{2.67}$$

the backward euler ($p=0$, $a_0=1$, $b_{-1}=1$),

$$y^{n+1} = y^n + \Delta t \left[\frac{dy}{dt}\right]^{n+1} \tag{2.68}$$

and the trapezoidal method ($p=0$, $a_0=1$, $b_0=½$, $b_{-1}=½$)

$$y^{n+1} = y^n + \frac{1}{2}\Delta t \left[\frac{dy^{n+1}}{dt} + \frac{dy^{n}}{dt}\right] \tag{2.69}$$

are all special cases of the finite difference method.

If $b_{-1} = 0$, the method is said to be an explicit method, whereas for all other cases, the method is called an implicit method. In an implicit method of solution the function value at the new time level cannot be computed explicitly out of the function value at the old time level. It will be demonstrated that the explicit methods are not unconditionally stable. With the multistep method every order of accuracy can be obtained, using more and more time points.

- **Stability**

Dalquist (Dalquist 1959) demonstrated that only the implicit finite difference methods up to order two are unconditional stable or A-stable. The order of the finite difference method is defined as the number of time levels, which are taken into consideration to compute a variable at the new time level. Hence, if an unconditionally stable scheme is demanded, the highest possible order of accuracy is equal to two.

- **Consistency**

Examining the consistency, it can be proven by expansion in Taylor series that properly posed finite differences are consistent with the differential equations (Vreugdenhill 1985).

Consistency and stability are vital conditions to be met. However, one would also like to have a method whereby the truncation error is small, even for larger time and space steps. This can be achieved by using a difference equation with a high order truncation error. The order of the truncation error is said to be equal to u if the truncation error is proportional with Δx^u or Δt^u. Although the number of computation per step generally increases for higher order methods, the savings in total computation time increases for higher order methods. Taking into account the stability criteria, the most accurate unconditional stable difference scheme has an truncation error of second order. More accurate difference schemes cannot be unconditionally stable.

The advantage of a more accurate method of solution is that larger time steps can be taken, so that the computation becomes faster. However, the time step which is used in the numerical computation is not always determined by accuracy considerations. For example, if automatic control systems are included, the maximum time step can be described by the controller rather than by accuracy constraints. In that case, the possible gain of higher order methods will be limited.

Although no attention has been paid to the development of an optimal set of difference equations, as this would go beyond the scope of this study, some remarks about the development of the ideal set of difference equations have to be made. The exponential fitting method of Liniger (Stelling 1979) would reach the ideal method as this method is unconditionally stable and always of second order accuracy. Another, and probably more promising, potential improvement of the numerical integration method for the De Saint Venant Equations would be the introduction of a variable time step, whereby the step size is selected during the ongoing computation by the model itself, on basis of a predefined accuracy constraint. However, problems may arise with respect to the stability of the computation. Furthermore, it seems not efficient to compute for every time step a new computational time step, as this would cost too much computation time.

2.6.4 The Preissmann scheme.

The Preissmann scheme, a well known finite difference method in computational hydraulics, has been used in the MODIS model. The consistency, stability and accuracy of the Preissmann scheme have been extensively described in a vast amount of publications e.g. (Preissmann 1961), (Abbott 1979, pp 183), (Cunge et. al. 1979) and (Stelling 1979). The Preissmann scheme is unconditionally stable and of second order accuracy in space and usually of first order accuracy in time. The scheme is depicted in Fig. 2.4.
Basically, the computation of space and time derivatives are equal to the trapezoidal method Eq.(2.69), and read on a grid as shown in Fig. 2.4 as follows:

$$\frac{\partial Q}{\partial x} \approx \frac{Q_j^n - Q_{j-1}^n}{\Delta x} + \theta \frac{(\Delta Q_j - \Delta Q_{j-1})}{\Delta x} \tag{2.70}$$

$$\frac{\partial Q}{\partial t} \approx (1-\psi)\frac{\Delta Q_{j-1}}{\Delta t} + \psi \frac{\Delta Q}{\Delta t} \tag{2.71}$$

Where:

$$\Delta Q_j - Q_j^{n+1} - Q_j^n \tag{2.72}$$

n = superscript for time level
j = subscript for space location
θ = a weighting coefficient for distributing the space derivatives over two successive time levels
ψ = a weighting coefficient for distributing terms in space.

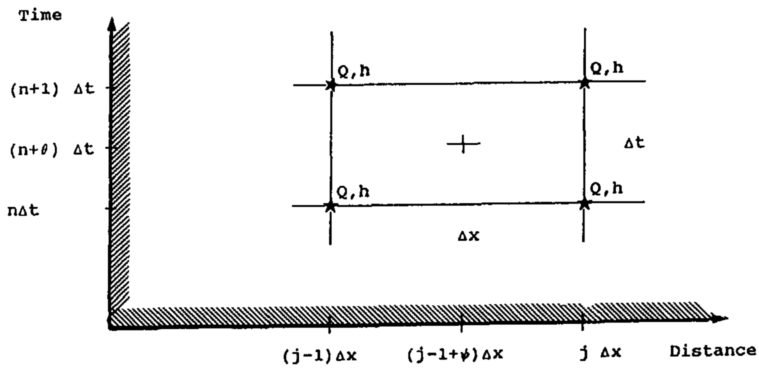


Fig. 2.4 The Preissmann scheme

The major feature of the Preissmann scheme compared with the Abbott-Ionescu scheme (Cunge 1979) or Cranck-Nicolson scheme (Vreugdenhil 1985) is its simple structure with both a flow and a geometrical variable in each grid point. This implies a simple treatment of boundary conditions and a simple incorporation of structures and bifurcation points. Furthermore, if $\psi = 0.5$, the method will always be of second order accuracy in space, even for variable branch lengths.

2.6.5 User defined parameters

The user of a numerical computer model himself has to define values of some computational parameters. By doing so, the computation speed, memory allocation and accuracy of the calculation are influenced. A balance has to be found between computation speed and memory allocation on the one hand, and accuracy on the other. The most apparent user defined parameters are the mesh size Δx and the time step Δt. The computations are most accurate when the value of both parameters satisfy the Courant-Lewy-Friederichs condition:

$$Cr - (v \pm c) \bullet \frac{\Delta t}{\Delta x} - 1 \tag{2.73}$$

In that case, the finite difference method reaches the method of characteristics. Furthermore, higher accuracy will be obtained if smaller time and space steps are used. For studies dealing with long waves and gradually varied unsteady flow, larger steps can generally be used than for rapidly changing processes. A practical method to derive sound space and time steps is to repeat a computation with a reduced step size until no more significant changes in results are found.

Finally, the user can manipulate the rate of numerical diffusion, a special type of truncation error, by varying the time interpolation coefficient θ from 0.5 to 1.0. When the time interpolation coefficient is smaller than 0.5 the method is not unconditionally stable anymore. When the time interpolation coefficient equals 0.5, the most accurate solutions are found as the method becomes of second order accuracy in time. However, there are practical reasons, e.g. damping of undesired fluctuations, for using a slightly uncentered time interpolation coefficient (e.g. 0.5).

The computation is not mass conservative for time interpolation coefficients greater than 0.5. This characteristic can be used to reach a steady state in a closed system with an excess of water. The excess of water in the system can be drained by setting θ equal to 1.

2.7 Summary and Conclusions

The mathematical description of gradually varied unsteady flow is provided by the De Saint Venant Equations. These are two partial differential equations of first order, expressing the conservation laws of mass and momentum. Unfortunately, analytical solutions are difficult to find. Three methods are available to find approximated solutions. Each method has its own field of application.

The analytical methods are extremely useful for a preliminary study of the unsteady flow, and to check the outcome of the other methods. Two analytical solutions were derived for simplified equations in order to compute the response time of an open canal after a change in discharge. It was found that the response time can be considerable for larger canal systems. The response time is mainly determined by the volume required to transit from one steady state into another. This volume generally increases almost linear with the variation of inflow discharge and, consequently, the response time is hardly influenced by the rate of change in discharge at all. The response time can be expected to increase when the resistance increases, or the bed slope becomes milder, or the outflow structure creates more set up.

The most accurate method to analyze the De Saint Venant Equations is the method of characteristics. Due to the graphical solution technique of the characteristic equations, the method also provides insight in the unsteady flow phenomena, and numerical integration. The method of characteristics converts the original partial differential equations into a set of ordinary difference equations without simplifications. Physically, the primary characteristics represent the celerity of an infinitesimal disturbance, which is the maximum celerity of a disturbance.

Although the method of characteristics is the most accurate one, it is not the most convenient method for numerical integration. Finite difference methods prove to be more easy in use, mainly because a fixed grid can be used without having to interpolate, and because larger time steps can be used without violating the stability conditions. It can be demonstrated that the most accurate unconditionally stable finite difference method has a second order of accuracy in time. The Preissmann scheme, which underlies the computer model which has been developed, approaches the most accurate finite difference method.

In this chapter emphasis has been put on the computation of unsteady flow in open canals. In the next chapter attention will be paid to the developed modelling package which computes the unsteady flow in canals that are (real time) controlled. With this modelling package irrigation canal systems can be modelled and the (unsteady) flow phenomena in the system can be analyzed numerically.

3 The flow model

3.1 Introduction

To study the hydraulic performance of controlled irrigation canals, a new model has been developed. This model is called MODIS, an acronym of "*Mo*delling *D*rainage and *I*rrigation *S*ystems". The MODIS model distinguishes itself from existing unsteady flow models by an accurate computation of flow through a wide variety of structures, and the possibility to control these structures according to various operational scenarios. Furthermore, the MODIS model includes a post processor to compute operation performance indicators reflecting the operational performance of the system. These characteristics make the MODIS model a unique model. In appendix F a description of the model is presented.

This chapter is structured as follows. First, the historical development of flow models will be reviewed in order to position the MODIS model in the variety of existing flow models. As flow models do already exist, it was decided that the MODIS model could use one of the existing models as a computational base. To select the best available existing flow model for further development and to review the capabilities of the existing flow models, a comparative study of existing hydrodynamic flow models has been conducted. The results of this study are presented in paragraph 3.3. In subsequent paragraphs the adapted modules of the model for the canal flow (paragraph 3.4), structure flow (paragraph 3.5), control of regulators (paragraph 3.6), and the module for the operation performance parameters (paragraph 3.7) will be discussed. The chapter is closed by a summery and conclusions.

3.2 History of flow models

In the old days, scale models or simplified equations had to be used in order to find solutions of the De Saint Venant Equations, which are known for more than a century. Numerical integration is another possibility to find solutions, but this requires a vast amount of repetitive computations. Nevertheless, the first numerical model was used without having computer facilities. This was in 1926, when Lorentz build the Zuiderzee model to compute the effect of closing the "Zuiderzee" estuaria (Lorentz 1926). The introduction of the computer made it possible to execute the required computations in an acceptable period of time. By now, numerical flow models (also called hydrodynamic flow models or simply flow models) have replaced models (also called hydrodynamic flow models or simply flow models) have replaced most other solution methods.

It is common to distinguish different generations of flow models. The first generation, then, is associated with models which do the same thing as human beings did without having computers. Later on it was observed that a procedure that was 'human friendly' was usually very sub-optimal, compared to machine-friendly methods. The second generation lasted from 1960 till 1970. During this time models were built for a specific problem. An example of a second generation model is the model of the Mekong Delta built in the mid 1960s. At that time, Delft Hydraulics Laboratory was competing for the job with a scale model of the Mekong Delta, and they lost it. The third generation models were 'Modelling Systems', built by specialized modelling centres building models on the basis of already programmed standard procedures. As the number of users increased, more investments could be made to develop fast and reliable standard routines.

Fourth generation flow models became possible with the introduction of IBM's PC/AT computers. The standardization of hardware and software (standard Fortran) made it possible to produce Modelling packages for a "mass market". A modelling package is used to make a model for a specific project. The fourth generation models make use of routines developed for the third generation models but more attention is paid to the user-friendliness. The development costs of fourth generation models are in the order of million dollars, whereas the purchase price is in the order of ten thousand dollar.

In the future Abbott (Abbott 1990) is expecting fifth generation models. Fifth generation models are more than simulation models. They encapsulate simulation models, but also optimisation procedures, expert systems and data bases with regulations and constraints. Abbott calls such a model a hydroinformatics system.

Summarized, one can state that the first three generations were mainly focused on the computational aspects. Fourth generation models are mainly an extension in the direction of user-friendliness and presentation of results. Future fifth generation models will integrate the simulation module in a hydroinformatics system, in which optimization will play an important role. The MODIS model is essentially a fourth generation model. It is a modelling package simulating the water flow in irrigation systems. Besides, it has some characteristics of a fifth generation model as it not only simulates the flow but also interprets the results of simulations. In this way it not only suitable for (irrigation) engineers but also for operators of irrigation systems.

3.3 Comparative study

3.3.1 Introduction

In the Netherlands, numerous flow models have been developed for the computation of one dimensional gradually varied unsteady flow in rivers and drainage channels. These models can be used as a basis for the model required for the simulation and evaluation of unsteady flow in controlled (irrigation) canals. To select the best available model out of the existing models, and to review the potentiality of the existing models, a comparative study based on vocal interviews with the developers or frequent model users has been conducted. Seven flow models were selected and compared on subsequently input, output, computation, software and miscellaneous features.

3.3.2 Results

The outcome of the study has been published (Schuurmans 1988) and is presented in Table 3.1. One year later, a similar study has been conducted by SAMWAT (Cooperation of water Boards). The results they obtained fortify the results found earlier.

Table 3.1 should be read a bit carefully, as the answers given are not always reliable, those related to the computational aspects in particular. Many users of flow models were not familiar with the numerical terminology and the mathematical backgrounds of their model.

3.3.3 Discussion

Examining Table 3.1, it is noticed that on many aspects the models are similar. All models are based on the De Saint Venant Equations, and they all apply the same kind of numerical solution of these equations, namely finite differences. The output facilities of the models are also, more or less, the same. They all produce tabulated and graphical output. Furthermore, nearly all models have been written in Fortran computer language.

Within the framework of similarity, many differences exist. The method of defining the canal system, for example, varies considerably in complexity. Some models automatically generate a computational grid over the user defined canal configuration, whereas others lack this feature. Without having an automatic grid generation, already in the modelling phase, the user has to be aware of numerical aspects, as it is laborious to alter the numerical space step in a later stage. Furthermore, not all models have (extensive) error and consistency control facilities.

Table 3.1 Comparison of flow models (Schuurmans 1988)

Criterion \ Model name	Rubi con	Ewas	Ribasi	Knota	Hydra	Wa flow	Net flow	Du flow
INPUT								
Interactive data input	no	no	yes	no	yes	no	yes	yes
File input with the help of an editor	yes	yes	yes	yes	yes	yes	yes	yes
File input unformatted (no specific position for each input variable	yes	no	no	yes	yes	yes	yes	yes
Error checking of input data	yes	no	yes	no	no	no	yes	no
Interpolation facilities for input data	yes	no	yes	no	yes	yes	yes	yes
Maximum number of branches	mem.	100	75	100	1500	mem	mem	200
Maximum number of branches connected to one node	mem	4	mem	4	mem	4	4	mem
Name input possible for nodes and branches	yes	no	no	no	yes	no	yes	no
Restriction for ratio of branch lengths	no	yes	yes	yes	no	no	no	no
Possible to add user defined structures	yes	no	no	no	no	no	yes	no
Possible to operate structures in time	yes	no	no	no	yes	yes	yes	yes
Possible to place structures parallel	yes	no	no	yes	no	yes	yes	yes
Cross-sectional data variable with water depth	yes	no	yes	no	no	yes	yes	no
Easy modelling of trapezoidal cross-sections	no	yes	no	yes	yes	no	no	no
Possible to continue with result of a previous computation	yes	no	no	no	yes	yes	yes	no
OUTPUT								
Formatted output of input data	yes	yes	yes	yes	yes	no	yes	yes
Graph of output data on plotter	yes	yes	yes	no	yes	yes	yes	no
Output at any location at any time	yes	no	yes	no	yes	yes	yes	yes
Output as function of time and place	yes	no	yes	no	yes	yes	yes	yes
COMPUTATION								
Steady computation possible	no	no	no	no	yes	no	no	no
Unsteady computation possible	yes	yes	yes	yes	yes	yes	yes	yes
Complete De Saint Venant Equations	yes	yes	yes	yes	yes	yes	yes	yes
Implicit numerical scheme	yes	yes	yes	yes	yes	yes	yes	yes
Name of scheme	Pr	Vr	?	?	?	CN	CN	Pr
Method of solution of equations	Gaus	?	?	Gaus	Gaus	?	?	Gaus
Variable time step during a simulation	yes	no	no	no	no	no	yes	no
Easy variation of space step	yes	no	no	no	no	yes	yes	no

Criterion \ Model name	Rubi con	Ewas	Ribasi	Knota	Hydra	Wa flow	Net flow	Du flow
Water level and discharge computed at each node	yes	no	yes	yes	yes	no	no	yes
SOFTWARE								
Model built up of well defined subsystems	yes	yes	no	no	no	no	no	yes
Computer language of source code	F'77	F'66	F'77	F'77	F'77	F'77	F'77	F'77 & B
Users manual of program available	yes	no	yes	no	yes	yes	yes	yes
System documentation available	yes	no	no	no	no	yes	yes	no
OTHERS								
Model running on a Personal Computer	yes	no	yes	yes	no	yes	no	yes
Minimum required RAM-memory	256	-	520	-	640	-	-	640
Lines of source code *1000	10	?	6	1.5	100	6	30	?
Year of development	'84	'78	'80	'78	'81	'84	'76	'89
Price of executable version (US$)	8000	free	5000	250	not sold	8000	not sold	125
Price of source code (US$)	15000	free	not sold	250	not sold	not sold	not sold	250

Legend

mem	= memory limitation	?	= not known
CN	= Cranck Nicholson	Vr	= Vreugdenhil
Pr	= Preissmann	Gaus	= Gauss-seidel iteration
B	= Quick Basic	F'66	= Fortran 66
F'77	= Fortran 77		

Although all reviewed models use the finite differences technique to solve the De Saint Venant Equations, no model uses exactly the same numerical scheme. Even if the same type of numerical scheme is used, e.g. a Preissmann scheme, still small differences will exist. Apparently, there is no generally accepted optimal numerical scheme for the numerical integration of the De Saint Venant Equations.

3.3.4 Selection

On the basis of a multi-criteria analysis, presented in Table 3.2, Rubicon was selected as the best available model for further development. For the analysis, the same criteria were used as for Table 3.1. The main features of Rubicon are its modular structure and detailed software documentation, which makes it possible to extend the model. Furthermore, the model is reasonably user-friendly due to the unformatted model input, which implies that input data have to follow a pre-described sequence; they do not have fixed positions. Finally, the model has extensive error and consistency checking facilities, an automatic generation of a computational grid, and it is available on a Personal Computer.

The flow model that has been developed by using Rubicon as a base model, is named MODIS. MODIS is an acronym for "Modelling Drainage and Irrigation Systems". A number of papers describing the MODIS model have been published (Schuurmans 1990c), (Schuurmans 1991a), Appendix F. In the following paragraphs the computational heart of the MODIS model and its modifications will be discussed. The computational heart comprises four modules: a module for the computation of the unsteady canal flow, a module for the flow through structures, a module for control of the regulators, and a module for the computation of operation performance parameters. Each of them will subsequently be discussed.

Table 3.2 Weighing the flow models

Model / Criteria as per Table 3.1	Rubicon	Ewas	Ribasi	Knota	Hydra	Waflow	Netflow	Duflow
Input	+	-	+	-	+	+	+	+
Output	+	0	+	-	+	+	+	+
Computation	+	0	+	0	+	+	+	+
Software	+	-	-	-	0	+	+	0
Others	+	-	+	+	-	+	-	+
Total	5+	3-	3+	2-	2+	5+	3+	4+

+ : Good
0 : Average
- : Poor

3.4 The unsteady canal flow module

3.4.1 General

The mathematical equations and the method of solutions of unsteady canal flow have been extensively discussed in the previous chapter. It was concluded that the most convenient method of solution is numerical integration of the De Saint Venant Equations by finite differences. The applied numerical scheme should satisfy the conditions of consistency and convergency, whereby the latter can be replaced by stability. The Preissmann scheme, a well known and widely applied unconditionally stable scheme, is used in the MODIS model. The Preissmann scheme is of second order accuracy in space and usually of first order accuracy in time. The basic principles of the Preissmann scheme were discussed in paragraph 2.6.4. In this paragraph, the finite difference equations, which are solved by the flow model, are presented. In addition, the treatment of the non-linear terms are discussed. The convective acceleration term was originally computed explicitly, but in the MODIS model it has been rewritten in an implicit form. Finally, dry bed flow, supercritical flow and leakage losses are discussed. For more detailed information about the MODIS model and its computational background, reference is made to its user's guide (Appendix F).

3.4.2 The difference equations

Both the equation of continuity and the equation of momentum of the De Saint Venant Equations are partial differential equations of the first order. To solve this set of two partial differential equations, the partial differentials are replaced by finite difference equations, which can be solved numerically. In the transformed equations two unknowns can be distilled for every computational grid point, namely ΔQ and Δh representing the increment during one time step Δt. (In the equations presented in the previous chapter, Eq.(2.1) & (2.2), the dependent variables were Q and A rather than Q and h, see also Annex A). The new values for Q and h at the new time level become equal to the values at the previous time level plus the increments.

▪ Continuity equation

The equation of continuity, expressed in Q and h as dependent variables reads (Appendix A):

$$\frac{\partial Q}{\partial x} + b_s \frac{\partial h}{\partial t} = q \qquad (3.1)$$

The discretized form of this equation using the Preissmann scheme reads:

$$\frac{Q_j - Q_{j-1}}{\Delta x} + \theta \frac{\Delta Q_j - \Delta Q_{j-1}}{\Delta x} + (1-\psi)\, b_{s(j-1)}^{n+1/2} \frac{\Delta h_{j-1}}{\Delta t} + \psi\, b_{s(j)}^{n+1/2} \frac{\Delta h_j}{\Delta t} = q_{j-1/2}^{n+1/2} \qquad (3.2)$$

▪ Momentum equation

The momentum equation is discretized in a similar way. Again the dependent variables of Eq.(2.2) are replaced by Q and h rather then Q and A (Appendix A). The resulting equation is written as:

$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x}(\beta \frac{Q^2}{A}) + gA \frac{\partial h}{\partial x} + \frac{g\, Q\, |\, Q\, |}{k^2 A\, R^{4/3}} = 0 \qquad (3.3)$$

and is discretized according to the Preissmann scheme as:

$$
\begin{aligned}
&(1-\psi)\,\frac{\Delta Q_{j-1}}{\Delta t} + \psi\,\frac{\Delta Q_j}{\Delta t} \qquad + \\
&\frac{\left[(\frac{\beta}{A})^{n+1/2}\;Q^n\,(Q^n+\Delta Q)\right]_j - \left[(\frac{\beta}{A})^{n+1/2}\;Q^n\,(Q^n+\Delta Q)\right]_{j-1}}{\Delta x} \qquad + \\
&\%^{n+1/2}_{j-1/2}\left[\frac{h_j - h_{j-1}}{\Delta x} + \theta\,\frac{\Delta h_j - \Delta h_{j-1}}{\Delta x}\right] \qquad + \\
&(1-\psi)\,\frac{g}{(k^2 A\,R^{4/3})^{n+1/2}_{j-1}}\;|\,Q^n_{j-1}\,|\,(Q^n_{j-1} + \Delta Q_{j-1}) \qquad + \\
&\psi\,\frac{g}{(k^2 A\,R^{4/3})^{n+1/2}_{j}}\;|\,Q^n_j\,|\,(Q^n_j + \Delta Q_j) \;-\; 0
\end{aligned}
\tag{3.4}
$$

Many variants of the Preissmann scheme exist; they mainly relate to the treatment of non-linear terms or semi-constant coefficients like the convective acceleration and the resistance term. These non-linear terms have to be linearized in order to solve the unknowns in a matrix computation. In the MODIS model, the values of the non-linear terms are determined by interpolating between the values at the old and new time level, starting with the values at the old time level. The number of iterations can be specified by the user, but a value of two is recommended. The CHAIN program of the Hydraulics Research Station Wallingford computes the value of the coefficients by extrapolation of the value at the old time level and the previous old time level. This is more accurate than evaluating the value at the old time level only, but less accurate than using an iteration step (Wallingford 1978).
In general, it is impossible to prove that a certain discretization is the best one as an objective reference does not exist. Apparently, this is the reason of existence of many variants of the Preissmann scheme. Nonetheless, it can sometimes be made plausible that a certain discretization is the best one.

3.4.3 Resistance term

Consider the following differential equation, which can be interpreted as the equation of momentum, whereby only the local acceleration and the resistance (= $K\,Q^2$) are taken into account,

$$\frac{\partial Q}{\partial t} + K\,Q^2 - 0 \tag{3.5}$$

The analytical solution of the differential equation is known and reads:

$$Q = \frac{Q_0}{1 + K Q_0 t} \tag{3.6}$$

Where Q_0 is the discharge on t =0, K is a resistance coefficient and Q is the discharge.
If the following discretization of the resistance term would be applied,

$$K Q^2 \approx K Q^{n+1} Q^n \tag{3.7}$$

then the difference equation would read:

$$\frac{Q^{n+1} - Q^n}{\Delta t} + K Q^{n+1} Q^n = 0 \tag{3.8}$$

and its solution becomes:

$$Q^{n+1} = \frac{Q^n}{1 + K Q^n \Delta t} \tag{3.9}$$

By comparing the solution of both equations it will be noticed that they are identical. Hence, it is made plausible that the above written discretization of the resistance term, Eq.(2.7), is the best one, as any other discretization would not result in an identical solution. Moreover, the resistance term is written implicitly, which favours the stability of the difference equation.

3.4.4 The convective acceleration term

In irrigation canals the convective acceleration term ($\frac{\delta}{\delta x}(\beta \frac{Q^2}{A})$) remains usually small. Therefore, it does not seem very important to compute the convective acceleration term accurately. However, the term should not create instabilities, and besides there are always some situations were the convective acceleration term becomes important (Cunge, 1979, pp 99-103). Stelling (Stelling 1984) made plausible that the following schematization is the best one (β = 1),

$$\frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right) \approx \frac{\left[\frac{Q^{n+1}\,Q^n}{A^{n+1/2}}\right]_{j-1} - \left[\frac{Q^{n+1}\,Q^n}{A^{n+1/2}}\right]_j}{\Delta x} \tag{3.10}$$

In MODIS the above presented implicit discretization, also referred to as Verwey's variant of the Preissmann scheme (Cunge 1979), has been used. It is remarkable that Mr Verwey, who was involved in the development of Rubicon, the base program underlying MODIS, did not use the implicit discretization in the Rubicon base model.

The difference between the explicit and implicit formulation of the convective acceleration term has been investigated for a sudden variation in discharge from 30 to 140 m^3/s. Fig. 3.1. It can be noticed that the implicit formulation provides slightly less fluctuations, but the differences are very small.

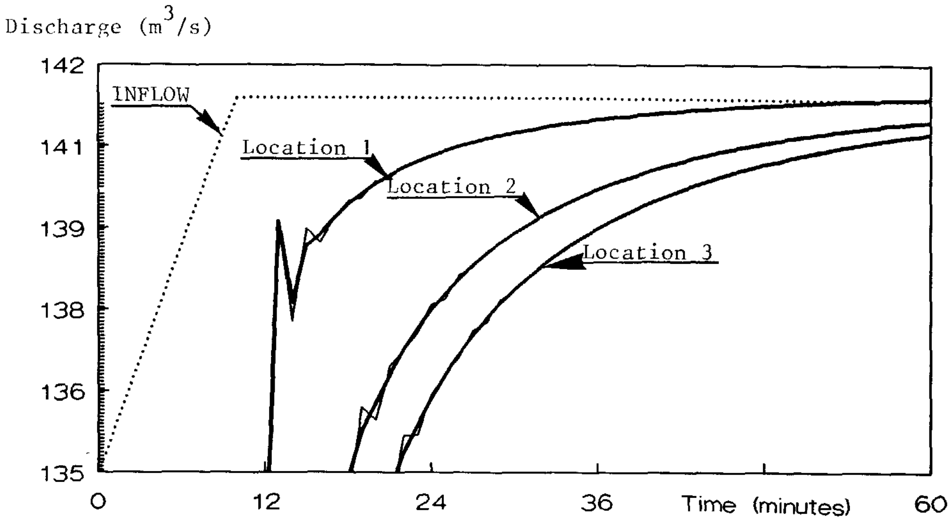


Fig. 3.1 Implicit and explicit formulation of convective acceleration term. Bold line represents the implicit formulation and the normal line represents the explicit formulation at three different locations.

3.4.5 Dependable coefficients

For the evaluation of dependable, or semi-constant coefficients, the implicit midpoint rule should be used rather than the trapezium rule. This implies that in the following equation:

$$\frac{\partial u}{\partial t} + f(u) - 0 \qquad (3.11)$$

the function f should be discretized by;

$$\frac{u^{n+1} - u^n}{\Delta t} + f\left[\frac{1}{2}(u^{n+1} + u^n)\right] \qquad (3.12)$$

rather then by

$$\frac{u^{n+1} - u^n}{\Delta t} + \frac{1}{2}\left(f(u^{n+1}) + f(u^n)\right) \qquad (3.13)$$

It is clear that both discretizations are not identical but only by looking to the original equation, can the correct discretization be derived (Cunge 1979, pp 103).

3.4.6 Dry bed flow

In principle. the MODIS model cannot simulate dry bed flow. This is an inevitable result of the limitations of the De Saint Venant Equations. In order to avoid abortion of the computation in case of dry bed flow, a special routine has been applied in MODIS. Every time this routine is activated, the user is warned that this very routine has been applied.

Abortion is avoided by creating a so-called "Preissmann slot" at the bottom of each profile. A "Preissmann slot" is an artificial profile, under the canal bed. It was found out that the best results were obtained when the bed width was kept equal to the bottom width whereas a small cross-sectional area and a high resistance were assumed. Consequently, the slot does hardly contribute to flow conveyance, and only when the water level is below the bed level.

It was found that a Preissmann slot could only postpone and not prevent abnormal termination of the program. To prevent the slot from "falling dry", every time the water level drops below the bottom level, the water level is raised up to 0.01 m above the bottom level and a small base flow of 0.0001 m^3/s is assumed. These additional adjustments proved to work well.

3.4.7 Supercritical flow

The model cannot simulate supercritical canal flow. This is not because the De Saint Venant Equations are not valid for supercritical flow conditions, but because of the applied solution procedure, the double sweep procedure, whereby the unknowns are solved first downstream. When supercritical flow occurs, the solution should first be found upstream and proceed in downstream direction.

Locally, e.g. downstream of structures, supercritical flow is allowed if it is directly followed by a hydraulic jump. In that case the supercritical flow is incorporated in the structure equation.

3.4.8 Seepage losses

Seepage losses can be modelled in the canal system by extracting flow at some locations along the canal. The amount of flow can be specified as a function of e.g. the wetted perimeter. However, no special routine has been written to model seepage losses automatically in between every two grid points. No special attention has been paid to seepage losses, as in this study the emphasis is put on the relation between canal operation and hydraulic performance. Seepage losses are generally but slightly related to canal operation and are hardly affected by unsteady flow (Manz 1990).

3.4.9 Model tests

Several tests have been conducted to demonstrate the correctness of the model. These tests mainly concerned the unsteady flow in canal system without any structures.

In (Contractor & Schuurmans 1991), two tests carried out by the ASCE Task Committee on Irrigation Canal System hydraulic Modeling, were described. The first, proposed by Holly, see also (Cunge et. al. 1980), checked the volume conservation of various models. The second one, proposed by Contractor, evaluated the variation of the discharge in time for a ramp discharge at the head of the canal for various Courant numbers. As a reference, a computation based on the method of characteristics was used. The first test showed that for two iterations, the model is 100% mass conservative and the second one showed that the MODIS model is very accurate for small Courant numbers.

Presently, more tests are being conducted in order to demonstrate the model's correctness for controlled canal systems using real life laboratory measurements as a reference.

3.5 Flow through structures

3.5.1 Introduction

If a structure is placed in the canal system, the momentum equation is replaced by the stage discharge relationship of the structure. The previously discussed De Saint Venant Equations cannot be used to describe discontinuities like those that occur around regulators. The "normal" stage discharge curves can be used for unsteady flow computations, because the reaction time of flow through regulators can be neglected compared to the response time of the canal system. However, some modifications are required to incorporate the structure equation in the overall implicit computation. The mathematical methods, needed to avoid potential pitfalls, will be discussed.

3.5.2 Analytical description

As stated before, the "normal" stage discharge relationships of flow through structures can be incorporated in a numerical unsteady flow model. As the upstream and downstream water levels may fluctuate during a computational run, the flow condition (free or submerged) can change during a run. Hence, the model should be able to handle all flow conditions and check continuously which flow condition is applicable. The outer limits of a flow condition are usually determined empirically, and they are unique for each type of regulator. Therefore, the user himself must be able to specify these outer limits.

Furthermore, empirically determined discharge coefficients are commonly found in stage discharge equations. The values of these coefficients are not constant but dependable on various parameters such as the shape and dimensions of the regulator and the upstream water level head (Bos, 1976), (Ackers et.al.,1978). The model should be capable to handle the same variable coefficients as found in literature.

In the MODIS model the following types of structures can be modelled everywhere along the canal system: orifices, weirs, pipes, pumps, local loss structures, and Neyrtec baffle distributors. Furthermore, the user can write his own structure by using the fortran defined function facility (MODIS user's guide, Appendix G). Nearly all structures have been redefined in MODIS compared to the definitions in its base model Rubicon.

3.5.3 Numerical computation

The previous discussion concentrates on the analytical equations used in the model. Although the stage discharge equations of regulators are not differential equations, problems may arise. The computation of the flow through regulators does in itself not lead to instabilities, but it may break the implicit computation of the canal flow. This implies that new values upstream and downstream of a structure are not linked. Consequently, instabilities might occur. To overcome this problem, the flow through a regulator should be incorporated in the overall implicit computation. The following procedure has been applied for the MODIS model to incorporate the structure computation in the implicit canal computation.

The equation of the regulator is a normal function denoted by:

$$Q = f(h_1, h_2) \tag{3.14}$$

To fit the structure equation in the same format as the momentum equation, the following procedure has been applied. First the function is linearized,

$$Q = Q_0 + \frac{\partial Q}{\partial h_1} dh_1 + \frac{\partial Q}{\partial h_2} dh_2 \tag{3.15}$$

Where the discharge at time level n is equal to Q_0. The variation of Q during the period of time Δt equals,

$$\Delta Q = \frac{\partial Q}{\partial h_1} \Delta h_1 + \frac{\partial Q}{\partial h_2} \Delta h_2 \tag{3.16}$$

Due to the linearization an error is introduced. In other words, the new values Q, h_1 and h_2 will not exactly fit in the stage discharge curve of the structure. The error ε is equal to:

$$\varepsilon = Q^{n+1} - f(h_1,h_2)^{n+1} \tag{3.17}$$

Where Q(n+1) represents the numerically computed discharge and $f(h_1,h_2)$ is the discharge which follows from the stage discharge relationship. To avoid accumulation of errors, the error made can be corrected in the following computation by changing the linearized equation into:

$$\Delta Q = \frac{\partial Q}{\partial h_1} \Delta h_1 + \frac{\partial Q}{\partial h_2} \Delta h_2 - \varepsilon \tag{3.18}$$

In steady situations ε will be equal to zero.

To make the most accurate discretization (of second order), the derivatives of Q with respect to the water levels are evaluated at time level n + ½. The second order of accuracy can be demonstrated by developing the function Q^{n+1} and Q^n in Taylor series using $Q^{n+½}$ as a reference point. Again, the implicit midpoint rule has been used rather than the trapezium rule. This implies that the value at $Q^{n+½}$ is computed by:

$$Q^{n+\frac{1}{2}} = f(h_1^{n+\frac{1}{2}}, h_2^{n+\frac{1}{2}}) \tag{3.19}$$

and not by,

$$Q^{n+\frac{1}{2}} = \frac{f(h_1,h_2)^n + f(h_1,h_2)^{n+1}}{2} \tag{3.20}$$

as has been applied in Rubicon.

In MODIS model the derivatives of the discharges are computed numerically, by varying the upstream and downstream water levels 0.01 m and computing the resulting variation in Q. The advantages of this numerical computation of the derivatives are twofold. First, discharge coefficients, which are a function of the water levels are automatically included in the derivatives, and, secondly, non standard structures, such as Fortran defined structures and the Neyrtec distributors, can easily be included in the computer package.

The disadvantage of the numerical computation of the derivatives is that the computation time increases compared to analytical expressions of the derivatives. In practice, the discharge according to the stage discharge curve is computed 7 times to reach an accurate solution of the flow through a structure.

3.5.4 Modelling a watermanagement system

To model a watermanagement system the canal system has to be defined in the model by using nodes and branches. A branch represents a conveyance element with a cross-section, resistance and bed slope. Nodes are used to link branches and to indicate a branch end. By defining the nodes and branches, the user does not have to bother about computational aspects, as the model itself generates a computational grid over the user's defined system. (see also MODIS user's guide, Appendix G)

Structures can be located everywhere along a branch, and can be placed both in series and in parallel. This latter feature can be used to define composite structures. The type of structure, e.g. weir, pump, orifice, pipe, is specified by defining the structure under the appropriate name. The (real time) operation of regulators is discussed in the following paragraph.

3.6 Control of regulators

3.6.1 Introduction

To describe the flow phenomena in a canal system with regulators, one should not only consider the De Saint Venant Equations, but also the equations of the regulators. If the regulators are (real time) controlled, again the (dynamic) behaviour of the system will change, and is determined by an interaction of the controllers, the regulators and the canal system. To predict the behaviour of a controlled system, a model, which incorporates the controllers, the regulators and the canal system, is needed. (Fig. 3.2).

The behaviour of a real time controlled canal system is rather complex, as the behaviour of the individual components is already complex; see also paragraph 4.2. Therefore, a (computer) model to analyze the system is indispensable. Presently, no modelling packages exist for real time controlled water management systems; see also paragraph 1.3.

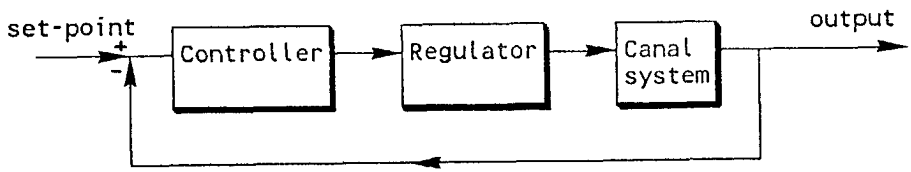


Fig. 3.2 Real time control system (Schuurmans 1991b)

In this paragraph a short classification of control systems will be presented and their implementation in the MODIS model will be discussed. Finally, some attention will be paid to its numerical computation.

3.6.2 Classification of control systems

Control of regulators is needed to reach or to maintain a desired state of the system. The hydraulic state of the system is defined by the water levels and flow rates, which might vary in time and place. For an overview of the existing variety of control systems, reference is made to the lecture notes "Control of watermanagement systems" (Brouwer et. al. 1990). Any control system comprises the following basic elements:

1) The control loop can be open or closed. In case of closed loop control, also referred to as real time control (Fig. 3.2), various types of controllers or control algorithms can be used to maintain the control variable on its set-point.
2) The control variable can be a water level, a discharge or a volume.
3) The mode of operation can be manual or automatic. In automatic operation, the regulators in the system are operated without intervention of human operators.
4) The control system can be selfregulating or non-selfregulating. A control system is called self-regulating if the system returns to its desired state, after a disturbance without outside manipulations. Thus, if a self-regulating control system is applied to irrigation, the inflow discharge will be adjusted if water is withdrawn from the system. Self-regulation should not be confused with automation! The main advantages of a self-regulating control system are that managerial tasks can be considerably reduced and that the water distribution can be more flexible. For on-demand systems, self-regulation is a must, but also for semi-demand or arranged deliveries, self-regulation can be advantageous.
5) Three levels of control or levels of information can be distinguished:local, regional and global. Local control is applied when the controlled variable is measured in the vicinity of the regulator, and regional control is applied when the controlled variable is measured at one or more locations in the region of the

regulator. When the regulators are controlled on the basis of information distributed over the entire system, global or central control is used.

6) The operation strategy of the regulators is defined as the variation of the set-points or target levels in time. The value of the set-points can be determined a priori or during the ongoing process. In the later case one distinguishes: a heuristic approach (with or without an experts system), a rule based strategy (if..then..else..), and finally the use of mathematical optimization techniques.

MODIS model can handle all listed elements and, therefore, it can simulate all control systems, except, as yet, global control. Furthermore, the operation strategy, the set of set-points in time, cannot be determined by an optimization model or expert system. Instead, watermanagers using MODIS have to adapt a heuristic approach to optimize their strategy.

It is obvious that it is irrelevant for the behaviour of water flow in the system whether operation is carried out manually or automatically. Automatic control as a function of time and on/off control can be simulated by defining e.g. the gate settings a priori as a function of time. It is also possible to redefine the gate settings at predefined moments in time, taking into account the actual state of the system. This latter approach has been followed at the "Senior advanced course on Appropriate Modernization and Management of Irrigation Systems" (Schuurmans & Huyskes 1990) where the participants simulated manual operation by resetting the regulators after observing the actual water distribution. In MODIS it is also possible to simulate real time control (closed loop control), whereby control is based on the actual state of the system following a control algorithm.

3.6.3 Implementation in MODIS

Normally, the user specifies fixed structure parameters, e.g. a certain value for the gate opening height. Control of a regulator implies that the gate opening height has to be adjusted during a computational run. In that case, the gate opening height can be specified as a function of time (open loop control), or as a function of geometrical or flow variables (closed loop).

In case of closed loop control, the gate opening height can be specified as a tabulated function of e.g. a water level, but it is also possible to make reference to a controller, whereby the level of control (local or regional), and the type of controller has to be specified.

Two types of controllers have been implemented in the MODIS model: a multi-speed step controller, whereby the speed can be adjusted as a function of the deviation, and a Proportional Integral Differential (PID-) controller. In formula the output signal u_t of a PID controller reads:

$$u_t = K_P \cdot e_t + K_I \cdot \sum_{t=0}^{t} e_t + K_D \cdot (e_t - e_{(t-dt)}) \tag{3.21}$$

Where K_P is the proportional gain factor, K_I is the integral gain factor, K_D is the differential gain factor and e_t is the deviation from the target level.If a controller is applied, certain control parameters will have to be specified, e.g. gain factors, speed of gate movement, dead band and minimum and maximum gate openings. Furthermore, the target level has to be given. The value of the target level, in turn, can also be specified as a constant, a function of time or a function of a geometrical variable like a water level. By doing so all types of canal control systems, such as upstream control, downstream control, BIVAL control, and ELFLOW -control can be modelled.

3.6.4 Numerical computation

If the value of the controlled variable has been specified as a function of time, the value of the parameters will be determined at the old time level that is at $t = n\,\Delta t$. In this way, a fast response is obtained. At the other hand, if the value of the controlled variable, has been specified as a function of a geometrical variable, its value will determined by the value of the geometrical variable at $n+½$, as this is more accurate. In reality, the value is first determined in first iteration step at n, and in the following iterations at $n+½$.

If a controller is applied, the value is adjusted at the old time level only, and only in the first iteration. This has been done for two reasons. The first reason is that in real life, it is also most common to adjust the controlled parameter at fixed intervals, based on the actual situation. If these intervals are equal to the time step applied in the numerical computation, the simulation will be most accurate if in the model the parameter is also adjusted on the basis of the actual situation that is on $t = n\,\Delta t$. The second reason is that if a parameter is adjusted more than once during a computational step, the controllers will not behave as defined. For example, the integral controller should integrate only once during a time step.

Finally, the output signals of the controllers are bounded by minimum and maximum values. If the outer limits are reached, the integral controller of the PID-controller should not further count up the deviations, otherwise the system will react too retardedly later on.

3.7 Performance indicators

3.7.1 Introduction

A computer model produces numbers, which represent information. When an unsteady flow model is used, this information embodies the hydraulic state of the system in certain locations at certain intervals of time. In order to interpret the model results, these data have to be converted into performance indicators.

As to the hydraulic design, one is usually interested in maximum and minimum values. Instead of looking to minima and maxima only, one can also look at the time during which user defined minima and maxima have been exceeded in order to indicate the performance of a water level regulator.

To evaluate and compare operation alternatives simulated with a computer model, operation performance parameters are needed to characterize the quality of operation rather than information describing the hydraulic state of the system. These operation performance parameters have to be easily interpretable, comparative and diagnostic.

Efficiencies are important design and operation parameters. However, it was found that an efficiency does not indicate the performance of a system, because the efficiency should be related to the intended or design efficiencies in order to be able to indicate the systems performance. Lately, the ICID working group on irrigation efficiencies has been converted into a working group on irrigation performance. Performance is a broader term than efficiencies and it compares an actual situation with a desired or ideal situation.

In literature, several operational performance parameters are found. Examples are the Operation Performance Ratio (OPR) defined by (Lenton 1982) and used by (IIMI 1987), (Makin 1986) and (Francis & Elawad 1989). The Operation Performance Ratio is then defined as the ratio between the actual flow rate and the intended flow rate. The drawbacks of the OPR are that only the flow rate is considered and not the moment of delivery, and, furthermore, that the effect of unsteady flow, whereby the flow rate is variable in time, has not been taken into account. (Fig. 3). To quantify the variation of the OPR in time and space, statistical parameters have to be used. For example, assuming a normal distribution results in a mean OPR and its standard variation. Apart from the question whether a normal distribution can be applied, the problem of interpreting the statistical parameters remains.

New performance indicators were defined which do cover the unsteady flow phenomenon and the moment of supply (Schuurmans 1989), but statistical parameters were not used. The Delivery Performance Ratio (DPR) specifies the extent to which an offtake receives its intended supply. The intended supply has to be defined by the user in terms of begin and end time of supply, an intended discharge and an allowable variation in discharge. The second one is called the operation efficiency (e_o) and specifies the amount of water lost by inappropriate allocation of the water to an offtake.

These performance parameters were developed for a computer model whereby data, describing the hydraulic state of the system, become available in great numbers. Furthermore, the operation efficiencies do not only cover canal leakage losses but also the losses due to improper operation.

3.7.2 Performance of individual offtakes

For the evaluation of the simulated water distribution to an individual offtake, two operation performance parameters were defined. The first one is called the Delivery Performance Ratio (DPR), and the second one is called the operation efficiency (e_o). In formulae these parameters read:

$$DPR - \frac{V_e}{V_i} \bullet 100\% \tag{3.22}$$

$$e_o - \frac{V_e}{V_a} \bullet 100\% \tag{3.23}$$

Where,

DPR	=	Delivery performance ratio	(-)
e_o	=	Operation efficiency	(-)
V_e	=	Volume effectively delivered	(m^3)
V_i	=	Volume intended to be delivered	(m^3)
V_a	=	Volume actually delivered	(m^3)

As can be read from the presented formulae, three volumes of water are distinguished: An intended volume to be supplied, an actual volume supplied, and an effective volume supplied.

The intended volume to be delivered (Vi) is defined by the user in terms of begin and end time of supply, an intended flow rate and an allowable range of variation in flow rate. (Fig.3.3a).

The actual volume of water received by an offtake (Va) is calculated by integration of the supplied flow rate and time. (Fig. 3.3b).

The actual volume supplied is considered to be an effective volume (Ve) only when the moment of supply is within the user's defined period of supply and when the flow

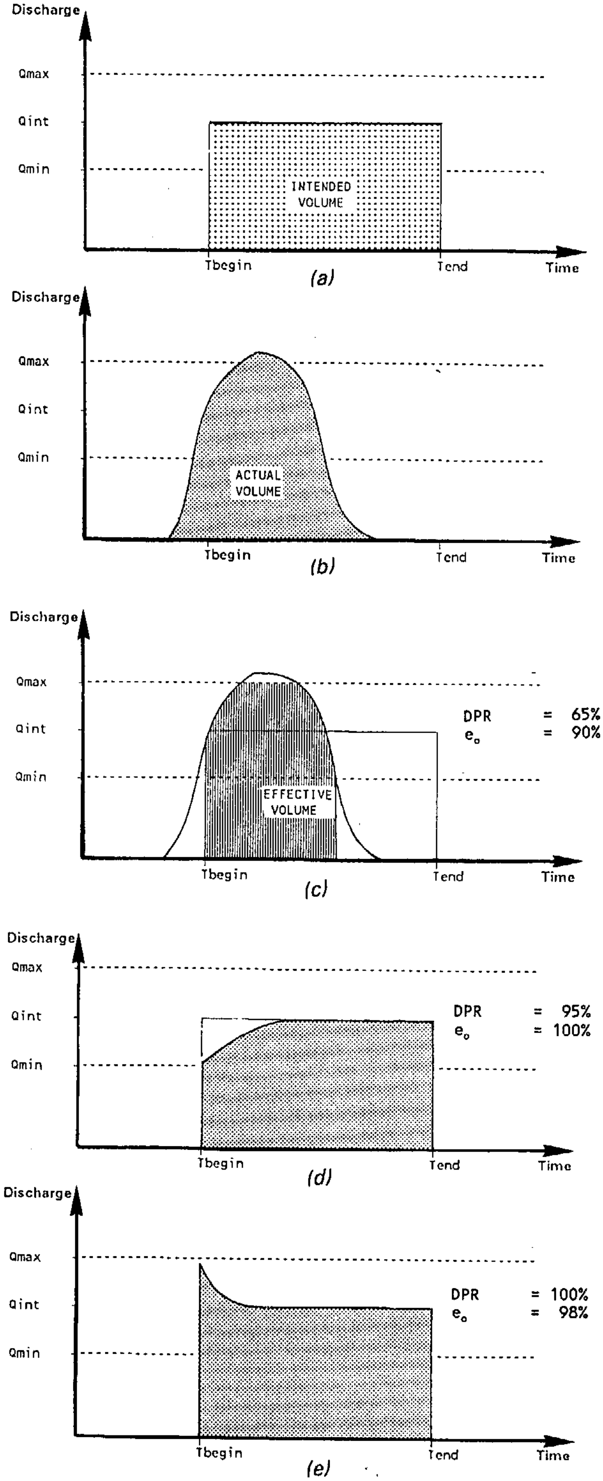


Fig. 3.3 Operation performance parameters

rate is within the allowable range of flow rates. A too high flow rate might lead to spillage, whereas a too small flow rate might not be handled at all. Furthermore, the effective volume can never exceed the intended volume Vi. (Fig. 3.3c).

The allowable range of flow rates has to be defined by the user. If one does not know what is desired, e.g. what is the allowable range of flow rates, it is per definition not possible to define the performance. If the allowable range is wide, the performance will generally be good, whereas for more narrow ranges the performance will generally decrease.

■ **Interpretation of performance parameters**

An operation efficiency of 100% indicates that no water has been spilled but this does not mean that the offtake received enough water. Therefore, both parameters should be considered in the evaluation. The performance parameters are diagnostic parameters. For example, if both parameters are less than 100%, one knows that there is room for improvement, without the necessity of an increase in the actual water supply. In reality, a compromise has to be found between the delivery performance ratio and the operation efficiency.To illustrate the use of the defined performance parameters, some examples of possible combinations of DPR and e_o are presented. (Fig. 3.3d & e). In Fig 3.3d no water is spilled, but the received supply is

less than the intended supply resulting in a DPR of less than 100%. In Fig 3.3e, the operation efficiency is less than 100% because the total volume received is more than the intended volume, and thus the effective volume equals the intended volume.

3.7.3 Performance of a complete system

To evaluate the operational performance of a canal system including all its offtakes, overall performance parameters are needed. The overall Delivery Performance Ratio is calculated by a weighed average of the DPR of the individual offtakes and reads:

$$DPR_{overall} = \frac{\sum_{n=1}^{n=p} V_{e,n}}{\sum_{n=1}^{n=p} V_{i,n}} = \frac{\sum_{n=1}^{n=p} DPR_{,n} \; V_{i,n}}{\sum_{n=1}^{n=p} V_{i,n}} \qquad (3.24)$$

Where $DPR_{overall}$ is the overall Delivery Performance Ratio, $V_{e,n}$ is the effective volume received by offtake n (m^3), $V_{i,n}$ is the intended volume to be received by offtake n (m^3), and p is the number of offtakes involved.

The overall operation efficiency incorporates the operational losses of the individual offtakes, these losses due to filling up the canal to its operational level, and the leakage losses and/or spill losses if these occur. In formula it reads,

$$e_{0,overall} = \frac{\sum_{n=1}^{n=p} V_{e,n}}{V_{a,intake}} \tag{3.25}$$

Where $e_{0,overall}$ is the overall operation efficiency, $V_{a,intake}$ is the actual intake volume at the head of the main canal (m^3), $V_{e,n}$ is the effective volume received by offtake n (m^3), and p is the number of offtakes involved.

The overall delivery performance ratio becomes equal to the overall operation efficiency if the sum of intended volumes is equal to the volume taken in. In that case the overall performance of the canal system can be characterized by one single figure.

- **Seepage losses**

In this study no special attention has been paid to seepage losses. Nonetheless, it is very well possible to incorporate seepage losses in the model by extracting certain amounts of water at certain locations. The losses due to seepage can be accounted for by the overall operation efficiency only.

By not paying attention to seepage losses, it can be clearly demonstrated how much water can be lost by improper operation of the offtakes and regulators. Furthermore, the relation between seepage and operational performance does hardly exist (Manz 1990).

3.8 Summary and conclusions

To study the hydraulic performance of controlled canal systems for unsteady flow conditions, an irrigation flow model named MODIS has been developed. MODIS used Rubicon as a calculation base model. Rubicon had been selected as the model most appropriate for further development. The MODIS model differs from normal simulation models by: (1) An accurate simulation of a wide range of standard irrigation structures. (2) the possibility of simulating all types of (real time) control scenarios, (3) a post-processor which computes easily interpretable and diagnostic operation performance parameters. In subsequent paragraphs the methods of computation, applied to the various modules, have been discussed.

The model's main limitations are that it cannot simulate supercritical flow and that some mathematical tricks have to be applied to keep the model running on dry beds. Furthermore, seepage can be modelled, but not very elegantly.

The use of flow models can be advantageous, but some caution is warranted. One should not forget that a numerical simulation model is a complex tool which requires skilled users, which must be able to transform model results into real life results. If numerical instability occurs the discrepancy between both will be clear and, therefore, less dangerous. Numerical diffusion at the other hand, is more difficult to detect and, therefore, more dangerous.

At present, there is not much experience with the use of unsteady flow models in the field of evaluation of the hydraulic performance of irrigation systems. In the next chapter a couple of cases will be discussed in order to illustrate possible applications, and the potentials and limitations of the MODIS model.

4 Model Applications

"Accurate hydraulic simulation models are cumbersome to use because of the size of the programs, the many hydraulic variables, and the complex mathematics required in unsteady flow analysis" (Plusquellec 1988).

4.1 Present use of hydrodynamic flow models

The fact that flow models are (becoming) available should not be the reason of actually using these models. An uncontrolled use of hydrodynamic flow models should be avoided. In many situations, steady state analysis can already contribute to an improved performance. Although unsteady flow models can also be used for steady state analysis, a more simple steady state hydraulic model should be preferred in those cases.

There are many situations in which a flow model can be very useful, but sometimes confusing and misleading arguments are used for not using them. For example, in the audiovisual production "Improving the operation of canal irrigation systems" of the World Bank the following phrases were found:
"The simulation programs estimate canal reactions to various flow changes and gate movements. However, accurate hydraulic simulation models are cumbersome to use because of the size of the programs, the many hydraulic variables, and the complex mathematics required in unsteady flow analysis. The accuracy of such programs is limited by the difficulty of correctly estimating hydraulic values in the field. For example the roughness of a canal may fluctuate widely during an irrigation season due to weed growth. Old unlined canals are difficult to model because of irregular and variable dimensions. Accurate hydraulic simulations require the formulation of gate discharge equations. Discharge coefficients must be calibrated in the field for individual gates under a wide range of flows". (Plusquellec 1988).

These phrases, concerning the use of flow models, are confusing and not always applicable. In the first place, a simulations program, like MODIS, can run on PC's and, by now, even on lap-top computers. Furthermore, the input data required for unsteady flow computations are the same as the data required for steady flow computations. The hydraulic computations are indeed complex and that is precisely the reason of using a computer program. However, the user of the program does not have to bother about the complexity of the hydraulics as long as his program is (used) correct(ly). Concerning the accuracy of the input data, Plusquellec is right by stating that it is a tedious job to find the correct parameters. However, correctly calibrated values are only needed if the program is used for the actual operation of the system. For other applications, such as design, modernization and training of designers and operators it is usually not needed to calibrate the program: the necessary insight and comparative results can be obtained by using realistic design parameters.

The most well known applications of unsteady flow models are dealing with the development of control algorithms for computer controlled canal system, whereby stability is one of the main subjects of investigation. In the early 1960s, the French company Sogreah, directed by dr. A. Preissmann, developed, among others, hydraulic simulation models for the development of the BIVAL system (patented by Sogreah) (Chevereau 1987). Dynamic regulation, however, a central control system implemented in the Canal de Provence, France, was studied and constructed on a "small scale" old supply canal rather than using a hydrodynamic flow model (Rogier 1987). In 1967 the Bureau of Reclamation and the University of California at Berkeley entered into a cooperative agreement to develop and test a mathematical simulation model for canal control. Results of this work eventually led to the development of the Electronic Filter Level Offset (ELFLO) system (Ploss 1987). Other examples of real time control systems, developed (partly) with unsteady flow models are the HyFlo system (Harder 1972), the CARDD system (Burt, 1983 & 1989), and Gate Stroking (USBR 1979). Other model applications dealing with automatic control of irrigation systems are (Zimbelman & Bedworth 1983) and (Balogun et al 1988).

Unsteady flow models are not only used for the development and design of closed loop control systems, but also to study the (manual) operation of watermanagement systems. For these applications modelling packages were used, which, however, were not able to simulate closed loop controlled systems. *"There are still basic problems which need to be overcome in order to advance quickly with canal automation research. One of such problems is the lack of easy-to-use computer simulation models in which new control algorithms may be tried out".* (Burt 1989). Some models do have some capabilities of simulating closed loop controlled systems. For example, the CHAIN model of Hydraulics Research Station Wallingford which could simulate local speed controlled gates (Wallingford 1978).

In Canada, the Irrigation Conveyance System Simulation, (ICSS), model has been developed as an aid in the identification and examination of the impacts of the irrigation conveyance system's physical and operational characteristics on water transfer efficiencies (Manz 1989). Presently, Manz is still continuing research with the ICSS model (Manz, 1990). The ICSS model is not capable of handling automatically controlled gates and only runs on a mainframe computer.

In Sri Lanka, the International Irrigation Management Institute (IIMI), with the assistance of the Societe Grenobloise d'Etudes et d'Applications Hydraulics (SOGREAH), has since September 1987 used mathematical modelling as a methodology to investigate main canal operations and to conduct research on the interactions between design and management of an irrigation system (Sally et.al. 1989). The MISTRAL-SIMUTRA model they are using cannot simulate real time control systems. Much time and effort have been put into calibration of the model (Berthery et.al. 1989). Recently, SOGREAH has been replaced by the French firm CEMAGREF, which is using another flow model.

Other examples of model applications concerning the operation of irrigation systems are listed: (Rofe 1984), (Theaembert 1976), (Hamilton & de Vries 1986), (Poppel & Weijland 1986) and (Pereira et.al. 1990). The list of model applications is certainly not complete, but one could state that actual research in this field is rather limited, not fundamental and scattered all over the world. It is to be expected that, due to the present availability of models and hardware, much more research will be conducted in the near future.

In the following paragraphs some case studies conducted with the MODIS model are presented. These cases have already been published before and are of different nature. The first one is entitled: "Computer Simulations of regional controlled irrigation canals" and deals with stability and performance of different control systems. The second one reads: "Operational performance of canal control systems" and quantifies the effects in performance of manual upstream control, automatic upstream control and automatic downstream control systems. The last case, "Identification of a control system for a canal with night storage" is a practical case dealing with the modernization of a 110 km long irrigation canal in Jordan with only day irrigation and in-line canal storage.

Other cases have only been published in the form of master theses at Delft University of Technology and at the International Hydraulic and Environmental Institute (IHE) and are not presented here (Huyskes 1990), (Monem 1990), (Khalajh 1991).

4.2 Computer simulations of regional controlled irrigation canals

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Abstract A hydrodynamic flow model was used to investigate the performance and stability of regional canal control systems. The reliability of the numerical approach was checked with the method of linearized equations. Once regional control systems can be easily modelled, their performances can be compared with those of other control systems and the best one can be selected on a rational basis.

1 Introduction

Canal control systems facilitate a reliable and equal water distribution according to scheduled deliveries. In the past decades, numerous new and sophisticated canal control systems have been developed (Zimbelman 1983) (Chevereau 1987). To select the most appropriate one, it is needed to investigate their performances for a specific canal system. In order to do so, a modelling package in which various control systems can be simulated and evaluated would be extremely useful.

At Delft University of Technology, MODIS modelling package has been developed to calculate the unsteady flow in irrigation systems. Local automatic control systems were already incorporated in the modelling package. Recently, a study has been conducted to extend the existing control features with regional control systems. In this paper it is described how regional control has been implemented. Furthermore, a number of runs, which were made with the regional control facility, are presented. The results of the simulation have been compared with results obtained with the method of linearized equations.

The paper is structured as follows. At first the reasons for applying regional control are discussed. Thereafter, a description of a regional control system is given. Two methods can be used to investigate its stability and performance, the first one is linearizing and the other is called discretizing. The latter method has been applied in the numerical model. The results of both methods are presented and discussed. Finally, the conclusions concerning the applicability of regional control simulations are presented.

2 Local versus regional control

Local downstream control and regional downstream control are, contrary to upstream control, self-regulating control systems from a water supply point of view. A control system is called a self-regulating control system when the system returns to its desired state after a disturbance, which does not require any outside scheduling action. Thus, if a self-regulating control system is applied, the inflow discharge will be adjusted if water is withdrawn from the system. Self-regulation should not be confused with automation. In automatic control, the regulators in the system are automatically operated without intervention of human operators, but this does not necessarily make the system self-regulating.

The main advantages of a self-regulating control system are that managerial tasks can be considerably reduced and that the water distribution can be more flexible. For on-demand systems, self-regulation is a must, but also for semi-demand or arranged deliveries, self-regulation can be advantageous.

Local downstream control is a well-known example of a self-regulating control system. The advantage of local control is that mechanical automatic regulators, whose functioning is independent of external power supply, can be applied. The Avio and Avis gates, manufactured by Alsthom Fluide, are examples of mechanical automatic downstream control gates. Local downstream control has also some disadvantages. Horizontal canal embankments are required along the pools, and the (low head) gravity offtakes have to be located just downstream from the water level regulators, where the water level is kept constant, or more expensive types of offtake structures that can accommodate the water level variations are required. Furthermore, the system has a considerable response time, during which water is supplied from, (or supplied to), a buffer reservoir. This reservoir, in turn, is created by the horizontal embankments along the pools.

To overcome the disadvantages of local downstream control, regional downstream control can be applied. In regional downstream control the water level is measured downstream at one or more locations in the region of the regulator. By controlling a water level situated further downstream, the length of the horizontal embankments along a pool can be reduced. Therefore, regional downstream control can be applied to steep canals and to the modernization of originally (non self-regulating) upstream control systems. The main disadvantages of regional downstream control are that a communication system and external power supply are required.

The reduction of the length of the horizontal embankments for regional control causes a reduction of the available buffer capacity, but, at the same time, the required buffer capacity is also diminished, because of the faster response time. For example, the buffer capacity of a constant volume controlled system is zero, and if the water level is controlled at the downstream end of the pool, the buffer capacity will even be negative. A negative buffer storage implies that there has to be a (temporary) deviation between the actual and the target value of the controlled variable.

3 Description of regional downstream control

Regional downstream control is a real time control system. Real time control refers to the fact that the process of monitoring and controlling is performed during the ongoing process. To do so, four entities are required: (1) a sensor to monitor the ongoing process, (2) a regulator to manipulate the process, (3) a controller that exerts the regulator, and finally, (4) a communication system between the sensor regulator and controller. The regulator and the controller will be discussed more in detail as these elements were defined in the numerical model.

3.1 Regulator

The water level regulator is often of orifice type. The stage discharge relationship of an orifice reads:

$$Q = c_e \cdot \mu \cdot B \cdot a \cdot \sqrt{2g \cdot \Delta h} \qquad (1)$$

Where, c_e discharge coefficient (-), μ contraction coefficient (-), B width of the orifice (m), a opening height of the orifice (m), g acceleration due to gravity (m/s^2) and Δh is $h_1 - h_2$ for submerged flow (upstream water level minus water level just downstream of the orifice) and $h_1 - h_3$ for free flow (upstream water level minus water level at the vena contracta).

3.2 Controller

The controller determines how the regulator reacts on disturbances in the system. The input of the controller is a deviation e which is equal to the actual value of the controlled variable minus its target value. The output signal of the controller is a new gate setting of the regulator.

The most simple controller is the so-called "step controller", whereby the output signal equals zero, when the deviation is within a "dead band". If the value of the controlled variable is outside the "dead band" the gate will be exerted at regular intervals with a fixed speed.

A more sophisticated, and often applied, controller is the PID (Proportional Integral Differential) controller, whereby the output signal is based on actual and previous deviations (Cool 1985). In formula the output signal u of a PID controller reads:

$$u_t = K_P \bullet e_t + K_I \bullet \sum_{t=0}^{t} e_t + K_D \bullet (e_t - e_{(t-dt)}) \tag{2}$$

Where K_P is the proportional gain factor, K_I is the integral gain factor, K_D is the differential gain factor and e_t is the deviation from the target level.

Apart from the step and PID controller, other types of controllers exist. For instance, BIVAL regional control (Cunge 1980) uses a step controller combined with a variable target level. The value of the target level is determined by the discharge at a pivot point. The pivot point is the point where the water level is controlled. In formula the target level used for the BIVAL controller reads (Chevereau 1987):

$$Y_{target} = Y_{target,Q_{max}} + D \bullet \frac{(Q_{max} - Q_{pivot})}{Q_{max}} \tag{3}$$

Where, $Y_{target\ ,\ Q_{max}}$ target water level for maximum discharge (m), D maximum decrement (m), Q_{max} maximum discharge at pivot point (m^3/s) and Q_{pivot} actual discharge at pivot point (m^3/s).

3.3 Location of the pivot point

In a water level controlled pool a pivot point can always be distinguished. The pivot point is the point where the water level is kept constant and around which the water levels in a pool will turn if the discharge in a pool is varied. (Fig. 1).

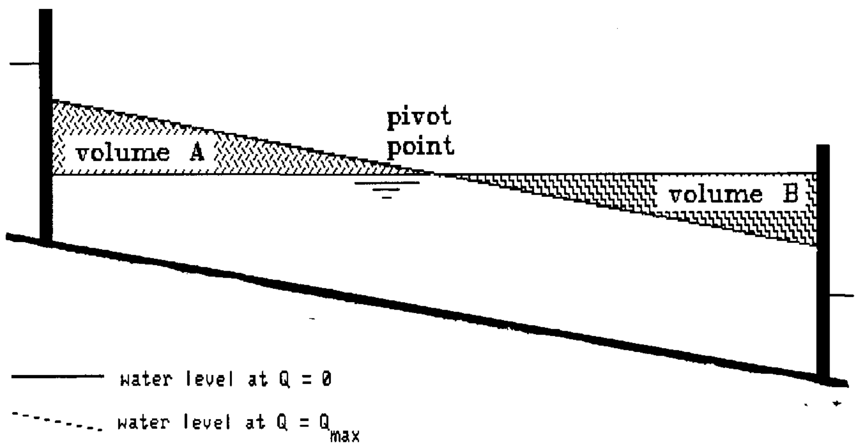


Fig. 1 Location of the pivot point

In formula, the location of the pivot point is described as (Chevereau 1987):

$$Y_{pivot} = a\,Y_{us} + (1 - a)\,Y_{ds} \tag{4}$$

Where Y_{us} is the water level at the upstream end of the pool and Y_{ds} is the water level at the downstream end of the pool.

By varying the factor a from 1 to 0, the pivot point is moved from upstream to downstream. In local downstream control, the pivot point is located just downstream of the regulator ($a \approx 1$). In BIVAL control (Cunge 1980), and constant volume control, the pivot point is located somewhere in the middle of a pool ($a \approx 0.5$) and in ELFLOW and CARDD (Zimbelman 1983) the pivot point is located at the downstream end of a pool ($a \approx 0$).

4 Methodology of investigation

4.1 Mathematical description of the controlled system

To study the stability and performance of regional control systems mathematically, one should consider the partial differential equations which describe the controlled system. The controlled system consist of the canals system, regulators and controllers. (Fig. 2a).

The gradually varied unsteady flow in the open canal system is described by the Barré de Saint-Venant equations (Cunge 1980). These equations cannot be used to describe discontinuities such as structures. Therefore, structure equations should be added and linked to the de Saint-Venant equations. If the regulators are real time controlled, controllers, which link the actual state of the system with the operation of the regulators, should be added.

The mathematical description of a controlled system is more complex than the sum of its components (Cool 1985). Therefore, it is not surprising that no analytical solutions of the equations are found, as it is already impossible to find analytical solutions of the de Saint Venant Equations only.

Two methods are available to find approximated solutions. Both methods are based on the principle of replacing the original equations by more simple equations (Fig. 2b & 2c). The first method is called linearizing and the second one discretizing. When a hydrodynamic computer model is used, the method of discretizing is applied.

4.2 Method of linearized equations

Analytical solutions can be found if the partial differential equations, describing the controlled system, are linearized. Although this does not yield an exact solution, the method of linearized equations serve rather well for a stability analysis. The method is, however, less suitable to investigate the performance of the system. Usually, simplifications are introduced, such as the assumption of a rectangular cross-section and simple controllers. Often it is assumed that the controller exactly maintains the target value. Hâncu (Hâncu 1974) incorporated proportional controllers, which already yields a rather complex stability criterion.

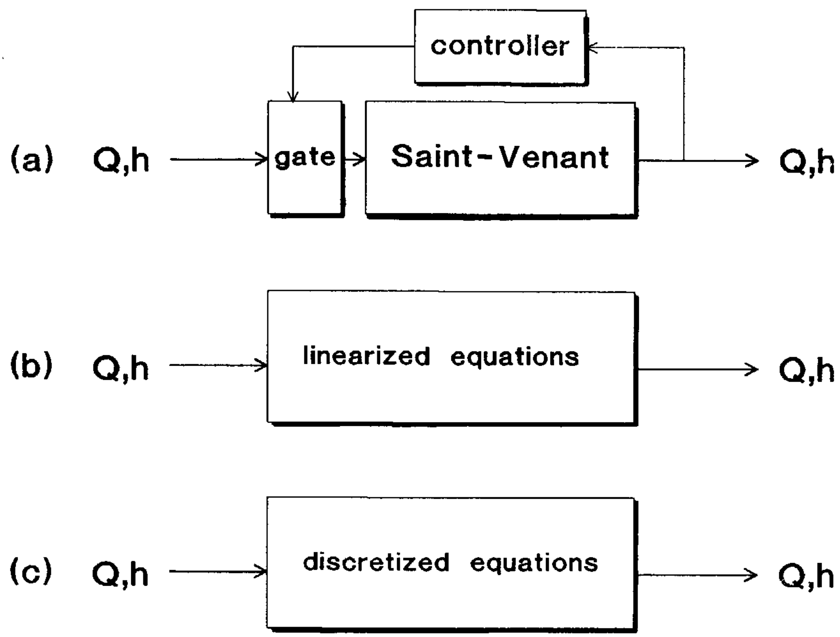


Fig. 2 Representation of a controlled system and its simplified equivalents

Once the linearized equations have been found, the stability of the system can be examined by investigating whether a disturbance in the system is amplified or damped in time. In literature, this classical stability analysis has been carried out by various authors e.g. (Hâncu 1974) and (Preissmann 1974). The main advantages of the analytical stability analysis are that the boundaries of stability can be derived and that insight is gained in the physical process.

4.3 Numerical approach

Instead of using the method of linearized equations, a numerical approach can be followed to solve the equations which describe the controlled system. In order to do so, the equations have to be discretized, and as a result again approximated solutions are found (Fig. 2c). Numerical diffusion and numerical instability are manifested proves of the approximated nature of numerical solutions. Due to the repetitive character of the numerical solution, the method is extremely suitable for a computer.

The advantages of the numerical method are that it is simple in use, especially for non-mathematically experienced engineers and that no simplifications have to be introduced concerning the configuration of the system or the types of controllers. Besides, non-linear effects can be incorporated.

The main disadvantage of the numerical method of solution is that only unique numerical solutions can be found. As a result, it is not possible to derive generally valid stability criteria, which can be found with the method of linearized equations.

To apply the numerical method, a computer model is needed. There are numerous computer packages on which a canal system can be modelled, and by means of which the unsteady flow phenomenon in the system can be calculated. However, there is hardly any modelling package on which closed loop controlled systems can be modelled (Schilling 1987). At the Delft University of Technology, MODIS hydrodynamic modelling package was developed which allows the simulation of real time control systems (Schuurmans 1991).

Recently, the authors of this paper extended the local controllers facilities of the computer model to regional controllers (Hoitink 1990). This new module fits nicely in the existing package and is easy in use. With this module all types of regional control systems can be modelled, such BIVAL- control and ELFLOW-control.

An example of the input file for regional control is depicted in Fig. 3. In the first block, the location and dimensions of the regulator are specified. Instead of specifying a value for the gate opening height (last column of Fig. 3), the gate opening is regionally automatically controlled (RA). The name of the controller is specified by the user and here the name "BIVAL1" was selected.

In the next input block the controller is presented. Subsequently, the name of the controller, the names of the water level sensor points, the type of controller, the initial opening height, the target water level to be maintained, the alpha factor (Eq. 4), the gain factors of the PID-controller (K_p, K_i and K_d), and the minimum and maximum gate opening height are specified.

*ORIF -	Definition of orifice structures							
+								
+Name	Branch Location	Flow	Free/	Effective	Flow	Opening	Sill	Gate
+of	name	Direction	Drowned	discharge	contraction	width	level	opening
+struct.				coef.	coef.			height
ORIF1	BRANCH1 10.50	PX	1.0	0.90	0.63	10.00	11.00	RA$BIVAL1
=								
*END	of structure definition							

*AURC -	Definition of automatic regional controllers											
+												
+Name	First	Second	Type	Initial	Target	alpha	Gain parameters			Min.	Max.
+of	sensor	sensor	of	opening	water	for	kp	ki	kd	opening	opening
+contr.	point	point	control	Struct.	Level	bival				height	height
+											
BIVAL1	POINT1	POINT2	PID	0.00	11.50	0.67	7.5	1.0	0.5	0.00	1.50
=											
*END	of automatic controllers										

Fig. 3 Example of input file of regional controlled regulator

To review the behaviour of various local and regional control systems a number of simulations were made. The results of the simulations were compared with the results derived with the method of linearized equations.

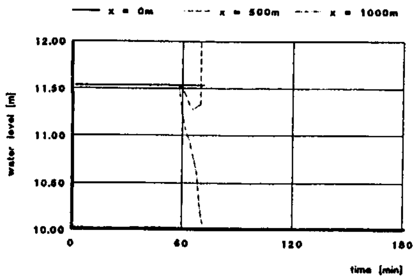
5 Results

The results of the computer simulations are depicted in Fig. 4 to 7. Originally it was intended to check the model results on its stability conditions found in literature, but it was found that this was almost impossible, due to many simplifications underlying the existing stability criteria. In Table 1, some of the existing stability condition are presented. (Alsthom 1990), (Hâncu 1974) and (Preissmann 1974).

Table 1 Stability criteria of downstream control systems

Control system	α	Stability criterion	Remarks
Local downstream control	1	$iL > \frac{u^2}{g}$	No decrement assumed by the regulator
BIVAL control	0.67	$iL > \frac{5u^2}{g}$	for Fr > 0.06
ElFlow	0.0	always stable	constant level assumed

These stability criteria do not include the correct equations of the controllers, and, therefore, they serve as an indication only.



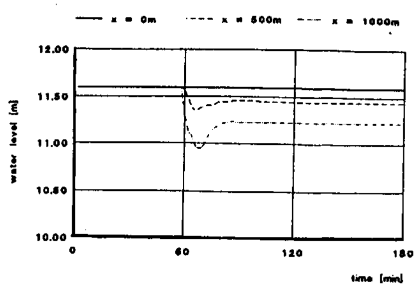


Fig. 4 *Unstable downstream control* *Fig. 5* *Stable downstream control*

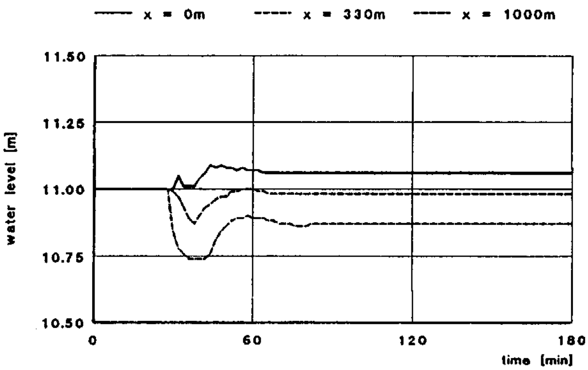


Fig. 6 *Simulation of a Bival controlled canal*

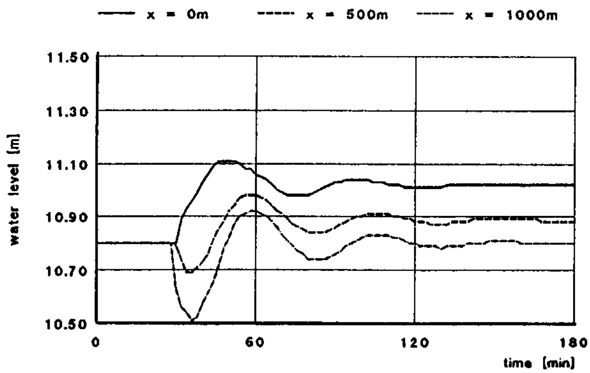


Fig. 7 *Simulation of an ELFLOW controlled canal*

The computer simulations were carried out on a canal with the following dimensions: Length : 1000 m, bottom width : 10 m, side slope : 1ver : 2hor and a roughness coefficient (Strickler): 40 $m^{1/3}/s$. The initial state was a discharge of 0 m^3/s and horizontal water levels.

After 1 hour, a disturbance was initiated downstream by a sudden increase of the offtake discharge from 0 to 15 m^3/s. The step function was selected, as in this function all frequencies are incorporated. The computational time step was equal to the measuring frequency, namely 2 minutes. Furthermore, a space step of 100 m was used.

First, local downstream control was simulated for a canal with a bed slope of 2 cm/km. This could be done by applying the regional control module using an α-value of 1. However, to check the analytically obtained stability criterion, downstream control was simulated by assuming a constant upstream water level as a boundary condition. If the target water depth was 1.54 m, the system proved to be unstable. (Fig. 4). This is in accordance with the analytical results presented in Table 1. When the target depth was raised only 6 cm up to 1.60 m, the system became stable. The water velocity (u) becomes less and the stability criterion of Table 1 is met. (Fig. 5).

Thereafter, BIVAL control was simulated by using an α-factor of 0.67. The canal bed slope was taken equal to 50 cm/km and at the downstream end a gate was assumed. After some time, the downstream gate was suddenly raised 0.30 m.The analytically derived stability criterion could not be tested, because this criterion was derived by assuming a constant water level at the pivot point. For the computer simulations a PI-controller was applied with the following gain parameters (Shinskey 1979): K_P = 1.25, and K_I = 0.9. The water level regulator was of orifice type with its sill level at the bottom of the canal and a width of 10 m. The simulation results show that the water level almost returns to its target level after a disturbance and that the upstream level is raised whereas the downstream level is lowered. (Fig. 6). The reason that the target level is not exactly met is that the water level sensors are located only at both pool ends and not in the pivot point itself.

Finally, ELFLOW control was simulated by using an α-factor of 0. The canal bed slope was taken equal to 50 cm/km and at the downstream end a gate was assumed. After some time, the gate was suddenly raised 0.30 m. For the simulations, again a PI-controller, with the following gain parameters K_P = 0.5 and K_I = 0.25, was applied. The water level regulator used was of orifice type with the sill level at the bottom of the canal and a width of 10 m. The results showed that, after some oscillations, the water level returned to its target level and all the other water levels were raised. (Fig. 7). The initial drop of the controlled water level is inevitable, due to the lack of a buffer reservoir.

6 Conclusions and recommendations.

If regional control is used, one should assure its proper performance and stability. The existing stability criteria, derived with the method of linearized equations, are only indicative because many simplifications are made with respect to the canal system, regulator and controllers.

The performance and stability can be analyzed with a numerical model. In the model no major simplifications have to be introduced and it is easy in use, especially for non-mathematically minded engineers. It was almost impossible to test the reliability of this numerical approach, as no good reference data were available. Therefore, it is planned to collect reference data of physical scale models.

Now the regional control systems have been incorporated in the MODIS model, the performance of nearly all types of control systems can be investigated and quantified. In doing so, the best control system for a particular project can be selected on a rational basis.

7 References

Alsthom 1990 — Additional technical documents, GEC ALSTHOM, La Courneuve Cedex, January 1990.

Chevereau 1987 — Chevereau G. and S. Schwartz-Benezeth, "BIVAL systems for downstream control", Proceedings of the symposium: Planning, operation, rehabilitation and automation of irrigation water delivery systems, ASCE, Portland, Oregon, USA, 1987.

Cool 1985 — Cool J.C., F.J. Schijff, T.J. Viersma, "Regeltechniek", ISBN 90-6674-002-7, Delta press, Amerongen, the Netherlands, 1985.

Cunge 1980 — Cunge J.A., F.M. Holly, A. Verwey, "Practical aspects of computational river hydraulics", Pitman advanced publishing program, London, UK, 1980.

Hâncu 1974 — Hâncu S, E. Rus, "Stabilité du mouvement de l'eau dans les canaux d'adduction des systèmes d'irrigation à fonctionnement automatique", Institute d'études et de recherches pour améliorations foncières, Revised Roum. Sci. Techn.-Méc. Appl., part 19, nr. 3, page 431-450, Bucarest, Roumania, 1974.

Hoitink 1990 — Hoitink B.P.A., "Regional control of irrigation systems, a study of stability and performance of regional downstream control canals", Delft University of Technology, Faculty of Civil Engineering, 1990.

Preissmann 1974 — Preissmann A, G. Chevereau, "Remarques sur le choix des méthodes d'appréciation de la qualité des divers systèmes de régulation de canaux à surface libre", Proceedings of the symposium on "Fluid motions stability in hydraulic systems with automatic regulators" in Bucarest, Roumania, 1974.

Schilling 1987 — Schilling W., "Real Time Control of Urban Drainage Systems, The state-of-the-art", IAWPRC/IAHR Joint Committee, 1987.

Schuurmans 1991 — Schuurmans W., "A model for real time controlled canal systems", proceedings of the European Conference Advances in water resources technology, Athens, 1991.

Shinskey 1979 Shinskey F.G., "Process-control systems", McGraw-Hill book company ISBN 0-07-0556891-X, New York, U.S.A, 1979.

Zimbelman 1983 Zimbelman D.D., D.D. Bedworth, "Computer Control for Irrigation-Canal Systems", Journal of Irrigation and Drainage Engineering, Vol 109, No. 1, March, 1983.

4.3 Operational performance of canal control systems

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Abstract Three canal control systems (manual upstream control, automatic upstream control and automatic downstream control) were studied for the main canal of the Shoeibieh irrigation project in Iran. The canal and the selected control systems were modelled in a hydrodynamic modelling package. The operational performances of the canal control systems were investigated and quantified for two water delivery schedules, one with small but frequent adjustments, and the other one with large, but infrequent adjustments. Based on the results of the study, automatic upstream control with a "small step" delivery schedule has been selected for the project.

1 Introduction

The aim of the study was to quantify the effect of alternative canal control systems and delivery schedules on the operational performance. By doing so, a rationally based decision for the most appropriate canal control system and a delivery schedule for the Karun Irrigation Project could be made.

The study has been applied to the Shoeibieh region of the Karun Irrigation Project, a new irrigation project located in the southern part of Iran (Maherani 1990). The total project area covers 160,000 ha with different cropping patterns (sugercane and "cropping pattern A"). These cropping patterns have been established, following the local agricultural practices and demands. The project area is divided into four regions, one of which is the Shoeibieh region, serving 42,000 ha. The selected region is fed by a 48 km long conveyance and a main canal with a capacity of 54 m^3/s. (Fig. 1).

The paper is structured as follows. First, two water delivery schedules and three different canal control systems for the main canal of the Shoeibieh region are presented. Thereafter, the hydrodynamic model, used to study the alternative systems, is discussed and the performance parameters, which were used to judge the results of simulation, are defined. Computer simulations were carried out for two conditions: ordinary conditions with sufficient supply and extraordinary conditions

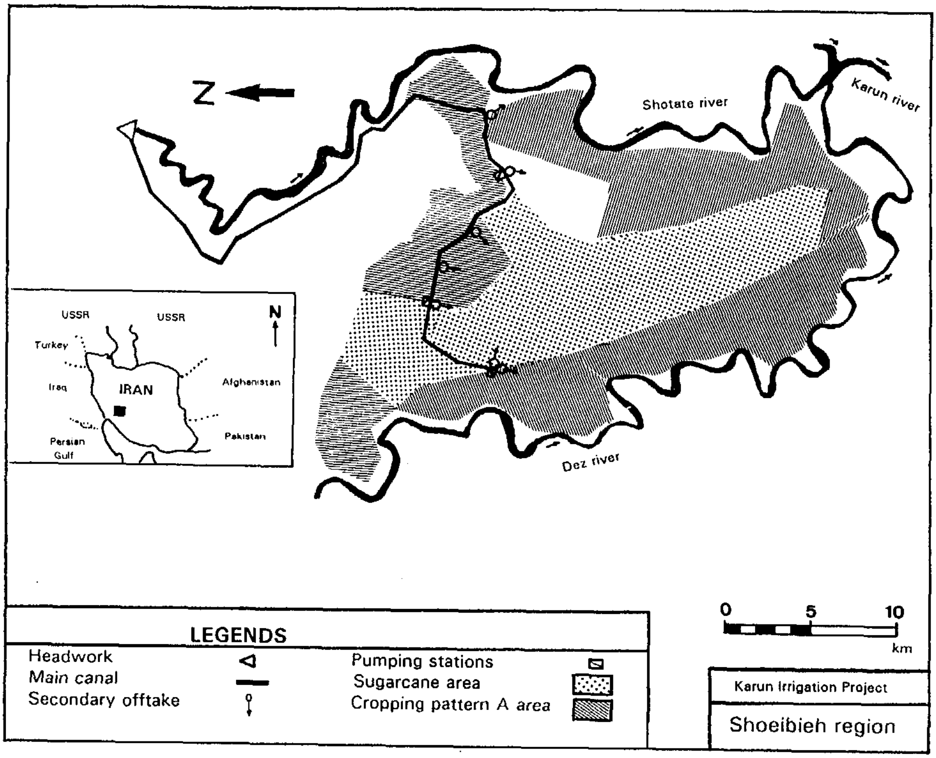


Fig. 1 The Shoeibieh region of the Karun Irrigation Project

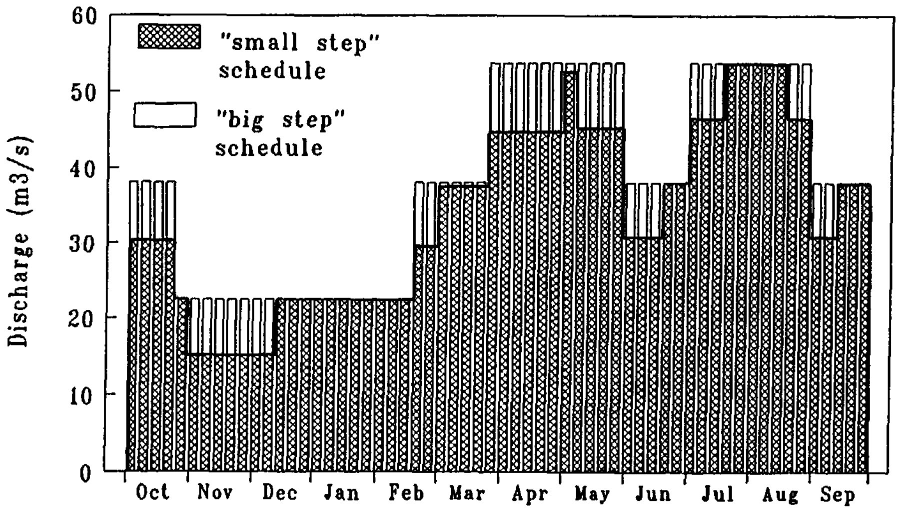


Fig. 2 The "small step" and "large step" delivery schedule for the main canal

with both a surplus and a deficiency in supply. The results of the computer simulations are presented by means of the previously defined performance parameters. Finally, the conclusions and recommendations of the study are presented.

2 Water delivery schedules and canal control systems

2.1 Water delivery schedules

The arranged method of water distribution, 24 hours per day and 7 days a week, has been selected for the project of the possible on-demand, the semi-demand and arranged method. Arranged distribution implies that the water distribution is determined by the irrigation authorities in advance of the irrigation season. Two delivery schedules were derived for the main canal of the project area. The first one is referred to as the "small step" schedule, with frequent and small adjustments, and the second one is called the "large step" schedule, with infrequent but large adjustments. The advantage of the "large step" schedule is that it is easy in operation as gate adjustments have to be made only 6 times per year. For the "small step" schedule, frequent gate adjustments are required, but the water losses are smaller as the water requirements are followed more closely. (Fig. 2). In the "large step" schedule which is more easy to operate, the scheduled losses are 16%, whereas for the "small step" schedule, which is more troublesome to operate, the scheduled losses are negligible.

2.2 Canal control systems

To facilitate the water distribution among the secondary offtakes according to the derived schedules, a canal control system is needed. For the project with both pump and gravity offtakes, a local water level control system is preferred. By controlling the water levels in the vicinity of the (gravity) offtakes, the offtake discharges can be easily controlled. Two types of local water level control systems can be distinguished, namely local upstream and local downstream control. Furthermore, the water level regulators can be operated manually or automatically. For the project three alternative control systems were considered: (1) manual upstream control, (2) automatic upstream control and (3) automatic downstream control. The types of regulators applied for each control system are collected in Table 1.

After the delivery schedules and the control systems were defined, their relationship was investigated. By modelling each control system and evaluating the performance of both the "small step" and "large step" delivery schedule, this relationship could be examined and quantified.

Table 1 **Selected canal control systems and related regulators**

No.	Control system	Mode of operation	Water level regulator	Offtake structure
1	Upstream	Manual	Radial gate	Flat sliding gate plus measuring device
2	Upstream	Automatic	AMIL gate	Baffle distributor
3	Downstream	Automatic	AVIO gate	Baffle distributor

3 Method of investigation

3.1 The hydrodynamic model

MODIS hydrodynamic modelling package was used to define a model of the studied project area. For more information about the modelling package, reference is made to an accompanying paper and MODIS user's guide (MODIS 1990). Three models were made, one for each control system. (Fig. 3). The 48 km main canal, with 8 gravity offtakes and 3 pump offtakes, was modelled by using a space step of 500 m and a time step of 10 minutes.

To simulate the behaviour of the AMIL and AVIO gates, orifice types of gates were modelled in which the gate opening height was controlled by a PID-controller. (Cool 1985). Manual operation of the offtake structures (and the water level regulators in case of manual upstream control), was simulated by specifying gate opening heights as a function of time.

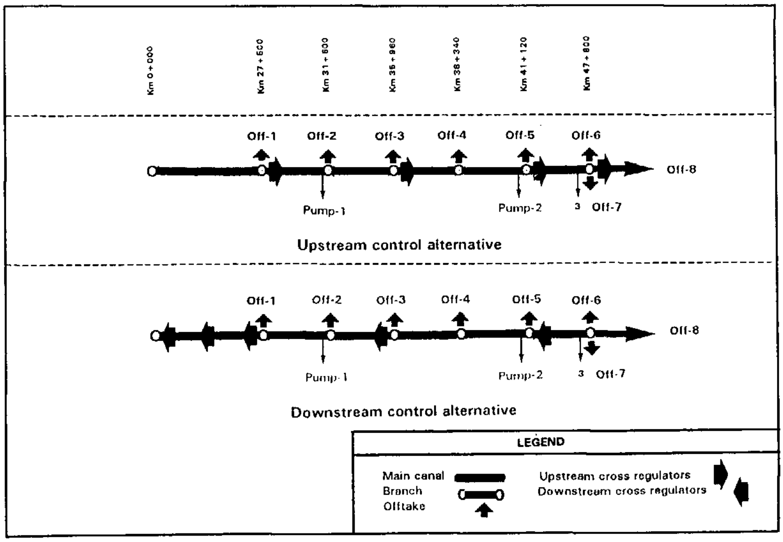


Fig. 3 Modelled upstream and downstream controlled systems

3.2 Evaluation of the performance of individual offtakes

For the evaluation of the simulated water distribution, two operation performance parameters were defined. The first one is called the Delivery Performance Ratio (DPR), which specifies the extent to which an offtake receives its intended supply. The second one is called the operation efficiency (e_o) and specifies the amount of water lost by inappropriate allocation of the water to an offtake. In formulae these parameters read,

$$DPR = \frac{V_e}{V_i} \cdot 100\% \tag{1}$$

$$e_o = \frac{V_e}{V_a} \cdot 100\% \tag{2}$$

Where:

DPR	=	Delivery performance ratio	(-)
e_o	=	Operation efficiency	(-)
V_e	=	Volume effectively delivered	(m^3)
V_i	=	Volume intended to be delivered	(m^3)
V_a	=	Volume actually delivered	(m^3)

The intended volume to be delivered (Vi) is defined by the user in terms of begin and end time of supply, an intended flow rate and an allowable range of variation in flow rate. (Fig. 4). The actual volume of water received by an offtake (Va), is considered to be an effective volume (Ve) only when the moment of supply is within the user's defined period and the flow rate is within the allowable range with a maximum value of Vi. (Fig. 5). One should realize that the operational efficiency of individual offtakes is only related to operational losses and not to leakage losses. Leakage losses are assumed to be relatively constant, irrespective of the operation method followed, and are deemed to be accounted for in the scheduled deliveries for the purpose of this study.

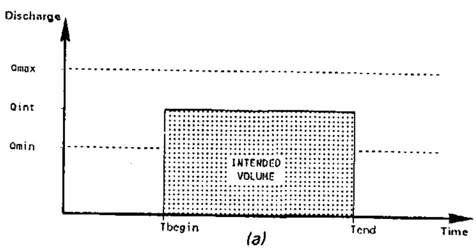


Fig. 4 Intended supply

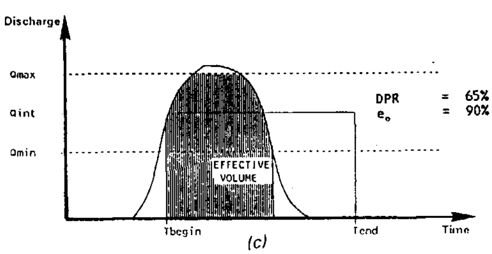


Fig. 5 actual supply

3.3 Evaluation of the performance of the complete system

To evaluate the operational performance of the main canal, including all its offtakes, overall performance parameters are needed. The overall Delivery Performance Ratio is calculated by a weighed average of the DPR of the individual offtakes and reads:

$$DPR_{overall} = \frac{\sum_{n=1}^{n=p} V_{e,n}}{\sum_{n=1}^{n=p} V_{i,n}} \tag{3}$$

Where $DPR_{overall}$ is the overall Delivery Performance Ratio, $V_{e,n}$ is the effective volume received by offtake n (m^3), $V_{i,n}$ is the intended volume to be received by offtake n (m^3), and p is the number of offtakes involved.

The overall operation efficiency incorporates the operational losses of the individual offtakes, the losses due to filling up the canal to its operational level, and the leakage losses and/or spill losses, if these occur. In formula it reads,

$$e_{0,overall} = \frac{\sum_{n=1}^{n=p} V_{e,n}}{V_{a,intake}} \tag{4}$$

Where $e_{0,overall}$ is the overall operation efficiency, $V_{a,intake}$ is the actual intake volume at the head of the main canal (m^3), $V_{e,n}$ is the effective volume received by offtake n (m^3), and p is the number of offtakes involved.

The overall delivery performance ratio becomes equal to the overall operation efficiency, if the sum of intended volumes is equal to the volume taken in. In that case the overall performance of the canal system can be characterized by one single figure.

3.4 Periods of simulation and evaluation

To investigate the operational performance of the selected control systems for both the "small step" and "large step" delivery schedules, ideally a complete year should be simulated. However, this would take too much time and effort as the model calculates with time steps of 10 minutes. Therefore, only the most interesting period has been simulated and this is the period of unsteady flow. The results obtained for the period of unsteady flow have been converted into results for periods of time equal to the period between gate adjustments. After a change in inflow rate has been made and the gates have been set in their new position, the system will transform into a new steady state. Once the new steady state has been reached, no more variation in time will occur. So, the period of simulation was based on the largest response time of all three canal control systems considered (this equals 20 hours). The response time is defined as the time needed to transit from one steady state into another steady state. As this time is theoretically infinitive, the time at which 90% of the change has been achieved is denoted as the response time.

For the evaluation of the results, a period of three weeks respectively two months should be considered for the "small step" and "large step" schedules. The performance parameters found for the simulation period of 20 hours can be converted into a longer period of time, assuming a perfect performance (DPR and e_0 of 100%) once a new steady state is reached. (Fig. 6).

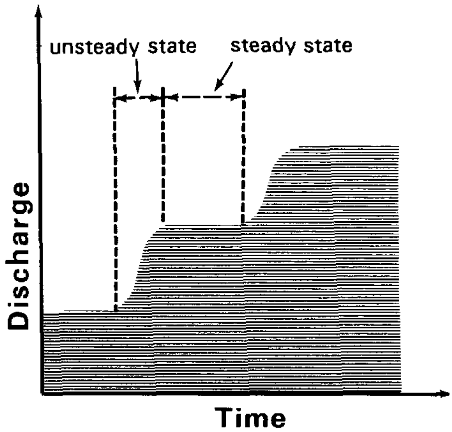


Fig. 6 *Period of unsteady state and period of steady state*

4 Results of simulation

4.1 Normal conditions

Once the system had been modelled and the operation performance parameters had been defined, the performance of each control system was investigated for normal conditions (actual inflow equals expected inflow). Various trials, operation strategies for the offtake structures, and for the water level regulators in the manual upstream control case, were executed. The results showed that during the 20 hour simulation period, the performance differed for each offtake, which reflects a non-uniform or unequal water distribution.

The overall results of the simulations are depicted in Table 2. In this table, the overall performances (DPR which equals e_0) of the main canal are given for each canal control system and delivery schedule. Both the performance parameters found in the first trial (simultaneous operation of all regulators) and the performance parameters found in the best trial are depicted. The best trial was achieved by changing the settings and timings of the regulators operations, following a trial and improvement procedure, until no further improvements were found. It can be read from the table, that in some cases the results of both trials can vary considerably. Only when automatic downstream control is applied, the results between the trials are identical.

One should not forget that the values, found by the computer simulation, reflect the operational losses only and, therefore, they should be expected to be higher than those which will be found in practice, because in the computer simulation the instructions are followed perfectly and the stage discharge curves of the regulators are known exactly.

Table 2 Results of model simulations for overall performance

		20 hours period		2 months	3 weeks
		"Large step" schedule	"Small step" schedule	"Large step" schedule	"Small step" schedule
Control system	Performance	DPR = e_o	DPR = e_o	DPR = e_o	DPR = e_o
Manual upstream	First trial	80%	87%	99.6%	99.4%
	Best trial	86%	96%		
Automatic upstream	First trial	88%	96%	99.8%	99.5%
	Best trial	89%	97%		
Automatic	First trial	100%	100%	100%	100%
	Best trial	100%	100%		

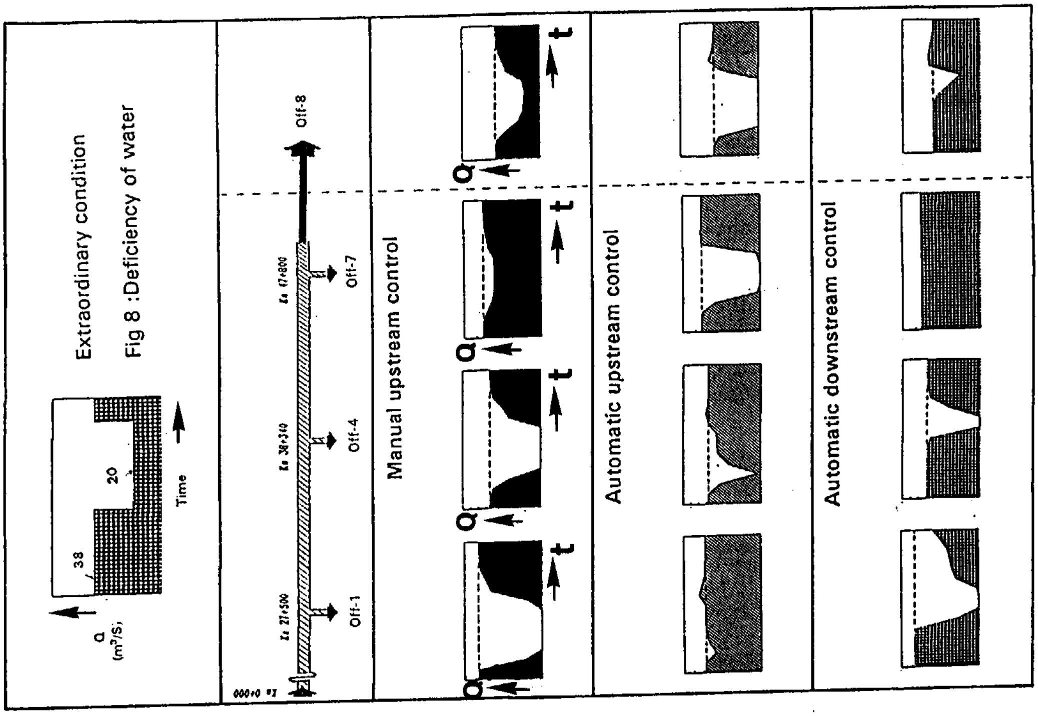
4.2 Exceptional cases

In addition to simulations carried out for normal conditions, the water distribution has been evaluated for two exceptional cases. The first exceptional case is a surplus in intake discharge at the head works and the second one is a deficiency in intake discharge. The results of these simulations are depicted graphically in Fig. 7 & 8.

Surplus of water

In case of a manual upstream control system, in which the operators do not react on the surplus of water, the surplus of water will be distributed more or less proportionally over the system, due to the application of underflow structures in both the main canal and at the offtakes.

In the automatic upstream control system, the excess of water will be transported downstream. This is because the automatic water level regulators maintain a (nearly) constant water level, so that the offtake discharges remain (nearly)



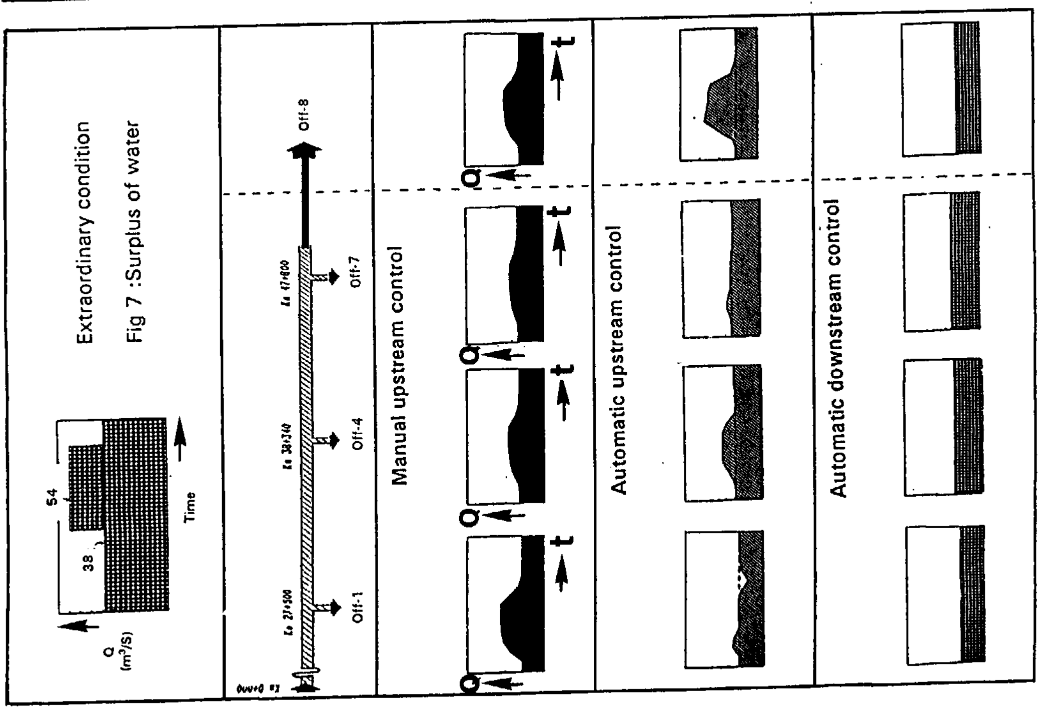


Fig. 7 & 8 Water distribution for abnormal conditions constant. If the system is equipped with automatic downstream control regulators, a surplus of water cannot enter the system. Hence, this type of condition cannot occur in a downstream control system.

Deficiency of water
When a considerable deficiency of water occurs, the manual upstream control system reacts as follows. The deficiencies are divided over the offtakes due to the proportional character of the bifurcation points. (Not that it is a proportionally controlled system, but when the regulators are not operated, the system shows some proportional behaviour). The offtakes located upstream, however, do suffer most. This is because the system is not operated during the period of shortage, so that the water levels in the upstream sections will lower, and consequently the offtake discharges in this section will reduce considerably. In the automatic upstream control system, the downstream offtakes suffer most from the water deficiencies. Theoretically, the upstream offtakes should not be affected, but "in practice" it is seen that the upstream offtakes, which are not located in the vicinity of the regulators, do suffer. In downstream control, one can see from the graphs that, contrary to upstream control, the upstream offtakes are facing shortage of water, whereas the downstream offtakes are hardly affected at all.

5 Discussion of the results

With respect to the Shoeibieh irrigation project, where the system is operated once in three weeks or once in two months only, it is noticed that the differences in operational performance of the control systems and delivery schedules will fade away and that all systems will reach a perfect operational performance, provided that the inflow rate is constant and according to schedule. The effect of the unsteady flow phenomenon, caused by operating the system, is negligible and, consequently, all canal control systems show an equal and perfect performance. In general, it can be stated that the impact of unsteady flow on the irrigation water distribution, caused by operation, will be significant only when the ratio of the response time and the period of operational adjustments is in the order of 1.

Reviewing the behaviour of the canal control systems for (unexpected) extraordinary circumstances, it is noticed that the manually upstream controlled system gives the best spreading of harmful effects. The automatic upstream control system shows the tendency of transporting the negative effects to the downstream users, whereas in the downstream control system the upstream users are most unfortunate.

The effect of unsteady flow in response to operation would be far more significant if the system was operated more frequently, for instance in case of 12 hours irrigation instead of 24 hours a day. In that case the downstream control system will give the best performance, followed by the automatic upstream control system. The manual upstream control systems will give the lowest performances. Furthermore, the differences between the first trial and the best trial indicate, that it would be rather difficult to identify the moments in time and magnitude of adjustments, which will achieve the best performance in the manual upstream control system. For only if there is hardly any difference in performance for the first and the best trial, it is apparently relatively simple to operate the system satisfactorily. As it was already difficult to find a sound operation strategy in a computer model environment, for manual upstream control, practical operation in the field, will become even more troublesome. Consequently, higher operation losses can be expected in real life. This is true for all control systems, but the relative increase is expected to be highest for manual upstream control systems.

6 Conclusions and recommendations

Two sources of water management losses can be distinguished. The first one is referred to as "scheduled losses" and is a result of the fact that the scheduled delivery does not exactly follow the water requirements (including the conveyance and tertiary unit efficiency). The second one is called "operational losses" and is a result of the fact that the actual deliveries to the offtakes are not exactly equal to the scheduled deliveries.

In a computer model environment, where the stage discharge curves of all regulators are known exactly and where the operational instruction are followed precisely, the operational losses are a result only of the unsteady flow phenomena in the canal system. The unsteady flow phenomena can be studied with a hydrodynamic flow model; as a result the operational losses can be quantified. If the period between gate adjustments is long, in comparison with the response time of the system, the operational losses due to the unsteady flow phenomena can be neglected. This happened in the case studied, in which the duration of unsteady flow phenomena was negligible in relation to the total irrigation period and, consequently, the operational losses were negligible, provided that the inflow into the system follows the schedule accurately.

In addition to using a hydrodynamic flow model to study the effect of unsteady flow on the water distribution for normal operational conditions, the model can be used to review the system's behaviour for extraordinary conditions when the inflow into the system does not accurately follows the schedule. In practice, these circumstances frequently occur and it should be noticed that in these situations the system does not always react as was expected theoretically. For instance, in manual upstream control all the offtakes were affected by variations in intake discharge.

Based on the results of the study, the automatic upstream control system was selected the most appropriate control system for the Shoeibieh region of the Karun

Irrigation Project. This decision was based on the consideration that the automatic upstream control system is easier to operate than the manual upstream control system. The downstream control system is easiest in operation but the gain in performance was considered not being worth the higher investment costs involved in downstream control, due to the horizontal embankments and more expensive regulators. The operational losses were negligible for all control systems considered, because of the relatively short response time of the system in relation to the period between gate adjustments.

Furthermore, the "small step" delivery schedule has been selected. The scheduled losses are much lower for the "small step" schedule compared to the "large step" schedule. As the automatic upstream control system is relatively easy to operate, the operational losses are expected to remain small, even in the real life environment.

7 Literature

MODIS 1990 — Schuurmans W, *MODIS user's guide*, Delft University of technology, Faculty of Civil Engineering, department of irrigation, Delft, the Netherlands.

Cool 1985 — Cool J.C., F.J. Schijff, T.J. Viersma, *Regeltechniek*, Delta Press B.V., Amerongen, the Netherlands.

Maherani 1990 — Maherani M., *Automation of Karun Irrigation Project*, International Institute for Hydraulic and Environmental Engineering, Delft, the Netherlands

4.4 Identification of a control system for a canal with night storage

reprint of an article for the Journal of Irrigation and Drainage Engineering of the ASCE, 1991.

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key words: night storage, canal control systems, unsteady flow model

Abstract. The performance of three local control systems for an irrigation canal with night storage was investigated by using a hydrodynamic flow model. It was found that all the control systems were able to control the system if the set-points of the controllers varied in time. However, the frequency and preciseness of the required set-point adjustments varies considerably per control system. The magnitude and timing of set-points adjustments can only be determined with the help of a simulation model.

1 Introduction

For irrigation schemes with only day irrigation, night storage is nearly always indispensable. One possibility to create night storage is to use the canal for in-line storage. By doing so, no separate night storage reservoirs are required. However, control of the canal system will become much more difficult due to the fluctuating water levels.

In this article a methodology is presented to derive a suitable control system and accompanying operation strategy for irrigation canals with in-line canal storage. This will be done by presenting a case study in which a local control system for a 110 km long irrigation canal with 36 cross regulators and no separate night storage reservoirs outside the canal was identified.

The paper is structured as follows. At first, a description of the canal system and its present operation practice is presented. Thereafter, the applied method of investigation is presented. A daily water balance was set up to demonstrate the storage capacity of the canal system versus the required storage capacity. Then, the canal system was modelled in a computer model and three local control systems were simulated. The results of the simulations were evaluated by using pre-defined performance indicators. Finally, the conclusions and recommendations of the study are presented.

2 Description of the system

The King Abdullah Canal (former East Ghor Main Canal) is a 110 km long irrigation canal in Jordan with a capacity of only 15 m^3/s. (Fig. 1). The canal feeds nearly 2/3 of the entire Jordan farm land and provides drinking water for the capital Amman. The lined canal has a trapezoidal uniform cross-section with a depth varying from 1.85 m to 2.73 m. The bottom slope varies from 18 cm per km for the Northern branch (0-65 km), via 28 cm per km for the Southern branch (65-96 km), to 10 cm per km for the remaining 14 km of the Southern branch (96-110 km).

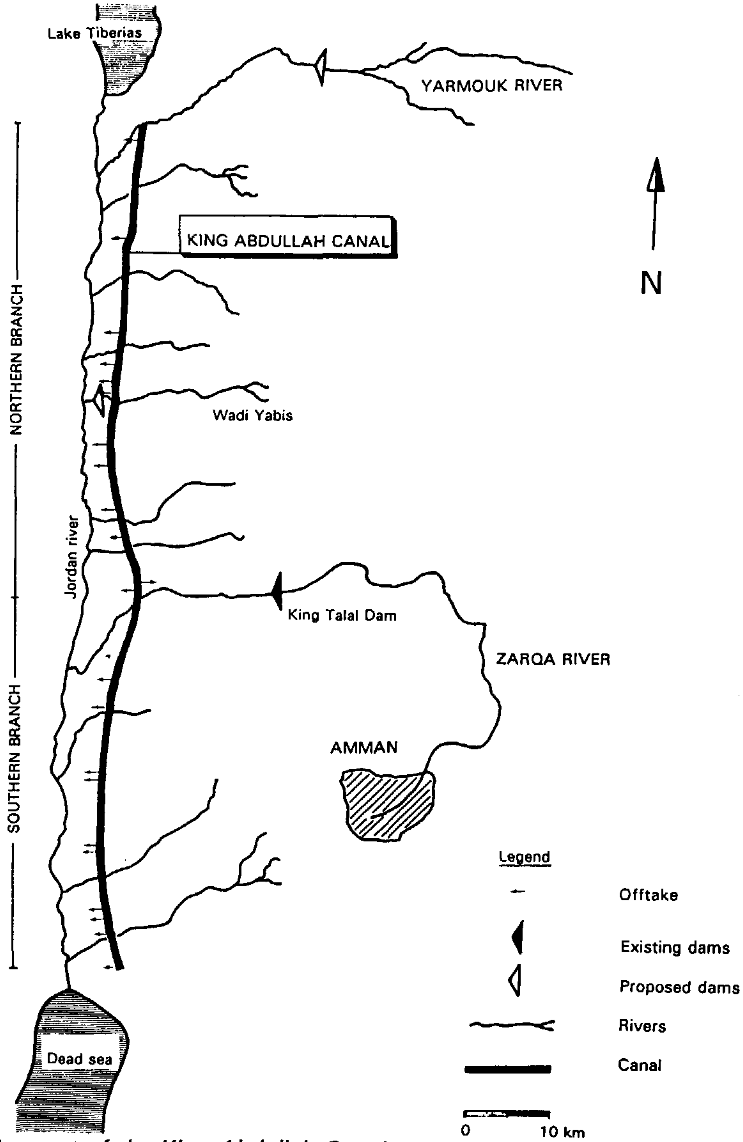

Fig. 1 Lay-out of the King Abdullah Canal

Irrigation water is subtracted from the canal by 22 "high head gravity" or by pump offtakes during only 8 hours a day, whereas drinking water for Amman is provided during 24 hours a day. Requests for irrigation water have to be made one day in advance. The amount of water allocated to the offtakes depends on the requested and the available amounts of water. The requested amount has its peak in May. The Northern branch is fed by the Yarmouk river, which has a strongly seasonal character with floods in January and low flows in May. The Southern branch receives a rather constant inflow from a reservoir and occasional spillage from the Northern branch.

To control the water flow 36 cross regulators, each comprising a central gate with symmetrical weirs at each side, were constructed in the main canal. The operational instructions of the manually operated regulators are determined by an experienced chief operator.

To improve insight in operational behaviour of the canal, and to improve the water distribution performance, a study has been conducted to find a suitable control system and accompanying operation strategy for the 36 regulators in the main canal. The control system should be as simple as possible with the least possible hardware modifications, and allow for the conveyance of floods to reservoirs (e.g. Wadi Yabis) located along the canal system. Moreover, the required hardware modifications should be kept to a minimum.

3 Method of investigation

At first, it should be investigated whether the canal storage capacity is sufficient to store the irrigation water during the night. The canal storage capacity can be determined by comparing the maximum and minimum water levels, for steady flow conditions, in each pool in the period of maximum demand.

In Fig. 2, both the cumulative available storage capacity and the required storage capacity for the period of maximum irrigation demand for the King Abdullah Canal are depicted. It can be read that the total canal storage capacity is sufficient for the Northern branch, but insufficient for the middle reach of the Southern branch. The latter's capacity has to be increased, e.g. by raising the embankment level, before any control system can be selected. Furthermore, it can be noticed that the individual pools are not "self-supporting", but that the storage capacity of several pools is needed to fulfil the irrigation requirements of one offtake. In order to make this storage available on the right time, careful operation of the regulators, both in magnitude and time, is needed.

For this purpose, a control system and accompanying operation strategy is needed for the cross regulators. In the case study three existing local control systems were selected for further investigation: upstream, downstream, and mixed control, all of which can be operated both manually and automatically. To check the applicability of the control systems, the systems will have to be simulated and evaluated on a computer model which can calculate the unsteady flow phenomenon.

The most critical event is the period of maximum water requirement as in that period the required storage capacity is highest. Another critical event can be the conveyance of floods. In the case study, the system had to convey floods through the Northern branch.

For this study the MODIS model (an acronym of Modelling Drainage and Irrigation Systems) has been used to simulate and evaluate the selected control systems, as the MODIS model is able to simulate and evaluate controlled irrigation canals. (Schuurmans 1991).

The entire canal system, including all its regulators, syphons and offtakes, was modelled by using two branches: a Northern and a Southern one. The model generated a computational grid along the branches with a step size of 500 m, and calculated with a time step of 2½ minutes.

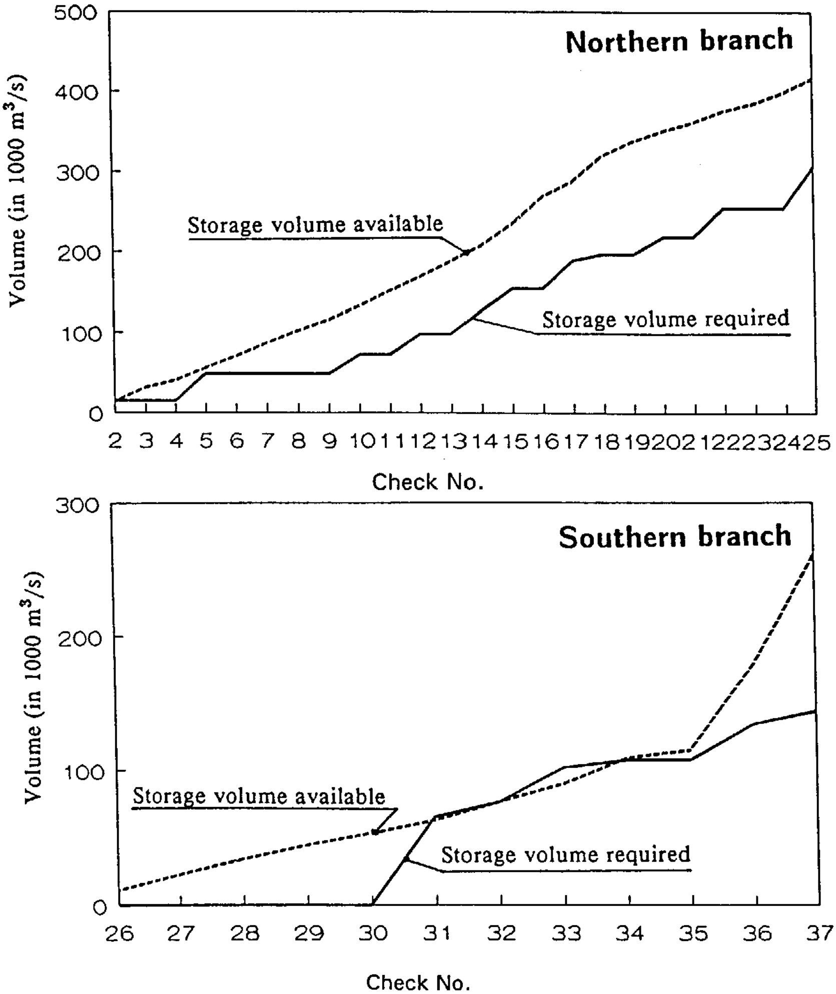

Fig.2 *Cumulative storage capacity and storage requirement of the King Abdullah Canal*

After the system was modelled tests had to be carried out to check the validity of the model. One of the tests simulated a flood wave through the system. (Fig. 3). The propagation time of the wave was checked with an analytical diffusion formula. Moreover, volume conservation of the wave was checked. It was found that the response time of the uncontrolled system was more than 3 days. The long response time, combined with the limited night storage capacity, already indicates the difficulties in controlling the system. The model was not calibrated, because the system has to function for a range of resistance values.

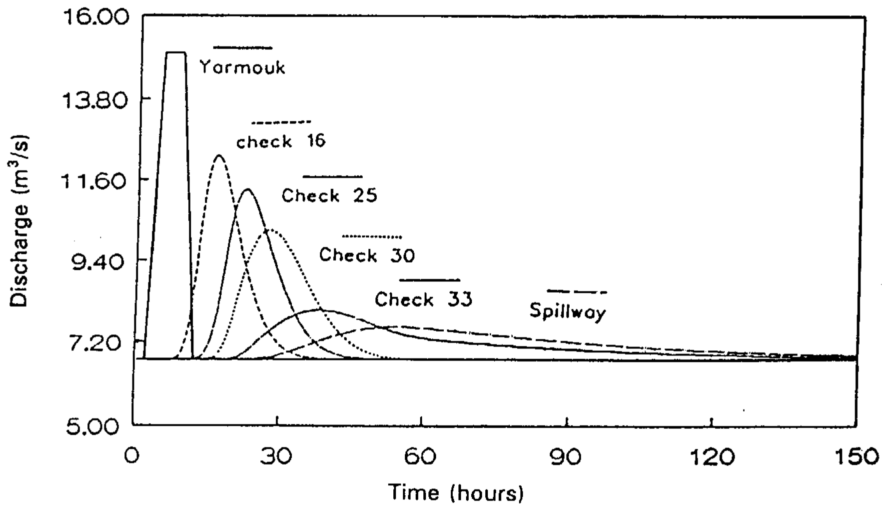

Fig.3 *Propagation of flood wave through the canal*

After the validity of the numerical model has been confirmed, each control system can be modelled by using the closed loop control facilities of the MODIS model. This implies that the height of the gate opening of each regulator is controlled on the basis of the water level upstream from the regulator in case of upstream control, downstream from the regulator in case of downstream control and both upstream and downstream from the regulator in case of mixed control.

In the case study two types of controllers, a Proportional Integral Differential (PID) and a speed controller were modelled. As the simple speed controller showed a nearly equal performance to the PID-controller, the speed controller with a dead band of 0.10 m, and a gate movement speed of 0.15 cm/min was used for the rest of the simulations. (This is a result of the gradual variations in the system and of the fact that many gates appeared to be completely opened during the irrigation period).

The set-points, or target water levels to be maintained, could be varied in the model as a function of time. The variation of the set-points in time is called the operation strategy, and was derived for each control system by a heuristic approach. No use could be made of optimization techniques as at the present state-of-the-art these techniques can only be applied to simplified equations. In this case the hydrodynamics were considered to be too predominant to be neglected.

To allow for a fast interpretation of the simulation results, evaluation parameters have to be formulated in advance. In the case study the most apparent performance indicator was, that, during the irrigation period, the water depth in front of the offtakes should not drop below a minimum depth of 0.70 m. Furthermore, the water levels should not overtop the canal embankments. If the first indicator is met, a perfect performance will be achieved, as no pumps will fall dry. If the embankments are not overtopped, no hardware modifications will be required.

4 Results of the simulations

■ Upstream control

Fig. 4 shows the maximum and minimum water levels in the Northern branch of the canal for upstream control with fixed set-points. The water levels in the tail end drop below the minimum levels, whereas the variation of the water levels at the head reach is minimal, furthermore, the water level is rather constant in those pools with no offtake. This implies that available storage capacity is not used beneficially. To make the stored water available to those pools with insufficient storage capacity, two solutions were worked out.

The first and most simple one was to increase the minimum opening height of the regulators during the irrigation period. The results showed a considerable improvement, but did not lead to a perfect performance. (A perfect performance is obtained when the water depths in front of the offtakes do not drop below the minimum required depth of 0.70 m).

The other solution was to lower the set-points during the simulation period. By doing so, the water stored in the upper reaches was made available to the lower reaches. The magnitude and timing of the set-point adjustments was derived by adopting a "trial and error" approach. The final results showed a perfect performance and are depicted in Fig. 5.

For both the Northern and Southern branch set-points adjustments, with a magnitude ranging from 0.20 m to 0.10 m, appeared to be needed every hour. It should be noted that the magnitude and timing of adjustments is unique for each situation and, consequently, a model is needed to derive the control strategy. However, the most critical period of maximum requirements has been simulated, so that for other situations the operation strategy will be simpler. The upstream control system is very suitable to convey floods through the canal.

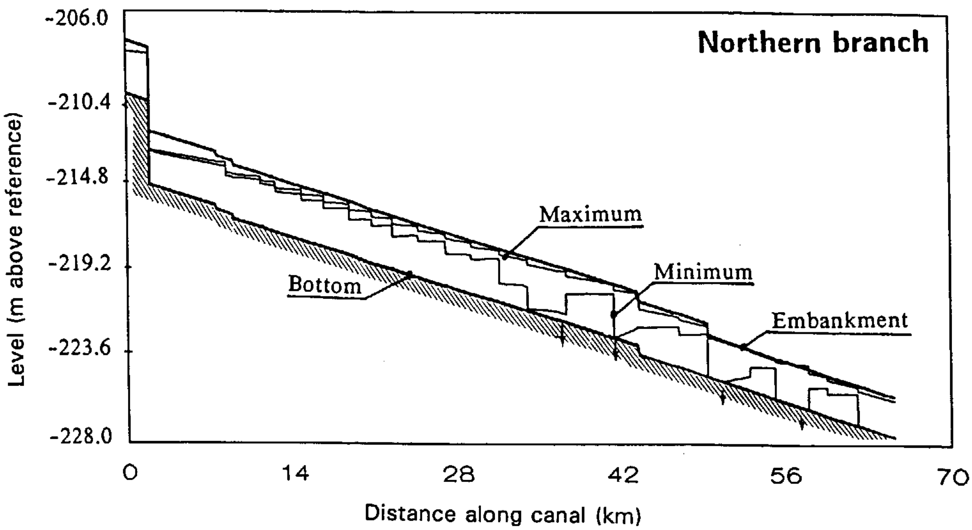

Fig.4 Upstream control of the Northern branch using fixed set-points

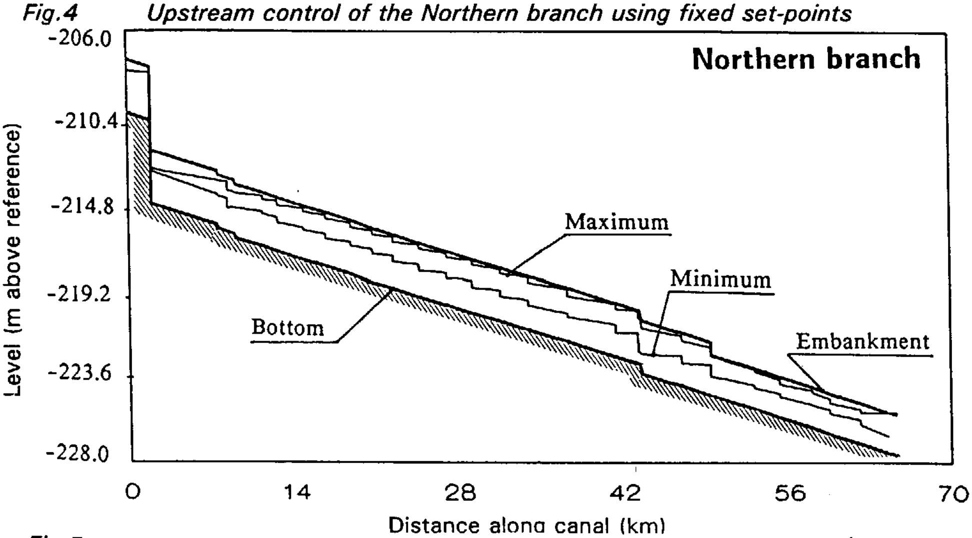

Fig.5 Upstream control of the Northern branch using variable set-points

- **Downstream control**

Fig. 6 depicts the maximum and minimum water levels in the Northern branch in case of downstream control with fixed set-points. The water levels at the tail end of the Northern branch are falling below the minimum water depth of 0.70 m. One would expect just the opposite, namely water shortage in the upper sections of a downstream controlled canal.

The reason for this phenomenon, water shortage downstream, is, that the gates do not open sufficiently and, hence, hamper the conveyance. The fact that the gates do not open sufficiently can be explained as follows. The normal water depth downstream from a regulator is determined by the discharge through the canal. If the target level downstream from the gate is too low, the desired water level will be reached with too small a discharge. As a result the regulator will not be opened further as the target level has already been reached. However, the demand downstream might very well exceed the discharge through the regulator. The result yields a falling water level in the lower reaches of the canal.

To solve this problem, the target water levels will have to be raised, so that the regulators will open more and pass higher discharges. This, in turn, will lead either to higher horizontal embankments along the canal or to variable target levels during the day. The disadvantage of the first option is that it will be costly, whereas the second solution will provide less safety in case the water offtake downstream is suddenly stopped during the irrigation period. The results of the latter alternative are presented in Fig. 7. Note that only some target levels need to be raised. Yet, another possibility would have been to use regional downstream control, e.g. ELFLOW-control, whereby the set-point is located at the tail end of the reach downstream the regulator. However, this last option was rejected as being a too sophisticated one.

For the steeper Southern branch, downstream control was less attractive. Not only because of the horizontal canal embankments required for downstream control, but also because of the fact that most offtakes are located in the middle reaches of the Southern branch. Sufficient storage capacity is available at the tail end, which, in case of downstream control, would not be totally used.

The main advantage of downstream control is that it is self-regulating for irrigation demands. The canal system does not only have to delivery irrigation water, but also has to convey floods. Unfortunately, downstream control systems cannot convey floods through the canal, as the system does not react on upstream water levels. To overcome this difficulty, mixed control was introduced.

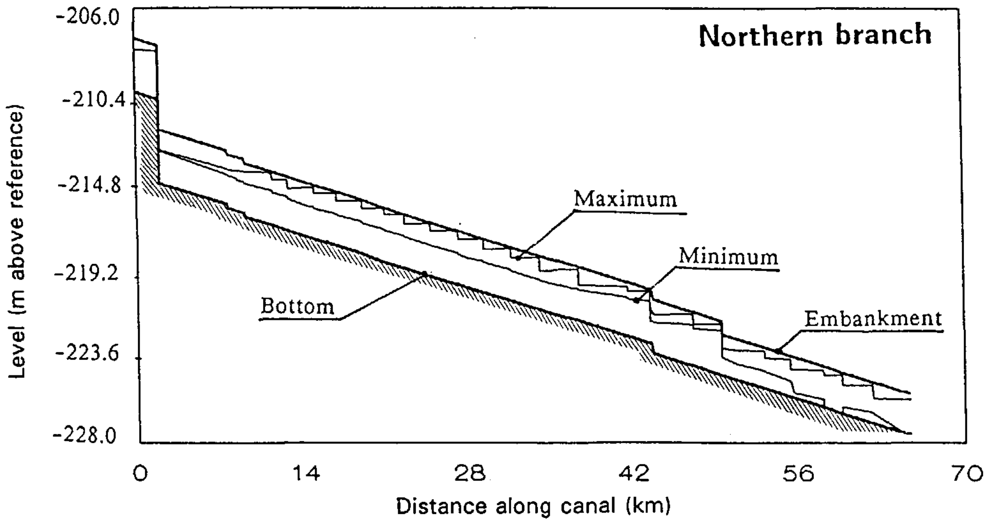

Fig.6 *Downstream control of the Northern branch using fixed set-points*

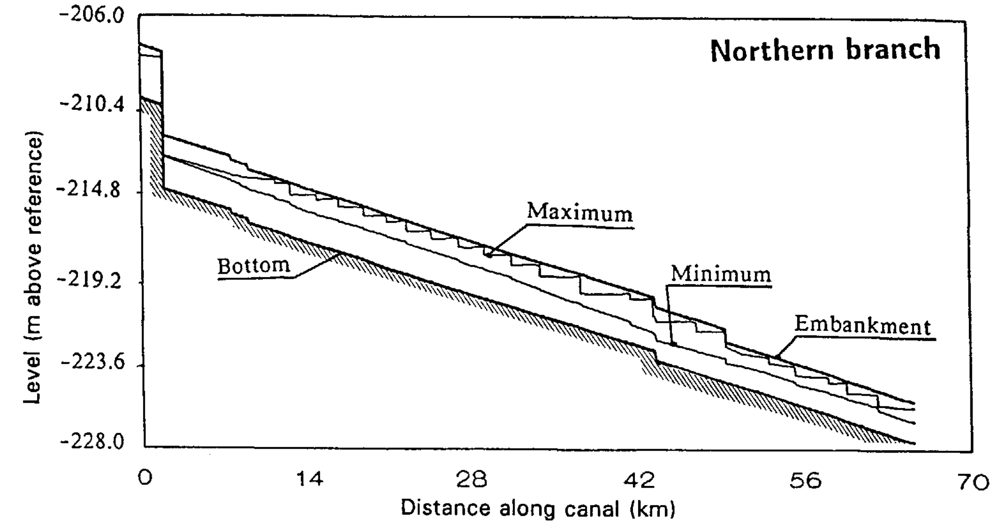

Fig. 7 *Downstream control of the Northern branch using variable set-points*

■ **Mixed control**

The mixed control system combines downstream control and upstream control. This means that on both sides of the regulator the water levels are sensored. If the upstream water level exceeds a maximum upstream level the gate will always be opened, and if the upstream water level is below the upstream target level, the regulator will react to the downstream water level like a downstream control system. To prevent the upstream sections from falling dry, one can also specify a minimum upstream water level, below which the gate will always be closed. However, in our simulation the upstream sections never fell dry.

The results of the mixed control simulations are equal to the results of the downstream control simulations. (Fig. 7). If a flood enters the canal system, the system will react like an upstream control system.

If mixed control is implemented, the target levels should be carefully determined in order to avoid interaction between the downstream target level of one regulator and the upstream target level of the next one. To avoid interaction, the upstream target level should always be higher than the downstream target level of the regulator located upstream. However, if the difference between both levels is too small, interaction will occur, whereas the night storage capacity will reduce if the difference between both levels is too large.

■ **Miscellaneous aspects**

Manual versus automatic operation

The regulators can be operated both manually and automatically. Automatic operation is inevitable if the operation frequency becomes too high, e.g. several times per hour. The simulations showed that for the Northern branch with downstream control, an operation frequency of once per hour yields good results. This implies that it is possible to operate the regulators manually. Further simulation can be made to demonstrate the frequency for each regulator. The minimum operation frequency of the steeper Southern branch turned out to be about once per five minutes, which implies that manual operation is impossible for this branch.

Dry bed flow

Most hydrodynamic simulation models cannot handle dry bed flow, as this would constitute a discontinuity in the computation. Normally, dry bed flow does not occur, or does not have to be simulated. However, for a canal with night storage, some pools may fall dry during a simulation. In order to avoid abortion of the computation, the MODIS model automatically generates a so called "Preissmann slot" at the bottom of each cross-sections. The bottom width of this slot is equal to the bed width of the cross-section to avoid discontinuities in the computation. The "Preissmann slot" turned out not to be sufficient. Therefore, a special routine, which prevents the slot from falling dry, was written and linked to the model. If the water drops below the bottom level the water depth will be artificially increased to 0.01 m and a small base flow of 0.0001 m^3/s will be added. With this additional measure the model showed no difficulties in simulating "dry bed flow" and the amount of artificially created water remains small.

Decision support system

By now, the model has only been used as a simulation model for the design of the control system. The model can also be used for the operation of the system. In the design phase, control strategies have been derived for the most critical events only. In reality, these events seldom occur and the control strategy might become different. To derive a sound operation strategy, the model can be used as a decision support system.

5 Summary and Conclusion

Control of a canal with both a conveyance and a storage function is difficult. Most existing control systems are based on the principle of maintaining a target level or target volume. If a canal is used for storage the water levels and volume will have to vary, instead of being constant. Nevertheless, existing control systems can be used, especially when the target water levels are altered during the day. The variation of the target levels in time is called the operation strategy and it can be derived by using a simulation model. In this study, the operation strategies were derived by applying a heuristic approach. The use of optimization techniques would take too much computation time, or would require unrealistic simplifications.

Downstream control is most suitable to control a canal with night storage. One would expect that the head reaches will face water shortage in case of downstream control, but this will not occur when the downstream target levels are not high enough, because the regulators will not pass sufficient discharge. It was found that the operation strategy of a downstream control canal is relatively simple and easy to derive, compared to an upstream controlled canal. This is a result of the fact that downstream control is a self-regulating control system from an irrigation point of view.

If the canal has to convey floods too, upstream target levels will be indispensable. The mixed control system is therefore most suitable for canals with night storage, and for canals which have to convey floods.

The regulators can be operated manually or automatically. Automatic operation becomes a must when the frequency of operation becomes too high e.g. every 5 minutes. With a simulation model the required operation frequency can be assessed and, consequently, the need for automation can be demonstrated.

A study as presented in this article could only be executed by means of a hydrodynamic flow simulation model, which allows for the simulation of automatically closed loop controllers. In this respect, the MODIS model proved to be very convenient.

References

Haskoning, Jouzy & Partners (1991), *Rehabilitation and upgrading of the King Abdullah Canal, Hydraulic model for calculation and analysis of the water in unsteady flow conditions*, Nijmegen, the Netherlands.

Schuurmans W. (1991), *Description and Evaluation of Program MODIS*, Proceedings of the National Conference on Irrigation and Drainage Engineering, ASCE, Honolulu, Hawaii.

4.5 Concluding remarks

4.5.1 Lessons from the case studies

Some general conclusions can be drawn from the case studies with respect to the use of the model and the impact of unsteady flow in general.

■ General

At first it should be stated that the model proved to be able to simulate the water distribution in canal networks for a wide range of canal control systems. In some cases the behaviour of a certain control system could have been predict without using a simulation model like the MODIS model. In other cases, like the exceptional cases presented in paragraph 4.3 and the cases presented in paragraph 4.4, the effect of a control system could hardly be predicted without using a model. Moreover, the effects could be quantified, by using a simulation model. This allows for a rational decision for the most suitable design and operation alternative.

With respect to the impact of unsteady flow, it can be stated that this will increase with the ratio of the system response time and operation interval. Although, the model can simulate unsteady flow, a number of indirect effects of unsteady flow cannot be simulated. One of the most important unknown parameters is the behaviour of the canal operators.

In addition to the general recommendations, specific recommendations and considerations with respect to the design and operation of irrigation system can be presented.

■ Design

The use of orifices functioning as water level regulators can lead to operational problems, not only because they are sensitive for discharge variations, but also because they are obstructing the flow and as a result they increase the system response time considerably. Weirs, at the other hand, are less sensitive for discharge variations and can reduce the system response time.

The operational losses due to improper operation of the offtakes are affecting the conveyance efficiency. The operational losses should be compensated by increasing the period of supply rather than by increasing the flow rate of supply. This, because the unsteady flow causes a delay in time between sending and receiving. Increasing the flow rate does not reduce the time lag. If the operation efficiency is compensated by increasing the duration of inflow supply, it will not affect the design capacities of the system.

In the model simulations, no significant difference could be found between the operational performance of automatically upstream controlled systems and manually upstream controlled systems. However, it is more difficult to derive the optimal operation strategy for manually operated control systems. In general, it can therefore be stated that the operational losses for subsequently self-regulating automatically controlled systems, non-selfregulating automatically controlled systems and manually controlled systems will increase.

For automatically operated control systems model simulations are needed to determine or verify the control algorithm and its parameters. Furthermore, it can be worthwhile to examine the operation efficiency, delivery performance ratio and operational instructions of the offtakes. However, it is not necessary to simulate self-regulating control systems in order to find the operation efficiency, delivery performance ratio and operational instructions for normal conditions. The time lag between delivery and supply does not affect the performance of self-regulating control systems.

Simulating the operation of a system in the design phase reduces the risk of design errors which do not necessarily have to be related to unsteady flow.

■ **Operation**

Operational losses found with model simulations can be expected to be lower than those found in a real life environment, because less exact data are available in a real life environment. In the model environment all system parameters and gate characteristics are exactly defined.

Although operational losses of a few percentages are small compared to e.g. tertiary unit losses, these water savings might already justify the investment costs of more expensive (e.g. automatic self-regulating) control systems.

Although a fine-tuned delivery schedule follows the water requirements more closely and thus increases the water use efficiency compared to more rigid schedules, the operational efforts to realize a fine tuned schedule should not be underestimated. The resulting operational losses will diminish the effect of the reduced schedule losses. With a simulation model the operational losses for ideal conditions can be calculated.

Even in automated, but non self-regulating, control systems a flow model is useful to determine the timing of offtake adjustments. If the system is self-regulating, operation of the offtakes becomes much easier and there is no need to use flow models for the operation. However, for the design of these systems, flow models are needed to demonstrate the validity of the design. Furthermore, if the set-points are variable in time, flow models might be needed to determine the operation strategy of the set-points.

If the gates of manually operated systems are reset in such a way that they realize a new water distribution, the initial water distribution may deviate in time due the retarded response of the system. Instead of specifying the discharges to be delivered and water levels to be maintained, one can also define the operational instructions in terms of new gate positions. If the stage discharge relationships of the regulators are not precisely known, it is important to specify also the period of time after which they have to check and, if necessary, correct the gate settings. If the volume of water lost during the transition state becomes significant, that is, if the duration of constant supply is short compared to the response time of the system. the moment in time of adjustment should be specified too. The proper timing of adjustments can be assessed with the help of a flow model.

4.5.2 Topics for further research

In the previous paragraphs some research topics conducted with the MODIS model have been discussed. At present research in this field is continued on the following topics.

- **Development of new control algorithms**

In the last few decades many automatic closed loop control algorithms for regulators have been developed on heuristic basis rather than on a rational one. At present a research project has been initiated by the Faculty of Civil Engineering, The Faculty of Mechanical Engineering and Cal Poly (Technical University of California) to develop and test new and existing regional downstream control algorithms. The new control algorithms should preferably have both an adaptive and a predictive nature. The tests will be carried out on both the MODIS model and a scale model of Cal Poly in California.

- **Verification**

So far, only theoretical cases have been used to check the correctness of the MODIS model. In the near future, the correctness of the MODIS model will be verified by data derived from two scale models. The first scale model is scale model in California before-mentioned, and the other one is a scale model of Alsthom Fluides in Paris.

- **Field experience**

Upto now, all model applications have been desk studies, and no experience is available with field applications. It is obvious that problems will have to be solved with respect to the calibration of the model, and with the definition and assessment of field operation performance indicators. The field operation performance indicators are needed to measure the impact of the model on the irrigation performance. In cooperation with IIMI-Pakistan research in this field is conducted for an irrigation scheme in Punjab, Pakistan.

■ **Model improvements**
A model is never finished. At present, the MODIS model can be used to simulate and evaluate operational instructions, but it cannot generate operational instructions. It would be interesting to investigate to which extent the model can be used to generate realistic operational instructions. Possible options are to combine the model with an optimization model which makes use of simplified flow equations, or to embed the model into an expert system.

Another modification to be made is to construct, in addition to the existing file input structure, an interactive data input menu for the MODIS model A possible negative side effect of menu supported input is that it might stimulate uncontrolled use of the model by inexperienced user.

To facilitate the before-mentioned model modifications, COW (Centre for Operational Watermanagement), which will be closely related to the section Watermanagement of the Delft University of Technology, has been established.

5 Summary and conclusions

5.1 Reasons underlying the development of the MODIS model

This study deals with the hydraulic performance of controlled irrigation canals. A proper hydraulic performance of the conveyance and distribution network is a pre-requirement for successful irrigation. To ensure a good hydraulic performance, the hydraulic behaviour of the system has to be accurately calculated. Well known canal resistance formulae, such as Strickler/Manning, are only valid for uniform steady state conditions. These conditions seldom occur due to the influence of structures, the effect of operation of structures, and/or variable inflow rates. In reality, the flow phenomenon is non-uniform and unsteady, which implies a retarded response and diffusion. The equations for the hydraulic description of the system for non-uniform unsteady flow conditions were derived more than a century ago. Before computers were available, it was extremely laborious to find solutions of the so-called De Saint Venant Equations. Now that computers and computer models have become available, it is relatively easy to make an accurate calculation of the hydraulics of the canal system.

It was found that, although many hydrodynamic computer packages were available for river systems, none of them was tailored for controlled irrigation systems. Most apparent deficiencies of existing packages were the lack of an accurate and extensive library of standard irrigation structures and the impossibility to operate these structures according to different operation concepts. The lack of easy-to-use computer package for controlled watermanagement systems has been noted in literature (Schilling 1987), (Burt 1989).

Therefore, a new model has been developed in the first phase of this study to investigate the hydraulic performance of controlled irrigation canals. This new model is named MODIS (Modeling Drainage and Irrigation Systems) and can model a variety of canal network configurations and a wide range of structures. Furthermore, various operation concepts can be simulated. To allow for an objective, diagnostic evaluation of the model results, a module for the computation of operation performance parameters has been incorporated in the modeling package. The MODIS model is extensively described in various papers (Schuurmans 1990c), (Schuurmans 1991a) and (Schuurmans 1991b). Moreover, a detailed user's guide and software documentation of the program do exist (appendix F).

5.2 How to use the MODIS model

The developed flow model is a complex mathematical tool. Although the model is user-friendly, experience and background knowledge of the model is needed to get reliable results. To solve the De Saint-Venant Equations and the structure and controller equations numerically, the partial differentials have to be approximated by finite differences.

Many numerical schemes have been developed, in search for an accurate, stable and efficient scheme. Due to guaranteed stability of the implicit schemes these schemes are highly preferred and normally used in practice. In the MODIS model the Preissmann scheme is used, because this is one of the most convenient schemes for one dimensional flow problems. The applied numerical scheme determines the character of the hydrodynamic flow model, as to its stability, accuracy and flexibility. The method of solution of the resulting set of equations determines memory space (RAM-Memory) and computation speed.

The accuracy of the computation can be influenced by changing the mesh size Δx, the time step Δt and the interpolation factor between two successive time steps Θ. The most accurate solutions are found if the Courant number Cr (relation between the mesh size Δx, the time step Δt and the critical wave celerity) is close to unity. However, this is not required in order to assure stability, as an implicit scheme is used. The value of the time interpolation coefficient Θ should be chosen as close to 0.5 as possible. In practice a value of 0.55 is used to create some numerical damping in order to damp small undesired osculations.

The most practical way to evaluate the accuracy is to repeat the calculation with another time step or mesh size and to compare the results of computation. The accuracy of the computations is often limited by the accuracy of the required field data. This does not imply that the accuracy of the numerical computation becomes insignificant compared to the accuracy limitations of the input data, but that the required accuracy has to be related to the aim of the computation.

Special hydraulic conditions, which cannot be handled by the model, may occur. Examples are dry bed flow and supercritical flow. The user should know how to handle in these situations and how to interpret the results if "tricks" are used to overcome the problems.

As to the input data of a model, one should realize that no more input data are required for an unsteady flow model than for a steady one. The required accuracy of the input data is closely related to the model application. Calibration of a model, to demonstrate the validity of a design or for the training of designers and operators in the effects of the unsteady flow phenomenon, is not needed (nor possible).

5.3 Impact of unsteady flow on irrigation system performance

With the help of the MODIS model, the hydraulic behaviour of irrigation canal systems, especially its unsteady component, can examined. Unsteady flow can, for example, be caused by a change made in flow rate at the head of canal. After some time, the change in flow rate has arrived downstream and has become more gradual upon propagation. Gates along the canal have to be continually adjusted to maintain a desired water level or offtake discharge. If gates are not set properly and on the right time, water will either be spilled or shortage of water will occur downstream. In general, the unsteady flow phenomenon will affect the water distribution in the following four ways.

■ **Transient state**
During the period of time that the system is in transient state, the water distribution will be disturbed. Unsteady flow in irrigation canals is nearly always a transient phenomenon due to the predominant influence of friction. The duration of this transient flow phenomenon, or the system response time, can be considerable for open canals. The system response time can be defined as the period of time which is needed to transform from one steady state into another steady state. It is more convenient to use the required time before e.g. 90% of the variation has been achieved, because the period of time required to reach a new steady state is theoretically infinite. To determine the systems response time approximation formulae or flow models can be used. The approximation formulae, presented in chapter two, allow for a fast and fairly accurate computation of the system response time for simple canal systems. However, they are not valid for systems with many structures and certainly not for controlled systems. It is important to note that the response times of controlled systems are totally different from those of the uncontrolled ones.

The impact of unsteady flow on the water distribution will become significant if the operation time interval is short compared to the system response time. This situation can be expected for on-demand or semi-demand irrigation systems, in which flow rate changes are frequently made, for systems with only day irrigation, and for systems with variable inflow rates.

■ **Troublesome operation**
Another, more indirect, effect of unsteady flow on the water distribution is that operators might set their gates in wrong positions. An initial water distribution will deviate in time due to the retarded response of the system. This could cause frustration of bona-fide canal operators who cannot succeed in fulfilling the required water distribution. The effect of wrong gate settings on the water distribution can be much more significant than the previously discussed effect of unsteady flow during transient state, as the gates might have been in a wrong position for a longer period of time.

■ **Scheduling**
Unsteady flow also affects the water distribution if a rigid schedule and not a "fine tuned" delivery schedule is applied, because one cannot realize the latter one. It is obvious that there is a relationship between the applied canal control system and the flexibility of making changes. Automated systems are easier to operate than manually operated systems, but it is the self-regulating systems which provide the most flexibility. This implies that fine tuned delivery schedules are more often applied in automatically operated self-regulating control systems than in manually operated non self-regulating control systems

■ **Reliability**
Farmers expect to receive their water share in a certain quantity and at a certain moment in time. Due to unsteady flow, the water might arrive in another quantity and at another moment in time. Consequently, the reliability of supply will be reduced. The side effects of an unreliable supply, such as illegal offtakes and demolition of the system, cannot be identified with an hydraulic model, but may force the water operation in a downward directed spiral.

5.4 Model applications

5.4.1 General

Flow models are widely available nowadays, and in the near future flow models for irrigation systems are expected to become widespread too. The progressive development of fast and reliable Personal Computers during the last decade enables the use of these models all over the world including Developing Countries. The models can be used to study the day-to-day operation of irrigation systems in order to verify the possibility of a safe, reliable and equal water distribution. The operational problem should not only be addressed in the operational phase of a system but already in the design or modernization phase.

5.4.2 Design

It is easiest to use a flow model in the design phase, because calibration is not needed (nor possible) and the requirements related to the user-friendliness of the model are easy to achieve compared with other applications. It is important to address problems, related to the daily operation, already in the design phase of a project, because in this phase one is most flexible to alter the design. The flow model should not be used in the first place to determine system capacities, but rather to identify operation bottlenecks and to make a quantitative comparison between alternative operation and control systems. By doing so, a rational based selection can be made between, for example, manually and automated control systems, or between self-regulating and non self-regulating control systems. The flow model adds a new dimension, of operational water control evaluation, to the existing design procedures, but does not replace the existing design methodology.

Many existing irrigation schemes are troublesome to operate. To improve their functioning, a balance between modernization of the infrastructure and improved operation rules for the existing infrastructure has to be found. As in the developed model both hardware and software modifications can be simulated and evaluated, the model is extremely useful to derive this optimal balance and assess the required hardware modifications.

5.4.3 Operation

In the operation phase it is less easy to use a flow model than in the design phase, because calibration is probably needed and because the requirements related to the model's user-friendliness are high. If a system never reaches a steady state, (as is the case with systems dependant on variable inflows and with systems with day irrigation supply), the use of unsteady flow models, to determine the operation rules and setting in order to assure a reliable and equal water distribution, will be most apparent.

To improve the operation, a flow model could be used as a decision support system for the operator. In case of off-line computer aided control, the model simulates and evaluates decisions of the operator. In that way, the operator can check the effectiveness of his operational decisions in advance. The existing MODIS model cannot be used for on-line computer aided supervisory control as it does not generate decisions.

5.4.4 Training

In order to make canal operators and designers aware of the impact of unsteady flow, the model can be used as a simulator comparable with flight simulators used for pilots. By using the model as a simulation model the designers, who usually do not operate the system, become aware of the operational problems and operators come to understand why it is so difficult to operate a system.

The problems related to the operation of a system with long response times was clearly demonstrated in a "Senior Advanced Course on Appropriate Modernization and Management" using the MODIS model (Schuurmans & Huyskes 1990b). In this workshop on canal operation, the participants of the course had to reset a number of check gates and offtakes in order to achieve a new intended water distribution. They were allowed to reset the gates every 4 hours and they could examine the resulting water distribution. Most of the participants did not succeed in matching the actual supply with the intended supply. In fact, they completely forgot the unsteady flow phenomenon, and did not realize that the system needs time to reach a new steady state. By simulating the operation of the system, however, they managed to gain a good understanding of the hydraulic operational problems and learned how to handle them.

5.5 Concluding remarks

Referring to the objective of this study,

"The development of a user-friendly computer model which can calculate the non-uniform unsteady flow phenomenon in controlled irrigation systems and compute operation performance indicators. The potentials and limitations of the model so developed has to be demonstrated with a number of applications.",

it can be concluded that such a model has been developed indeed. With the developed MODIS model a number a case studies have been conducted, which demonstrated the applicability of the MODIS model. The model proved to be very convenient to address operational problems of irrigation system in both the operational phase and design phase of irrigation schemes. The operation performance indicators, incorporated in the MODIS model, proved to be suitable for a quantitative analysis of design and operation alternatives.

Samenvatting & Conclusies

Dutch translation of chapter 5

Onderliggende redenen voor de ontwikkeling van het model MODIS

Deze studie behandelt het hydraulisch gedrag van gestuurde irrigatiekanalen. Een goed hydraulisch gedrag van het transport- en distributienetwerk van een irrigatiesysteem is een noodzakelijke voorwaarde voor succesvol irrigeren. Om een goed hydraulisch gedrag te verzekeren is een nauwkeurige berekening van het systeemgedrag nodig. De bekende weerstandsformules, zoals de Manning/Strickler formule, is niet algemeen geldig maar alleen voor uniforme permanente stromingscondities. Deze condities komen in de praktijk zelden voor door de invloed van kunstwerken, de bediening van kunstwerken en/of door variërende instroming. In werkelijkheid is de stroming niet uniform en niet permanent, wat een vertraagde reactie en diffusie tot gevolg heeft. De vergelijkingen die het hydraulisch gedrag voor niet uniforme niet permanente stromingscondities beschrijven zijn al meer dan een eeuw oud. Echter, voordat computers beschikbaar waren was het zeer arbeidsintensief om oplossingen van deze zogenaamde De Saint Venant Vergelijkingen te vinden. Nu computers beschikbaar zijn is het relatief eenvoudig om een nauwkeurige berekening van de stroming in het kanalensysteem te maken.

Er werd geconstateerd dat, hoewel veel hydrodynamische stromingsmodellen beschikbaar zijn voor rivieren, geen van deze modellen geschikt was voor bestuurde irrigatiesystemen. De meest in het oog springende tekortkomingen van bestaande pakketten was het gebrek aan een nauwkeurige en uitgebreide bibliotheek van standaard irrigatiekunstwerken en de onmogelijkheid om deze kunstwerken te besturen volgens verschillende besturingsconcepten. Het gebrek aan eenvoudig te gebruiken computer pakketten voor bestuurde waterbeheersingssystemen is ook in de literatuur geconstateerd (Schilling 1987), (Burt 1989).

Daarom is in de eerste fase van de studie een nieuw model ontwikkeld om het gedrag van gestuurde irrigatiekanalen te onderzoeken. Dit nieuwe model is genaamd MODIS (Modelleren van Drainage en Irrigatie Systemen) en kan een verscheidenheid van kanaal configuraties en kunstwerken modelleren. Daarnaast kunnen verschillende sturingsconcepten worden gesimuleerd. Om de model resultaten objectief en diagnostisch te kunnen interpreteren is een module voor de berekening van besturingsgedrag parameters in het modellen pakket opgenomen. Het model MODIS is uitgebreid beschreven in verschillende papers. (Schuurmans 1990c), (Schuurmans 1991a) en (Schuurmans 1991b). Daarnaast bestaat er een uitgebreide gebruikershandleiding (Appendix F) en systeem documentatie van het programma.

Hoe het model te gebruiken

Het ontwikkelde model is een ingewikkeld stuk gereedschap. Hoewel het model gebruikersvriendelijk is, zijn ervaring and achtergrondkennis nodig om betrouwbare uitkomsten te krijgen. Om de De Saint Venant Vergelijkingen, de kunstwerk- en de besturingsvergelijkingen op te lossen worden de partiële differentiaal vergelijkingen met eindige differenties benaderd aan de hand van een numeriek schema.

Veel numerieke schema's zijn ontwikkeld op zoek naar nauwkeurige, stabiele en efficiënte rekenschema's. Vanwege de gegarandeerde stabiliteit verdienen impliciete schema's veruit de voorkeur en deze worden in de praktijk dan ook meestal gebruikt. In het ontwikkelde model MODIS is het Preissmann schema gebruikt omdat het een van de meest geschikte schema's is voor één dimensionale stroming problemen. Het gebruikte numerieke schema bepaalt het karakter van het stromingsmodel, zoals stabiliteit, nauwkeurigheid en flexibiliteit. De oplossingsmethode van de resulterende vergelijkingen bepaalt het benodigde RAM-geheugen van de computer en de berekeningssnelheid.

De nauwkeurigheid van de numerieke berekening kan worden beïnvloed door de stap grootte Δx, de tijdstap Δt en de tijdinterpolatiefactor Θ te variëren. De meest nauwkeurige oplossingen worden gevonden als het Courant getal Cr (relatie tussen de stap grootte Δx, de tijdstap Δt en de kritische golfsnelheid) dicht bij één ligt. Dit is echter geen noodzakelijke voorwaarde voor de stabiliteit indien een impliciet schema wordt gebruikt. De waarde van de tijdinterpolatiefactor Θ moet zo dicht mogelijk bij 0.5 liggen. In de praktijk wordt een waarde van 0.55 gebruikt om ongewenste fluctuaties uit te dempen.

De meest eenvoudige manier om de nauwkeurigheid van de berekening te evalueren is om de berekening te herhalen met een kleinere stap grootte of tijdstap en de resultaten van beiden berekeningen met elkaar te vergelijken. De nauwkeurigheid van de berekening is vaak begrenst door de nauwkeurigheid van de benodigde invoergegevens. Dit betekent niet dat de nauwkeurigheid van de numerieke berekening ondergeschikt is aan de nauwkeurigheid van de invoergegevens, maar dat de benodigde nauwkeurigheid moet worden gerelateerd aan het doel van de berekening.

Er kunnen speciale hydraulische condities optreden die het model niet aankan. Voorbeelden hiervan zijn de stroming over een droog kanaalbed en superkritische stroming. De gebruiker moet weten hoe met deze situaties om te gaan en hoe de berekeningsresultaten moeten worden geïnterpreteerd als wiskundige trucs worden gebruikt om de berekening te continueren.

Wat betreft de benodigde invoer gegevens, moet men realiseren dat er voor een permanent stromingsmodel niet meer invoergegevens nodig zijn dan voor een niet-permanent stromingsmodel. De benodigde nauwkeurigheid van de invoer gegevens is sterk afhankelijk van de toepassing. Kalibratie van een model is niet nodig (en ook niet mogelijk) om de juistheid van een ontwerp aan te tonen, of om ontwerpers en beheerders te trainen in de effecten van niet permanente stroming.

Invloed van niet permanente stroming op het gedrag van irrigatie systemen

Met behulp van het model MODIS kan het hydraulisch gedrag van een irrigatiesysteem en met name de niet permanente component worden bestudeerd. Niet permanente stroming kan b.v. worden veroorzaakt door een verandering van het instroomdebiet aan het begin van het kanaal. De debietsverandering arriveert enige tijd later benedenstrooms en is onderwijl geleidelijker geworden. Kunstwerken langs het kanaal moeten continu worden bijgesteld om de gewenste waterstanden en aftapdebieten te handhaven. Indien de kunstwerken niet op het juiste moment in de juiste positie staan, zal benedenstrooms oftewel water worden verspilt of er treden water tekorten op. In het algemeen zal de niet permanente stroming de waterverdeling op de volgende vier wijzen beïnvloeden.

■ **Overgangstoestand**

Gedurende de tijd dat het systeem in een overgangstoestand is van de ene permanente toestand naar een andere, zal de waterverdeling zijn verstoord. Niet-permanente stroming in irrigatiekanalen is bijna altijd een overgangsverschijnsel ten gevolge van de overheersende invloed van de wrijving. De tijdsduur van dit overgangsverschijnsel, oftewel de systeem reactietijd, kan worden gedefinieerd als de tijd die het systeem nodig heeft om van de ene permanente toestand in een andere permanente toestand te komen. Het is beter om de tijd te gebruiken die nodig is om b.v. 90% van de verandering tot stand te brengen, omdat de tijd die nodig is om in een andere permanente toestand te komen theoretisch oneindig lang is. Om de systeem reactietijd te bepalen kunnen analytische formules worden gebruikt. De benaderingsformules, gepresenteerd in hoofdstuk 2, kunnen worden gebruikt voor een snelle en redelijk nauwkeurige berekening van de systeem reactietijd van eenvoudige kanalenstelsels. Echter, deze zijn niet geldig voor systemen met veel kunstwerken en zeker niet voor bestuurde systemen. Het is belangrijk om te realiseren dat de systeem reactietijd van een onbestuurd kanaal totaal verschillend is van die van een bestuurd kanaal.

De invloed van de niet-permanente stroming op de waterverdeling zal significant worden indien het besturingsinterval kort is in verhouding tot de systeem reactietijd. Deze situatie kan worden verwacht bij, systemen met water levering op aanvraag of semi-aanvraag waarbij het instroomdebiet vaak verandert, bij systemen met alleen dag irrigatie en bij systeem waarbij het instroomdebiet varieert.

■ **Lastig beheer**

Een ander, meer indirect, gevolg van niet permanente stroming op de waterverdeling is dat beheerders de kunstwerken in een verkeerde positie kunnen zetten. Een initiële waterverdeling zal in de loop van de tijd veranderen doordat het systeem vertraagd reageert. Dit kan leiden tot frustraties bij bona-fide kanaal beheerders die niet in staat zijn de gewenste waterverdeling te realiseren. Het gevolg van een verkeerde stand van de kunstwerken kan veel groter zijn dan de eerder genoemde gevolgen van niet permanente stroming gedurende de overgangstoestand, omdat de kunstwerken veel langer in een verkeerde positie kunnen staan.

■ **Planning**

Niet permanente stroming beïnvloed de water verdeling ook indien een "vast" in plaats van een nauwkeurig water verdelingsschema word toegepast, omdat men een precieze water verdeling niet kan verwezenlijken. Het is duidelijk dat er een verband bestaat tussen het toegepaste besturingssysteem en de flexibiliteit om veranderingen te bewerkstelligen. Geautomatiseerde systemen zijn eenvoudiger te bedienen dan hand bestuurde systemen, maar de zelf-regulerende systemen zijn het meest flexibel. Dit betekend dat precieze waterverdelingsschema's vaker worden gebruikt in combinatie met automatisch bediende zelf-regelende besturingssystemen, dan in combinatie met handbediende niet zelf-regelende besturingssystemen.

■ **Betrouwbaarheid**

Boeren verwachten hun water deel in een bepaalde hoeveelheid en op een bepaald tijdstip te ontvangen. Door de niet permanente stroming kan het water in een andere hoeveelheid en op een ander tijdstip bij de gebruiker arriveren. Dientengevolge vermindert de betrouwbaarheid van de waterlevering. De neveneffecten van een onbetrouwbare watertoelevering ten gevolge van niet-permanente stroming, zoals illegale aftappingen en vernielingen aan het systeem, kunnen niet worden geïdentificeerd met een hydraulisch model, maar kunnen het waterbeheer wel negatief beïnvloeden.

Model Toepassingen

■ **Algemeen**

Stromingsmodellen zijn tegenwoordig overal verkrijgbaar en in de nabije toekomst zullen stromingsmodellen voor irrigatiesystemen ook wijd verspreid zijn. De enorme ontwikkeling van snelle en betrouwbare Personal Computers in de laatste decennia heeft het mogelijk gemaakt om deze modellen overal ter wereld, inclusief Derde

Wereld landen, te gebruiken. De stromimgsmodellen kunnen worden gebruikt om het dagelijks beheer van irrigatiesystemen te bestuderen om zodoende de mogelijkheden van een veilige, betrouwbare en eerlijke waterverdeling te verifiëren. Niet alleen in de beheersfase maar reeds in ontwerp- of moderniseringsfasen moet aandacht worden besteed aan het operationeel beheer.

- **Ontwerp**

Het is het eenvoudigst om het model in de ontwerpfase te gebruiken, omdat kalibratie dan niet nodig (en ook niet mogelijk) is en omdat aan de eisen t.a.v. gebruikersvriendelijkheid eenvoudig kan worden voldaan in vergelijking met andere toepassingen. Het is belangrijk om problemen met betrekking tot het dagelijks beheer reeds in de ontwerpfase van een project te onderkennen, omdat dan het meest flexibel is om het ontwerp aan te passen. Het stromingsmodel moet niet in de eerste plaats worden gebruikt om de systeem capaciteiten vast te stellen, maar eerder om operationele problemen op te sporen en om een kwantitatieve vergelijking van alternatieve operatie mogelijkheden en besturingssystemen te maken. Op deze wijze kan een rationele afweging worden gemaakt tussen b.v. handmatig en automatisch gestuurde systemen, of tussen zelf regelende en niet zelf-regelende systemen. Het stromingsmodel voegt een nieuwe dimensie aan de bestaande ontwerp procedures toe, n.l. een evaluatie van het operationeel beheer, maar komt niet in de plaats van de bestaande ontwerp methodiek.

- **Beheer**

In de beheersfase is het gebruik van een stromingsmodel minder makkelijk dan in de ontwerpfase, omdat waarschijnlijk kalibratie nodig is en omdat de eisen t.a.v de gebruikersvriendelijkheid hoog zijn. Indien het systeem nooit in een permanente stromingstoestand komt, (dat is het geval bij systemen met een variërende instroming en bij systemen met alleen dag irrigatie), is het gebruik van stromingsmodellen absoluut noodzakelijk voor het bepalen van de beheersregels en klepstanden om zodoende een betrouwbare en eerlijke waterverdeling te garanderen.

Om het beheer te verbeteren kan een stromingsmodel worden gebruikt als een beslissingsondersteunend systeem voor de beheerder. In het geval dat off-line computer ondersteunde besturing wordt toegepast, wordt het model gebruikt voor het simuleren en evalueren van voorgenomen beslissingen van de waterbeheerder. Op die manier kan de beheerder de effectiviteit van zijn beslissen van te voren controleren. Het bestaande model MODIS kan (nog) niet worden gebruikt voor on-line computer ondersteund supervisory besturing omdat het geen beslissingen genereert.

- **Training**

Om de kanaal beheerders en ontwerpers bewust te maken van de invloed van de niet-permanente stroming kan het model als simulator worden gebruikt, net zoals vliegsimulators voor piloten worden gebruikt. Door het model als simulator te gebruiken worden ontwerpers, die meestal niet betrokken zijn bij het beheer, zich bewust van de beheersproblemen en begrijpen beheerders waarom het zo lastig is om het systeem te opereren.

De problemen met betrekking tot het beheren van een systeem met een lange systeem reactietijd werd duidelijk geïllustreerd in een cursus (Senior Advanced Course on Appropriate Modernization and Management) waarbij gebruik werd gemaakt van het model MODIS (Schuurmans & Huyskes 1990b). De deelnemers moesten een aantal kunstwerken (niveau regelaars en aftapkunstwerken) instellen om zodoende een nieuwe, vooraf gedefinieerde, waterverdeling te bereiken. De kunstwerken mochten om de vier uur worden bijstellen en de resulterende waterverdeling kan worden aanschouwd. De meeste deelnemers slaagden er niet in om de waterverdeling overeen te laten komen met de gewenste waterverdeling. In feite vergaten ze volledig het niet-permanente stromingsverschijnsel: ze realiseerden zich niet dat het systeem tijd nodig heeft om in een nieuwe permanente toestand te komen. Echter, door het beheer te simuleren slaagden ze erin om een goed begrip van de operationele problemen te krijgen en leerden ze hoe deze te overwinnen.

Slotbeschouwing

Refererend aan de doelstelling van deze studie,

"De ontwikkeling van een gebruikersvriendelijk model die de niet uniforme, niet permanente stromingsverschijnsel in bestuurde irrigatiesystemen en besturingsgedrag parameters kan berekenen. De mogelijkheden en beperkingen van het ontwikkelde model moet worden aangetoond d.m.v. een aantal toepassingen.",

kan worden geconcludeerd dat het model inderdaad ontwikkeld is. Met het ontwikkelde model MODIS zijn een aantal case studies uitgevoerd, welke de mogelijkheden van het model hebben aangetoond. Het model blijkt bij uitstek geschikt te zijn om operationele problemen van irrigatiesystemen te onderzoeken, zowel in de ontwerpfase als in de beheersfase. De besturingsgedrag indicatoren die in het model MODIS zijn opgenomen, blijken geschikt te zijn voor een kwantitatieve analyse van ontwerp en besturings alternatieven.

References

Abbott 1975 — see Yevjevich 1975, Vol I, p. 63-88.

Abbott 1979 — Abbott, M.B., *Computational Hydraulics*, Elements of the Theory of Free Surface Flow, Pitman, London, 1979.

Abbott 1990 — Abbott, M.B., *The hydraulics of the informational revolution: hydroinformatics*, fourth draft, IHE, Delft, the Netherlands, 1990.

Abernethy 1989 — Abernethy, Charles L, Performance criteria for irrigation systems, Proceedings of the International Conference *Irrigation, Theory and Practice*, ISBN 0-7273-0904-8, Pentech press, London, UK, 1989.

Abramowitz 1970 — Abramowitz, A., I. Stegun, *Handbook of Mathematical Functions* , Dover Publications, Inc., New York, USA, 1970.

Almering 1982 — Almering, J.H.J. e.a., *Analyse*, Delftse Uitgevers Maatschappij, 4e druk, Delft, the Netherlands, 1982.

Balogun 1988 — Balogun, O.S., M. Hubbard and J.J. de Vries, Automatic control of canal flow using linear quadratic regulator theory, *Journal of Irrigation and Drainage Engineering*, ASCE, Vol 114, 1988.

Berthery 1989 — Berthery, Daniel, Hilmy Sally, & Jayantha Arumugam, Presentation of the Mistral-Simutra software package & Application of Mistral-Simutra to the Kalankuttiya branch canal (Mahaweli System H), Sri Lanka", *IIMI Working Paper 9*, Diagna Village, via Kandy, Sri Lanka, 1989.

Booij 1980 — Booij, N. , Report on the ICES subsystem Flows, *Report no. 78-3,* University of Technology, Delft, the Netherlands, 1980.

Bos 1976 — Bos, M.G., *Discharge measurements structures*, Laboratory for Hydraulics and catchment Hydrology, Report no 4, the Netherlands, 1976.

Bouman 1978 — Bouman, J.E.G. and E. Schultz, Berekening van de niet stationaire stroming in waterlopen in stedelijk en landelijk gebied, *Flevobericht RIJP nr. 127*, the Netherlands, 1978.

Brouwer 1990 — Brouwer, Robert, Paul Ankum, Fons Nelen en Wytze Schuurmans, *Control of watermanagement systems*, Lecture Notes, University of Technology, Faculty of Civil Engineering, Department of watermanagement, Delft, the Netherlands, 1990.

Burt 1983 — Burt, Charles M., *Regulation of sloping canals by automatic downstream control*, presentation at the 1983 Winter Meeting American Society of Agricultural Engineers, 1983.

Burt 1989	Burt, Charles M., *Canal Automation Providing On-Demand Water Deliveries for Efficient Irrigation*, Department of Agricultural Engineering, California Polytechnic State University (Cal Poly), San Luis Obispo, CA 93407, USA, 1989.
Chow 1959	Chow, Ven Te, *Open-channel hydraulics*, Mc Graw Hill Book company, New York, USA, 1959.
Cranck 1975	Cranck, J., *The Mathematics of Diffusion*, Claredon Press, Oxford, UK, 1975.
Cunge 1980	Cunge, J.A., F.M. Holly, A Verwey, *Practical Aspects of Computational River Hydraulics*, Pitman Publishing Limited, London, UK, 1980.
Dahlquist 1959	Dahlquist, G., Stability and Error bounds in the Equations, *Trans. of the Royal Inst. of Technology, No. 130,* pp 1-86, 1959
Duflow 1989	Booij, N., Noorman, W. Spaans et al., *User's manual of Duflow*, draft version, International Hydraulic and Environmental Institute & University of Technology, Delft, the Netherlands, 1989.
Farley 1987	Farley, R.E., *Software Engineering Concepts*, McGraw-Hill Book Company, New York, USA, 1987.
FAO 1971	Food and Agricultural Organisation, *On automated irrigation*, FAO Rome, Italy, 1971
Francis 1989	Francis, Marcus R., Elawad Omer M.A., Diagnostic investigations and rehabilitation of canals in the Geriza Irrigation Scheme, Sudan, Proceedings of Asian Regional Symposium *Modernization and Rehabilitation of Irrigation and Drainage Schemes*, Manila, The Philippines, 1989.
Hamilton 1986	Hamilton, D.L., J.J. de Vries, Microcomputer simulation of canal operation, *Journal of Irrigation and Drainage Engineering, ASCE, vol 112*, August 1986.
Harder 1972	Harder, J.A., J.S. Michael, C.P. Buyalski, Automatic downstream control of canal check gates by the Hydraulic Filter Level Offset (HyFLO) Method, Proceedings of the *Eighth ICID Congress*, Varna, 1972.
Hydra 1985	Heidemij, *Users Guide HYDRA, computerprogramma voor het berekenen van hydraulische netwerken*, Arnhem, the Netherlands, 1981.
IIMI 1987	IIMI, *Study of Irrigation Management in Indonesia, phase I*, Technical Assistance agreement No. 673 INO, IIMI-ADB-GOI, Diagna Village, via Kandy, Sri Lanka, 1987.
Lax 1954	Lax, P.D., Weak solutions of non-linear hyperbolic equations and their numerical applications, *Comm. Pure and Applied Mathematics, No. 7*, 1954.
Lenton 1982	Lenton, Roberto L., *Management tools for improving irrigation performance* Lecture notes presented at the special course on "Water Management in Irrigation Systems", Central Water

Commission.

Lorentz 1926 — Lorentz, H.A., *Verslag Staatscommissie Zuiderzee, 1918-1926"*, Algemene landsdrukkerij, Den Haag, the Netherlands, 1926.

Mahmood 1975 — see Yevjevich 1975.

Makin, 1987 — Makin, I.W., Warabandi in Punjab India, *Report OD/96* of Hydraulics Research Wallingford in collaboration with Irrigation and Power Research Institute Amritsar, Punjab, India, 1987.

Manz 1989 — Manz, David H., Computer simulation of manually operated irrigation conveyance systems typical of Alberta, Canada, Proceedings of the International Conference *Irrigation, Theory and Practice*, ISBN 0-7273-0904-8, Pentech press, London, UK, 1989.

Manz 1990 — Manz, David H., Use of ICSS model for prediction of conveyance system operational characteristics, Proceedings of the symposium of the 14th ICID congress *Real time scheduling of water deliveries*, Mayar Printers, New Delhi, India.

Miller 1975 — see Yevjevich 1975

Mitchell 1969 — Mitchell, A.R. , *Computational Methods in Partial Differential Equations*, John Wiley and Sons, 1969.

Pereira 1990 — Pereira, Luis Santos, Manuel Rijo & J.L. Teixeira, From upstream to real time management: Research for modelling in a surface irrigation system, Proceedings of the symposium of the 14th ICID congress *Real time scheduling of water deliveries*, Mayar Printers, New Delhi, India.

Ploss 1987 — Ploss, Lowell F., Canal automation using the Electronic Filter Level Offset(EL-FLO) system, Proceedings of the Symposium *Planning, Operation, Rehabilitation and Automation of Irrigation Water Delivery Systems*, ASCE, Portland Oregon, USA, 1987

Plusquellec 1988 — Plusquellec, Hervé, *Improving the operation of canal irrigation systems*, an audiovisual production of the Economic Development Institute of the World Bank, Washington DC 20433, USA, 1988.

Poppel 1986 — Poppel, J.W.P.M., Weijland H.B., *Towards Farmer-minded irrigation Projects*, Vol. 1 & 2, University of Technology, Delft, the Netherlands, 1986.

Preissmann 1961 — Preissmann, A, *Propagation des intumescences dans les caneaux et rivières*, Proceedings of the First Congress of the French Association for Computation, Grenoble, France, 1961.

Ribasi 1984 — DHV Consulting Engineers, *Gebruikershandleiding Ribasi 3.1*, Amersfoort, the Netherlands, 1985.

Richtmeyer 1967 — Richtmeyer, R.D. & K.W. Morton, *Difference methods for Initial Value Problems*, 2nd edition, Interscience, New York, USA, 1967.

Rofe 1984 — Rofe, B.H., Flow study for operation of a canal regulator under tidal conditions, Proceedings of the 1st International Conference *Hydraulic Design in Water Resources Engineering*, Southampton, UK, 1984.

Rogier 1987 Rogier, D, C. Coeuret, J. Bremond, *Dynamic Regulation on the Canal de Provence*, ASCE, 1987.

Rouse 1957 Rouse, H, S. Ince, *History of Hydraulics*, Iowa Institute of Hydraulic Research, USA, 1957.

Rubicon 1984 Haskoning, *Users Guide Hydrodynamic Modelling System Rubicon*, Nijmegen, the Netherlands, 1984.

Sally 1989 Sally, Hilmy, Daniel Berthery, Frederic Certain & André Durbec, Calibration of the Kirindi Oya RBMC mathematical flow simulation model, Description of the field measurements campaign and preliminary results, *IIMI Working Paper 10*, Diagna Village, via Kandy, Sri Lanka, 1989.

SAMWAT 1988 SAMWAT (SAMen werken op het gebied van het onderzoek ten behoeve van het WATerbeheer.) Working group of TNO which prepared a datafile of all computer models for watermangement available in the Netherlands, Delft, the Netherlands, 1989.

Schilling 1987 Schilling, W., *Real Time Control of Urban Drainage Systems*, The state-of-the-art", IAWPRC/IAHR Joint Committee, 1987

Schoemaker 1976 Schoemaker, H.J., Non-uniform flow, shifts in limits of stability, *Publication 171*, Hydraulics Laboratory, Delft, the Netherlands, 1976.

Schoemaker 1979 Schoemaker, H.J., Side weirs along prismatic canals, *Publication 218*, Hydraulics Laboratory, Delft, the Netherlands, 1979.

Schoemaker 1989 Schoemaker, Private communications during the course of the study, 1989-1991.

Schuurmans 1987 Schuurmans, Wytze, *Rehabilitation of secondary unit S1AK*, a first step to the modernisation of the Ganges Kobadak Irrigation Project, University of Technology, Delft, the Netherlands, 1987.

Schuurmans 1988 Schuurmans, Wytze, *Study of existing hydrodynamic flow models*, ISBN. 90-800089-3-1, University of Technology, Delft, the Netherlands, 1988.

Schuurmans 1989 Schuurmans, Wytze, Impact of unsteady flow on the irrigation water distribution, Proceedings of the International Conference *Irrigation, Theory and Practice*, ISBN 0-7273-0904-8, Pentech press, London, UK, 1989.

Schuurmans 1990a Schuurmans, Wytze, Analytical formulae for the response time of irrigation canals, Journal *Irrigation and Drainage systems, Vol 4, No. 1*, pp. 37-58, Kluwer Academic Publishers, Dordrecht, the Netherlands, 1991.

Schuurmans 1990b Schuurmans, Wytze and Eric J, Huyskes, *Operational Performance of conventional irrigation systems*, lecture notes for the Senior Advanced Course on Appropriate Modernization and Management of Irrigation Systems, Delft, the Netherlands, 1990.

Schuurmans 1990c Schuurmans, Wytze, Computer simulations of automatically controlled watermanagement systems, Proceedings of the First

	Junior European Conference *Applications of Operations Research to Real Time Control of Water Resources Systems*, ISBN 3 906484 04 1, Dübendorf, Switzerland, 1990.
Schuurmans 1991a	Schuurmans, Wytze, A model for real time controlled canal systems, Proceedings of the European conference *Advances in Water Resources Technology*, Balkema Publishers, ISBN 90-6191 184 2, Rotterdam, the Netherlands, 1991.
Schuurmans 1991b	Schuurmans, Wytze, Description and Evaluation of Program MODIS, Proceedings of the National ASCE Conference *Irrigation and Drainage*, ISBN 0-87262-811-6, New York, USA, 1991.
Stoker 1959	Stoker, J.J., *Water Waves, The Mathematical Theory with Applications*, Interscience Publisher inc New York, 1959.
Sommerfeld 1949	Sommerfeld, A., *Partial Differential Equations in Physics*, Academic Press inc.,Publishers, New York, USA, 1949.
Stelling 1979	Stelling, Guus S., Nauwkeurigheids aspecten van IMPLIC, Delft Hydraulics Laboratory, *Report W 367*, Delft, the Netherlands, 1979.
Stelling 1984	Stelling, Guus S., On the construction of computational methods for shallow water flow problems, *Rijkswaterstaat Communications No. 35*, the Netherlands, 1984.
Thaembert 1976	Thaembert, R.L., Mathematical model of water allocation methods, Proceedings of the *10th ICID congress, Question 35*, 1976.
USBR 1979	Falvey, Henri T., Luning P.C., *Gate Stroking*, Bureau of Reclamation, Technical Information Service, Operations Division, Springfield, Virginia 22151, 1979.
Vreugdenhil 1973	Vreugdenhil, C.B., Hydraulic Research for Water Management, Committee for Hydrological Research TNO, *Verslagen en mededelingen no. 18*, 's-Gravenhage, the Netherlands, 1973.
Vreugdenhil 1980	Vreugdenhil, C.B., *Waterloopkundige berekeningen II*, Delft University of Technology, Delft, the Netherlands, 1980.
Vreugdenhil 1985	Vreugdenhil, C.B., *Numerieke berekeningen in waterbouwkunde en hydrologie*, University of Technology, Delft, the Netherlands, 1985.
Volp 1987	Volp, C., *Design and Evaluation of Water Management Systems*, University of Technology, Delft, the Netherlands, 1987.
Wallingford 1978	Wallingford, Numerical model study of the east ghor main canal, Jordan, *Report No. EX 834*, Hydraulics Research Station Wallingford, Oxon ox10 8BA, Wallingford, UK, 1978.
Yevjevich 1975	Yevjevich, V., K. Mahmood, W.A. Miller, *Unsteady flow in open channels*, Water Resources Publications, Fort Collins, Colorado, USA, 1975.
Zimbelman 1983	Zimbelman, D.D., D.D. Bedworth, Algorithm for the automatic control of an open channel water distribution system, *Journal of Irrigation and Drainage Engineering, ASCE, vol 109*, March 1983.

List of symbols

A	= area of cross section
a	= hydraulic exponent of Bakhmeteff
A_s	= storage surface area
b	= canal bed width
b_s	= storage width of canal
b	= hydraulic exponent of Bakhmeteff
C	= Chezy resistance coefficient
c	= critical velocity
Cr	= Courant number
D	= diffusion coefficient
e	= 2.71828
exp	= exponential function
Fr	= Froude number (= v/c)
g	= gravitational acceleration
H	= mean water depth (= A/B)
h	= water level
I	= function build up of accumulative distribution functions
K	= discharge coefficient
k	= Strickler resistance coefficient (=1/n)
M	= bed resistance term
m	= side slope (m hor : 1 ver)
n	= Manning resistance coefficient (=1/k)
n	= ratio of H and H_0
P	= cumulative distribution function
P	= wetted perimeter of cross-section
p	= hydraulic exponent of Bakhmeteff
Q	= flow rate
Q	= complementary cumulative probability function
q	= flow rate per unit width
q	= lateral inflow per unit length
q	= flow rate of offtake
R	= hydraulic radius
r	= hydraulic exponent of Bakhmeteff
S	= storage volume
s	= canal bed slope
s	= variable
T	= response time
T	= width of canal at water surface level
t	= time
τ	= dimensionless time variable

t	=	time
t'	=	time on new reference system
u	=	exponent
u	=	fluid velocity
u	=	celerity of a bore
v	=	mean fluid velocity (=Q/A)
v_l	=	velocity of lateral inflow
W	=	gravity force minus resistance
x	=	space coordinate in flow direction
x'	=	space coordinate in flow direction of new reference system
y	=	water depth according to parabola profile
Z	=	normal or Gaussian probability function
z	=	bottom level
z	=	sill level of structures
α	=	coefficient in canal routing approximation
α	=	interpolation coefficient between two successive space steps
ß	=	Boussinesq coefficient for velocity distribution
π	=	3.14159
φ	=	celerity of diffusion or celerity of moving reference system
λ	=	coefficient in the diffusion coefficient
λ	=	dimensionless resistance factor
μ	=	variable of cumulated distribution function
ρ	=	density of water
τ	=	shear stress
τ	=	dimensionless time variable
ξ	=	dimensionless space variable
η	=	coefficient
θ	=	interpolation coefficient between two successive time levels
θ	=	angle of lateral inflow
δ	=	partial differential
d	=	normal differential
Δ	=	small difference
υ	=	time integration variable
κ	=	coefficient of diffusion celerity

Subscripts:

x	=	derivative with respect to x
t	=	derivative with respect to t
in	=	inflow
out	=	outflow
0	=	original value
1	=	new value
m	=	mean value

List of publications

In response to this study the following publications have been written.

Schuurmans, W. (1988), *Study of Existing Hydrodynamic Flow Models*, University of Technology, ISBN 90-800089-3-1, Delft, the Netherlands.

Schuurmans, W. and A.J.M. Nelen (1989), Real time Control Systems for the Watermanagement in low lands (the Netherlands), Proceedings of the International symposium *Water Management in Shallow sea and low lands*, Saga, Japan.

Schuurmans, W. (1989), Impact of unsteady flow on irrigation water distribution, Proceedings of the International Conference *Irrigation, Theory and Practice*, ISBN 0-7273-0904-8, Pentech press, London, UK.

Schuurmans, W. and R. Brouwer (1990), Use of Real time scheduling computer programs in The Netherlands, Proceedings of the 14th ICID-congress symposium *Real time scheduling of water deliveries*, Mayar Printers, New Delhi, India.

Schuurmans, W. (1990), Analytical formulae for the response time of irrigation canals, *Irrigation and Drainage Systems*, Vol 4, No. 1, Kluwer Academic Publishers, Dordrecht, the Netherlands.

Schuurmans, W. (1990), Computer simulations of automatically controlled watermanagement systems, Proceedings of the First Junior European Conference *Applications of Operations Research to Real Time Control of Water Resources Systems*, ISBN 3 906484 04 1, Dübendorf, Switzerland.

Schuurmans, W. and E.J. Huyskes (1990), Operational Performance of Conventional Irrigation Systems, lecture notes for the *Senior Advanced Course on Appropriate Modernization and Management of Irrigation Systems*, Delft, the Netherlands.

Schuurmans, W. and E.J. Huyskes (1990), Operational Performance of Automatically Controlled Irrigation Systems, lecture notes for the *Senior Advanced Course on Appropriate Modernization and Management of Irrigation Systems*, Delft, the Netherlands.

Schuurmans, W. (1991), A model for real time controlled canal systems, Proceedings of the European conference *Advances in Water Resources Technology*, Balkema Publishers, ISBN 90-6191 184 2, Rotterdam, the Netherlands.

Schuurmans, W and B. Hoitink (1991), Computer simulations of regional controlled irrigation canals, Proceedings of the European conference *Advances in Water Resources Technology*, Balkema Publishers, ISBN 90-6191 184 2, Rotterdam, the Netherlands.

Schuurmans, W. and Meharani M. (1991), Operational performance of canal control systems, *Water Resources Management*, Vol 5, No 2, Kluwer Academic Publishers, Dordrecht, the Netherlands.

Schuurmans, W. (1991), Description and Evaluation of Program MODIS, Proceedings of the National ASCE Conference *Irrigation and Drainage*, ISBN 0-87262-811-6, New York, USA.

Contractor, D.N. and W Schuurmans (1991), Informed use and potential pitfalls of canal models, Proceedings of the National ASCE Conference *Irrigation and Drainage*, ISBN 0-87262-811-6, New York, USA.

Rogers, David C., Wytze Schuurmans and James W. Keith, (1991), Canal Model Evaluation and Comparison Criteria, Proceedings of the National ASCE Conference *Irrigation and Drainage*, ISBN 0-87262-811-6, New York, USA.

Clemmens, A.J., F.M. Holly, and W Schuurmans, (1991), Description and Evaluation of Program: DUFLOW, Proceedings of the National ASCE Conference *Irrigation and Drainage*, ISBN 0-87262-811-6, New York, USA.

Schuurmans, W. (1991), Noodzaak en toepassingen van sturingssimulaties (Need and applications of control simulations), Proceedings of the Symposium Sturing in het Waterbeheer, *SAMWAT raport No. 6*, ISBN 90-6743-188-5, 's-Gravenhage, the Netherlands.

Brouwer, R., P. Ankum, A.J.M. Nelen, and W. Schuurmans (1990), Lecture notes for the Msc Course *Control of Watermanagement Systems,* Delft, the Netherlands.

Schuurmans, W, P. Wonink (1991), Identification of a control system for a canal with night storage, *Journal of Irrigation and Drainage Engineering*, ASCE.

APPENDIXES

A De Saint Venant Equations

One dimensional flow is described by two dependent variables; a flow variable (e.g. Q) and a geometrical variable (e.g. A). The dependent variables are dependent on two independent variables, namely place x and time t. This means that Q and A vary in time and place.
The variation is described by physical conservation laws, such as conservation of mass, momentum and energy. For the description of the flow phenomena, mass conservation and either momentum or energy conservation should be obeyed. For continuous processes, such as gradually varied unsteady flow momentum conservation and energy conservation are equivalent. However, for discontinuous processes such as a moving hydraulic jump, momentum conservation can only be applied. Therefore, mass conservation and momentum conservation have been used in this study to derive the equations describing the one dimensional flow phenomena.

A.1 Conservation of mass

The law of conservation of mass may be stated very simply and quite generally, as follows:

Consider a system of control surfaces enveloping a control volume such that an inside and outside of the control volume are uniquely defined. Then the net mass of fluid passing from outside to inside through the control surfaces equals the net increase of mass of the control volume. (Abbot,1979)

Mass entering - Mass leaving = Accumulation of mass

If this law is applied to a control volume as shown in Fig. A.1, and assuming that the dependent variables are continuous differentiable functions, the terms become equal to:

$$\text{Mass entering} = \rho\, Q\, \Delta t \tag{A.1}$$

$$\text{Mass leaving} = \left[\rho\, Q + \frac{\partial}{\partial x}(\rho\, Q)\, \Delta x\right] \Delta t \tag{A.2}$$

$$\text{Mass accumulation} = \frac{\partial}{\partial t}(\rho\, A)\, \Delta x\, \Delta t \tag{A.3}$$

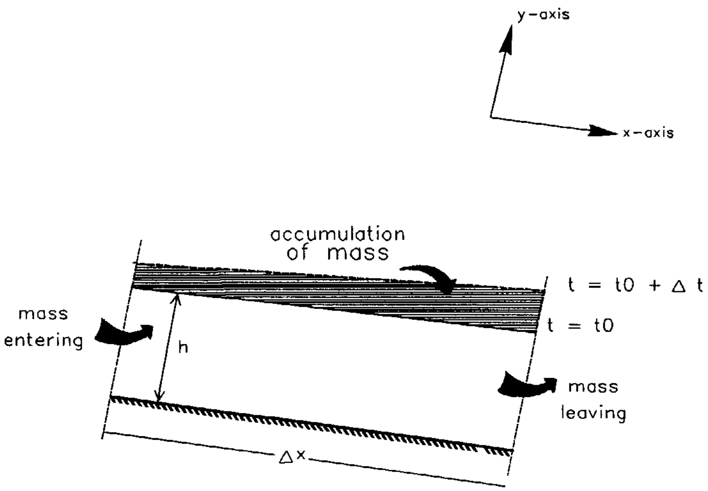

Fig. A.1 Conservation of mass

The resulting equation reads:

$$-\frac{\partial}{\partial x}(\rho\, Q)\,\Delta x\,\Delta t = \frac{\partial}{\partial t}(\rho A)\,\Delta t\,\Delta x \qquad \text{(A.4)}$$

When the water is assumed to be incompressible and well mixed (ρ = constant in time and place) the law of conservation of mass can be simplified by conservation of volume.

$$-\frac{\partial Q}{\partial x}\,\Delta x\,\Delta t = \frac{\partial A}{\partial t}\,\Delta t\,\Delta x \qquad \text{(A.5)}$$

dividing by Δx and Δt yields:

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = 0 \qquad \text{(A.6)}$$

Instead of using the geometrical variable A, Eq. (A.6) can also be written by using the canal surface width T and water level h as geometrical variables. The relationship between A, T and h reads:

$$\frac{\partial A}{\partial t} = \frac{dA}{dh}\,\frac{\partial h}{\partial t} = T\,\frac{\partial h}{\partial t} \qquad \text{(A.7)}$$

Beware that the canal surface width T is a function of the water level h. In case of irregular shaped cross-sections, such as cross-sections which include overbank areas, a distinction is made between the storage width B_s which include the overbanks and a smaller canal top width T which is used in the momentum equation assuming that the overbank area conveys virtually no discharge.

A.2 Conservation of momentum

A.2.1 General

Momentum is a description of a state of motion of a system of masses. Energy is another description of a state of motion of a system. Because of the fact that only one description of a state of motion is required, it is sufficient to consider only conservation of momentum to describe the motion of a system of masses.

Momentum is defined by the vector sum of all (mass * velocity) products (Abbott,1979). This is the amount of motion "embodied" in a state. A force acting on a mass can change the amount of motion which follows from Newton's second law (Force = mass * acceleration). The component product of (force * time) is called impulse. Momentum is thus a vector describing the state of motion, whereas impulse is a vector affecting the state of motion.

The law of conservation of momentum may be stated very simply, as follows:

Consider a system of control surfaces enveloping a control volume of masses such that an inside and an outside are uniquely defined. Then the net inflow of momentum into the system plus the net impulse working on the system equals the net increase of momentum of the system. (Abbott,1979)

Net momentum entering control volume	+	**Net impulse acting on control volume**	=	**Net increase of momentum of control volume**

This law has been applied to a control volume as presented in Fig. A.2. The following equations follow:

A.2.2 Momentum entering control volume

□ Momentum in x-direction **entering** control volume during a period of time Δt

$$\int_0^A \rho u^2 \, dA \; \Delta t = \rho \beta v^2 A \; \Delta t \qquad \text{(A.8)}$$

In which ß is the Boussinesq momentum correction coefficient defined by:

$$\beta \equiv \frac{\int_0^A u^2 \, dA}{v^2 A} \qquad \text{(A.9)}$$

Where v is the mean flow velocity.

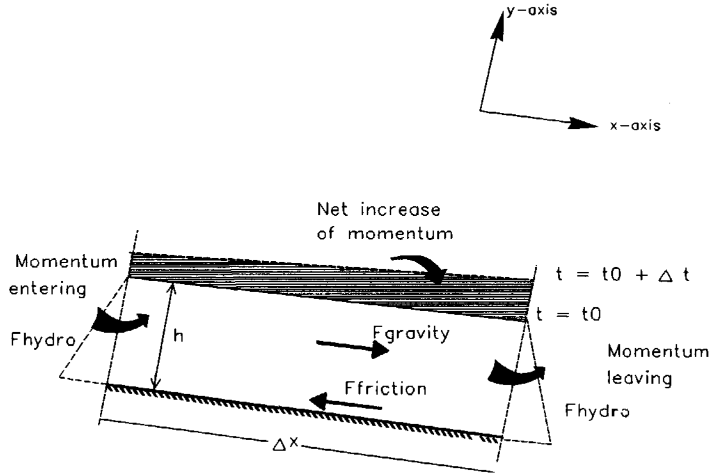

Fig. A.2 Conservation of momentum

□ Momentum in x-direction **leaving** control volume during Δt

$$\rho\beta v^2 A \;\Delta t + \frac{\partial}{\partial x}(\rho\beta v^2 A)\;\Delta x\;\Delta t \qquad \text{(A.10)}$$

So, the **net inflow** of momentum in x-direction during Δt is equal to,

$$-\frac{\partial}{\partial x}(\rho\beta v^2 A)\;\Delta x\;\Delta t = -\rho\frac{\partial\left[\beta\frac{Q^2}{A}\right]}{\partial x}\;\Delta x\;\Delta t \qquad \text{(A.11)}$$

A.2.3 Momentum induced by impulse acting in x-direction on control volume

□ Impulse due to **gravity** force during Δt

$$\int_0^A \rho\, g \sin(s)\;\Delta x\; dA\;\Delta t = \rho\, A\, g\sin(s)\;\Delta x\;\Delta t \qquad \text{(A.12)}$$

□ Impulse due to **frictional** force during Δt

$$\int_0^P -\tau\, dP\;\Delta x\;\Delta t = -\tau P\,\Delta x\;\Delta t \qquad \text{(A.13)}$$

□ Impulse due to **hydrostatic** pressure on **inflow** side during Δt

$$\int_0^{h(x)} \rho\, g \cos(s)\, (h_{(x)}-y)\, b_{(x,y)}\, dy\, \Delta t \tag{A.14}$$

□ Impulse due to **hydrostatic** pressure on **outflow** side during Δt

$$-\int_0^{h(x)} \rho\, g \cos(s) \left(h(x)-y\right) b_{(x,y)}\, dy\, \Delta t \tag{A.15}$$

□ Impulse due to **hydrostatic** pressure on (gradually) varying sides during Δt

$$\int_0^{h(x)} \rho\, g \cos(s) \left(h(x)-y\right) \frac{\partial b_{(x,y)}}{\partial x}\, \Delta x\, dy\, \Delta t \tag{A.16}$$

Thus, the **net** impulse due to **hydrostatic** pressure becomes:

$$\left(\int_0^{h(x)} \rho\, g \cos(s) \left(h(x)-y\right) b\, dy \Delta t\right)_{x=x_1} - \left(\int_0^{h(x)} \rho\, g \cos(s) \left(h(x)-y\right) b\, dy \Delta t\right)_{x=x_1+\Delta x} + \int_0^{h(x)} \rho\, g \cos(s) \left(h(x)-y\right) \frac{\partial b}{\partial x}\, \Delta x \Delta t \tag{A.17}$$

Which can be written as:

$$-\frac{\partial}{\partial x}\left(\int_0^{h(x)} \rho\, g \cos(s) \left(h(x)-y\right) b\, dy\right) \Delta x \Delta t + \int_0^{h(x)} \rho\, g \cos(s) \left(h(x)-y\right) \frac{\partial b}{\partial x}\, dy \Delta x \Delta t \tag{A.18}$$

Eq. (A.18) can be further simplified by applying Leibnitz's theorem for differentiation of an integral. This theorem is stated as follows (Abramowitz & Stegun 1970) (p. 11):

$$\frac{d}{dc}\int_{a(c)}^{b(c)} f(x,c)\, dx = \int_{a(c)}^{b(c)} \frac{\partial}{\partial c} f(x,c)\, dx + f(b,c)\frac{db}{dc} - f(a,c)\frac{da}{dc} \tag{A.19}$$

Applying this theorem to the first term of Eq. A.18 yields:

$$-\frac{\partial}{\partial x}\left(\int_0^{h(x)} \rho\, g \cos(s) \left(h(x)-y\right) b\, dy\right) \Delta x \Delta t = -\int_0^{h(x)} \rho\, g \cos(s) \frac{\partial}{\partial x}\left((h(x)-y)b\right) dy \Delta x \Delta t - \rho\, g \cos(s)\left(h(x)-h(x)\right) b \frac{\partial h}{\partial x} \Delta x \Delta t \tag{A.20}$$

The second term of Eq. (A.20) equals zero, and the first term can be written as:

$$-\int_0^{h(x)} \rho\, g \cos(s) \frac{\partial}{\partial x}\left((h(x)-y)b\right) dy \Delta x \Delta t = -\int_0^{h(x)} \rho\, g \cos(s)\left(h(x)-y\right)\frac{\partial b}{\partial x} dy \Delta x \Delta t - \int_0^{h(x)} \rho\, g \cos(s)\, b \frac{\partial h}{\partial x} dy \Delta x \Delta t \tag{A.21}$$

Thus, Eq. (A.18), representing the net impulse due to hydrostatic pressure, can be written as:

$$-\int_0^{h(x)} \rho\, g \cos(s)\, (h(x)-y)\, \frac{\partial b}{\partial x}\, dy\ \Delta x \Delta t - \frac{\partial h}{\partial x}\int_0^{h(x)} \rho\, g \cos(s)\, b\, dy \Delta x \Delta t$$
$$+ \int_0^{h(x)} \rho\, g \cos(s)\, \big(h(x)-y\big)\, \frac{\partial b}{\partial x}\, dy \Delta x \Delta t \qquad (A.22)$$

The variation in cross-section is compensated by the forces acting on the slopes in the x-direction, and the resulting net impuls thus reads:

$$- \frac{\partial h}{\partial x} \rho\, g \cos(s) \int_0^{h(x)} b\, dy\ \Delta t = - \rho\, g \cos(s)\, A\, \frac{\partial h}{\partial x}\ \Delta t \qquad (A.23)$$

A.2.4 Net increase of momentum of control volume

Momentum in x-direction contained in the control volume on $t = t_0$ is equal to,

$$\int_0^A \rho\, u\, dA\ \Delta x = \rho\, Q\ \Delta x \qquad (A.24)$$

Momentum in x-direction contained in the control volume on $t = t_o + \Delta t$ is equal to,

$$\rho\, Q\ \Delta x + \frac{\partial}{\partial t}(\rho\, Q)\ \Delta x\ \Delta t \qquad (A.25)$$

Thus the net increase of momentum in the control volume during Δt yields,

$$\frac{\partial}{\partial t}(\rho\, Q)\ \Delta x\ \Delta t \qquad (A.26)$$

A.2.5 Summarized

The following expressions were found:

- Net momentum entering:

$$- \frac{\partial}{\partial x}(\rho \beta v^2 A)\ \Delta x\ \Delta t = - \rho \frac{\partial \left[\beta \dfrac{Q^2}{A}\right]}{\partial x}\ \Delta x\ \Delta t \qquad (A.27)$$

- Momentum induced by impulse:

$$\rho\, g \sin(s)\, A\ \Delta x\ \Delta t - \tau P\ \Delta x\ \Delta t - \rho\, g \cos(s)\, A\, \frac{\partial h}{\partial x}\ \Delta x\ \Delta t \qquad (A.28)$$

□ Net increase of momentum:

$$\frac{\partial}{\partial t}(\rho\, Q)\, \Delta x\ \Delta t \qquad (A.29)$$

Substitution of the given expressions in the law of conservation of momentum leads to the following equation:

$$\rho \Delta t \Delta x \left[-\frac{\partial}{\partial x}(\beta \frac{Q^2}{A}) \ - g\cos(s) A \frac{\partial h}{\partial x} - \frac{\tau}{\rho} P + g A \sin(s) \right] = \rho \frac{\partial Q}{\partial t} \Delta t\, \Delta x \qquad (A.30)$$

Rearranging the terms and dividing by $(-\rho\ \Delta x\ \Delta t)$ result in the following equation:

$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x}\left[\beta \frac{Q^2}{A}\right] + g\cos(s) A \frac{\partial h}{\partial x} + \frac{\tau}{\rho} P - g \sin(s) A = 0 \qquad (A.31)$$

A.2.6 Resistance

In the previous equations, the resistance force has been expressed by using a mean shear stress. The hydraulics of the last century have given a number of resistance formulae that express the shear stress in terms of v, h and the physical properties of the canal, assuming turbulent, steady flow conditions. It is common practice to use these equations also for gradually varying unsteady flow. The general resistance equation reads:

$$\tau = \rho\, v\, |\, v\, |\, \lambda \qquad (A.32)$$

Using Chezy resistance formula the dimensionless resistance factor λ is expressed by:

$$\lambda = \frac{g}{C^2} \qquad (A.33)$$

Where C = Chezy resistance coefficient ($m^{½}/s$).
In irrigation practices the Manning formula is often used as in handbooks (Chow 1959) values for Manning n are given for various canals. The relationship between Chezy C and Manning n ($s/m^{⅓}$), or Strickler k (= 1/n)) reads:

$$C = \frac{1}{n} R^{1/6} = k\, R^{1/6} \qquad (A.34)$$

Where n = Manning resistance coefficient and k = Strickler resistance coefficient.
So, the expression for the resistance force expressed by a mean shear stress can be replaced by (P = A/R)

$$\frac{\tau}{\rho} P = \frac{P \rho g v\, |\, v\, |}{\rho\, k^2 R^{1/3}} = \frac{A g v\, |\, v\, |}{k^2 R^{4/3}} = \frac{g Q\, |\, Q\, |}{A\, k^2 R^{4/3}} \qquad (A.35)$$

The momentum equation is then written as

$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x}\left[\beta \frac{Q^2}{A}\right] + g\cos(s)A\frac{\partial h}{\partial x} + \frac{g\,Q\,|\,Q\,|}{k^2 R^{4/3} A} - g\sin(s)A = 0 \qquad \text{(A.36)}$$

I *II* *III* *IV* *V*

The terminology applied reads:

I = Local acceleration
II = Convective acceleration
III = Variation of water depth
IV = Resistance force
V = Driving force by gravity

A.2.7 Simplified wave models

Sometimes not the complete momentum equation in combination with the continuity equation is used to describe gradually varied unsteady flow. Simplified equations have been developed by making assumptions regarding the relative importance of the various terms. In Table A.1 the names of different wave models are presented.

Table A.1 Simplified wave models

Wave Model	Terms of Eq. (A.36) incorporated
Kinematic wave	IV + V
Diffusion wave	III + IV + V
Dynamic wave	I + II + III + IV + V
Gravity wave	I + II + III

A.3 Lateral inflow

Lateral inflow will affect both mass conservation equation and momentum conservation equation. Define a lateral inflow of q per unit length, which is entering the main canal making an angle θ with the main canal. The inflow discharge is defined positive for inflow and negative for outflow. The velocity of the inflow discharge is equal to v_l

A.3.1 Conservation of mass

The equation of conservation of mass is affected by the lateral inflow. Only the mass entering the control volume is affected and reads with lateral inflow (q is positive for inflow):

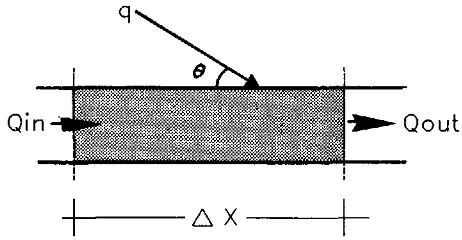

Fig. A.3 Lateral inflow

$$Massentering = \rho\, Q\, \Delta t + \rho\, q\, \Delta x\, \Delta t \qquad (A.37)$$

Substituting this term in the law of conservation of masses yields:

$$-\rho \frac{\partial Q}{\partial x} \Delta x\, \Delta t + \rho\, q\, \Delta x\, \Delta t = \rho \frac{\partial A}{\partial t} \Delta x\, \Delta t \qquad (A.38)$$

Dividing by (ρ Δx Δt) and rewriting the equation yields

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = q \qquad (A.39)$$

A.3.2 Conservation of momentum

Only the term "momentum entering the control volume" is affected by lateral inflow and only then when the inflow discharge has a velocity component in the same direction as the flow in the canal. In case of perpendicular lateral inflow the momentum equation remains unchanged. Assume that the inflow discharge q makes an angle θ with the streamlines of the main canal and has a velocity v_l. The velocity component in the direction of the streamlines is thus equal to $v_l \cos(\theta)$. The momentum entering the control volume becomes:

$$\rho\, \beta \left[\frac{Q^2}{A}\right] \Delta t + \rho\, q\, \Delta x\, v_l \cos(\theta)\, \Delta t \qquad (A.40)$$

Substitution of this term into the law of conservation of momentum yields

$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x}\left[\beta \frac{Q^2}{A}\right] + g\cos(s) A \frac{\partial h}{\partial x} + \frac{g Q\,|\,Q\,|}{k^2 R^{4/3} A} - g\sin(s) A = q\, v_l \cos(\theta) \tag{A.41}$$

A.4 Other notations of the De Saint Venant Equations

A.4.1 Water depth and bed slope

The "variation of the water depth" term and the "bed slope" term can be incorporated in one single term if the canal bed slope is small (so that cos(s) ≈ 1 and sin (s) ≈ s). Remembering that on a horizontal reference system:

$$s = -\frac{dz}{dx} \tag{A.42}$$

The momentum equation can be written as:

$$g A \frac{\partial h}{\partial x} - g A s = g A \left(\frac{\partial h}{\partial x} + \frac{dz}{dx}\right) = g A \frac{\partial \eta}{\partial x} \tag{A.43}$$

Where η is the water level on a horizontal reference system. The collection of the terms into one water level term is often applied in flow models in order to reduce the number of computations. However, it should be kept in mind that this is only correct for mild bed slopes.

A.4.2 Convective acceleration

The convective acceleration term can be rewritten to a non-conservative form, which reads (β is assumed to be constant):

$$\frac{\partial}{\partial x}\left[\beta \frac{Q^2}{A}\right] = \frac{2\beta Q}{A}\frac{\partial Q}{\partial x} - \frac{\beta Q^2}{A^2}\frac{\partial A}{\partial x} \tag{A.44}$$

A.4.3 Other dependent variables

The De Saint Venant Equations can be written in Q and A and c as dependent variables only. The "water depth variation term" can be transformed as follows:

$$g A \cos(s) \frac{\partial h}{\partial x} = g A \cos(s) \frac{1}{T}\frac{\partial A}{\partial x} \tag{A.45}$$

Remembering that

$$c^2 = \sqrt{g\cos(s)\frac{A}{T}} \qquad (A.46)$$

The term becomes,

$$gA\cos(s)\,\frac{\partial h}{\partial x} = c^2\,\frac{\partial A}{\partial x} \qquad (A.47)$$

Defining W as,

$$W = gA\left[\sin(s) - \frac{gQ\,|\,Q\,|}{k^2 R^{4/3} A^2}\right] \qquad (A.48)$$

and using the non-conservative form of the convective acceleration term, the motion term of De Saint Venant Equations can be written as:

$$\frac{\partial Q}{\partial t} + 2\beta\,\frac{Q}{A}\,\frac{\partial Q}{\partial x} + c^2(1-\beta Fr^2)\,\frac{\partial A}{\partial x} = W \qquad (A.49)$$

This form of De Saint Venant Equations is compact and convenient for further manipulations.

A.4.5 Value of ß

ß is defined as the momentum correction coefficient and referred to as Boussinesq coefficient. ß is equal to unity when the flow is uniform along the cross-section. In all other cases ß is greater than unity. When the canal profile consist of a main canal with flood plains, a significant deviation of uniform flow exists. In most irrigation canals, ß can be taken equal to unity. By varying the value of β, the effect of non-uniform water velocity distribution can be investigated. By setting β equal to zero, the convective acceleration term is neglected. This can, for example, be used to set the water level equal to the energy level upstream of a structure.

B Derivation of the diffusion equation

B.1 De Saint Venant Equations

The De Saint Venant Equations, with no lateral inflow, expressed in Q and A read (Appendix A):

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = 0 \qquad \text{B.1}$$

$$\frac{\partial Q}{\partial t} + 2\beta \frac{Q}{A}\frac{\partial Q}{\partial x} + c^2(1 - \beta\, Fr^2)\frac{\partial A}{\partial x} = W \qquad \text{B.2}$$

Where

$$c = \sqrt{g \cos(s) \frac{A}{T}} \qquad \text{B.3}$$

$$Fr = \frac{v}{c} = \frac{Q}{A\, c} \qquad \text{B.4}$$

$$W = g A \left\{ \sin(s) - \frac{Q \mid Q \mid}{k^2 R^{4/3} A^2} \right\} \qquad \text{B.5}$$

$$\beta \equiv \frac{\int_0^A u^2 dA}{v^2 A} \qquad \text{B.6}$$

The two partial differential equations of first order, Eq. (B.1) and Eq. (B.2), can be combined in one partial differential equation of second order. Taking the derivative of Eq. B.1 with respect to x and the derivative of Eq. B.2 with respect to t, and neglecting some small terms yields,

$$\frac{\partial^2 Q}{\partial x^2} + \frac{\partial^2 A}{\partial t \partial x} = 0 \qquad \text{B.7}$$

$$\frac{\partial^2 Q}{\partial t^2} + 2\beta \frac{Q}{A}\frac{\partial^2 Q}{\partial x \partial t} + c^2(1 - \beta\, Fr^2)\frac{\partial^2 A}{\partial x \partial t} = \frac{\partial W}{\partial t} \qquad \text{B.8}$$

When, $\frac{\partial^2 A}{\partial t \partial x} = \frac{\partial^2 A}{\partial x \partial t}$ this term can be replaced by $-\frac{\partial^2 Q}{\partial t^2}$. The partial differential equation, expressed in Q, thus obtained reads:

$$\frac{\partial^2 Q}{\partial t^2} + 2\beta \frac{Q}{A} \frac{\partial^2 Q}{\partial x \partial t} - c^2(1 - \beta Fr^2) \frac{\partial^2 Q}{\partial x^2} = \frac{\partial W}{\partial t} \qquad \text{B.9}$$

B.2 Resistance term

The aim is to find an expression for the derivative of W, which is a function of A and R. An expression for the derivative W can be found, when the geometrical variables are expressed as power functions of some well defined linear measure. The mean water depth has been chosen, as a linear measure. (Appendix E). When n is defined as:

$$n = \frac{y}{y_0} \qquad \text{B.10}$$

then the geometrical variables A and R can be defined by,

$$A = A_0 n^a \qquad \text{B.11}$$

$$R = R_0 n^r \qquad \text{B.12}$$

The applied exponents a and r are chosen for the expected interval of variation of H, say between H_{min} and H_{max}. The value of a and r are found to be:

$$a = \frac{\ln(\frac{A_{max}}{A_{min}})}{\ln(\frac{H_{max}}{H_{min}})} \qquad \text{B.13}$$

$$r = \frac{\ln(\frac{R_{max}}{R_{min}})}{\ln(\frac{H_{max}}{H_{min}})} \qquad \text{B.14}$$

So, R can be written as,

$$R = n^r = (A^{\frac{1}{a}})^r = A^{\frac{r}{a}} \qquad \text{B.15}$$

Now, that the geometrical variables are expressed as power function, the derivative of W with respect to time, can be determined. The term W, expressed as a function of A and Q, read:

The derivative of W with respect to time can be written as:

$$W = g A \sin(s) - \frac{g Q \mid Q \mid}{k^2 A^{\frac{4r}{3a}} A}$$ B.16

$$\frac{dW}{dt} = \frac{\partial W}{\partial A}\frac{dA}{dt} + \frac{\partial W}{\partial Q}\frac{dQ}{dt}$$ B.17

Where,

$$\frac{\partial W}{\partial A} = g \sin(s) + (1 + \frac{4r}{3a}) \frac{g Q \mid Q \mid}{k^2 R^{4/3} A^2}$$ B.18

and

$$\frac{\partial W}{\partial Q} = -2 \frac{g Q}{k^2 R^{4/3} A}$$ B.19

When M is defined by,

$$M = \frac{Q \mid Q \mid}{\sin(s)\, k^2 A^2 R^{4/3}}$$ B.20

The derivative of W with respect to time is written as,

$$\frac{dW}{dt} = g \sin(s) \left[\left(1 + M + \frac{4r}{3a} M\right) \frac{dA}{dt} - 2 M \frac{A}{Q} \frac{dQ}{dt}\right]$$ B.21

Remembering that, according to the continuity equation $\frac{dA}{dt} = -\frac{dQ}{dt}$, this can be written as:

$$\frac{dW}{dt} = -g \sin(s) \left[\left(1 + M + \frac{4r}{3a} M\right) \frac{dQ}{dx} + 2 M \frac{A}{Q} \frac{dQ}{dt}\right]$$ B.22

B.3 Moving reference system

When the equation is transformed to a moving reference system, a simple and accurate form of the diffusion equation can be derived. This moving reference system is equal to:

$$x' = x - \varphi t$$ B.23

$$t' = t$$ B.24

Where the celerity φ is taken equal to:

On this new reference system, the derivatives of Q are transformed into:

$$\frac{\partial Q}{\partial x} = \frac{\partial Q}{\partial x'} \qquad \text{B.26}$$

$$\frac{\partial^2 Q}{\partial x^2} = \frac{\partial^2 Q}{\partial x'^2} \qquad \text{B.27}$$

$$\frac{\partial Q}{\partial t} = \frac{\partial Q}{\partial t'} - \varphi \frac{\partial Q}{\partial x'} \qquad \text{B.28}$$

$$\frac{\partial^2 Q}{\partial x \partial t} = \frac{\partial^2 Q}{\partial x' \partial t'} - \varphi \frac{\partial^2 Q}{\partial x'^2} \qquad \text{B.29}$$

$$\frac{\partial^2 Q}{\partial t^2} = \frac{\partial^2 Q}{\partial t'^2} - 2\varphi \frac{\partial^2 Q}{\partial x' \partial t'} + \varphi^2 \frac{\partial Q^2}{\partial x'^2} \qquad \text{B.30}$$

The obtained partial differential equation of second order,

$$\frac{\partial^2 Q}{\partial t^2} + 2\beta \frac{Q}{A} \frac{\partial^2 Q}{\partial x \partial t} - c^2(1 - \beta Fr^2) \frac{\partial^2 Q}{\partial x^2} = \frac{\partial W}{\partial t} \qquad \text{B.31}$$

Reads on the new reference system,

$$\frac{\partial^2 Q}{\partial t'^2} - 2\varphi \frac{\partial^2 Q}{\partial x' \partial t'} + \varphi^2 \frac{\partial^2 Q}{\partial x'^2} + 2\beta \frac{Q}{A} \left[\frac{\partial^2 Q}{\partial x' \partial t'} - \varphi \frac{\partial^2 Q}{\partial x'^2} \right] - c^2(1 - \beta Fr^2) \frac{\partial^2 Q}{\partial x'^2} = \frac{\partial W}{\partial t'} \qquad \text{B.32}$$

Neglecting the acceleration terms $\frac{\partial^2 Q}{\partial x' \partial t'}$ and $\frac{\partial^2 Q}{\partial t'^2}$, this equation reduces to,

$$\left[\varphi^2 - 2\beta \frac{Q}{A} \varphi - c^2(1 - \beta Fr^2) \right] \frac{\partial^2 Q}{\partial x'^2} = \frac{\partial W}{\partial t'} \qquad \text{B.33}$$

When M is set equal to unity, which is allowed in the absence of a backwater curve, the derivative of the resistance term becomes in this new reference system,

$$\frac{dW}{dt'} = -g \sin(s) \left[2\varphi \frac{dQ}{dx'} + 2\frac{A}{Q} \frac{dQ}{dt'} - 2\varphi \frac{dQ}{dx'} \right] \qquad \text{B.34}$$

or,

$$\frac{dW}{dt'} = -2g \sin(s) \frac{A}{Q} \frac{dQ}{dt'} \qquad \text{B.35}$$

The obtained diffusion equation reads,

$$\left[\varphi^2 - 2\beta \frac{Q}{A}\varphi - c^2(1-\beta Fr^2)\right] \frac{\partial^2 Q}{\partial x'^2} = -2g\sin(s)\frac{A}{Q}\frac{\partial Q}{\partial t'} \qquad \text{B.36}$$

Rearranging the terms yields,

$$\frac{-\varphi^2 + 2\beta \frac{Q}{A}\varphi + c^2(1-\beta Fr^2)}{2g\sin(s)} \frac{Q}{A}\frac{\partial^2 Q}{\partial x'^2} = \frac{\partial Q}{\partial t'} \qquad \text{B.37}$$

When a diffusion coefficient D is introduced, the diffusion equation reads simply,

$$D\frac{\partial^2 Q}{\partial x'^2} = \frac{\partial Q}{\partial t'} \qquad \text{B.38}$$

Where D is equal to,

$$D = \frac{-\varphi^2 + 2\beta \frac{Q}{A}\varphi + c^2(1-\beta Fr^2)}{2g\sin(s)} \frac{Q}{A} \qquad \text{B.39}$$

Remembering that φ reads,

$$\varphi = \left[1 + \frac{2r}{3a}\right]\frac{Q}{A} \qquad \text{B.40}$$

and assuming that $\beta = 1$, the diffusion coefficient can be written as,

$$D = \frac{\left[(1+\frac{2r}{3a})\frac{Q}{A}\right]\left[(1-\frac{2r}{3a})\frac{Q}{A}\right] + c^2(1-\beta Fr^2)}{2g\sin(s)} \frac{Q}{A} \qquad \text{B.41}$$

Which is equal to,

$$D = \frac{\left[1-(\frac{2r}{3a})^2\right](\frac{Q}{A})^2 + c^2(1-Fr^2)}{2g\sin(s)} \frac{Q}{A} = \frac{Q\,c^2}{2gA\sin(s)}\left[1-(\frac{2r}{3a}Fr)^2\right] \qquad \text{B.42}$$

Substituting $c^2 = g\frac{A}{T}\cos(s)$, leads to the final notation for the diffusion coefficient:

$$D = \frac{Q}{2\,T\,tg(s)}\left[1-(\frac{2r}{3a}Fr)^2\right] \qquad \text{B.43}$$

From this formula it can be read that the diffusion coefficient becomes negative when

$Fr > \frac{3a}{2r} \approx 1.5$, which implies that the canal flow in trapezoidal canals becomes unstable

for Fr greater than approximately 2.25 (for rectangular profiles this value equals about 1.5). The diffusion equation can also be re-transformed to the normal reference system. In that case the diffusion equation reads,

$$D \frac{\partial^2 Q}{\partial x^2} = \varphi \frac{\partial Q}{\partial x} + \frac{\partial Q}{\partial t} \qquad \text{B.44}$$

B.4 Solution of the diffusion equation

The solution for an instantaneous variation in discharge (unit step function) is found by integrating the normal probability function $Z(\xi,\tau)$ with respect to time and yields,

$$Q = Q_0 + \Delta Q \; I(\xi,\tau) \;, \qquad \text{B.45}$$

Where, the function I is defined by,

$$I(\xi,\tau) = \int_o^{\tau} Z(\xi,\tau) d\tau \qquad \text{B.46}$$

and where the function Z is called the Gaussian probability function defined by,

$$Z(\xi,\tau) = \frac{1}{\sqrt{2\pi}} \frac{1}{\sqrt{2\tau}} e^{\left(-1/2 \frac{(\xi-\tau)^2}{2\tau}\right)} \qquad \text{B.47}$$

Whereby ξ is a dimensionless parameter for the distance and τ is a dimensionless parameter for the time. So, I reads,

$$I(\xi,\tau) = \int_0^{\tau} \frac{1}{\sqrt{2\pi}} \frac{1}{\sqrt{2\tau}} e^{\left(-1/2 \frac{(\xi-\tau)^2}{2\tau}\right)} d\tau \qquad \text{B.48}$$

The problem is to find an analytical expression for the function I, which implies working out the integral of Z to time. When we replace the integration variable τ for $\frac{1}{2}u^2$, so that,

$$\frac{d\tau}{\sqrt{2\tau}} = du \qquad \text{B.49}$$

Then Eq. (B.48) can be replaced by,

$$I(\xi,u) = \int_0^{1/2u^2} \frac{1}{\sqrt{2\pi}} e^{-1/2(\frac{\xi}{u}-\frac{u}{2})^2} du \qquad \text{B.50}$$

The expression for I thus obtained, looks similar to the (extensively tabulated) cumulative probability function Q, which is defined by,

$$Q(x) = \int_x^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}g^2} dg \qquad \text{B.51}$$

If g is replaced by $(\frac{\xi}{u}-\frac{u}{2})$ then dg becomes,

$$dg = -\left[\frac{\xi}{u^2} + \frac{1}{2}\right] du \qquad \text{B.52}$$

and the integration boundaries are replaced by:

$$\begin{aligned} g = x &\rightarrow u = u_1 \\ g = \infty &\rightarrow u = 0 \end{aligned} \qquad \text{B.53}$$

The cumulative probability function now reads,

$$Q(\frac{\xi}{u}-\frac{u}{2}) = \int_{u_1}^{0} \frac{1}{\sqrt{2\pi}} e^{-1/2(\frac{\xi}{u}-\frac{u}{2})^2} -(\frac{\xi}{u^2}+\frac{1}{2})du \qquad \text{B.54}$$

By changing the boundaries of integration, the sign of the integral is reversed,

$$Q(\frac{\xi}{u}-\frac{u}{2}) = \int_{0}^{u_1} \frac{1}{\sqrt{2\pi}} e^{-1/2(\frac{\xi}{u}-\frac{u}{2})^2} (\frac{\xi}{u^2}+\frac{1}{2})du \qquad \text{B.55}$$

In a similar way, an expression for the function $Q(\frac{\xi}{u}+\frac{u}{2})$ can be derived and yields,

$$Q(\frac{\xi}{u}+\frac{u}{2}) = \int_{0}^{u_1} \frac{1}{\sqrt{2\pi}} e^{-1/2(\frac{\xi}{u}+\frac{u}{2})^2} (\frac{\xi}{u^2}-\frac{1}{2})du \qquad \text{B.56}$$

The exponents of the exponential function can be worked out and read,

$$-\frac{1}{2}(\frac{\xi}{u}-\frac{u}{2})^2 = -\frac{1}{2}(\frac{\xi}{u}^2+\frac{u}{2}^2)+\frac{\xi}{2} \qquad \text{B.57}$$

$$-\frac{1}{2}(\frac{\xi}{u}+\frac{u}{2})^2 = -\frac{1}{2}(\frac{\xi}{u}^2+\frac{u}{2}^2)-\frac{\xi}{2} \qquad \text{B.58}$$

Substitution of these expression in the function Q leads to,

$$Q(\frac{\xi}{u}-\frac{u}{2}) = e^{\frac{\xi}{2}} \int_{0}^{u_1} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}((\frac{\xi}{u})^2+(\frac{u}{2})^2)} (\frac{\xi}{u^2}+\frac{1}{2})du \qquad \text{B.59}$$

which is equal to,

$$Q(\frac{\xi}{u}-\frac{u}{2}) = e^{\frac{\xi}{2}} \int_{0}^{u_1} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}((\frac{\xi}{u})^2+(\frac{u}{2})^2)} (\frac{\xi}{u^2})du + \frac{1}{2}e^{\frac{\xi}{2}} \int_{0}^{u_1} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}((\frac{\xi}{u})^2+(\frac{u}{2})^2)} du \qquad \text{B.60}$$

In the same way $Q(\frac{\xi}{u}+\frac{u}{2})$ can be written as:

$$Q(\frac{\xi}{u}+\frac{u}{2}) = e^{-\frac{\xi}{2}} \int_0^{u_1} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}((\frac{\xi}{u})^2+(\frac{u}{2})^2)} (\frac{\xi}{u^2}) du - \frac{1}{2} e^{-\frac{\xi}{2}} \int_0^{u_1} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}((\frac{\xi}{u})^2+(\frac{u}{2})^2)} du \qquad \text{B.61}$$

Multiplying the function $Q(\frac{\xi}{u}-\frac{u}{2})$ with $e^{-\frac{\xi}{2}}$, and the function $Q(\frac{\xi}{u}+\frac{u}{2})$ with $e^{\frac{\xi}{2}}$ and summing the resulting functions yields,

$$e^{-\frac{\xi}{2}} Q(\frac{\xi}{u}-\frac{u}{2}) + e^{\frac{\xi}{2}} Q(\frac{\xi}{u}+\frac{u}{2}) = \int_0^{u_1} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}\left((\frac{\xi}{u})^2+(\frac{u}{2})^2\right)} du \qquad \text{B.62}$$

When this equation is multiplied with $e^{\frac{\xi}{2}}$, the following equity is found,

$$Q(\frac{\xi}{u}-\frac{u}{2}) + e^{\xi} Q(\frac{\xi}{u}+\frac{u}{2}) = \int_0^{u_1} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\xi}{u}-\frac{u}{2}\right)^2} du \qquad \text{B.63}$$

This equality is the one where we were searching for:

$$I(\xi,u) = \int_0^{1/2u^2} \frac{1}{\sqrt{2\pi}} e^{-1/2(\frac{\xi}{u}-\frac{u}{2})^2} du \qquad \text{B.64}$$

So the function I can be written as,

$$I(\xi,u) = Q(\frac{\xi}{u}-\frac{u}{2}) + e^{\xi} Q(\frac{\xi}{u}+\frac{u}{2}) \qquad \text{B.65}$$

Expressed in the coordinates ξ and τ this becomes,

$$I(\xi,\tau) = Q(\xi-\frac{\tau}{\sqrt{2\tau}}) + e^{\xi} Q(\xi+\frac{\tau}{\sqrt{2\tau}}) \qquad \text{B.66}$$

C Hydraulics exponents of Bakhmeteff

The geometrical variables of irrigation canals are difficult to calculate as a result of the trapezoidal canal profiles. Therefore rectangular profiles are usually assumed in analytical formulas, whereas in numerical computations the geometrical variables are usually discretized as function of the water level. If the trapezoidal profile is approximated with a parabola (or a power function), the geometrical variables can easily be calculated. This reduces the computation time and makes it possible to incorporate the shape of an arbitrary profile in analytical formulas.

It was Boris Alexandrovitsch Bakhmeteff (1880-1951) who used hydraulic exponents for the calculation of back-water curves in open canals. Inspired by the work of Bakhmeteff, Schoemaker proved that all geometrical variables can be approximated by power profiles of the hydraulic water depth H over a range of the mean water depth with an error of less than 1%. (Schoemaker 1989)

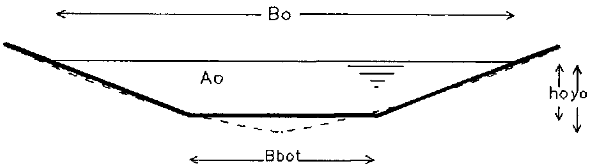

Fig. C.1 Parabolic canal profile

□ **Geometrical variables**

Given a trapezoidal profile with a bottom width of B_{bot}, a side slope of m (1 hor: m ver) and a hydraulic mean depth H (= A/T). This profile can be approximated by a parabolic profile with a water depth y and reference water level y_0.

If n is defined as,

$$n = \frac{y}{y_0} \qquad \text{C.1}$$

then the following geometrical variables can be defined,

$$A = A_0 n^a \qquad \text{C.2}$$

$$B = B_0 n^b \qquad \text{C.3}$$

$$P = P_0 n^p \qquad \text{C.4}$$

$$R = R_0 n^r \qquad \text{C.5}$$

Where

A	=	cross sectional area	m^2
B	=	storage width	m
P	=	wetted perimeter	m
R	=	hydraulic radius	m
0	=	index indicating a reference value	-
n	=	ratio of actual and reference water depth	-
a,b,p,r	=	hydraulics exponents	-

The parabolic profile is completely determined by the coefficient n and the hydraulic exponents. The parabolic approximation is characterized by the linear relationship between y and H. This can be demonstrated as follows,

$$H = \frac{A}{B} = \frac{A_0 n^a}{B_0 n^b} = H_0 n^{a-b} = H_0 n = \frac{H_0}{y_0} y \qquad \text{C.6}$$

The relationship between a and b ($a = b + 1$) follows from,

$$\frac{dA}{dy} = B(y) \quad \frac{d(A_0 n^a)}{d(y_0 n)} = \frac{A_0}{y_0} a n^{a-1} = B_0 n^b \qquad \text{C.7}$$

In addition it is found that,

$$\frac{A_0 a}{y_0} = B_0 \qquad y_0 = a \frac{A_0}{B_0} = a H_0 \qquad \text{C.8}$$

So n defined as y/y_0, can also be defined as H/H_0.

□ **Hydraulics exponents**

The hydraulics exponents a,b,p and r, can be found by using the definition for n. When the maximum and minimum values of A, B, P and R are known, the hydraulic exponents read:

$$a = \frac{\ln(\frac{A_{max}}{A_{min}})}{\ln(\frac{H_{max}}{H_{min}})} \quad \text{C.9}$$

$$b = \frac{\ln(\frac{B_{max}}{B_{min}})}{\ln(\frac{H_{max}}{H_{min}})} \quad \text{C.10}$$

$$p = \frac{\ln(\frac{P_{max}}{P_{min}})}{\ln(\frac{H_{max}}{H_{min}})} \quad \text{C.11}$$

$$r = \frac{\ln(\frac{R_{max}}{R_{min}})}{\ln(\frac{H_{max}}{H_{min}})} \quad \text{C.12}$$

Whereby the subscripts max and min refer to the maximum and minimum value respectively. Beware that the closer the maximum and minimum values are together the more accurate the profile is approximated.

The exponent "a" characterizes the increase of storage capacity as function of the water level. For a rectangular profile a is found to be equal to unity. For trapezoidal profiles, a is usually varying between 1.3 to 1.6. The coefficient r is close to unity which implies that the hydraulic radius can usually be approximated by the hydraulic mean depth.

The power functions of the geometrical variables, with known reference values and known coefficients, can be substituted in the analytical expressions. In that way it becomes possible to calculate the derivatives of the geometrical variables to space and time.

D Derivation of the characteristics

D.1 General

The characteristics are defined as lines in the x-t field along which the partial derivatives, with respect to time and place, do not exist. This property is used to derive the characteristic relationships. The set of equations, specifying the relationship between A and Q in matrix notation, read:

$$\begin{bmatrix} c^2(1-\beta Fr^2) & 0 & 2\beta\frac{Q}{A} & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & dx & dt \\ dx & dt & 0 & 0 \end{bmatrix} * \begin{bmatrix} \frac{\partial A}{\partial x} \\ \frac{\partial A}{\partial t} \\ \frac{\partial Q}{\partial x} \\ \frac{\partial Q}{\partial t} \end{bmatrix} = \begin{bmatrix} W \\ 0 \\ DQ \\ DA \end{bmatrix} \qquad \text{D.1}$$

The solution for the partial differentials can be found using Cramer's rule. For example, the derivative of A with respect to space is equal to,

$$\frac{\partial A}{\partial x} = \frac{D'}{D} \qquad \text{D.2}$$

Where D is the determinat of the matrix, and D' is the first determinant. If the partial derivatives do not exist, then there should be no solution for Eq. D.2. This implies that both determinants D and D' should be equal to zero. The primary characteristics are found by setting D equal to zero, and the secondary characteristics are found when D' (or D'', D''', D'''') are set equal to zero.

D.2 Primary characteristics

The primary characteristics are found when D is set to 0. The following equation follows:

$$D = dx * \begin{bmatrix} c^2(1-\beta Fr^2) & 0 & 1 \\ 0 & 1 & 0 \\ dx & dt & 0 \end{bmatrix} - dt * \begin{bmatrix} c^2(1-\beta Fr^2) & 0 & 2\beta\frac{Q}{A} \\ 0 & 1 & 1 \\ dx & dt & 0 \end{bmatrix} = 0 \qquad \text{D.3}$$

Which is equal to,

$$D = dx \begin{bmatrix} c^2(1-\beta Fr^2) & 1 \\ dx & 0 \end{bmatrix} - dt\, c^2(1-\beta Fr^2) \begin{bmatrix} 1 & 1 \\ dt & o \end{bmatrix} - dt\, 2\beta \frac{Q}{A} \begin{bmatrix} 0 & 1 \\ dx & dt \end{bmatrix} \qquad \text{D.4}$$

or,

$$D = -dx^2 + dt^2 * c^2(1-\beta Fr^2) + dt\, dx * 2\beta \frac{Q}{A} \qquad \text{D.5}$$

Dividing by $-dt^2$ gives an equation in dx/dt,

$$D = \frac{dx^2}{dt} - c^2(1-\beta Fr^2) - \frac{dx}{dt} 2\beta \frac{Q}{A} = 0 \qquad \text{D.6}$$

Which can be rewritten as,

$$D = \left(\frac{dx}{dt}\right)^2 - 2\beta \frac{Q}{A} \frac{dx}{dt} - c^2(1-\beta Fr^2) = 0 \qquad \text{D.7}$$

Using the square root formula, the solution for dx/dt can be found,

$$\frac{dx}{dt} = \frac{2\beta \frac{Q}{A} \pm \sqrt{\beta^2 \left(\frac{Q}{A}\right)^2 + 4c^2(1-\beta Fr^2)}}{2} \qquad \text{D.8}$$

If $\beta = 1$, then this equation can be simplified by:

$$\frac{dx}{dt} = \frac{Q}{A} \pm c = c(Fr \pm 1) \qquad \text{D.9}$$

The found expressions are called the primary characteristics, which give the lines in the x-t field along which the partial differentials do not exist.

D.3 Secondary characteristics

The secondary characteristics are obtained by working out the secondary determinants such as D', D'' et cetera. Working out the secondary determinants, will lead to the same equations, which are the called the secondary characteristics. Working out D''' = 0 gives,

$$D''' = \begin{bmatrix} c^2(1-\beta Fr^2) & 0 & W & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & DQ & dt \\ dx & dt & DA & 0 \end{bmatrix} = 0 \qquad \text{D.10}$$

Which can be written as,

$$D''' = \begin{vmatrix} c^2(1-\beta Fr^2) & W & 1 \\ 0 & DQ & dt \\ dx & DA & 0 \end{vmatrix} = 0 \qquad \text{D.11}$$

or,

$$c^2(1-\beta Fr^2)\begin{vmatrix} DQ & dt \\ DA & 0 \end{vmatrix} + dx\begin{vmatrix} W & 1 \\ DQ & dt \end{vmatrix} = 0 \qquad \text{D.12}$$

Which can be written as:

$$-c^2(1-\beta Fr^2)\,dt\,DA + dx\,W\,dt - dx\,DQ = 0 \qquad \text{D.13}$$

Hence DQ reads,

$$DQ = -c^2(1-\beta Fr^2)\,DA\,\frac{dt}{dx} + W\,dt \qquad \text{D.14}$$

Remembering that an expression for dt/dx has already been found and assuming $\beta = 1$, the secondary characteristics can be simplified by

$$DQ = -c^2(1-Fr)(1+Fr)\,DA\,\frac{dt}{dx} + W\,dt = \frac{c^2(Fr-1)(Fr+1)}{c(Fr\pm 1)}\,Da + W\,dt \qquad \text{D.15}$$

which is identical to

$$DQ = c(Fr \mp 1)\,DA + W\,dt \qquad \text{D.16}$$

This equation represents, lines in the Q-A field, and are called the secondary characteristics.

D-4

E Celerity of a bore

The celerity of a bore cannot be derived from the De Saint Venant equations, as the assumption of hydrostatic pressure is not valid. Furthermore the dependent variables are not continuous. The propagation celerity can be derived by using a mass and momentum balance, whereby the boundaries are taken so far removed from the discontinuity, that hydrostatic water pressure can be assumed at the boundaries, and so close, that the resistance, compared to the difference in hydrostatic pressure, can be neglected.

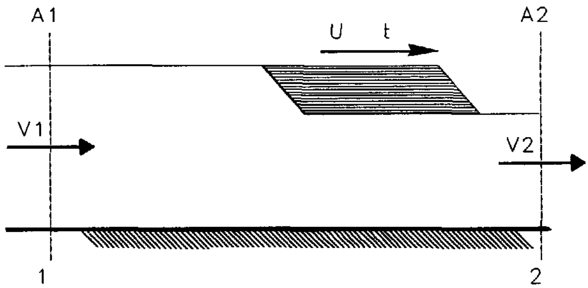

Fig. E.1 Celerity of a bore

E.1 Continuity of masses

The equation of continuity of mass reads,

mass entering - mass leaving = increase of mass

When the density of water is assumed to be constant, and a moving reference system with celerity u is used, the equation of mass yields:

$$A_1 (v_1 - u) = A_2 (v_2 - u) \qquad \text{E.1}$$

which can be rewritten as:

$$A_1 (v_1 - u)^2 = \frac{A_2^2}{A_1} (v_2 - u) \qquad \text{E.2}$$

This expression is later on substituted in the equation of momentum.

E.2 Equation of momentum

The equation of momentum is defined as:

net momentum entering control volume	+	**net impulse working on control volume**	=	**net increase of momentum of control volume**

(Momentum is equal to the product of mass and velocity). If this law is applied to the system, as shown in Fig. E.1, the following expressions can be derived.

E.2.1 Net momentum entering the system per time

The net momentum entering the control volume is equal to,

$$\rho A_1 (v_1 - u)^2 - \rho A_2 (v_2 - u)^2 \tag{E.3}$$

which can be written by using the law of conservation of masses,

$$\rho A_2 (v_2 - u)^2 \left[\frac{A_2 - A_1}{A_1}\right] \tag{E.4}$$

E.2.2 Net impulse working on the system due to hydrostatic pressure per time

$$\rho\, g \cos(s) \left(\int_0^{h_1} b(y)\,(h_1 - y)\ dy - \int_0^{h_2} b(y)\, h_2 - y)\ dy \right) \tag{E.5}$$

The integrals can be worked out by using power functions for the geometrical variables B and A (see also appendix "Hydraulic exponents of Bakhmeteff"),

$$A = A_0 n^a \qquad\qquad B = B_0 n^b \tag{E.6}$$

Where n is defined by,

$$n = \frac{y}{y_0} \tag{E.7}$$

and where y is the water depth in the approximated canal profile. The subscript 0 refers to a reference value. Using the power functions, the integral becomes:

$$\int_0^{h_1} B(y)\,(h_1 - y)\,dy \approx \int_0^{y_0 n_1} B_0 n^b\,(y_0 n_1 - y_0 n)\,d(y_0 n) = \int_0^{n_1} B_0\, y_0^2 (n_1 n^b - n^{b+1})\,dn \tag{E.8}$$

Which is equal to,

$$\int_0^{n_1} B_0\, y_0^2 (n_1 n^b - n^{b+1})\, dn = B_0\, y_0^2 \left[\frac{1}{b+1} - \frac{1}{b+2}\right] n_1^{b+2} \tag{E.9}$$

This equation can be expressed in A instead of B, considering the following relationship between A and B,

$$A = A_0 n^a = \int_0^{h_1} B(y)\, dY = \int_0^{n_1} B_0 n^b y_0\, dn = \frac{B_0 y_0}{b+1} n^{b+1} \tag{E.10}$$

So,

$$A_0 = \frac{b_0 y_0}{a}, \qquad a = b + 1 \tag{E.11}$$

The integral can thus be written as,

$$B_0\, y_0^2 \left[\frac{1}{b+1} - \frac{1}{b+2}\right] n_1^{b+2} = \frac{A_0^2 a^2}{B_0} \left[\frac{1}{a} - \frac{1}{a+1}\right] n_1^{a+1} = \frac{A_0^2}{B_0} \frac{a}{a+1} n_1^{a+1} \tag{E.12}$$

The net impulse is thus equal to,

$$\rho\, g \cos(s) \frac{A_0^2}{B_0} \frac{a}{a+1} (n_1^{a+1} - n_2^{a+1}) = c_0^2 A_0 \frac{a}{a+1} (n_1^{a+1} - n_2^{a+1}) \tag{E.13}$$

E.2.3 Net increase of momentum per time

The net increase of momentum is zero, with respect to the moving reference system.

E.2.4 Momentum equation

Now that, all terms have been determined, the momentum equation can be written as,

$$\rho A_2 (v_2 - u)^2 \left[\frac{A_2 - A_1}{A_1}\right] + \rho\, c_0^2 A_0 \frac{a}{a+1} (n_1^{a+1} - n_2^{a+1}) = 0 \tag{E.14}$$

Remembering that A = A_0 n^a, this can be written as,

$$A_0 n_2^a (v_2 - u)^2 \left[\frac{n_2^a - n_1^a}{n_1^a}\right] + c_0^2 A_0 \frac{a}{a+1} (n_1^{a+1} - n_2^{a+1}) = 0 \tag{E.15}$$

Which is identical to,

$$(v_2 - u)^2 = c_0^2 \frac{n_1^a}{n_2^a} \frac{a}{a+1} \frac{n_1^{a+1} - n_2^{a+1}}{n_1^a - n_2^a} \tag{E.16}$$

The celerity of a bore thus becomes

$$u = v_2 \pm c_0\, f(a,n) \qquad \text{E.17}$$

where, the function f is equal to:

$$f(a,n) = \sqrt{\frac{n_1^a}{n_2^a}\,\frac{a}{a+1}\,\frac{n_1^{a+1} - n_2^{a+1}}{n_1^a - n_2^a}} \qquad \text{E.18}$$

When the head of the bore becomes infinite (n_2-n_1 -> 0), the function f becomes equal to one. In general, the celerity of a bore is in between the critical celerity upstream and downstream of the bore.

F Description and evaluation of program MODIS

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Abstract MODIS is an implicit hydrodynamic modeling package that has been developed to investigate the hydraulic performance of dynamic controlled irrigation systems. The model's most apparent features are its accurate computation of a wide range of standard structures and its many operation possibilities. Furthermore, the model is able to compute performance indicators which allow for a fast and diagnostic interpretation of the model results. The user interface is not menu driven and the program is not public domain. The program is most suitable for experienced users and for large systems.

Introduction

The name MODIS is an acronym of "Modeling Drainage and Irrigation Systems", and has been developed at Delft University of Technology in 1990. The computational base program underlying MODIS is the river modeling package named Rubicon. The main reason for its development was the fact that no models tailored for controlled irrigation canals were to be found. The main limitations of existing programs are the lack of an accurate computation of standard irrigation structures and the limited operation possibilities of these structures. The "Task Group on Real Time Control of Urban Drainage Systems" also comes up with the conclusions that "*Models for the state of the system ... have been developed in a great number for static, non-controllable systems. However, hardly any model has been described allowing to simulate automatic regulators and external control input during the simulated process*". (Schilling 1987). The same conclusion has been drawn two years later: "*.. a key to efficient research on canal automation is the existence of an easy-to-use, accurate, and flexible unsteady flow canal hydraulic simulation program. This research project did not find such a program*". (Burt, 1989)

In this paper a description of the program is presented following the "Canal model evaluation and comparison criteria" (Rogers et al 1991). Moreover, an application is presented to illustrate its use. The application deals with the modernization of a 110 km long irrigation canal in Jordan.

Technical Merit

Computational accuracy

The model solves the complete De Saint Venant equations. The applied numerical solution technique is based on finite differences by using the Preissmann implicit scheme. This implies that the numerical method is of second order accuracy in place and, usually, of first order in time (depending on the value of the time interpolation coefficient). The values of the non-linear terms are determined by interpolating between the values at the old and new time level, starting with the values at the old time level. The number of iterations can be specified by the user, but a value of two is recommended.

Numerical solution criteria

The numerical method is mass and momentum conservative if at least two iterations are used. By using an implicit scheme (the four point implicit Preissmann scheme), stability is guaranteed for any Courant number. The numerical solution converges to the real solution if the solution is stable, because the difference equations have proven to be consistent with the differential equations (Cunge et. al 1980).

Robustness

Robustness has been given priority to accuracy. If accuracy considerations are violated a warning message appears. For example, if input errors are encountered the model will continue reading the input data, and afterwards print the encountered errors in an echo file of the input file. In this way input errors are well traced and can be quickly corrected. To avoid program termination in case of dry bed flow, a Preissmann slot is automatically added to trapezoidal cross-sections. A special routine prevents the slot from falling dry by continuously checking if the water levels are lower than the bed levels. If so, the water depths at those locations are artificially increased to 0.01 m above the bottom level and a base flow of 0.001 m^3/s is generated. (A warning message is printed whenever this routine is activated).

Initial conditions

To solve the De Saint Venant equations initial and boundary conditions are needed. The initial condition requires water levels and discharges at every computational point at the beginning of the computation. The user has only to specify initial conditions at the branch ends, as the program interpolates the values for intermediate grid points located in that branch. It is also possible to use the outcome of a previous computation as an initial state for further computations. This feature can be used, for example, to use a pre-calculated steady state as an initial state.

Internal and external boundary condition analysis

The canal layout is modeled by using branches and nodes. Branches represent conveyance elements such as pools or reaches. Nodes are only used to link branches together and to indicate a branch end. In addition nodes can be used to model reservoirs whereby the storage area can be specified as a function of the water level.

External boundary conditions are imposed on nodes indicating branch ends. The user can choose between a water level, discharge, or stage-discharge relationship as boundary condition. The water level and discharge can be specified either as constants or as functions of time (stage hydrographs and discharge hydrographs).

Internal boundary conditions are needed to link branches, and there water level compatibility is assumed. Structures can be specified anywhere along a branch without having to specify boundary conditions.

The boundary conditions are rewritten internally in the same linearized format as the mass and momentum equations, and thus fully incorporated in the implicit solution procedure. The boundary conditions can be specified as fixed values, as time series, or as a function of a user written fortran routine.

Special hydraulic conditions

The model has a special routine to avoid dry bed flow in order to keep the model running. However, no special equations are incorporated to calculate advances on a dry bed. Rapid flow changes and bore waves too are in principal not covered by the De Saint Venant equations, which are valid for gradually varied unsteady flow only. However, the error made is small and the model can handle rapid changes quite well as long as the Courant number is chosen sufficiently close to one and the time interpolation coefficient is somewhat greater than ½ (Contractor & Schuurmans 1991).

Supercritical flow cannot be handled. This is not because the De Saint Venant equations are not valid for supercritical flow, but it is due to the (double sweep) matrix solution algorithm. Hydraulic jumps can only be handled in the vicinity of structures, where the De Saint Venant equations have been replaced by structure equations. Reversal of flow directions will cause no problems. It is even possible to use different structure parameters for flow in a positive and in negative direction.

Modeling capabilities

System configuration

The system configuration is modeled by using branches and nodes. No restrictions are imposed on the branch lengths, and both branched and looped canal networks can be handled. The model generates a computational grid along every branch using a Δx increment, specified by the user. The user can add additional computational grid points to the canal system. These user defined grid points are needed to specify the branch characteristics such as its profile and elevation. Every grid point has an elevation and a cross-section. The cross-section consists of a profile shape, Boussinesq coefficient(s) and resistance coefficient(s). All construction elements such as branches, nodes, grid points and cross-sections have user defined names instead of numbers.

Structures have to be located within a branch. As more than one structure can be placed within a branch, a branch can consist of various pools separated by structures. Moreover, it is possible to model composite structures by locating several structures at the same location.

Frictional resistance
The friction term of the De Saint Venant Equations is represented by Strickler/ Manning resistance formula. The resistance value can be varied in height and with the longitudinal distance.

Boundary Condition types
Structures are not treated as boundary conditions in the MODIS model, as they are placed within a branch. When a structure is encountered, the momentum equation of the De Saint Venant Equations is replaced by the structure equation which is rewritten in the same format as the momentum equation and thus fully incorporated in the implicit solution procedure.

The standard structure library incorporated in the MODIS model comprises: pumps, weirs, orifices, pipes, head loss structures, and Neyrtec baffle distributors. Furthermore, the user can add his own written Fortran defined structures, but this requires some knowledge of Fortran and the program. Structures can be placed in series and in parallel. The latter facility is used to define compound structures.

Special attention has been paid to an accurate computation of structure flow. As the upstream and downstream water level can fluctuate during a computational run, the flow condition can also fluctuate, e.g. from free to submerged flow, or from orifice flow to weir flow. The model continuously checks which flow condition is to be applied, taking the actual state of the system into account. Some default values for shifting from one flow condition to another are incorporated, but the user can also define these boundaries himself. Moreover, the value of various structure coefficients (e.g. the contraction coefficient or effective discharge coefficient) depends on the hydraulic conditions. The values of these coefficients can be specified by the user either as constants or as a function of hydraulic parameters.

Turnouts
Turnouts are treated in the same way as the structures described in the previous section. The user has to define a branch first, and then to locate a structure inside that branch. At the branch end a fixed water level could be specified as a boundary condition. Only if the outflow rate is predefined, lateral inflow/outflow facilities can be used which do not require an additional branch.

Operations duplication
The MODIS model is able to simulate all types of structure operation. Various structure parameters such as width, sill level, and gate opening height can be given a constant value or can be specified as functions. There can be time functions, but also functions of e.g. an upstream water level. Pumps are switched on if the actual water level exceeds a user defined level, and switched off if the actual water level drops below another lower user defined level. (These levels in turn can be specified as a function of time). Moreover, the capacity of the pump can be specified as a function of the head.

Automatic control
It is obvious that it is irrelevant for the behaviour of the system whether operation is carried out manually or automatically. Automatic control as a function of time and on/off control can

therefore be modelled in the same way as described in 3.5. In MODIS it is also possible to simulate real time control (closed loop control) whereby control is based on the actual state of the system following a control algorithm.

Two types of control algorithms have been implemented in the MODIS model: a multiple speed controller and a Proportional Integral Differential (PID) controller. For each controller control parameters have to be specified. These control parameters, such as gain factors, can be specified as constants, but also as a function of time or a hydraulic variable. Thus, the target levels of the controllers can be adjusted in time and the speed of gate movement can be specified as a function of the deviation from the set-point. Finally, different levels of control can be defined such as local and regional control.

As a result, all types of existing canal control systems, such as upstream control, downstream control, mixed control, BIVAL control, and ELFLOW-control can be modelled in the MODIS model. User defined control algorithms can be added to the model by using the Fortran function facility.

Miscellaneous limitations

Model limitations are mainly related to memory limitations. The user can tailor its model for a specific project by changing the maximum number of branches, nodes, structures, functions, controllers, et cetera. In order to do so, the user has to redefine some maximum parameters and to re-compile the complete model package. There are no limitations to physical dimensions. Both trapezoidal and irregular cross-sections can be modelled. The minimum computational time step is 1 second. The user can define the required format of the output data and thus presents the water levels as precise as he wants.

User considerations

User interface

The MODIS model is controlled from an operation template which is shown on the screen. By moving a highlighted bar through the template to a specific item and by pressing a help key, information about the selected item is provided. The program consists of several sub programs which have to be run in sequence. Every subprogram has its own input file and produces both an echo file with error messages, and an output file. Each input data file is first checked on possible errors before execution of the program. The error messages, divided into warnings, errors, and fatal errors, are printed in an echo file of the input file.

The input data (in metric units only) can be found in input files. Interactive data input is not possible. An input file comprises input tables supported by explanatory comment lines. The input data has to be specified in the input tables following a pre-described sequence, but no fixed column position is needed. Both names and numbers can be used to denote structures, branches, nodes, functions, controllers et cetera.

To facilitate easy data input, special information characters can be used to reduce the amount of input data. For example, the " = " symbol means that the value is equal to the value of the same column specified above. The "?" symbol stands for interpolation of data specified in the same column above and below. Furthermore, all editor features such as "find and replace", "copy", and "move" are available. Practice has shown that the use of input files instead of a menu driven input might be overwhelming for the first time user, whereas the more experienced user finds its way easily and quickly.

The final computational results, as functions of time or place, can be presented both graphically and in tables. Possible output parameters are: water levels, water depths, discharges, wetted cross-sectional area, flow width, storage width, Boussinesq coefficient, hydraulic radius, resistance, Froude number. The coloured graphs can be printed on various types of screens and on a wide variety of printers and plotters.

To facilitate an easy interpretation, it is possible to print only maximum, mean, and minimum values at certain locations during a specific periods of time. Furthermore, the percentage of time in which user defined minimum and maximum values have been exceeded can be printed.

To evaluate the water distribution in irrigation systems simulated by the MODIS model, operation performance indicators which are computed by the model can be used. They consist of a delivery performance ratio (DPR), which specifies to which extent the user intended distribution is satisfied, and an operation efficiency (e_o), which indicates how much water is lost due to inappropriate operation and leakage (Schuurmans & Maherani, 1991).

Documentation and support

The program package is supported by a user's manual, in which also examples are presented. Furthermore, an updated software documentation is available containing an alphabetic list of all parameters, all subroutines and Hipo diagrams of each subprogram. Purchasers of the program can get assistance both from Delft University of Technology and from Haskoning Consulting Engineers who developed the base river modeling package "Rubicon" out of which MODIS was developed.

Direct costs

The executable version of the program package costs about *f* 20,000 and to obtain the source code an additional amount is needed, depending on the applications. The program can run on any IBM-compatible computer with 640 KB Ram memory, equipped with a mathematical co-processor and with a hard disk. The program performs most well on an AT-computer with a 80286 or higher processor. For the graphical output, the commercial "HALO" graphical package is needed.

Indirect costs

It takes one day to get familiar with the program and to know where to find what in the user's guide. To define a model of the irrigation system using the MODIS package requires another few days, depending on the size of the system. Apart from calibration (if needed) most of the time is involved in determining which runs have to be made and how to interpret the results. This requires a skilled watermanagement engineer for operation rather than a computer specialist. To find a sound solution several simulations are usually required. Interpretation of the model results is fastened with the help of the model interpretation parameters which also do have some diagnostic importance. The model execution time depends on the size of the model. Even for large irrigation systems with hundreds of grid points and more than a hundred structures it will take less than a second to proceed one computational step on a 386 machine. This implies that the total simulation time required to simulate a few days is a matter of minutes.

Summary and conclusions

A great number of flow models which calculate the unsteady flow phenomena in one dimensional canal systems do exist. Each program has its own characteristics and limitations. The MODIS model was developed especially to study the hydraulic performance of controlled irrigation canals. In that respect special attention has been paid to an accurate computation of structure flow and a wide range of operation concepts. Although the program is not menu driven, it has proven to be convenient for more experienced users and large canal systems. MODIS is a commercial program package and not public domain.

References

Burt, Charles (1989), *Canal Automation Providing On-Demand Water Deliveries for Efficient Irrigation*, Department of Agricultural Engineering, California Polytechnic State University (Cal Poly), San Luis Obispo, CA 93407, USA.

Cunge, J.A., F.M. Holly, A. Verwey (1980), *Practical Aspects of Computational River Hydraulics*, Pitman Publishing Limited, London, UK.

Contractor D.N., W. Schuurmans (1991), Informed use and potential of canal models, Proceedings of the ASCE National Conference *Irrigation and Drainage*, ISBN 0-87262-811-6, Hawaii, USA.

Rogers D.C., J. Keith and W. Schuurmans (1991), Canal model evaluation and comparison criteria, Proceedings of the ASCE National Conference *Irrigation and Drainage*, ISBN 0-87262-811-6, Hawaii, USA.

Schilling W. (1987), *Real Time Control of Urban Drainage Systems, The state-of-the art*, IAWPRC/IAHR Joint Committee.

Schuurmans W., M. Maherani (1991), Operational performance of canal control systems, *Water Resources Management*, Vol 5, No. 2.