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DOI
10.1109/LCSYS.2017.2778009

Publication date
2018

Document Version
Accepted author manuscript

Published in
IEEE Control Systems Letters

Citation (APA)
Baldi, S. (2018). Cooperative output regulation of heterogeneous unknown systems via passification-based adaptation. *IEEE Control Systems Letters*, *2*(1), 151-156. https://doi.org/10.1109/LCSYS.2017.2778009

Cooperative output regulation of heterogeneous unknown systems via passification-based adaptation

Simone Baldi

***Abstract*—While several robust cooperative output regulation approaches for heterogeneous systems have been proposed (with fixed-gain distributed controllers), the design of adaptive-gain distributed controllers becomes relevant in dealing with larger uncertainty than robust approaches. This work addresses the adaptive cooperative output regulation problem for heterogeneous systems with unknown linear dynamics, where possibly large system uncertainty would make fixed-gain robust approaches not applicable. A passification method is adopted to design adaptive-gain distributed controllers solving the problem. The proposed method includes two steps: in the first step, a distributed observer of the exogenous signal is designed for those systems that cannot access the exosystem, and a reference model is designed whose output can converge to the exogenous signal; in the second step, command generator tracking is achieved via adaptive laws that make the output of each system converge to the output of the reference model, and thus to the exogenous signal. Stability analysis is provided via a Lyapunov approach, and a numerical example illustrates the effectiveness of the approach.**

***Index Terms*—Cooperative output regulation, distributed adaptive control, passification method.**

I. Introduction

A wide range of multi-agent coordination missions such as output synchronization, leader-following, formation keeping, and many more tasks, can be formulated as a distributed output regulation problem [1]. The main idea behind cooperative output regulation is that the systems can be divided into two groups: the first group of systems can access the signals generated by the exosystem, while the second cannot. As a result, the regulation problem cannot be solved by the decentralized approach: typically, some distributed observer of the exogenous reference signal must be devised. Most approaches to cooperative output regulation problem can be divided into two families: the internal model approach [2], and the feedforward approach [3]. Recent advances in the field include: removing the assumption that all systems knows the matrix of the exosystem [4]; reducing the communication burden by exchanging partial state information [5]; addressing switching communication topologies [6], [7]. Despite these advances in cooperative output regulation, only few works have been focusing on the problem of cooperative output regulation when the systems might present large uncertainty. Since the very beginning, researchers in cooperative output regulation have recognized the need for addressing parameter uncertainties in system matrices [8], from which a fruitful line of research on heterogeneous systems stemmed, aiming to solve the cooperative output regulation problem when the agents differ from each other: [9] addresses cooperative control problems in heterogeneous harmonic oscillators coupled by diffusive links. It is shown that, in the presence of parameter uncertainties arising from heterogeneity, structural requirements are needed for robust output synchronization.

Currently, the cooperative output regulation problem for heterogeneous uncertain systems is addressed by combining the distributed observer and the feedforward method in such a way to solve a robust cooperative output regulation problem with fixed-gain control [10], [11]. However, with some exceptions like a class of minimum phase systems in [12], the regulator equations underlying the cooperative problem might have no fixed-gain solution if uncertainty is too large [13]. To address large uncertainty, adaptive-gain distributed control becomes of utmost importance. The passification technique has been shown to be an effective tool to deal with large uncertainty, which has been applied to single agents with input/output communication delays [14] and adaptive synchronization [15]. However, in synchronization approaches via passification, there is no distinction between the group of systems that can access the exogenous signals, and those that cannot: therefore, the distributed observer design is not addressed [16], [17]. In [18], a sliding-mode design is used in place of the distributed observer to address cooperative regulation.

As a result, we can summarize this overview of the state of the art by saying that there are mature robust cooperative output regulation approaches for heterogeneous systems in the presence of small uncertainty, but the study of adaptive cooperative output regulation approaches for heterogeneous systems in the presence of large uncertainty is not equally mature: to be more specific, only limited classes of uncertainty have been addressed via adaptive cooperative control, namely unknown (but identical) control directions [19], unknown leader parameters [20], nonlinear systems in output feedback form with unknown (but identical) parameters [21], and nonlinear systems in the parametric strict-feedback form [22].

In this work we address the cooperative output regulation problem for heterogeneous unknown linear systems: we use the term ‘unknown’ in place of ‘uncertain’ to stress that the system matrices are not known a priori and thus possibly subject to large uncertainty, so that fixed-gain distributed control would not be applicable. We use the passification method to solve this problem. The proposed method involves two steps: in the first step, we design a distributed observer of the exogenous signal for those agents that cannot access the exosystem, and we define a reference model whose output can converge to the exogenous signal; in the second step, command generator

The research leading to these results has been partially funded by the European Commission FP7-ICT-2013.3.4, Advanced computing, embedded and control systems, under contract #611538 (LOCAL4GLOBAL).

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tracking is achieved via adaptive laws that make the output of each system converge to the output of the reference model, and thus to the exogenous signal. As a result, the adaptive gains can handle large heterogeneity. Stability analysis is provided via a Lyapunov approach, and a numerical example illustrates the effectiveness of the approach. It is worth mentioning that, differently from distributed adaptive approaches based on model reference adaptive control (MRAC) [23], which require restrictive system matching conditions based on state feedback which may limit their applications [24, Chap. 4], we provide less restrictive matching conditions based on regulator equations for tracking a given class of exogenous signals.

The rest of the paper is organized as follows: in Section II we give the preliminaries for the adaptive command generator tracking (ACGT) approach for a single system; in Section III we formulate the problem and design the distributed observer and distributed controller; a numerical example is provided in Section IV, while Section V concludes the work.

Notation: The notation in this paper is standard. The transpose of a matrix or of a vector is indicated with X' and x' respectively. The trace of a square matrix X is $tr[X]$. A vector signal $x \in \mathbb{R}^n$ is said to belong to $\mathcal{L}_2$ class ($x \in \mathcal{L}_2$), if $\int_0^t \|x(\tau)\|^2 \, \mathrm{d}\tau < \infty$, $\forall t \geq 0$. A vector signal $x \in \mathbb{R}^n$ is said to belong to $\mathcal{L}_\infty$ class ($x \in \mathcal{L}_\infty$), if $\max_{t\geq 0} \|x(t)\| < \infty$, $\forall t \geq 0$. A directed graph is indicated with the pair $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{N}$ is a finite nonempty set of nodes, and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is a set of ordered pair of nodes, called edges. Let $\mathcal{N}_i$ denote the subset of $\mathcal{V}$ which consists of all the neighbors of node i. The adjacency matrix $\mathcal{A} = [a_{ij}]$ of a weighted graph is defined as $a_{ii} = 0$ and $a_{ij} > 0$ if $(j, i) \in \mathcal{E}$, where $i \neq j$.

II. Adaptive command generator tracking with exosystem

The task of adaptive command generator tracking (ACGT) is to make the output of a high-order linear system follow the output of a low-order reference model [25, Chap. 9]. The approach has been proposed as a way to achieve robustness in adaptive control in the presence of unmodelled dynamics [26], [27], [28]. Historically, ACGT has been formulated for constant reference signals [28]. In this section we extend the approach to deal with more general classes of exosystems: such an architecture will be defined as ACGT with exosystem. Consider the high-order system

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned} \tag{1}$$

with $x \in \mathbb{R}^{\overline{n}}$, $u \in \mathbb{R}^p$, $y \in \mathbb{R}^p$, and the matrices A, B, C are unknown matrices of appropriate dimensions. In addition, consider the low-order reference model

$$\begin{aligned} \dot{x}_m &= A_m x_m + B_m r \\ y_m &= C_m x_m \end{aligned} \tag{2}$$

with $x_m \in \mathbb{R}^n$, $r \in \mathbb{R}^p$, $y_m \in \mathbb{R}^p$, and the matrices A_m, B_m, C_m are known matrices of appropriate dimensions. As common in (cooperative) output regulation literature, we assume the reference to be generated by the exogenous system

$$\begin{aligned} \dot{v} &= Sv \\ r &= Rv \end{aligned} \tag{3}$$

with $v \in \mathbb{R}^n$, and S, R also known matrices. In order to have a well-posed problem, the following assumptions, classical in cooperative output regulation literature, are made.

Assumption 1: S has no eigenvalues with negative real part.

Assumption 2: The pairs (A, B) and (A_m, B_m) are stabilizable, and the pairs (C, A) and (C_m, A_m) are detectable.

The control objective is to find an adaptive control law $u(\cdot)$ that, without using the knowledge of the matrices A, B, and C, can guarantee $y - r \to 0$.

To this purpose, define the ideal input

$$u^* = K^* x_m + L^* r + F^*(y^* - y_m) \tag{4}$$

where K^*, L^*, F^* are ideal unknown gains such that $y^* \to r$, being y^* the output of the ideal closed-loop system formed by systems (1) and (2) with ideal input (4). Let us split the tracking in three stages: the first stage is deriving the reference model guaranteeing $y_m \to r$; the second stage are the matching conditions achieving $y^* \to r$, and the third stage is convergence of the closed-loop dynamics $y \to y_m$.

1) Reference model conditions: In order to solve the first stage ($y_m \to r$), a well-posedness assumption is made.

Assumption 3: The reference model (2) is such that the following regulator equations have solution for a matrix Π

$$\begin{aligned} \Pi S &= A_m \Pi + B_m R \\ 0 &= C_m \Pi - R. \end{aligned} \tag{5}$$

It is well known that the regulator equations are necessary and sufficient for tracking [29]: Assumptions 1 and 2 imply that the solution to (5) is well posed, in the sense that after defining $\tilde{x}_m = x_m - \Pi v$, (5) lead to the dynamics

$$\begin{aligned} \dot{\tilde{x}}_m &= A_m \tilde{x}_m \\ y_m - r &= C_m \tilde{x}_m \end{aligned} \tag{6}$$

resulting in $y_m \to r$, provided that A_m is Hurwitz.

2) Matching conditions: We first show that in general it is impossible to attain $y^* \to y_m$ (and thus $y^* \to r$) for arbitrary r: let us define $\overline{x} = [x^* \; x_m]$, being x^* the state of the ideal closed-loop system. The dynamics of $\overline{x}$ can be written as

$$\begin{aligned} \dot{\overline{x}} &= \underbrace{\begin{bmatrix} A + BF^*C & BK^* - BF^*C_m \\ 0 & A_m \end{bmatrix}}_{\overline{A}_{cl}} \overline{x} + \underbrace{\begin{bmatrix} BL^* \\ B_m \end{bmatrix}}_{\overline{B}_{cl}} r \\ y^* - y_m &= \underbrace{\begin{bmatrix} C & -C_m \end{bmatrix}}_{\overline{C}_{cl}} \overline{x} \end{aligned} \tag{7}$$

whose explicit solution is

$$y^*(t) - y_m(t) = \overline{C}_{cl} e^{\overline{A}_{cl} t} \overline{x}(0) + \overline{C}_{cl} \int_0^t e^{\overline{A}_{cl}(t-\tau)} \overline{B}_{cl} r(\tau) d\tau. \tag{8}$$

While the first term in (8) goes to zero if the matrix $\overline{A}_{cl}$ is Hurwitz, the second term cannot go to zero for arbitrary r.

If tracking of y_m is not possible for arbitrary r, it becomes possible for some classes of reference signals: therefore, the exogenous system (3) is used to characterize the class of reference signals for which the objective $y^* \to r$ can be attained. This leads us to a second set of regulator equations

$$\begin{aligned}\overline{\Pi}S &= \begin{bmatrix} A+BF^*C & BK^*-BF^*C_m \\ 0 & A_m \end{bmatrix}\overline{\Pi} + \begin{bmatrix} BL^*R \\ B_mR \end{bmatrix} \\ 0 &= [C \quad 0]\,\overline{\Pi} - R \end{aligned} \tag{9}$$

where $\overline{\Pi}$ can be partitioned as $\overline{\Pi} = [\Pi_0' \ \Pi']'$, with Π being the same as in (5), because the solution to the Sylvester equation in (5) is unique when the spectra of A_m and S are disjoint [29]. We will refer to (9) as 'matching conditions' because they give the conditions for which the ideal output y^* of the system to be controlled matches the reference signal r (and thus the output of the reference model y_m). Note that (9) are based on feedback from the output y and the auxiliary variable x_m, according to the controller (4): however, being tailored to the exogenous system defined by (S, R), (9) are different than matching conditions in MRAC, which can guarantee $y \to y_m$, but not $y \to r$ [24, Chaps. 4 and 5].

3) Closed-loop dynamics: Now that we have all the conditions guaranteeing $y^* \to r$ (for the ideal closed-loop system), we need to study the actual closed-loop dynamics and show $y \to y_m \to r$. To this purpose, let us define $\tilde{x} = x - x^*$, and the actual input

$$u = Kx_m + Lr + F(y - y_m) \tag{10}$$

where K, L, and F are the estimates of K^*, L^*, and F^*. We are now ready to write the actual closed-loop dynamics formed by systems (1) and (2) with the actual input (10)

$$\begin{aligned}\begin{bmatrix} \dot{\tilde{x}} \\ \dot{\tilde{x}}_m \end{bmatrix} &= \underbrace{\begin{bmatrix} A+BF^*C & 0 \\ 0 & A_m \end{bmatrix}}_{\overline{A}}\begin{bmatrix} \tilde{x} \\ \tilde{x}_m \end{bmatrix} \\ &\quad + \begin{bmatrix} B \\ 0 \end{bmatrix}(\tilde{K}x_m + \tilde{L}r + \tilde{F}(y-y_m)) \\ y - y_m &= [C \quad -C_m]\begin{bmatrix} \tilde{x} \\ \tilde{x}_m \end{bmatrix} + \zeta \end{aligned} \tag{11}$$

where $\tilde{K} = K - K^*$, $\tilde{L} = L - L^*$, $\tilde{F} = F - F^*$ and ζ is a term decaying exponentially to zero since $y - y_m = y - y^* + r - y_m + \zeta$, and under Assumption 3 y^* converges to r exponentially. The resulting adaptive laws are

$$\begin{aligned}\dot{K} &= -\gamma(y-y_m)x_m' \\ \dot{L} &= -\gamma(y-y_m)r' \\ \dot{F} &= -\gamma(y-y_m)(y-y_m)' \end{aligned} \tag{12}$$

where $\gamma > 0$ is an adaptive gain. The derivation of (12), as well as stability and convergence analysis of the closed-loop dynamics (11), will be provided in Sect. III. Before concluding this section, some remarks follow.

Remark 1: The equations in (5) depend bilinearly on the matrices A_m, B_m, C_m and Π: the following procedure can be adopted to obtain a suitable reference model (2) satisfying Assumption 3. Select the structure of the reference model (2) in the observable canonical form

$$\begin{aligned}\dot{x}_m &= \begin{bmatrix} 0 & -d_n \\ & \vdots \\ I_{n-1} & -d_1 \end{bmatrix} x_m + \begin{bmatrix} n_n \\ \vdots \\ n_1 \end{bmatrix} r \\ y_m &= [0 \ \cdots \ 0 \ 1]\,x_m \end{aligned} \tag{13}$$

where $d_1, \cdots, d_n$ are selected by the designer in order to obtain a Hurwitz matrix. Then, (5) will depend linearly on the matrices Π and B_m. For example, for the bidimensional exogenous oscillating system

$$\begin{aligned}\dot{v} &= \begin{bmatrix} 0 & 1 \\ -\omega^2 & 0 \end{bmatrix} v \\ r &= [0 \ \ 1]\,v \end{aligned} \tag{14}$$

and the matrix

$$\Pi = \begin{bmatrix} p_1 & p_2 \\ p_3 & p_4 \end{bmatrix} \tag{15}$$

we obtain the solution

$$\begin{aligned} p_1 = -\omega^2 \quad p_2 = 0 \quad p_3 = 0 \quad p_4 = 1 \\ -d_2 + n_2 = -\omega^2 \\ -d_1 + n_1 = 0 \end{aligned} \tag{16}$$

where $d_1 > 0$ and $d_2 > 0$ for Hurwitz conditions, and n_1, n_2 can be solved from (16).

Remark 2: The following consideration should be made on the dimension of the reference model (2): in MRAC the reference model has the same dimension of the system to be controlled, while ACGT literature suggests to take the dimension of the reference model smaller than the dimension of the system, without indicating the exact dimension [30]. With the proposed approach, the dimension of (2) is designated to be the same as the dimension of the exogenous system (3).

Remark 3: The decomposition of the tracking problem $y \to r$ into $y \to y_m \to r$) arises from the architecture of the ACGT, in which the output of the system to be controlled is asked to follow the output of a reference model: as a consequence, the closed-loop dynamics which have been derived involve not only the dynamics of x (or x^*), but also the dynamics of x_m.

III. Adaptive Cooperative Output Regulation

The ACGT with exosystem can be extended to a cooperative output regulation framework. Let us consider a family of unknown heterogeneous N systems in the form

$$\begin{aligned}\dot{x}_i &= A_i x_i + B_i u_i \\ y_i &= C_i x_i, \qquad i \in \{1, \ldots, N\} \end{aligned} \tag{17}$$

with $x_i \in \mathbb{R}^{\overline{n}_i}$, $u_i \in \mathbb{R}^p$, $y_i \in \mathbb{R}^p$, and the matrices A_i, B_i, C_i are unknown matrices of appropriate dimensions. The state of the systems in (17) might be of different dimensions $\overline{n}_i$, i.e. the state dimension can be heterogeneous as well. Let the systems in (17) be connected according to an adjacency matrix $\mathcal{A} = [a_{ij}]$, $i, j \in \{0, \ldots, N\}$. The index 0 is associated to the exosystem (3), therefore $a_{i0} > 0$ if and only if system i can access the reference signal r and the exogenous state v. The following connectivity assumption is made:

Assumption 4: Let $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ be the directed graph associated to $\mathcal{A}$: then, there exists a directed path from node 0 to every system $i \in \{1, \ldots, N\}$ in the network.

We can now provide the problem formulation and its solution.

*Problem 1 (**Adaptive Cooperative Output Regulation**):* Given the unknown heterogeneous systems (17), the exogenous system (3), and the directed graph $\mathcal{G}$, find an adaptive distributed control strategy $u_i(\cdot)$, $i \in \{1, \ldots, N\}$ that, without using the knowledge of A_i, B_i, C_i, $i \in \{1, \ldots, N\}$, guarantees bounded closed-loop signals and

$$\lim_{t\to\infty} y_i(t) - r(t) = 0, \quad i \in \{1, \ldots, N\} \tag{18}$$

Theorem 1: Under Assumptions 1-4, the adaptive cooperative output regulation problem is solved by the following distributed control

$$\begin{aligned} u_i &= K_i x_{m_i} + L_i r_i + F_i(y_i - y_{m_i}) \\ \dot{\eta}_i &= S\eta_i + \mu\left(\sum_{j\in\mathcal{N}_i} a_{ij}(\eta_j - \eta_i) + a_{i0}(v - \eta_j)\right) \\ \dot{x}_{m_i} &= A_m x_{m_i} + B_m r_i \\ y_{m_i} &= C_m x_{m_i} \quad r_i = R\eta_i \\ \dot{K}_i &= -\gamma(y_i - y_{m_i})x'_{m_i} \\ \dot{L}_i &= -\gamma(y_i - y_{m_i})r'_i \\ \dot{F}_i &= -\gamma(y_i - y_{m_i})(y_i - y_{m_i})' \end{aligned} \tag{19}$$

where $\eta_i \in \mathbb{R}^n$ and μ is a design positive number.

Proof: In [3] it is shown that under Assumptions 1 and 4 the distributed observer guarantees that $\eta_i \to v$, $i = 1, \ldots, N$ exponentially. In addition convergence rate can be increased by increasing μ. This implies, with reference to the closed-loop system (11), that the error between r and $r_i = R\eta_i$ has the same effect of the exponentially decaying term ζ. Therefore, stability and convergence analysis of the multi-agent system can be carried out via stability and convergence analysis of the closed-loop dynamics (11). In line with the passification method [15], we assume the system (11) to be strictly positive real (SPR): assuming (11) to be SPR is not more restrictive than the basic passification-based condition of having $(A + BF^*C, B, C)$ SPR [15]. In fact, the block-diagonal structure of $\overline{A}$ in (11) implies that if $(A + BF^*C, B, C)$ is SPR and A_m is Hurwitz, then (11) is SPR. Note that A_m is Hurwitz by design. The SPR condition can be relaxed to the *hyper minimum-phase* (HMP) condition [15], [31], which is necessary and sufficient for output feedback strict passification [32]: such relaxation is not shown here for compactness, but the simulations in Section IV are also carried out for HMP systems. The SPR condition leads to the Kalman-Yakubovich Lemma [15]

$$\begin{aligned} P_1(A + BF^*C) + (A + BF^*C)'P_1 &< 0 \\ P_1 B &= C' \\ P_2 A_m + A'_m P_2 &< 0 \\ P_{12} B &= -C'_m \end{aligned} \tag{20}$$

for a matrix P partitioned as follows

$$P = \begin{bmatrix} P_1 & P'_{12} \\ P_{12} & P_2 \end{bmatrix}. \tag{21}$$

The stability analysis starts from the Lyapunov function

$$\begin{aligned} V(\tilde{x}, \tilde{x}_m, \tilde{K}, \tilde{L}, \tilde{F}) = [\tilde{x}' \ \tilde{x}'_m]\, P \begin{bmatrix} \tilde{x} \\ \tilde{x}_m \end{bmatrix} + tr\left(\tilde{K}\gamma^{-1}\tilde{K}'\right) \\ + tr\left(\tilde{L}\gamma^{-1}\tilde{L}'\right) + tr\left(\tilde{F}\gamma^{-1}\tilde{F}'\right) \end{aligned} \tag{22}$$

with time derivative (time index t is omitted for compactness)

$$\begin{aligned} \dot{V} &= [\tilde{x}' \ \tilde{x}'_m]\left(P\overline{A} + \overline{A}'P\right)\begin{bmatrix} \tilde{x} \\ \tilde{x}_m \end{bmatrix} + \\ &\quad 2\,[\tilde{x}' \ \tilde{x}'_m]\, P \begin{bmatrix} B \\ 0 \end{bmatrix}\left(\tilde{K}x_m + \tilde{L}r + \tilde{F}(y - y_m)\right) + \\ &\quad 2tr\left(\tilde{K}'\gamma^{-1}\dot{\tilde{K}}\right) + 2tr\left(\tilde{L}'\gamma^{-1}\dot{\tilde{L}}\right) + 2tr\left(\tilde{F}'\gamma^{-1}\dot{\tilde{F}}\right) \\ &< 2\left(\begin{bmatrix} B \\ 0 \end{bmatrix}' P \begin{bmatrix} \tilde{x} \\ \tilde{x}_m \end{bmatrix} x'_m + \gamma^{-1}\dot{\tilde{K}}'\right)\tilde{K} + \\ &\quad 2\left(\begin{bmatrix} B \\ 0 \end{bmatrix}' P \begin{bmatrix} \tilde{x} \\ \tilde{x}_m \end{bmatrix} (y - y_m)' + \gamma^{-1}\dot{\tilde{F}}'\right)\tilde{F} + \\ &\quad 2\left(\begin{bmatrix} B \\ 0 \end{bmatrix}' P \begin{bmatrix} \tilde{x} \\ \tilde{x}_m \end{bmatrix} r' + \gamma^{-1}\dot{\tilde{L}}'\right)\tilde{L} < 0 \end{aligned} \tag{23}$$

where we have used the Kalman-Yakubovich Lemma (20), and the property $a'b = tr(b'a)$. Using standard Lyapunov arguments we can prove boundedness of all closed-loop signals and convergence of $\tilde{x}$, $\tilde{x}_m$ to 0. In fact, since $V > 0$ and $\dot{V} \leq 0$, it follows that $V(t)$ has a limit, i.e.,

$$\lim_{t\to\infty} V(\tilde{x}(t), \tilde{x}_m(t), \tilde{K}(t), \tilde{L}(t), \tilde{F}(t)) = V_\infty < \infty \tag{24}$$

and V, $\tilde{x}$, $\tilde{x}_m$, $\tilde{K}$, $\tilde{L}$, $\tilde{F} \in \mathcal{L}_\infty$. In addition, by integrating $\dot{V}$ it follows that for some $Q > 0$

$$\begin{aligned} \int_0^\infty [\tilde{x}'(\tau) \ \tilde{x}'_m(\tau)]\, Q \begin{bmatrix} \tilde{x}(\tau) \\ \tilde{x}_m(\tau) \end{bmatrix} \mathrm{d}\tau \leq \\ V(\tilde{x}(0), \tilde{x}_m(0), \tilde{K}(0), \tilde{L}(0), \tilde{F}(0)) - V_\infty \end{aligned} \tag{25}$$

from which we establish that $\tilde{x}$, $\tilde{x}_m \in \mathcal{L}_2$. Finally, since $\dot{V}$ is uniformly continuous in time (this is satisfied because $\ddot{V}$ is finite), the Barbalat's lemma implies $\dot{V} \to 0$ as $t \to \infty$ and hence $\tilde{x}$, $\tilde{x}_m \to 0$, from which we derive $y \to y_m \to r$. The proof is completed by repeating the analysis above to all systems in (17).

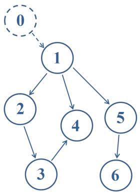
Some remarks follow.

Remark 4: The proposed distributed controller (19) exploits feedback from the output y_i and auxiliary variables that are calculated locally (x_{m_i} and y_{m_i}) or in a distributed way (η_i and r_i): no state feedback is used. In fact, (19) contains: a distributed observer η_i for v; a copy of the reference model driven by η_i; adaptive laws for the control gains.

Remark 5: While in most works on cooperative output regulation for uncertain systems the control gains are fixed and designed through robust control considerations, the last three equations of (19) reveal that in the proposed approach the control gains can vary in order to adapt to the uncertainty *of system i. As a result, one can deal with larger uncertainties than robust approaches.*

Remark 6: *In practice, noises in the feedback loop might drastically change the performance of the proposed adaptive controller: this case is not been addressed due to lack of space. Nevertheless, in view of the decentralized reasoning behind the proof of Theorem 1, the proposed approach inherits all the robustness properties of the passification method as studied in literature [32]. In addition, tools like leakage, dead-zone and projection, widely known in adaptive literature [33], can be added to the adaptive law so as to prevent an undesirable system performance under noisy conditions.*

IV. Numerical example



	d_1	d_2	d_3	n_1	n_2	n_3
#1	1	2	1.5	1	1.5	1.5
#2	0.75	2.5	1.25	0.5	1	0.5
#3	1.25	2	2	1.25	1	1.5
#4	0.5	1	0.25	0.75	0.75	1
#5	0.75	1	0.5	1.5	2	1
#6	1.5	2.5	3	1	0.5	1.5

Fig. 1: The leader-follower directed communication graph

Simulations are carried out on the directed graph shown in Fig. 1, where node 0 acts as the exosystem and node 1 is the only node that can access the exogenous signals. The unknown systems (17) are taken as third-order linear systems in the observable canonical form, in such a way that the transfer function has numerator $n_1 s^2 + n_2 s + n_3$ and denominator $s^3 + d_1 s^2 + d_2 s + d_3$. The coefficients for each system are reported in Table 1: the systems are heterogeneous and have been selected in such a way that the SPR condition is verified. In addition, the systems are unknown to the designer, i.e. the value of their coefficients in Table 1 is not used for control design. The exosystem (3) is taken as a harmonic oscillator

$$\begin{aligned} \dot{v}_1 &= v_2 \\ \dot{v}_2 &= -\omega^2 v_1 \\ r &= v_2 \end{aligned}$$

with $\omega = 0.7$. Therefore, the reference model (2) is selected as a second-order linear system in the observable canonical form (13), with $d_1 = 1$, $d_2 = 0.5$, and $n_1 = d_1$, $n_2 = d_2 - \omega^2$.

The adaptive gain is taken as $\gamma = 10$, and the distributed observer gain is taken as $\mu = 1$: all estimated control gains K, L and F are initialized to 0.

The resulting adaptive cooperative output regulation is shown in Fig. 2. In particular, it can be seen that all outputs y_i converge to the corresponding y_{m_i}: at the same time all r_i are converging to r. Also, note also that all outputs of the reference model y_{m_i} converge to the reference signal r as predicted by the regulator equation of Assumption 3. Finally, Fig. 3 shows the adaptive control gains for all systems: because of the heterogeneity, each system has different control gains.

To show that the SPR condition can be relaxed to HMP, we change the sign to all d_2 in Table 1: this leads to having two unstable poles and thus non-SPR systems: however, the systems are hyper minimum-phase as defined in [15], and thus the passification approach holds true: in fact, convergence of the outputs is shown in Fig. 4 (adaptive gains are not shown for lack of space).

V. Conclusions

This work has addressed the cooperative output regulation problem for heterogeneous unknown linear systems. In contrast with state-of-the-art approaches for heterogeneous 'uncertain' linear systems based on robust control, here the term 'unknown' was used to stress that the agents are possibly subject to larger uncertainty than fixed-gain robust approaches can typically handle. A passification method was used to design a distributed adaptive controller and solve the problem: the main feature of the proposed adaptive controller is that the controller gains are not fixed, but can vary in order to adapt to the system uncertainty. Stability analysis has been provided via a Lyapunov approach, and a numerical example illustrates the effectiveness of the approach. Future work could include: handling switching topologies by using adaptive switching strategies similarly to [34], [35]; removing the assumption that each follower knows the matrix of the exosystem, as in

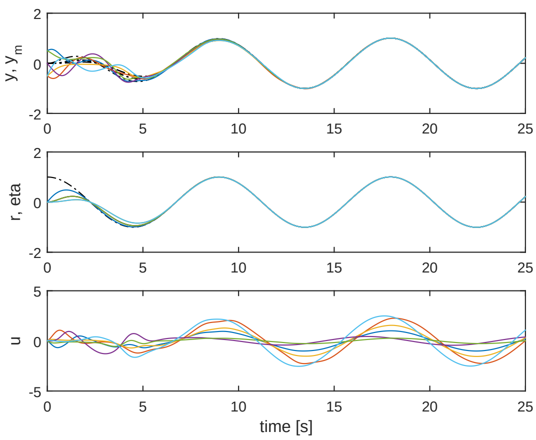


Fig. 2: Outputs, references and inputs for all SPR systems.

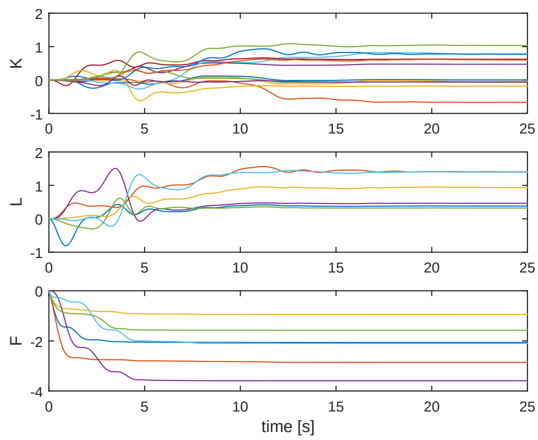


Fig. 3: Adaptive gains for all SPR systems.

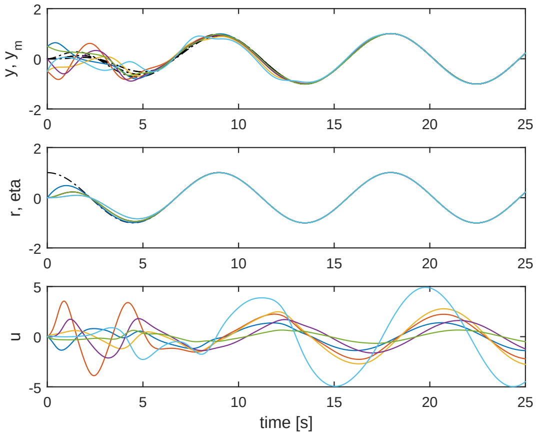


Fig. 4: Outputs, references and inputs for all non-SPR systems.

[4]; extension to nonlinear systems via nonlinear passification methods, similar to [15].

ACKNOWLEDGMENTS

The author is grateful to Profs. B. Andrievsky and A. Fradkov for fruitful discussions on the passification method.

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