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Design formulas for granular filters

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WL | delft hydraulics

Design formulas for granular filters

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1 Introduction

1.1 Reason of the study

WL|Delft Hydraulics has been commissioned by the Road and Hydraulic Engineering Division of Rijkswaterstaat (order AK 31507497 dated January 17, 2000) to derive design formulas for conditions in granular filters consisting of a toplayer on base material
The study has been carried out within the framework of research on filter design formulas, in which respect is referred to earlier studies (Verheij, 1998; Verheij, 1999a; Verheij, 1999b).

The project has been carried out by WL|Delft Hydraulics with technical assistance of Geodelft. Ir. H.J.Verheij and ir. H.A.H.Petit of WL|Delft Hydraulics carried out the study with Mr.Verheij being in charge of the project management; Dr. H. den Adel of GeoDelft provided technical contributions. Dr. G.J.C.M. Hoffmans was the representative of the Road and Hydraulic Engineering Division of Rijkswaterstaat and provided very useful information in order to derive a practical model relation for designing geometrically open filters (Hoffmans, 1996b).

1.2 Objective and approach

Up to now the prediction potential of new design formulas was checked with experimental data. However, the prediction potential was poor, except for those formulas which include the presence of an assumed base term for the shear stress. The base term was introduced in the foregoing studies in analogy with a base term in the formula representing the flow conditions in the filter.
Since measurements point out that a base term is necessary to arrive at an agreement between theory and measurements, the theory without such a base term is not correct. Therefore, it was decided to derive formulas for the flow and the shear stress in which such a base term is incorporated using mathematical methods. Such formulas will be derived for granular filters consisting of a toplayer on base material for laminar as well as turbulent flow conditions in the filter.

Summarizing, the objectives of the study are:

1. To derive shear stress equations for the flow conditions in the filter for laminar as well as turbulent conditions,
2. To develop filter design formulas comparable with the Bakker/Konter-formula,
3. To calibrate the new formulas with available data of small-scale experiments.

The afore-mentioned objectives will be treated in the next chapters. In the last chapter conclusions will be presented.

2 Derivation of shear stress formulas

2.1 General

The situation of a horizontal one-layer filter will be considered: a filterlayer with a thickness d above base material. Above the filter structure water is flowing. At the interface flow-filterlayer the flow velocity is u_s.
Figure 2.1 shows the situation and the various parameters.

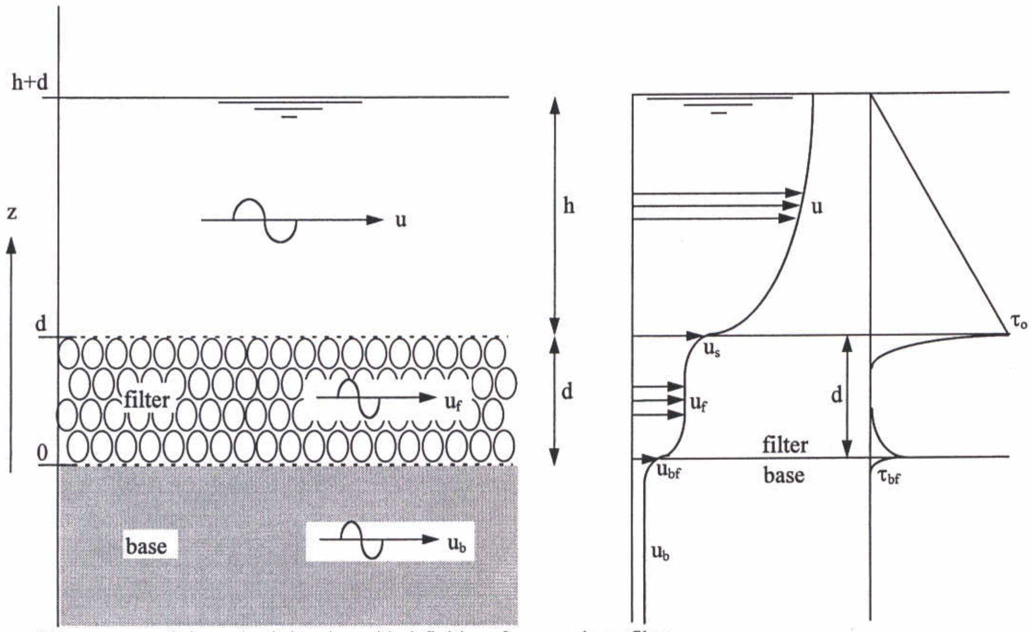


Figure 2.1 Schematized situation with definitions for a one-layer filter

Note: The shear stress in the filter increases more gradually than in the base material. In the filter the length scale equals with D_{f15}, in the base material with D_{b15}. Subsequently, this influences the flow velocity at the interface at z = 0.

2.2 Derivation shear stress equations

The basis of the shear stress derivation are three equations, viz.:

- the Navier-Stokes equation for uniform flow in a filter,
- the Forchheimer equation relating the flow velocity in a poreus medium and the hydraulic gradient, and
- a simple hypothesis relating flow velocity and shear stress.

The Navier-Stokes equation for uniform flow in filters reads:

$$\frac{\partial \tau}{\partial z} + F + \rho g i = 0 \tag{2.1}$$

With the Forchheimer equation $F = -\rho g\left(au + bu^2\right)$ (2.2)
follows:

$$\frac{\partial \tau}{\partial z} - \rho g a u - \rho g b u^2 + \rho g i = 0 \tag{2.3}$$

The shear stress is assumed to be related to the flow velocity by:

$$\tau = \mu \frac{\partial u}{\partial z} \tag{2.4}$$

where μ is the product of density and viscosity.

Substituting results into:

$$\mu \frac{\partial^2 u}{\partial z^2} - \rho g a u - \rho g b u^2 + \rho g i = 0 \tag{2.5}$$

Eq.(2.6) is the starting point for the following derivations.

$$\frac{\partial^2 u}{\partial z^2} - \alpha u - \beta u^2 + \gamma = 0 \tag{2.6}$$

with: $\alpha = \frac{\rho g a}{\mu}, \beta = \frac{\rho g b}{\mu} \textit{ and } \gamma = \frac{\rho g i}{\mu}$

2.3 Laminar flow conditions

Assuming laminar conditions, e.g. $\beta = 0$ and $\mu = \rho \upsilon_w$ (with υ_w = kinematic viscosity) in (2.6), the resulting differential equation reads:

$$\frac{\partial^2 u}{\partial z^2} - \alpha u + \gamma = 0 \tag{2.7}$$

with: $\alpha = \frac{ga}{\upsilon} \textit{ and } \gamma = \frac{gi}{\upsilon}$

This equation can be solved resulting into:

$$u(z) = C_1 e^{-z\sqrt{\alpha}} + C_2 e^{z\sqrt{\alpha}} + \gamma / \alpha \tag{2.8}$$

At the boundaries of the filter layer we have the boundary conditions (assumptions):

$$\text{at } z = 0: \qquad u = u_{bf} \tag{2.9}$$

$$\text{at } z = d: \qquad u = u_s \tag{2.10}$$

From these boundary conditions we find for the integration constants in the general solution the following values:

$$C_1 = \frac{\left(u_{bf} - \frac{\gamma}{\alpha}\right)e^{d\sqrt{\alpha}} - \left(u_s - \frac{\gamma}{\alpha}\right)}{e^{d\sqrt{\alpha}} - e^{-d\sqrt{\alpha}}} \tag{2.11}$$

$$C_2 = \frac{\left(u_s - \frac{\gamma}{\alpha}\right) - \left(u_{bf} - \frac{\gamma}{\alpha}\right)e^{d\sqrt{\alpha}}}{e^{d\sqrt{\alpha}} - e^{-d\sqrt{\alpha}}} \tag{2.12}$$

Finally, the stresses in the filter can be described with:

$$\tau \approx \tau_0 e^{(z-d)\sqrt{\alpha}} + \tau_{base}\, e^{-z\sqrt{\alpha}} \tag{2.13}$$

This equation equals a formula presented by Hoffmans in a fax (Hoffmans, 1999).

At the interface filter-base material the shear stress reads as (by assuming $\lambda = \sqrt{\alpha}$) :

$$\tau(0) = \tau_{base} + \tau_0 e^{-\lambda d} \tag{2.14}$$

Details about the derivation are presented in Appendix A.

2.4 Turbulent flow conditions

Assuming turbulent conditions, e.g. a = 0 in eq.(2.2) and:

$$\tau = \alpha_l \rho u \frac{\partial u}{\partial z} \tag{2.15}$$

the resulting differential equation reads:

$$u\frac{\partial^2 u}{\partial z^2} + \left(\frac{\partial u}{\partial z}\right)^2 - \frac{gb}{\alpha_l}u^2 + \frac{g\,i}{\alpha_l} = 0 \tag{2.16}$$

By substitution $u = \sqrt{v}$ into (2.16) the differential equation reduces to:

$$\frac{\partial^2 v}{\partial z^2} - \frac{2gb}{\alpha_l}v + \frac{2g\,i}{\alpha_l} = 0\,. \tag{2.17}$$

This equation has the general solution:

$$v = C_3 e^{-\xi z} + C_4 e^{\xi z} + \delta \tag{2.18}$$

with $\xi = \sqrt{\frac{2gb}{\alpha_l}}$ and $\delta = \frac{i}{b}$.

The general solution of equation (2.18) can now be given as $u = \sqrt{v}$:

$$u(z) = \sqrt{C_3 e^{-z\xi} + C_4 e^{z\xi} + \delta} \tag{2.19}$$

For C_3 and C_4 similar equations may be presented as eq.(2.11) and (2.12):

$$C_3 = \frac{\left(u_{bf}^2 - \delta\right)e^{\xi d} - \left(u_s^2 - \delta\right)}{e^{\xi d} - e^{-\xi d}} \tag{2.20}$$

and

$$C_4 = \frac{\left(u_s^2 - \delta\right) - \left(u_{bf}^2 - \delta\right)e^{-\xi d}}{e^{\xi d} - e^{-\xi d}} \tag{2.21}$$

Finally, at the interface filter-base material the shear stress reads as:

$$\tau \approx \tau_0 e^{\xi(z-d)} + \tau_{base} e^{-\xi z} \tag{2.22}$$

$$\tau(0) = \tau_{base} + \tau_0 e^{-\xi d} \tag{2.23}$$

Reference is made to Appendix A for details on the derivation.

2.5 Value of damping parameters λ and ξ

The coefficients a and b in the Forchheimer equation are defined as:

$$a = \frac{c_0 \upsilon_w (1-n)^2}{n^3 g D_{f15}^2} \quad with \quad c_0 = 160 \tag{2.24}$$

$$b = \frac{c_7}{n^2 g D_{f15}} \quad with \quad c_7 = 2.2$$

This results for the term λd in case of laminar flow into:

$$\lambda d = d\sqrt{\alpha} = \frac{d}{D_{f15}}\sqrt{\frac{c_0(1-n)^2}{n^3}} \tag{2.25}$$

With $c_0 = 160$ and $n = 0.4$:

$$\lambda d = 30\frac{d}{D_{f15}} \tag{2.26}$$

For turbulent flow conditions, where we defined:

$$\xi = \sqrt{\frac{2bg}{\alpha_l}}$$

the result is (see also Verheij, 1998):

$$\xi d = \sqrt{\frac{2d^2}{\alpha_l}\frac{c_7}{n^2 D_{f15}}} \tag{2.27}$$

Den Adel (1995) relates μ in eq.(2.4) to u according: $\mu = \alpha_f D_{f15}\rho u$ (in analogy with Shimizu et al, 1990).
Comparing this with the applied relation eq.(2.15)): $\mu = \alpha_l \rho u$
it may be concluded that: $\alpha_l = \alpha_f D_{f15}$

Substituting $\alpha_f D_f$ in eq.(2.27) instead of α_l results in:

$$\xi d = \frac{d}{D_{f15}}\sqrt{\frac{2c_7}{\alpha_f n^2}} \tag{2.28}$$

Den Adel also estimated a value of *0.9* for α_f. Substituting this value into the equation for λd together with $c_7 = 2.2$ and $n = 0.4$ gives:

$$\xi d = 5.5\frac{d}{D_{f15}} \tag{2.29}$$

3 Derivation of filter design formulas

Applying the shear stress equations derived in Chapter 2, filter design formulas for laminar and turbulent flow conditions in a granular filter may be derived. At first, a design formula with average load and strength conditions will be discussed. Secondly, formulas are derived on the basis of the Hoffmans/Grass approach which takes into account statistical distributions of load and strength.
The details are presented in Appendix B, while in Figures 3.1 and 3.2 additional information is presented.

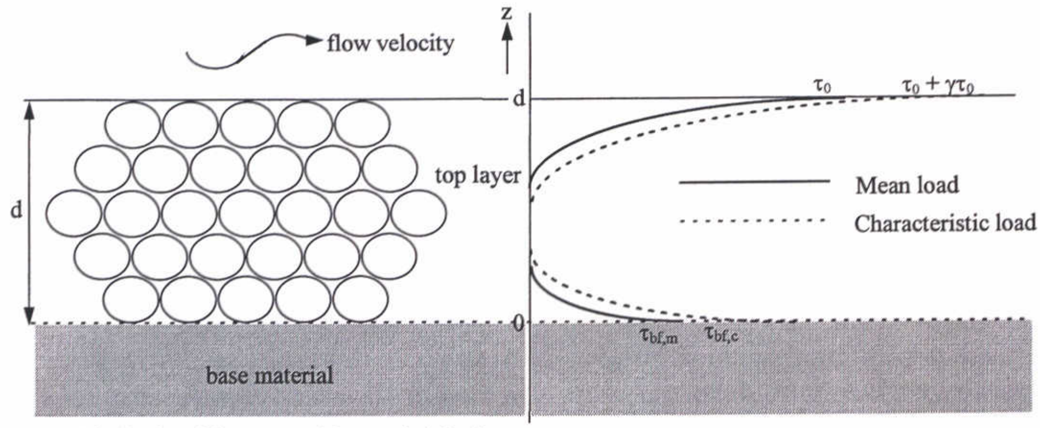


Figure 3.1 Distribution of the mean and characteristic load

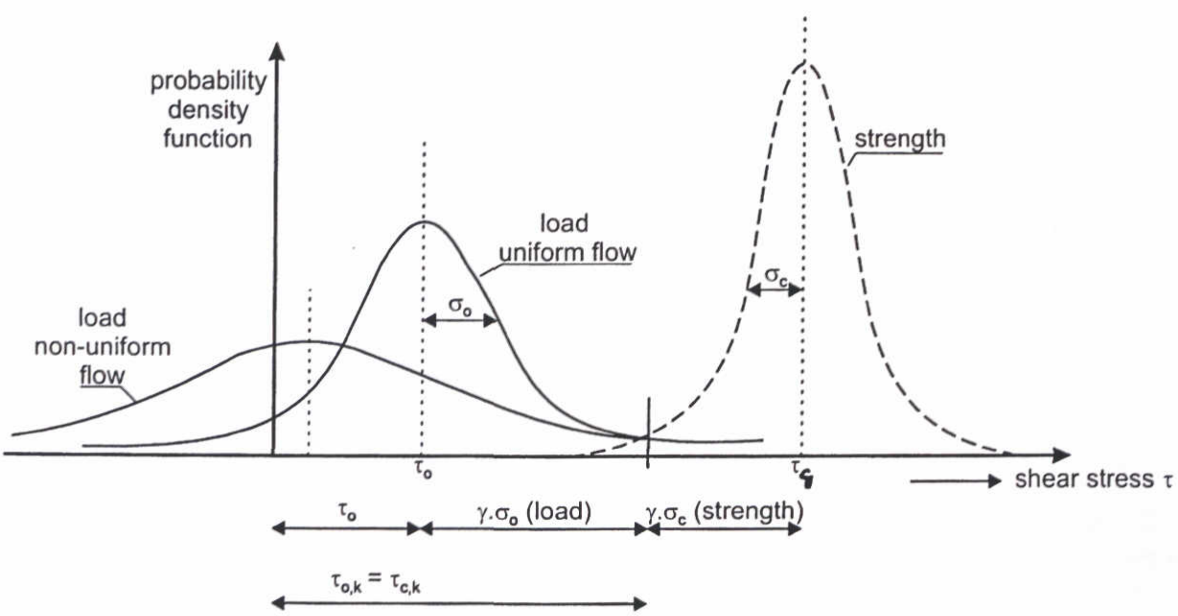


Figure 3.2 Probability functions of the loading and strength parameters

3.1 Average load and strength conditions

Laminar flow conditions

Equilibrium of load and strength at the interface between filter and base material reads as:

$$\tau_{load} = \tau_{strength} \tag{3.1}$$

With the shear stress (see eq.(2.14)):

$$\tau_{load} = \tau_{base} + \tau_0 e^{-\lambda d} \tag{3.2}$$

and the strength of the base material

$$\tau_{strength} = \tau_{c,G} = \psi_{c,G,b} \Delta_b \rho g D_{b50} \tag{3.3}$$

and assuming $\tau_{base} = \eta \tau_0$ (3.4)

this results in:

$$\frac{D_{f15}}{D_{b50}} = \frac{D_{f15}}{D_{f50}} \frac{1}{\eta + e^{-\lambda d}} \frac{\psi_{c,G,b} \Delta_b}{\psi_{c,G,f} \Delta_f} \tag{3.5}$$

For large values of λd this changes into:

$$\frac{D_{f15}}{D_{b50}} = \frac{D_{f15}}{D_{f50}} \frac{1}{\eta} \frac{\psi_{c,G,b} \Delta_b}{\psi_{c,G,f} \Delta_f} \tag{3.6}$$

The value of η will be determined in Chapter 5.

turbulent flow conditions

The resulting equation is the same as for laminar conditions, because the structure of the formula for the shear stress is identical (not the values of λd and ξd):

$$\frac{D_{f15}}{D_{b50}} = \frac{D_{f15}}{D_{f50}} \frac{1}{\eta + e^{-\xi d}} \frac{\psi_{c,G,b} \Delta_b}{\psi_{c,G,f} \Delta_f} \tag{3.7}$$

which for large values of ξd this changes into:

$$\frac{D_{f15}}{D_{b50}} = \frac{D_{f15}}{D_{f50}} \frac{1}{\eta} \frac{\psi_{c,G,b} \Delta_b}{\psi_{c,G,f} \Delta_f} \qquad (3.6)$$

3.2 Characteristic load and strength conditions

Laminar flow conditions

Similar to the preceeding derivation, however applying Grass' concept, so replacing τ_0 by $\tau_0 + \gamma_t\,\sigma_0$ (see Figure 3.1) and substituting τ_{load} for $\tau(0)$ eq.(2.14) can be written as:

$$\tau_{load} = (\tau_0 + \gamma_t \sigma_0)(\eta + e^{-\lambda d}) \tag{3.8}$$

Note 1: $\tau_{base} = \eta(\tau_0 + \gamma_t \sigma_0)$

Note 2: The value of σ_0 is not equal to 0, because spatial fluctuations in the load are still present, but the value will be less than the value for turbulent flow because of the lack of turbulence.

The strength of the base material may be written as:

$$\tau_{strength} = \tau_{c,G} - \gamma_b \sigma_{c,b} = \tau_{c,G} - \gamma_b \alpha_{c,b} \tau_{c,G} = \tau_{c,G}(1 - \gamma_b \alpha_{c,b}) = \psi_{c,G,b} \Delta_b \rho g D_{b50}(1 - \gamma_b \alpha_{c,b}) \tag{3.9}$$

Substituting (3.8) and (3.9) into (3.1) and re-arranging results into:

$$\frac{D_{t50}}{D_{b50}} = \frac{1}{\eta + e^{-\lambda d}} \frac{1 - \gamma_b \alpha_{c,b}}{1 - \gamma_t \alpha_{c,t}} \frac{\psi_{c,G,b} \Delta_b}{\psi_{c,G,t} \Delta_t} \tag{3.10}$$

This equation is a general formula with the implicit assumptions of Gaussian distributions of load and strength, and a relationship based on uniform flow.

Assuming $\alpha_{c,b} = \alpha_{c,t}$ and $\gamma_b = \gamma_t$ and multiplying both sides with D_{t15}/D_{t50} results in:

$$\frac{D_{t15}}{D_{b50}} = \frac{D_{t15}}{D_{t50}} \frac{1}{\eta + e^{-\lambda d}} \frac{\psi_{c,G,b} \Delta_b}{\psi_{c,G,t} \Delta_t} \tag{3.11}$$

which changes for a one layer system into:

$$\frac{D_{f15}}{D_{b50}} = \frac{D_{f15}}{D_{f50}} \frac{1}{\eta + e^{-\lambda d}} \frac{\psi_{c,G,b} \Delta_b}{\psi_{c,G,f} \Delta_f} \tag{3.12}$$

For large values of λd the same equation results as for laminar flow conditions and average load conditions:

$$\frac{D_{f15}}{D_{b50}} = \frac{D_{f15}}{D_{f50}} \frac{1}{\eta} \frac{\psi_{c,G,b} \Delta_b}{\psi_{c,G,f} \Delta_f} \tag{3.6}$$

Note that this expression does not depend on neither α_c nor γ and so this relation is independent of the gradation of either toplayer or base material as long as these gradations and the damage levels γ are the same or nearly the same.

turbulent flow conditions

In analogy with the result for laminar conditions also for turbulent conditions the same equation results (except ξd in stead of λd):

$$\frac{D_{f15}}{D_{b50}} = \frac{D_{f15}}{D_{f50}} \frac{1}{\eta + e^{-\xi d}} \frac{\psi_{c,G,b}\Delta_b}{\psi_{c,G,f}\Delta_f} \qquad (3.13)$$

For large values of ξd results:

$$\frac{D_{f15}}{D_{b50}} = \frac{D_{f15}}{D_{f50}} \frac{1}{\eta} \frac{\psi_{c,G,b}\Delta_b}{\psi_{c,G,f}\Delta_f} \qquad (3.6)$$

These results are quite remarkable:

1. There is no influence in the ratio D_{f15}/D_{b50} for either type of flow
2. The assumption: simultaneous instability of both filter and base material leads to a ratio D_{f15}/D_{b50} which depends on the fluctuations in strength only.
3. Although characteristic values for both load and strength are introduced, the resulting ratio D_{f15}/D_{b50} is under certain conditions independent of the fluctuations in load and strength. The reason is that instability of base and filter material has been assumed at the same moment as well as equal values of the erosion level γ and the ratios of α between standard deviation and mean value.

4 Discussion of results

The following aspects will be discussed in this chapter:

- applicability of the results
- comparison with other filter formulas
- value of u_s (z = d)
- value of u(z = 0)
- influence gradation

applicability of the results

The parameters δd and λd can be expressed into the material parameters of the filter. The values are equal to respectively about 5.5 and 30.d/D_{f15} (see Section 2.5), which means that for conditions in the filter in between laminar flow and turbulent flow, the value of the exponent is somewhere in the range of (5.5 to 30)*d/D_{f15}

The foregoing means that even for a filter that consists of only one layer of grains, the influence of the open water flow is absolutely negligible. The derived formulas predict an influence of the shear stress on a scale of D_{f15}/30 or D_{f15}/5.5. Such a length scale is much smaller than the filter grains and so the continuum assumptions as in Forchheimers equation are no longer valid.

As a conclusion, the theory predicts that the shear stress has little penetration depth. The consequence is that the derived equations are valid for thin as well as thick filters. Therefore, the flow in a granular bed protection is uniform. It may be considered as being driven by a constant pressure gradient.

Nevertheless, mathematically the obtained solution is correct, but not fysically with respect to the shear stresses near the interfaces. However, the shear stresses are only a tool to derive a filter design formula to predict a stable filter.
The resulting formula, for, instance (3.6):

$$\frac{D_{f15}}{D_{b50}} = \frac{D_{f15}}{D_{f50}} \frac{1}{\eta} \frac{\psi_{c,G,b}\Delta_b}{\psi_{c,G,t}\Delta_t} \qquad (3.6)$$

can be written as (assuming $\psi_{c,G,b} = \psi_{c,G,f}$ and $\Delta_b = \Delta_f$):

$$\frac{D_{f15}}{D_{b50}} = \frac{1}{\eta} \frac{D_{f15}}{D_{f50}} \qquad (4.1)$$

or

$$\frac{D_{f50}}{D_{b50}} = \frac{1}{\eta} \qquad (4.2)$$

The ressemblance with traditional design formulas with constant ratios for D_{f50}/D_{b50}, for instance:

$$\frac{D_{f50}}{D_{b50}} \leq 5\ to\ 10 \tag{4.3}$$

is large. Comparing (4.1) and (4.2) results in $\eta = 0.10$ to 0.20.
In Chapter 5 the value of η will be calibrated resulting in $\eta = 0.01$, which is much smaller than a value of 5 to 10. However, the reason for this difference is that eq.(4.3) holds for geometrical sandtight filters which are overdimensioned with respect to the hydraulic load at the interface, while eq.(4.2) is derived applying instantaneous instability of filter and base material and implicitly takes into account the lower hydraulic load at the interface filter/base material.

Summarizing: the final result is according to what may be expected, although the way to derive this result is not quite correct.

The obtained results for a one layer system consisting of a filter layer on base material (see Figure 2.1) may be applied also to a filter structure consisting of two filter layers. For laminar flow conditions the set of equations can be solved mathematically, however, an analytical solution for turbulent flow conditions is not possible (of course, numerically a solution can be obtained).

Finally, it should be noted that the parameters C_1 and C_3 influence the behaviour of the flow near the interface filter-base material, while the parameters C_2 and C_4 influence the behaviour of the flow near the interface flow-filter.

Comparison with other filter design formulas

Earlier the so-called Bakker/Konter-formula, B/K-formula for short, has been derived (Bakker et al, 1994):

$$\frac{D_{f15}}{D_{b50}} = \frac{2.2}{e^2 C_0} \frac{\psi_b \Delta_b}{\psi_t \Delta_t} \frac{R}{D_{t50}} \tag{4.4}$$

with

$$C_0 = f\left(\frac{d}{D_{t50}}, \frac{R}{D_{t50}}\right) \tag{4.5}$$

The B/K-formula depends on the water depth h (or R), and subsequently, is also related to the large eddies in the flow. In the present study it is assumed:

$$\tau_{base} = \eta \tau_0 \tag{4.6}$$

which, in principle, assumes also a dependency with the large eddies in the flow with a length scale equal to the water depth h or the hydraulic radius R.

Eq.(4.4) can also be written as (assuming again $\psi_b = \psi_f$ and $\Delta_b = \Delta_f$):

$$\frac{D_{f15}}{D_{b50}} = \frac{2.2}{e^2 C_0} \frac{R}{D_{f50}} = \frac{2.2}{e^2 C_0} \frac{R}{D_{f15}} \frac{D_{f15}}{D_{f50}} \tag{4.7}$$

Furthermore, a formula of Stephenson (1979) exists:

$$\frac{D_f}{D_b} = 8\frac{\psi_b \Delta_b}{\psi_f \Delta_f}\frac{R}{D_f} \approx 8\frac{R}{D_f} \tag{4.8}$$

which can be written as:

$$\frac{D_{f15}}{D_{b50}} = 8\frac{R}{D_{f50}}\frac{D_{f15}}{D_{f50}} \tag{4.9}$$

The structure of the resulting equations (4.7) and (4.9) is similar as eq.(4.1), although in (4.7) and (4.9) the ratio depends on R/D_{f50}.

Comparing the eq.(4.7) and (4.9) with eq.(4.1) the results for η is:

$$\text{Bakker/Konter: } \eta = \frac{e^2 C_0}{2.2}\frac{D_{f15}}{R} = \frac{e^2 C_0}{2.2}\frac{D_{f15}}{D_{f50}}\frac{D_{f50}}{R} \tag{4.10}$$

$$\text{Stephenson: } \eta = \frac{1}{8}\frac{D_{f50}}{R} \tag{4.11}$$

Also Hoffmans (1996a; see also Van Os, 1998) presented implicitly a relation for η:

$$\tau_{base} = \tau_0 \frac{\beta}{c_t}\frac{D_{f50}}{h} \tag{4.12}$$

or:

$$\eta = \frac{\beta}{c_t}\frac{D_{f50}}{h} = \frac{\beta}{c_t}\frac{D_{f50}}{R} \tag{4.13}$$

Summarizing, we have various formulas for the parameter η. In Chapter 5 figures will be presented and a conclusion will be drawn with respect to the value of η.

<u>value of u_s</u>

The shear stress at the interface flow/filter layer depends on the flow velocity u_s:

$$\tau_0 = \rho \upsilon_w \sqrt{\alpha}\left(u_s - u_f\right) \tag{4.14}$$

or

$$u_s = \frac{\tau_0}{\rho \upsilon_w \sqrt{\alpha}} + u_f \tag{4.15}$$

In the flow the relationship holds:

$$\tau_0 = \rho u_*^2 \tag{4.16}$$

Substituting this result into the equation for u_s:

$$u_s = \frac{\rho u_*^2}{\rho \upsilon_w \sqrt{\alpha}} + u_f \qquad (4.17)$$

With $u_f = i/a_f$ and $u_*^2 = ghi$ and substituting a value for $\sqrt{\alpha}$ (see eq(2.24)) it is also possible to change the equation for u_s into:

$$u_s = u_*^2 \left(\frac{1}{\sqrt{g \upsilon_w a}} + \frac{1}{gha} \right) \qquad (4.18)$$

This means that the flow velocity at the interface flow-filter layer is larger than u_*. This is according our expectations, because at a permeable bed the flow velocity is higher than at an impermeable bed.

The above equations are only valid for laminar flow conditions, however, the same holds for turbulent flow.

Hauer (1996) presents a relationship between u_s and u_* based on Japanese test results and theoretical considerations:

$$u_s = (1.5 \text{ to } 5)\, u_* \qquad (4.19)$$

The first value has been derived by Hoffmans (1996b) for Japanese tests with glass beads.

Conclusion: The presence of the term ρgi is essential, because it is responsible for the average flow, not only in the water above the filter but also inside the filter. It creates an extra driving force which is independent of the depth.

Nevertheless, the value of u_s is an average value based on continuum principles. In reality, the interface flow/filter layer consists of grains and open areas in between. Locally, in the open areas the flow velocity will be higher than u_s. As a result the value of u(0) will be locally much higher than the predicted value as derived from continuum equations.

value of u(0)

At the interface filter-base material the flow velocity is assumed to be equal to:

$$u(z=0) = u_{bf} \qquad (4.20)$$

Different formulas may be assumed for u_{bf}, for instance:

$$u(z=0) = \frac{1}{2}\left(u_f + u_b\right) \qquad (4.21)$$

However, this assumption is not quite true. In stead of this assumption the following assumption will be used:

$$u(z=0) = u_b + \Delta u \qquad (4.22)$$

with Δu = part of the difference between u_f and u_b.

Assuming the flow reduces over a distance equal to $(½D_{f50} + ½D_{b50})$ the following value for Δu can be derived:

$$\Delta u = \left(u_f - u_b\right)\frac{D_{b50}}{D_{b50} + D_{f50}} \tag{4.23}$$

Subsequently resulting in:

$$u\left(z=0\right) = u_b + \left(u_f - u_b\right)\frac{D_{b50}}{D_{b50} + D_{f50}} \tag{4.24}$$

The value of $(u_f - u_b)$ depends on the permeability ratio of filter and base material, and thus on the particle diameters. Smaller particles result in lesser permeability and thus a smaller flow velocity. The term $(u_f - u_b)$ can be described by:

$$u_f - u_b = u_f\left(1 - \frac{D_{b15}^2}{D_{f15}^2}\right) \tag{4.25}$$

Furthermore, the value of u_b may also be expressed as a function of u_f and material diameters:

$$u_b = u_f\frac{D_{b15}^2}{D_{f15}^2} \tag{4.26}$$

Substituting (4.25) and (4.26) into (4.24) finally results into an equation which maybe used instead of the assumption that the flow velocity at the interface is equal to u_{bf}:

$$u\left(z=0\right) = u_f\frac{D_{b15}^2}{D_{f15}^2} + u_f\left(1 - \frac{D_{b15}^2}{D_{f15}^2}\right)\frac{D_{b50}}{D_{b50} + D_{f50}} \tag{4.27}$$

After some rearranging, and assuming $D_{b15}/D_{f15} = D_{b50}/D_{f50}$, the following equation results:

$$u\left(z=0\right) = \sqrt{u_f u_b} \tag{4.28}$$

Eq.(4.28) may also be derived analytically. However, the derivation is not presented here, because it is considered to be out of the scope of this study.

influence of gradation

Eq.(3.10):

$$\frac{D_{t50}}{D_{b50}} = \frac{1}{\eta + e^{-\lambda d}}\,\frac{1-\gamma_b\alpha_{c,b}}{1-\gamma_t\alpha_{c,t}}\,\frac{\psi_{c,G,b}\Delta_b}{\psi_{c,G,t}\Delta_t} \tag{3.10}$$

is a tool to predict the maximally allowed ratio D_f/D_b as a function of the gradation of filter and base material. This can be explained as follows:

- If the base material is broadly graded the value of $\alpha_{c,b}$ is larger than $\alpha_{c,f}$. As a result the required D_f/D_b is lower than for base and filter materials which are comparable in gradation.
- If only the filter material is broadly graded, the value of $\alpha_{c,f}$ is larger than $\alpha_{c,b}$, so the maximum value of D_f/D_b is higher than for similar graded materials.

These predictions are in agreement with what we expect. A broadly graded base material has more fines than a more uniform base material. The filter must prevent erosion of the fines as well, by reducing the water velocity. This can only be achieved by more fines in the filter, so on average smaller particles: D_f/D_b is lower than for uniform materials. A broadly graded filter material has a lot of fines, which reduce the water velocity in the filter and so the load on the base material. As a result the broadly graded filter material is allowed to have an average grain size which is larger than for a uniform material.

5 Calibration

The value of η will be calibrated with experimental results mentioned in Van Huijstee et al (1991) as far as they concern simultaneous instability of filter and base material. In Table 5.1 the experimental results are mentioned.

Measured D_{f15}/D_{b50}	D_{f15}/D_{f50}	ψ_b	ψ_f	η
132	0.667	0.047	0.035	0.0068
132	0.667	0.047	0.035	0.0068
99	0.75	0.047	0.035	0.0102
53	0.75	0.028	0.035	0.0235
53	0.75	0.028	0.035	0.0235
99	0.75	0.047	0.035	0.0102
40	0.602	0.022	0.035	0.0095
150	0.618	0.043	0.035	0.0051
214	0.789	0.043	0.035	0.0045

Table 5.1

In Table 5.1 also the value of η is presented which is calculated with:

$$\eta = \frac{D_{f15} / D_{f50}}{D_{f15} / D_{b50}} \frac{\psi_{c,G,b}\Delta_b}{\psi_{c,G,t}\Delta_t} \tag{5.1}$$

This equation is derived from eq.(3.6).
From Table 5.1 can be calculated that the average value for η is:

$$\eta = 0.011$$

however, the range is large (η between 0.005 and 0.024).

Applying (5.1) implies that the exponential term $e^{-\lambda d}$ or $e^{-\xi d}$ is ignored. This is possible beacuse the values are very small compared to the value of η which can be shown as follows.
Suppose $d/D_{f15} = 3$ ($d/D_{f50} = 2$ and $D_{f50}/D_{f15} = 1.5$) then $\xi d = 16.5$ and $\lambda d = 90$ and, subsequently:

$$e^{-\lambda d} = 6.8.10^{-8} \text{ and } e^{-\xi d} = 8.2.\ 10^{-40}$$

Both values are much smaller than the calibrated value of $\eta = 0.011$. Thus, neglecting the exponentional term is allowed.

The value of η is defined with:

$$\tau_{base} = \eta\ \tau_0 \tag{5.2}$$

The base term represents the quasi-steady groundwater flow driven by the flow above the toplayer. Instantaneous fluctuations are no longer present. With eq.(A.9) and (A.11) the ratio τ_{base}/τ_0 can be derived:

$$\frac{\tau_{base}}{\tau_0} = \frac{\sqrt{u_f u_b}}{u_s - u_f} \tag{5.3}$$

Assuming $u_s = O(10^{-1})$, $u_f = O(10^{-2})$ and $u_b = O(10^{-3})$ a ratio can be determined of:

$$\frac{\tau_{base}}{\tau_0} \approx 0.03$$

Similarly, this can be done for turbulent flow conditions with the eq.(A.29) and (A.31), resulting in:

$$\frac{\tau_{base}}{\tau_0} = \frac{u_f^2}{0.5\,u_s^2} \approx 0.02 \tag{5.4}$$

Summarizing, the order of magnitude is what we expect.

We may compare the result for η also with the values which can be calculated with the other design formulas presented in Chapter 4.
Firstly, the Bakker/Konter-formula: substituting C_0 = 15 (Bakker et al, 1995), e = 0.24 and assuming D_{f15}/D_{f50} = 0.67 and D_{f50}/R = 10 to 30, the result is:

$$\eta = 0.26\frac{D_{f50}}{R} = 0.009 \; to \; 0.026$$

The result for the Stephenson formula reads:

$$\eta = 0.125\frac{D_{f50}}{R} = 0.004 \; to \; 0.013$$

Finally, the Hoffmans formula with $\beta = 0.02$ (bases on Shimizu's tests) and $c_t = 0.08$ (based on Forchheimer) reads (Hoffmans, 1996b):

$$\eta = 0.25\frac{D_{f50}}{R} = 0.008 \; to \; 0.025$$

With respect to the Bakker/Konter-formula the result was expected, because the underlying data set is the same. However, with respect to the Stephenson formula and Hoffmans formulas the datasets are different. Hoffmans, for instance, uses the dataset of Shimizu.
In addition to the Hoffmans formula is should be noted that experiments carried out by Verheij (1997) with values for D_{f50} and R of 15.2 mm and 0.35 m respectively, results into $\eta = 0.011$.

Conclusion: An average value of η equal to about η = 0.01 seems to be correct. However, it is recommended to validate the value of η with other experimental results.

6 Conclusions

The desk study allows the following conclusions:

- Formulas for the shear stress at the interface filter-base material including a base term can be derived:

average load condition: $\tau(0) = \tau_{base} + \tau_0 e^{-\lambda d}$ or $\tau(0) = \tau_{base} + \tau_0 e^{-\xi d}$

$$\text{with } \tau_{base} = \eta \tau_0$$

characteristic load condition:

$$\tau(0) = \tau_{base} + (\tau_0 + \gamma_t \sigma_0) e^{-\lambda d} \text{ or } \tau(0) = \tau_{base} + (\tau_0 + \gamma_t \sigma_0) e^{-\xi d}$$

$$\text{with } \tau_{base} = \eta(\tau_0 + \gamma_t \sigma_0)$$

The formulas are valid for laminar and turbulent flow conditions for thin and thick filters. The values of ξ and λ are different.

- For average load and strength conditions the filter design formula reads:

$$\frac{D_{f15}}{D_{b50}} = \frac{D_{f15}}{D_{f50}} \frac{1}{\eta} \frac{\psi_{c,G,b} \Delta_b}{\psi_{c,G,f} \Delta_f}$$

The parameter η is a damping parameter and related to the shear stress at the interface of flow and filter layer.

For characteristic load and strength conditions the formula is:

$$\frac{D_{f15}}{D_{b50}} = \frac{D_{f15}}{D_{f50}} \frac{1}{\eta + e^{-\lambda d}} \frac{1 - \gamma_b \alpha_{c,b}}{1 - \gamma_f \alpha_{c,f}} \frac{\psi_{c,G,b} \Delta_b}{\psi_{c,G,f} \Delta_f}$$

These relations are independent on the type of flow; they hold for both laminar and turbulent flow.

- The value of η has been calibrated with experimental results which resulted into an average value of:

$\eta = 0.01$

This value is within the expected order of magnitude and ressembles very well with results of other formulas based on other datasets.
However, it is recommended to validate the value of η.

- The obtained results are quite remarkable:
    1. There is no influence in the ratio D_{f15}/D_{b50} for either type of flow
    2. The assumption: simultaneous instability of both filter and base material leads to a ratio D_{f15}/D_{b50} which depends on the fluctuations in strength only.
    3. Although characteristic values for both load and strength are introduced, the resulting ratio D_{f15}/D_{b50} is under certain conditions independent of the fluctuations in load and strength. The reason is that instability of base and filter material has been assumed at the same moment as well as equal values of the erosion level γ and the ratios of α between standard deviation and mean value.

References

Adel, H. den, et al (1995): *Implementatie filteronderzoek*, Grondmechanica Delft, rapport CO349410/16, Delft

Bakker, K.J. et al (1994): *Design relationship for filters in bed protection*, Journal of Hydraulic Engeneering, ASCE Vol.120, no 9.

Hauer, M. and T.van der Meulen (1996): Parallel stroming in een geometrisch open filterconstructie. TU Delft

Hoffmans, G.J.C.M. (1996a): *Analyse Bakker/Konter-formule* Rijkswaterstaat, Dienst Weg- en Waterbouwkunde, memo, Delft.

Hoffmans, G.J.C.M. (1996b): *Geometrisch open filters onder invloed van stroming en turbulentie.* Rijkswaterstaat, Dienst Weg- en Waterbouwkunde, bureaustudie, Delft.

Hoffmans, G.J.C.M. (1999): *Commentaar op rapport Q2552 A design formula for laminar flow conditions in granular filters.* Rijkswaterstaat, Dienst Weg- en Waterbouwkunde, fax d.d. 20 september 1999, Delft.

Huijstee, J.J.A. van, en H.J.Verheij (1991): *Verruiming ontwerpregels voor filters in bodemverdedigingen, gelijktijdige instabiliteit van toplaag en filterlaag.* Waterloopkundig Laboratorium , verslag modelonderzoek Q572, Delft.

Os, P.van (1998): *Hydraulische belasting op een geometrisch open filterconstructie*, Technische Universiteit afstudeerrapport, Delft.

Shimizu, Y., T.Tsujimoto en H. Nakagawa (1990): *Experiment and macroscopic modelling of flow in highly permeable porous medium under free surface flow.* Journal of Hydroscience and Hydraulic Engineering 1990 (8) No.1 69-78.

Stephenson, D. (1979): *Rockfill in hydraulic engineering*. Elsevier Scientific Publishing Company.

Verheij, H.J. (1997): *Stroming in granulaire filters, orienterende experimenten.* Waterloopkundig Laboratorium, meetverslag Q2331, Delft.

Verheij, H.J. (1998): *Evaluation of recent desk study results on granular filters*. Delft Hydraulics, desk study Q2416, May 1998, Delft

Verheij, H.J. (1999a): *Analysis and prediction potential of new design formulas for granular filters.* Delft Hydraulics, desk study Q2506, March 1999, Delft

Verheij, H.J. (1999b): *A design formula for laminar flow conditions in granular filters.* Delft Hydraulics, desk study Q2552, November 1999, Delft

A Derivation of shear stress formulas

1 Laminar flow conditions

In the absence of turbulence the stationary flow equation in the filter is given by:

$$\rho \nu \frac{\partial^2 u}{\partial z^2} - \rho g a u + \rho g i = 0 \tag{A.1}$$

which can be simplified to

$$\frac{\partial^2 u}{\partial z^2} - \alpha u + \gamma = 0 \tag{A.2}$$

where $\alpha = ga/\upsilon$ and $\gamma = g i/\upsilon$.

This differential equation has a general solution:

$$u(z) = C_1 e^{-z\sqrt{\alpha}} + C_2 e^{z\sqrt{\alpha}} + \gamma/\alpha \tag{A.3}$$

At the boundaries of the filter layer we have the boundary conditions (assumptions):

$$\text{at } z = 0: \qquad u = u_{bf} \tag{A.4}$$

$$\text{at } z = d: \qquad u = u_s \tag{A.5}$$

We can thus write the boundary conditions as:

$$C_1 + C_2 + \gamma/\alpha = u_{bf} \tag{A.6}$$

and

$$C_1 e^{-d\sqrt{\alpha}} + C_2 e^{d\sqrt{\alpha}} + \gamma/\alpha = u_s \tag{A.7}$$

From these boundary conditions we find for the integration constants in the general solution the following values:

$$C_1 = \frac{u_{bf} e^{d\sqrt{\alpha}} - u_s - \frac{\gamma}{\alpha}\left(e^{d\sqrt{\alpha}} - 1\right)}{e^{d\sqrt{\alpha}} - e^{-d\sqrt{\alpha}}} = \frac{\left(u_{bf} - \frac{\gamma}{\alpha}\right) e^{d\sqrt{\alpha}} - \left(u_s - \frac{\gamma}{\alpha}\right)}{e^{d\sqrt{\alpha}} - e^{-d\sqrt{\alpha}}} \tag{A.8}$$

$$C_2 = \frac{u_s - u_{bf} e^{-d\sqrt{\alpha}} - \frac{\gamma}{\alpha}\left(1 - e^{-d\sqrt{\alpha}}\right)}{e^{d\sqrt{\alpha}} - e^{-d\sqrt{\alpha}}} = \frac{\left(u_s - \frac{\gamma}{\alpha}\right) - \left(u_{bf} - \frac{\gamma}{\alpha}\right) e^{d\sqrt{\alpha}}}{e^{d\sqrt{\alpha}} - e^{-d\sqrt{\alpha}}} \tag{A.9}$$

for the stresses at the bottom of the layer we therefore find:

$$\frac{\tau_{base}}{\rho\nu} := \frac{\tau(z=0)}{\rho\nu} = \left.\frac{\partial u}{\partial z}\right|_{z=0} =$$

$$\sqrt{\alpha}\left(C_2 - C_1\right) = \sqrt{\alpha}\,\frac{2u_s - u_{bf}\left(e^{d\sqrt{\alpha}} + e^{-d\sqrt{\alpha}}\right) - \frac{\gamma}{\alpha}\left(2 - e^{d\sqrt{\alpha}} - e^{-d\sqrt{\alpha}}\right)}{e^{d\sqrt{\alpha}} - e^{-d\sqrt{\alpha}}}$$

(A.10)

For large values of $d\sqrt{\alpha}$ this can be approximated by:

$$\frac{\tau_{base}}{\rho\nu} \approx \sqrt{\alpha}\left(-u_{bf} + \frac{\gamma}{\alpha}\right) \tag{A.11}$$

In the case of laminar flow $\frac{\gamma}{\alpha} \approx u_f$:

$$\frac{\tau_{base}}{\rho\nu} \approx \sqrt{\alpha}\left(u_f - u_{bf}\right) \tag{A.12}$$

At the top of the layer we find:

$$\frac{\tau_0}{\rho\nu} := \frac{\tau(z=d)}{\rho\nu} = \left.\frac{\partial u}{\partial z}\right|_{z=d} =$$

$$\sqrt{\alpha}\left(C_2 e^{d\sqrt{\alpha}} - C_1 e^{-d\sqrt{\alpha}}\right) = \sqrt{\alpha}\,\frac{u_s\left(e^{d\sqrt{\alpha}} + e^{-d\sqrt{\alpha}}\right) - 2u_{bf} + \frac{\gamma}{\alpha}\left(2 - e^{d\sqrt{\alpha}} - e^{-d\sqrt{\alpha}}\right)}{e^{d\sqrt{\alpha}} - e^{-d\sqrt{\alpha}}}$$

(A.13)

For large values of $d\sqrt{\alpha}$ this can be approximated by:

$$\frac{\tau_0}{\rho\nu} \approx \sqrt{\alpha}\left(u_s - \frac{\gamma}{\alpha}\right) \approx \sqrt{\alpha}\left(u_s - u_f\right) \tag{A.14}$$

Using (A.10) and (A.13) we can determine C_1 and C_2 in terms of the stresses at the bottom and the top of the layer:

$$C_1 = \frac{\tau_{base}\, e^{d\sqrt{\alpha}} - \tau_0}{\rho\nu\sqrt{\alpha}\left(e^{-d\sqrt{\alpha}} - e^{d\sqrt{\alpha}}\right)} \text{ and } C_2 = \frac{\tau_{base}\, e^{-d\sqrt{\alpha}} - \tau_0}{\rho\nu\sqrt{\alpha}\left(e^{-d\sqrt{\alpha}} - e^{d\sqrt{\alpha}}\right)} \tag{A.15}$$

For large values of $d\sqrt{\alpha}$ we find the approximating values:

$$C_1 \approx \frac{-\tau_{base}}{\rho\nu\sqrt{\alpha}} \text{ and } C_2 \approx \frac{\tau_0}{\rho\nu\sqrt{\alpha}} e^{-d\sqrt{\alpha}} \tag{A.16}$$

Since $\frac{\tau}{\rho\nu}=\frac{\partial u}{\partial z}=\sqrt{\alpha}\left(C_2 e^{z\sqrt{\alpha}}-C_1 e^{-z\sqrt{\alpha}}\right)$, (A.17)

the total (approximating) solution for the stresses can now be given as:

$$\tau \approx \tau_0 e^{(z-d)\sqrt{\alpha}}+\tau_{base}\, e^{-z\sqrt{\alpha}} \tag{A.18}$$

2 Turbulent flow conditions

In the case of turbulent flow we use the assumption that the turbulence viscousity can be approximated using

$$\mu=\alpha_l \rho u \tag{A.19}$$

The (only significant) stress component can now be written as:

$$\tau=\alpha_l \rho u \frac{\partial u}{\partial z} \tag{A.20}$$

The flow equation now becomes:

$$u\frac{\partial^2 u}{\partial z^2}+\left(\frac{\partial u}{\partial z}\right)^2-\frac{ga}{\alpha_l}u-\frac{gb}{\alpha_l}u^2+\frac{g\,i}{\alpha_l}=0 \tag{A.21}$$

We assume the turbulent contribution in the Forchheimer terms to be dominant and therefore ignore the linear term (by setting a = 0 in (A.21)).

$$u\frac{\partial^2 u}{\partial z^2}+\left(\frac{\partial u}{\partial z}\right)^2-\frac{gb}{\alpha_l}u^2+\frac{g\,i}{\alpha_l}=0 \tag{A.22}$$

By substitution $u=\sqrt{v}$ into (A.22) the differential equation reduces to:

$$\frac{\partial^2 v}{\partial z^2}-\frac{2gb}{\alpha_l}v+\frac{2g\,i}{\alpha_l}=0. \tag{A.23}$$

This equation has the general solution:

$$v=C_3 e^{-\xi z}+C_4 e^{\xi z}+\delta \tag{A.24}$$

where we defined: $\xi=\sqrt{\frac{2gb}{\alpha_l}}$ and $\delta=\frac{i}{b}$.

The general solution of equation (A.22) can now be given as $u=\sqrt{v}$.

Again as in the laminar case we have the following boundary conditions for the velocity:

$$\text{at } z = 0: \qquad u = u_{bf} \tag{A.25}$$

$$\text{at } z = d: \qquad u = u_s \tag{A.26}$$

We implicitly assumed $u \geq 0$ and can thus write these boundary conditions as:

$$u_{bf}^2 = C_3 + C_4 + \delta \tag{A.27}$$

and

$$u_s^2 = C_3 e^{-\xi d} + C_4 e^{\xi d} + \delta \tag{A.28}$$

From these we can find:

$$C_3 = \frac{u_{bf}^2 e^{\xi d} - u_s^2 - \delta\left(e^{\xi d} - 1\right)}{e^{\xi d} - e^{-\xi d}} = \frac{\left(u_{bf}^2 - \delta\right)e^{\xi d} - \left(u_s^2 - \delta\right)}{e^{\xi d} - e^{-\xi d}} \tag{A.29}$$

and

$$C_4 = \frac{u_s^2 - u_{bf}^2 e^{-\xi d} - \delta\left(1 - e^{-\xi d}\right)}{e^{\xi d} - e^{-\xi d}} = \frac{\left(u_s^2 - \delta\right) - \left(u_{bf}^2 - \delta\right)e^{-\xi d}}{e^{\xi d} - e^{-\xi d}} \tag{A.30}$$

For the stress at the bottom of the layer we now find:

$$\tau_{base} = \alpha_l \rho u \left.\frac{\partial u}{\partial z}\right|_{z=0} = \tfrac{1}{2}\alpha_l \rho \left.\frac{\partial v}{\partial z}\right|_{z=0} = \tfrac{1}{2}\alpha_l \rho \xi \left(C_4 - C_3\right) =$$

$$\tfrac{1}{2}\alpha_l \rho \xi \frac{2u_s^2 - u_{bf}^2\left(e^{\xi d} + e^{-\xi d}\right) - \delta\left(2 - e^{\xi d} - e^{-\xi d}\right)}{e^{\xi d} - e^{-\xi d}} \tag{A.31}$$

For large values of ξd (say >10) this can be approximated by:

$$\tau_{base} \approx -\alpha_l \rho \xi \left(\delta + \tfrac{1}{2}u_{bf}^2\right) = -\rho\sqrt{2gb\alpha_l}\left(\frac{i}{b} + \tfrac{1}{2}u_{bf}^2\right) \tag{A.32}$$

For the stress at the top of the layer:

$$\tau_0 = \alpha_l \rho u \left.\frac{\partial u}{\partial z}\right|_{z=d} = \tfrac{1}{2}\alpha_l \rho \left.\frac{\partial v}{\partial z}\right|_{z=d} = \tfrac{1}{2}\alpha_l \rho \xi \left(C_3 e^{-\xi d} - C_4 e^{\xi d}\right) =$$

$$\tfrac{1}{2}\alpha_l \rho \xi \frac{u_s^2\left(e^{\xi d} + e^{-\xi d}\right) - 2u_{bf}^2 + \delta\left(2 - e^{\xi d} - e^{-\xi d}\right)}{e^{\xi d} - e^{-\xi d}} \tag{A.33}$$

For large values of ξd this can be approximated by:

$$\tau_0 \approx \tfrac{1}{2}\alpha_l \rho \xi \left(u_s^2 - \delta\right) = \tfrac{1}{2}\rho\sqrt{2gb\alpha_l}\left(u_s^2 - \frac{i}{b}\right) \tag{A.34}$$

Without approximations we can find from (A.31) and (A.33):

$$C_3 = \frac{2}{\alpha_l \rho \xi} \frac{\tau_0 - \tau_{base} e^{\xi d}}{e^{\xi d} - e^{-\xi d}} \text{ and } C_4 = \frac{2}{\alpha_l \rho \xi} \frac{\tau_0 - \tau_{base} e^{-\xi d}}{e^{\xi d} - e^{-\xi d}} \qquad \text{(A.35)}$$

For large values of ξd we can use the approximations:

$$C_3 \approx \frac{-2}{\alpha_l \rho \xi} \tau_{base} \qquad \text{and} \qquad C_4 \approx \frac{2}{\alpha_l \rho \xi} \tau_0 e^{-\xi d} \qquad \text{(A.36)}$$

For the stress inside the layer we can than use:

$$\tau \approx \tau_0 e^{\xi(z-d)} + \tau_{base} e^{-\xi z} \qquad \text{(A.37)}$$

B Derivation of filter design relations

I Average load and strength

At the interface filter/base material equilibrium is assumed of load and strenght:

$$\tau_{load} = \tau_{strength} \tag{B.1}$$

The shear stress equation reads:

$$\tau_{load} = \tau_{base} + \tau_0 e^{-\lambda d} \tag{B.2}$$

The strength of the base material may be written as:

$$\tau_{strength} = \tau_{c,G} = \psi_{c,G,b} \Delta_b \rho g D_{b50} \tag{B.3}$$

Assuming $\tau_{base} = \eta \tau_0$ (B.4)

Substituting (B.2), (B.3) and (B.4) into (B.1) results in:

$$\tau_0 \left(\eta + e^{-\lambda d} \right) = \psi_{c,G,b} \rho g \Delta_b D_{b50} \tag{B.5}$$

and after re-arranging:

$$\frac{1}{D_{b50}} = \psi_{c,G,b} \rho g \Delta_b \frac{1}{\tau_0 \left(\eta + e^{-\lambda d} \right)} \tag{B.6}$$

Multiplying both sides with D_{t50} and assuming instantenous instability of filterlayer and base material:

$$\frac{D_{t50}}{D_{b50}} = D_{t50} \, \psi_{c,G,b} \rho g \Delta_b \frac{1}{\tau_0 \left(\eta + e^{-\lambda d} \right)} \tag{B.7}$$

Substituting for D_{t50}:

$$D_{t50} = \frac{\tau_0}{\psi_{c,G,t} \rho g \Delta_t} \tag{B.8}$$

at the rigth hand side of (B.8) and performing some re-arranging, finally results in an explicit general relationship:

$$\frac{D_{t50}}{D_{b50}} = \frac{1}{\eta + e^{-\lambda d}} \frac{\psi_{c,G,b}\Delta_b}{\psi_{c,G,t}\Delta_t} \tag{B.9}$$

For large values of λd this results into:

$$\frac{D_{t50}}{D_{b50}} = \frac{1}{\eta} \frac{\psi_{c,G,b}\Delta_b}{\psi_{c,G,t}\Delta_t} \tag{B.10}$$

Formula (B.10) can easily be changed by multiplying both sides with D_{t15}/D_{t50} in:

$$\frac{D_{t15}}{D_{b50}} = \frac{D_{t15}}{D_{t50}} \frac{1}{\eta} \frac{\psi_{c,G,b}\Delta_b}{\psi_{c,G,t}\Delta_t} \tag{B.11}$$

For a one layer system (toplayer directly on base material) the subscript *t* may be replaced by *f* resulting finally in the new formula:

$$\frac{D_{f15}}{D_{b50}} = \frac{D_{f15}}{D_{f50}} \frac{1}{\eta} \frac{\psi_{c,G,b}\Delta_b}{\psi_{c,G,t}\Delta_t} \tag{B.12}$$

2 Characteristic load and strength

The load is given by:

$$\tau_{load} = \tau_0\left(\eta + e^{-\lambda d}\right) \tag{B.13}$$

which can be written by applying Grass' concept, so replacing τ_0 by $\tau_0 + \gamma_t\,\sigma_0$ and substituting τ_{load} for $\tau(0)$ as:

$$\tau_{load} = \left(\tau_0 + \gamma_t\sigma_0\right)\left(\eta + e^{-\lambda d}\right) \tag{B.14}$$

The strength of the base material may be written as:

$$\tau_{strength} = \tau_{c,G} - \gamma_b\sigma_{c,b} = \tau_{c,G} - \gamma_b\alpha_{c,b}\tau_{c,G} = \tau_{c,G}\left(1 - \gamma_b\alpha_{c,b}\right) = \psi_{c,G,b}\Delta_b\rho g D_{b50}\left(1 - \gamma_b\alpha_{c,b}\right) \tag{B.15}$$

Substituting (B.14) and (B.15) into (B.1) results in:

$$\left(\tau_0 + \gamma_t\sigma_0\right)\left(\eta + e^{-\lambda d}\right) = \psi_{c,G,b}\rho g\Delta_b D_{b50}\left(1 - \gamma_b\alpha_{c,b}\right) \tag{B.16}$$

and after re-arranging:

$$\frac{1}{D_{b50}} = \psi_{c,G,b}\rho g\Delta_b\left(1-\gamma_b\alpha_{c,b}\right)\frac{1}{\left(\tau_0+\gamma_t\sigma_0\right)\left(\eta+e^{-\lambda d}\right)} \tag{B.17}$$

Multiplying both sides with D_{t50}:

$$\frac{D_{t50}}{D_{b50}} = D_{t50}\,\psi_{c,G,b}\rho g\Delta_b\left(1-\gamma_b\alpha_{c,b}\right)\frac{1}{\left(\tau_0+\gamma_t\sigma_0\right)\left(\eta+e^{-\lambda d}\right)} \tag{B.18}$$

Substituting for D_{t50}:

$$D_{t50} = \frac{\tau_0+\gamma_t\sigma_0}{\psi_{c,G,t}\rho g\Delta_t\left(1-\alpha_{c,t}\gamma_t\right)} \tag{B.19}$$

at the rigth hand side of (B.19) and performing some re-arranging, finally results in an explicit general relationship:

$$\frac{D_{t50}}{D_{b50}} = \frac{1}{\eta+e^{-\lambda d}}\,\frac{1-\gamma_b\alpha_{c,b}}{1-\gamma_t\alpha_{c,t}}\,\frac{\psi_{c,G,b}\Delta_b}{\psi_{c,G,t}\Delta_t} \tag{B.20}$$

This equation is a general formula with the implicit assumptions of Gaussian distributions of load and strength, and a relationship based on uniform flow.

Assuming $\alpha_{c,b} = \alpha_{c,t}$ and $\gamma_b = \gamma_t$ results in:

$$\frac{D_{t50}}{D_{b50}} = \frac{1}{\eta+e^{-\lambda d}}\,\frac{\psi_{c,G,b}\Delta_b}{\psi_{c,G,t}\Delta_t} \tag{B.21}$$

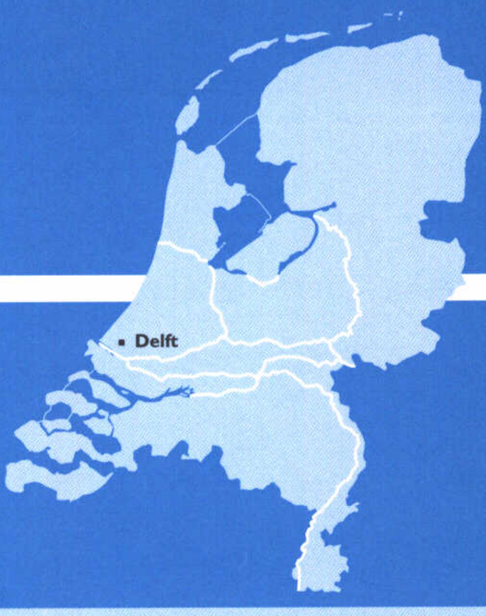
Note that this expression does not depend on α_c and so this relation is independent of the gradation of either toplayer or base material as long as these gradations are the same or nearly the same.

Formula (B.21) can easily be changed by multiplying both sides with D_{t15}/D_{t50} into:

$$\frac{D_{t15}}{D_{b50}} = \frac{D_{t15}}{D_{t50}}\,\frac{1}{\eta+e^{-\lambda d}}\,\frac{\psi_{c,G,b}\Delta_b}{\psi_{c,G,t}\Delta_t} \tag{B.22}$$

For a one layer system (toplayer directly on base material) the subscript t may be replaced by f resulting finally in the new formula:

$$\frac{D_{f15}}{D_{b50}} = \frac{D_{f15}}{D_{f50}}\,\frac{1}{\eta+e^{-\lambda d}}\,\frac{\psi_{c,G,b}\Delta_b}{\psi_{c,G,t}\Delta_t} \tag{B.23}$$



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