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State-of-the-art

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SHIP COLLISION AVOIDANCE METHODS: STATE-OF-THE-ART

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Abstract: Collision prevention is critical for navigation safety at sea. At early ages, researchers aimed at developing navigational assistance systems for enhancing situational awareness of human operators as human is at the core of collision avoidance. Recently, autonomous vehicles have gained a remarkable amount of attention with a focus on solving collision problems by machines. This results in two groups of studies, both working on preventing collisions but with different focuses: one aims at conflict detection, and the other focuses on conflict resolution.

This paper offers a comprehensive overview of collision prevention techniques based on the three basic processes of determining evasive solutions, namely, motion prediction, conflict detection, and conflict resolution. The strengths and weaknesses of different methods for these three fundamental processes are discussed. Limitations and new challenges are highlighted. Moreover, this review points out the differences between the research for manned and unmanned ships and how the research in the two domains can learn from each other. A potential roadmap for the transition from existing manned ships to fully unmanned ships is provided in the end.

Keywords: Collision avoidance; conflict detection, conflict resolution; human-machine interactions; Autonomous Surface Vehicle; manned and unmanned ships

1. Introduction

1.1 Background

Ship collision is an imperative task for navigation safety at sea. Due to the high frequency and severe consequences of collisions, both practitioners and researchers have paid much attention to related research. Various kinds of techniques aiming at preventing collision accidents have been developed. From numerous accident reports and investigations, researchers share the common knowledge, i.e. human factor is the main cause of ship collision accidents (Chauvin, Lardjane, Morel, Clostermann, & Langard, 2013). Therefore, existing collision prevention technologies are mainly from two perspectives, i.e., assisting human on board and eliminating human factors.

Enhancing the situation awareness of Officers On Watch (OOW) on board is a classical research subject from the 1950s (Tam, Bucknall, & Greig, 2009). Many techniques were applied to support the OOWs on board, e.g., ship domain, automatic radar plotting aid, etc. To eliminate the human factor, researchers turn to develop autonomous systems which can find collision-free solutions automatically and replace the role of the human in collision prevention. The unmanned ship with an autonomous system is usually defined as an Autonomous Surface Vehicle (ASV). With the fast development of robotics, artificial intelligence, etc., ASVs have gained a remarkable amount of attention in recent years.

Today, two groups of studies have been developing in parallel to achieve a smarter/autonomous navigation system. Although the focuses and goals of these two studies are different, many scholars believe the studies for manned ships can benefit the research for the development of unmanned ships, and vice versa (Lopez-Santander & Lawry, 2016). However, there is a lack of studies elaborating on this statement. In this article, we want to bridge this gap, which helps peer researchers with different background learn from each other with respect to collision prevention.

1.2 Related works

There are many related literature reviews which collect techniques for collision avoidance, such as (Campbell, Naeem, & Irwin, 2012; Z. X. Liu, Zhang, Yu, & Yuan, 2016; Polvara, Sharma, Wan, Manning, & Sutton, 2017; Tam et al., 2009; Tu, Zhang, Rachmawati, Rajabally, & Huang, 2018). However, these reviews have not pointed out the links between the state-of-the-art methods for manned and unmanned ships.

Firstly, the reviews are either from the perspective of supporting the human in collision avoidance (Tam et al., 2009; Tu et al., 2018) or from the perspective of developing ASVs (Campbell et al., 2012; Z. X. Liu et al., 2016; Polvara et al., 2017). The discussion across these two groups of studies is missing in these reviews.

Secondly, these studies usually are for different aims, in which collision avoidance methods are seldom collected or even ignored. For instance, review (Tu et al., 2018) described the collision risk assessment, but it neglected the techniques for conflict resolution; In (Campbell et al., 2012) and (Z. X. Liu et al., 2016), the authors addressed the developments of ASVs in details, while conflict detection and obstacle avoidance were of less focus; paper (Polvara et al., 2017) focused on the techniques for path planning and only included a few of studies related to reacting collision avoidance for unmanned ships.

Thirdly, as the quantity of related literature increasing dramatically, an update is needed for the peer-researchers' convenience. Literature review (Tam et al., 2009) addressed the limitations of ship collision avoidance methods proposed from an early age, in particular from the 1950s to early 2000s. Together with the limitations addressed in other reviews, the identified limitations are concluded as follows: (1) no environmental factors are taken into account in collision prevention (Polvara et al., 2017; Tam et al., 2009); (2) incorporating regulations in collision prevention algorithms is still a challenge (Campbell et al., 2012; Z. X. Liu et al., 2016; Polvara et al., 2017), e.g., International Regulations for Preventing Collisions at Sea (COLREGs); (3) the prevention only considers static obstacles or semi-dynamic obstacles which moves without changes on headings (Z. X. Liu et al., 2016; Tam et al., 2009); (4) highly ideal motion model is used in collision avoidance (Polvara et al., 2017; Tam et al., 2009); (5) balancing efficiency and effectiveness is ignored (Z. X. Liu et al., 2016). These limitations are widely accepted and used in recent articles and literature reviews. However, as new methods and techniques are springing up, some limitations have been overcome while new challenges have raised.

1.3 Contributions

This paper aims at collecting developments of collision prevention techniques either for manned ships or unmanned ships. A comparative evaluation of these techniques is provided, highlighting their potentials in the development of smart navigation assistance system and autonomous system. Compared with existing reviews, the main contributions of our paper are twofold:

(1) The knowledge of ship collision avoidance techniques is updated with detailed comparisons of the strengths and weakness of methods in three processes of collision avoidance, namely motion prediction, conflict detection, and conflict resolution;
(2) The bridge between the studies for manned and unmanned ships has been discussed, and the potential road from the existing manned ships to full unmanned ships is highlighted.

1.4 Outline

This paper is organized as follows: Section 2 introduces the framework that we use to review the collision prevention methods; Section 3, 4, and 5 conduct comprehensive surveys of motion prediction, conflict detection, and conflict resolution, respectively. Section 6 discusses the further developments of existing techniques for collision avoidance and the essential steps from manned ships to unmanned ships. Finally, conclusions are summarized in Section 7.

2. Framework for review and evaluation of existing methods

2.1 Research scope

The process of avoiding collisions is named as collision avoidance. The techniques involved in this process are called collision prevention techniques. Various categorizations of collision prevention techniques have been presented in (Z. X. Liu et al., 2016; Tam et al., 2009), i.e., route planning, path planning, and reactive collision avoidance. In this article, we distinguish these studies as follows: route planning takes place on large scale map, e.g., weather routing, etc.; path planning aims at finding a collision-free path on a local map considering static obstacles; reactive collision avoidance focuses on avoidance of moving obstacles or obstacles unknown in prior, which is the focus of this review.

The scope of this review narrows down to the reactive collision avoidance for both manned and unmanned ships. Specifically, we collect two types of research: 1) the prevention techniques for manned ships, which support the OOW on board, e.g., collision warning and searching for evasive actions; and 2) the methods applied in ASVs that drive the ship to deviate from the predefined path for collision avoidance.

Therefore, we re-defined collision avoidance for both manned and unmanned ships as follows:

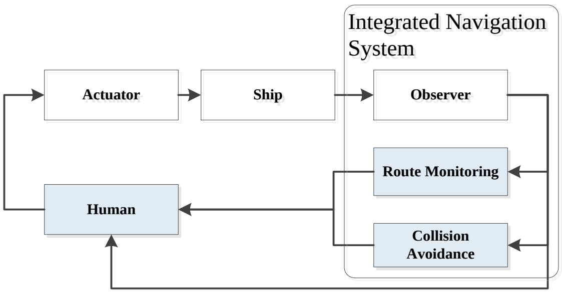
Definition: Collision Avoidance (CA) is a process in which one ship (manned or unmanned) departs from its planned trajectory to avoid a potential undesired physical contact at a certain time in the future.

The ship under control is called Own-Ship (OS). Obstacles include stationary obstacles and moving obstacles (Target-Ships, TSs).

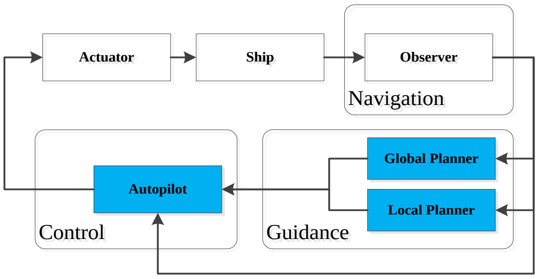
2.2 General framework of ship collision avoidance

According to the definition of collision avoidance, the collision prevention problem contains two sub-problems: "*conflict detection*" and "*conflict resolution*". Solving the conflict detection problem is to determine whether the ship is in danger and when to take evasive actions. Solving the conflict resolution problem is to answer the question of what actions should be taken to prevent collision (Kuchar & Yang, 2000).

For manned ships, modern bridge system such as an Integrated Navigation System (INS), is designed to support collision avoidance mainly during conflict detection stage. Its main function is to offer information to navigators and to send an alarm if necessary. Human, who decides to/not to take actions, plays a major role in conflict resolution. For an ASV, a Guidance Navigation Control (GNC) system takes the whole responsibility for collision prevention. The Guidance system is engaged to detect and solve the conflict at the same time, which decides *When* and *How* to take evasive actions. The other two sub-systems offer information to support the guidance system and implement the planned actions, which are the Navigation system and the Control system, respectively. The data/information flows in a manned ship, and an unmanned ship during collision avoidance are separately presented in Fig. 1.



(1) the decision process in a manned ship



(2) the decision process in an autonomous ship

Fig. 1 Structure of Navigation System in manned and unmanned ships

Based on Fig. 1, one can see that either for the navigation system in manned ships or unmanned ships, some essential modules are needed to reach a collision-free solution for the ship. When the ship observes the positions of Target Ships (TSs) at present, it estimates the possible positions of these ships in the future and their corresponding collision risks. Based on the estimations, the OS might decide to keep its current route or to find a new collision-free solution.

The process of collision prevention and its information flows in the manned and unmanned ship can be abstracted as Fig. 2. Five components are included: (1) Observer, which contains various sensors offering data to support other modules; (2) Motion Prediction module, which estimates the future trajectories of the Own Ship (OS) and the obstacles; (3) Conflict Detection module, which checks collision risk and launches collision warning if necessary; (4) Conflict Resolution module, which determines the evasive solutions and then, (5) Actuator, which implements the solutions.

The "Motion Prediction", "Conflict Detection" and "Conflict Resolution" are the main focuses of this paper, which are investigated in Section 3-5. In particular, the following questions are discussed: *what methods can be used to predict the trajectory of obstacles; how is the collision risk measured and used for early alarm; and what approaches are used to determine the actions to prevent the approaching dangers*. Other modules, such as "Observer" and "Actuator", are also necessary for collision prevention, but they are not included in the scope of this review. We presume the observers can offer accurate information about the states of the system; the limitations on actuators have been considered in "Conflict Resolution" and the actuators can implement the collision-free solutions and follow the reference. Readers who are interested in the observers and actuators for collision prevention are referred to (Z. X. Liu et al., 2016).

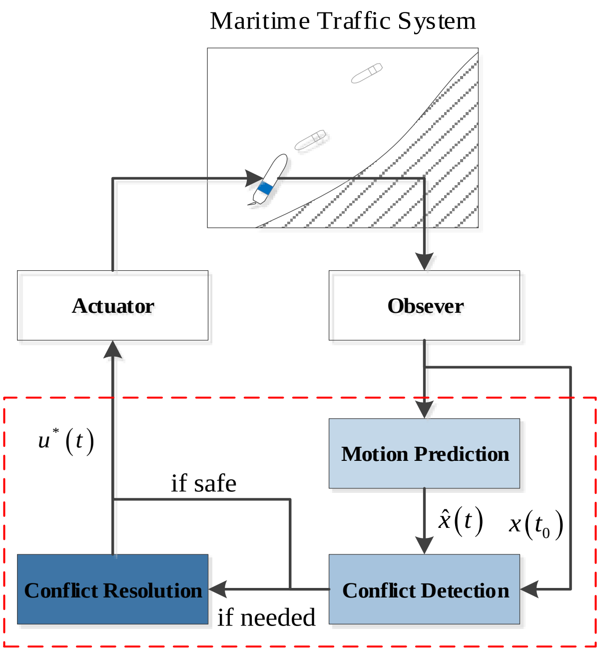


Fig. 2 The information flow of collision prevention on board ship.

2.3 Methodology

The scope of this review is on "ship collision avoidance" which contains three sub-modules: motion prediction, conflict detection, and conflict resolution. Although each module covers numerous studies, we only collect the studies aiming at avoiding collisions with three steps, as shown in Fig. 3:

(1) Firstly, we search in databases of "Web of Knowledge" and "Scopus" to collect journal and conference papers with the following keywords in title, keywords, and abstract: "ship", "vessel", "unmanned surface vehicle", "USV", "autonomous surface vehicle", and "ASV", "collision avoidance", "collision prevention", "avoid collision", "prevent collision", "navigation safety". The research with a series of keywords which indicate that it is out of our scope is excluded, such as "underwater", "aircraft", "car", "collision protection", "estimation of collision damage", "ship-

bridge collision", "ship-iceberg", etc. The searching result is narrowed down by limiting the language to "English", and research domain to "engineering". At this step, 304 pieces of record are obtained until Mar. 1st, 2019.

(2) A further literature filtering is performed to identify the studies that are not completely fitting our scopes. According to the scope described in Section 2.1, some records are removed, e.g., the studies relating with sharing navigation experience, the studies only considering path planning or formation control, the studies focusing on the construction of ship domain, the studies that do not consider moving obstacles. In the end, 90 pieces of records are obtained.

(3) After reading the selected papers, we add some papers as the complement to our database. Three types of studies are added: the papers that are cited in the 90 papers but not included in our database; the studies which were published before 2000 but are classical and are sources of some methods; the papers published in 2019 but have not appeared in the database.

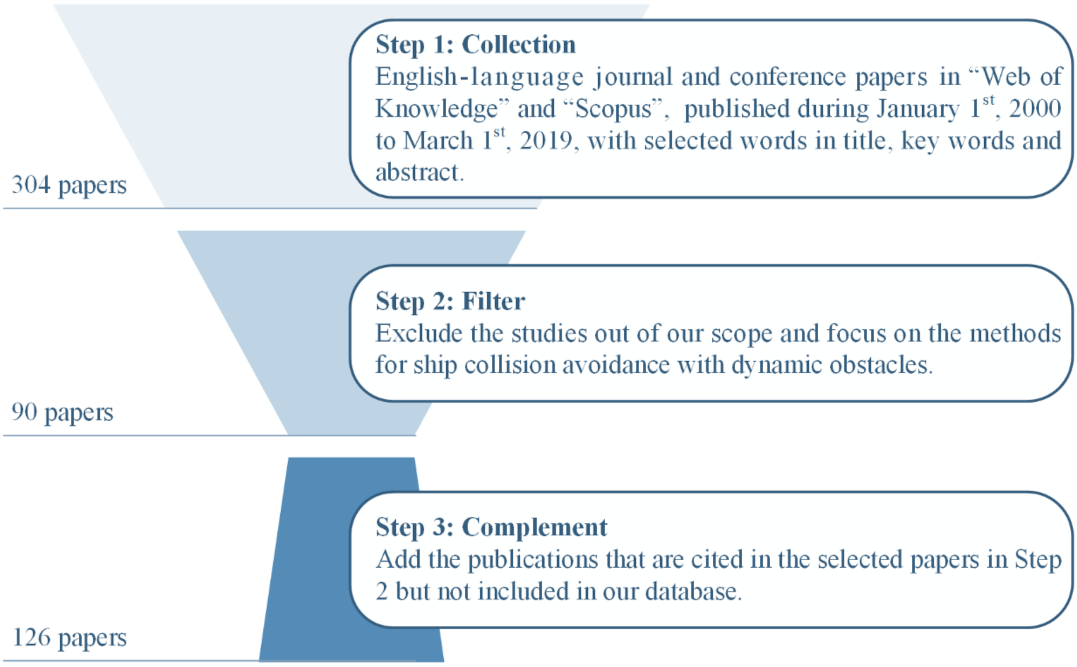


Fig. 3 Literature collection steps.

3. Motion Prediction

Motion prediction is a fundamental module for ship collision avoidance, which contains a process that predicts the trajectories of the OS and obstacles. When the OS encounters with potential dangers, the predicted trajectories are used to determine the collision risk for conflict detection. Moreover, when the OS determines a resolution, the predicted trajectories are also needed in the collision risk check.

In this section, we first present the popular ship motion models used in prediction, followed by existing techniques used in predictions. A summary of the trends of research on prediction in collision prevention is also presented.

3.1 Ship motion models in prediction

The motion prediction usually relies on the mathematical expression of the system, i.e., motion models of the ship. In this article, the behaviors of the ship in the horizontal plane are focused, i.e., surge, sway, and yaw. Thus, the workspace of the ship is the horizontal space, i.e., $W = \mathrm{R}^2$, and the configuration space (C-space) consists of position and orientation, i.e., $C = \mathrm{R}^2 \times \mathrm{S}^1$. Detail introductions of motion models with 6 degrees of freedom are provided in (Thor I. Fossen, 2011).

According to the constraints used in modelings, the motion models are categorized as holonomic models (constraints on configurations only) and non-holonomic models.

3.1.1 Holonomic model

The simplest way to describe the ship's motion is based on the assumption that the ship is a holonomic vehicle which moves freely in a horizontal plane. Two versions of formulations are found.

$$\begin{cases} \dot{x} = u_x \\ \dot{y} = u_y \end{cases} \; or \; \begin{cases} \dot{x}(t) = u\cos\psi \\ \dot{y}(t) = u\sin\psi \end{cases}, \tag{1}$$

where u_x and u_y are components of ship speed w.r.t. the earth; u is resultant speed and ψ is heading of the ship.

In the trajectory prediction of the TSs, Equation (1) is noted as constant velocity model (X. R. Li & Jilkov, 2003) which is widely used, then u and ψ are the observed speed and course of the TSs. Examples can be found in (Benjamin, Leonard, Curcio, & Newman, 2006; Degre & Lefevre, 1981; Lazarowska, 2017; Lenart, 1983; Pedersen, Inoue, & Tsugane, 2003; Szlapczynski & Szlapczynska, 2015, 2017a).

3.1.2 Kinematic model

The holonomic model ignores constraints on tangent C-space, e.g., acceleration of linear and angular speed. Thus, in some studies, kinematic motion models are proposed. A standard form of kinematic models is shown as follows:

$$\begin{cases} \dot{x} = u \cdot \cos\psi \\ \dot{y} = u \cdot \sin\psi \\ \dot{u} = a_t \\ \dot{\psi} = a_n / u \end{cases}, \tag{2}$$

where (x, y), u, ψ are the position of the ship, speed, and heading angle; a_t and a_n are tangential and normal accelerations, respectively. This model comprises various kinematic models (X. R. Li & Jilkov, 2003), such as "*unicycle*" model, "*dubins car*" model, and "*simple car*" model. In these models, "*dubins car*" model (a_t=0) (e.g., (Hvamb, 2015; Vincent, 1977)) and "*simple car*" model (e.g., (S. Fossen, 2018)) have been used in maritime studies.

3.1.3 Dynamic model

Kinematic models ignore the ship's mass that has great impacts on ship motion. The accelerations in each direction which have complex mechanisms are not properly addressed, as well. Therefore, researchers introduced kinetics relations into a kinematic model to increase the accuracy of prediction.

(1) Vectorial representation for marine vehicle

A widely used dynamic model is described in a compact vectorial setting, which contains two formulations: one describes the kinematic relations; the other shows the kinetic equations. The kinematic formulation is $\dot{\eta} = R(\psi)\nu$, where $R(\psi)$ is a rotation matrix, $\eta = [x, y, \psi]^{\mathrm{T}}$, and $\nu = [u, v, r]^{\mathrm{T}}$. The kinetic relation is then formulated as (T. I. Fossen, 2002):

$$M\dot{\nu} + C(\nu)\dot{\nu} + D(\nu)\nu + g(\eta) = \tau + w(t), \tag{3}$$

where M, C, and D denote the mass, Coriolis, and damping matrices; g and w are vectors of restoring forces and disturbance; τ denotes an input vector contains surge force, sway force and yaw moment. Considering that most merchant ships are under-actuated, i.e., the degree of controls is smaller than system states, some researchers also used an under-actuated model, such as:

$$M\dot{v}+C(v)\dot{v}+D(v)v+g(\eta)=Bf\ . \tag{4}$$

where B is the actuator configuration matrix, f *is* the input vector contains thruster and rudder angle. The applications can be found in (Abdelaal, Franzle, & Hahn, 2018; Eriksen, Breivik, Pettersen, & Wiig, 2016; Moe & Pettersen, 2016; Soltan, Ashrafiuon, & Muske, 2010; Martin S. Wiig, Pettersen, & Krogstad, 2017). This form is widely used in designing controllers and observers in ASVs.

(2) Mathematic Model Groups (MMG)

MMG model is another dynamic model, which is used in maneuverability prediction. Instead of using the forces as inputs, the MMG employs rudder angle and propeller revolutions as the inputs and models the responses of hydrodynamic forces to different inputs by empirical formula method. In this way, more details of rudders and propellers are considered, e.g., the specifications of rudders and propellers. Thus, this model is usually used in the theoretical analysis of ship maneuverability. Details of MMG refers to (Yasukawa & Yoshimura, 2015). The application of this model in collision prevention presented in (He et al., 2017; S. J. Li, Liu, & Negenborn, 2019; Y. Xue, Lee, & Han, 2009).

This model can produce a relatively accurate trajectory, while the cost is high. It requires a better understanding of ship hull, rudder, and propeller. Moreover, since the relationships between control inputs and forces are nonlinear and complicated, this model is less popular in the design of ship controllers and observers. Researchers prefer to use some simplified model based on MMG to predict ship's trajectory.

(3) Other well-known simplified dynamic models

Since the ship dynamics models are complicated, researchers often employ some simplified models in the design of collision avoidance approaches. Although these simplified models are less precise, they are more applicable (X. R. Li & Jilkov, 2003).

One simplifying technique is to ignore some less important terms in the aforementioned models (more details in (Z. X. Liu et al., 2016)). One group of simplified models uses first-order/second-order response equation to describe the dynamics of rotations and assumes the surge speed is constant and no sway speed (Fang, Tsai, & Fang, 2017; C. Liu, Negenborn, Chu, & Zheng, 2017; J. F. Zhang, Zhang, Yan, Haugen, & Soares, 2015). Here, the first-order response equation is also noted as Nomoto equation. The simplified model is similar to "*dubins car*" adding a damping factor in rotation. Some models considered the simplified response of surge speed w.r.t. rudder angle and propeller revolution (X. Wang, Liu, & Cai, 2017).

Another frequent used simplifying technique is called successively linearization, which is based on Taylor expansion. Researchers linearized the ship motion model around an estimated trajectory (L. Chen, Hopman, & Negenborn, 2018; Huarong Zheng, 2016). As a result, the motion model has a relatively simple form, and the predicted trajectory approximates to the real trajectory. As the real input deviates from the initial setting, the errors of prediction might increase.

The above simplifications are based on the mathematical expression of ship dynamics, while the other simplifications are based on simulation/experiment data. Specifically, researchers either use a simulator to generate the responses of the ship with different inputs or collect the experimental data of the ship's response with different inputs. Then they use regression methods to find equations that fit the data best (Miloh & Pachter, 1989; Szlapczynski & Krata, 2018).

3.2 Prediction of trajectory

3.2.1 Prediction of the OS's trajectory

In an ideal case that the control inputs and motion model of the OS are known, the prediction of the OS's trajectory turns to be solving the ordinary differential equations in Section 3.1. The simplest way is to assume that the OS is a holonomic vehicle, which is popular in many collision prevention studies. However, the errors between the predicted trajectory and the real trajectory are huge due to this unrealistic assumption. Thus, some researchers consider the non-holonomic constraints and use kinematic models in prediction to make the predicted trajectory closer to the real trajectory. One advantage is its concise form, while the accuracy of the prediction is still the issue. Therefore, nowadays, many researchers employ either the dynamic model or simplified dynamic model in trajectory prediction. Due to the complicated form of the equation, the analytical solutions are usually infeasible, and the numerical method is usually needed, e.g., Runge-Kutta methods, etc.

In other cases, researchers face with more practical problems, such as uncertainties on motion models and parameters. Then, some model identifications are needed to obtain the motion model. Moreover, a challenging issue is considering noise and errors in predictions. In this case, studies usually applied Kalman filter (or its variations) in trajectory prediction.

3.2.2 Prediction of the TS's trajectory

Since the information of the TS is insufficient for the OS, e.g., parameters of motion model, inputs to the system, etc., the prediction of the TS is more challenging than that of the OS. Due to these uncertainties, researchers usually prefer to use simple models, such as the holonomic model and kinematic model. The simplest way to predict the trajectory of the TS is based on the assumption that the TS keeps its velocity and neglects environmental disturbance. It is widely used but less accurate for collision avoidance. A more reasonable approach is considering the uncertainties of models, inputs, and disturbance. The methods for predicting the trajectory of the TS can be categorized into three groups according to the knowledge of the TS.

Physics-based methods predict the motion of the ship only depending on the laws of physics, while the existing studies either ignore the control inputs or treat the maneuvers as white noise. KF is a preferred technique used to consider these noises and give the best guess of the ship's trajectory in many studies. Together with the KF, holonomic models (Candeloro, Lekkas, & Sørensen, 2017) or kinematic models (e.g., "*simple car*" model (Shah et al., 2015)) are employed. To handle the nonlinearities and uncertainties of these motion models, the variations of the Kalman filter are used, e.g., extended Kalman Filter (S. Fossen, 2018), Particle Filter, Interacting Multiple Model Kalman filter, probabilistic filter (Eriksen, Wilthil, Flåten, Brekke, & Breivik, 2018), etc. Although these methods can predict the trajectory of the ship in a short period, they cannot predict the changes in trajectory due to the changes of maneuvers (Lefèvre, Vasquez, & Laugier, 2014).

Maneuver-based methods take the maneuvers of the ship into account, i.e., navigational intention, which is learned/estimated from historical traffic data or by the protocols for ship encounter situations, e.g., COLREGs. Algorithms learn the behavioral patterns of ships in a certain area from massive traffic data and then use these patterns to support the prediction (Scheepens, van de Wetering, & van Wijk, 2014). Some popular learning models are neural network (Simsir, Amasyalı, Bal, Çelebi, & Ertugrul, 2014), Gaussian process (Rong, Teixeira, & Guedes Soares, 2019), Hidden Markov Model (Peel & Good, 2011), etc. More details of these models are addressed in the paper (Tu et al., 2018).

Interaction-aware methods consider the interactions between ships in prediction. Specifically, communications between ships are included. The OS and TS would broadcast (J. F. Zhang et al., 2015), exchange, or negotiate their maneuver intentions (e.g., intended course (Q. Hu, Yang, Chen, & Xiao, 2008; D. Kim, Hirayama, & Okimoto, 2017; D. G. Kim, Hirayama, & Park, 2014)) or the trajectory information, i.e., ships can exchange their planned trajectories (L. Chen, Y. Huang, H. Zheng, J.J. Hopman, & Negenborn, 2019; H. R. Zheng, R. R. Negenborn, & G. Lodewijks, 2017). The exchanged trajectories are the trajectories of the ships estimated by themselves who have a better knowledge about their own dynamics and intentions. In return, the predicted trajectories are more accurate than those predicted by other methods.

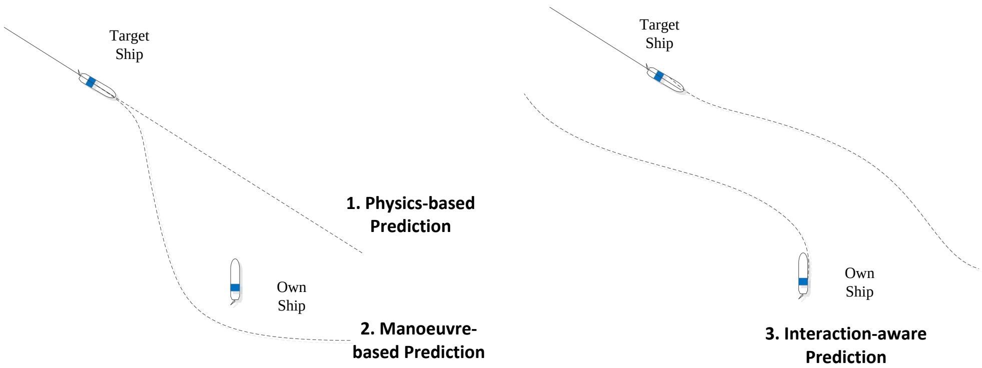


Fig. 4 An illustration of different predictions methods

The asymmetry of information is eliminated gradually in these three methods. An illustration of different prediction methods is shown in Fig. 4. The physics-based prediction provides the trajectory according to its historical data only. Since the TS is sailing to the southeast with constant speed and course, the physics-based method suggests that the TS would continue this movement in the future. The manoeuver-based method, however, first recognizes the pattern of the TS, e.g., the give-way intention, and predicts that the TS would perform starboard turn that is suggested by COLREGs. Then, the trajectory is predicted based on the recognized pattern. Different from previous methods, the Interaction-aware method requires the exchange of information among ships, e.g., the OS and the TS broadcast their trajectories. Thus the predicted trajectory is the trajectory offered by the TSs.

3.3 Discussion on Motion Prediction

In the conclusions drawn in existing review papers, the studies of collision avoidance are suffering from some ideal assumptions that the ships (the OS and the TSs) are holonomic, the TS moves without changing on their headings and speeds, and environmental disturbances are ignored (Z. X. Liu et al., 2016; Tam et al., 2009). Based on these assumptions, the physical-based method is widely used, and the predicted trajectories are usually shaped as straight lines that are unrealistic.

In recent years, many researchers engaged in overcoming these drawbacks. Some achievements are listed as follows.

Firstly, researchers have applied various non-holonomic models in the prediction of the OS’s trajectory, such as kinematic models, dynamics models, and simplified dynamics models. Given control inputs, the trajectory of the OS is usually calculated by the Runge-Kutta method. In some studies, environmental disturbances are considered, where the distribution of disturbance is assumed to be known, e.g., (Huarong Zheng, 2016). In return, the trajectories of the OS can be bounded by tubes or funnels.

Secondly, the developments of the prediction of the TS are currently towards the usages of the maneuver-based methods and interaction-aware methods that are capable to incorporate more information in the prediction. Manoeuver-based methods estimate the steering intentions first and then predict the trajectory, e.g., (Cho, Han, & Kim, 2018). However, the errors of the estimated intention could not be completely eliminated by existing methods, and collision avoidance is sensitive to these errors. For instance, when two ships encounter in a close range, any misestimation would result in a collision. Therefore, some researchers preferred to use the interaction-aware method to predict the TS's trajectory, which allows cooperation among ships. A simple way is exchanging their intentions among ships, such intended course (D. Kim et al., 2017) or turning points (J. F. Zhang et al., 2015); alternatively, the broadcasting trajectory is another way, which is more accurate than the above methods since the ship has better knowledge about its own dynamics than other ships, examples can be found in literature (L. Chen et al., 2018; Huang, Chen, & van Gelder, 2019). However, in this way, communication burdens are increased.

By these new methods, many collision avoidance techniques do not hold the assumption that the TS sails with constant speed and heading. However, some problems remain, e.g., the uncertainty of the motion model, limited knowledge about environmental disturbance acting on the ship, performance of communications, etc. These uncertainties should be analyzed and bounded for the subsequent collision avoidance.

4. Conflict Detection

Conflict detection refers to determinations of *whether* and *when* evasive actions should be taken by the OOW. The core of this process contains a collision risk assessment, which triggers an event that either requires human to notice collision dangers or asks the human/machine to find a collision-free solution. Conflict detection in practice includes:

(1) Identify potential collisions and launch an alarm for the OOWs on board or operators in Vessel Traffic Service (VTS) (Goerlandt, Montewka, Kuzmin, & Kujala, 2015; Kao, Lee, Chang, & Ko, 2006);
(2) Trigger the autonomous system on board to find evasive actions (Kuwata, Wolf, Zarzhitsky, & Huntsberger, 2014);
(3) Evaluate the risk of alternative paths or evasive actions (Johansen, Perez, & Cristofaro, 2016).

The first two applications relate to risk-informed decision making (Zio & Pedroni, 2012) and the last one focuses on the risk-based decision making that minimizes the risk of actions. Moreover, conflict detection has another application of identifying collision candidates. Readers can find more details in (P. Chen, Huang, Mou, & van Gelder, 2019).

Collision risk in this article refers to the probability/likelihood of collision. Two main categories are identified from the literature, namely "**expert-based method**" and "**model-based method**". The first group of studies directly utilizes experts' knowledge to assess collision risk. As a result, the measured risk reflects the belief of experts about collision event. The second group of methods assesses the probability of collision event based on a simplified model describing the physical process of collision. Consequently, the measured risk is a conditional probability of collision. In each group, the representations of the collision risk to human are various, which includes **numerical representation** and **graphical representation**. Some researchers prefer to present the risk as a number, which is considered to be a numerical representation; while the others show the risk in a two-dimensional map, e.g., rings of warning, action lines, etc., that is treated as a graphical representation. In the following sections, we will overview the methods of collision detection in these four categories.

4.1 Expert-based method

4.1.1 Collision Risk Index

This category of methods usually sets off a collision alarm based on a numerical value called collision risk index (CRI). When this index violates a pre-set threshold, a collision alarm is launched. Researchers usually invite experts, such as captains, pilots, etc., and learn their experiences to determine the measures, models, and thresholds. As a result, the meaning of collision alarm usually can be interpreted as *the situation when most of the experts believe the ship is in danger now*.

The most popular method in practice is to utilize two indices to measure the risk, namely Distance to Closest Point of Approach (DCPA) and Time to CPA (TCPA). Specifically, researchers use a polynomial equation to combine the value of DCPA and TCPA into one number, which is the CRI. In some studies, researchers introduce data processing first and then use the results in the equation. The data processing refers to the square of measures (Kearon, 1979), non-dimenisionalization considering ship length and speed (Lee & Rhee, 2001), etc. A generalized form of CRI measurement is shown as follow:

$$CRI_1 = w_1 f\left(DCPA\right) + w_2 f\left(TCPA\right),$$

where w_1 and w_2 are the weights. The determination of these weights is based on the experts' knowledge. To integrate experts' knowledge in the measurement, various techniques have been used, e.g., Fuzzy Theory (Lee & Rhee, 2001), Probit Regression (Chin & Debnath, 2009), etc.

Many researchers follow this way of thinking while making some improvements. Firstly, more risk indicators (RI) are introduced, which show more details of encounters, such as relative distance (B. Li & Pang, 2013), relative bearing (Y. X. Zhao, Li, & Shi, 2016), ratio of speeds (Gang, Wang, Sun, Zhou, & Zhang, 2016), ship domain (Ahn, Rhee, & You, 2012), etc. Secondly, more techniques are used to construct the CRI measurement, such as Multilayer Perceptron (Ahn et al., 2012), Analytical Hierarchical Process and Evidential Reasoning (Y. X. Zhao et al., 2016), Support Vector Machine (Gang et al., 2016), and Dempster–Shafer evidence theory (B. Li & Pang, 2013), etc. Thirdly, researchers aware that the risk measurement needs to adapt to different scenarios(M. Baldauf, Benedict, Fischer, Motz, & Schroder-Hinrichs, 2011). For instance, different nature-environment conditions, like wave conditions, visibility, day/night condition, operation area, etc., and different encounter types (Hilgert & Baldauf, 1997). Paper (Goerlandt et al., 2015) showed a demonstrate that using the improved CRI measurement in collision alert, where the authors utilized 16 indicators in five different encounter scenarios and the RIs and their weights are adapted according to experts' preference via fuzzy theory. This type of risk measurement is generalized as:

$$CRI_2 = \frac{\sum_i w_i f_i\left(RI_i\right)}{\sum_i w_i}.$$

Different from the polynomial models, some researchers accept non-linear relationships to measure the CRI. Some of them keep DCPA and TCPA as RIs, while the other introduces new RIs. For example, in (Lisowski, 2002), DCPA, TCPA, and relative distance (d_{ij}) are used and safety distance is added. The form of measurement is shown as follow, which contains a Euclidean norm:

$$CRI_3 = \left[w_1 \left(\frac{DCPA}{d_s}\right)^2 + w_2 \left(\frac{TCPA}{T_s}\right)^2 + w_3 \left(\frac{d_{ij}}{d_s}\right)^2 \right]^{-\frac{1}{2}}.$$

Based on this model, paper (Szlapczynski, 2006) introduced ship domain to replace a constant safety distance. In (Mou, van der Tak, & Ligteringen, 2010), researchers incorporate the basic risk level (r_{basic})

in the measurement which shows the risk level in a certain water area. Their CRI is increasing exponentially as the decrease of DCPA and TCPA, which is formulated as:

$$CRI_4 = r_{basic} e^{-TCPA/10} e^{-|DCPA|} \cdot \mathrm{F}_{angle},$$

where F_{angle} is scale factor determined by different encounter scenarios. This idea is developed in (Ren, Mou, Yan, & Zhang, 2011) and a Fuzzy logic is introduced to determine the coefficients. Instead of using formulations, some researchers design a pre-set matrix table to determine CRI value (Ożoga & Montewka, 2018). The other group of researchers abandons T/DCPA in measurement and uses some observable variables as RIs, such as relative distance (d_{ij}), relative speed (v_{ij}), etc. In (Perera & Soares, 2015; Wen et al., 2015), researchers found the inner product of v_{ij} and d_{ij} shows the tendency of relative movement (converge or diverge) and the magnitude shows the speed of this movement. Both are useful to describe the collision risk. Thus, the CRI is measured as:

$$CRI_5 = \cos\left(v_{ij}, d_{ij}\right) = \frac{v_{ij} d_{ij}}{\left\|d_{ij}\right\| \left\|v_{ij}\right\|}.$$

If CRI is positive and approaching 1, two ships are converging, and the collision is likely to happen; otherwise, they are relatively safe. In (W. Zhang, Montewka, & Goerlandt, 2015; W. B. Zhang, Goerlandt, Montewka, & Kujala, 2015), v_{ij}, d_{ij}, and encounter angle (θ_{ij}) are used to measure the risk, which is formulated as:

$$CRI_6 = k d_{ij}^{-1} v_{ij} \left(m \sin\left(\theta_{ij}\right) + n \sin\left(2\theta_{ij}\right)\right),$$

where k, m, and n are parameters determined via supervised learning. In particular, selected data samples from some encounter scenarios are used to determine a group of parameters which fits the data best.

4.1.2 Warning Rings by Ship domain

This category of methods usually visualized the collision risk by a set of warning rings surrounding the OS. When a target ship enters or will enter this region (i.e., CPA inside of this region), a collision alarm is triggered. The determination of the warning ring is related to the concept called **ship domain,** a region surrounding one ship that the OOW prefers to keep it clear from obstacles (Fujii & Tanaka, 1971). More details of ship domain refer to (Szlapczynski & Szlapczynska, 2017b). According to the definition of ship domain, this concept reflects the experts' belief about the minimum safety region. Therefore, ship domain is not suitable for performing as a collision criterion directly. If, in particular, one ship used ship domain as the ring of warning, the collision alarm might be too late for a ship to keep its ship domain clear (Davis, Dove, & Stockel, 1980). However, it can be used to detect collision danger in indirect ways. The following three ways are popular in the literature.

(1) Type I: Trajectory of the TS w.r.t. ship domain.

Collision danger is determined by comparing ship domain and predicted trajectory. When the predicted trajectory of the TS crosses the domain, then a collision alarm is triggered. Since ship domain is a psychological barrier (Fujii & Tanaka, 1971) and does not have a hard boundary between safety and danger (J. S. Zhao, Wu, & Wang, 1993), researchers suggested to use the fuzzy ship domain with fuzzy boundaries, see (J. Zhao, Tan, Price, & Wilson, 1994; J. S. Zhao et al., 1993), to fit the OOW's judgments. This idea is popular in recent years. For example, paper (Pietrzykowski, 2008) investigated ship fuzzy domain in narrow fairways; In (Pietrzykowski & Uriasz, 2008), the authors proposed a ship fuzzy domain adapted to various encounter scenarios. The collision risk level, then, is presented by the degree of the predicted trajectories violating the fuzzy boundaries.

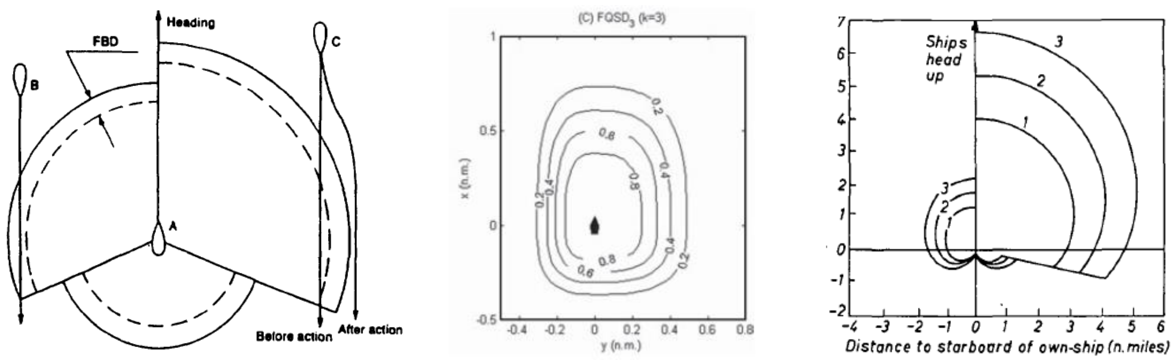
(2) Type II: Position of the TS w.r.t. ship domain.

This type of studies triggers collision alarms by comparing the position of the TS with ship domain (or expanded domain (Tam & Bucknall, 2010; L. Zhang & Meng, 2019)). Some researchers follow the idea of fuzzy ship domain and mark fuzzy boundaries with different risk levels. This type of method is called Spatial Collision Risk (SCR) (N. Wang, 2010). The SCR level is determined by experts' knowledge using Fuzzy set theory (N. Wang, 2012), specifically Gaussian membership function. The SCR of one TS depends on the position of the TS. As the TS approaches the OS, the SCR rise to 1. Then, researchers suggest the OOW take evasive actions when the SCR is too large. However, how to determine a threshold for SCR is not mentioned in the literature. Instead of using fuzzy ship domain, paper (L. Zhang & Meng, 2019) employed a probabilistic ship domain based on historical AIS data to assess the collision risk.

The other researchers developed an expanded ship domain, called 'arena' (Davis et al., 1980), where the violation of this region implies the OS needs to take evasive actions. The original arena is enlarged from ship domain considering questionnaires from the experts and navigational regulations. Later on, some researchers incorporated maneuvering performance in formulating the arena, such as (Colley, Curtis, & Stockel, 1983).

(3) Type III: Overlapping of ship domain.

In this type of methods, the overlapping of ship domain is used to launch collision alarms. Two modes have been found in the literature: some researchers proposed the ship should take evasive actions as long as two ship domains are overlapping, such as (Kijima & Furukawa, 2003); while, the other suggests to estimate the secant line of two ship domains first and then trigger an alarm if the length of the secant line is increasing and the change of slope is smaller than 1 degree, e.g., (Kao et al., 2006).



(1) Fuzzy ship domain (2) Spatial collision risk (3) Ship arena

Fig. 5 Fuzzy ship domain, spatial collision risk, and ship arena

Three similarities of the three types of studies are collected as follows: (1) these studies are relying on the determination of ship domain; (2) the boundary of ship domain is modified to represent different risk levels, where the experts' knowledge are usually employed; (3) they are visualized in a map for the OOW to understand the risk level.

4.2 Model-based method

This category of methods usually has a given encounter scenario and address some truths of given scenarios to the OOW by one indicator or risk map. These truths refer to collision happen/not happen or the probability of collision in given scenarios. The violation of risk threshold means "*provided the scenario is true, the probability of collision is unacceptably high*".

4.2.1 Binary collision criteria

Binary collision criteria usually offer a deterministic result to users about collision event, i.e., happen/not happen, based on a given scenario. DCPA is a popular criterion, which is based on the scenario that the TS keeps its velocity constantly, and the shape of the ships is a circle. In this scenario, if the DCPA is smaller than the assumed safe distance, the collision is going to happen; otherwise it not. Although this scenario is highly unrealistic, it is widely used for manned and unmanned ships (Shah et al., 2015). Besides, it is also an important risk indicator in CRI measurement (see Section 4.1.1).

4.2.2 Probability of collision

Since the process of collision is not deterministic, it is natural to use probability to describe the collision risk. In particular, many uncertainties might influence the result of dangerous encounters, e.g., sensors errors, environmental disturbances, reactions of the TS, errors in prediction, etc. However, considering all of these uncertainties is impossible, especially some of them are difficult to measure. Alternatively, researchers usually presume the system is known, and only one/part of uncertainties are considered. Moreover, the probability distributions are assumed to be known in advance. Hence, the probability of collision with given uncertainties can be calculated. In (Shah et al., 2015), Monte-Carlo simulation is employed to assess the probability. In (Park & Kim, 2016), a concept of probability flow is employed to calculate the probability of collision which can save some computational time. These uncertainties might come from sensors and environment disturbance (Park & Kim, 2016; Shah et al., 2015). However, how to set a threshold to launch collision alarm is not mentioned.

4.2.3 Dangerous Region

One group of methods aims at collecting a set of the OS's speed or course that is leading to collisions with the TS and presenting this set in the figure to the OOW. A collision alarm is triggered when the current velocity of the OS is inside of this set. Researchers have given various names to this set, e.g., Collision Threat Parameter Area (CTPA) (Lenart, 1983), Collision Danger Sector (CDS) (Pedersen et al., 2003), Velocity Obstacle (VO) (Huang, van Gelder, & Wen, 2018), etc. In this article, this group is named as Dangerous Region in Velocity-space (DR-Vspace). At early stages, to construct the CTPA/CDS set, the researchers accepted some restrict assumptions, e.g., the TS keeps a constant speed and course and the TS and the OS are shaped as circles, etc. (Lenart, 1983; Pedersen et al., 2003). Recently, researchers released some of these settings and expanded its applications in maritime practice. For instance, in (Szlapczynski & Szlapczynska, 2015), ship domain is introduced; in (Huang et al., 2018), the TS is allowed to change its speed and course; in (Huang et al., 2019), the maneuverability of the ship is considered, etc. Since the VO set collecting dangerous velocities in solution space, some researchers proposed to use the percentage of the safe solutions, e.g., velocity (Huang & van Gelder, 2019) or course (You & Rhee, 2016), to represent the collision risk.

The other group of studies directly presents a dangerous area of one TS to the OS in the workspace. The dangerous area is usually placed at CPA. A collision is launched when the current velocity of the OS might lead to the OS violate this area. Since these methods using workspace instead of velocity space, we name this group of studies as DR in work-space (DR-Wspace). One representative method is called Predicted Area of Dangers (PAD) (Zhao-lin, 1988) or Projected Obstacle Area (POA) (Gerhart et al., 2006). Other used concepts include: Obstacle Zone by Target (OZT) whose the size and the position are determined by a joint probability (Fukuto & Imazu, 2013; Kayano & Kumagai, 2017), Fuzzy Collision Danger Domain (FCDD) that considers multiple factors to determine its size (Su, Chang, & Cheng, 2012), etc.

4.2.4 Action Lines

Another group of studies focuses on identifying an action line surrounding the OS in geographical space. This line indicates the last chance for the OS to avoid collision via a fix evasive action, e.g., a hard-port turn, etc. This concept is similar to the above-mentioned arena in Section 4.1.2. However, the determination of the action line depends on simulations rather than experts' judgment. These studies presume that the TS keeps its initial speed and the OS takes a fixed evasive action, e.g., a hard-port turn.

By multiple simulations, a set of initial positions of the TS that the OS can avoid collision with via the fixed action is found, i.e., action line (Szlapczynski, Krata, & Szlapczynska, 2018) or critical distance (Krata & Montewka, 2015). A similar idea is presented in (Michael Baldauf, Mehdi, Fischer, & Gluch, 2017), in which the line is named as the last line of defense. By repeating the simulations with different fixed evasive actions, a series of action lines are obtained, which can be used for collision alarm.

The common points of Dangerous Region methods and Action Line methods are (1) they are not dependent on experts' judgment; (2) they strongly depend on pre-set scenarios; (3) the output of these methods is presented in a graphical figure.

4.3 Comparison

Measuring collision risk during an encounter is challenging. The challenges mainly come from the uncertainties in the encounters, e.g., the intentions of the OOWs in TSs, the performance of machines (OS and TS), the environmental conditions, etc. As a result, obtaining an accurate collision risk at each time step is difficult if not impossible. However, risk measurement is indispensable to support human and machine in collision prevention, e.g., collision alert, trigger a collision avoidance. Thus, the above methods have been proposed to construct the collision risk, which basically follows two lines of thinking.

The first line of thinking considers the uncertainties by employing experienced experts to assess the collision risk. The measured risk presents a general belief of a group of experts, and it can reflect part of the collision. More importantly, it allows experts to share their situational awareness with OOWs or VTS operators (Goerlandt et al., 2015). Thus, it is usually served as a **risk-inform tool** which helps the OOW to identify the potential dangers that are believed to be dangerous by most experts. However, these methods also suffer from some drawbacks: (1) eliminations of bias from experts are challenging; (2) the risk only reflects the belief of experts rather than the physical process of the specific collision, e.g., collision is going to happen or not.

The other eliminates the uncertainty by simplifying a real encounter into an ideal scenario considering parts of uncertainties or no uncertainty. This type of research then evaluates the risk of given scenarios. The ideal scenario can be the worst case, normal case, or the most possible case (Kuchar & Yang, 2000). This type of research might not give the OOW a real probability of collision, but it provides some facts about the given scenarios, which is also helpful in collision avoidance. The main advantage of these methods is offering the users a clear conclusion which is easy for users to use it, e.g., DCPA, last possible time to take certain evasive action (Michael Baldauf et al., 2017), etc. However, the drawbacks of these methods are also obvious: (1) the simplified scenario cannot reflect the complete environment that the OS faces and cannot cover all scenarios; (2) it is difficult to meet various demands of different users. For instance, some people are more sensitive to dangers and they might require a lower threshold of risk, whereas, others are risk takers and need a higher threshold. Although the assumptions of these methods are unrealistic, these methods can help the OOW to know some facts of given scenarios, which can also be helpful for **risk-based decision making**. Thus, these methods are widely used in navigation practice and expert-based risk assessment, e.g., DCPA, CTPA, etc.

Two representations of collision risk are presented in this article. One uses a digital number to represent the risk, the other uses a graphical form to show the risk. When the risk is presented in digital numbers, it can be used to compare with each other. For instance, a higher number refers to high risk and vice versa. On the other hand, the graphics-based form can categorize the TS into various groups, while the TSs in the same group are not comparable by this method. The graphics-based form is more intuitive for users (Szlapczynski & Szlapczynska, 2017b), and the graphics risk is usually integrated into the maps to support users to be aware of the surrounding situation.

4.4 Discussion

Since the binary-based method is relatively simple and offers some facts about the identified scenario, this method is widely used in conflict detection for the ASVs. However, as the other review mentioned, an open question is incorporating collision prevention with the navigational regulation, i.e., COLREGs (Hilgert & Baldauf, 1997). The conflict detection is employed to trigger evasive actions, which also need to comply with regulations. However, the regulations are general and designed for the human. Thus, teaching the ASV to perform as human, e.g., assess collision danger incorporating with regulation, the role of human cannot be ignored. In this perspective, the experts' knowledge is necessary to be introduced. However, this problem still needs more research (Woerner, Benjamin, Novitzky, & Leonard, 2018).

In the manned ship, conflict detection is mainly servicing as reminders for the OOWs, which needs to be adapted to human's performance. For example, the navigators who are risk taker might accept some risky scenarios, while others might see these scenarios as dangers. Thus, the expert-based methods are widely presented in studies for supporting the manned ship, and Fuzzy methods are popular in this type of studies. However, this type of collision criteria cannot express more details about the collision process. It cannot give an explicit judgment about collision happening or not to users. Thus, researchers suggest using some model-based methods. One representative method tries to find the last moment to execute certain maneuvers. To summarize, the expert-based method can offer customized services for different navigators, which can meet users' preference. On the other hand, the model-based method usually addresses some relative objective fact, which shows the hard boundary of safety and danger. These two methods can be combined to offer a better service to manned and unmanned ships.

5. Conflict Resolution

Conflict resolution is the core of collision prevention, which determines collision-free solutions for the ship. Many methods have been developed for this purpose. However, most of them are similar, even though these methods appear to be quite disparate (Siegwart, Nourbakhsh, & Scaramuzza, 2011). In this section, similar methods are collected in the same groups, and six groups of ship collision avoidance techniques are identified, namely

(1) **Rule-based method** uses "If-then" rules to guide the collision avoidance process;
(2) **Virtual vector method** generates a virtual vector field to determine the motion of the ships;
(3) **Discretization of solutions with collision check method** searches the discrete solution-space and find a collision-free solution or an optimal solution.
(4) **Continuous solutions with collision constraints method** formulate collision as constraints and find the optimal solution in continuous space;
(5) **Re-planning method** formulates the collision avoidance as a path planning problem and searches collision-free path in free configuration space;
(6) **Hybrid method** combines some of the previous methods in collision avoidance.

In the following, the details of these groups and their application in maritime are presented.

5.1 Main Algorithms

Some terminologies used in the description of these algorithms are addressed here. The workspace of the ship is defined as a horizontal plane consists of positions. Configuration space (C-space) is the set of all possible configurations of the OS, which consist of free space C_{free} and obstacle space C_{obs} where the configurations lead to collisions.

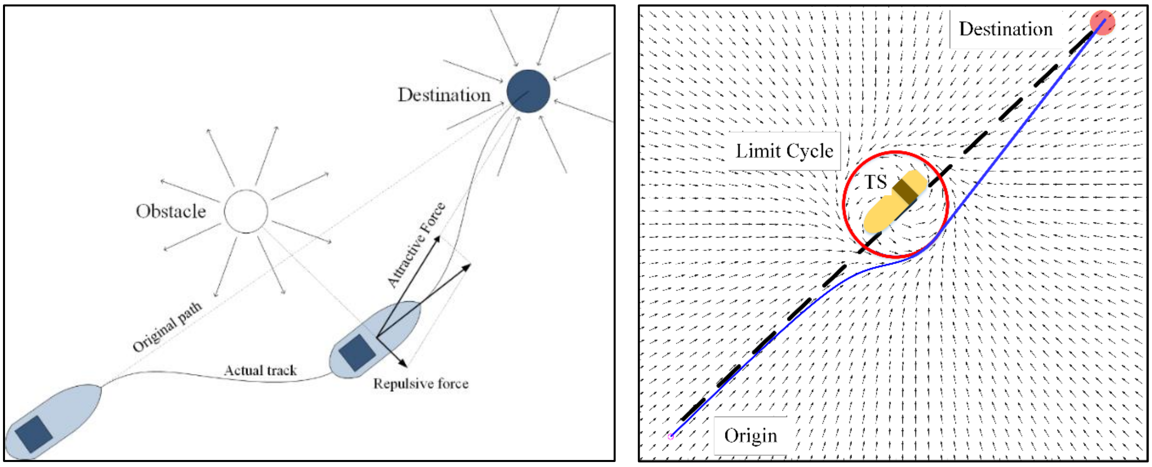
5.1.1 Rule-based Method

Rule-based methods use a set of pre-set rules to guide collision avoidance. For example, when a ship encounters with other ships, the ship will turn 75 degrees (or 30 degrees) to the starboard side (right-hand side) (Naeem, Irwin, & Yang, 2012; Tam & Bucknall, 2013) or enlarge rudder angle until the trajectory is collision-free (Fang et al., 2017).

Obviously, a single rule cannot handle all kinds of encounters in a dynamic environment. Thus, multiple rules are considered. A widely used way is incorporating International Regulations for Preventing Collisions at Sea" (COLREGs) and good seamanship in the rule system. This system is expected to suggest rule-compliant actions for the OS in various scenarios, which is usually based on Neural networks (Praczyk, 2015), Fuzzy logic and Bayesian network (Perera, Carvalho, & Soares, 2012). Since the enumeration of rules for all scenarios is impossible, this method does not guarantee collision-free. If a case is not studied in advance, this method might not find out a proper solution.

5.1.2 Virtual vector field Method

Virtual vector method generates a virtual field to determine the OS's motion. Two specific algorithms are found: Artificial Potential Field (**APF**) and Limited Cycle Method (**LCM**).



(1) Artificial Potential Field (APF) (2) Limited Cycle Method (LCM)

Fig. 6 Illustration of virtual vector field Methods (1) APF and (2) LCM.

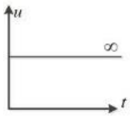
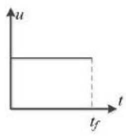
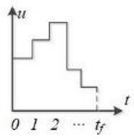
APF (Khatib, 1985) or virtual force field (Borenstein & Koren, 1989) generates a repulsive potential field around the obstacles and an attractive potential at the destination. The sum of these potential fields determines the resultant virtual force to guide the motion of the vehicle. This algorithm does not directly provide a collision-free path but a direction of motion, which is not designed for a dynamic environment at the beginning. To avoid collisions in a dynamic environment, researchers improve this basic algorithm by considering two factors: the velocity of obstacle and the maximal deceleration of the vehicle (Ge & Cui, 2002). Specifically, the repulsive potential of the obstacle is enlarged by considering these factors. This technique was also applied in ship collision avoidance, see (H. Lyu & Yin, 2018; H. G. Lyu & Yin, 2017). One main disadvantage of this method is that the ship might be trapped in a local minimum. The conditions leading to local minima have been concluded in (Zeng & Bone, 2013). Additionally, the dynamics of ships are not fully taken into account, where the ship is assumed to be holonomic.

LCM uses a stable limit cycle for motion planning. The stable limit cycle has a feature that all the neighboring points are attracted to the cycle. Researchers place the center of a limit cycle at the center of the obstacle, which encompasses the whole obstacle. A series of trajectories converge to the cycle is generated. Then, the OS follows one of these trajectories until the destination is clear. The ellipse and circle-shaped cycles are used in the (Soltan, Ashrafiuon, & Muske, 2009; Soltan et al., 2010) for ship motion planning in rough sea conditions (Mahini, DiWilliams, Burke, & Ashrafiuon, 2013). However, this algorithm usually requires the obstacles are relatively static, or the speed of the OS is much greater than the obstacles; and it only avoids one obstacle at each time. These issues might be problems when the OS encounters multiple obstacles, or an obstacle's speed is larger than or equal to the OS.

5.1.3 Resolution search in discretizing solution-space with collision check

Another group of methods discretize the solution-space of the ship and eliminate the dangerous solutions by collision check. The collision-free solution is chosen from the rest. These studies assume the motion model of the OS is known, and the environmental disturbance is temporarily neglected. Then, given a control input and its duration time, the trajectory of the OS is deterministic, and the collision check becomes possible. These types of methods discretize the solution-space first, and then different algorithms differ in duration time of the inputs, see Table 1.

Table 1. Three groups of methods using discretization of solution-space

Name of methods	Inputs	Duration	Illustration	Applications on board
Decision Disc (DD)	(ψ,u)	$[0,\infty)$		(Benjamin et al., 2006; Degre & Lefevre, 1981; Kuwata et al., 2014; Lenart, 1983; Pedersen et al., 2003; Szlapczynski, 2008; Szlapczynski & Krata, 2018)
Dynamic Window (DW)	(u,r)	$[0,t_f]$		(Loe, 2008; Serigstad, 2017)
Discrete inputs Optimization (DIO)	$(\psi,u)/\delta$	$[0,t_f]$		(Johansen et al., 2016; S. Li, Liu, Cao, & Zhang, 2018)
Lattice-based Search (LBS)	(ψ,u)	$\{0,1,\ldots,t_f\}$		(Shah et al., 2015; Švec, Thakur, Raboin, Shah, & Gupta, 2013)
Brute-force search (BFS)	ψ	$\{0,1,\ldots,t_f\}$		(J. F. Zhang et al., 2015)

(1) Each control input keeps constant in the future

Decision Disc (**DD**) approach chooses course and speed as control inputs to the ship and presents the solution-space as a disc. The control input is assumed to be unchanged in the future. Thus, each input represents a unique trajectory. If this trajectory is collision-free, the control is reserved; otherwise, the control is rejected. Then, the collision-free controls are directly presented to the officers (Degre & Lefevre, 1981), or an optimal solution is chosen from those collision-free solutions by optimization (Benjamin et al., 2006). In this process, DCPA is usually employed for the collision check. This method has different names in different studies, e.g., Collision Threat Parameter Area (CTPA), Collision Danger Sector (CDS), etc. (Lenart, 1983; Pedersen et al., 2003). When being used in practice, this method also incorporates with ship domain (Szlapczynski, 2008), restricted waterways, worse environmental scenarios (Szlapczynski & Krata, 2018), etc. A similar technique is also applied in an ASV (Kuwata et al., 2014), where the irregular shape of the obstacles, sensing errors, and the COLREGs are considered. The main disadvantage of these studies is neglecting the kinematic and dynamic constraints of the OS, which might lead the method to fail to avoid collisions in close range encounters.

(2) Each control input keeps constant in a given time window

Dynamic Window approach (**DW**) (Fox, Burgard, & Thrun, 1997) chooses velocity tuples (u,r) as inputs (*u* is linear velocity, and *r* is angular velocity) and presumes the chosen input is fixed in a given time step. The construction of the dynamic window includes two steps: firstly, all the tuples that the OS can reach in given time step are selected as an initial dynamic window, in which velocity and acceleration constraints are considered; secondly, the initial dynamic window is reduced by keeping those tuples that ensure the vehicle can stop before hitting with obstacles. The remaining tuples contain all admissible velocity, which constitutes the dynamic window. The optimal collision-free velocity tuple is searched in this dynamic window. The limitations of original DW include: susceptible to local minimum, assumption on circular arcs path, frozen environment during decision time step, etc. (Martinez-Gomez, 2010) Some improved algorithms solve these problems: (Brock & Khatib, 1999) incorporated connectivity of free space to avoid local minimum in somehow; (Seder & Petrovic, 2007) extended DW to deal with moving obstacles; (Loe, 2008) demonstrated DW with non-circular paths and applied the method in ASVs; (Serigstad, 2017) modified the original DW considering the dynamic of ASVs. However, either the original DW or the variations hold a critical assumption that the static state is always a safe state for the vehicle, which might not be held in practice, especially when a ship is crossing a clutter intersection.

Discrete-Inputs Optimization **(DIO)** use a group of solution candidates as representatives of the solution-space and find a collision-free solution by optimization. To avoid the obstacles, the cost function assigns a big cost to the trajectory closes to the obstacle. The ship obtains an optimal solution and the corresponding trajectory by minimizing the cost function (Bertaska et al., 2015). An example of this method is shown in (Johansen et al., 2016) where Model Predictive Control (MPC) with soft collision avoidance constraints in the objective function is used. Instead of applying this optimal trajectory, MPC applies the optimal control at the first stage and update the optimization to obtain a new optimal solution/trajectory, more examples see (S. Li et al., 2018; Sun, Wang, Fan, Mu, & Qiu, 2018). For these methods, how to balance the optimality and computational time is a challenging topic.

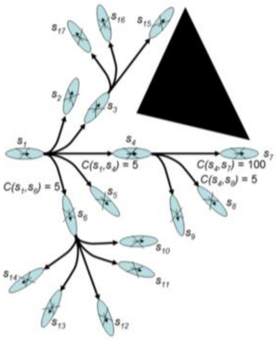


Fig. 7 Lattice-based graph (the figure is from (Kushleyev & Likhachev, 2009))

(3) Control input changes at each time step

Lattice-based Search **(LGS)** method allows control inputs to change at each time steps (Kushleyev & Likhachev, 2009; Pivtoraiko, Knepper, & Kelly, 2009). Since this type of algorithms is time-consuming, researchers usually search some representative candidates rather than the whole solution-space in each step. In a time step, these control candidates will generate a graph that consists of the trajectories of the OS with different controls. This graph can be reused to generate all the paths of the OS in the next time step, which saves the computational time. In return, a lattice-based graph is obtained, see Fig. 7. By searching this graph, a collision-free trajectory and the corresponding inputs in each time step are obtained. Papers (Shah et al., 2015) and (Švec et al., 2013) applied this method in ship collision avoidance. The solution-space is defined as (u_d, ψ_d) where u_d and ψ_d are desired surge speed and course of the ship. Ship dynamics are considered to generate the graph in their research. The main disadvantage is that searching all the branches in the graph are computationally expensive, which might not be used for real-time collision avoidance. To reduce the computation, (Shah et al., 2015) considered a small discrete solution-space with 30 solutions and employed risk assessment to assign a priority search direction in the graph.

Brute-force search **(BFS)** method is also one of the options. Solution-space and time horizon are discretized into cells. Each cell contains a combination of solutions and its duration time. Correspondingly, a path of the vehicle can be generated, in which constraints on kinematic and dynamic can be considered. A collision check is used to abandon or keep the cell. Since, the computational complexity is a big issue, especially for online decision making, some modifications are needed. For example, in (J. F. Zhang et al., 2015), only one input is considered (i.e., desired course) and the course and time horizon are searched in the range [30°, 60°] and [180 sec, 900 sec]. The search starts in the time direction and stops when the first collision-free cell is found. The main drawbacks are that it has high computational complexity, and the solution is not optimal.

5.1.4 Resolution search in continuous solution-space with collision constraints

Some methods are not relying on the discretization of the solution-space and find the collision-free solutions in a continuous solution-space. Two groups of methods are found according to the order of collision check.

(1) Identify collision-free solution-space first

This group of methods usually conduct a collision check first and then find an optimal solution in a collision-free space. Instead of checking the solutions one by one, these methods use a polygon/circle to represent an obstacle and then conduct additional operations to formulate a set of control inputs that lead to collisions. Then, an optimal solution can be obtained accordingly.

Velocity obstacle (**VO**) algorithm is one popular algorithm proposed in (Fiorini & Shiller, 1998). A circle is used to represent an obstacle, and researchers assume the obstacle moves with constant speed and course. In return, they formulated a set of velocities that lead to the relative velocity point to the obstacles as a VO set which is shaped like a cone. Due to this simple shape, each VO set can be easily formulated by three linear constraints (Alonso-Mora, Beardsley, & Siegwart, 2018). The idea of VO algorithm has been adopted in maritime research, e.g., (Zhuang et al., 2016). Three key disadvantages of the original VO algorithm that pointed out by researchers are: (1) the algorithm hold the assumption that the velocity of obstacle has to be constant in the future; (2) the dynamic constraints on the OS and the TS are usually out of consideration; (3) the shape of obstacle is assumed to be regular and convex, e.g., circle, ellipse, etc.

Non-linear velocity obstacle (**NLVO**) (Large, Sekhavat, Shiller, & Laugier, 2002) extended the VO algorithm, where the motion of obstacle is not necessary to be linear but known in advance. Different from the VO algorithm, the NLVO algorithm aims at finding the velocities that lead the OS to violate the safety area around the TS at time *t*. Those velocities are collected in a sub-set, and the envelope of these sub-sets that results in collisions at different time slices is the NLVO set. In the maritime domain, paper (Huang et al., 2018) proposed to use NLVO algorithm and its development to support ship collision prevention. These studies, however, still assume the vehicle is holonomic. A generalized velocity obstacle (**GVO**) is proposed to consider the non-holonomic constraints and dynamic constraints (Bareiss & van den Berg, 2015; Wilkie, Berg, & Manocha, 2009), which has been also applied in the maritime environment (Huang et al., 2019).

Vision Cone (**VC**) method is an enlarged collision cone (Chakravarthy & Ghose, 1998) with a buffer angle (Savkin & Wang, 2013). The courses are chosen as inputs, and two collision-free courses are identified by the vision cone, which are the boundaries of the cone. The optimal collision-free course is chosen from two collision-free courses. This algorithm can deal with moving obstacles whose velocity is constant. In (Y. Z. Xue, Clelland, Lee, & Han, 2011) and (Fan, Sun, & Wang, 2019), this idea has been applied in ASV. In (Martin S. Wiig et al., 2017) and (M. S. Wiig, Pettersen, & Savkin, 2017), the authors expanded the algorithm for the under-actuated unmanned ship. The algorithm guarantees safety, while it only deals with one obstacle and requires the speed of obstacle smaller than the speed of the OS.

(2) Optimization with Model Predictive Control in discretizing and finite time horizon

Another group of methods usually conduct a collision check and optimization together. Specifically, the collision check is formulated as a hard constraint in the optimization, e.g., $x(t) \in C_{free}$. A general form of this optimization is formulated as:

$$
\begin{aligned}
&\min_{u \in U} J(u, x) \\
\text{s.t.: } & x(k+1) = f\left(x(k), u(k), k\right); \\
& x(k) \in C_{free}, \forall k \in \{0,1,2,\ldots t_f\}; \\
& x(0) = x_0;
\end{aligned}
\tag{5}
$$

where J is a cost function depends on the control inputs and state of the system; $f(x,u,t)$ is the dynamics of the ship; C_{free} is collision-free configuration; x_0 is the initial state of the system (the ship).

In this optimization problem, time is discretized, the dynamics model of the OS is discrete, and control input at each time step is variable that can be used to optimize the cost function (L. Chen et al., 2018; H. Zheng, R. R. Negenborn, & G. Lodewijks, 2017; H. R. Zheng et al., 2017). The constraints of this optimal control problem include kinematic constraints, the dynamics of the vehicle, collision-free conditions, etc. The solution of this optimization problem is collision-free solutions for the OS, which is executable and stratifies collision-check. In literature, this method is usually combined with a receding horizon scheme. Thus, we name this group of methods as MPC-based Collision Avoidance **(MPC-CA)**. In (L. Chen et al., 2018), a linearized dynamic model of ASV and a linear objective function (infinity norm) were used in collision check, which helps to solve the optimization problem efficiently. Some researchers consider the non-linear dynamic model and nonlinear function of collision-check. In (Abdelaal et al., 2018), the authors used two circles to represent ships and solve the optimization problem via a direct multiple shooting method. Paper (Ferranti, Negenborn, Keviczky, & Alonso-Mora, 2018) used circles and ellipses to present the OS and obstacles, respectively, and solve the optimization problem via commercial solver. Though these studies show that this problem can be solved in a reasonable time, the solution is a local minimum due to the nonlinearity and non-convexity of the problem. Additionally, as time is discretized, collision-check between time steps is needed, which is noted as safety verification.

5.1.5 Re-planning method

Instead of searching the collision-free solution in solution-space, **re-planning method** searches solutions in workspace directly. The re-planning is triggered when the collision criteria reach pre-set thresholds (L. Hu et al., 2017). Two groups of methods are found that the first group replies on the graph searching methods while the other is not.

For the first group, at each time step, a graph searching algorithm is triggered to find an optimal collision-free path (Candeloro et al., 2017). Various algorithms based on this method have proposed. Two main differences among the algorithms are the assignment of cost in each cell/node and the approaches used to search the optimal solution. Some researchers assign a high cost to the cells surrounding the moving obstacles considering the speed of the obstacle, e.g., (Lazarowska, 2017; Y. C. Liu, Liu, Song, & Bucknall, 2017; Smierzchalski & Michalewicz, 2000), while the other assigns an area surrounding the Projected Obstacle Area (POA) with a high cost, e.g., (Gerhart et al., 2006). Based on the map of costs, researchers use searching algorithms to find an optimal path with lower cost, such as fast marching method (FMM)(Y. C. Liu et al., 2017), particle swarm optimization (Kang, Chen, Zhu, Wang, & Xie, 2018), etc.

Another group of re-planning methods directly use evolutionary algorithms to find a path, which does not directly depend on the graphical map. The employed evolutionary algorithms include evolutionary set algorithms (Szlapczynski, 2011), ant colony algorithm (Lazarowska, 2014), genetic algorithms (Tsou, 2016), etc.

5.1.6 Hybrid of algorithms

From previous sections, we have introduced the algorithms we found in the literature of ship collision avoidance. In fact, in maritime practice, researchers usually combine those algorithms to perform collision avoidance. For instance, the rule-based method is unusually combined in other algorithms, such as VO, DW, DD, etc., to make sure the behavior of the ship complies with regulations. In (Kuwata et al., 2014), the authors used VO algorithm and COLREGs to exclude the velocity resulting collisions and the velocity violating regulations. In (Song et al., 2018), VO algorithm was combined with APF which services as a global planner. In (H. G. Lyu & Yin, 2017), the rules-based method incorporates with APF for guiding the unmanned ship.

5.2 Comparison

The details of the comparison between the aforementioned methods are listed in Table A1 and Table A2 in Appendix A. This section mainly addresses the general feature of each group of methods.

5.2.1 Comparison across different algorithms

In general, Rule-based methods are simple and easy to conduct. However, these methods cannot enumerate all the scenarios, especially when encountering with multiple obstacles. For example, the navigational rules only address the obligations of ships in two-ship encounter scenario, while the ship might meet more than two ships and more complex environmental conditions.

Virtual vector methods contain APF and LCM, which usually ignore the ship's dynamics during motion planning. The APF method might get trapped in a local minimum, which is addressed in many studies. LCM method only considers one obstacle or one group of obstacles in each time, and the obstacles need to be stationary or have a relatively low speed. Nevertheless, researchers found that this method performs well with stationary obstacles in rough seas (Mahini et al., 2013). Both APF and LCM only show one solution to the ship, whereas this solution might not be optimal.

Discretization inputs with collision check is a big group of methods. These methods could take ship dynamics into account. Specifically, researchers assume the dynamics of the ship is known and predict the trajectory of the OS with different inputs. However, the calculation of each input is time-consuming. To solve this issue, some simplifications have been introduced. Some methods only consider the change of input at the first time step, see DW, DD, etc. These methods not only offer one optimal collision-free solution but also present a set of unsafe solutions at the first time step. Others only select several representative inputs and consider changes in more time steps, such as LGS and BFS. These methods offer one collision-free solution to the ship from the selected inputs. A common challenge of these algorithms is the balance between efficiency and effectiveness. A small grid would benefit a better solution, but it is time-consuming; A big grid saves computational time, but it might skip the optimal solution. Moreover, the quality of prediction is quite important for collision check, while a few of these studies discuss the impacts of uncertainties on these techniques.

Continuous inputs method is not relying on the discretization of the inputs. One group of algorithms collects all the solutions leading to collisions in a set and then conducts optimization, e.g., VO, NLVO, GVO, etc. The other considers the collision check as one constraint in optimization and directly solve the optimal solution by solvers, e.g., MPC-CA. The difference between these algorithms are forms of solutions. VO algorithm and its variations can visualize and present the unsafe solutions to users, which can help them to understand the choice of the optimal solution. Moreover, the ship is required to keep one solution in the whole prediction horizon. On the contrary, MPC methods directly offer one optimal solution which allows the ship to change course and speed in the prediction horizon. However, the optimal solution is calculated by solvers, which is not much transparent to users.

Re-planning method follows the idea of path planning, which constructs a cost map firstly and then searches an optimal path on the map. The method, however, cannot avoid the situation that a ship violates an inevitable collision state around the dynamic obstacle. An inevitable collision state is a state (positions) that one ship cannot avoid collision with others no matter what actions the ship performs. This method usually offers one optimal path for users, but the dynamics of the ship is ignored.

5.2.2 Remarks

The data flow in a conflict resolution module can be refined as Fig. 8. Inputs of this module include map information, trigger event from ‘conflict detection’ module, obstacles’ trajectories from ‘motion prediction’ module, the state of the system, etc. The output of the resolution module is the control input of the system (ship), which is determined by the algorithms presented in Section 5.1.

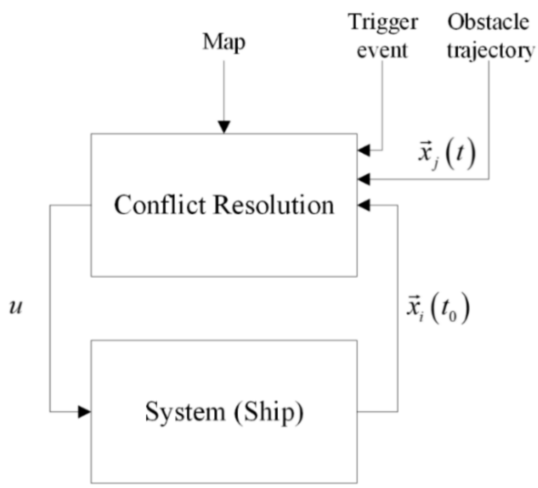


Fig. 8 Representation of information flow between the conflict resolution module and ship module

The given map shows the moving boundary of the ship, which contains navigable waters and static obstacles, such as rocks, islands, coastlines, etc. One popular way to represent the map is by using an occupancy map. The occupancy map transfers the Electronic Navigational Charts (ENC) into a bitmap, which is popular when using graph searching methods. The other popular way is using polygons to represent the obstacles in ENC, including convex polygons and non-convex polygons. Convex polygons, such as circles, ellipses, hexagons, and squares, are widely used to simplify the control problems. An overview of the representations of maps in existing methods is given in Table A2.

The trigger event in Fig. 8 is used to activate the collision-resolution module. In some algorithms, collision-resolution is computationally expensive and is only triggered when the predefined trigger event occurs, e.g., DD methods (Kuwata, Wolf, Zarzhitsky, & Huntsberger, 2011; Kuwata et al., 2014).

From the perspective of a ship, the trajectories of the obstacles are generated by one out of three types of methods in the 'motion prediction' module, i.e., physics-based method, maneuver-based method, and interaction-aware method. When we consider the collision avoidance from the perspective of the entire traffic system, different control architectures have been used to coordinate the behavior of the ships (Negenborn & Maestre, 2014). A 'decentralized' control system assumes that each ship solves collision problems independently, in which physical-based or maneuver-based methods are widely used. A 'distributed' control system allows ships to communicate and cooperate with each other, where interaction-aware methods are used. A 'centralized' control system has one central controller that decides on trajectories and control inputs for all the ships. The control architectures of existing research are presented in Table A2.

The solutions of 'conflict resolution' module follow three forms depending on the employed algorithms:

(1) the algorithm offers a collision-free maneuver, such as course, speed, velocity, yaw rate, etc. Most rule-based methods, APF, the original DW, DD, VO, and NLVO. result in this mode of solution.
(2) the algorithm offers one collision-free trajectory, while the maneuvers to follow the trajectory are not provided. The re-planning algorithms and LCM usually belong to this mode. These modes, however, ignore or simplified the ship dynamics, which might result in infeasible or unsafe solutions in practice.
(3) the algorithm offers a collision-free trajectory and the relevant controls. These algorithms incorporate ship dynamics, such as DIO, LBS, BFS, GVO, and MPC-CA. Additionally, some algorithms only offer one solution but not optimal, e.g., rule-based methods, BFS, APF, and LCM; some algorithms offer one optimal solution, e.g., re-planning methods, MPC-CA, and VC; other algorithms not only offer one solution but a set of alternative solutions, e.g., DW, DD, VO, and GVO.

The details of the solution form are presented in Table A1.

5.3 Discussion

5.3.1 Trends of conflict resolutions in recent years

In recent years, many techniques have been proposed and used to develop ASVs, which enriches the tools for solving the collision avoidance problem. Correspondingly, some highly unrealistic and strong assumptions, which were criticized by researchers, have been released. Some important factors, like environmental disturbance and regulations, are also discussed in some studies, e.g., (Szlapczynski & Krata, 2018; Huarong Zheng, 2016).

In the beginning, the dynamics of the OS is usually ignored, and the OS is seen as a holonomic vehicle; besides, the moving obstacles are assumed to be semi-dynamic. Recently, many algorithms are capable to consider the dynamic constraints, e.g., MPC-CA, GVO, DW, etc. In those methods, the motion model of the OS is known or identified in advance. Besides, with the improvements in prediction techniques (in Section 3), many algorithms do not request the moving obstacle to keep its initial speed and course.

In the previous review articles, researchers argued that environmental disturbance is a critical factor in collision avoidance, which is less discussed in relevant studies. In recent years, many studies try to take this factor into account. Two lines of thinking have been found in these studies to handle the disturbances. One considers the trajectory errors due to the environmental disturbance. Specifically, the disturbance might drive the OS to depart from its collision-free trajectory. By assuming a perfect knowledge about the bound environmental disturbances, the possible trajectories of the OS can be calculated or bounded in a tube. Then, the solution that leads to all these possible trajectories/tubes away from the obstacles is the collision-free solution under environmental disturbance, see (Johansen et al., 2016; Huarong Zheng, 2016). The other one concentrates on the changes of solution-space caused by environmental disturbances, e.g., some safe solution might get unsafe in a harsh environment. Researchers try to figure out all the unsafe/infeasible solutions due to the harsh conditions, e.g., (Szlapczynski & Krata, 2018). Specifically, researchers apply guidelines of navigation in adverse weather & sea conditions and simulators to identify the solutions that are not recommended or result in an excessive rolling. Then, these solutions are marked as dangerous solutions and blocked from the solution space.

Another pitfall had been highlighted in previous research was the lack of consideration of navigation regulation, e.g., COLREGs. In recent year, researchers introduce part of navigation rules in collision avoidance, e.g., (Johansen et al., 2016; Kuwata et al., 2014; H. Lyu & Yin, 2018; Perera et al., 2012), etc. Some popular rules are frequently used in finding a rule-compliant collision-free solution, i.e., Rule 6, 8, 13-19 from COLREGs. These rules clearly address the obligations of ships in two-ship scenarios. It seems that building a complete regulation-compliant ASV is close. However, the regulation is written for the human, which makes rules are open to some interpretation and difficulty to "translate" in machine language (Woerner et al., 2018). Thus, incorporating all the rules from COLREGs and good seamanship in an autonomous system is still an open question. Quantifying the entire regulations and good seamanship for the ASV still need more efforts in the future (He et al., 2017; Woerner et al., 2018).

5.3.2 Using studies of ASVs for supporting conflict resolution in the manned ship

Most of the conflict resolutions are developed for automatic collision avoidance. However, they have great potential to support the human on board: (1) offering one optimal/feasible solution; (2) checking the safety of human's inputted solutions; (3) showing all the unsafe solutions. The algorithms, such as BFS, LGS, MPC-CA, etc., can offer a feasible/optimal collision-free solution to the OOWs directly. Other algorithms, like DIO, can support human to check the chosen-solutions by the OOWs. The other algorithms which collect dangerous solutions and present them to users, such as DW and VO, can not only give an optimal solution but also validate the chosen-solutions. Although these techniques can facilitate the collision prevention process with the OOWs, not all the algorithms can directly apply to manned situations. The reasons are as follows:

Firstly, collision-free solutions that some algorithms find are not friendly for the OOW. For instance, MPC-CA offers a series of forces acting on the ship for collision prevention, while the navigators might not know the effects of these forces and how to steer the ship to generate such forces. Secondly, although some algorithms can offer a readable solution for human, they usually use ideal motion models, e.g., VO, NLVO, DD, etc. Consequently, collision-free solutions may fail in certain situations. Specifically, when the relative distance is not large enough to ignore the errors between real dynamics and ideal dynamics, the solutions these methods find may still lead to collisions (Huang et al., 2019).

Some studies containing two levels of controllers, a lower-level controller, and a higher-level controller, present an option to solve this problem, such as DW (Loe, 2008), LBG (Shah et al., 2015), GVO (Huang et al., 2019), etc. The lower level controller generates a series of commands on actuators according to the inputted desired forces, while the higher-level controller calculates the desired forces for tracking the reference and outputs the forces to the lower controller. The higher-level controller might perform a link between human and machine (Huang et al., 2019). For example, the OOWs do not need to read or implement the desired forces by themselves but use the higher-level controller. The ASV with higher-level controller presents the selected collision-free solution to human is a readable way, such as an optimal trajectory (by MPC-CA), speed and course (by DD, VO, etc.), or speed and yaw rate (by DW). The human, then, can read and intervene in the machine via changing the collision-free reference. This design makes the interaction between OOWs and ASV possible.

6. Discussion

6.1 Developments of collision avoidance in maritime research

In the related review papers, the authors have concluded some research gaps in collision avoidance techniques. Many efforts have been put to fill those gaps and have led to some changes. The details of these changes are discussed in previous sections, e.g., Section 3.3, 4.3, and 5.3. This section provides a summary:

- Environmental factors are taken into account in some studies. Specifically, researchers consider prediction errors when assuming the uncertainties are bounded or exclude maneuvers that are unsafe in harsh environments.
- Some rules from COLREGs have been considered. However, more efforts are needed to apply the whole COLREGs rules and good seamanship in ship collision avoidance, especially in multiple-ship encounters;
- Some algorithms can handle collision avoidance with dynamic obstacles, but the trajectories of the TSs have to be known or bounded;
- The dynamics of ships have been taken into consideration in many studies. Nevertheless, these models are usually known and deterministic;
- Efficiency and effectiveness have been considered, while less discussion on the trade-offs between them in different scenarios.

Together with these changes, we conclude the following limitations of existing techniques and challenges in developing new collision avoidance methods for both manned and unmanned ships:

(1) Uncertainties of ship motion models are usually ignored.

Most existing studies use deterministic dynamics models in collision avoidance. However, uncertainties are inevitable. Specifically, these uncertainties mainly come from unmodeled dynamics, changes of parameters due to different working conditions, uncertainties of external disturbances, etc.

In existing studies, we observed that some studies had discussed the environmental disturbances, but few of them consider the uncertainties by parameters and unmodeled dynamics. However, these factors cannot be ignored since the parameters and dynamics would change when a ship has different loading conditions or working environments. How to handle these uncertainties is challenging but essential for developing automatic collision avoidance systems on board.

(2) Developing rule-compliant navigation systems is still an open question.

Although many studies have implemented several rules when deciding collision avoidance actions, the research on the development of a completely rule-compliant system is still blank. Firstly, many complicated scenarios in practice might activate multiple rules in COLREGs. Choosing the most suitable rule is difficult for collision avoidance systems. For instance, in a multiple-ship scenario, the ship might have two conflict obligations, e.g., “give way” and “stand on”; the ship might encounter with multiple types of ships (e.g., fishing ship, sailing ships) which apply different rules. Secondly, the navigation rules, e.g., COLREGs, is written for the OOWs in human’s language (Woerner et al., 2018), which is a guideline without quantifying information for the machine. For instance, the ship is asked to keep at a safe speed, while the value of safe speed is not addressed in the rules.

In fact, the implementation of COLREGs strongly relies on experts' knowledge that is also noticed as good seamanship (He et al., 2017). A few studies focus on incorporating the seamanship in the navigation system, e.g., (Benjamin et al., 2006; He et al., 2017; Woerner et al., 2018). However, more efforts are needed before the navigation system can completely interpret the whole COLREGs in various scenarios.

(3) Discussion on working conditions of methods is lacking.

Since the existing collision avoidance methods are suffering from various problems, each method has its working conditions. If the working conditions are matched, the method can offer collision-free solutions; otherwise, the collision-free solution is not guaranteed. However, only a few studies discuss their working conditions. The working conditions include but not limited to the initial distance between ships that one algorithm can find a collision-free solution, the maximum of computation time that find a collision-free solution, the errors in solutions, the maximum of tolerance to environmental disturbance, etc. Further studies are needed to offer a unique criteria system to judge the working conditions of various algorithms, which is helpful for their application in practice.

(4) Safety verification is often overlooked when proposing a new method.

Many collision avoidance methods are working properly in designed simulations and case-based testing, but they lack systematic verifications. For instance, some collision prevention techniques utilized linearization techniques to simplify the problem, but this inevitably includes errors, e.g., GVO. Some algorithms use the discrete trajectories of the TS to check collision at each time, which does not guarantee the safety of the ship between time slots, e.g., MPC-CA, etc.

Instead of checking a finite set of scenarios in simulators, a framework that provides analytical proofs of safety is needed (Schwarting, Alonso-Mora, & Rus, 2018). Some formal verification methods developed for autonomous car/aircraft can be introduced in the future, such as reachability analysis (Althof, 2010), funnel libraries (Majumdar & Tedrake, 2017), etc.

(5) The balance between effectiveness and efficiency of the methods should be considered.

The balance between efficiency and effectiveness is less of a focus on existing collision avoidance. These two benchmarks sometimes are conflicting, and researchers usually sacrifice the effectiveness to increase efficiency. For instance, some methods only check several solutions instead of searching the whole solution-space, e.g., LGS, DIO, etc.; some researchers assume the solutions will last for certain horizon, e.g., DD, VO, DW, etc. These studies simplify the method for efficiency, but they do not provide a discussion of the influence of simplification.

(6) Modeling environmental disturbances need further studies

Although researchers offered some ideas to handle the influence of environmental disturbances on ship collision avoidance, it is still a long way to completely solve this problem. The main challenges are handling two types of uncertainties: (1) the uncertainty of environmental disturbances at a specific position and time, e.g., surrounding the ships; (2) the uncertainties of the responses of the ship under different disturbances. Since weather forecasting models are usually stochastic, meteorologists use stochastic-dynamic models for predictions (Palmer et al., 2005). However, in CA studies, the disturbance is modeled with a fixed noise value, e.g., white noise, due to the vacancy of a predicting model that estimates the dynamic disturbances surrounding the ship. Besides, the responses of a ship in various disturbances with different loads are unmodelled properly. Therefore, for avoiding collisions in various environmental disturbances, it still needs a better understanding of environmental disturbances and their impacts on ship's dynamics.

6.2 Trends in the technology development for ship collision avoidance

While reading new emerging literature, we find some interesting ideas, which desired further developments. The potential developments of ship collision avoidance are concluded from the perspective of the three modules of collision avoidance, as follows:

(1) Motion prediction:

The development of prediction methods shows a trend from ignoring uncertainties to considering /eliminating uncertainties. Researchers use various methods to find the tubes/boundaries of the predicted trajectories and narrow down the tubes.

For the prediction of the OS, one main uncertainty comes from dynamics modeling. At early ages, researchers use a holonomic model to predict the trajectory, which usually is unrealistic and contains huge errors. Recently, researchers employed various deterministic non-holonomic models to predict the trajectory of the OS in collision avoidance. However, as we discussed in Section 6.1, the dynamics of the ship might change due to different loads, speed changing, disturbances, etc. For modeling these uncertainties, some online parameter-identification methods offer solutions, e.g., support vector machines (C. Liu, Zheng, Negenborn, Chu, & Xie, (Accepted); Xu & Guedes Soares, 2016; Zhu, Hahn, Wen, & Sun, 2019), etc.

For the prediction of the TS, the main uncertainties come from the intention of the TSs and environmental disturbances. At early stages, researchers presume the moving obstacle to move linearly and to neglect the uncertainties. Recently, these uncertainties are taken into account. Specifically, some studies use physical-based methods to consider environmental disturbance when making predictions, where the environmental disturbances are considered as white noise. Additionally, some researchers suggested improving the accuracy of prediction by estimation of maneuvering intentions, in which some techniques from artificial intelligence are needed. Moreover, some studies assume communication among the ships to eliminate the uncertainty of the TSs' intentions.

(2) Conflict detection:

The developments of conflict detection methods (both expert-based methods and model-based methods) share a similar trend: from a generalized method for all scenarios to explicit models for specific ships in specific scenarios. The expert-based method relies on experts' judgments. It changes from "using a general formulation to estimate collision risk in all scenarios" to "using an adapted formulation to estimate risk in different scenarios". Meanwhile, the model-based method considers more specific motion model and environmental disturbance to conduct a more reliable result for a specific ship.

Additionally, the applications of conflict detection methods are diversifying. The measured risk not only supports the conflict detection at present but also helps the ship to find a low conflict route (Shah et al., 2015). Researchers found that choosing low-risk solutions all the time might reduce the conflict rate along the whole route. Moreover, various risk measurments also facilitate waterway safety management (P. Chen et al., 2019). The risk measurements have been also applied to post-collision analysis, e.g., identifying collision candidates (P. Chen, Huang, Mou, & van Gelder, 2018; Weibin Zhang, Goerlandt, Kujala, & Wang, 2016).

(3) Conflict resolution:

Most collision avoidance techniques use simplified motion models and offer one feasible solution to the ship. In recent year, some studies adopted a specific motion model in collision avoidance and offered various options that support the decision making of the OOW. For example, researchers incorporate optimization into collision avoidance, which can offer an optimal solution (L. Chen et al., 2018); some studies eliminate dangerous solutions (Huang et al., 2019), etc. More and more forms of collision-avoidance solutions enrich their applications in practice.

Additionally, we also observed following developing trends in conflict resolution:

Validation of methods in simulators attracts more and more attention. The collision avoidance techniques for the ships are still based on ideal settings. The techniques are demonstrated by simulations or in a laboratory environment. Although these settings are different from the real operating environment, there are necessary steps for validation of a technique before it can be used in the real world. It is also necessary to test the extreme conditions of these algorithms in the simulators, which is feasible and economical.

Hybrid conflict resolutions method that can handle different scenarios is the trend. Some methods that are suitable for close-range encounter require a huge computation resource, which might not be suitable in collision avoidance in long distance. On the contrary, the methods suitable for long-range collision avoidance might result in collisions due to the neglect of dynamics in the close-range encounters. As a result, the hybrid conflict resolutions method that combines different algorithms for different scenarios becomes a promising alternative.

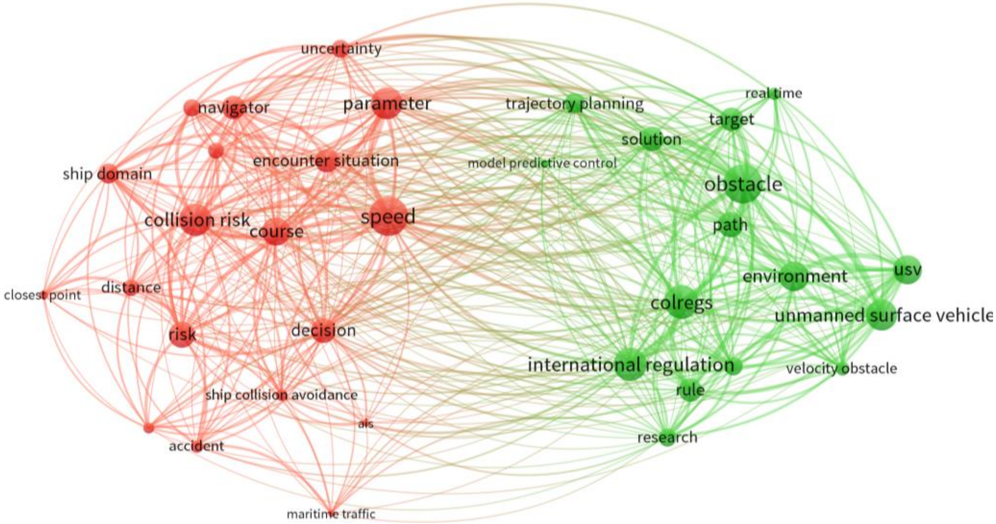
The ship domain becomes popular in representing the ship's shape in collision avoidance for applying in practice and incorporating regulations. Many studies firstly would use a simple geometric shape to represent the ship in the development of prevention techniques. However, in practice, the ship's shape is irregular, and the ship usually keeps different buffers in different directions for multiple reasons, e.g., rule compliance, etc. This buffer is the so-called ship domain in maritime studies (Szlapczynski & Szlapczynska, 2017b). Recent papers using the ship domain in collision avoidance methods have been presented in (He et al., 2017; Z. Liu, Wu, & Zheng, 2019; Szlapczynski & Szlapczynska, 2015, 2017a).

6.3 The road from manned ships to unmanned ships

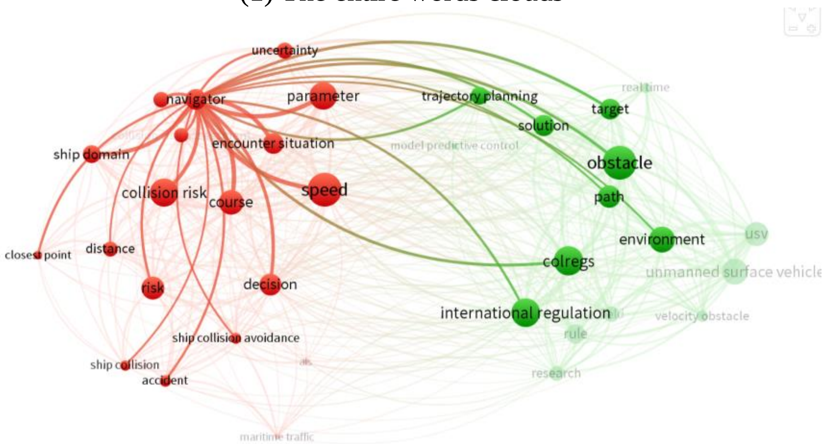
From the perspective of collision avoidance, manned ships and unmanned ships are similar. Specifically, they share many common processes, namely motion prediction, conflict detection, and conflict resolution. However, the focuses of the existing studies for manned ships and unmanned ships are slightly different. For manned ships, research focuses more on detecting collision dangers to remind the OOW, while how to support the OOW find a conflict resolution is less of the focus. For unmanned ships, studies mainly concentrate on finding a collision-free solution, and the conflict detection relies on some relatively simple criteria, e.g., relative distance. These two groups of research have great potential to be complementary.

A more intuitive demonstration of these difference has been presented in Fig.9, where we present the occurrence of keywords in the abstract and title of collected articles (from 2000). In Fig.9, panel (1) shows the entire word clouds of collected articles; panel (2) shows the cloud associating to supporting navigators, i.e., the manned-ship study; panel (3) shows the cloud working on the unmanned ship, i.e., the unmanned-ship study.

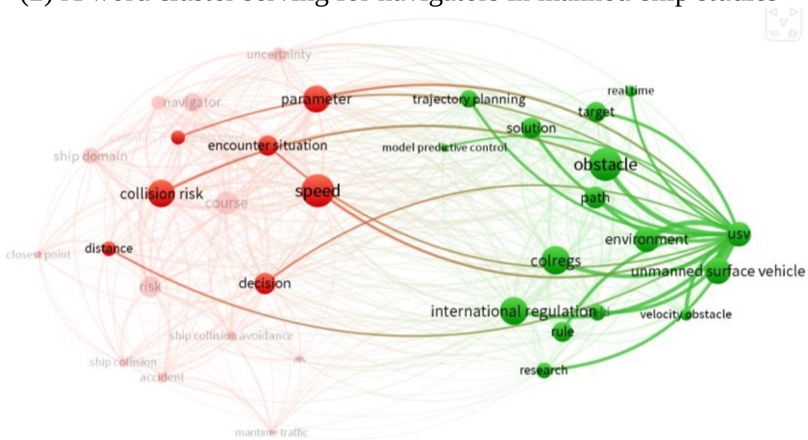
In Fig.9 (1) and (2), the manned-ship studies (on the left-hand side of the figure) are more related to collision risk assessment, i.e. conflict detection. The keyword "navigator" is strongly connected to risk-related terms, such as “risk”, “collision risk”, “ship domain”, “closest point”, etc., which is compliant with our collected methods in Section 4. In Fig.9 (1) and (3), the word cluster of “USV” is apart from “collision risk” assessment and close to “trajectory planning”, “model predictive control”, etc. It implies that the focus of this group is on finding a real-time trajectory, i.e. conflict resolution.



(1) The entire words clouds[1]



(2) A word cluster serving for navigators in manned-ship studies



(3) A word cluster serving for unmanned-ship studies

[1] These word maps are generated based on VOSviewer version 1.6.10.

Fig. 9 Word clouds of articles working on ship collision avoidance.

Although the studies of the manned ship and the unmanned ship are complementing one another, existing studies for either manned ships or unmanned ship cannot directly bring the existing shipping industry to the unmanned era. In fact, even the development of autonomy in waterborne transport is speeding up recently, the autonomy of the vehicle cannot be achieved in an overnight. From fully manned ships to autonomous ships, there are numerous challenges that need to be overcome. Inspired by (UK, 2017), we introduce six levels of controls which link the manned ships and autonomous ships w.r.t. collision avoidance. Each control level might match one or several types of maritime autonomous surface ship (MASS) defined by IMO[2] (IMO, 2018). The details are shown in Table 2.

Level 0 refers to a situation that no machine is involving in collision avoidance. Since most of the ships are requested to equip certain navigational assistant system on board, e.g., INS, etc., we consider this level has been passed.

Table 2 Six levels of control from manned ship to unmanned ship

Level	**Implications in Collision Avoidance (CA)**	**MASS Types**	
Level 0	No machine is involved and the human fully takes responsibility to detect dangers and take evasive actions.	-	-
Level 1	The human directly controls the ship and machines offer certain service in conflict detection, which is the existing level of a merchant ship.	I	Human on board.
Level 2	The human directly controls the ship and machines offer supports both in detection and resolution, i.e., available solutions and validate chosen solutions.	I/II	Human in the offshore center & on board.
Level 3	Machines operate the ship under the monitor of the human, which support the human to understand the choice of the solutions. The human can indirectly control the ship via machines or directly controls the ship via the on-board operators.	II/III	Humans in the offshore center or on board
Level 4	Machines can control the ship independently while it informs the human and sends an alarm when it in an emergency issue. Then, the human can indirectly control the ship via machines.	III	Humans in the offshore center
Level 5	Machines control the ship autonomously and humans cannot direct or indirect control the ship during each voyage.	IV	Humans in the offshore center

Level 1, the machine offer supports in conflict detection, which is the main scope of existing studies for the manned ships, i.e., ships in MASS type I. It offers various supporting tools for conflict detection, i.e., sharing the situational awareness of experts with the OOWs. The aim of these studies is training navigational assistance systems which mainly in charge of supporting the OOW to take evasive actions in time.

Level 2, the machine expands the function of the assistance systems with conflict resolution, which might occur in MASS type I or II. A few researchers have presented some prototypes on this theme, e.g., CTPA, CDS, etc. However, these prototypes usually ignore the ship's dynamics and used the holonomic model in collision prevention. As a result, the prototypes do not work well in close range. Here, the

[2] Four types of Maritime Autonomous Surface Ship (MASS) defined by IMO are:
Type I, ship with automated processes and decision support; Type II, remotely controlled ship with seafarers on board; Type III, remotely controlled ship without seafarers on board; Type IV, fully autonomous ship.

studies for ASV can provide some good references, e.g., the design of two-level controllers. However, more attention should be paid to the uncertainties in the systems, especially the uncertainties of parameters in motion models, where these parameters are difficult to calculate for each ship. In this level, the ship is still controlled by human operators on board or in the offshore center, and the machine offers supports.

Level 3, the control of the ship is switched to machines, and human operators authorize the machine to take actions, which might occur in MASS type II or III. This level requires a deeper interaction between human and machine, which is seldom discussed in the existing literature. Many collision prevention algorithms are proposed to find one solution to the ship, regardless of the interaction between human and machine: how to present the solution in a way that human can easily understand and implement; how to support human to modify the collision actions without leading to a worse situation; how to validate the safety of human's choices, etc. This is a critical step for improving the autonomy of the ships, and it is also a strong reason to convince human to trust the machines.

Level 4, the machine takes the full responsibility of collision avoidance, and human operators supervise the machine if necessary, which could occur in MASS type III. This level requires the machine to be aware of emergencies in which the machine might not guarantee the safety of the ship, and the human's intervention becomes necessary. To achieve this, it requires researchers to test the extreme conditions of collision prevention algorithms, i.e., safety verification. These studies are not yet included in most collision prevention studies in maritime research.

Level 5 is the fully autonomous ships, and human operators are only informed without any form of interventions, which could occur in MASS type IV. This level has received numerous attentions from researchers and highlighted by societies. However, to achieve a fully autonomous system needs long-term developments. The challenges are the uncertainties of model and parameters. Moreover, how to comply with the various regulations in complicate scenarios is also one open question. Some studies in the manned ships might offer a line of thinking to help the ASVs to be rule-compliant, i.e., to incorporate the experts' judgments in collision prevention.

From Level 1 to Level 5, the interactions between human operators and the machine are gradually increasing. Level 2, 3, and 4 ask the machine contains more functions than collision detection, specifically, supporting the OOW find one collision-free solution, checking the safety of the inputted solution by the OOW, eliminating unsafe solutions, supporting the human to understand the solution selected by the machine, etc. These improvements are essential for testing the reliability of autonomous systems, increasing the trust between the human and the machine, and reducing the workload of the human. We believe they (Level 2-4) are the key steps to the autonomous era, which are less focused nor discussed.

In brief, propelling the autonomous shipping is not only continuing the existing studies on the manned ship and the unmanned ship (Level 1 and Level 5), but also filling the gaps between them (Level 2-4), specifically, making the unmanned ships user-friendly for human operators, exploring more functions in the existing manned ships, etc.

7. Conclusion

This article provides a comprehensive overview of the techniques used for ship collision avoidance both for manned ships and unmanned ships. Three processes of collision avoidance are identified, i.e., motion prediction, conflict detection, and conflict resolution. We then analyzed existing methods from these three aspects and identified new trends in ship collision avoidance studies.

Besides the techniques that have been mentioned in other review papers, the current paper has included an overall presentation of the state-of-the-art methods. For motion prediction, communication, and cooperation between ships provide the intentions of the TSs, which makes the prediction more accurate and enables the ship to avoid obstacles in a dynamic environment. For conflict detection, researchers designed innovative risk assessment methods to look more into specific scenarios. For conflict resolution, many studies used dynamic models instead of simplified models in collision avoidance to make the methods closer to the behavior of a real ship. Additionally, COLREGs rules have been considered in some simple encounter situations.

Realizing recent changes in the developments of these new methods, at the same time, we find some limitations of existing collision-avoidance studies, which we have categorized into: (1) uncertainties which are resulting from un-modeled dynamics and model parameters are being ignored; (2) complete COLREGs-rule compliant collision avoidance systems are still being a big challenge; (3) working conditions of existing methods which are usually not analyzed in these studies; (4) safety verification of collision avoidance methods which is missing; (5) balance between effectiveness and efficiency of the methods should be considered; (6) modelling the environmental disturbance is still one challenge.

Future research directions to develop reliable ship collision avoidance systems are also provided in this paper through the analysis of existing techniques. The quality of prediction is essential for subsequent conflict detection and resolution. Existing research using tubes to take uncertainties into consideration shows good performance. However, the boundaries of the predicted trajectories need further studies. Conflict detection relying on collision risk assessment is helpful not only in improving situational awareness of the OOW but also in reducing the conflict rate. Thus, better risk assessment methods are expected. Collision resolution needs more studies in safety validations and extreme conditions for different methods. A combination of various collision avoidance methods to handle different encounter scenarios is one of the future research directions.

The roadmap from manned ships to unmanned ships w.r.t. collision prevention is also discussed in this paper, which contains six levels. Existing studies for the manned ships and unmanned ships have different focuses, but they are somehow complementary. Learning from manned ships can make the unmanned ships capable to deal with complicated situations, while the techniques for autonomous collision avoidance can support the decision making of the OOW. We claim that there should not be fixed boundaries between manned and unmanned ships. We believe that enhancing the interactions between human and autonomous systems is the key to the autonomous era.

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List of Abbreviations

APF	Artificial Potential Field	GNC	Guidance Navigation Control
ASV	Autonomous Surface Vehicle	INS	Integrated Navigation System
BFS	Brute-force search	KF	Kalman Filter
CA	Collision Avoidance	LCM	Limited Cycle Method
CDS	Collision Danger Section	LGS	Lattice-based Search
COLREGs	Convention on the International Regulations for Preventing Collisions at Sea	MASS	Maritime Autonomous Surface Ship
		MMG	Mathematic Model Groups
CPA	Closest point of approach	MPC	Model Predictive Control
DCPA	Distance to CPA	MPC-CA	MPC based collision avoidance
TCPA	Time to CPA	OOW	Officer on watch
CRI	Collision Risk Index	OS	Own Ship
CTPA	Collision threat parameter area	PAD	Projected Area of Dangers
DD	Decision Disc	POA	projected obstacle area
DR	Dangerous Region	RI	Risk Indicator
DR-Vspace	DR in Velocity space	SCR	Spatial Collision Risk
DR-Wspace	DR in Workspace	TS	Target Ship
DIO	Discrete Inputs Optimization	VC	Vision Cone
DW	Dynamic Window	VTS	Vessel Traffic Service
ENC	Electronic Navigational Chart	VO	Velocity Obstacle
FCDD	Fuzzy Collision Danger Domain	GVO	Generalized VO
FMM	Fast Marching Method	NLVO	Nonlinear VO

Appendix A

Table A1 Comparison of different collision avoidance methods

	Name	Motion Model	Number of TSs	Solution form	Optimal	Description	Ref.
Rule-based	Single-rule	NA	Single	One maneuver	-	Simple, but only works in limited cases, e.g., open sea.	(Fang et al., 2017; Naeem et al., 2012)
	Multiple-rule	NA	Single	One maneuver (u and ψ)	-	Simple, but might not figure out all the possible scenarios.	(Perera et al., 2012)
Virtual Vector	APF	NA	Multiple	One maneuver (ψ)	-	Simple, but easy goes to a local minimum, not for a dynamic environment, and ignore dynamics.	(H. Lyu & Yin, 2018; H. G. Lyu & Yin, 2017)
	LCM	NA	Single	One trajectory	-	Simple, but only works with one high-speed obstacle and ignore dynamics.	(Mahini et al., 2013; Soltan et al., 2009, 2010)
Discrete Inputs	DW	K./D.	Multiple	One maneuver (*u* and *r*)	√	Popular and offer alternative solutions, but not for a dynamic environment. Stopping is not always safe.	(Loe, 2008; Serigstad, 2017)
	DD	H.	Multiple	One maneuver (u and ψ)	√	offer alternative solutions; ignore dynamics	(Benjamin et al., 2006; Kuwata et al., 2014; Szlapczynski & Krata, 2018)
	DIO	D./ H.	Multiple	One trajectory & control inputs	√	The balance between efficiency and effectiveness is challenging.	(Johansen et al., 2016; D. Kim et al., 2017; S. Li et al., 2018) (D. Kim et al., 2017)
	LBS	D.	Multiple	One trajectory & control inputs	√	Considering traffic congestion to reduce the search burden, while computing burden is still a challenging issue.	(Shah et al., 2015; Švec et al., 2013)
	BFS	D.	Multiple	One trajectory & control inputs	-	The process is time-consuming and the solution is not optimal.	(J. F. Zhang et al., 2015)
Continuous Inputs	VO	H.	Multiple	One maneuver	√	Simple, efficient, and offer alternative solutions but ignore ship dynamics.	(Huang et al., 2018; Y. X. Zhao et al., 2016; Zhuang et al., 2016)
	GVO	D.	Multiple	One trajectory & control inputs	√	Capable of considering dynamic constraints and offer alternative solutions, but need to accept errors due to linearization.	(Huang et al., 2019; L. Chen et al., 2019)
	VC	D.	Single	One trajectory & control inputs	√	Simple and effective, but each time only can avoid one obstacle with low speed.	(Martin S. Wiig et al., 2017; M. S. Wiig et al., 2017; Y. Z. Xue et al., 2011)
	MPC-CA	D.	Multiple	One trajectory & control inputs	√	Local minimal and computing time is dependent on solvers.	(Abdelaal et al., 2018; L. Chen et al., 2018; Ferranti et al., 2018; L. Chen et al., 2019; H. Zheng, Negenborn, & Lodewijks, 2018; H. R. Zheng et al., 2017)
Re-planning	Graph search	NA	Multiple	One trajectory	√	Strongly depend on fresh frequency and velocity of the obstacle. Might lead to an inevitable collision state.	(Lazarowska, 2017; Y. C. Liu et al., 2017)
	EA	NA	Multiple	One trajectory	√	Ignore ship dynamics and might lead to an inevitable collision state.	(Lazarowska, 2014; Szlapczynski, 2013; Szlapczynski, 2014; Tsou, 2016; Tsou & Hsueh, 2010)

NA: ship dynamics are not directly considered in constructing a collision-free solution, but during the implementation of the solution, the dynamics are considered. H. is Holonomic model; K is kinematic model; D is dynamic model. '√' refers to that the optimization is incorporable; '-' means the optimization is not considered.

Table A2 Comparison of different collision avoidance methods

	Methods	Geometry of TSs	Map	Control architecture*	Remarks	Ref.
Rule-based	Single-rule	-	-	Decentralized	Suitable for emergent actions.	(Fang et al., 2017; Naeem et al., 2012)
	Multiple-rule	-	-	Decentralized	In compliance with COLREGs.	(Perera et al., 2012)
Virtual	APF	Circle	-	Decentralized	Treat TSs to be semi-dynamic in finding solutions.	(H. Lyu & Yin, 2018; H. G. Lyu & Yin, 2017)
	LCM	Circle/ Ellipse	-	Decentralized	Under-actuated ship. TSs' movement is known.	(Mahini et al., 2013; Soltan et al., 2009, 2010)
Discrete Inputs	DW	Polygon	Polygon	Decentralized	Under-actuated ship; Two-level controller; Semi-dynamic TSs	(Loe, 2008; Serigstad, 2017)
	DD	Circle	-	Decentralized	Find an optimal maneuver; easily to be rule-compliant; Semi-dynamic TSs	(Benjamin et al., 2006)
		Ship domain	Occupancy map	Decentralized	Considering rules, restrict waters, and etc.; Semi-dynamic TSs	(Degre & Lefevre, 1981; Kuwata et al., 2014; Lenart, 1983; Pedersen et al., 2003; Szlapczynski & Krata, 2018)
	DIO	Circle	-	Decentralized	Collision condition is formulated as a soft constraint; Semi-dynamic TSs; The solution is (u, ψ).	(Johansen et al., 2016)
		Circle	-	Distributed	Collision condition is a soft constraint; Solution is (δ).	(S. Li et al., 2018)
		Circle	-	Distributed	Collision condition is a soft constraint. Negotiations among ships. Solution is (ψ).	(D. Kim et al., 2017)
	LBS	Polygon	-	Decentralized	Adaptive sampling use spatio-temporal complexity. KF filter to predict TSs' trajectory.	(Shah et al., 2015; Švec et al., 2013)
	BFS	Circle	-	Distributed	Ships broadcast their intentions.	(J. F. Zhang et al., 2015)
Continuous Inputs	VO	Circle	-	Decentralized	TS keeps u and ψ ; Reciprocal VO.	(Y. X. Zhao et al., 2016; Zhuang et al., 2016)
		Circle	Polygon	Distributed	TS's trajectory is known; Non-Linear VO.	(Huang et al., 2018)
	GVO	Circle	Polygon	Distributed	Two-level controller; fewer maneuvers.	(Huang et al., 2019; L. Chen et al., 2019)
	VC	Circle	-	Decentralized	Under-actuated ship; Semi-dynamic TSs.	(Martin S. Wiig et al., 2017; M. S. Wiig et al., 2017)
		Circle	Polygon	Decentralized	Combined with APF.	(Y. Z. Xue et al., 2011)
	MPC-CA	Circle	-	Decentralized	NMPC; Semi-dynamic TSs; Under-actuated ship.	(Abdelaal et al., 2018)
		Ellipse	Polygon	Centralized & Distributed	NMPC.	(Ferranti et al., 2018)
		Circle	Polygon	Decentralized & Distributed	Waterborne-AGV and ADMM.	(Huarong Zheng, 2016; H. Zheng et al., 2018; H. R. Zheng et al., 2017)
		Polygon (square)	Polygon	Distributed	Demonstration in canal networks.	(L. Chen et al., 2018; L. Chen et al., 2019)
Re-planning	Graph search	Polygon (close to circle)	Occupancy map	Decentralized	Fast Marching Method; using KF algorithm to predict trajectory.	(Y. C. Liu et al., 2017)
		Hexagon	Polygons	Decentralized	Search in trajectory database. Semi-dynamic TSs	(Lazarowska, 2017)
	EA	PAD (hexagon)	Polygon (from ENC)	Decentralized	Gene algorithm; Semi-dynamic TSs	(Tsou, 2016; Tsou & Hsueh, 2010)
		Hexagon	Polygons	Decentralized	Ant Colony Optimisation; Semi-dynamic TSs	(Lazarowska, 2014)
		Ship domain	Occupancy map	Centralized	Evolutionary sets;	(Szlapczynski, 2013; Szlapczynski, 2014)

Note: 'Control architecture' column follows the categorization in (Negenborn & Maestre, 2014). 'Map' column indicates how the obstacle in ENC is considered, specifically, '-': ignoring navigational charts during CA (i.e. CA in open sea); Polygon: using polygons to represent obstacles in the chart; Occupancy map: using a bitmap to represent navigational charts.

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