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The Design of a Shaft Cross Coupling System for the Engines of an STOL Aircraft

by

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SUMMARY

The design of a system for mechanically cross coupling the engines of an STOL transport aircraft has been studied, and in particular, the initial design problems of selecting the operating speed and assessing the weight of the system has been investigated.

It is found that a minimum weight system will exist, but that, in practice, the speed limitations provided by the shaft support and other bearings in the system will prevent this being employed, and will determine the weight of the system.

CONTENTS

FIGURES

1. INTRODUCTION

Recent studies of multi-engined, turbofan powered STOL transport aircraft[1], indicate that difficulties arise in obtaining adequate low speed handling performance in the event of a single engine failure.

One method which has been proposed for overcoming this difficulty is to mechanically cross couple the adjacent engines on each wing, on the reasonable assumption that engine failure due to gas generator malfunction is much more likely than fan failure.

A similar system, using shafts for cross coupling, has been used in the Brequet 941 aircraft, discussed in detail by Bruyere[2].

The weight penalty associated with such a system, assumes greater than usual significance, since cross coupling is essentially a standby system, not contributing normally to the operational efficiency of the aircraft. This weight penalty is, in general, related to the horsepower which is to be transmitted, the distance between the engines being coupled, and the speed at which the shafts and associated gearing of the system operates.

In this report, these initial design problems of selecting a system speed and assessing the associated system weight for the cross coupling system of a STOL transport aircraft recently studied at Cranfield Institute of Technology, are examined.

1.1 System Layout

The layout of the proposed system for coupling the adjacent pair of engines on each wing of a four engined STOL aircraft is shown in Figure 1. The horsepower to be transmitted is 6000, and the distance between the adjacent engines is approximately 20 feet. The engine fan cruise speed is 3500 r.p.m.

The system is comprised of:-

a) a bevel gearing stage contained within the fan or propeller reduction gearbox of each engine, the output shafts leading up the engine pylons.
b) transfer bevel gearboxes at the top of each pylon, to turn the drive axis parallel with the wing front spar.
c) a cross shaft, supported periodically in bearings, running inside the leading edge of the wing.

Flexible couplings are provided at the ends of the shafts, where they meet the gearboxes; the shafts are of single lengths of thin-walled, circular tube cross section, with no intermediate joints between the couplings. The cross shaft support bearings incorporate damping devices to enable the shaft to run through critical speeds, if necessary.

1.2 Design requirements

The design objectives are to determine the total weight as a function of operating speed of the system, to enable a rational choice of system speed to be made, subject to the following conditions:-

a) the shaft must not operate continuously at a critical whirling speed, since vibration and fatigue problems are likely to occur under these conditions

b) the shaft must accommodate some flexural curvature without damage to allow for the effects of wing deflection and possible misalignment of the support bearings.

c) the shear and bending stresses in the shaft must not exceed the capabilities of the shaft material under the fatigue conditions imposed by the alternating bending of condition b) above, and the applied torque. Local buckling of the shaft must also be avoided.

Other factors which may influence the choice of operating speed are that power losses in the system tend to increase as the speed increases, and that maintenance and lubrication of the shaft support bearings in particular and the system in general, may become more demanding as the speed increases.

It is necessary to assess the weight of the various components of the system in terms of speed and torque.

2. SHAFT DESIGN

2.1 Speed limitation

For the thin walled shaft shown in Figure 1, the fundamental critical speed will be given by

$$n_c = (k+1)^2 \frac{15\pi}{\sqrt{2}} \frac{d}{l} \sqrt{\frac{E_s g}{\rho_s l^2}}$$

where
- n_c = critical speed (r.p.m.)
- l = total length of shaft
- E_s = Youngs modulus
- d = diameter of shaft
- ρ_s = weight density of shaft material
- k = number of intermediate equally spaced supports.

In practice, the critical speeds tend to be somewhat lower than that above, due to support flexibility and damping at the intermediate bearings.

For higher critical speeds

$$n_c = z^2(k+1)^2 \frac{15\pi}{\sqrt{2}} \frac{d}{l}\sqrt{\frac{E_s g}{\rho_s l^2}}$$

$(z = 1,2,3, \ldots)$

The speed limitations for the shaft are then

$$n < (k+1)^2 \frac{15\pi}{\sqrt{2}} \frac{d}{l}\sqrt{\frac{E_s g}{\rho_s l^2}} \qquad \ldots (1)$$

for subcritical operation, or

$$(z+1)^2(k+1)^2 \frac{15\pi}{\sqrt{2}} \frac{d}{l}\sqrt{\frac{E_s g}{\rho_s l^2}} > n > z^2(k+1)^2 \frac{15\pi}{2} \frac{d}{l}\sqrt{\frac{E_s g}{\rho_s l^2}} \qquad \ldots (2)$$

for supercritical operation.

2.2 Stress constraints

The maximum 'static' shear stress at which the shaft material may operate is limited by considerations of

a) fatigue, in conjunction with the alternating bending stresses and torsional stresses on the shaft.

b) the need to cater for overload torques, with suitable safety factors.

c) local buckling of the wall of the shaft.

For a given shaft material, it will be possible to specify the 'static' shear stress acceptable and the associated alternating bending stress, and to evaluate the torsional fatigue performance under these conditions, using for example the techniques of reference 3.

The consequence of imposing a static shear stress limitation, for a given horsepower to be transmitted, leads to a minimum speed limitation on the shaft, i.e.

$$n \geqslant \frac{33000H}{\pi^2 \tau_{max} l^3} \left(\frac{l}{d}\right)^3 \frac{d}{t} \qquad \ldots (3)$$

where τ_{max} is the specified static shear stress (in lbf/ft^2 units here)

H = maximum horsepower to be transmitted

The consequence of imposing an alternating bending stress limitation for a shaft flexed into a uniform radius of curvature with a mid span deflection of m, leads to a shaft diameter limitation, i.e.

$$\frac{d}{l} \leqslant \frac{l}{4m} \frac{\sigma_{max}}{E_s} \qquad \ldots (4)$$

where $\pm \sigma_{max}$ is the maximum alternating stress acceptable.

The critical shear stress for buckling of circular tubes in torsion is given by (frem ref.4)

$$\tau_{CR} = \frac{0.272 \times 2\sqrt{2}}{(1-\nu^2)} E(\frac{t}{d})^{3/2}$$

assuming elastic material behaviour.

For a shaft, a factor F must be applied to this, to ensure that the buckling stress is not achieved in practice, leading to the condition that

$$\frac{\tau_{CR}}{F} \geqslant \tau_{max}$$

i.e.

$$\frac{t}{d} \geqslant 1.14(\frac{\tau_{max}}{E_s} F)^{2/3} \quad \ldots (5)$$

where $F \geqslant 1$ and is to be specified

ν = Poisson's ratio, taken as 0.3

Since a thin walled shaft has been assumed, a further condition that, say,

$$\frac{t}{d} \leqslant 0.08 \quad \ldots (6)$$

is necessary to ensure that the equations developed so far do not become too inaccurate.

Equations (1) to (6) thus define a design space for the shaft, when the appropriate required operating conditions and material properties are defined.

For the aircraft system investigated, a steel cross shaft was assumed, and the following values were taken:-

τ_{max} = 40,000 lbf/ins^2

σ_{max} = $\pm$20,000 lbf/ins^2

m/l = 1/100

E_s = 30 x 10^6 lbf/ins^2

l = 20 ft.

H = 6000

F = 1.50

From these values, equations (1) to (6) may be evaluated to produce the shaft design spaces shown in Figs. 2 and 3. Fig.2 is for a shaft operating below its fundamental critical speed, and Fig.3 is for a shaft operating at supercritical speeds, between the fundamental and second critical modes.

3. SYSTEM WEIGHT

The total weight of the cross coupling system is made up from

a) the shaft weight, which can be estimated accurately

b) the weight of the shaft couplings and the shaft supports, which can be estimated reasonably accurately from typical design layouts.

c) the bevel gearboxes.

3.1 Bevel gearing

The total weight of gearboxes is difficult to estimate analytically, in general terms, because of the wide variations in bearing and shaft layout which are possible. The weight of a pair of bevel gears, to transmit a given torque, can however be estimated from their design equations. The total weight of a complete gearbox can then be estimated using empirically derived methods.

3.2 Weight of bevel gears

The simplified design equations for bevel gears (reference 5) can be expressed as

$$S_c \geqslant \left(\frac{2T}{I} \, \frac{K_m}{K_v} \, C_p^{\,2} \, \frac{1}{Fd^2} \right)^{1/2}$$

for contact stress, and for tooth bending stress

$$S_T \geqslant \frac{2T}{J} \, \frac{K_s K_m}{K_v} \, \frac{P_d}{dF}$$

where T = applied pinion torque

S_c = allowable contact stress

S_T = allowable tooth bending stress

d = pinion diameter at the larger end

F = facewidth

P_d = diametral pitch = N_p/d

C_p = material elastic coefficient

I, J = geometry factors dependent on the number of pinion teeth and the gear ratio.

N_p = number of pinion teeth.

For bevel gears having $F = \frac{1}{3}$ x front cone distance which is usual, and 90° shaft angle

$$F = \frac{d}{6A} \quad \text{where } A = \sin\left[\tan^{-1} \frac{1}{m_g}\right]$$

and m_g = gear ratio.

Also, $K_s = (d/N_p)^{1/4}$

The smallest pinion diameter to transmit a given torque is the smallest value of d which simultaneously satisfies both of the above equations, when the inequality signs are replaced by equalities.

This pinion diameter is given by

$$d = \left\{ \frac{12AT}{I} \, \frac{K_m}{K_v} \cdot \frac{C_p{}^2}{S_c{}^2} \right\}^{1/2} \qquad \ldots (5)$$

and the associated pinion torque by

$$T = \frac{J^{12} S_t{}^{12}}{I^{11} N_p{}^9} \, \frac{K_v}{12AK_m} \left(\frac{C_p}{S_c} \right)^{22} \qquad \ldots (6)$$

In Reference 5 are presented values of I and J for 35° spiral angle bevel gears, which is the type of gear which would be used in this application.

It can be shown that the weight of a pair of bevel gears if given by

$$W_g = C\, m_g(m_g + 1)\rho_g d^3 \qquad \ldots (7)$$

where ρ_g is the weight density of the gear material used (assuming both pinion and wheel are made from the same material)

C is the shape factor - dependent on the amount of material removed from the centre of the gear blanks.

Thus combining equations (5), (6) and (7), the weight of a pair of bevel gears to transmit a given torque may be estimated.

For the system under investigation, casehardened steel gears were assumed, and the following values for material and loading factors taken:-

$K_v = 1.0$, $K_m = 1.10$, $C_p = 2800$

$S_t = 30{,}000$ lbf/in^2 , $S_c = 20{,}000$ lbf/in^2.

Figure 4 shows the relationship between gearset weight and pinion torque obtained by this method.

To obtain the weight of a complete gearbox it is necessary to use an empirical method. In reference 6, it is suggested that, for helicopter gearboxes, the weight is distributed in the following proportions

Gears and supporting shafting	50 per cent
Casing	30 per cent
Bearings	20 per cent

Using these results and taking a value of C = 0.053, the gear weights for the system may be calculated for a range of shaft speeds.

It has been assumed that the lower (engine) bevel gears are contained within the engine gearbox, and that the casing for these gears is not therefore part of the cross coupling system weight.

3.3 Shaft weight

The shaft weight, for a given size of shaft is readily calculated.

Typical design layouts for this system indicate that each flexible coupling weighs about 5 per cent of the shaft weight and that a typical support bearing and its housing assembly weighs about 7 lbf.

Using the above values, the system weights corresponding to each shaft speed and size have been plotted on figure 2 and 4. Figure 5 shows, typically, how the system weight is proportioned between the various components of the system, in this case for the subcritical shaft speed system of figure 2 at t/d = 0.04.

4. DISCUSSION

Several interesting features are apparent from figures 2, 3 and 4.

a) it is clear that a minimum weight will exist for the system, under the constraints so far imposed but that it will occur at too high a shaft speed to be of practical significance in this instance. At larger values of (shaft length/H.P. transmitted) than obtain in the case here, the minimum system weight may occur at more moderate speeds however.

b) the weight advantage to be gained from using the supercritical shaft system is small, particularly at moderate shaft speeds. This is because the support bearing weight is only a small percentage of the total system weight in this case (see figure 5). Figure 6 shows the typical weight advantage of the supercritical system over the subcritical system. Nevertheless, the number of support bearings required is halved, and this will improve the reliability and maintainability of the system.

c) on figure 3 are shown curves indicating the suggested limiting speeds from reference 7, for grease lubricated ballraces, and for ballraces having an oil lubrication system. Similar curves will apply of course to the subcritical system of figure 2.

Grease lubrication for the shaft support bearings is attractive because of its simplicity and avoidance of sealing problems. Nevertheless, it can be seen that the adoption of this type of bearing and lubrication involves a severe weight penalty.

Oil lubricated bearings on the other hand, permit a lower weight system to be adopted, because higher speeds can be used but will involve providing a lubrication system for the shaft support bearings, with the attendant possible reliability and maintenance problems associated with such a system.

It can be seen that the choice of bearings and type of lubrication, is thus likely to be a crucial factor when selecting the speed at which the system is to operate.

d) for the degree of shaft flexure specified for this system, the bending stress limitation has not provided a serious constraint in selecting an operating speed. For systems requiring a greater degree of shaft flexure, or where the shaft is curved, the bending stress limitation may well provide a constraint on the minimum speed of the system however and shaft materials having a low modulus are required in these circumstances (see reference 2).

5. CONCLUSIONS

The selection of shaft speed for cross coupling the engines of a typical STOL transport aircraft and the associated system weight, has been investigated.

It has been found that, for the A71 aircraft, requiring 6000 H.P. to be transmitted over a distance of 20 ft, a shaft speed of 9500 r.p.m. is likely to represent a practical speed limit, providing a system weight of 460 lbf. The cross coupling shaft diameter for this system is 2.10 ins with a wall thickness of 0.168 ins.

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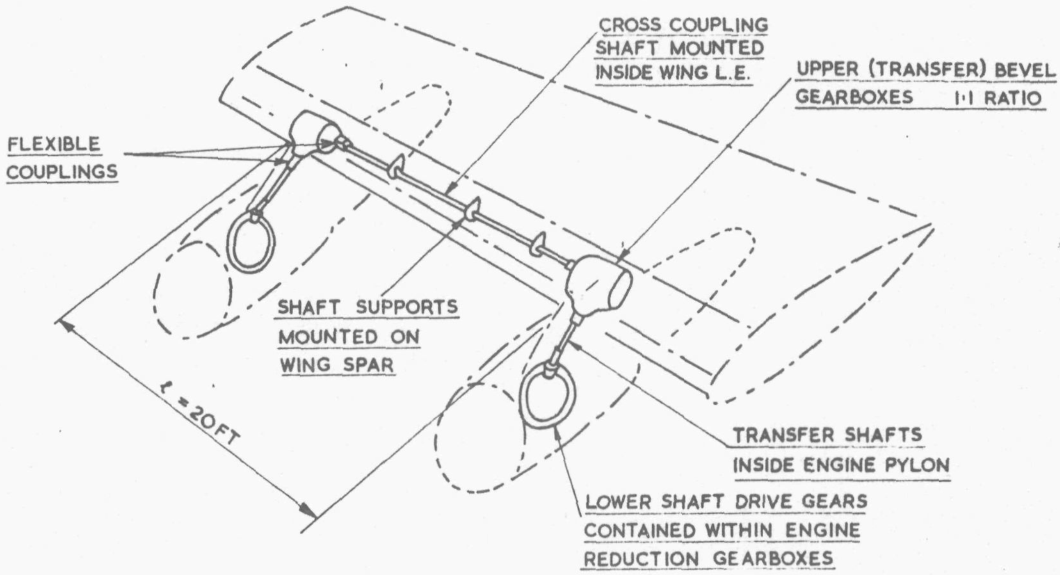

FIG. 1 LAYOUT OF SYSTEM

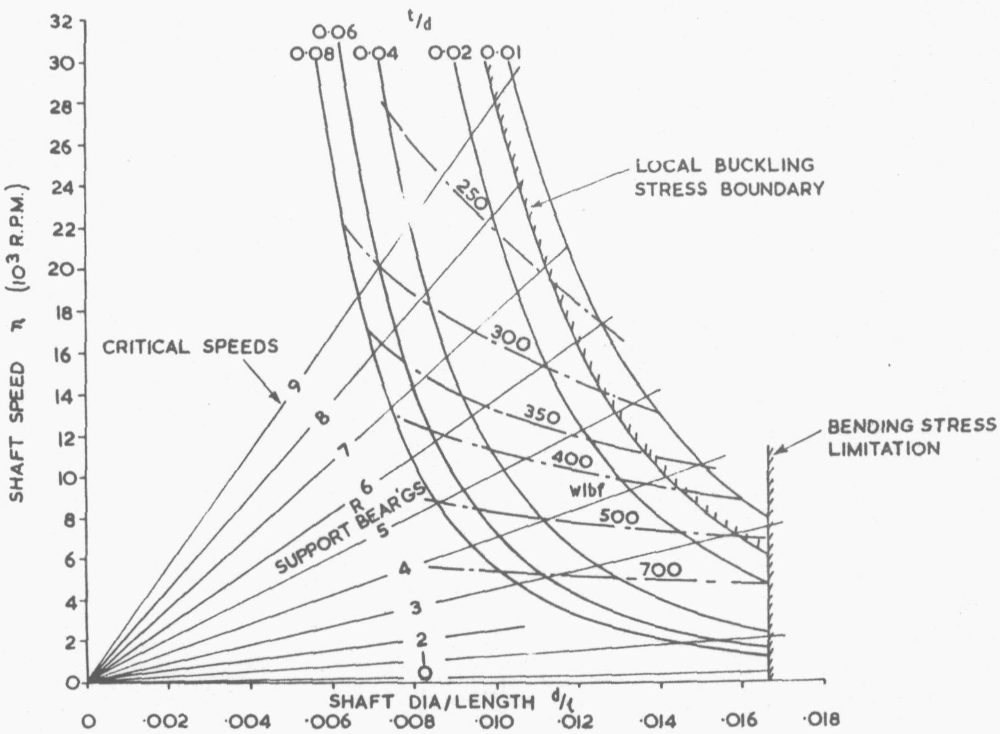

FIG. 2 DESIGN DIAGRAM FOR TRANSMISSION SHAFT, SUBCRITICAL SPEEDS

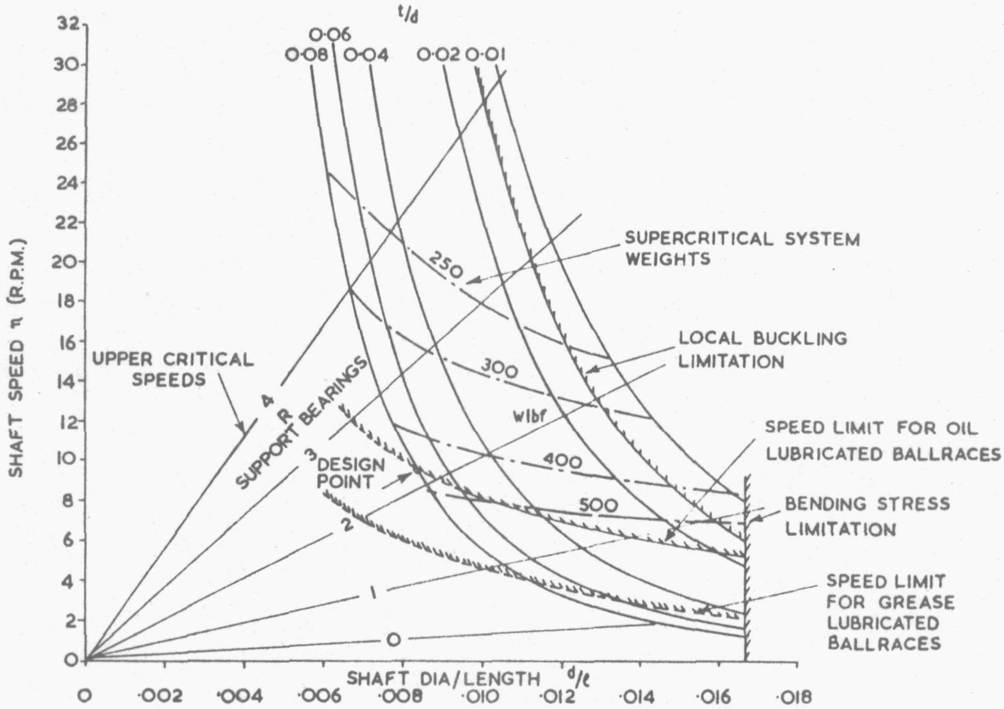

FIG. 3 DESIGN DIAGRAM FOR TRANSMISSION SHAFT, SUPERCRITICAL SPEED

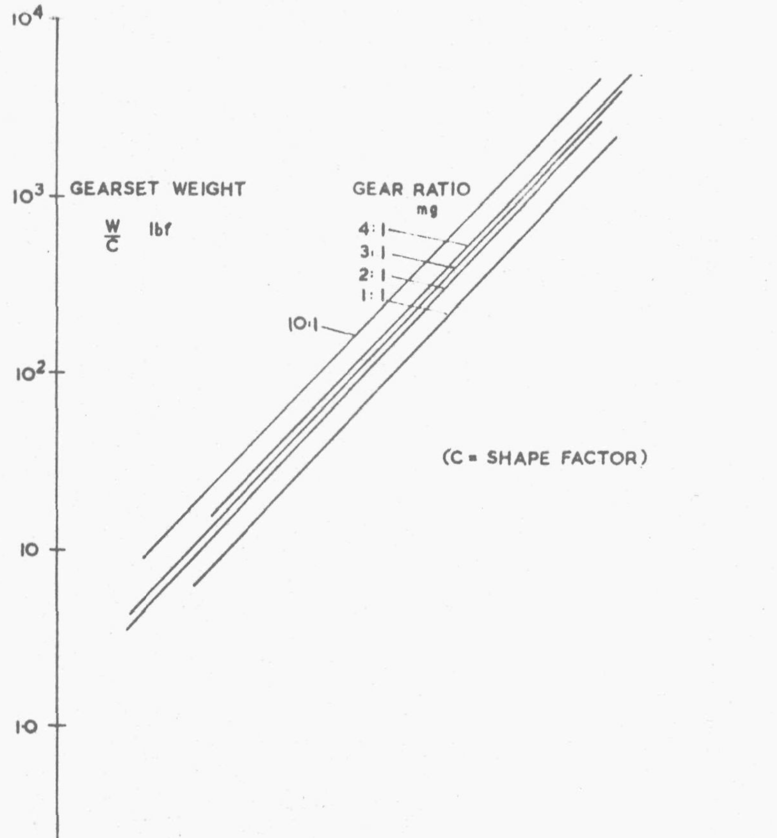

FIG. 4 GEARSET WEIGHT – PINION TORQUE FOR 90° SPIRAL BEVEL GEARS

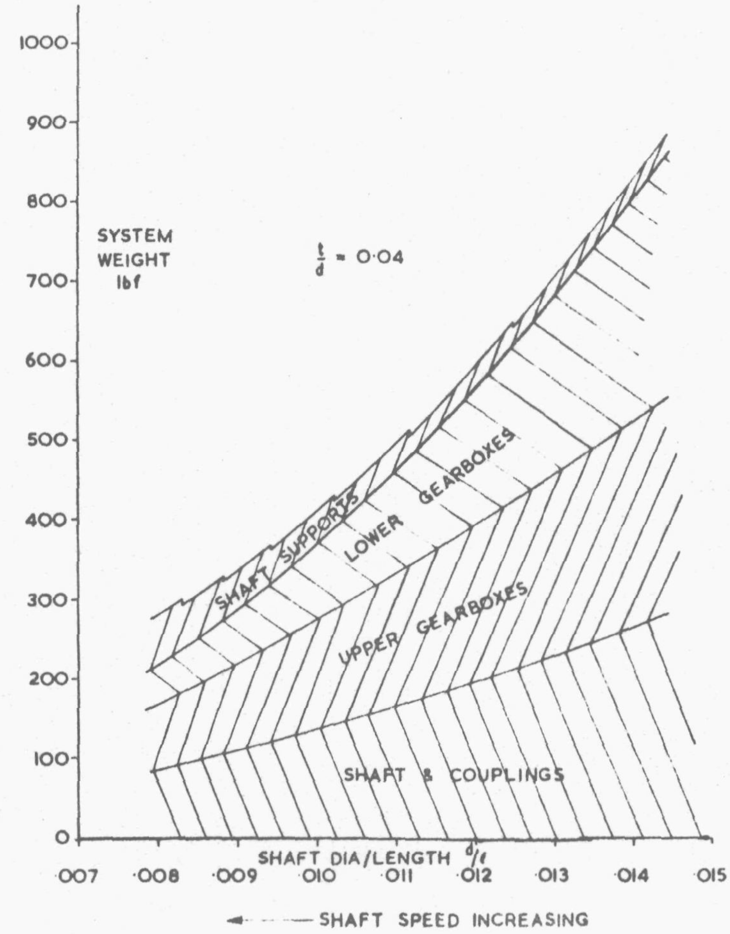

FIG. 5 TYPICAL WEIGHT BREAKDOWN

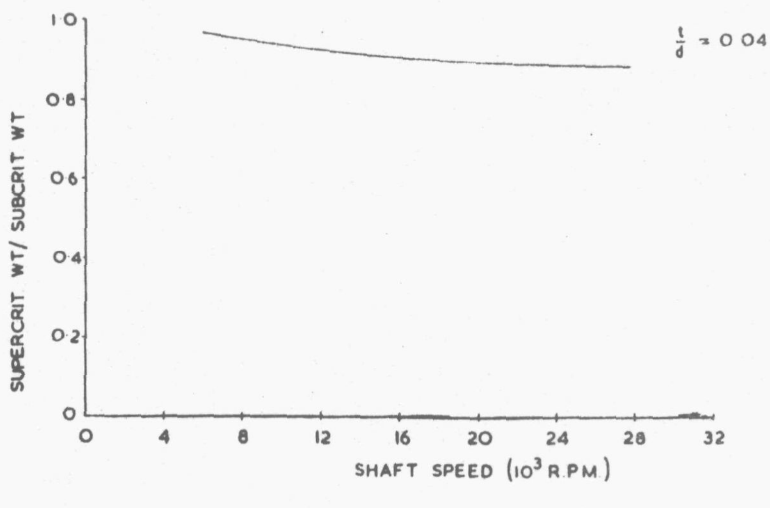

FIG. 6 TYPICAL WEIGHT ADVANTAGE FOR SUPERCRITICAL SYSTEM