Joint Parameters Estimation Using 3D Tensor MUSIC in the Presence of Phase Residual

Xu, Shengzhi; Yarovoy, Alexander

DOI
10.1109/RADAR.2018.8557322

Publication date
2018

Document Version
Final published version

Published in
2018 International Conference on Radar, RADAR 2018

Citation (APA)
Xu, S., & Yarovoy, A. (2018). Joint Parameters Estimation Using 3D Tensor MUSIC in the Presence of Phase Residual. In V. Clarkson (Ed.), *2018 International Conference on Radar, RADAR 2018* Article 8557322 IEEE. https://doi.org/10.1109/RADAR.2018.8557322

Joint Parameters Estimation Using 3D Tensor MUSIC in the Presence of Phase Residual

Shengzhi Xu, Alexander Yarovoy
Microwave Sensing, Signals and Systems (MS3)
Delft University of Technology (TU Delft)
2628 CD Delft, the Netherlands
Email: s.xu-4@tudelft.nl, a.yarovoy@tudelft.nl

***Abstract*—This paper investigates the joint range, Doppler and Direction-of-Arrival (DOA) estimation with a wideband phased array in the presence of phase residual which brought by the range-Doppler couplings. 3D MUSIC algorithm is adopted and a compensation approach is proposed to eliminate the influence of the phase residual on the estimation accuracy. Tensor Decomposition (TD) is applied to obtain the noise subspace. Therefore, the spatial smoothing technique can be avoided. Simulation data validate the improvement of joint parameters estimation performance using the proposed method.**

I. Introduction

Detection and localization of moving targets are important in many fields such as automotive radar, ground moving target indication (GMTI), underwater acoustic array. After deramping for frequency-modulated-continuous-waveform (FMCW) signal or range compression for pulse radar, the parameters of moving targets can be jointly revealed by the multi-dimensional sinusoids data. The spectral analysis tool and spectral estimator can be applied to extract these parameters. Many traditional single frequency estimators have been proposed and can be extended to multiple dimensional frequency estimator. One of the most famous estimators is Fourier transform, which is simple and efficient. However, Fourier transform is criticized due to its bad resolution performance, especially when the data size is limited. Subspace-based methods, like two dimensional multiple signal classification (MUSIC) [1] and two dimensional estimation of signal parameters via rotational invariance technique (ESPRIT) [2], explore the orthogonality between signal subspace and noise space and achieve resolution limits beyond Rayleigh criterions. Owing to the excellent performance of the resolution and accuracy, algorithms from this group are promising in the real application. Some other optimization estimators are proposed recently as well, but most of them suffer the heavy computational burden or are sensitive to the initial guess.

In recent years, when wideband radar signal became widely used due to the requirement of improving range resolution, the range migration problem has attracted significant attention. This problem becomes especially severe for radars with large operational bandwidth and fast moving targets [3]. A variety of algorithms have been proposed to specifically tackle this problem. The iterative adaptive algorithms (IAA) presented in [4] provide super-resolution estimation by alternatively calculating covariance matrix and estimation results. This algorithm is extended to the wideband waveform case with range migration problem in [3] and a fast implementation is further proposed in [5]. The problem for IAA is that the algorithm consumes huge memory and time when the raw data dimension is large and the scanning area is divided into dense bins, which makes the algorithm not practical for the real-time application. The Keystone transform (KT) and matched filter in [6] were used to eliminate range walk problem and Radon Fourier transform (RFT) was proposed in [7] to consider even higher order coupling problems by line or curve searching in the time-frequency domain. However, since these approaches need large raw data size to do interpolation and coherent integration, they could not provide the comparably high resolution as other super-resolution algorithms mentioned in [1], [2]. Implementation of RFT is also time-consuming for line searching in multi-dimensional data. Although many waveform design methods are proposed to solve range migration problem [8], these algorithms increase the system complexity and the resolution is not as high as super-resolution algorithms. It is worth noting that, the definition of conventional range migration, which shown as a coupling term in the data model, is based on the Rayleigh criterions. However, the resolution of subspace-based methods has broken such criterions, so the coupling term will always decrease the performance of these methods more or less even if the target migrates less than one range resolution cell in one coherent processing interval (CPI). As such, for the purpose of not misleading, the term of phase residual will be adopted rather than range migration in the following.

In this paper, we propose a 3D MUSIC-based algorithm for the joint estimation of range, Doppler and DOA using wideband pulse Doppler radar. In addition, an embedded compensation approach is proposed to suppress the influence of the phase residual phenomenon. Besides, TD is applied here to extract the noise subspace. Thus, the spatial smoothing technique for improving the detectability of the coherent sources can be avoided. The rest of the paper is organized as follows. In section II, the signal model for the wideband phased array is presented. In section III, compensated 3D TD-MUSIC is introduced to estimate the range, Doppler and DOA jointly. The simulation results are shown in section IV and finally, conclusions are drawn in section V.

II. Wideband Model

A. Element Received Signal Model

A pulse-Doppler radar with wideband waveform is considered herein. Assume there are I fast moving scatterers presented in the far-field and the reflection of these scatterers are received by a uniform linear array (ULA). The received signal for a single element is considered at first. The received data model $\mathbf{Y} \in \mathbb{C}^{K\times M}$ can be conveniently expressed after FT on the fast-time as

$$\mathbf{Y} = \sum_{i=1}^{I} x_i \mathbf{A}_i + \mathbf{N} \tag{1}$$

where $i = 1, 2, ..., I$, x_i, represent the number index of scatterers and the complex amplitude of ith scatterer, respectively, $\mathbf{A}_i$ is a $K \times M$ matrix containing the signature of ith scatterer and $\mathbf{N}$ is additional white Gaussian noise with power σ^2. The signature $\mathbf{A}_i$ involved in (1) has been studied in [9] and is shown to be the product of a two-dimensional (2D) sinusoids with cross-coupling terms.

$$A_i = \exp\left[j2\pi\left(-\tau_i \frac{B}{K}k + \frac{2vf_c}{c}T_r m + \frac{2v_i}{c}\frac{B}{K}T_r km\right)\right] \tag{2}$$

where τ_i denotes the initial round-trip delay of ith scatterer, v_i denotes the velocity of ith scatterer in the range direction, B denotes the bandwidth, f_c denotes the carrier frequency, T_r denotes the pulse repetition interval, $k = 0, 2, ..., K-1$ denotes fast-time/frequency index, $m = 0, 1, ..., M-1$ denotes slow time/frequency index, and K is the number of sampling points in fast time and M is the total number of pulses, respectively.

In (2), the first two components represent a fast-time frequency sampled at a rate B/K and a Doppler frequency $2vf_c/c$ associated with slow time sampling T_r. The third term brought by the wide bandwidth and the radial movements of the scatterers is the cross-coupling between fast-time and slow-time.

B. Array model

The array data model can be established then. The steering vector of ULA $\mathbf{a}_i^{\theta} \in \mathbb{C}^{L\times 1}$ is

$$\mathbf{a}_i^{\theta} = [1, e^{j2\pi\frac{d}{\lambda}\sin\theta_i}, \ldots, e^{j2\pi\frac{(L-1)d}{\lambda}\sin\theta_i}]^T \tag{3}$$

where $l = 0, 1, \ldots, L-1$, L, d, λ, and θ_i, respectively, represent the index of elements, the number of elements, the inter-space of the neighbouring elements, the wavelength of centre frequency and the DOA of ith scatterer. By using the steering vector, the data model of a single element is extended to array data model as

$$\begin{aligned} s_{l,k,m} &= \sum_{i}^{I} x_i \left\{\exp\left(j2\pi\frac{ld}{\lambda}\sin\theta_i\right)\right. \\ &\times \exp\left[j2\pi\left(-\tau_i\frac{B}{K}k + \frac{2v_i f_c}{c}T_r m\right)\right] \\ &\times \left.\exp\left(j2\pi\frac{2v_i}{c}\frac{B}{K}T_r km\right)\right\} \end{aligned} \tag{4}$$

The discrete data are stacked and further simplified by using the notations $\boldsymbol{\omega}_i^d \in \mathbb{C}^{M\times 1}$ and $\boldsymbol{\omega}_i^r \in \mathbb{C}^{K\times 1}$ as

$$\begin{aligned} \boldsymbol{\omega}_i^d &= [1, e^{j2\pi\frac{2v_i f_c}{c}T_r}, \ldots, e^{j2\pi\frac{2v_i f_c}{c}T_r(M-1)}]^T \\ \boldsymbol{\omega}_i^r &= [1, e^{-j2\pi\tau_i\frac{B}{K}}, \ldots, e^{-j2\pi\tau_i\frac{B}{K}(K-1)}]^T \end{aligned} \tag{5}$$

and

$$[\boldsymbol{\Omega}_i^{dr}]_{m,k} = \exp\left(j2\pi\frac{2v_i}{c}\frac{B}{K}T_r km\right) \tag{6}$$

where $[\cdot]_{m,k}$ denotes the entry in the $(m+1)$th row and the $(k+1)$th column. The array model $\boldsymbol{\mathcal{S}} \in \mathbb{C}^{L\times M\times K}$ then can be written in a compact way as

$$\boldsymbol{\mathcal{S}} = \sum_{i}^{I} x_i(\mathbf{a}_i^{\theta} \circ \boldsymbol{\omega}_i^d \circ \boldsymbol{\omega}_i^r) \odot (\mathbf{1}_L \circ \boldsymbol{\Omega}_i^{dr}) \tag{7}$$

where $\mathbf{1}_L = [1, 1, ...1]^T \in \mathbb{R}^{L\times 1}$, $\circ$ and $\odot$ denote the outer product and Hadamard product, respectively. The array data model in the presence of noise is rewritten as

$$\boldsymbol{\mathcal{Y}} = \boldsymbol{\mathcal{S}} + \boldsymbol{\mathcal{N}} \tag{8}$$

where $\boldsymbol{\mathcal{N}}_{l,m,k} \in \mathbb{C}^{L\times M\times K}$ represent the white Gaussian discrete noise.

III.TD AND COMPENSATED 3D MUSIC ALGORITHM

In this section, the TD-based 3D MUSIC algorithm with phase residual compensation is presented. One of the key step of MUSIC algorithm is the extraction of the noise subspace. Usually, the spatial smoothing technique is applied to obtain the smoothed covariance matrix to increase the detectability of the coherent sources. However, the usage of spatial smoothing decreases the effective raw data size. TD is adopted here to extract the noise subspace to avoid this side effect. Unlike the method in [10], in which the author formulates the multi-dimensional covariance matrix, TD is directly performed. Here, the number of the sources are assumed to be known as I. In fact, although the rank-one components of TD are not mutually orthogonal, they could represent the whole signal subspace. The orthogonalization process is applied to orthogonalize and unitize these components. Next, the noise subspace can be easily obtained from orthogonal complement subspace of the signal subspace. However, the received signals cannot be extracted exactly as rank-one components due to the coupling terms in the data model. Thus a compensation is made before TD to adjust the phase of the signal reflection of one target to a rank one component. As the targets with different velocity have different coupling phase residuals, the compensation has to be implemented in each velocity bin. Therefore, the velocity term is chosen as the first scanning domain to reduce the computational complexity.

The compensation term for the coupling at the velocity scanning bin v_p is written as coupling component $\boldsymbol{\mathcal{D}} \in \mathbb{C}^{L\times M\times K}$ as

$$\boldsymbol{\mathcal{D}} = \mathbf{1}_L \circ \boldsymbol{\Omega}_p^{dr} \tag{9}$$

Then multiply the conjugate compensation term with raw data elementwisely as

$$\hat{\boldsymbol{\mathcal{Y}}} = \boldsymbol{\mathcal{Y}} \odot \boldsymbol{\mathcal{D}}^* \tag{10}$$

where $(\cdot)^*$ means complex conjugate of a matrix. Since the compensation term is just a phase shift, it will not increase the noise power. The new data which the coupling term is removed for the velocity v_p are obtained. Then the range and angle scanning can be performed in current velocity bin.

After removing the phase residual in velocity bin v_p, TD is applied to extract I rank-one component. It is worth noting that these rank-one components cannot represent all the targets signal subspace. Only the targets with velocity v_p, whose the phase residuals are compensated, is rank-one component among them.

Then by outer product and orthogonalization process, the signal subspace is reformed as the unit orthogonal column matrix. After that, the noise subspace is obtained by orthogonal projection as

$$\mathbf{U}_n\mathbf{U}_n^H = \mathbf{I} - \mathbf{U}_s\mathbf{U}_s^H \tag{11}$$

where $\mathbf{I}$ is the identity matrix. The steering function vector is formulate for scanning bin $[\theta_p, v_q, r_h]$ as

$$\boldsymbol{\alpha} = \mathbf{a}_p^\theta \otimes \boldsymbol{\omega}_q^d \otimes \boldsymbol{\omega}_h^r \tag{12}$$

The MUSIC spectrum is

$$\boldsymbol{\mathcal{P}}_{p,q,h} = \frac{1}{\boldsymbol{\alpha}^H(\mathbf{I} - \mathbf{U_s}\mathbf{U_s}^H)\boldsymbol{\alpha}} \tag{13}$$

Above all, the algorithm is concluded in Algorithm (1)

Algorithm 1 compensated 3D TD-MUSIC

for v_p in $[-v_m,\, v_m]$ **do**
  $\hat{\boldsymbol{\mathcal{Y}}} := \boldsymbol{\mathcal{Y}} \odot \boldsymbol{\mathcal{D}}^*$
  $\hat{\boldsymbol{\mathcal{Y}}} =: \sum_i^I \mathbf{a}_i \circ \mathbf{b}_i \circ \mathbf{c}_i$ # Tensor Decomposition
  $\mathbf{U}_s(i) := \mathbf{a}_i \otimes \mathbf{b}_i \otimes \mathbf{c}_i$
  $\mathbf{U}_s(i) := \dfrac{\mathbf{U}_s(i)}{||\mathbf{U}_s(i)||}$
  $\mathbf{U}_s :=$ orth$(\mathbf{U}_s)$ # Orthogonalization
  $\mathbf{U}_n\mathbf{U}_n^H := \mathbf{I} - \mathbf{U}_s\mathbf{U}_s^H$
  for θ_q in $[-\theta_m,\, \theta_m]$ **do**
    for r_h in $[0,\, R_m]$ **do**
      $\boldsymbol{\alpha} := \boldsymbol{\omega}_p^d \otimes \boldsymbol{\omega}_q^\theta \otimes \boldsymbol{\omega}_h^r$
      $\boldsymbol{\mathcal{P}} := \dfrac{1}{\boldsymbol{\alpha}^H(\mathbf{I} - \mathbf{U_s}\mathbf{U_s}^H)\boldsymbol{\alpha}}$
    endfor
  endfor
endfor

IV. Simulation

In this section, the influence of bandwidth and target radial velocity on estimation error is analyzed and the ability of joint Doppler and DOA estimation using the proposed method is evaluated. The parameters of the simulation are shown in Table (I). Here the range resolution of the system is $\frac{c}{2B} = 0.15m$. It is obvious that the coupling term will decrease the performance of the MUSIC algorithm because it destroys the rank-one structure of the signal component. Part of the signal energy will leak into noise part. Therefore, the signal subspace cannot represent the true signal parameters. For the estimation error example please refer [11].

TABLE I: System Parameters

Parameters	Values
Carrier frequency	10 GHz
Number of antenna elements	8
Number of fast-time samplings	64
Number of pulses for CPI	8
Distance between elements	15 mm
Bandwidth	1 GHz
PRI	0.1 ms
SNR	10 dB

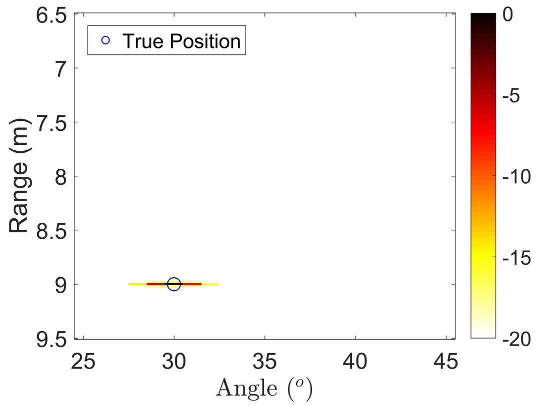
Three targets at the coordinates (range (m), angle (degrees), velocity(m/s)) $(9, 30, 60)$, $(7, 40, 55)$ and $(8, 35, 50)$ are correspondingly set to simulate multiple moving targets. In this simulation, we assume the number of targets is known. However, the coupling terms usually make it very difficult to correctly estimate the number of sources. Thanks to the fact that the dynamic noise subspace is guaranteed to be orthogonal to the steering function vector, the number of the sources is allowed to be slightly overestimated. The estimation of the model rank will be discussed detailly in the future publication. There are many algorithms and tools of tensor decomposition available and in our simulation, the nonlinear least square (NLS) is adopted [12]. The simulation results using 3D TD-MUSIC compensation are shown in Fig. 1(a) and Fig. 2(a), respectively. From the figures, one can see that all the peaks corresponding to the targets reveal in the right position with high resolution. All the results are normalized and restricted in 20 dB for better observation.

For comparison, the results using same 3D TD-MUSIC algorithm without compensation are shown in Fig. 1(b) and Fig. 2(b), the peaks corresponding to the targets appear at the biased position and the closed targets appear as ghost targets. According to the system parameters, although the target may not migrate more than one range resolution cell, the coupling terms influence the resolution and accuracy significantly. Thus, the phase residual phenomenon should not be ingnored in the super-resolution algorithms.

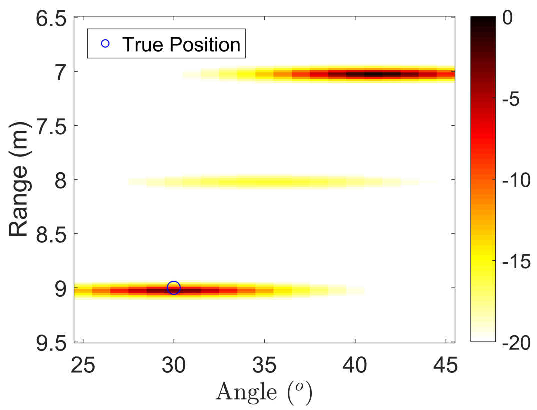
The simulation results successfully validate the improvement of estimation performance on accuracy and resolution. The time for one slice map in Fig. 1(a) is around one minute, while the time for FT is less than one second. Although 3D TD is more computationally intensive than conventional FT, it provides much higher resolutions on estimation. This algorithm could be a subsidiary to provide better estimation in the local spectrum after implementing FT. Moreover, with the parallel processing and more powerful hardware techniques, TD-MUSIC could be a very promising algorithm in the future.

V. Conclusion

In this paper, we have proposed the 3D TD-MUSIC algorithm with the coupling phase residual compensation for joint estimation of range, Doppler and DOA by wideband radar. The influence of the coupling phase on the parameters estimation is removed at first by applying a compensation process for each velocity scanning bin. Then TD is applied to decompose the 3-dimensional raw data, and corresponding orthogonal signal subspace is obtained by orthogonalizing the outer product of rank one component. Finally, the 3D MUSIC algorithms are used to estimate the range, Doppler and DOA jointly. The simulation results validate the improvements of proposed methods with high resolution in joint range, Doppler and DOA estimation.

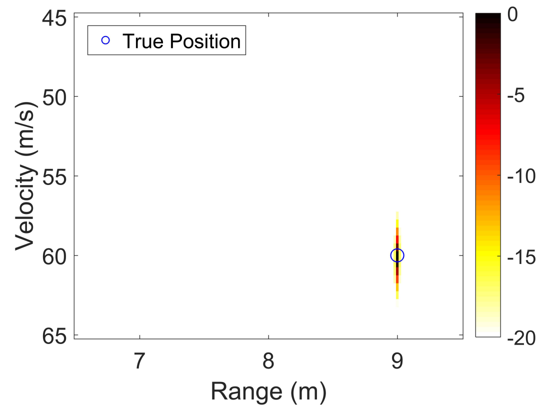


(a)

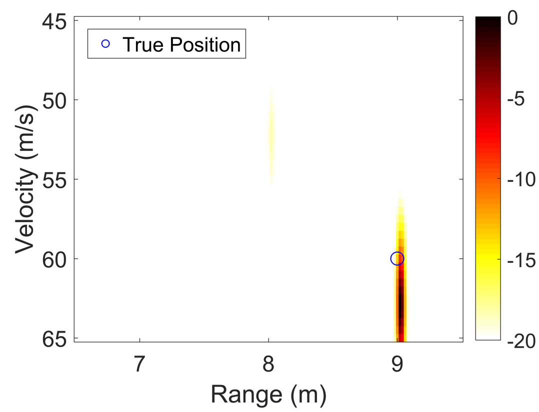


(b)

Fig. 1: Range-angle map at velocity 60 m/s (a) with phase compensation, (b) without phase compensation



(a)



(b)

Fig. 2: Range-velocity map at angle 30° (a) with phase compensation, (b) without phase compensation

ACKNOWLEDGMENT

The authors would like to thank China Scholarship Council (CSC) for the funding support.

REFERENCES

[1] R. O. Schmidt, "A signal subspace approach to multiple emitter location spectral estimation," *Ph. D. Thesis, Stanford University*, 1981.

[2] R. Roy, A. Paulraj, and T. Kailath, "Estimation of signal parameters via rotational invariance techniques-esprit," in *30th Annual Technical Symposium*. International Society for Optics and Photonics, 1986, pp. 94–101.

[3] N. Petrov and F. L. Chevalier, "Iterative adaptive approach for unambiguous wideband radar target detection," in *Radar Conference (EuRAD), 2015 European*, Sept 2015, pp. 45–48.

[4] W. Roberts, P. Stoica, J. Li, T. Yardibi, and F. A. Sadjadi, "Iterative adaptive approaches to mimo radar imaging," *IEEE Journal of Selected Topics in Signal Processing*, vol. 4, no. 1, pp. 5–20, Feb 2010.

[5] N. Petrov and F. L. Chevalier, "Fast implementation of iterative adaptive approach for wideband unambiguous radar detection," in *Signal Processing Conference (EUSIPCO), 2015 23rd European*, Aug 2015, pp. 1207–1211.

[6] Z. Sun, X. Li, W. Yi, G. Cui, and L. Kong, "Detection of weak maneuvering target based on keystone transform and matched filtering process," *Signal Processing*, vol. 140, pp. 127 – 138, 2017. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S0165168417301846

[7] J. Yu, J. Xu, Y. N. Peng, and X. G. Xia, "Radon-fourier transform for radar target detection (iii): Optimality and fast implementations," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 48, no. 2, pp. 991–1004, APRIL 2012.

[8] F. Roos, D. Ellenrieder, N. Appenrodt, J. Dickmann, and C. Waldschmidt, "Range migration compensation for chirp-sequence based radar," in *2016 German Microwave Conference (GeMiC)*, March 2016, pp. 317–320.

[9] F. Deudon, S. Bidon, O. Besson, and J. Y. Tourneret, "Velocity dealiased spectral estimators of range migrating targets using a single low-prf wideband waveform," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 49, no. 1, pp. 244–265, Jan 2013.

[10] C. L. Liu and P. P. Vaidyanathan, "Tensor music in multidimensional sparse arrays," in *2015 49th Asilomar Conference on Signals, Systems and Computers*, Nov 2015, pp. 1783–1787.

[11] S. Xu and A. Yarovoy, "Joint doppler and doa estimation using 2d music in presence of phase residual," in *2017 European Radar Conference (EURAD)*, Oct 2017, pp. 203–206.

[12] N. Vervliet, O. Debals, and L. D. Lathauwer, "Tensorlab 3.0 — numerical optimization strategies for large-scale constrained and coupled matrix/tensor factorization," in *2016 50th Asilomar Conference on Signals, Systems and Computers*, Nov 2016, pp. 1733–1738.