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COLREGs-aware Trajectory Optimization for Autonomous Surface Vessels

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Abstract: This paper presents a rule-compliant trajectory optimization method for the guidance and control of autonomous surface vessels. The method builds on Model Predictive Contouring Control and incorporates the International Regulations for Preventing Collisions at Sea—known as COLREGs—relevant for motion planning. We use these traffic rules to derive a trajectory optimization algorithm that guarantees safe navigation in mixed-traffic conditions, that is, in traffic environments with human operated vessels. The choice of an optimization-based approach enables the formalization of abstract verbal expressions, such as traffic rules, and their incorporation in the trajectory optimization algorithm along with the dynamics and other constraints that dictate the system's evolution over a sufficiently long receding horizon. The ability to plan considering different types of constraints over a long horizon in a unified manner leads to a proactive motion planner that mimics rule-compliant maneuvering behavior. The efficacy of the derived algorithm is validated in different simulation scenarios.

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Keywords: Autonomous Surface Vessels, Model Predictive Control, Traffic Regulations

1. INTRODUCTION

Over the past few years, we have witnessed our society rapidly moving towards an increased level of automation. While the automotive industry has had the leading role in this trend, the maritime domain is also progressing towards developing and utilizing autonomous maritime systems in many applications including transportation, environmental monitoring, or search and rescue missions. This shift towards autonomy is motivated by numerous potential benefits, such as greater efficiency, reduced operational costs, and increased safety. Autonomous maritime navigation has the potential to significantly reduce the risk of collisions that often lead to human casualties, damaged property, and devastating environmental disasters.

Ongoing research regarding safety in autonomous maritime navigation has focused on the problem of incorporating the International Regulations for Preventing Collisions at Sea (COLREGs) in existing motion planning algorithms. Methods of subsets of controls, such as Velocity Obstacles (Kuwata et al., 2013) and its extensions (Kufalor et al., 2018; Cho et al., 2020), as well as methods of physical analogies, such as Artificial Potential Fields (Lyu and Yin, 2019) have been studied thoroughly to work along with COLREGs as they are methods of low-computational complexity. These algorithms, however, are reactive and difficult to combine with traffic regulations, often resulting in rule violations. Search-based methods like A* (Švec et al., 2014; Agrawal and Dolan, 2015; He et al., 2022), and RRT* (Chiang and Tapia, 2018; Enevoldsen et al., 2021) have also been employed. They search for a dynamically feasible path in a joint time-state space by either creating artificial costs or obstacles in the discrete grid map in order

to resemble rule-compliant maneuvers. These methods are still not suitable to encompass the complete set of traffic regulations and may even ignore some of the rules in multi-vessel situations (Chiang and Tapia, 2018). Recently, learning-based methods have also been investigated in conjunction with the traffic rules (Meyer et al., 2020; Xu et al., 2020), though drawbacks in these methods often include poor generalization ability, convergence to local minima, and lack of formal guarantees.

A popular category for motion planning under COLREGs includes methods based on Model Predictive Control (MPC). The main benefit of these methods is the possibility to combine constraints of different nature under one scheme. Among the limitations, the most important ones include deadlocks (due to the local nature of the computed path) and high computational demands (depending on the complexity of the formulated problem). To circumvent these limitations, Johansen et al. (2016) and some extension works (Hagen et al., 2018; Kufalor et al., 2019; Tengedal et al., 2020) established a sample-based MPC approach that considers a finite space of control inputs. Unlike typical MPC formulations, these methods do not identify the best action at every time step during trajectory generation. Conventional gradient-based approaches have also been studied as well (Abdelaal et al., 2018; Eriksen et al., 2020; Du et al., 2021) having the benefit of exploring the entire control input space. However, all aforementioned approaches rely on a heuristic cost function (that either combines hazard metrics or creates repulsive fields based on the geometrical situation) and consider only a subset of the necessary regulations which may lead to rule violations.

Our work relies on a specific MPC formulation, namely, on Model Predictive Contouring Control (MPCC), which has been adopted in various autonomous vehicle applications (Schwartz et al., 2018; Ferranti et al., 2018; Brito et al., 2019; de Groot et al., 2021). While MPCC has shown good results in local motion planning, it still bears the limitation of avoiding dynamic obstacles in an unstructured manner as there is no consideration of traffic rules. We show how this local planner can be extended to incorporate the “rules of the road”. We apply our method to marine vessels considering every rule relevant to local motion planning explicitly, that is, every relevant rule is formulated as a constraint in the MPCC problem. We show, in different navigation scenarios among other traffic participants, how our algorithm can locally provide safe and rule-compliant trajectories by utilizing these traffic regulations.

2. PROBLEM FORMULATION

We assume that the Autonomous Surface Vessel (ASV) moves in a planar workspace $\mathcal{W} = \mathbb{R}^2$ with state denoted as $\mathbf{z} \in \mathcal{Z} \subseteq \mathbb{R}^n$ and control input as $\boldsymbol{\tau} \in \mathcal{T} \subseteq \mathbb{R}^m$. The evolution of the ASV’s state is expressed by the following discrete, nonlinear system:

$$\mathbf{z}(t+1) = \mathbf{f}(\mathbf{z}(t), \boldsymbol{\tau}(t)), \quad t = 0, 1, \dots \quad (1)$$

We assume a planar motion for the n_{obs} neighboring vessels as well, and that their state $\mathbf{z}^i \in \mathcal{Z}^i \subseteq \mathbb{R}^{n_i}$, $i = \{1, \dots, n_{\text{obs}}\}$, is known to sufficient precision within an area around the ASV via a suitable perception framework. We consider the subset of COLREGs 1-18 that describes navigation of vessels “in sight of one another” and that are relevant to motion planning. The state of the ASV is constrained by these rules expressed mathematically as a set of state constraints denoted as $\mathcal{Z}^R(\mathbf{z}, \mathbf{z}^i) \subseteq \mathbb{R}^n$. At each planning cycle, given the initial states \mathbf{z}_0 and \mathbf{z}_0^i of the ASV and vessel i respectively as their current states, $\mathbf{z}(t)$ and $\mathbf{z}^i(t)$, as well as a reference path parameterized by path parameter s and initialized at s_0 , our goal is to formulate an MPCC problem over a finite time horizon N with the set of states $\mathbf{z}_{0:N} \in \mathcal{Z}$, set of inputs $\boldsymbol{\tau}_{0:N-1} \in \mathcal{T}$ and path parameter $s_{0:N}$ as decision variables:

$$\min_{\mathbf{z}, \boldsymbol{\tau}, s} \sum_{k=0}^N J(\mathbf{z}_k, \boldsymbol{\tau}_k, s_k) \quad (2a)$$

$$\text{s.t.: } \mathbf{z}_{k+1} = \mathbf{f}(\mathbf{z}_k, \boldsymbol{\tau}_k), \quad k = 0, \dots, N \quad (2b)$$

$$s_{k+1} = g(\mathbf{z}_k, s_k), \quad k = 0, \dots, N \quad (2c)$$

$$\mathbf{z}_k \in \mathcal{Z} \cap \mathcal{Z}^R(\mathbf{z}_0, \mathbf{z}_0^i), \quad k = 0, \dots, N \quad (2d)$$

$$\boldsymbol{\tau}_k \in \mathcal{T}, \quad k = 0, \dots, N-1 \quad (2e)$$

where we denote the predicted variables with subscript k . The solution to the receding horizon problem is the optimal input sequence $\boldsymbol{\tau}_{0:N-1}^*$ that minimizes cost function (2a) under system dynamics (2b), path evolution (2c), state constraints (2d) and input constraints (2e). The cost function (2a) is designed such that the ASV will achieve navigation objectives according to the path evolution (2c) discussed in Section 3.2 and compliance to a subset of the rules discussed in Section 3.3. The dynamics (2b) and physical limitations of the state and inputs (2d), (2e) are detailed in Section 3.1 and the rule-compliance constraints (2d) that serve the task of dynamic collision avoidance in a structured manner are derived in Section 3.3.

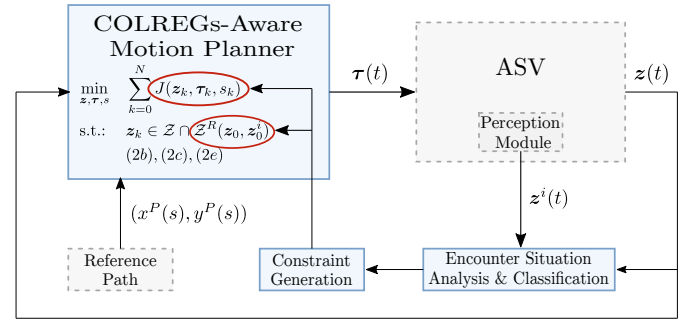


Fig. 1. Schematic overview of the COLREGs-aware trajectory optimization method

3. COLREGS-AWARE TRAJECTORY OPTIMIZATION

An overview of our COLREGs-compliant navigation architecture is provided in Figure 1. We encode the traffic rules in an algorithmic framework for situational awareness, which is necessary for rule-compliant decision making. The “Constraint Generation” module generates a set of mathematical constraints that are suitable for a receding horizon problem and can guarantee a rule-compliant motion based on the module “Encounter Situation Analysis & Classification” that assigns a specific traffic role to the vessels. The latter is not detailed in this work for the sake of brevity. Our “COLREGs-Aware Motion Planner” module optimizes the ASVs’ trajectory and generates the corresponding control command. As a result, alternatively to previous works (Schwartz et al., 2018; Brito et al., 2019; Ferranti et al., 2018), we consider dynamic collision avoidance implicitly by enforcing compliance to the traffic rules. In the remainder of this section we detail the main modules of our architecture.

3.1 ASV Model

We adopt a 3-DoF kinematic model¹. The ASV’s configuration is described by its position $\mathbf{p} = (x, y)^\top$ and orientation ψ . We then denote as $\mathbf{z} = (x, y, \psi)^\top \in \mathcal{Z}$ the system’s state and as $\boldsymbol{\tau} = (u, 0, r)^\top \in \mathcal{T}$ the control input, where u and r denote the surge and yaw velocities, respectively. We make the simplifying assumption that the sway velocity is zero as in Eriksen et al. (2020). The evolution of the system’s state is expressed by the following continuous-time, nonlinear system:

$$\dot{\mathbf{z}} = \underbrace{\begin{bmatrix} \cos(\psi) & -\sin(\psi) & 0 \\ \sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\mathbf{R}(\mathbf{z})} \boldsymbol{\tau} \quad (3)$$

In addition, $u \in [u_{\min}, u_{\max}]$, $r \in [r_{\min}, r_{\max}]$, and u_{\min} , u_{\max} , r_{\min} , r_{\max} are the minimum and maximum longitudinal and angular velocity inputs, respectively. The rest of the states are subject to limitations imposed by the traffic rules and discussed in Section 3.3. Lastly, the continuous dynamics (3) are discretized using a Runge-Kutta method in the form (2b) for the purpose of solving the receding horizon problem (2).

¹ MPCC can accommodate more complex models if needed.

way to implement this rule is to impose constraints on the angular acceleration \dot{r} and the longitudinal acceleration \dot{u} to be larger than a certain value. However, as explained in Eriksen et al. (2020), this can result in a highly non-convex (and even non-connected) search space and, consequently, in a hard-to-solve nonlinear optimization problem. Moreover, these variables are not included in (2). To circumvent these problems, we consider this rule in the design of constraints for Rules 13-15 later in this section. These constraints will cause the ASV to alter its course in a sufficient, rule compliant manner.

Rule 8.c states that “if there is sufficient room, alteration of course alone may be the most effective action to avoid a close-quarters situation”. This is already considered in (8) where we can tune weight q_u accordingly to track the reference speed. According to Rule 8.e, though, the vessel “shall slacken her speed or take all way off by stopping or reversing her means of propulsion”. This means that the objective described in term (8) might interfere with collision avoidance as it describes a conflicting goal for the motion planner. The problem can be overcome by switching the value of the tuning parameter q_u of cost term (8) according to the vessel role as $q_u \in \{q_{uSO}, q_{uGW}, q_{uEGW}\}$ with $q_{uEGW} \ll q_{uGW} < q_{uSO}$.

Next, we consider Rules 13-15, which describe the maneuver a vessel should follow in the *Overtaking*, *Head-On situation*, and *Crossing situation* respectively. In most MPC-based works these constraints are implemented as soft constraints via a heuristic cost function. In this work, instead, the goal is to implement these rules as hard constraints to guarantee a rule-compliant behavior. Moreover, the design should not cause problems with feasibility and allow the solution of (2) in real time. Thus, we aim to design a set of affine constraints for each pairwise situation that will result to a convex polytope in the receding horizon problem (2). We can then have strict rule-compliance guarantees in multi-vessel situations without complicating the solution of the optimization problem.

To design these constraints, we assume knowledge of the current state \mathbf{z}^i of vessel i as well as knowledge of its length l^i and beam w^i via a suitable perception module. Based on this information, we can create bounding boxes for the other vessels enlarged by the ASV's footprint radius ρ , as illustrated in Figure 3. The position of the vertices of the enlarged circumscribed rectangle are:

$$\mathbf{p}_0^{i,j} = \mathbf{p}_0^i + \mathbf{R}^{xy}(\mathbf{z}^i) \begin{pmatrix} \pm(l^i/2 + \rho) \\ \pm(w^i/2 + \rho) \end{pmatrix} \quad (9)$$

for each vertex $j = sb, ss, pb, ps$ shown in Figure 3. For the ASV, we assume that it is circumscribed by a circle $\mathcal{C}_k(\mathbf{p}_k, \rho)$ of the previously discussed radius ρ .

When the ASV has a GW role, we can define the affine constraints (illustrated in Figure 3.a) to guarantee that the ASV will follow a maneuver that is compliant with Rules 13-16. These affine constraints are the anti-clockwise tangent lines to the ASV's circumscribing circle $\mathcal{C}_k(\mathbf{p}_k, \rho)$ that pass through each vertex $\mathbf{p}_k^{i,j}$ and the respective tangent point $\mathbf{q}_k^{i,j} = \mathbf{q}_k^{i,j}(\mathcal{C}_k(\mathbf{p}_k, \rho), \mathbf{p}_k^{i,j})$. To render the constraints affine at each time t , we use the current positions of the ASV and the other vessel which we denote as \mathbf{p}_0 and \mathbf{p}_0^i respectively. We can then use the latter to

compute the tangent point as $\mathbf{q}_0^{i,j} = \mathbf{q}_0^{i,j}(\mathcal{C}_0(\mathbf{p}_0, \rho), \mathbf{p}_0^{i,j})$. For each vessel i and for each of its vertices j of the circumscribed rectangle, the constraint is given by:

$$\mathbf{A}_0^{i\top}(\mathbf{q}_0^{i,j}, \mathbf{p}_0^{i,j})\mathbf{p}_k \leq b_0^i(\mathbf{q}_0^{i,j}, \mathbf{p}_0^{i,j}) \quad (10a)$$

with

$$\mathbf{A}_0^{i\top} = [\mathbf{q}_{y,0}^{i,j} - \mathbf{p}_{y,0}^{i,j} \quad \mathbf{p}_{x,0}^{i,j} - \mathbf{q}_{x,0}^{i,j}] \quad (10b)$$

$$b_0^i(\mathbf{q}_0^{i,j}, \mathbf{p}_0^{i,j}) = \mathbf{q}_{y,0}^{i,j}(\mathbf{p}_{x,0}^{i,j} - \mathbf{q}_{x,0}^{i,j}) - \mathbf{q}_{x,0}^{i,j}(\mathbf{p}_{y,0}^{i,j} - \mathbf{q}_{y,0}^{i,j}) \quad (10c)$$

In case the ASV has an EGW role, a different constraint should be defined to ensure compliance with Rule 17. This is designed as illustrated in Figure 3.b so that the ASV follows a maneuver according to Rule 17. In order to design this constraint we first compute the point $\mathbf{p}_k^{E_i}$ as the intersection of the line that passes through points \mathbf{p}_k and \mathbf{p}_k^i with circle $\mathcal{C}_k(\mathbf{p}_k, \rho)$ thus, $\mathbf{p}_k^{E_i} = \mathbf{p}_k^{E_i}(\mathcal{C}_k(\mathbf{p}_k, \rho), \mathbf{p}_k^i)$. To keep these constraints affine as well, we use again the current positions of the ASV and the other vessel, \mathbf{p}_0 and \mathbf{p}_0^i respectively. The constraint can be then defined as:

$$\mathbf{A}_0^{E_i\top}(\mathbf{p}_0, \mathbf{p}_0^i)\mathbf{p}_k \leq b_0^i(\mathbf{p}_0, \mathbf{p}_0^i) \quad (11a)$$

with

$$\mathbf{A}_0^{E_i\top} = [\mathbf{p}_{x,0}^i - \mathbf{p}_{x,0} \quad \mathbf{p}_{y,0}^i - \mathbf{p}_{y,0}] \quad (11b)$$

$$b_0^i(\mathbf{p}_0, \mathbf{p}_0^i) = (\mathbf{p}_{x,0}^i - \mathbf{p}_{x,0})\mathbf{p}_{x,0}^{E_i} + (\mathbf{p}_{y,0}^i - \mathbf{p}_{y,0})\mathbf{p}_{y,0}^{E_i} \quad (11c)$$

Lastly, if the ASV has a SO role, no constraints are imposed and the vessel is required to maintain its course and speed according to Rule 17.

4. SIMULATION RESULTS

This section validates the efficacy of our algorithm in different scenarios. The selected scenarios cover all possible vessel encounters in which the ASV needs to take action and emphasize rule compliance in each situation. Our framework is implemented in ROS and the solver used is generated with Forces Pro (Domahidi and Jerez, 2014).

Figure 4 demonstrates the ASV's maneuver in an Overtaking situation where the ASV has a GW role. As described in Rule 13, the ASV turns to starboard while it keeps out of the way of the Obstacle Vessel. Figure 5 illustrates a Starboard-Crossing situation in which the ASV has a GW role. In this scenario the ASV takes a collision avoidance maneuver to its starboard and avoids crossing ahead of the other vessel according to Rule 15. Figure 6 shows the ASV in a Head-On situation and a GW role. In compliance with Rule 14, the ASV changes course to starboard so that each vessel passes on the port side of the other while it keeps out of the way of the Obstacle Vessel. Lastly, Figure 7 presents a Port-Crossing situation where the ASV normally would have a SO role, but the Obstacle Vessel does not comply with the rules and does not take action to avoid collision. In this case, the ASV has an EGW role and needs to take action to avoid collision while it does not alter its course to port for a vessel on its own port side.

In the aforementioned Figures, the initial position of each vessel is illustrated with a more transparent color shade which progressively becomes opaque until the last recorded position. In every scenario, the ASV autonomously performs maneuvers that are clear and readily apparent thus complying with Rule 8. Animations of the simulation scenarios can be found in Tsolakis et al. (2022).

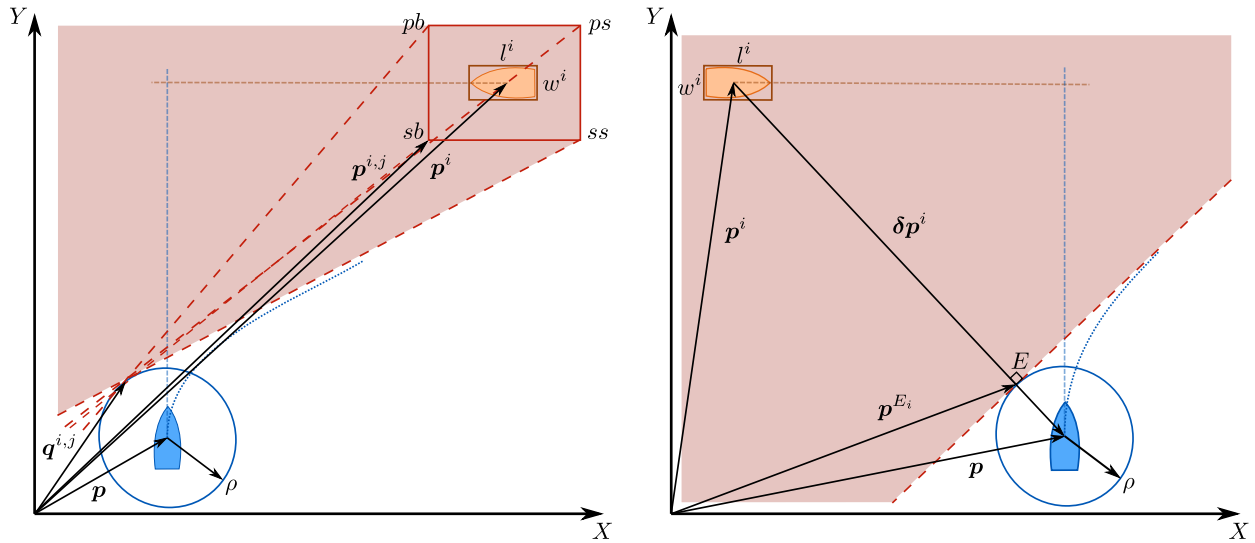


Fig. 3. Collision avoidance affine constraints when the ASV has a) Give-Way role and b) Emergency Give-Way role

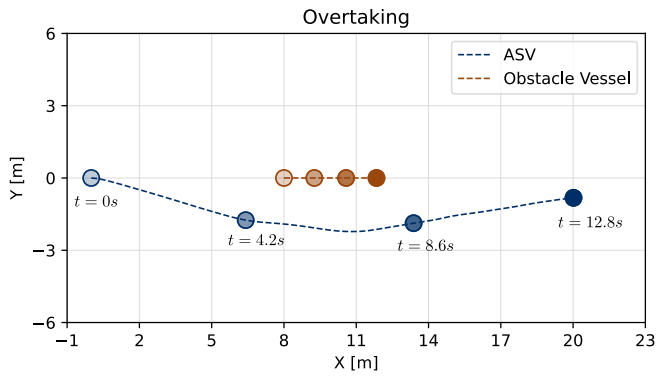


Fig. 4. Overtaking situation with the ASV in Give-Way role, turning to starboard while it keeps out of the way of the Obstacle Vessel as described in Rule 13.

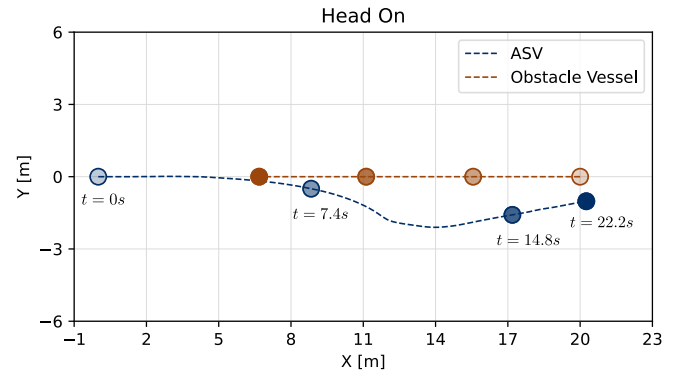


Fig. 6. Head-On situation with the ASV in Give-Way role, turning to starboard so that each vessel passes on the port side of the other as described in Rule 14.

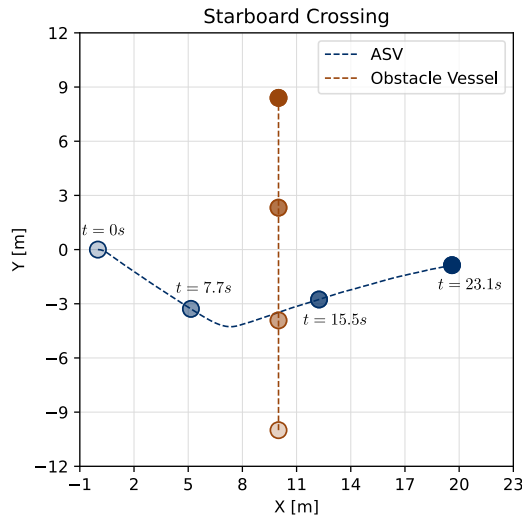


Fig. 5. Starboard Crossing situation with the ASV in Give-Way role, turning to starboard while it keeps out of the way of the Obstacle Vessel and avoids crossing ahead of the other vessel as described in Rule 15.

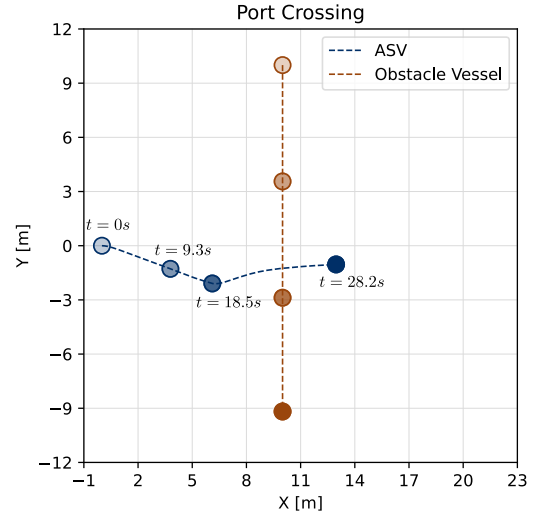


Fig. 7. Port Crossing situation with the ASV in Stand-On role, but obliged to take a collision avoiding maneuver to its starboard and then passing behind the non-compliant vessel as described in Rule 17.

5. CONCLUSIONS & FUTURE WORK

This paper proposed a trajectory optimization algorithm for COLREGs-compliant collision avoidance. The efficacy of the proposed algorithm was validated via different simulation scenarios specifically chosen to showcase rule-compliant collision avoidance maneuvers that comply with COLREGs. For future work, we will extend our method to consider the kinetic model of vessels including model uncertainties. Moreover, performance analysis will be realized to characterize the effect of sampling time and receding horizon length in the proposed method.

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