Corrigendum to
‘A Banach–Dieudonné theorem for the space of bounded continuous functions on a separable metric space with the strict topology’ (Topology and its Applications (2016) 209 (181–188), (S0166864116301213) (10.1016/j.topol.2016.06.003))

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Corrigendum

Corrigendum to: 'A Banach–Dieudonné theorem for the space of bounded continuous functions on a separable metric space with the strict topology' [Topol. Appl. (2016) 181–188]

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A R T I C L E I N F O

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The author regrets a mistake made in Kraaij [2]. We summarize the results which remain valid and those whose validity is now unclear.

1. An overview of the status of the main results

Let $\mathcal{X}$ be a separable metric space. On $\mathcal{X}$ we consider the space of bounded continuous functions $(C_b(\mathcal{X}), \beta)$ equipped with the strict topology, cf. Sentilles [3]. In addition, let $\mathcal{M}_\tau(\mathcal{X}) = (C_b(\mathcal{X}), \beta)'$ be the space of τ-additive Borel measures on $\mathcal{X}$ and let σ be the weak topology on $\mathcal{M}_\tau(\mathcal{X})$ induced by $C_b(\mathcal{X})$.

In Kraaij [2], four additional topologies were considered on $\mathcal{M}_\tau(\mathcal{X})$:

- σ^{lf}, the finest locally convex topology on $\mathcal{M}_\tau(\mathcal{X})$ that coincides with σ on all β-equicontinuous sets in $\mathcal{M}_\tau(\mathcal{X})$,
- σ^f, the finest topology on $\mathcal{M}_\tau(\mathcal{X})$ that coincides with σ on all β-equicontinuous sets in $\mathcal{M}_\tau(\mathcal{X})$,
- $k\sigma$ the finest topology on $\mathcal{M}_\tau(\mathcal{X})$ that coincides with σ on all weakly compact sets in $\mathcal{M}_\tau(\mathcal{X})$,
- β° the polar topology on $\mathcal{M}_\tau(\mathcal{X})$ generated using all pre-compact sets in $(C_b(\mathcal{X}), \beta)$, cf. Köthe [1].

The main result of Kraaij [2] is Theorem 1.7 that states that $\sigma^{lf} = \sigma^f = k\sigma = \beta^\circ$. The following result remains true:

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Proposition 1.1. *$\sigma^f = k\sigma$ and $\sigma^{lf} = \beta^\circ$.*

As a consequence of the missing identification $\sigma^f = \sigma^{lf}$, it is unclear whether $(C_b(\mathcal{X}), \beta)$ is infra-Pták by using Proposition 1.2. This in turn leads to the failure of establishing Corollaries 1.10, 1.11 and 1.12. Proposition 1.6 and Lemma's 1.8 and 1.9 are established using results in the literature and remain valid as it is.

2. The mistake and an overview of its consequences in the proof sections

The proof that $\sigma^f = \sigma^{lf}$ was based on the observation that as $\sigma^{lf} \subseteq \sigma^f$ it suffices to verify that σ^f is locally convex.

This was carried out in two steps.

Step 1: σ^f was explicitly identified as a quotient topology $\mathcal{T}$ and it was shown that $k\sigma = \mathcal{T} = \sigma^f$. This part remains valid.

Step 2: The explicit characterization $\mathcal{T}$ was then used to establish that $\mathcal{T}$ is locally convex. This part contains an error in the proof of Lemma 2.7. As a consequence, is unclear whether Lemma 2.8 and Proposition 2.6 remain true.

The error: The proof that addition is a continuous map for $\mathcal{T}$ is mistaken, the proof that scalar multiplication is continuous remains valid.

The exact mistake in Lemma 2.7 is the claim that $H \subseteq U$. Let $\oplus : \mathcal{M}_{\tau,+}^2 \times \mathcal{M}_\tau \to \mathcal{M}_\tau^2$ be the map defined by $\oplus(\mu, \nu, \rho) = (\mu + \rho, \nu + \rho)$. Let A, B be σ_+ open subsets in $\mathcal{M}_{\tau,+}$ for and let C be σ open in $\mathcal{M}_\tau$. Finally let $(\mu_0, \nu_0) \in A \times B$.

The sets H and U were defined as

$$H := (\mu_0 + C) \times (v_0 + C),$$
$$U := \oplus(A \times B \times C).$$

The issue is that $\oplus$ adds the set C interpreted as the diagonal $\{(\rho, \rho) \,|\, \rho \in C\}$ to the set $A \times B$, whereas the construction to obtain H adds the much larger product space $C \times C$ to (μ_0, ν_0). As a consequence the claim $H \subseteq U$ remains unproven.

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