Delft University of Technology

Rail network robustness

The role of rapid development and a polycentric structure in withstanding random and targeted attacks

Cats, Oded; Krishnakumari, Panchamy; Tundulyasaree, Krissada

Important note
To cite this publication, please use the final published version (if applicable).
Please check the document version above.

1 **METROPOLITAN RAIL NETWORK ROBUSTNESS: INVESTIGATING THE**
2 **ROLE OF RAPID DEVELOPMENT AND A POLYCENTRIC STRUCTURE**

3 **Oded Cats**
4 Department of Transportation and Planning
5 Delft University of Technology
6 P.O. Box 5048, 2600 GA Delft, The Netherlands
7 Phone number: +31 15 27 85279
8 Email: o.cats@tudelft.nl

9 **Panchamy Krishnakumari**
10 Department of Transportation and Planning
11 Delft University of Technology
12 P.O. Box 5048, 2600 GA Delft, The Netherlands
13 Phone number: +31 15 27 85279
14 Email: p.k.krishnakumari@tudelft.nl

15 **Krissada Tundulyasaree**
16 Department of Transportation and Planning
17 Delft University of Technology
18 P.O. Box 5048, 2600 GA Delft, The Netherlands
19 Phone number: +31 15 27 85279
20 Email: k.tundulyasaree@student.tudelft.nl

Cats, Krishnakumari and Tundulyasaree

Abstract

1   In large-scale urban agglomerations, heavy rail in the form of metro and commuter train serves as the backbone of
2   the metropolitan public transport network (MPTN). Transport systems, including MPTN, are subject to recurrent
3   disruptions that may result in severe consequences for network performance and society at large. The objective of
4   this paper is to compare the robustness of selected public transport networks which exhibit different properties to
5   both random and targeted attacks and gain insights on the role of network structure and development pattern. We
6   adopt a complex network theory approach, inveistgating network performance under alternative sequential
7   disruption scenarios corresponding to the successive closure of either stations or track segments. Targeted attacks
8   are simulated by the removal of the most central network elements. Network performance is measured both in terms
9   of the capacity of the network to function in terms of connectivity as well as the impedance induced that remain
10   connected. Two exemplary networks are selected – the polycentric Randstad region and the largest and rapidly
11   developing Shanghai metro network. The results indicate that both networks are highly vulnerable to targeted
12   attacks with the Shanghai network exhibiting greater robustness in all scenarios. The connections between the urban
13   cores are most critical in the multi-centric Randstad network while the Shanghai network is most susceptible to
14   disruptions in the edge of its core, risking cutting off its long branches. Our findings provide more nuanced insights
15   on the relations between network structure and robustness.

Cats, Krishnakumari and Tundulyasaree

1 **1 INTRODUCTION**

2 Urban population increases rapidly worldwide as people are seeking better quality of life and job opportunities and
3 governments stimulate agglomeration effects through investments in infrastructure. Faced with this situation, local
4 authorities aim to improve mobility by densifying and expanding the respective public transport system. In large-
5 scale urban agglomerations, heavy rail in the form of metro and commuter train serves as the primary high-level
6 metropolitan public transport network (MPTN). Transport systems, including MPTN, are subject to recurrent
7 disruptions that may result in severe consequences for network performance and the metropolitan metabolism as a
8 whole. The robustness of critical infrastructure such as mass MPTNs is high on the planning and policy agenda (*1*).
9 Notwithstanding, there is lack of knowledge on how network structure and design philosophy impacts its robustness.
10 This is the topic of this study which adopts a complex network theory approach.
11
12 MPTN can be represented as a graph where stations correspond to nodes and track segments to links. Lin and Ban
13 (*2*) provide a review of the literature devoted to transport network topology and measures adopted from complex
14 network perspective since the early 2000s. A large number of studies that examined MPTN worldwide concluded
15 that they exhibit scale-free and small-world properties. The former implies that regardless of the size of the network
16 any node can be reached by a few steps while the latter implies that the degree of a graph follows a power-law
17 distribution with the number of connections decreasing exponentially – i.e. few stations with many connections and
18 many stations with few connections. The combination of these two properties is the blueprint of a hub and spoke
19 network that branches out as you move away from the central core which Roth et al. (*3*) found to characterize the
20 world's largest subway systems. This can be expected to happen if network evolution, the process in which new
21 stations and track segments are added to the network, follows a preferential attachment mechanism where new
22 stations are more likely to be connected to other stations which are already well-connected (i.e. what has become
23 known as the rich getting richer phenomenon).
24
25 The structure of the public transport network is intertwined with the urban and regional development and the
26 underlying policy making process. Metropolitan areas are often characterized as monocentric areas (as implied by
27 the etymology of the word metropolitan) with radial networks. Notwithstanding, MPTN vary in the extent to which
28 they are indeed concentrated around a single centre with secondary centres being the norm rather than the exception.
29 The extent to which an urban agglomeration exhibits monocentric or polycentric land-use and travel patterns
30 depends, among other things, on the underlying MPTN and the planning policy. The latter is also reflected in the
31 development of MPTN which is an important driver in realizing such plans. Examples of the latter are the light rail
32 projects in Paris and the on-going metro extension in Stockholm (*4*), both of which devised to stimulate a more
33 polycentric development. In several cases such as in the Randstad in the Netherlands and the Ruhr area in Germany,
34 an agglomeration of cities which is dominated by any one city, results with a polycentric structure that is then
35 reflected by a more distributed MPTN.
36
37 All of the abovementioned networks have evolved over many decades and their structure and developments reflects
38 the evolution in planning policies and realities and are the outcome of the decisions of a large number of planners
39 with none of them envisioning the current state of the network. In contrast, MPTN in large cities in China have
40 developed swiftly with unprecedented investments. These rapid developments are the result of a masterplan that is
41 overseen by the transport authority. For illustration, Shanghai, Guangzhou and Nanjing, which are the first, fourth
42 and sixth longest metro networks in the world in terms of track-km, have seen the first line inaugurated in 1993,
43 1997 and 2005, respectively. Metro systems in three more Chinese cities opened as recently as 2004-2005 and their
44 network already exceeds in length than the Paris network. This strikingly different planning trajectory implies less
45 path-dependence and reflects potentially a more cohesive top-down approach. It remains unknown what are the
46 consequences of these differences for network robustness.
47
48 The complex network theory approach enables the robustness analysis of MPTN and the relation between the
49 MPTN topology and its robustness. Derrible and Kennedy (*5, 6*) defined metrics that they believe are indicative of
50 network robustness. They postulated using the number of cyclic paths available in the network since it approximates
51 the possibility to use alternative routes under disruption. However, the relevance of this indicator in explaining
52 network performance in the event of disruptions through experiments was not established. According to their
53 criterion, the MPTN of Tokyo and Seoul are particularly robust followed by among others, Paris and London. Zhang
54 et al. (*7*) studied 17 generic network structures and concluded that redundancy is a key determinant of network
55 capability to withstand disasters. While some insights can be gained from analyzing a taxonomy of networks

Cats, Krishnakumari and Tundulyasaree

1   structures, real-world MPTN are complex and are comprised of a large number of diverse building blocks (e.g. hub-
2   and-spoke, grid, ring, diamond etc.) which cannot be represented as direct extrapolation of their fundamental
3   elements. Rodriguez-Nunez and Garcia-Palomares (*8*) and Jenelius and Cats (*9*) modelled the MPTN service
4   network concluded that lines that offer many transfer opportunities to other lines such as ring and cross-radial lines
5   are especially important in adding cycles and thus contributing to network robustness.

6
7   A systematic analysis of network robustness requires performing a full-scan of the impacts of failures on network
8   performance. In the field of MPTN, the studies of von Ferber et al. (*10*) and Zhang et al. (*11*) pave the way by
9   examining the impact of a sequential failure of network elements – either nodes (stations) or links (track segments).
10   In both studies, three strategies for determining the failure sequence were investigated – random, in descending
11   order of node degree and in descending order of betweenness centrality. The former reflects random failures while
12   the latter two are designed to examine MPTN ability to withstand targeted attacks. von Ferber et al. (*10*) applied the
13   analysis to the MPTNs of London and Paris. The results indicate that Paris is significantly more robust than London
14   in the event of node degree removal strategy while the networks perform very similar under the other two strategies.
15   However, the analysis was performed for the entire public transport network, treating metro and bus lines as if they
16   were indistinguishable. The results of the analysis in (*11*) for the network of Shanghai from 2010 cannot
17   unfortunately be directly compared due to differences in implementation and reporting.

18
19   The objective of this paper is to compare the robustness of selected public transport networks which exhibit different
20   properties to both random and targeted attacks and gain insights on the role of network structure and development
21   pattern. Two exemplary networks are selected – the polycentric Randstad region and the rapidly developing
22   Shanghai network.

23
24   This paper is structured as follows. In the following section we present our method and detail the network
25   representation, failure scenarios and the measures used for quantifying network robustness. In section 3 we present
26   our two case study heavy rail networks of Shanghai and Randstad. Results are reported and analyzed in the section
27   4. The conclusion and network design implications are then discussed in section 5.

28   ## 2 METHOD

29   ### 2.1 Network representation and centrality indicators

30   The MPTN infrastructure is represented as a graph by representing each station as a node and introducing a link
31   between each pair of stations that are directly connected by a track segment. This follows the so-called L-space
32   representation (*12*). The physical MPTN is thus represented as an undirected graph $G(N, E)$ where the set N denotes
33   rail stations and the link set $E \subseteq N \times N$ represents direct connections between stations. Each link may be operated
34   by one or several public transport lines. A is the adjanceny matrix of the graph G, where each entry in the matrix,
35   a_{ij}, equals one if there is a link connection stations $i, j \in N$ and is otherwise zero.

36
37   Even though the graph is unlabeled, network topology result with considerable differences among nodes and links in
38   terms of their network centrality. Two measures of network centrality are used in this study, degree and betweenness
39   and both are defined for both nodes and links. Degree is an indicator of local connectivity while betweenness
40   measures global connectivity. Node degree is the number of direct neighbors, hence directly connected stations, and
41   is defined as

42
43   $$k_i = \sum_{j \in N} a_{ij} \quad \forall i \in N \tag{1}$$

44
45   where k_i is the degree of node i. Following the definition proposed in (*10*), link degree is defined as the sum of
46   degrees of the nodes it connects minus its own contribution to their degrees, hence

47
48   $$k_e = k_{e^-} + k_{e^+} - 2 \quad \forall e \in E \tag{2}$$

49
50   Where e^- and e^+ denote the upstream and downstream nodes of link e, respectively.

51
52   Unlike degree centrality, betweenness centrality measures the role of the node or link in the network as a whole, not
53   only in relation to its direct neighbors. Betweenness centrality is defined here in relative terms, as the share of

1   shortest paths connecting origin-destination pairs in the network that traverse through a certain node or link. The
2   formula for node betweenness centrality is
3

4    $b_i = \sum_{j \in N; j \not\equiv i} \sum_{k \in N, k \not\equiv j} \frac{\sigma_{jk}(i)}{\sigma_{jk}}$ (3)

5

6   where σ_{jk} is the number of shortest paths between nodes $j, k \in N$ and $\sigma_{jk}(i)$ is the number of these paths that go
7   through node $i \in N$. And for link betweenness centrality
8

9    $b_e = \sum_{j \in N} \sum_{k \in N, k \neq j} \frac{\sigma_{jk}(e)}{\sigma_{jk}}$ (4)

10

11   where $\sigma_{jk}(e)$ is the number of shortest paths between nodes $j, k \in N$ that go through link $e \in E$.
12

13   The abovementioned indicators are used in determining the sequence in which disruption scenarios are simulated as
14   described in the following sub-section.

15   **2.2 Network element failure scenarios**

16   Network elements – either nodes or links – can be subject to both random and malicious failures. Both stations and
17   track segments may temporarily be closed due to construction or maintenance works, as well as unplanned closures
18   due to technical or mechanical failures such as vehicle breakdowns and switch or signal failure. Other causes of
19   unplanned disruptions are accidents, suicide attempts and terror-related threats and attacks. The latter typically affect
20   stations.
21

22   The approach taken in this study, inspired by (*10*), is to simulate a sequence of link or node failures. Each scenario
23   involves the sequential removals of either nodes or links. In the pursue of the most devastating effect, the
24   perpetrators of a targeted attack are assumed to target network elements that are most heavily loaded and that will
25   result with the most adverse conditions. Degree centrality and betweenness centrality are therefore used as a possible
26   attack strategy. Note that the removal of a node or link results with a new (reduced) network which affects the
27   centrality indicators for the remaining nodes and links. The metrics are therefore recalculated and updated after each
28   removal step in order to ensure that the most central network element is selected to be removed in the successive
29   step.
30

31   Random attacks involve the random removal of links or nodes, each one picked in random from the remaining set of
32   links or nodes. Crucially, the order in which links or nodes are removed may have great consequences for network
33   performance due to both the specific elements selected as well as path dependency in the removal sequence. Neither
34   the analysis in (*10*) nor (*11*) report how the random removal strategy was precisely performed. In order to attain
35   meaningful results, a number of simulation needs to be performed. At the same time, the number of possible
36   sequences is prohibitive even for fairly small networks. We assure statistically robust results by running a number of
37   random removal sequences and then determining how many such replications are needed to attain a statistically
38   significant result within a high (i.e. 95%) level of confidence.
39

40   In summary, six failure scenarios are analyzed; the combination of three successive removal strategies – random,
41   degree and betweenness, each of which applied to either nodes or links. The following sub-sections describes the
42   network robustness indicators calculated to assess the impacts of the failure scenarios.

43   **2.3 Network performance and robustness indicators**

44   System robustness is defined in the context of this study as system's ability to maintain its functionality under
45   disruptions. In the context of MPTN, the core function of the system is to enable users to travel efficiently between
46   different parts of the network. Various measures can be used to quantify network performance. Network robustness
47   is then assessed by comparing the network performance under disruption to the original undisrupted performance.
48

49   We use two indicators to describe network performance, pertaining to the ability to travel and the detour inflicted by
50   the disrupted situation. The former is assessed by identifying the largest sub-network that remains intact and
51   measuring its size in relation to the original (complete) network size, or mathematically
52

1     $S(r^n) = \frac{|N_r|}{|N|}$ or $S(r^e) = \frac{|E_r|}{|E|}$ (5)

2

3   where $S(\cdot)$ is the relative size of the largest connected sub-network in removal step r^n or r^e of removing nodes or
4   links, respectively. N_r (or E_r) is the set of nodes (or links) that remained in step r. The larger the sub-network is, the
5   more likely it is that one can travel between a given origin-destination pair.

6

7   Even if it is still possible to travel using the MPTN between a given origin and destination, the removal of nodes or
8   links may require performing detours and hence induce additional impedance. The second network performance
9   indicator is designed to assess this effect. The average shortest path length is commonly used as a network efficiency
10   indicator since it captures network transition capability. In order to facilitate the comparison of different networks as
11   given the objective of this study, the normalized average shortest path, $\hat{l}$, is calculated using the following equation:

12

13    $\hat{l} = \frac{2}{|N|(|N|-1)} \sum_{i \in N} \sum_{j \in N, i \neq j} l_{ij}$ (6)

14

15   where $\langle l \rangle$ is the mean shortest path length and l_{ij} is the length of a shortest path between nodes i and j (here, the
16   number of stations that need to be traversed).

17

18   Similar to the relative size metric, network performance under a given disruption state can be then assessed in terms
19   of its deterioration compared to the original state, denoted as $\tilde{l}$, as follows:

20

21    $\tilde{l}(r^n) = \frac{\hat{l}(r^n)}{\max\limits_{i,j} l_{ij}(0)}$ or $\tilde{l}(r^e) = \frac{\hat{l}(r^e)}{\max\limits_{i,j} l_{ij}(0)}$ (7)

22

23   Where $\hat{l}(\cdot)$ is the normalized average shortest path in a certain removal step r^n or r^e with the undisrtuped initial
24   phase denoted by zero. In the course of removal steps, the network may not remain intact anymore. The original
25   case here pertains to the longest rather than the average shortest path. A problem with the $\hat{l}$ metric arises when a pair
26   of nodes is totally disconnected as this will result in infinite value of the mean shortest path. In this study, the
27   infinite value is replaced by the 'diameter' of the network, i.e. the maximal shortest path length, in the initial
28   undisrupted network. Hence in case there is no path available between an OD pair, $l_{ij}(\cdot) = \max\limits_{i,j} l_{ij}(0)$.

29

30   The two network performance indicators can be calculated for the network resulting from each removal step. This
31   allows analyzing how performance losses evolve over the course of the sequence of removals and what was coined
32   by Cats et al. (*13*) the degrading rapidity. This is especially useful for identifying how long does the network remain
33   robust to disruption, whether there is a transition point and the overall trend. It is however also desirable to quantify
34   the overall robustness by considering the accumulated effect of disruption of different scenarios on different MPTN.
35   To this end, following (*13*), the integral over $S(\cdot)$ in relation to the sequence of increasingly disrupted networks
36   (from $|N_r| = \{|N|, |N| - 1, \dots, 0\}$, can be calculated. The larger the value the more robust the network is

37

38    $A = 100 \cdot \int_0^1 S(\cdot)$ (8)

39

40    $A = [0,1]$, where 0 is the most vulnerable and 1 is the most robust.

41

42   The following pseudocode describes the implementation of node removal strategies with the link removal strategies
43   following a similar procedure.

44

Algorithm 1: **Node removal strategies**
Input: **Rail station list *N*, edge list *E*, *strategy***
Output: **Network robustness measures**
$E_r \leftarrow E$, $N_r \leftarrow N$
while $\|N_r\|$ == 0 **do**
if strategy == degree
calculate node degree k of G(N_r, E_r)

Cats, Krishnakumari and Tundulyasaree

```
        [kr, order] ← DescendingSort(k)
        Nr ← Nr[order]
        i ← Nr[1]
    if strategy == betweenness
        calculate node betweenness b of G(Nr, Er)
        [br, order] ← DescendingSort(b)
        Nr ← Nr[order]
        i ← Nr[1]
    if strategy == random
        generate random number 1<m<|Nr|
        i ← Nr[m]
    Er ← Er − {aij, aki}              //Remove edges from the graph with node i as incoming or outgoing nodes
    Nr ← Nr − {i}                     //Remove node i from the graph
    calculate network robustness measures of updated G(Nr, Er)
```

1
2   The following section describes the networks to which this method is applied.

3   **3   APPLICATION**

4   The 2018 heavy rail networks of the Randstad area in the Netherlands and Shanghai, China are the two case study
5   networks. These two large networks are selected because they are exemplary of a polycentric development and a
6   rapid top-down development, respectively.
7
8   The Randstad area is a megapolis, an urban agglomeration in the west of the Netherlands which extends into four
9   provinces: North Holland, Flevoland, South Holland and Utrecht. The total population amounts to 8 million and
10   includes the four largest cities of the Netherlands: Amsterdam, Rotterdam, The Hague and Utrecht. The MPTN
11   consists of the inter-city and regional rail connections characterized by high frequencies and relatively short inter-
12   station distances as well as metro services which are centered around Amsterdam and Rotterdam, including a metro-
13   light rail line connecting the cities of Rotterdam and The Hague.
14
15   Shanghai is a fast growing city and is a global business hub with the metropolitan area encompassing more than 24
16   million inhabitants. Shanghai is characterized by a monocentric development pattern. The MPTN consists of the
17   rapidly developing metro network which is now the most extensive network in terms of track length in the world
18   with 200km of metro tracks currently under construction.
19
20   Table 1 summarizes key topological indicators for the two networks. Note that the Shanghai network analyzed by
21   (*11*) consisted of 240 stations and 264 links. Even though the Shanghai network includes 30% more stations than the
22   Randstad, the average number of intermediate stations that needs to be traversed when travelling between two
23   stations is almost the same and in the worst combination possible (diameter) even worse off in the Randstad. This
24   suggests that the Shanghai network is more efficient in terms of offering short travel alternatives between various
25   OD pairs.
26
27   Table 1: Summary table of network topological indicators

| Network	Number of nodes, $\|N\|$	Number of links, $\|E\|$	Average shortest path, $\bar{l}$	Diameter, $\max_{i,j} l(i,j)$
Randstad 2018	254	283	29.86	46
Shanghai 2018	329	377	31.19	41

28

29   **4   RESULTS**

30   An analysis of the characteristics and spatial distribution of centrality indicators in the two networks is first
31   presented in section 4.1. Thereafter, the results of the disruption simulation and robustness analysis detailed in
32   Section 2 for the 2018 networks of Randstad and Shanghai are presented in section 4.2.

Cats, Krishnakumari and Tundulyasaree

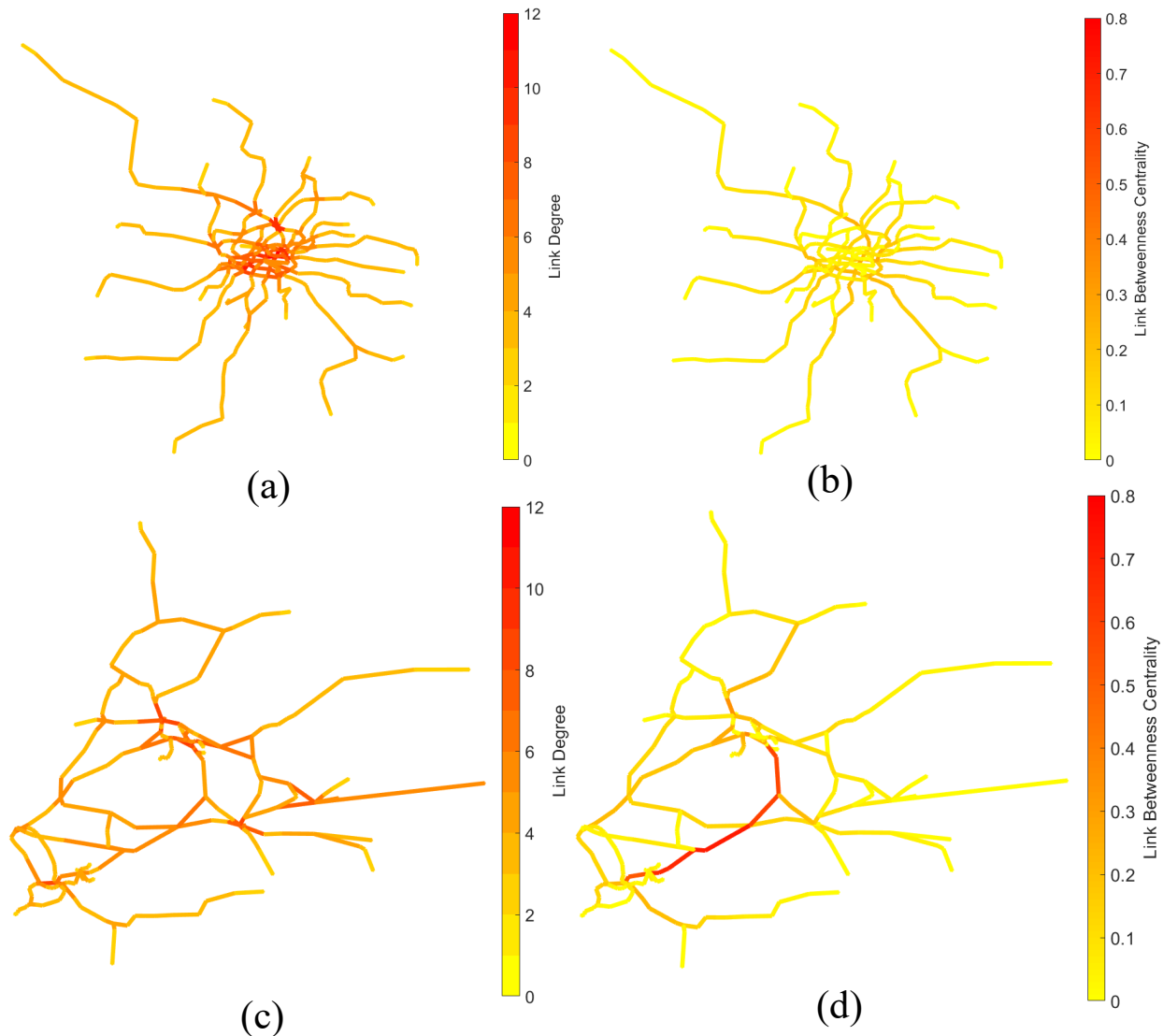1   **4.1 Network centrality indicators**

2   The simulated disruption scenarios are based on the dynamic update of degree and betweenness centrality rankings.
3   Furthermore, network robustness is expected to depend on network properties and the availability of alternative
4   routes, i.e. redundancy. It is therefore relevant to examine how the two networks – Shanghai and Randstad – differ
5   in terms of the the spatial variation in link and node centrality indicators as this can explain their performance under
6   disruptive situations. Figures 1 and 2 display the degree (left) and betweenness (right) centrality indicators for the
7   Shanghai (top) and Randstad (bottom) networks, for node and links, respectively.
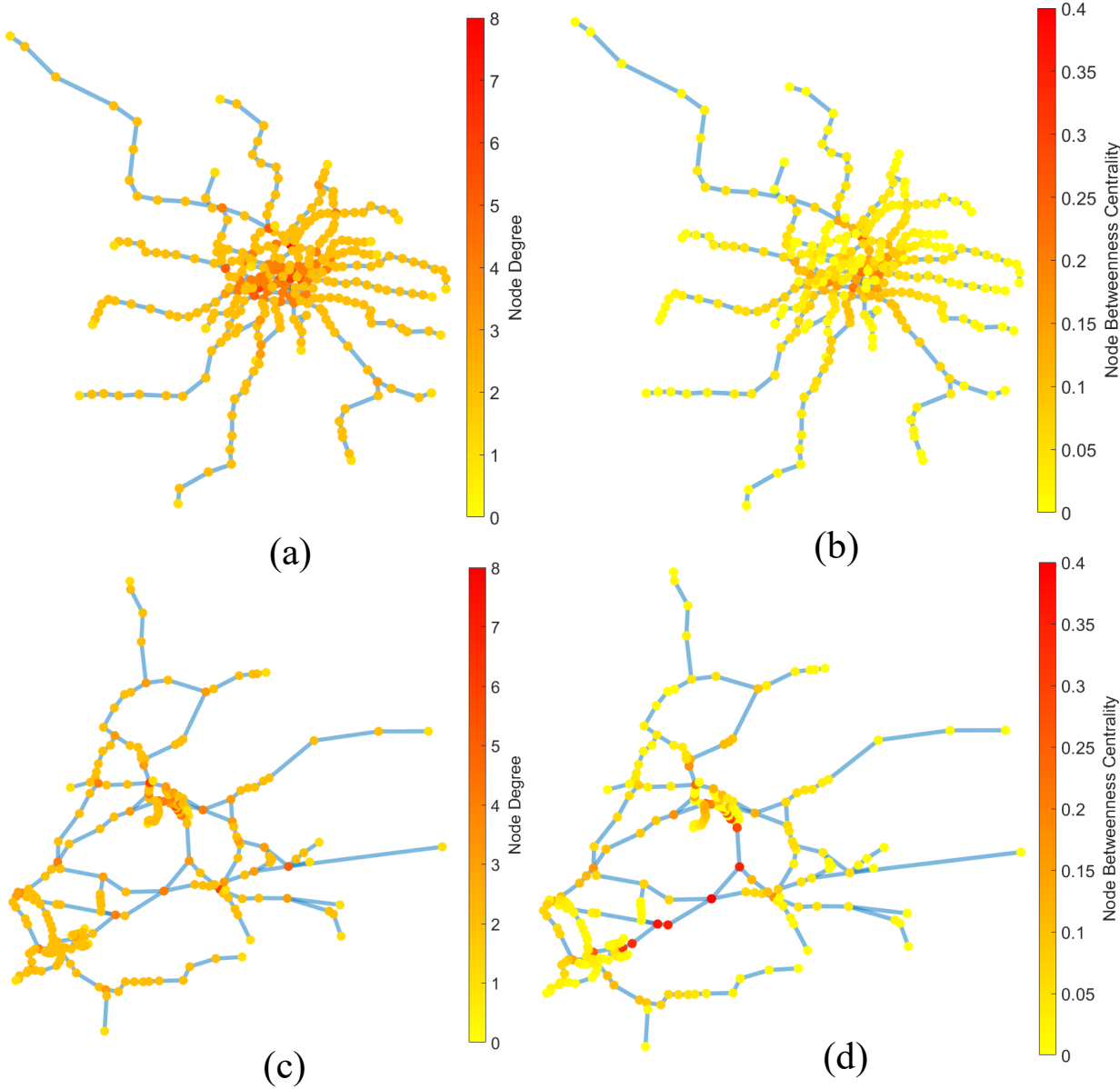
Figure 1: Node centrality indicators of the case study networks – (a) Shanghai, degree; (b) Shanghai, betweenness;
(c) Randstad, degree; (d) Randstad, betweenness.

12   The two networks have a profoundly different structure as can be observed in figures 1 and 2. The Shanghai network
13   has a typical radial structure, yet containing a finely meshed core with a large number of inner circuits. The
14   Randstad network in contrast has a less pronounced center and can be characterized as a grid network with some
15   additional cycles around its main cities. This differences in network structure are clearly reflected in the distribution
16   of node degree (Fig 1, left), where the highest node degrees are all concentrated in the geographical core of the

Cats, Krishnakumari and Tundulyasaree

1 Shanghai network, unlike their scattering in the Randstad case, corresponding to key rail intersections, often located
2 between rather than within dense urban areas. The same pattern can be seen in relation to link degree (Fig 2, left).


3
4 Figure 2: Link centrality indicators of the case study networks – (a) Shanghai, degree; (b) Shanghai, betweenness;
5 (c) Randstad, degree; (d) Randstad, betweenness.
6
7 Nodes with high betweenness centrality (Fig 1, right) are mostly located in the southeastern part of the Shanghai
8 core, reflecting the center of gravity of the network as the shortest paths between most OD pairs traverse this part of
9 the network. Conversely, nodes with high betweenness centrality in the Randstad network are situated along the
10 main northeast-southwest axis which connects Amsterdam and Rotterdam, the two largest cities in this polycentric
11 agglomeration. The same axis is clearly visible also when examining the link betweenness centrality (Fig 2, right). It
12 is evident that link betweenness centrality is much more evenly distributed in the case of Shanghai (note that the
13 metric is displayed in relative terms, all values summing up to 1 for each network).
14
15 The variability of the centrality indicators is further investigated by plotting and comparing the distributions of both
16 centrality indicators for both links and nodes for the two networks. While the node degree distributions are very
17 similar, the Shanghai network exhibits higher link degree values. This indicates that nodes with a high degree value
18 are more likely to be connected to other nodes with a high node degree in the Shanghai case than in the Randstad
19 case, creating 'cliques' of high degree nodes, creating a set of highly connected nodes in the core.
20

Cats, Krishnakumari and Tundulyasaree

1   Both node and link betweenness centrality are more skewed in the Randstad network than in the Shanghai network
2   (Fig 3, right). Few nodes and links in the Randstad network are highly central in terms of them constituting part of
3   the shortest path for a large share of the OD pairs, while the vast majority of nodes and links are on the shortest path
4   of only few OD pairs. Betweenness centrality in the Shanghai network is much more evenly distributed, potentially
5   making the network less dependent on any specific station or track segment.
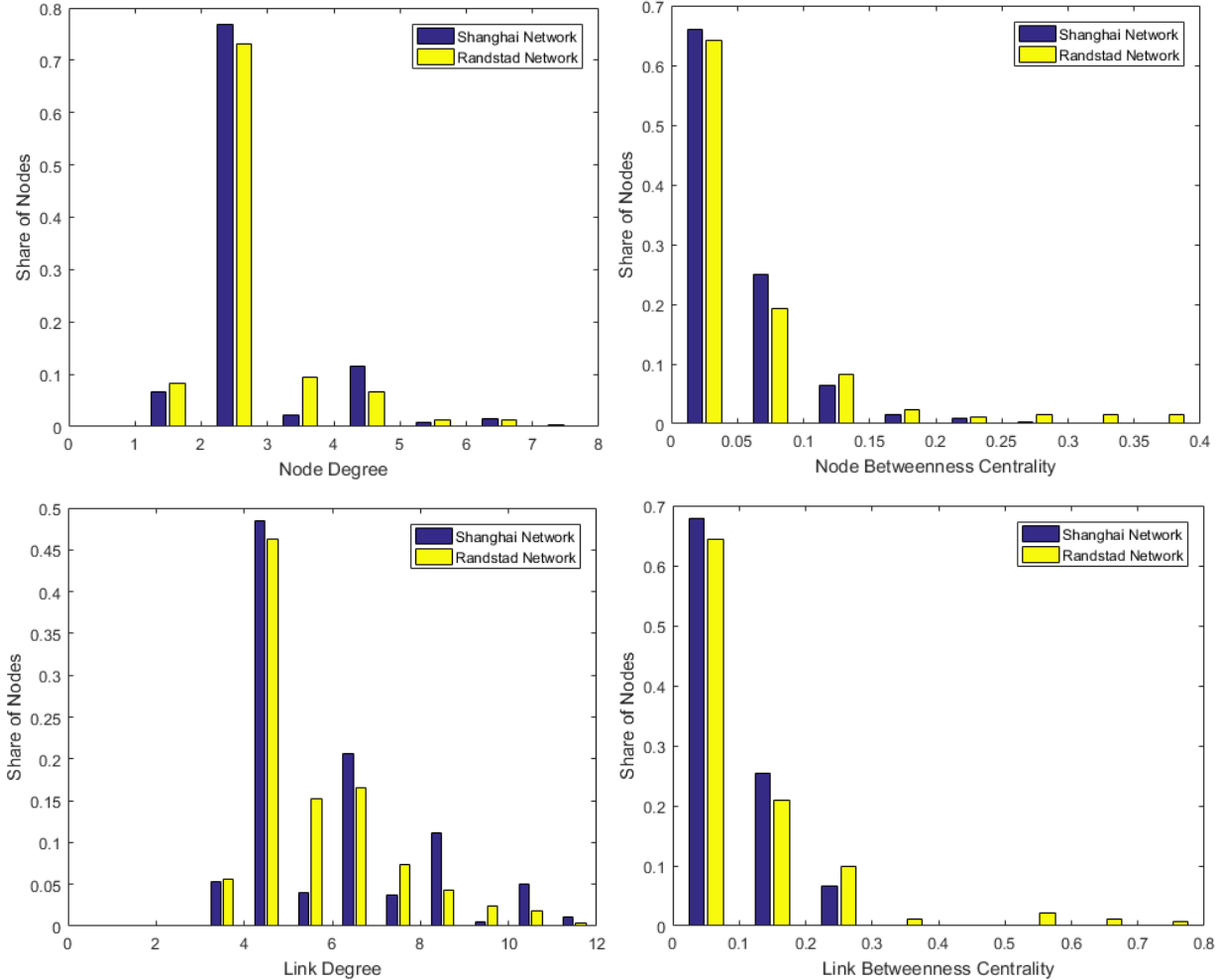

6
7   Figure 3: Comparison of node and link degree and betweenness centrality distributions for Shanghai and Randstad

8   **4.2 Network robustness comparison**

9   The results of the node and link removal strategies are summarized in Figure 4. Each curve corresponds to the
10   sequence of either link or node removal strategy by one of the removal sequences – based on degree (k),
11   betweenness (b) or random attacks (RA) – for either Randstad (R) or Shanghai (S). In all cases, the curve starts from
12   the value which corresponds to the initial undisrupted state of the network. In the case of the relative decrease in
13   network performance in terms of the size of the largest connected component, S, the intial value is one. The share of
14   this component is monotonically decreasing with a larger share of the nodes or links removed. Conversely, when
15   measuring changes in the average shortest path the initial value is zero and the value increases monotonically as a
16   one progresses in the sequential removal of network elements.

17
18   The relative size of the largest component (Fig 4, left) decreases quickly and abruptly for both networks when
19   removing nodes or links by order of importance, especially when removing based on betweenness. In Shanghai
20   network, S starts to drop when about 8% of the links are removed while for Randstad it drops earlier at 6.5%. In
21   contrast, a much slower and more gradual deterioration occurs when removing nodes or links randomly. After
22   removing about 40% of the nodes or 50% of the links, the sub-networks are extremely fragmented with no sub-
23   network consisting of more than 1% (3 nodes or links) of the original graph. Conversely, this does not happen until

Cats, Krishnakumari and Tundulyasaree

1   the removal of 80% of the nodes or links when those are removed at random. Both networks are more robust to link
2   removal than to node removal. This could be expected since the removal of a node involves the removal of all links
3   connected to this node, hence resulting with a faster breakdown. Interestingly, random removal can be more harmful
4   than targeted removal when few links are removed from the network (bottom, left). The pace of network
5   deterioration when removing links based on betweenness is twice as fast when removal is based on degree. Hence,
6   global connectivity is more important than local connectivity in determining the most critical links.
7
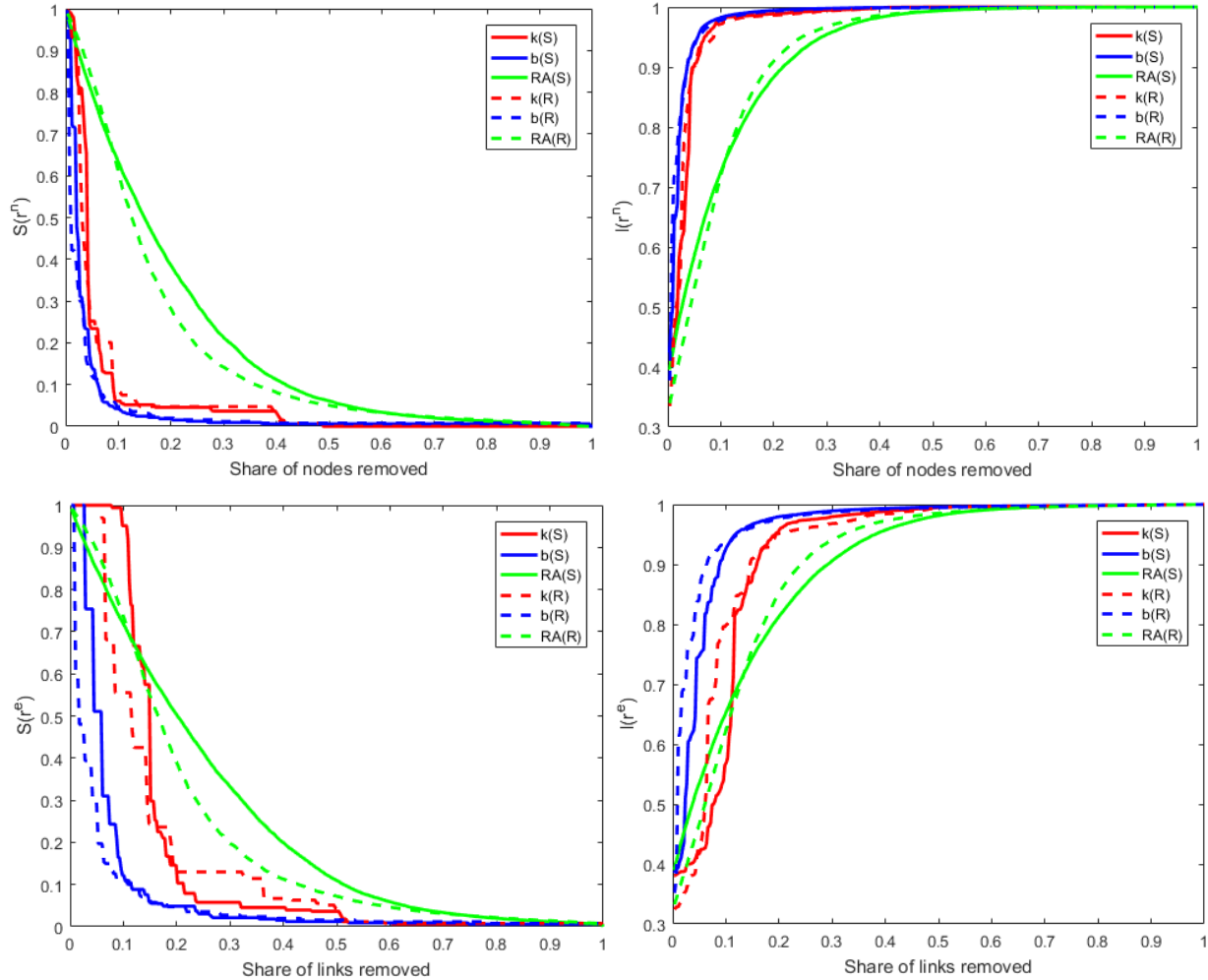
8
9   Figure 4: Shanghai (S) and Randstad (R) performance under alternative node (top) and link (bottom) removal
10 strategies (k – node centrality; b – betweenness centrality; RA –random) in terms of Relative size, S, (left) and
11 Shortest path (right).
12
13 In the case of node removal (top left), the patterns for the two networks is very similar. The performance of the
14 Shanghai network exhibits greater robustness than the Randstad network in the link removal scenarios, with larger S
15 values under most shares of links removed. For example, after the removal of 10% of the links with the highest
16 degree centrality, the Shanghai network is still connected, allowing travelling between any OD pair, while the
17 largest connected sub-network in the Randstad lost more than 40% of the original network elements. However, after
18 the removal of 15%, both networks are equally (dis)connected with $S \approx 15\%$.
19
20 Similar trends are observed for the normalized average shortest path length metric, $\tilde{l}$ (Fig 4, right). Again, both
21 networks are much more vulnerable to targeted attacks than to random attacks, with attacks targeting the network
22 elements with the highest betweenness centrality values being the most devastating. No significant differences
23 between the two networks are observed when removing nodes, while the Shanghai network prevails again as
24 superior to the Randstad network in terms of its robustness to link removal scenarios. In targeted removal scenarios,

Cats, Krishnakumari and Tundulyasaree

1 after only 5% of the nodes are removed, $\tilde{l}$ already approaches 0.9. Hence, even though both networks have not
2 broken down yet ($S = 1$), severe detours are needed, resulting in significantly longer paths. After removing 10% of
3 the nodes or 20% of the links based on either degree or betweenness centrality, the network break down as indicated
4 by the S metric is so severe, that the disconnected OD pairs dominate the shortest path calculations resulting with $\tilde{l}$
5 values approaching 1. This happens only after the removal of 40% of the nodes or 50% of the links when those are
6 removed at random.
7
8 The analysis of the curves plotted in Figure 4 shed light on the degrading rapidity of the two networks. In addition,
9 an aggregate metric of network robustness with respect to the largest connected component (Eq. 8), A, is calculated
10 for all scenarios. A is the integral of the curves plotted in Figure 4, left. Hence, an extremely vulnerable network
11 which breaks apart instantly after the first node or link has been removed will have $A = 0$. In contrast, the
12 hypothetical case of an extremely robust network that remains intact until the very last brick is removed will yield
13 $A = 1$.
14
15 In addition to the Randstad and Shanghai networks, we also calculate it for Paris and London based on the findings
16 reported in (*10*). We remind the reader that the latter considered the entire public transport network, resulting in
17 much larger graphs. The results show that the Randstad is consistently less robust than Shanghai. Both networks are
18 however much more vulnerable than London and especially Paris, albeit the latter two include also public transport
19 modes other than metro. The most pronounced differences are observed for link betweenness centrality removal.
20 While the Randstad and Shanghai are most vulnerable to betweenness removal scenarios, London and Paris are
21 more or equally vulnerable to degree-based removals. This is further discussed in the next section.
22
23 Table 1 : Aggregate metric of network robustness per scenario and network

Network	Link removal			Nodes removal		
	link degree (k)	*link betweenness centrality (b)*	*random attacks (RA)*	*node degree (k)*	*node betweenness centrality (b)*	*random attacks (RA)*
Shanghai	16.22	7.24	24.33	5.64	3.42	19.11
Randstad	14.95	5.06	20.76	5.54	2.97	16.82
London	20.95	27.20	27.45	5.45	8.71	29.31
Paris	47.12	55.93	56.04	10.77	10.67	37.93

24

5 DISCUSSION AND CONCLUSION

26 In this paper, a network vulnerability analysis is performed for both Shanghai and Randstad heavy rail network. The
27 two network possesses different structures as the former one is a recent and rapidly developing network serving a
28 monocentric metropolitan area while the latter is an exemplary of a polycentric urban agglomeration area that has
29 developed over many decades by a large number of planning authorities. Both link-based and node-based sequential
30 removal are applied to test the network and the performance of the degraded network is measured in terms of the
31 relative size of the largest component (S), the relative mean shortest path ($\tilde{l}$) and an aggregate robustness measure
32 (A). The first two measures are calculated for each phase in the iterative disruption strategy while the latter one is
33 calculated after the whole sequences of attacks. Random failures as well as targeted attacks based on degree and
34 betweenness centrality are examined.
35
36 The results are summarized in stylized schematic graphs in Figure 5, including the results for the Paris and London
37 public transport networks based on the findings reported in (*10*). A clear pattern can be observed – The Paris
38 network outperforms all other networks under all scenarios, exhibiting the most robust performance. In general, the
39 remaining networks can be ranked in the following descending order in terms of their robustness: London, Shanghai
40 and Randstad. While London is generally less vulnerable than Shanghai and Randstad in the event of targeted
41 attacks, it exercises a more abrupt pattern in case of degree-based targeted attacks or random attacks, resulting with
42 a more fragmented network than both Shanghai and Randstad beyond a certain share of the links has been removed.
43 This results with comparable overall robustness for the three networks in the case of random link removal (Table 2).
44

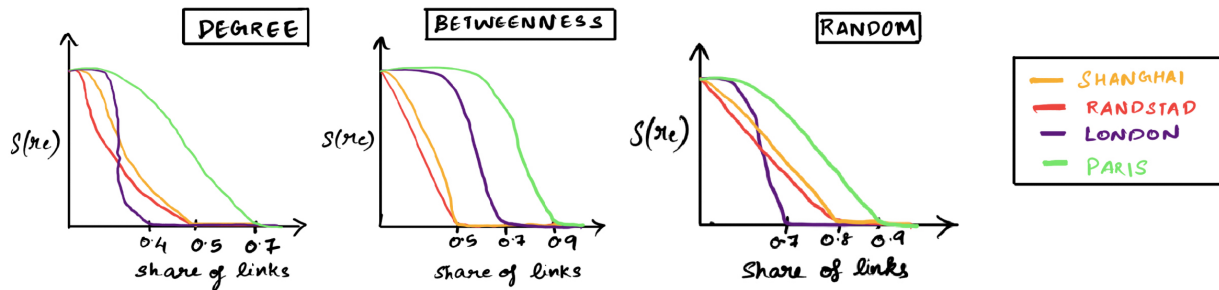Cats, Krishnakumari and Tundulyasaree

Figure 5: Schematic diagrams comparing the relative size deterioration for London, Paris, Shanghai and Randstad per link removal strategy.

The results of this study shed new light on some of the common conventions held in the network robustness discourse. The polycentric network of the Randstad was found the least robust, inferior to the monocentric networks of London, Paris and Shanghai. Unlike what might be expected, the network polycentricism does not yield in the case of the Randstad with a more distributed hub and spoke structure that is known to contribute to network robustness but rather results in few key stations and tracks that lie outside of the urban cores in a fork-like structure. The Shanghai network size provides opportunities for redundancy, the core of the network is finely meshed and the rapid and recent development – all within three decades – are all expected to be beneficial for network robustness. A closer inspection reveals however that the Shanghai network includes relatively more and longer branches which are vulnerable to targeted disruptions, especially in comparison to Paris which has many circuits due to short distances between stations and many peripheral hub connections. It is well-known that the connectivity of radial networks is highly vulnerable to disruption, isolating one branch from the remaining network. These results should be revisited by evaluating the redundancy offered by alternative public transport modes in the Randstad and Shanghai cases.

This study has several implications for network planning. First, the robustness of polycentric regions highly depends on the availability of routing alternatives between the most populated areas. Due to lower densities between the urban cores, networks serving polycentric areas may rely on a limited number of connecting corridors. This however has severe consequences for network robustness as their malfunction will result with an immediate breakdown and loss of a significant share of network functionality. Second, a finely meshed network core cannot substitute the availability of connections between outer hubs, in the absence of which long branches can become disconnected without any viable alternative.

Our findings suggest that the relation between network structure and its robustness is non-trivial. While in general more decentralized networks are more robust to targeted attacks, a polycentric urban agglomeration does not necessarily yield a more distributed network as measured in terms of centrality indicators. Furthermore, there is also no indication that a top-down planning style necessarily results with a more robust design. It may even be the case that a bottom-up and lengthy development may result with more fractal-like geometry due to underlying network evolutionary principles. Further research is needed in order to examine the generality of these results, related processes and their consequences for network design as well as the incorporation of demand and supply information.

AUTHORS' CONTRIBUTION:
The authors confirm contribution to the paper as follows: study conception and design: O. Cats; data collection: P. Krishnakumari; analysis and interpretation of results: K. Tundulyasaree, P. Krishnakumari, O. Cats; draft manuscript preparation: O. Cats. All authors reviewed the results and approved the final version of the manuscript.

REFERENCES
1. Homeland Security, U.S. Department of, 2010. Transportation Systems Sector-Specific Plan, an Annex to the National Infrastructure Protection Plan.
2. Lin, J., & Ban, Y., 2013. Complex Network Topology of Transportation Systems. Transport Reviews, 33(6), 658-685. doi:10.1080/01441647.2013.848955
3. Roth, C., Kang, S. M., Batty, M. and Barthelemy, M., 2012. A long-time limit for would subway networks. Journal of the Royal Society Interface 9, 2540-2550.

4. Cats, O., 2017. Topological evolution of a metropolitan rail transport network: The case of Stockholm. Journal of Transport Geography, 62, 172-183.

5. Derrible, S., Kennedy, C., 2010. Characterizing metro networks: state, form and structure. Transportation 37, 275-297.

6. Derrible, S., Kennedy, C. 2010. The complexity and robustness of metro networks. Physica A, 389, 17, 3678-3691.

7. Zhang X., Miller-Hooks E. and Denny K., 2015. Assessing the role of network topology in transportation network resilience. Journal of Transport Geography 46, 35-45.

8. Rodriguez-Nunez, E., Garcia-Palomares, J.C., 2014. Measuring the vulnerability of public transport networks. Journal of Transport Geography 35, 50-63.

9. Jenelius, E., Cats, O., 2015 The value of new public transport links for network robustness and redundancy. Transportmetrica A: Transport Science 11 (9), 819-835.

10. von Ferber, C., Berche, B., Holovatch, T., & Holovatch, Y., 2012. A tale of two cities. *Journal of Transportation Security, 5*(3), 199-216. doi:10.1007/s12198-012-0092-9

11. Zhang J., Xu X., Hing L., 2011. Networked analysis of the Shanghai subway network, in China. Physica A, 390, 4562-4570.

12. von Ferber, C., Holovatch, T., Holovatch, Y., & Palchykov, V., 2009. Public transport networks: empirical analysis and modeling. *The European Physical Journal B, 68*(2), 261-275.

13. Cats O., Koppenol G-J. and Warnier M., 2017. Robustness Assessment of Link Capacity Reduction for Complex Networks: Application for Public Transport Systems. Reliability Engineering & System Safety, 167, 544-553.