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A computational design exploration platform supporting the formulation of design concepts

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ABSTRACT

The comparison of various competing design concepts during conceptual architectural design is commonly needed for achieving a good final concept. For this, computational design exploration is a key approach. Unfortunately, most of existing research tends to skip this crucial process, and purely focuses on the late-stage design optimization based on a single concept that, they assume, has been good enough or accepted already. This paper focuses on information or knowledge extracted from a multi-objective design exploration for the formulation of a good geometrical building design concept. To better support the exploration process, a new integration plug-in is developed to integrate parametric modelling software and process integration and optimization software. Through a case study that investigates the daylight and energy performances of a large indoor space, this paper 1) tackles the importance of design exploration on the formulation of a good design concept; 2) presents and shows the usability of the new integration plug-in for supporting the exploration process.

Author Keywords

Multi-objective design exploration; Design concepts; Trade-off; Comparison; Top daylighting; Energy; Daylight

ACM Classification Keywords

J.6 COMPUTER-AIDED ENGINEERING

1 INTRODUCTION

There are multiple definitions of conceptual design and design concepts. According to Pahl [1], conceptual design is the phase in which the design requirements defined in the first phase are synthesized into a number of concept variants thus a final concept. O'Sullivan [2] includes into conceptual design the phase in which the designer takes specifications for an item to be designed and constructs a statement (subject to further modification) upon which the generation of many solutions is initiated. In this research, we define a *designňconcept* as a collection of ideas or principles that aim to achieve all the requirements of a design situation. Thus, to describe a design concept (which includes many potential solutions), there are at least two aspects that need to be specified: the design requirements and how to achieve them by applying the design ideas or principles. Moreover, similar to the definition in [1], we consider *conceptualň design* includes the phases of generating multiple potential design concepts, of evaluating and comparing the concepts for the most suitable one.

In computational design, parametric modelling [3, 4] is usually avoided during the conceptual design, especially in the very early phases. This is because the hierarchical structure of parametric models is considered to hinder the freedom of explorative thinking and to prematurely freeze ideation [5-7]. It is more widely applied, after a final design concept has been chosen (i.e. after the conceptual design), in combination with performance simulation for predicting various building performances. In this context, the design requirements are indicated by the performance criteria being considered; and the design ideas are implemented via the definition of parameters of the parametric model corresponding to the final concept. Thus, in some sense, the description of a design concept includes the formulation of an optimization problem, except for added values of design (such as cultural values, beauty, emotions, etc.). As such, a design concept is partially described or formulated by: a set of objective and constraint variables (i.e. output variables that are selected among performance criteria, to form an objective space); and a set of design variables (i.e. input variables that are selected among parameters of a parametric model, to form a design space).

In this paper, we claim that parametric modelling, together with performance simulation and computational design

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exploration, are also useful during the conceptual design, but in the relatively late phases. That is, when multiple competing design concepts are generated, they are useful for evaluating and comparing the different concepts, identifying the promising ones, and then re-formulating them or even ideating entirely new concepts. For this, the computational design exploration is a key approach.

Computational design exploration differs from design optimization. In this study, *design exploration* refers to the process of extracting information or knowledge and applying it to formulate or re-formulate a design concept. This process includes two levels of exploratory activities: 1) the exploration of information or knowledge hidden behind obtained data sets; and one step further, 2) the exploration of how to apply the extracted information or knowledge to support the human decision-making on ideation. This is an iterative process during which the definitions of objective variables, constraint objectives and design variables may change towards a better concept formulation. This change occurs in two ways: modification of existing design concept or creation of new design concept, which is similar to Gero's definition of design exploration [8]. In contrast, *design optimization* refers to the process that is only keen on searching for optimal design solution(s) by using various optimization algorithms, when the design concept has been well-defined. Thus, optimization typically occurs in a later stage, during which the definitions of objective variables, constraint objectives and design variables remain fixed.

The design exploration precedes the design optimization and it is more important. If one defined an improper or a bad design concept, he/she might probably get poor results no matter how advanced the optimization algorithm was. Unfortunately, most of existing research tends to skip the process of formulating a good design concept, and rather focuses on the late-stage optimization based on a single given design concept. As response, this paper focuses on the formulation of a good geometrical building design concept based on the information or knowledge extracted from a multi-objective design exploration.

Accordingly, a process integration platform is required that is suitable for multi-objective and multi-disciplinary design exploration and supports the decision-making on ideation. However, most of existing platforms integrating parametric modeling and simulation systems with a design exploration and optimization environment lack important features, like scalability, extensibility [9], ease in using advanced sampling techniques and post-processing tools etc. This largely limits their applications in conceptual architectural design. In our research, to better support the exploration process, a new integration plug-in is developed to integrate parametric modeling software (i.e. McNeel's Grasshopper – GH [10]) and process integration and optimization software (i.e. ESTECO's modeFRONTIER - MF [11]).

In a case study that investigates the daylight and energy performances of a large indoor space, three typical types of top daylighting design are explored; and eventually, an improved design concept comes up based on the design exploration. Through this case, the paper 1) highlights the importance of design exploration on the formulation of a good design concept; 2) shows the usability of the new integration plug-in for supporting the computational design exploration process.

2 METHODOLOGY

2.1 Computational framework

The overall computational framework is shown in Figure 1. It consists of two iterative loops: a design exploration loop (indicated by a large cycle: the outer loop, involving Stage 1, 2 and 3) and a design optimization loop (indicated by a small cycle: the inner loop involving Stage 1 and 4). Different from the traditional focus on design optimization, this paper focuses on design exploration.

Stage 1 - formulation of initial design concepts: it is involved in the process for one time (marked in dash lines in Figure 1). In this stage, multiple initial design concepts are created and ready for the subsequent design exploration. The tasks below are required:

- Formulation of design variables, parametric model(s)
- Formulation of objective variables, simulation model(s)

Stage 2 - computational design exploration: this is the key of the design exploration loop. It generates and collects data; explores the information or knowledge behind the obtained data (i.e. the first level of exploration); and explores how to apply the extracted information or knowledge to support the human decision-making on ideation (i.e. the second level of exploration). The tasks below are required:

- Workflow establishment
- Simulation run & Data storage
- Data analysis & Knowledge extraction
- Promising concept identification & Preference integration

Stage 3 - re-formulation of new design concepts: it is the consequence of the previous stage. In this stage, new design concepts are created and ready for the subsequent design

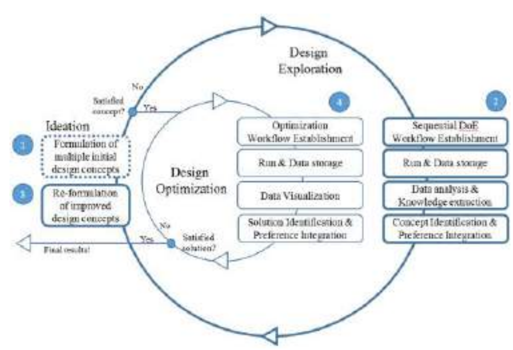

Figure 1. Overall computational framework

optimization (note that the optimization run is not included in this paper). The tasks below are required:

- Re-formulation of design variables, parametric model(s)
- Re-formulation of objective variables, simulation model(s)

2.2 Software Platform

To well support the computational framework, especially the computational design exploration, a process integration platform is required. It should be able to (1) connect with software familiar to architects and engineers; (2) cope with challenges related to computational power; and (3) include data analysis tools to facilitate analytic decision making etc.

Considering all these and other requirements, Grasshopper and modeFRONTIER have been chosen and integrated into one desired platform in this research (more information in Section 3). Grasshopper (integrated with Rhinoceros) is one of the most popular parametric modelling environments among architectural design professionals. It includes plug-ins for integrating various building performance simulations. modeFRONTIER is a process integration and automation platform for multi-objective and multi-disciplinary design exploration and optimization. It allows the integration with a variety of third party CAD and CAE tools; supports parallel computing; and offers a number of easy-to-use post-processing tools for data analysis and visualization.

2.3 Computational Design Exploration

Considering the relative importance of computational design exploration, as mentioned in Section 2.1, this section focuses on the computational process of Stages 2, for which the following four steps are involved.

Workflow Establishment

The parametric models and simulation models are set in GH, based on the formulation of multiple initial design concepts. The preliminary establishment of the workflow is facilitated by GH-MF integration. Through an *introspection* process, the input and output variables being investigated in GH are automatically propagated to MF. Thus, a modeFRONTIER workflow is preliminarily established, as shown in Figure 2. Moreover, some settings still need to be configured before the workflow is fully established. First, the domain and step of each input variable should be set properly; second, Design of Experiments (DoE) sampling strategies should be applied for guiding the choice of computer experiments or test designs; third, the sequential evaluation of previously defined test designs should be set. It is worth noting that DoE is very useful for design exploration. It helps to extract the most relevant qualitative information from a limited number of test designs. It is especially meaningful for the case that has relatively long simulation time. Uniform Latin Hypercube (ULH) [12] is one of the commonly used DoE sampling strategies in MF.

Simulation Run & Data Storage

Via various simulation engines, performances are predicted and the data are stored for later use. The simulation run and data storage is automated by GH-MF integration. During a *run* process, for each test design, MF automatically sends the input values to GH, and then receives the output values from GH when the simulation results are generated. Meanwhile, numerical simulation data are stored in MF database; images and 3D models are saved in the MF working directory. All the results can be browsed through MF user interface (more information in Section 3).

Data Analysis & Knowledge Extraction

Based on the data obtained, useful information (about data) is extracted by using statistical techniques. The information needed to know during design exploration includes *output-output* relationships and *input-output* relationships. The former refers to the inter-correlations between pairs of output variables; the latter refers to the impact of input variables on output variables. Considering the relatively large number of variables being considered, Multivariate Analysis (MVA) techniques are used to identify patterns and relationships among these variables. Self-Organizing Map (SOM) and Hierarchical Clustering (HC) [12] are among the handy MVA tools in MF.

The SOM is an unsupervised neural network for ordering of high-dimensional data in such a way that similar data are grouped spatially close to one another [13]. It represents multi-dimensional data in a two-dimensional space, which is very useful and directly interpretable. It can be used to identify the *output-output* relationships as well as other unexpected multiple variable correlations. The Cluster analysis tries to identify homogeneous subgroups of samples in a data set such that they both minimize within-group variation and maximize between-group variation [14]. The HC classifies large amounts of data into manageable and meaningful subgroups, which provides more abstract views to the inherent structure of the data. It facilitates to identify the *input-output* relationships by using a more refined data set.

Figure 2. modeFRONTIER workflow showing input and output variables

Based on the information obtained, useful knowledge (about building disciplines) is extracted via designers' interpretation. The knowledge to be extracted during design exploration may include reasons: why different criteria are correlated in some patterns and why different groups (or concepts) of designs perform differently? To obtain this knowledge, a good understanding of all input and output variable definitions is crucial, because it helps designers to interpret the information in building disciplinary contexts.

Promising Concept Identification & Preference Integration

Based on the knowledge obtained, the variables describing the most promising concept need to be identified, for the re-formulation of a new design concept. First, based on the correlation patterns between different criteria, the most contradictive criteria and/or the criteria with significantly different correlation patterns are selected as objective variables (not more than three); the remaining criteria can be selected as constraint variables. This ensures obtaining a meaningful set of Pareto trade-off solutions (from DoE test designs). Second, a special design variable that controls the switch among different groups (or concepts) of designs is crucial (see Section 4.3). Based on its effect on the selected objectives, the performance of each initial design concept is observed; and hence the design variables describing the most promising concept are identified.

It is worth noting that design preference integration is also important during the identification of the promising concept. Human designers or clients may have subjective preference on "soft" criteria (e.g. aesthetics) which are hard to evaluate numerically. The incorporation of design preference allows human to balance between visually preferred and high-performing concepts. For instance, one could select visually preferred but mid- or low-performing concepts for further research. In this sense, it is helpful to have a user-friendly interface that allows monitoring the variation of geometry while exploring data and simulation results, such as the interface provided by MF, as shown in Figure 3 (top right).

2.4 Re-formulation of New Concepts

There are two ways to re-formulate new concepts based on the most promising one. According to Gero [15], one option is to change the ranges of values for design variables (i.e. innovative design); while the other option is to introduce new design variables (i.e. creative design). In this paper, we are interested in the latter. Thus, new design variables are inspired via the analysis of the promising concept.

3 GH-MF INTEGRATION

The integration of Grasshopper and modeFRONTIER is useful for supporting computational design exploration. This potential has been shown in existing research [16-18], in which the GH-MF integration is provided by a previous version of customized nodes (both in GH and MF). To overcome some limitations in the last version (like manual operations, unstable process initiation and automation etc.), an improved version of the precedent is developed. The development has been implemented by using the myNODE tool [12] (a tool enabling the creation of custom nodes for the integration of external software in MF), and the Python tool (scriptable components in GH).

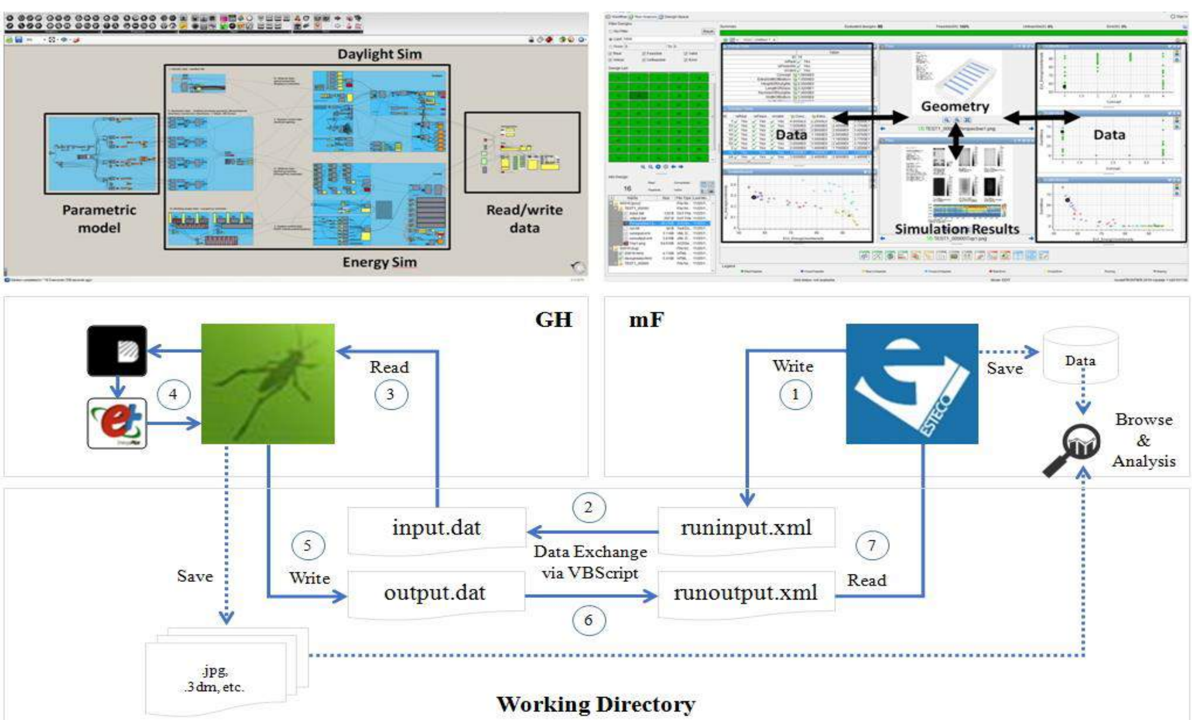

Figure 3. Grasshopper UI (top left); modeFRONTIER UI (top right); diagram of GH-MF integration (bottom)

Compare with the precedent, the new GH-MF integration streamlines the integration process through automatically recognizing input and output variables from GH in the *introspection* phase; facilitates the initiation of the *run* phase by one-click action; and improve the stability of the *run* phase by automatically calling and closing GH and Rhinoceros applications for each simulation run (instead of keeping them always alive which may increase the risk of crashing). Moreover, the new GH-MF integration is also supposed to work on Grid, for which, some bugs still need to be fixed. The communication between GH and MF is achieved via the automatic data exchange between .dat and .xml files, as shown in Figure 3 (bottom).

4 CASE STUDY

The case study investigates how to design the geometry of top-daylighting elements to improve daylight and energy performances of a large indoor space (i.e. 40m*70m*15m). It is structured as following. Three typical types of top-daylighting design are introduced as initial design concepts (Section 4.1). The formulation of the initial concepts is described, including the selection of objective variables (Section 4.2) and design variables (Section 4.3). Then, the computational design exploration process is followed; results about the obtained data, and results of the data analysis, knowledge extraction and design inspiration are showed (Section 4.4). Last, the performances of a new concept obtained are compared with that of the initial concepts (Section 4.4).

4.1 Initial Design Concepts

Top daylighting is a common and effective way of bringing light deep into a building, thus, it is often used in large single level space, such as indoor sports halls. Three typical types of top-daylighting design shown in Figure 4 (in the dash box) are initial design concepts. They are: skylights (Concept 1), roof monitors (Concept 2) and saw-tooth roofs (Concept 3). They perform differently in term of daylight and energy performances, due to different strategies of introducing light into the space. In addition, Concept 4 is the new concept proposed after the design exploration.

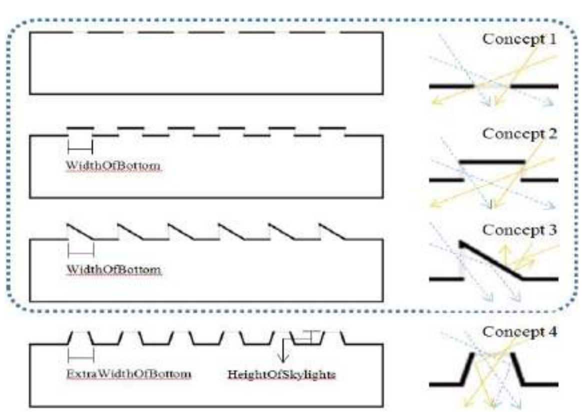

Figure 4. Initial design concepts (1-3); new concept (4)

Although there might be some general understanding of the initial concepts, it is not sufficient to support a good decision on which type we should choose. There are a lot of geometrical variations for each initial concept. The different geometrical variations perform differently though they belong to the same concept. Thus, there is a risk that one may choose a relatively high performing solution, but from a less promising concept. In that case, they may overlook a lot of better solutions or equally good trade-off solutions in a more promising concept. In this sense, it is crucial to have a clear view of the overall performance trend or pattern of each initial concept.

4.2 Objective Variables

Three categories of objective variables are selected for the formulation of the initial concepts, namely, energy, daylight and geometry related objectives. Energy related objectives include: Energy Use Intensity (EUI, energy used per square meter of floor area), Percentages of Cooling, Heating, Lighting, Equipment. Daylight related objectives include: Useful Daylight Illuminance (i.e. UDI(<100), UDI(100-2000) and UDI(>2000)); DaylitArea; OverlitArea; Average Uniformity. Here, UDI represents the percentage of floor area that meets the specified illuminance range at least 50% of the occupied time. DaylitArea represents the percentage of floor area that is above 300 lux for at least 50% of the occupied time (i.e. sDA). OverlitArea represents the percentage of floor area that is above 3000 lux for at least 5% of the occupied time, which indicates potential glare or overheating. Average Uniformity represents the annual average of the minimum-to-average uniformity ratio of illuminance. Geometry related objective is Area of Glass. It calculates the total area of the glass being used, which is often an interesting criterion associate with investment cost.

Moreover, Daysim [19] and EnergyPlus [20], as frequently used daylight and energy simulation engines, are chosen in this research. And the Grasshopper plug-ins Ladybug and Honeybee [21] are used to integrate parametric models with these simulation engines.

4.3 Design Variables

A special design variable called "Concept" is used to facilitate the simultaneous investigation of the multiple concepts. It controls the switch among different groups (or concepts) of designs; and determines the selection of other design variables that formulate the initial concepts. Normally, changing the value of a design variable will not affect the selection of other design variables. That is, the dimensions of the design space will be fixed, as the design variable changes its value. But, in this case, the variable "Concept" is a nominal variable, which is used for labelling the specific group (or concept) of designs without any quantitative meanings. The "values" of this variable include: Concept1, Concept2 and Concept3. When a specific "value" is chosen, different set of design variables will be selected to create the geometry of the corresponding

Type	Design variable	Range	Step
Share by Concept 1-4	NumberOfSkylights	2-10	1
	LengthOfGlass	30-38m	0.1 m
	WidthOfGlass	1-3 m	0.1 m
Share by Concept 2, 3	WidthOfBottom	1-3 m	0.1 m
Only for Concept 4	HeightOfSkylights	1-3 m	0.1 m
	ExtraWidthOfBottom	1-3 m	0.1 m

Table 1. Design variables and their ranges and steps.

initial concept. Thus, the specific "value" determines the dimensions of the design space being investigated. That is, by using the special variable "Concept", the dimensions of the design space are not fixed in this case, which is good for the simultaneous investigation of the multiple concepts.

Moreover, the design variables determined by the variable "Concept" are shown in Table I, together with their ranges and steps. Some of them are shared by multiple concepts; while some are only for a specific concept.

4.4 Results

Following the steps of computational design exploration (Section 2.3) and re-formulation of new concepts (Section 2.4), all the results obtained are reported in this section.

First, results about the obtained data are shown in Figure 5. They include: all the numerical data of design variables and objective variables; images showing daylight and energy simulation outcomes; and images showing geometries. This data are generated by applying ULH sampling strategy and running sequential simulations of the samplings. Second, results of the data analysis, knowledge extraction and promising concept identification are described in two parts: identification of promising objective variables and design variables. Self-Organizing Map and Hierarchical Clustering are used to analyze the data; then useful information and knowledge are extracted, based on which, a promising concept is identified. Last, results of the re-formulation of new concepts are shown. A new concept is formulated in a more informed manner, and its performances are compared with the performances of the initial concepts.

Identification of Promising Objective Variables

Self-Organizing Map is used to identify the *output-output* relationships. It is created by including all initial objective variables, based on the data sets related to initial concepts. As shown in Figure 6 (left), SOM maps of different variables are distributed on a hexagonal grid; and those with similar patterns are placed in adjacent positions. The color represents the values of variables; deep red means highest values and deep blue means lowest values. The level of similarity between patterns indicates the level of their correlation. Moreover, to facilitate observation, the general direction of achieving a desired performance goal of each objective variable is shown by a write arrow. For instance, EUI aims for a minimization goal, thus the direction of achieving this goal is from left bottom (high value) to top right (low value). Similar principles apply to others as well. By comparing the directions and patterns between pairs of initial objective variables, the following information and knowledge are obtained.

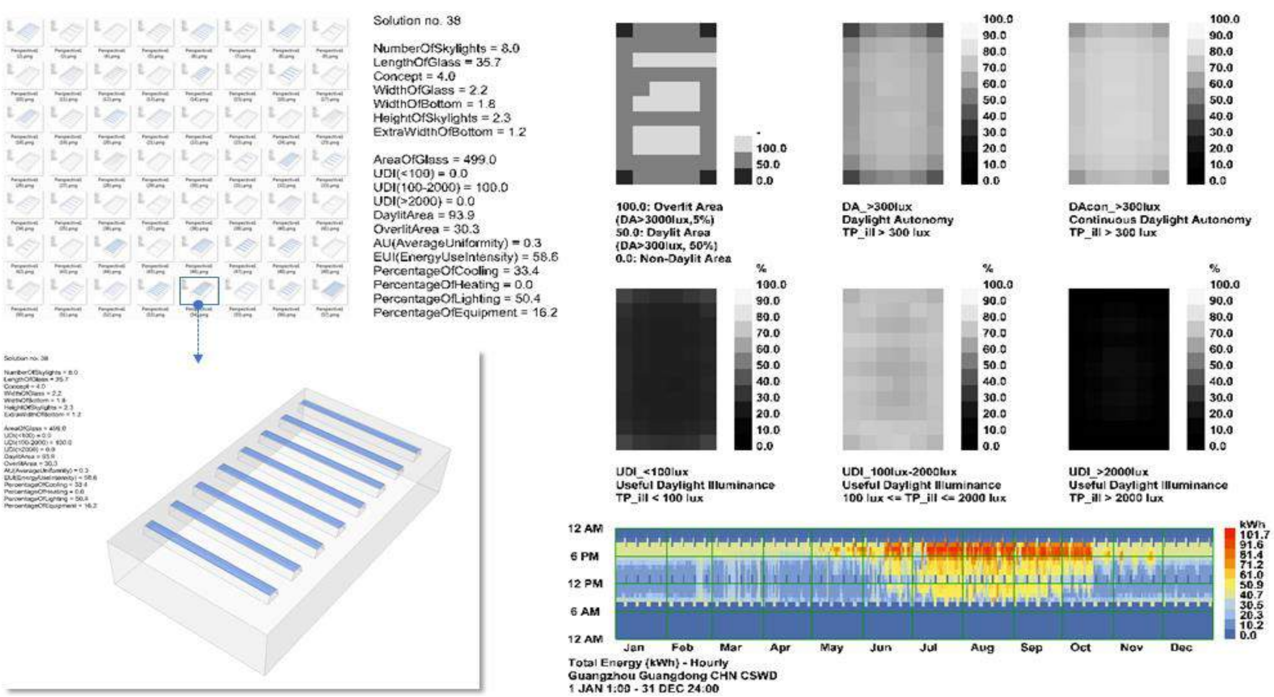

Figure 5. Geometry and simulation results

(1) *EUI* changes in the similar direction as *Percentageňofň Lighting*, but in the opposite direction as *Percentageňofň Lighting/Equipment/Heating*. By further checking the variable values, we notice that the increase of total energy use is mainly due to the increase of lighting energy use.

(2) *EUI* changes in the similar direction as *DaylitArea, UDI(<100),ň UDI(100-2000)*. This indicates that total energy use becomes high when more space has insufficient daylighting level and vice versa.

(3) *EUI* changes in the opposite direction as *OverlitArea*, and their patterns are very similar. This indicates that there might be an obvious trade-off between minimizing both total energy use and the risk of glare.

(4) *EUI* changes in the opposite direction as *Averageň Uniformity*, but their patterns are significant different. This indicates that there might be a less obvious trade-off between minimizing energy use and maximizing daylight uniformity.

(5) *EUI* changes in a different (not exactly opposite) direction as *AreaňofňGlass*, and the similarity level between their patterns is medium. This also indicates an interesting potential trade-off between minimizing both total energy use and investment cost.

Based on the above information and knowledge extracted, EUI, OverlitArea, Average Uniformity and Area of Glass (which may lead to interesting trade-off relations) are identified as promising objective variables.

Identification of Promising Design Variables

Hierarchical Clustering facilitates to identify the *input-output* relationships by using a clustered data set. The clusters are created by including the design variable "Concept" and the four objective variables identified, based on the data sets related to initial concepts. As shown in Figure 6 (right), the clusters are visualized by a clustering parallel coordinate chart which shows their distribution in all selected variables. Each cluster is represented by a colored band. The mean of each cluster is represented as a thick center line, whereas the confidence interval is represented as the band width. The selected variables are represented by parallel vertical lines. Moreover, to facilitate observation, the general direction of achieving a desired performance goal of each selected objective variable is shown by a black arrow. By using the concise visualization, the following information and knowledge are obtained.

(1) *Concept1* (i.e. green band) performs very well in *EUI*; but less well in *Averageň Uniformity*; and very bad in *OverlitArea* andň*AreaňofňGlass.* This may be associated with the widest angle of receiving daylight from all directions compare with other concepts.

(2) *Concept2* (i.e. blue band) performs very well in *Averageň Uniformity,ňOverlitAreaň*andň*AreaňofňGlass*; but relatively bad in *EUI.* This relatively balanced overall performance may benefit from the protruding roof elements blocking daylight from certain unwanted directions.

(3) *Concept3* (i.e. red band) performs very well in *OverlitAreaň*andň*AreaňofňGlass;ň*but very bad in *Averageň Uniformityň* and *EUI.* This may be associated with the asymmetric geometry of the saw-tooth roof which makes the daylight unevenly distributed.

Based on the above information and knowledge extracted, Concept2 is considered as a more promising initial concept out of three, with relatively balanced overall performance. The design variables describing the concept are identified.

Re-formulation of a New Concept

We are interested in getting a new and high-performing concept by adding new design variables, as mentioned in Section 2.4. Based on the concept identified, improving its EUI performance while maintaining its advantages in other performances is the key to improve its overall performance. Triggered by the previous analysis, we notice that good EUI, OverlitArea and Average Uniformity performances may be associated with the use of horizontal windows, protruding elements and symmetrical geometries; and that good Area of Glass performance can be achieved by searching within a certain concept, given its relatively wide band width.

Inspired by the above indications, a new concept (i.e. Concept4 shown in Figure 4) is proposed. It leaves the glazing exposed horizontal to the sun, blocks daylight by

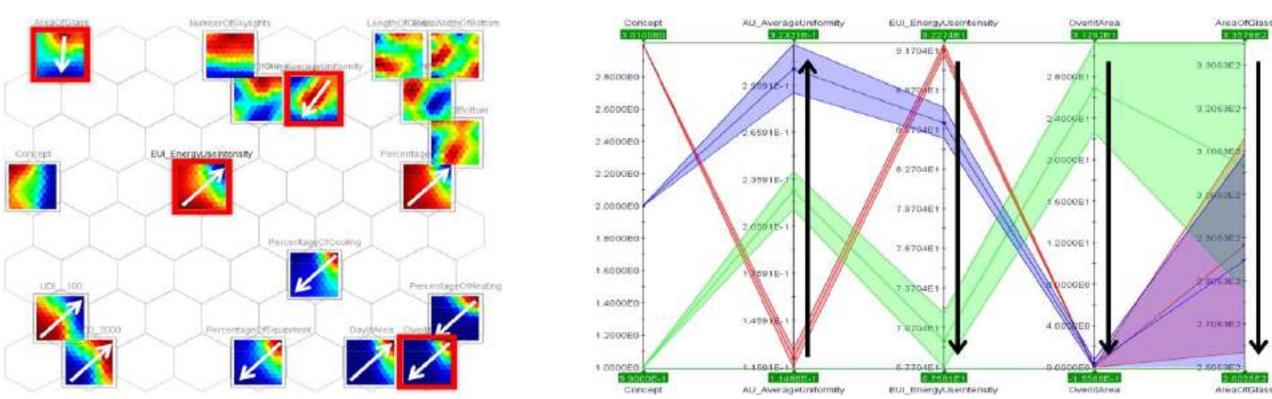

Figure 6. Self-Organizing Map (left); clustering parallel coordinate chart (right)

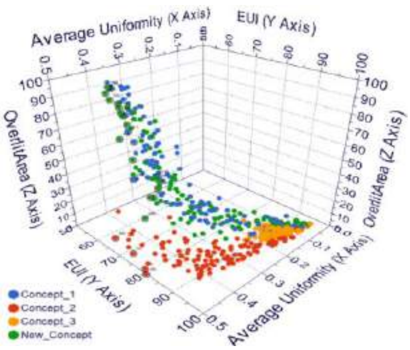

Figure 7. Objective space for new and initial concepts

using inclined protruding elements while remaining the symmetrical geometry. In this concept, the height and angle of the protrusion are important, thus HeightOfSkylights and ExtraWidthOfBottom are added to replace some original design variables in the initial promising concept. To better understand how the new concept performs, it is sampled by using the same sampling strategy and run. Data obtained are plotted in a 3D objective space, together with the data of the three initial concepts, as shown in Figure 7 (the objective Area of Glass is left out for the easy of visualization). The results show that, out of total 25 Pareto solutions, 15 solutions belong to the new concept, while Concept1 and Concept2 account for only 6 and 4 solutions respectively. This confirms the potential of the new concept in achieving better Pareto solutions, compared with the other concepts.

5 CONCLUSION

In conclusion, computational design exploration is crucial for formulating good design concepts in conceptual design stage. During the exploration, information and knowledge extraction plays an important role. To support this process, an integration and automation platform with powerful post-processing capability is highly useful. In this regard, GH-MF integration is a good option with great potential.

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