PROBABILISTIC DESIGN OF SEA DEFENCES

(Vrijling + Bakker)

nr. 13780501
maart 1980
Delft

Bijdrage aan de
Coastal Engineering Conference
te Sydney

PROBABILISTIC DESIGN OF SEA DEFENCES

by W.T. Bakker[1)] and J.K. Vrijling[2)].

0. Abstract

Designs of dikes and dunes according to current Dutch guidelines, based on a deterministic approach, are nog consistent with probabilistic philosophy. This statement is amplified in the present paper as a pilot investigation; a rough outline for a probabilistic method of dune and dike computation is given. Numerical comparison of probabilistic and deterministic methods is hampered due to the fact that the results according to the deterministic approach depend on engineering instinct in the choice of boundary conditions and because the probabilistic approach is not as yet operational. Illustrative computations show differences of a factor 1000 in failure probability, starting from dimensioning according to the same standards, following deterministic guidelines.

1. Introduction

Sea defences are constructed to safeguard the population against storm surges. The rich tradition in the field of dikes in Holland shows however, that complete safety is unattainable. Realizing this, a method to asses the probability of failure (or safety) of a system of sea defences has to be developed. All possible causes of failure have to be analysed and consequences determined. For this aim, the "fault three" is a good tool (figure 1).

1) Head of the advisory Department Flushing, Rijkswaterstaat, Netherlands.

2) Project Engineer, Delta Department, Rijkswaterstaat, Netherlands, Scientific officer, Dept. of Civil Engineering, Delft University of Technology

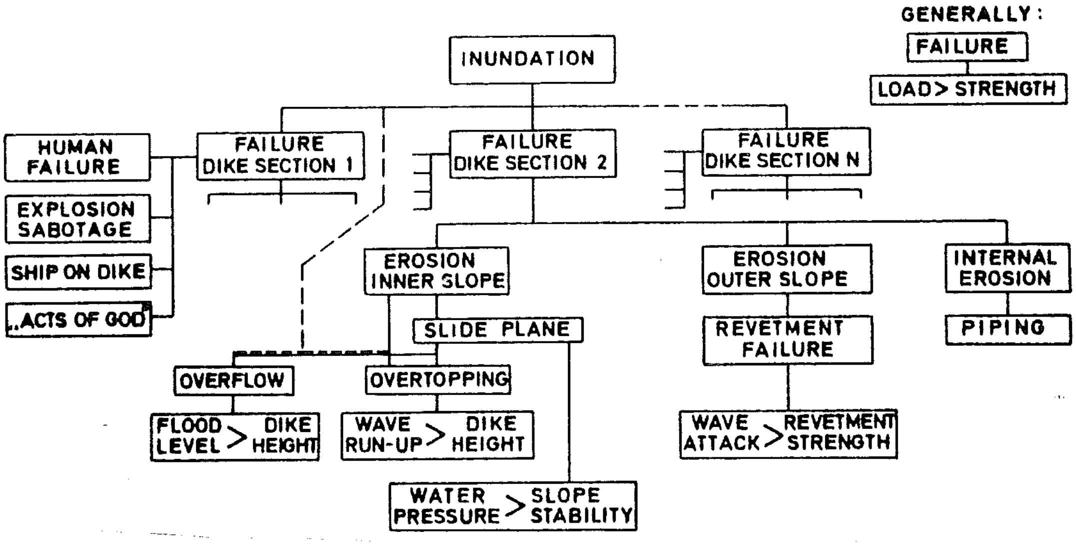

Fig. 1. Simplified fault tree for a dike circle, consisting of N sections.

The fault tree combines four categories of events, that may cause the inundation of a polder:

- human failure; management faults;
- aggresive human action;
- "acts of God";
- technical failure of structural elements.

Although all four categories of events are equally important for the overall safety of the polder, the engineers responsibility is mainly limited to the technical and structural aspects. Therefore this paper deals only with technical failure of structural elements.

In the fault tree, all possible modes of failure of elements can eventually lead to the failure of a dike section and to inundation. This reflects good engineering practice, where attention should be given to all failure mechanisms of the construction under design.
A common approach in the design of concrete or steel structures.
In dike and dune design, limit-state analysis is nog yet established, although it has many useful features in clarifying technical problems as will be shown in this paper.
The ultimate limit-state (u.l.s.) of a failure mechanism describes the situation, wherein the acting loads are just balanced by the strength of the construction. The probability of occurrence of this u.l.s. for each technical failure mechanism can be found from a "convolution integral" (CIRIA, 1976).

Starting from a probability density function (p.d.f.) of the boundary condition one finds with a transferfunction the p.d.f. of the loads on the structural element, called $f_L(\ell)$, being a function of the load ℓ. Combining the last-mentioned p.d.f. with the p.d.f. of the strength s of the structural section, called $f_S(s)$, gives the failure probability p_f of the element:

$$p_f = \int_0^\infty \int_0^\ell f_S(s)\, ds\, f_L(\ell)\, d\ell \qquad (1.1)$$

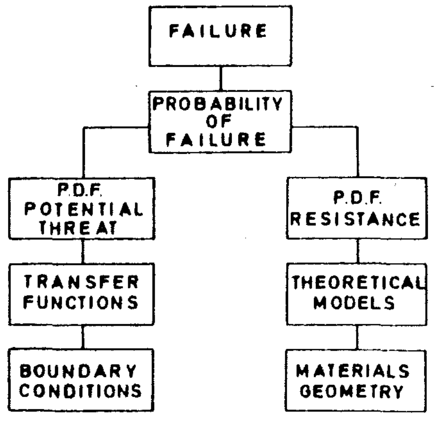

Fig. 2. The concept of the ultimate limit-state of a failure mechanism.

This concept is applicable in coastal engineering, when the narrow definitions of load and strength are widened to potential threat and resistance.
The adapted concept of a failure mechanism is given in figure 2.
First, all basic variables that play a role in the theoretical relationships on which the design of a particular element is based, have to be specified.
The main categories of basic variables are "resistance" and "potential threat". The category contains basic variables that can be defined as threatening boundary conditions for the construction e.g. wind velocity extremes, water levels or a ship's mass. The resistance of the construction is derived from the basic variables by means of theoretical models. The relations that are used to derive the potential threat from the boundary conditions are called transfer functions.
The safety margin between "potential threat" and "resistance" must guarantee a sufficienty low probability of failure.
Three different philosophies are currently available in construction practice:

1. deterministic philosophy;
2. quasi-probabilistic philosophy;
3. probabilistic philosophy.

The present dutch guidelines for dike and dune design follow a philosophy, that lies between the deterministic and the quasi-probabilistic approach. The ultimate potential threat is derived from extreme storm surge levels with a very low probability of exceedance (1% per century) and equated with the average resistance of the dike without any apparent safety margin. In this paper it will be shown that designs according these guidelines are not consistent with the probabilistic philosophy. Beside the ultimate limit-state, there are situations, where the ever continuing presence of a load causes a detoriation of constructional resistance in time, without any imminent danger of failure (e.g. fatigue, creep).

However, this detoriation of constructional resistance can cause an unexpected failure in extreme conditions. The serviceability of the construction can also diminish without leading to collapse (e.g. settlements, deformation).

The serviceability limit-state is principally treated in the same way as the ultimate limit-state. However, attention is rather given to loading situations that occur very frequently during the lifetime of the construction than to extreme conditions.

A point of great practical importance is that a serviceability limit-state, i.e. a detoriation of constructional resistance in time, can be solved in two ways:

1. improving the resistance of the construction to guarantee sufficient strength during the service life;
2. the detoriation of constructional resistance can be controlled by inspection and maintenance procedures.

The second solution, however, introduces a certain non-technical risk, because constructional safety depends on the care of other people.

In some fields of dike and dune design the application of the limit-state conception as described above is cumbersome because a theoretical description is not available. This is especially true for erosion and scour problems, which govern the design of dunes and dikes. Neither transfer functions to transform waves and tide into forces on grains nor a theoretical model for the stability of grains are known.

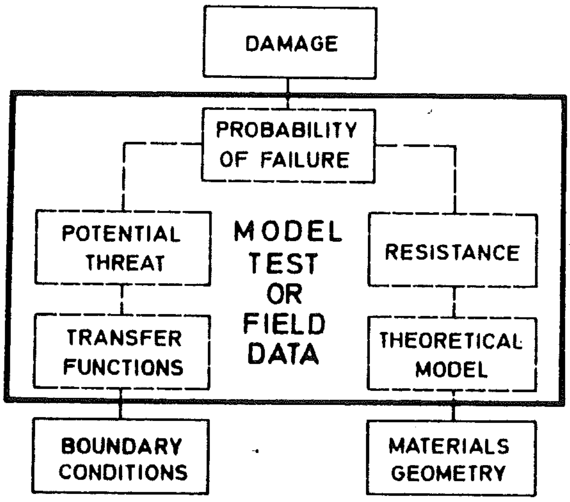

Fig. 3. The solution of a limit-state by black box approach.

To overcome this problem, a scheme to simulate all possible combinations of natural boundary conditions in a scale model of the construction and to correlate the damage done to the boundary conditions can developed (figure 3).
Of course, field data of boundary condition, resistance parameters and damage are preferred as base for correlation, if they are available in sufficient amount.

2. Calculation of dike height according to the present dutch guide lines

As an illustration of the concepts developed in the foregoing paragraph three simplified dike design according to the present dutch guidelines will be made. Afterwards the design will be checked with probabilistic methods.
As an example, two alternative designs situated in the mouth of the Eastern Scheldt, one facing Northwesterly storms and the other; a dike with a southeastern orientation (no wave attack), will be studied.
According to the guidelines (DELTA COMMITTEE REPORT, 1960) the starting point of the design of a dike that protects economically less important regions is a storm surge level with a frequency of exceedance of $2.5 * 10^{-4}$ p.a. This storm surge level is determined by statistical extrapolation of empirical data at NAP +5.50 m. The design wave is also found by statistical extrapolation from wave data (vide cross in figure 6):

$H_s = 5.00$ m $\quad$ $T_z = 7.7$ sec. $\quad$ $T_p = 12.0$ sec.

where:

H_s = significant wave height
T_z = mean zero crossing period
T_p = peak period of the wave spectrum.

To calculate the wave run-up, a transfer function of the following form[1] is applied (T.A.W., 1972):

$$r_{2\%} = 0.7\ T_p \sqrt{g\ H_s}\ \tan\alpha \qquad (2.1)$$

where:

g = acceleration of gravity

α = angle of the outer slope

$r_{2\%}$ = wave run-up, that is exceeded by 2% of the waves

A minimum wave run-up of 0.50 m always has to be accounted for.
Seiches and gust bump are estimated to have amplitudes of 0.24 and 0.25 m respectively. According to the guidelines, the amplitude of the gust bump B may be reduced if a combination with wave run-up occurs. The advised reduction R is:

$$R = \frac{B}{B + \xi . r_{2\%}} \qquad (2.2)$$

Now the minimum dike height can be calculated. However, three factors affect the height of the dike during its lifetime, i.e. secular change of the chartdatum (NAP), settlement of the dike and settlement of the deep soil. The first effect is estimated at 0.10 m per century (DELTA COMMITTEE, 1960). Soil-mechanical calculations have to provide an insight in the amount of settlement.

[1] In practical design calculations the wave run-up is evaluated with the formula $r_{2\%} = 8\ H_s \tan\alpha$, which gives smaller values.
Theoretically, last mentioned formula is only valid for a wave strepness $H/L \sim 0.05$. However, up T/L now, this theoretical error has not caused serious prothens due to run-up reducing phenomena.

The dike height is now determined by the addition of all phenomena (table 2.1).

Table 2.1	Transfer function	NW slope 1 : 6	NW slope 1 : 8	SE slope 1 : ?
storm surge level (NAP)		+ 5.50 m	+ 5.50 m	+ 5.50 m
wave run-up	$r_{2\%}=0.7\ TP\sqrt{gH_s}\ tg\ \alpha$	9.90 m	7.42 m	0.50 m
seiches	S	0.24 m	0.24 m	0.24 m
gust bump	$\frac{B}{B+\xi r_{2\%}} \cdot B$	0.03 m	0.04 m	0.18 m
design water level z (NAP)		+15.67 m	+13,20 m	+ 6.42 m
change of chart datum		0.15 m	0.15 m	0.15 m
settlement dike		0.10 m	0.10 m	0.10 m
settlement subsoil		0.50 m	0.50 m	0.50 m
dike height $\xi = \frac{1}{5}$ (NAP)		+16.42 m	+13.95 m	+ 7.17 m

2.1. Probabilistic calculation and evaluation of the dike height

In the designs of the previous paragraph all parameters, except the storm surge level and the waves, have been thought of as specified constants. For an advanced analysis, all parameters should be specified as stochastic, which implies that their exact magnitude is not known with certainty.

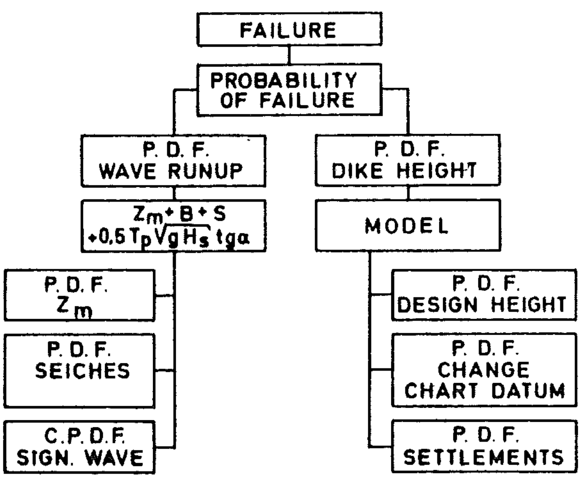

Fig. 4. Ultimate limit-state caused by wave run-up.

In figure 4 the ultimate limit-state caused by wave run-up is given in the schematical way developed in par. 1, whereby all relevant parameters are specified as probability density functions or distributions.

The distribution of the storm surge level z is based on the already mentioned set of empirical data. After a correction for the influence due to the Delta Works, the distribution has the form:

$$P_r\ (\underline{z} > z)=\exp(-2.3\ \frac{z-2.94}{0.696}) \qquad (2.3)$$

$z = [m]$

Due to the extrapolation of the distribution to very low probabilities of exceedance, some uncertainty is introduced which is supposed to be normally distributed. The standard deviation is defined as a function of the storm surge level (DELTA COMMITTEE, 1969).

$$\sigma_z = 0.11 \ (z - 2.25) \qquad \sigma_z = [m] \qquad (2.4)$$

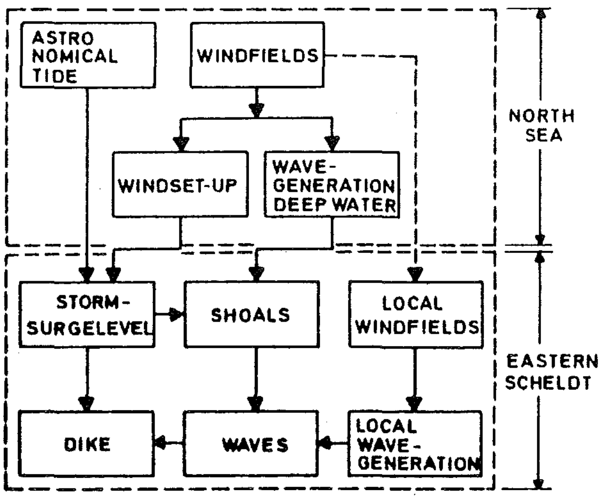

Fig. 5. Model to predict the sea state in the Eastern Scheldt.

A study of the sea states in the Eastern Scheldt has shown that the wave energy comes from two sources (VRIJLING and BRUINSMA, 1980).

1. Wave energy penetrates from the North Sea. This energy is reduced as a function of the water level by breaking on the shoals in front of the coast.
2. Local windfields generate wave energy in the area between the shoals and the dike.

This system analysis enabled the determination of the probability density function of the wave spectrum $S_{\eta\eta}$ as a function of the storm surge level z_m. Figure 6 shows the conditional probability density function of wave energy and storm surge level. Now the joint probability density function of wave spectra and storm surge levels can be determined:

$$p(S_{\eta\eta} . z) = p(z) . p(S_{\eta\eta}|\underline{z} = z)$$

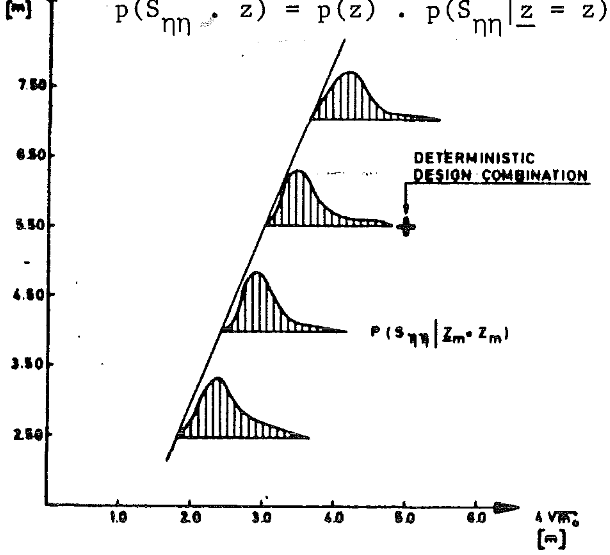

Fig. 6. Conditional probability density function of wave energy and storm surge level.

Seiches in dutch coastal waters are irregular waves with a period of 10 to 50 minutes, that show no correlation with the storm surge level.

Assuming a Rayleigh distribution for the relative maxima and estimating the number of maxima during a storm at N, the probability density function of the highest maximum is (BATTJES, 1972):

$$p(S_{max}) = \frac{S}{m_{o_s}} \cdot N \cdot \exp\left(-\frac{S^2}{2\, m_{o_s}}\right) \cdot \exp\left(-N \exp\left(-\frac{S^2}{2\, m_{o_s}}\right)\right) \qquad (2.5)$$

where:

$m_{o_s} = 0.08$ $\qquad\qquad$ $N = 12$

This function can be approximated by a Gaussian distribution with the parameters:

$\mu_s = 0.18$ m $\qquad\qquad$ $\sigma_s = 0.04$ m

Gust bumps are single pronounced elevations of sea level. Lacking statistical data, a Gaussian distribution is assumed with the parameters:

$\mu_b = 0.15$ m $\qquad\qquad$ $\sigma_b = 0.05$ m

Now the natural boundary conditions are reduced to three dimensions by combining the uncertainty of the exceedance curve of storm surge level, the seiche and the gust bump in one variable h. Assuming statistical independance the result is

$$\mu_h = \mu_s + \mu_b = 0.33 \text{ m} \qquad \sigma_h = \sqrt{\sigma_z^2 + \sigma_s^2 + \sigma_b^2} \qquad (2.6)$$

In the case studied here the transfer functions are trivial, except for the transformation from waves into wave run-up. A relation similar to (2.1) is used, which contains more spectral information (T.A.W., 1972):

$$r_{sign} = \frac{\gamma}{1.40} T_p \sqrt{g\, H_{sign}} \tan\alpha \text{ in which } \gamma = 0.48 + 0.37\, \varepsilon \qquad (2.7)$$

where:

$$\varepsilon = \frac{m_0\, m_4 - m_2^2}{m_0\, m_4} \qquad \text{spectral width}$$

m_n = n - th spectral moment

Further it is assumed that the wave run-up r is Rayleigh-distributed.

$$P_r\,(\underline{r} > r) = \exp\left(-2\left(\frac{r}{r_{sign}}\right)^2\right) \qquad (2.8)$$

Damage will only be done if the wave run-up exceeds the dike height several times during a storm. Based on the binominal distribution one finds for the probability distribution of wave run-up, which exceeds a given level at least m times in a storm containing N waves:

$$P(r_m) = 1 - \sum_{k=0}^{k=m-1} \frac{N!}{k!\ (N-k)!} \cdot P_r(\underline{r} > r)^k \cdot \{1 - P_r(\underline{r} > r)\}^{N-k} \qquad (2.9)$$

This distribution is conditional on the occurrence of the sea state $S_{\eta\eta}$.

For numerical reasons the three-dimensional space z, $S_{\eta\eta}$, h of natural boundary conditions is devided in small elements with dimensions Δz, $\Delta S_{\eta\eta}$ and Δh. The probability of occurrence P_r (z, $S_{\eta\eta}$, h) of a combination of boundary conditions falling within these elements is:

$$P_r(z,\ S_{\eta\eta},\ h) = p_r(z) \cdot p_r(S_{\eta\eta}|z) \cdot p_r(h|z) \qquad (2.10)$$

where $p_r(z)$, $p_r(S_{\eta\eta}|z)$ and $p_r(h|z)$ denote the respective (conditional probability densities times Δz, $\Delta S_{\eta\eta}$ and Δh respectively.
The potential threat, representative for this element now can be found by adding the storm surge level z and the seiche-gust bump combination h -giving together the still-water level z_m- and adding the wave m run-up r over and above that:

$$\tau = z + h + r \qquad (2.11)$$

The probability of occurrence is evaluated as:

$$P_r(\tau) = P_r(z,\ S_{\eta\eta},\ h) \cdot p(r_m|S_{\eta\eta})\ \Delta z \qquad (2.12)$$

Now the joint probability density function of still water levels and m-times exceeded wave run-up heights can be found by repeating the calculation for all possibilities combinations of boundary conditions.

The probability of exceedance of a specific potential threat can now be evaluated by integrating the two dimensional probability density function of still water levels and wave run-ups (figure 7).

$$P_r(\underline{\tau} > \tau) = \iint_{\underline{\tau} > \tau} p(z_m,\ r_m)\ d\ z_m\ d\ r_m \qquad (2.13)$$

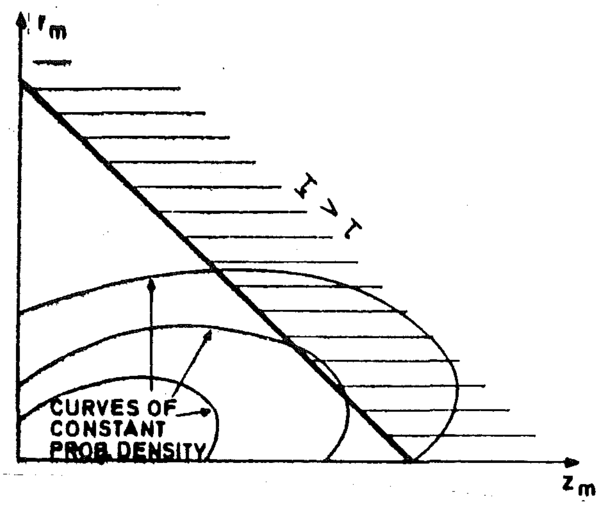

Fig. 7. Probability of wave run-up as part of the probability mountain.

The probability of exceedance curve, which is numerically evaluated, appears to be of the form:

$$P_r(\underline{\tau}>\tau)=\exp\{(8.36-\tau)/0.452\}$$
$$\tau= [m] \qquad (2.14)$$

Having derived the probability of exceedance curve, it is interesting to compare the potential threat with a probability of exceedance of $2.5 * 10^{-4}$ p.a. with the design level calculated according to the guidelines (table 2.2).

Table 2.2	slope 1 : 6	slope 1 : 8	SE	
guidelines	NAP +15.69	NAP +13.95	NAP + 6.42	heights
probabilistic	NAP +12.21	NAP +10.50	NAP + 5.95	in m

An even more interesting experiment is to evaluate the probability of failure of the simplified dike design of the proceeding paragraph. Because the real failure mechanisms are not expressed in mathematical form, failure will be arbitrarily defined as the exceedance of the dike height by at least 12 wave run-ups during a storm.

On the side of the "resistance", the height of the dike also presents some uncertainty. When a storm occurs at some time in the future the effects, that are accounted for in the difference between the planned dike height and the design water level, are realized to some uncertain extent. In fact, the theory of the serviceability limit-state developed in ch. 1 applies.

Table 2.3 heights in m levels: NAP	NW slope 1 : 6	NW slope 1 : 8	SE SE	σ	cp table 2.1
dike height	+16.44	+13.95	+ 7.17		
change of chartdatum	- 0.10	- 0.10	- 0.10	0.025	0.15
settlement dike	- 0.07	- 0.07	- 0.07	0.02	0.10
settlement subsoil	- 0.35	- 0.35	- 0.35	0.10	0.50
tolerance	-	-	-	0.10	-
	+15.92	+13.95	+ 6.65	0.14	0.75

This means, that the real dike height differs from the planned one unless a proper maintenance scheme is carried out. The real height can be approximated by normal distribution (table 2.3). Now the probability of failure caused by overtopping of the dike can be evaluated by integrating:

$$P_r(\text{failure}) = \int_0^\infty P_\tau\ (x) \cdot p_{\text{dike height}}\ (x) \cdot dx \qquad (2.15)$$

$$= \int_0^\infty e^{\frac{8.36-x}{0.452}} * \frac{1}{0.145\sqrt{2\pi}}\ e^{\frac{1}{2}\left(\frac{15.92-x}{0.145}\right)^2} dx$$

The probability of failure is given in table 2.4.

Table 2.4	NW slope 1 : 6	NW slope 1 : 8	SE
P_r (failure)	$5.8 * 10^{-8}$	$2.2 * 10^{-7}$	$4.1 * 10^{-5}$

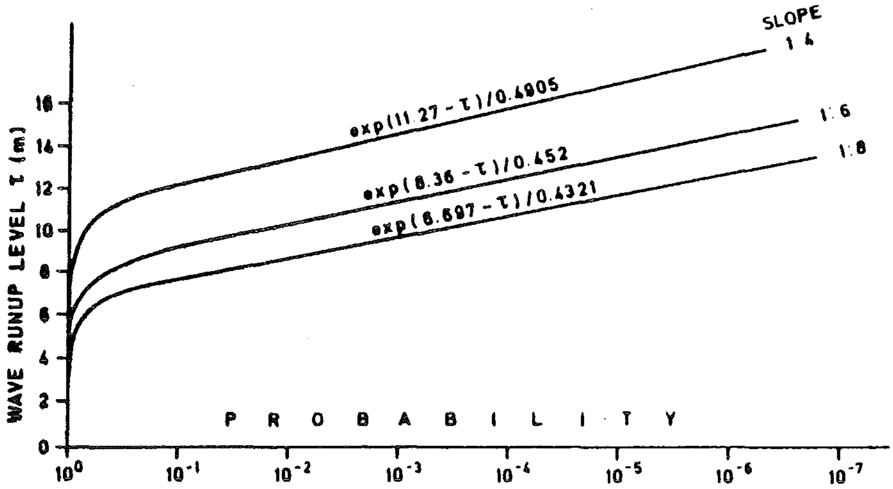

Fig. 8. Exceedance probability of wave run-up as function of revetment slope.

From the results (figure 8) it appears that the probability of overtopping of the dikes designed according the guidelines is not of the same order for various cases. Reducing the dike height by flattening the outer slope increased the chance of overtopping c.q. failure by a factor of nearly 4. The dike facing SE is even more unsafe. Here the potential threat, formed by the still water level without waves exceeding the dike height, will certainly cause inundation. So failure of this dike is at least 1000 times likelier than the dikes facing North West.
This illustrates that, under simplifying assumption, designs according the present dutch guidelines are not able to be compared without further attention.

3. Dune design

At the same place, where in chapter 2 a dike has been designed, a dune will now be constructed.

3.1. Calculation of dune breadth according to the present Dutch Guideline

The present Dutch "guideline for the Calculation of Dune Erosion during Storm Conditions" is based on the following assumption (v.d. GRAAFF (1977) and VELLINGA (1978)):

1°. During a storm surge the coastal profile is reshaped, to a uniform profile, the "stormprofile", described by the formula:

$$y = 0.415\ (\lambda\ x + 4.5)^{0.5} - 0.88 \qquad (3.1)$$

in which:
y = depth in meters below the maximum surge level
x = distance in meters from the point of the profile lying at maximum surge level
λ = coëfficiënt, lying between .8 and 1.25, dependent on the grain diameter; λ equals 1 when D_m = 200μ
The profile is given in figure 11.

2°. The depth d_b to which the storm profile applies is equal to 1.28 times the height of the significant wave H_s at the breaking point.
Thus defining the "width of spreading" B as the width of the stormprofile, over which the eroded sand from the dunes settles, one finds inversely from (3.1):

$$B = \frac{(2.41\ d_b + 2.12)^2}{\lambda} - \frac{4.5}{\lambda} \qquad (3.2)$$

3°. The inclination of the outer slope of the eroded dune is assumed to be 45°. The seaward side of the outer slope i.e. the "food" of the dune-coincides with the origin of the x, y-coordinate system.

4°. During the storm surge, the coastal profile is reshaped, in such a way that the total area of the eroded sand equals the area of the settled sand.

5°. Losses of sand, either to the regions outside the breaker zone, or in landward direction or by a longshore gradient of the littoral drift are neglected.

With respect to necessary dune dimensions, one should take into account a very low beach level in het initial situation before the surge; after the surge a dune breadth of 10 m at surge level should remain.
Starting from the same data as in chapter 2.1, i.e. H_s = 5.0 m, a surge level of N.A.P. +5.50 m and a beach level lying .30 m lower than the mean beach level, for a dune, which has to offer Delta protection during 10, 20 or 50 years respectively, one finds the dune dimensions given in table 3.1.

Table 3.1. (dune breadth in m)	MAINTENANCE PERIOD		
	10 y	20 y	50 y
storm erosion	44.58	44.58	44.58
yearly erosion	3.75	7.50	18.75
minimum body	10.00	10.00	10.00
dune dimension	58.33	62.08	73.33

3.2. Probabilistic calculation and evaluation of the dune breadth

3.2.1. General conciderations

The technical failure mechanisms, which can be distinguished for a dune, acting as sea defence, are principally the same as those for dikes:

a. failure outer slope
b. submerging
c. failure inner slope
d. internal failure

With respect to d, internal failure, apart from failure by hydrologic overpressure, one should also take rabbit holes into consideration as these may occur in the most landward side the dunes.

In the following, the failure mechanism mentioned in b and c are combined by assuming that there will be inundation when the water level rises above a certain given level z_s. This water level will be lower than the dune height, because of wave set-up and wave run-up.

It will be assumed, that a certain "body" of dune should remain, of which the width is assumed to be known, in order to avoid internal failure (figure 9).

The most important failure mechanism for dunes is the "failure" of the outer slope. Therefore this mechanism will be considered in the first place.

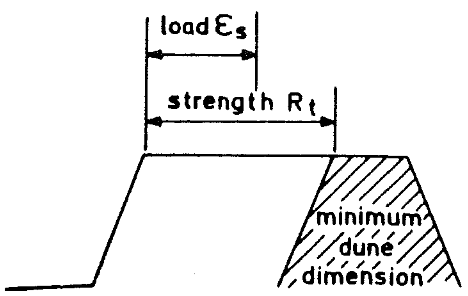

Fig. 9. Definitions of load and strength.

Generally, failure occurs when the load on a construction is larger than the strength. The simultaneous probability of this occurence is found by convolution of the probability density functions of load and strength. The following definitions of load and strength will be used (see also figure 9):

- The load ε_s is defined as the dune erosion, caused by surges combined with waves.
- The strength R_t at time t is defined as the dune breadth, above a certain minimum, necessary to avoid failure mechanisms other than erosion of the outer slope.

Consider first the strength R_t, composed of three components.

$$R_t = R_o + \underline{\varepsilon}_t + \underline{\varepsilon}_b \tag{3.3}$$

It depends upon:

a. the initial conditions R_o;

b. the effect $\underline{\varepsilon}_t$ of gradual erosion[1] and, in the event, periodical supply;

c. the effect on the erosion $\underline{\varepsilon}_b$ of the beach level at the moment of the occurrence of a severe storm.

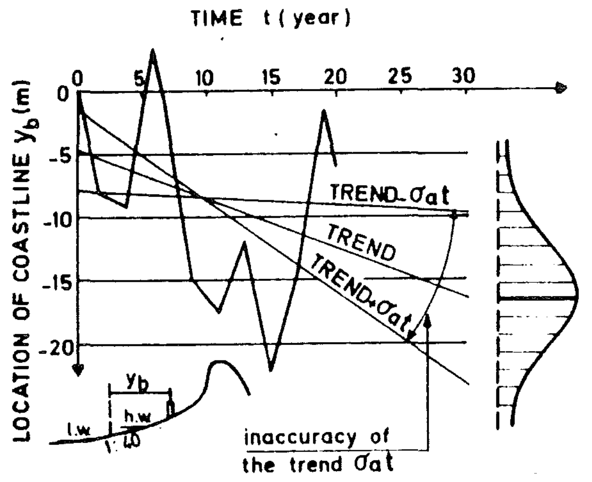

Fig. 10. Location of the coastline in the course at time.

With respect to gradual erosion, it is assumed that the future trend can be found in principle by linear extrapolation of the trend in the past:

$$\underline{\varepsilon}_t = \underline{a}\ t + \underline{b} \qquad (3.4)$$

Figure 10 gives an example: it shows the location of the coastline in a certain range, perpendicular to the coast, with respect to a reference pole; the "coastline" is defined as the mean between the high- and low-water line.

However, given this registration, the trend cannot be properly determined, and a uncertainty remains.

Therefore $\underline{a}$ and $\underline{b}$ are coefficients with a stochastical character.

$E(\underline{a})$, $\sigma(\underline{a})$ and $\sigma(\underline{b})$ are found by linear regression from the registration in the course of time, of the coastline; $E(\underline{b})$ is assumed zero and determines the reference, from which $\underline{\varepsilon}_t$ is taken.

The dune erosion ε_s during storm conditions depends upon the beach level ζ at the moment of the storm. In the mathematical model, this may be simulated by assigning to the strength R_t a normally distributed stochastical component $\underline{\varepsilon}_b$ with mean zero and standard deviation $\sigma(\underline{\varepsilon}_b)$, which is related to the standard deviation of the beach level $\sigma(\zeta)$ in the following way:

$$\sigma(\underline{\varepsilon}_b) = \frac{\partial \varepsilon_s}{\partial \zeta} \cdot \sigma(\underline{\zeta}) \qquad (3.5)$$

The used data and transfer functions are mentioned in table 3.2.

---

[1] In this paper the dimensioning of an eroding dune will be considered, as this case is more intricate than an accreting dune.

Table 3.2. Date and transfer functions used for finding $E(\underline{R}_t)$ and $\sigma(R_t)$.

$E(\underline{a})$		-0.375	m/y		
$\sigma(\underline{a})$		0.326	m/y	$\sigma(\underline{\zeta})$	0.30 m
$E(\underline{b})$		0		$\frac{\partial \underline{\varepsilon}_s}{\partial \zeta}$	11.79
$\sigma(\underline{b})$		7.21	m		
$E(\underline{\varepsilon}_t)$	$E(\underline{a}).t + E(\underline{b})$	$-0.375\ t$	m	$\sigma(\underline{\varepsilon}_b)$	3.54 m
$\sigma(\underline{\varepsilon}_t)$	$\sqrt{\sigma^2(a).t^2 + \sigma^2(\underline{b})}$	$\sqrt{0.326^2 t^2 + 7.21^2}$	m		

From (3.3) and (3.4) one finds, assuming $\underline{\varepsilon}_t$ and $\underline{\varepsilon}_b$ statistically independent:

$$E(\underline{R}_t) = R_o + E(\underline{a}) \ . \ t$$

$$\sigma^2(\underline{R}_t) = \sigma^2(\underline{a}) \ . \ t^2 + \sigma^2(\underline{b}) + \sigma^2(\underline{\varepsilon}_b) \tag{3.6}$$

Further considerations concerning the strength depend upon the applied strategy and are given in chapter 3.2.2. and 3.2.3. Consider next the load.

Replacing in figure 5 the dike by a dune and using the same schedule as explained in chapter 2.1 for all possible combinations of still water level z_m and wave heights H_s the dune erosion ε_s can be numerically calculated using the assumptions 1^o to 5^o, given in chapter 3.1.

Thus, one finds the probability distribution for the dune erosion by surge (of (2.14)):

$$P_r = (\underline{\varepsilon}_s > \varepsilon_s) = \iint_{\underline{\varepsilon}_s' > \varepsilon_s} p(z_m, H_s) \ d\ z_m \ d\ H_s \tag{3.7}$$

Starting from the initial profile, given in figure 11, the distribution found is shown in figure 12.

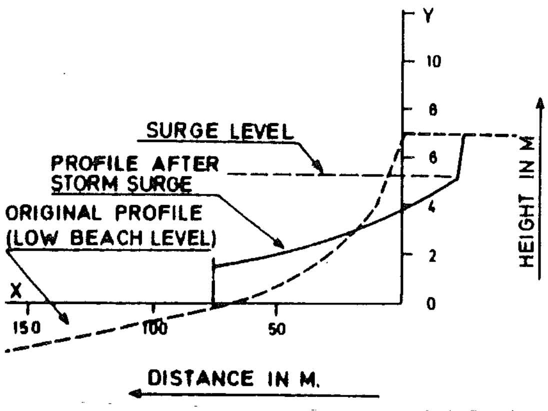

Fig. 11. Dune profile before and after storm surge.

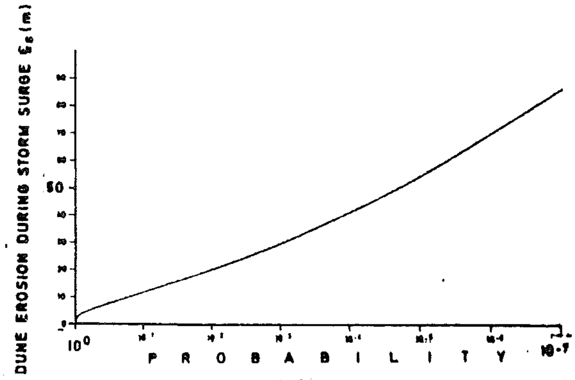

Fig. 12. Exceedance probability of dune erosion during storm surge.

Table 3.3. (values in m)	MAINTENANCE PERIOD		
	10y	20y	50y
$E(\underline{R}_t)$	R_o −3.75	R_o −7.50	R_o −18.75
$\sigma(\underline{R}_t)$	8.67	10.35	18.17

3.2.2. Non-intervention strategy

Now load and resistance will be combined.
In the numerical model, three examples have been elaborated starting from a period of non-intervention in the coastal processes of 10, 20 and 50 years respectively. Table 3.3 gives $E(\underline{R}_t)$ and $\underline{\sigma}(R_t)$ after these periods. Figure 13a shows an example of, on the one hand the probability density of the strength $\underline{R}_t$, and, on the other hand the exceedance probability of the load $\underline{\varepsilon}_s$. The figure shows two classes of failure: the ultimate limit-state, where the load surpasses the strength, and the serviceability limit-state, where the dune collapses, just because the gradual erosion is more than expected.
By convolution, one can calculate the total probability of failure as a function of the expected strength $E(\underline{R}_t)$ at time t (figure 13b).

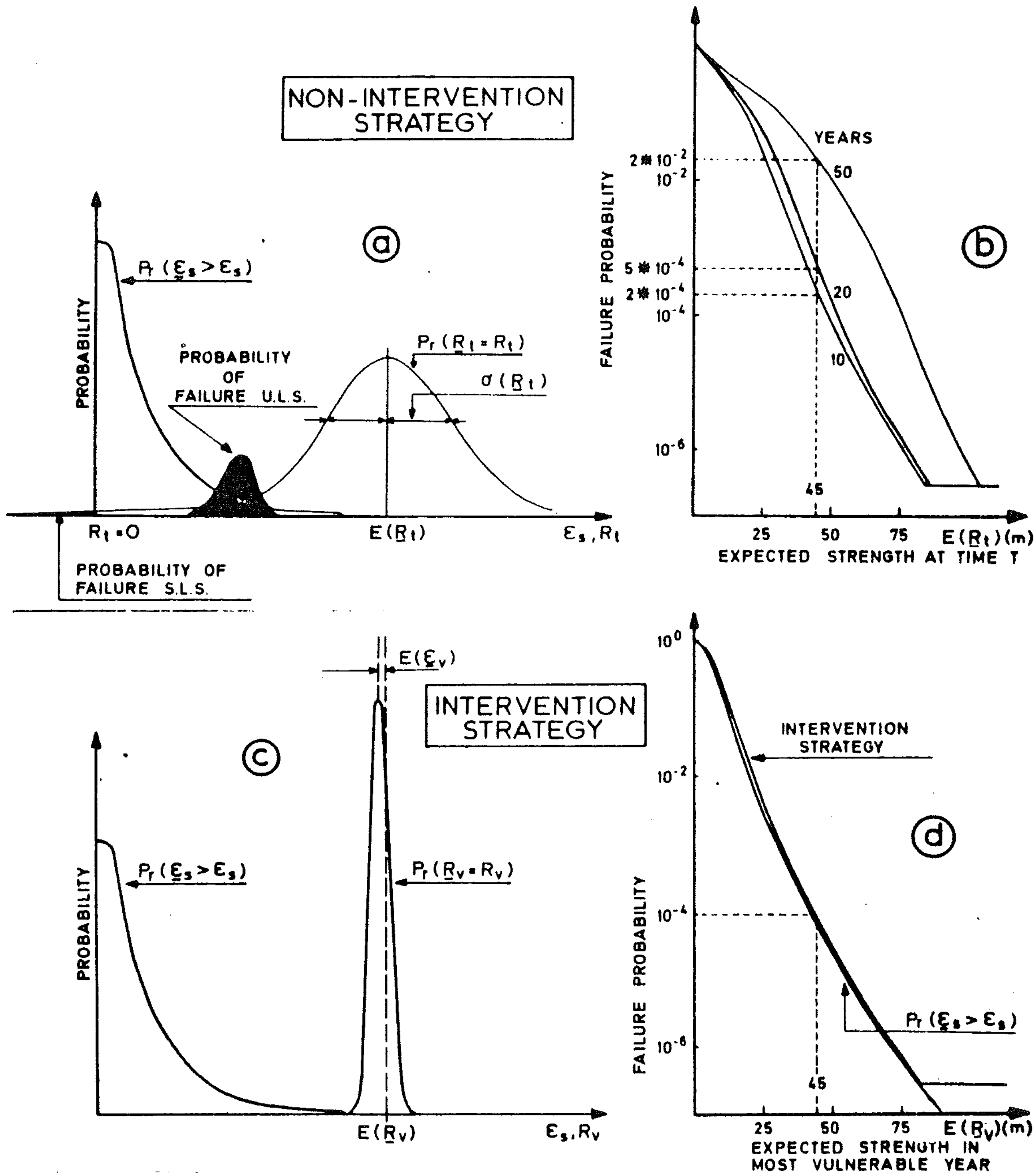

Fig. 13. Failure probability according to non-intervention and intervention theory.

The upper part of the curves has a Gaussian character, determined mainly by the serviceability limit-state (gradual erosion more than expected). The middle part has a more or less negative-exponential character, determined by the ultimate limit-state. The lowest part, the horizontal line, is determined by the risk of submergence of the dunes.
The results are discussed in the evaluation.

3.2.3. Intervention strategy

This strategy implies a sand supply to a seaward line 0 as soon as the dune foot surpasses a landward line L in landward direction (figure 14a, b). In the example treated, the line L has been chosen in such a way, that when the dune foot coincides with L, the strength necessary according to the deterministic approach was just available (i.e., that R_t equals 44.58 m, cf table 3.1.). It has been assumed, that if in one year measures, that the eroding dune foot surpasses the limit L in landward direction, it takes another year for preparing and carrying out the dune replenishment. The distance OL depends upon the desired expectation of the return period of sand supply. Starting from the assumptions given in figure 14, one finds a probability density of the location of the dune foot in the course of many years as sketched in figure 14c, either by a numerical Monte Carlo simulation, or by analytical computations (BAKKER, 1980). As it is the yearly probability of inundation that counts, the situation has to be considered in the year in which the dune is most vulnerable[1]. Figure 14c shows (interrupted line) the probability density of the location of the dune foot, under the condition, that it is measured in the year after the one, in which the landward surpassing of the line L was recorded and on the other hand before the supply to line 0 took place.

Figure 14d gives the same probability density, but now taking into account, that the rate of erosion $\underline{a}$ cannot be properly determined from the measurements, as stated in chapter 3.2.1. To this end, the computation of the last-mentioned probability density has been repeated for various values of the yearly expected erosion, assuming fixed values for the yearly variation of the erosion $\sigma(\underline{\varepsilon}_n)$ and for OL.

After this, these probability density distributions are combined again, by giving each a weight, proportional to the occurrence, probability -found from $E(\underline{a})$ and $\sigma(\underline{a})$, table 3.2- and adding. The result found does not depend very much on OL, when the expected return period is not too small. Analogus to figure 13a and 13b, figures 13c and 13d are constructed, where $\underline{R}_v$, the strength in the year the dune is the most vulnerable, replaces $\underline{R}_t$ from (3.3):

---

[1] This means that in other years a "hidden safety" will be present. In this way, the average safety over, say 100 years of a dune will be higher than the one for a dike (with respect to this aspect).

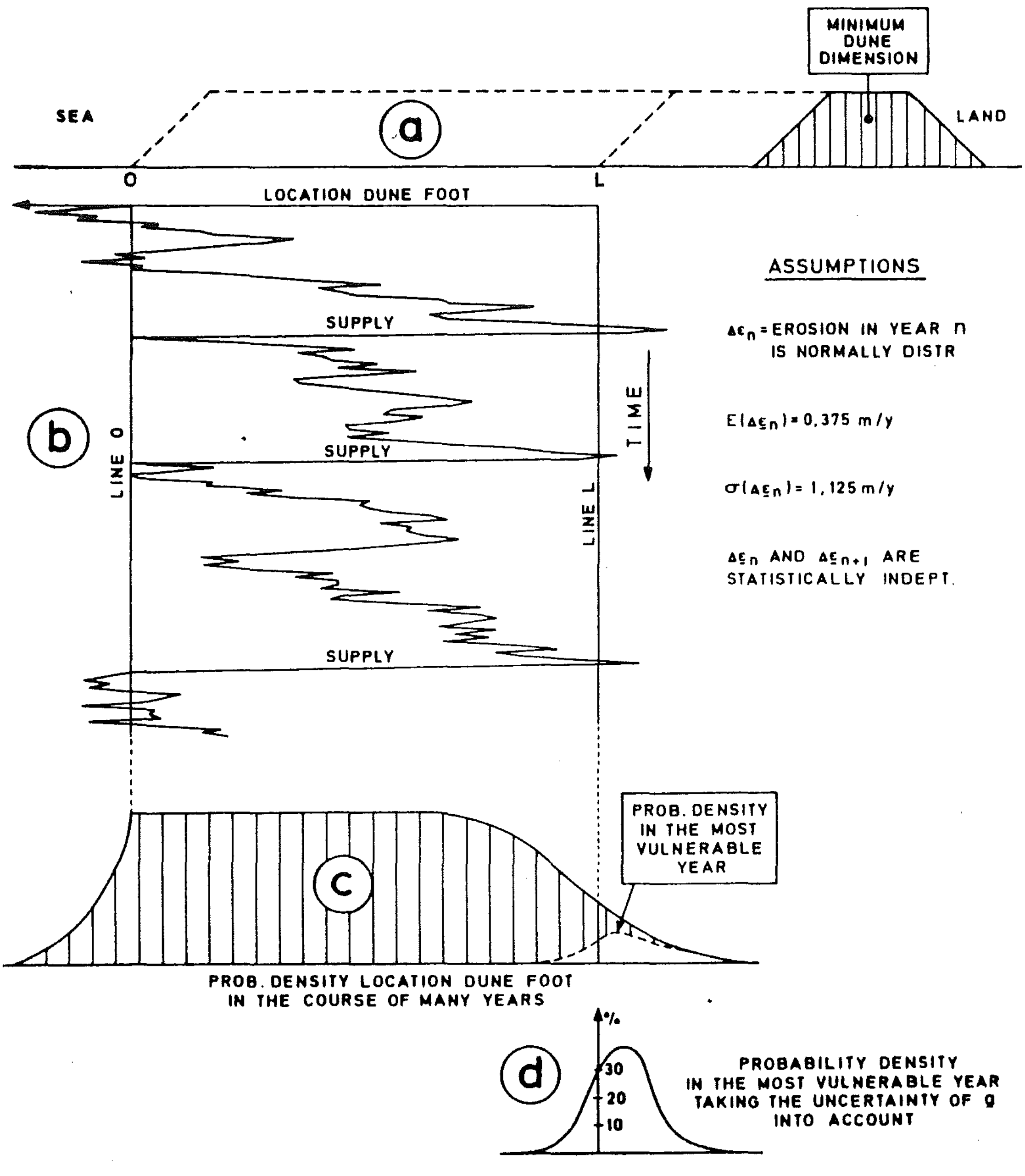

Fig. 14. Probability density of dune foot location in the year that the dune is most vulnerable.

$$\underline{R}_v = R_L + \underline{\varepsilon}_v + \underline{\varepsilon}_b \qquad (3.8)$$

in which R_L denotes the strength when the dune foot coincides with the line L; $\underline{\varepsilon}_v$ [1)] of which the probability density is shown in figure 14d, indicates the gradual erosion with respect to L in the most vulnerable year and $\underline{\varepsilon}_b$ has the same meaning as in (3.3). In the example, $E(\underline{\varepsilon}_v)$ and $\sigma(\underline{\varepsilon}_v)$ are found to be 1.05 m and 1.31 m respectively.

[1)] positive in seaward direction.

Evidently, the standard deviation of $\underline{R}_v$ in figure 13c is much less than that of $\underline{R}_t$ in figure 13a. Consequently, as figure 13d shows, the failure probability according to the intervention strategy is nearly determined by the probability of erosion, P_r $(\underline{\varepsilon}_s > \varepsilon_s)$, during storm surge.
It shows that for equal values of $E(\underline{R}_t)$ and $E(\underline{R}_v)$, the intervention strategy (with the given reaction time of 1 year) gives more safety than the non-intervention strategy.

3.3. Evaluation of deterministic and probabilistic approach of dune breadth computation

In chapter 3.1, a value of 44.58 m for the dune erosion during "super surge" with exceedance probability $2.5 * 10^{-4}$ was found.
Following the non-intervention strategy this value should be compares with the expected necessary strength $E(\underline{R}_t)$ at time t, given in figures 13a, b. Then, for t = 10, 20 and 50 years, figure 13b gives probabilities of failure of $2^5 * 10^{-4}$, $5 * 10^{-4}$ and $2 * 10^{-2}$ p.a. respectively, when $E(\underline{R}_t)$ equals 44.58 m.
Following the intervention theory, the wanted expectation of the strength $E(\underline{R}_v)$ determines the location of line L (c.q. (3.8)). When $E(\underline{R}_v)$ equals 44.58 m, figure 13d gives the probability of failure in the year the dune is the most vulnerable as $8 * 10^{-5}$ p.a.

Table 3.4.

	MAINTENANCE PERIOD 10y	20y	50y	
dune breadth in m acc. to guideline	58.33	62.08	73.33	exceedance prob. $2.5 * 10^{-4}$
failure probability for above-ment. dune	$2^5 * 10^{-4}$ $8 * 10^{-5}$	$5 * 10^{-4}$ $8 * 10^{-5}$	$2 * 10^{-2}$ $8 * 10^{-5}$	non-intervention intervention
dune breadth in m acc. prob. theory failure prob. $2.5 * 10^{-4}$	57.75 50.25	65.50 54.00	89.0 65.25	non-intervention intervention

In table 3.4 these values have been summarized. Furthermore this table gives the dune width following the guide line as given in table 3.1.

Another comparison can be made in a way similar to that in table 2.1, i.e. instead of fixing the strength $E(\underline{R}_t)$ and finding the failure probability, one fixes the failure probability and finds the necessary strength $E(\underline{R}_t)$. Choosing a probability of $2.5 * 10^{-4}$, as has been done for a dike (table 2.2), the results are summarized in table 3.4.

The intervention strategy results in smaller dune breadth for achieving the same safety as in the deterministic approach, because of the rather extreme choice of the wave boundary condition (H_s = 5 m where z = +5.50 m NAP, cf figure 6)[1]. As figure 13d shows, during storm surge the erosion $\underline{\varepsilon}_s$ with exceedance probability $2.5 * 10^{-4}$ p.a. equals 36.5 m instead of 44.58 m, as found from the deterministic approach (table 3.1); this illustrates the fact that the wave height H_s = 5 m is too high to be representative. Apart from this fact, it shows, that in the case non-intervention strategy is applied, the deterministic method gives values too low, especially for long return periods.

4. Discussion

The theory, developed in this paper is far from operational.

Further developments are in preparation in working groups of the Dutch Technical Advisory Committee on Water Defences. Questions which remain to be solved are:

a. The failure probability of a sea defence system, surrounding a protected area[2] should be less than the probability of a (super) surge, which the system in any case should be able to withstand[3].
   Standards will have to be made for the allowable risk to areas to be protected.

b. In the paper, only the probability of failure of the sea defence system in one cross-section is considered. Considering n cross-sections in a sea defence circle, the risk will be multiplied by a factor n, unless failure of one section implies failure of another. The length of the circle will affect the dimensions.

---

[1] It may be pointed out, that this wave height was originally a boundary condition for the storm surge barrier. Thus it is clear, that a "safe" value has been chosen. For dimensioning of dunes in the same region a less pessimistic estimate is usual. This illustrates that the accuracy and the result of a deterministic approach depends upon (subjective) "engineers instinct".

[2] This will be called a "sea defence circle".

[3] DELTACOMMITTEE (1960): to withstand: each surge 10^{-4} p.a.; failure probability: 8.10^{-6} p.a.; storm surge barrier project group (1979): failure probability storm surge barrier 10^{-7} p.a.

c. The assumption of failure of a dike by 12 wave run-ups is quite unsatisfactory: here soil mechanics and hydraulics (overflow discharge) will have to come into the picture.

d. With respect to dunes, the expectation and the variation of the width of spreading B (chapter 3.1 and 2^o) should replace eq. (3.3).
The failure mechanisms b, c and d (chapter 3.2.1) should be considered in more detail.

Considering the method as a whole, it may be pointed out that inaccuracy or uncertainty is translated into extra dimensions of the sea defence system. This gives an operational tool for steering coastal research, as the benefit of this research can be rated and wighted against the costs.

5. Conclusions

a. The probabilistic method is more consistent than the deterministic method.
   . Failure probability may differ considerably for various constructions, when using the same deterministic standards.

b. The probabilistic approach offers more opportunity for taking "hidden safeties" into account,
   . the unvertainty in structural strength, including the effect of various strategies of maintenance;
   . a better comparison is found for the safety of dunes and dikes.

c. The method gives a general approach to the goal pursued: safety for the hinterland.
   . it gives a better insight in the relationships between the various failure mechanisms and better evaluation of the failure mechanisms itself;
   . one is obliged to trace non-technical failure mechanisms.

d. The financial value of accuracy and maintenance can be determined. This gives a tool for steering coastal research.

6. Acknowledgement

This paper is based upon a report of a preliminary working group of the Dutch Technical Advisory Committee on Water Defences (1979).
The authors wish to thank the other contributers of this working group.
Also the effect of discussions in other groups is gratefully acknowledged.

References

- Bakker, W.T. (1980). Prob. design of dunes, acting as sea defence. Rijkswaterstaat, Department Coast and Sea, Nota WWKZ-80.V013.

- Battjes, J.A. (1972). Stat. eig. stat. processen. Ingenieur 84, nr. 27.

- Ciria (1976) Rationalisation safety factors struct. code. Rpt. 63.

- Deltacommittee (1960). Rept. Deltacommissie. Staatsdr. & Uitg. Bedr.

- Graaff, J.A. v.d. (1977). Dune erosion during storm surge. Coastal Engineering 1, nr. 2.

- T.A.W. (Technical Advisory Committee on Water Defences) (1972a). Wave run-up and wave overtopping.

- T.A.W. (1972b). Guidel. for Calc. of Dune Erosion during Storm cond.

- T.A.W. (1979). Report Prel. Group Probabilistic Method.

- Vellinga, P. (1978). Model tests on dune erosion. Proc. 16th ICCE Ch. 132.

- Vrijling, J.K. and Bruinsma, J. (1980). Hydraulic Boundary Conditions Symposium Hydraulic Aspects of Coastal Structures.