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Contents

OPTIMUM FIRE RESISTANCE

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ABSTRACT

Essentially the dimensioning of casings for the fire protection of steel structures is an economical optimization problem: if more insulation material is used the direct cost increases but on the other hand the loss expectation decreases. From this point of view two problems will be discussed in this paper: the so-called fundamental case from reliability theory and the centrically loaded Euler column. Results and conclusions may be useful for practical applications as well as further research.

NOTATION

C_T	total costs
C_I	cost of insulation
C_F	cost of failure due to fire
C	cost of insulation per unit thickness
r	ratio of failure cost to insulation cost
d	thickness of insulation material
$P(A)$	probability of event A
$\mu(x)$	mean value of the random variable x
$\sigma(x)$	standard deviation of the random variable x
$\sigma^2(x)$	variance of the random variable x
x_k	characteristic value of x (here always $x_k = \mu(x) \pm 1.64\sigma(x)$)
t	time
t_d	time of fire duration
t_o	a constant in the ISO fire curve
q	fire load density
D	fire duration at unit fire load
T	steel temperature – flash over
T_o	a constant in the ISO fire curve
T_c	temperature in the fire compartment
$\overline{T}_c$	time average of T_c
λ	thermal conductivity of insulation material – slenderness ratio
c_s	specific heat of steel
ϱ_s	mass density of steel
U/F	ratio of fire exposed surface (per unit length) to volume of steel (per unit length)
β	a random variable which expresses the uncertainty in t_d at given q
s_y	yield stress or 0.2 strain limit
s_p	limit of proportionality
E_0	modulus of elasticity
E_t	tangent modulus of elasticity
A_{ij}	coefficients in interpolation formulas relating material properties and temperature
α_i	a random variable which expresses the uncertainty of a material property at given temperature
L	external load
R	resistance
z	residual strength
F	steel area – failure
γ	factor of safety
u_0	initial deflection of the column
f	flange thickness of HE-section

Optimum fire resistance

1 Introduction

Often steel structures are protected against fire by a cladding or casing of insulation material. The dimensioning of the casing can be conceived as an optimization problem: if more insulation material is used the direct cost increases, but on the other hand the loss expectation decreases. For some value of the insulation thickness the sum of direct costs and loss expectation is a minimum.

The optimization problem stated can only be solved from the statistical point of view. The loss expectation is the product of the failure cost, the probability of fire and the probability of failure under fire conditions. To determine the failure probability it is necessary to treat most variables as random variables.

Of course, considerations of this type are seriously hampered by the lack of sufficient statistical data. This, however, should be a very bad reason to start thinking in a deterministic way. One can better follow a Bayesian approach (ref. [2], [3]) and attempt to quantify the uncertainties in the (statistical) parameters. In doing so it will be found necessary to introduce subjective judgements. However, in a deterministic procedure this is unavoidable too. Then, a Bayesian approach should be preferred, because the subjective elements can be introduced in a purer and more explicit way.

This paper gives a short description of a study on this subject carried out by two students as graduation work. The complete results are reported in ref. [1] (Dutch).

2 Formulation of the optimization problem

For a given structural member the optimization problem can be formulated mathematically as follows:

minimize

$$C_T = C_I + C_F \cdot P(F|T) \cdot P(T) \tag{1}$$

C_T = total cost
C_I = cost of insulation material
C_F = cost of failure due to fire
$P(T)$ = probability of flash-over
$P(F|T)$ = probability of failure due to flash-over.

The parameter to be varied is the insulation thickness d. It will be assumed that the relationship between the direct cost C_I and the insulation thickness d can be described by:

$$C_I = C \cdot d \tag{2}$$

C = cost of insulation material per unit thickness

It is usual (see for example ref. [4], [5]) to write the failure cost as:

$$C_F = r \cdot C_I$$

The ratio of direct cost to failure cost r is an indication of the relative importance of the structural member under consideration. The value of r has to be estimated for every special application. To give some idea, three numerical values are written down in Table 1.

Table 1 Values for r.

classification	r	example
small losses	500	roof girder
medium losses	5,000	frame member in medium-size building
high losses	50,000	column in high-rise building

The probability of a flash-over (real fire) depends to a great extent on the way the building is used (see for instance ref. [6], chapter 1). For this study the flash-over probability will be put at half a percent in the lifetime:

$$P(T) = 5 \cdot 10^{-3} \tag{4}$$

In formula (1) the most difficult quantity to determine is the probability of failure under fire conditions. It is with this problem that the next parts of this paper will deal.

3 The temperature-time relationship

In deterministic analysis the ISO standard fire curve (ref. [6]) is often used to describe the temperature-time relation in the fire compartment. Graphically this curve is represented in Fig. 1. The formula:

$$T_c = T_o \log\left(\frac{t}{t_o} + 1\right) \quad \text{where} \quad 0 < t < t_d \tag{5}$$

T_c = temperature in the fire compartment
t = time, measured in seconds
t_d = time of fire duration
T_o = 345 °C (a constant)
t_o = 7.5 s (a constant)

For practical purposes the part $t > t_d$ can be neglected.

The fire duration t_d depends on the amount of combustible material present in the fire compartment. Experiments have shown (ref. [6]) that the following rule is reasonably accurat:

$$t_d = D \cdot q \tag{6}$$

where

q = fire load per unit floor are (J/m^2)
$D = 3.6 \cdot 10^{-6}$ sm^2/J (a constant)

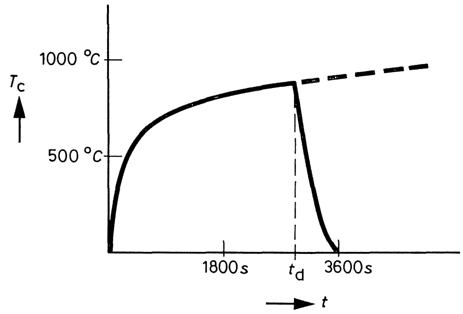

Fig. 1. ISO standard fire curve.

Given the external temperature-time-relationship the steel temperature in an encased beam or column can be calculated from the differential equation (ref. [1], [6]):

$$\frac{dT}{dt} = (T_c - T)\frac{\lambda}{c_s \varrho_s d}\left\{\frac{U}{F}\right\} \tag{7}$$

T_c = temperature in the fire compartment
T = steel temperature
t = time
λ = thermal conductivity of insulation material
c_s = specific heat of steel
ϱ_s = mass density of steel (7800 kg/m^3)
d = thickness of insulation material
U/F = ratio between surface exposed to fire (per unit length) and volume of steel (per unit length).

The ratio U/F is often referred to as the section factor. The coefficients λ and c_s generally depend on the temperature.

Fig. 2 presents a schematic diagram of the resultant steel temperature curve.

It will be clear that this way of determining the steel temperature is too cumbersome in the statistical analysis we have in mind to do. Therefore, some symplifying assumptions will be made. First of all we will suppose that the temperature in the compartment does not vary with time. As a value the average temperature according to the ISO-curve and the fire duration will be adopted (Table 2).

Table 2 Average room temperature versus fire duration.

fire duration t_d	900	1800	2700	3600	5400	7200	9000	10800	[sec]
average temp. $\bar{T}_c$	600	680	740	770	840	890	910	930	[°C]

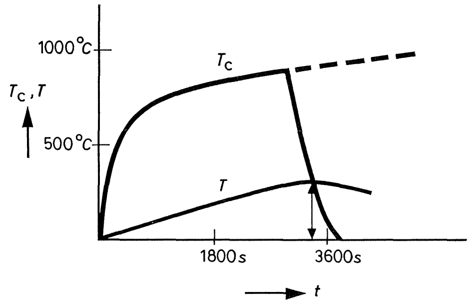

Fig. 2. Room and steel time-temperature relationship.

Furthermore we will assume λ and c_s to be temperature independent:

$$\lambda = 0.07 \text{ J/ms °C} \qquad c_s = 500 \text{ J/kg °C}$$

Now the solution of the differential equation (7) can be written explicitly:

$$T = \bar{T}_c \left\{ 1 - \exp\left(- \frac{\lambda}{c_s \varrho_s d} \cdot \frac{U}{F} \cdot t \right) \right\} \tag{8}$$

Calculations have shown that the errors introduced by these approximations are fully acceptable (ref. [1]).

Most variables introduced so far can be regarded as deterministic. An obvious exception is the fire load density q. Suppose q has a probability density function as indicated approximately in Fig. 3. The mean value of q is $\mu(q)$, the standard deviation $\sigma(q)$ (Numerical values will be assigned later on.) For our immediate purpose the exact type of the distribution does not matter.

In Fig. 3 the characteristic fire load density q_k is also indicated. A characteristic value is an unfavourable value with some low probability (2 or 5%) of being exceeded. Throughout this paper the characteristic value of any particular stochastic variable x will always be defined as:

$$x_k = \mu(x) \pm 1.64\sigma(x) \tag{9}$$

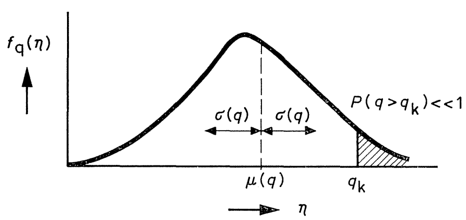

Fig. 3. Probability density function for the fire load density q.

If x has a normal distribution the probability of getting for x a more unfavorable value than x_k equals 5%.

Let us next consider the temperature-time relationship. Clearly, the formulas (5) and (6) only give a very rough approximation of reality. For example, the influence of the ventilation conditions and the nature of the fire load have been left out of consideration. Of course, it is very difficult to deal with those influences in a general way. But even in special cases there will remain many uncertainties: will the cupboard be open or closed at the time of the fire, will the room be in disorder or not, how long will it take for the fire brigade to arrive? etc.

A possible way out of these difficulties is to introduce a random variable which has to take into account all the uncertainties. In the study described here the following solution has been chosen:

- the average temperature in the compartment $\overline{T}_c$ is determined from the characteristic fire load density q_k, formulae (6) and Table 2
- the duration of the fire t_d to be used in equation (8) is given by:

$$t_d = \beta D q \tag{10}$$

β is the proposed random variable. It seems rasonable to take its mean value as equal to 1. In fact by doing so we say that equation (6) is correct on the average. The standard deviation will be taken as 0.3. This choice is a rather arbitrary one and should be considered as a personal judgement of the author on the uncertainties involved.

4 Material properties at high temperatures

As a consequence of the rise in temperature a reduction takes place in the material properties. Fig. 4 gives a number of stress-strain-relations for several temperatures (based on experiments [7]). The steel quality is Fe 360. The curves in Fig. 4 represent mean values, which accounts for the 280 MN/m^2 yield stress at $T = 0\,°C$.

It should be noted that the stress-strain-temperature relationship is not a unique one. So there is a difference between first raising the temperature and imposing the stress afterwards and vice versa. The former case is called active deformation, the

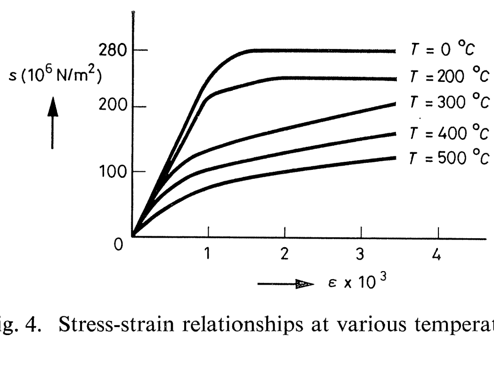

Fig. 4. Stress-strain relationships at various temperature levels.

latter case passive deformation. Fig. 4 gives passive s-ε-relations. Under fire conditions most deformations are passive.

In Fig. 5 the stress-strain relations have been drawn schematically. To establish a stress-strain-diagram at a given temperature, the following parameters can be chosen:

s_y = yield stress or 0.2 proof stress
s_p = limit of proportionality
E_0 = modulus of elasticity
E_t = tangent modulus of elasticity

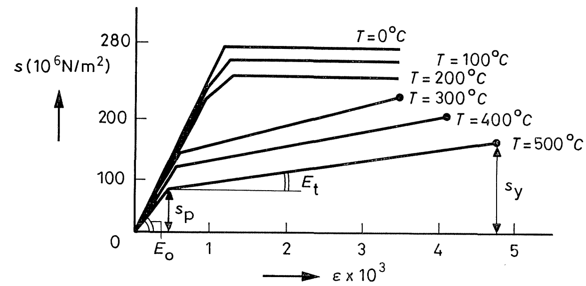

Fig. 5. Schematised stress-strain relationships.

For these quantities the following interpolation formulas will be used:

$$
\begin{aligned}
s_y &= \alpha_1(A_{11}+A_{12}T+A_{13}T^2)\\
s_p &= \alpha_2(A_{21}+A_{22}T+A_{23}T^2)\\
E_0 &= \alpha_3(A_{31}+A_{32}T+A_{33}T_2)\\
E_t &= \alpha_3(A_{11}+A_{22}T+A_{33}T^2)
\end{aligned}
\tag{11}
$$

where the A_{ij} are given by Table 3.

Table 3 Coefficients A_{ij}.

mat. constant	temp. domain	A_{i1} [N/m²]	A_{i2} [N/m² °C]	A_{i3} [N/m² °C²]
s_y	all T	$280\cdot10^6$	$-\ 130\cdot10^3$	-300
s_p	$T\leq 300\,°C$	$240\cdot10^6$	0	-100
s_p	$T>300\,°C$	$420\cdot10^6$	$-1200\cdot10^3$	$+100$
E_0	all T	$210\cdot10^9$	0	$-260\cdot10^3$
E_t	$T\leq 300\,°C$	$210\cdot10^9$	$-\ 617\cdot10^6$	0
E_t	$T>300\,°C$	$64\cdot10^9$	$-\ 175\cdot10^6$	$+150\cdot10^3$

For $\alpha_i = 1$ the formulas (11) correspond exactly to the diagrams in Fig. 5. The coefficients α_i serve to express the scatter in the material properties at a given temperature.

Table 4 Statistical properties of the coefficients α_i.

mat. constant	α_i	$\mu(\alpha_i)$	$\sigma(\alpha_i)$ at $T=0\,^\circ$C	$\sigma(\alpha_i)$ at $T=500\,^\circ$C
s_y	α_1	1	0.08	0.25
s_p	α_2	1	0.08	0.25
E_0	α_3	1	0.05	0.15
E_t	α_4	1	0.05	0.20

Of course little information is known about this scatter. Therefore, it is necessary to make some estimates. Table 4 gives values for $T = 0\,^\circ$C and $T = 500\,^\circ$C. For intermediate temperatures linear interpolation is adopted. The increasing standard deviations at high temperatures must also take into account the uncertainty in the mean values.

Note

When using Table 4 a difficulty may arise from the fact that T is itself a random variable. For simplicity the standard deviation of the material constants will always be determined at the mean temperature.

5 The fundamental case

As a first application a simple truss member, loaded in tension, will be optimized (Fig. 6). In reliability analysis such a tension member is referred to as a fundamental case. Characteristic for a fundamental case is the presence of only one stochastic strength parameter and one stochastic loading parameter.

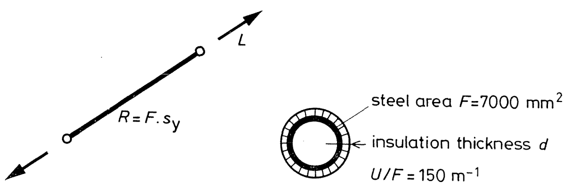

Fig. 6. The fundamental case.

The mean value and the standard deviation of the loading parameter L will be chosen as:

$$\mu(L) = 820 \text{ kN} \qquad \sigma(L) = 164 \text{ kN}$$

It will be assumed that under fire conditions the same load is present as under normal conditions (in reality some reduction might be possible).

The strength parameter R can be written as:

$$R = F \cdot s_y(T) \tag{12}$$

The steel area F will deterministically be fixed at 7000 mm^2. This value has been selected in such a way that under normal conditions the safety factor λ equals 1.5, where λ is defined as the ratio between the characteristic values of R and L. For the fire load density q the following statistical parameters will be adopted:

$$\mu(q) = 670 \text{ MJ/m}^2 \qquad \sigma(q) = 200 \text{ MJ/m}^2$$

Finally the section factor U/F will be fixed at 150 m^{-1}.

Now all the necessary data have been evaluated and the real analysis can begin. As a first step we concentrate on the strength R. From (8), (10), (11) and (12) it follows that R depends on three stochastic parameters, namely q, β and α_1:

$$R = R(q, \beta, \alpha_1) \tag{13}$$

So R is a stochastic variable itself. Following a mean-value first-order approximation (ref. [2]) the mean and variance of R can be determined from:

$$\mu(R) = R(\mu(q), \mu(\beta), \mu(\alpha_1)) \tag{14}$$

$$\sigma^2(R) = \left|\frac{\partial R}{\partial q}\right|^2 \cdot \sigma^2(q) + \left|\frac{\partial R}{\partial \beta}\right|^2 \cdot \sigma^2(\beta) + \left|\frac{\partial R}{\partial \alpha_1}\right|^2 \cdot \sigma^2(\alpha_1)$$

The partial derivatives must be evaluated at the point $q = \mu(q)$, $\beta = \mu(\beta)$, $\alpha_1 = \mu(\alpha_1)$.

Let us now define the residual strength z:

$$z = R - L \tag{15}$$

With the aid of z the required failure probability can be written as:

$$P(F|T) = P(z < 0)$$

In order to calculate this probability $P(z < 0)$ it is necessary to know the full statistical distribution of z. But so far we have not made any statement concerning the distribution type of the stochastic variables involved. Therefore, the distribution of z is also indeterminate. However, the failure probabilities are found to be of the order of magnitude of 10^{-2} to 10^{-3}. Fortunately in that region the failure probabilities are not very sensitive to the distribution types. So without introducing too large errors we may assume for z a normal distribution. The mean value and the standard deviation of that distribution can be found from:

$$\mu(z) = \mu(R) - \mu(L)$$

$$\sigma^2(z) = \sigma^2(R) + \sigma^2(L)$$

It is possible now to evaluate the failure probability as a function of the insulation thickness d (Fig. 10). The next step is the determination of the total cost for several values of the loss parameter r. The results are given in Fig. 7. The optimum insulation thickness d is seen to equal 10, 18 and 27 mm for $r = 500$, 5000 and 50.000 respectively. As a first conclusion it can be stated that the influence of the loss parameter is significant.

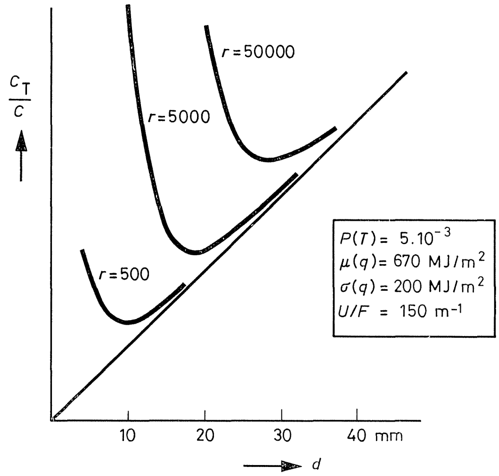

Fig. 7. Results for the fundamental case.

For the optima the failure probabilities prove to be proportional to the inverse of the product $r \cdot P(T)$:

$$P(F|T)_{\text{opt}} = \frac{1}{4r \cdot P(T)} \tag{16}$$

Calculations have shown [1], that the optimum failure probability (as opposed to the insulation thickness) is hardly sensitive to variations in the fire load and the section factor. This can be regarded as an interesting conclusion.

For the benefit of a deterministic analysis the corresponding factors of safety have also been determined. Starting from a characteristic fire load density the steel temperature could be calculated from (8). Next the strength under fire conditions was obtained by reducing the nominal strength in the same way as the mean strength was reduced in the statistical analysis.

The resulting safety factors (ratios of deterministic strength to characteristic load) are given in Table 5.

Table 5

r	γ_{opt}
500	0.85
5,000	1.15
50,000	1.30

A perhaps somewhat surprising result is the fact that for $r = 500$ the optimal safety factor is smaller than 1. However, it should be understood that values of γ smaller than 1 by no means quarantee failure. It is only an indication of a relatively high probability of failure (say larger than 1%). And if the economic consequences of the failure are small, as is the case for $r = 500$, such a high failure probability can turn out to be optimal.

6 The Euler column

With regard to the reliability under fire conditions special attention should be paid to columns in high-rise buildings. In the first place, a column failure will produce very high losses in general. Secondly, the failure mechanism is more complicated.

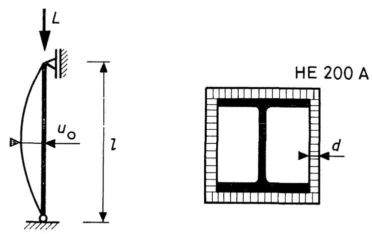

Fig. 8. The Euler column.

We will analyse the HE 200 A column shown in Fig. 8. With respect to the load L similar assumptions will be made as in the fundamental case. In Fig. 9 the relationship is given between the characteristic load L_k and the slenderness ratio λ. Fig. 9 is in accordance with the Netherlands building regulations (T.G.B. 1972).

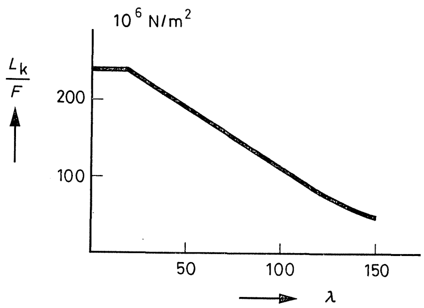

Fig. 9. Euler stress versus slenderness ratio λ.

It is a well known phenomena that members under compression never reach their theoretical maximum load-bearing capacity. (Euler load or, for small slendernesses, yield load) because of the inevitable disturbances. Disturbances may be present in many forms: eccentricities, initial deflections, inhomogenities, residual stresses, etc.

In this paper (as often in stability analysis) an artificial deflection will be introduced to represent all disturbances simultaniously. Of course this initial deflection u_0 must be regarded as a random variable. The mean value of u_0 will be fixed at $\mu(u_0) = 7$ mm for $\lambda = 50$ and $\mu(u_0) = 16$ mm for $\lambda = 160$. For intermediate values linear interpolation will be used. The standard deviation is estimated at $\sigma(u_0) = 0.3\mu(u_0)$.

Another important statistical parameter in the case of HE-A-columns is the flange thickness f. For the HE 200 A section the values $\mu(f) = 10$ mm and $\sigma(f) = 0.65$ mm seem reasonable (ref. [9]).

Starting from all the assumptions made so far, satisfactory agreement with the CECM test results (ref. [9], [10]) has been obtained (ref. [1]).

During a fire the rise in temperature will cause strain increases. These increases are passive. However, due to second order effects stress variations will occur and so active strain increases will be present too. Nevertheless we will solely use the passive s-ε-relations from Fig. 5. Furthermore we will consider these s-ε-relations as being purely elastic. In [1] has been shown that the errors introduced in this way are very small.

The above assumptions make it possible to reverse the problem: instead of looking for the critical temperature at a given load we can look for the maximum load-bearing capacity at a given temperature. From the arithmatical point of view this is advantageous.

The load-bearing capacity of the column is a function of a great number of stochastic variables:

$$R = R(q, \beta, u_0, f, \alpha_1, \alpha_2, \alpha_3, \alpha_4) \tag{18}$$

Analogous to the fundamental case the failure probability can be evaluated for a given insulation thickness d. In Fig. 10 results have been drawn for two different slenderness ratios: $\lambda = 60$ and $\lambda = 90$. (Input data such as fire load and section factor have been kept the same as in the fundamental case). From Fig. 10 it can be concluded

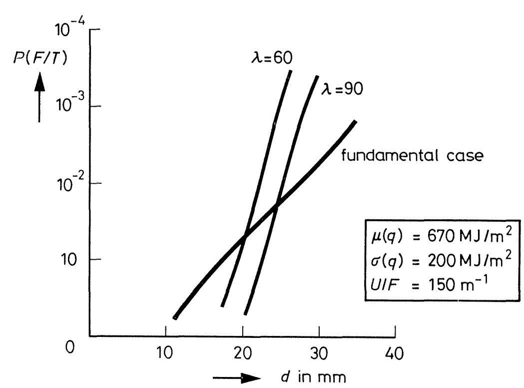

Fig. 10. Results for the Euler-column.

that the column is more sensitive to variations in d than the fundamental case is. This fact has two consequences:

- For the column the optimum insulation thickness is much less sensitive to the loss ratio r. For a slenderness ratio $\lambda = 60$ the loss parameters $r = 500$, 5000, 50000, result in $d = 18$, 20 and 24 mm respectively.

- The optimum failure probability is lower in the column case:

$$P(F|T)_{\mathrm{opt}} = \frac{1}{16rP(T)} \tag{18}$$

In Fig. 11 the optimum factors of safety are given. In contrast with the optimum failure probability the safety factor is not independent of the slenderness ratio. Both $P(F|T)_{\mathrm{opt}}$ and γ_{opt} prove to be rather insensitive to variations in fire load and section factor (ref. [1]).

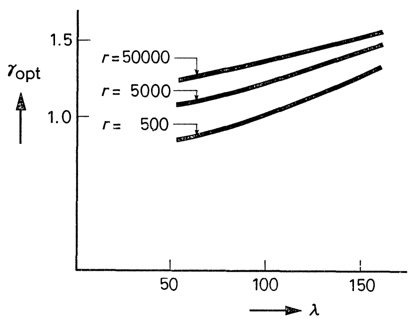

Fig. 11. Optimum load factor versus slenderness ratio.

To test the influence of the various assumptions made, alternative calculations were carried out (different standard deviations for the material properties, correlation between the material properties, different statistical parameters for β etc.). It has been shown ([1]) that the general picture of the results is not seriously affected by these variations.

7 The composition of the variance

Formula (14) gives the variance of the strength R as the sum of three terms. The first term comes from the fire load uncertainty, the second from the uncertainty in the fire duration and the third from the scatter in the yield stress. In Table 6 the contribution of these three terms is given. For high r values the yield-stress-part dominates. For lower values of r (thin insulation, high steel temperature) the significance of the q and β contributions increases.

Table 6 Composition of the variance of the strength R (fundamental case).

	$r = 500$	$r = 5{,}000$	$r = 50{,}000$
contribution of q	25 %	10 %	6 %
β	25 %	10 %	6 %
α_1	50 %	80 %	88 %

Table 7 gives the composition of the column strength variance for several slenderness ratio λ. As the influence of r is less important, the results are only given for $r = 5{,}000$. The scatter in the material properties (coefficients α_1 to α_4) proves to be of little significance in most cases. Only the yield stress makes some contribution in the small slenderness domain.

For slender columns the uncertainties in the temperature-time-relationship dominate.

In conclusion one could say that further research must concentrate on the statistical properties of the yield stress for moderate temperatures. Research on the temperature-time-relationship is valuable for slender columns and for low loss ratios.

Table 7 Composition of the variance of the strength R (Euler column, $r = 5000$).

	$\lambda = 50$	$\lambda = 70$	$\lambda = 90$	$\lambda = 120$	$\lambda = 150$
contribution of q	18 %	26 %	29 %	28 %	32 %
β	18 %	26 %	29 %	28 %	32 %
u_0	4 %	11 %	10 %	10 %	3 %
f	13 %	15 %	15 %	15 %	13 %
α_1	42 %	11 %	1 %	0 %	0 %
α_2	4 %	9 %	12 %	6 %	1 %
α_3	0 %	1 %	3 %	13 %	19 %
α_4	1 %	1 %	1 %	0 %	0 %

8 Comparison with Netherlands code requirements

According to Netherlands regulations the characteristic fire load density must be increased by 500 MJ/m^2 for low-rise buildings and by 1000 for high-rise buildings (dwellings only). The increase is meant to allow for the material losses. This way of design will be compared with the method outlined in this paper. In Table 8 the required insulation thickness for low-rise buildings is compared with $r = 5000$ and the high-rise buildings with $r = 50{,}000$.

Table 8 Comparision of the optimal insulation thickness with Dutch code requirements.

	low-rise buildings		high-rise buildings	
	optimum	code	optimum	code
fundamental case	$d = 18$ mm	23	28	34
column $\lambda = 50$	20	27	22	39
column $\lambda = 90$	23	34	27	48
column $\lambda = 120$	23	22	25	32

The code requirements prove to be rather conservative. Especially for $\lambda = 90$ there seems to be a clear case of over-design.

Note

When reading the figures in Table 8 it should be borne in mind that no load reduction has been applied.

Conclusions

1. It is possible to determine from economic criteria an optimum insulation thickness for the so-called fundamental case and for a centrically loaded Euler column.
2. In the fundamental case the optimum insulation thickness is heavily influenced by the loss ratio. The column is much less sensitive.
3. The failure probability corresponding to the optimum is proportional to the inverse of the loss ratio r and the flash-over probability $P(T)$. For columns the optimum failure probability is less than for the fundamental case. The slenderness ratio has no influence (for $\lambda > 50$).
4. The optimum safety factor depends on the loss ratio r and (for columns) on the slenderness ratio.
5. The fire load density and the section factor hardly affect the optimum failure probability and safety factor.
6. The most important statistical parameters are the mean value and the standard deviation of the yield stress. Especially in the low temperature range (say $T < 350\,^\circ\mathrm{C}$) more data should be extremely useful.
7. More insight into the temperature-time-relationship is important for low loss ratios and for slender columns.
8. For practical applications, information will be needed to evaluate the parameters r (loss ratio) and $P(T)$ (probability of flash-over).
9. Netherlands regulations require for columns a much thicker insulation than the economic criteria stated in this paper do. For the fundamental case the difference is much less.

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