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MEANDERING OF SMALL STREAMS IN ALLUVIUM

by

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SYNOPSIS

Investigations were made at the Hydraulics Research Station of channels which achieved a meander pattern in alluvium through natural processes at discharges between 0.2 and 3.2 cusecs. These followed an earlier programme in which the behaviour of straight channels was studied. Discharge, sediment load, hydraulic gradient and channel geometry were measured, and empirical correlations of the stream geometry with discharge and sediment load were deduced.

The movement of eroded bank material, the migration of meanders and the effect of sediment load on meander length were studied. It emerged that channels with small sediment loads remained straight and meandering occurred naturally at higher sediment loads. The ultimate pattern was also shown to be influenced by the initial channel condition. Flow resistance and channel shape were compared with a straight channel when about 60 percent of the energy loss was found to be due to the non-prismatic boundary of meandered channels. At the threshold of meandering, losses due to the development of shoals were found to be 25 percent of the total.

Dimensional analysis elucidated that no simple relation between meander length and discharge is completely satisfactory and none of the many empirical relations between meander length and discharge has any special significance.

A laboratory investigation of variable discharge hydrographs and a study of field discharge data revealed that the bank-full discharge or a flood with a recurrence interval of about a year, whichever is the lesser, generates the plan geometry.

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MEANDERING OF SMALL STREAMS IN ALLUVIUM

INTRODUCTION

Natural channels rarely remain straight for distances greater than about ten channel widths, (30), and thus it would appear that a straight alignment is unusual under natural conditions in erodible material. Inherent in the majority of, if not all, streams are forces which oppose the maintenance of a straight channel and may cause a meander or braided pattern to develop.

Although meandering channels have been studied in both field and laboratory for many years and numerous hypotheses have been put forward concerning their causation, there exists today no generally accepted explanation for their development. Werner (53) has summarised the early history from Von Baer who in 1858 suggested that the rotation of the earth produced secondary currents which caused unequal bank erosion and hence the development of meanders. Local disturbances due to variations in bank material, obstructions, tributary confluences etc., have also been put forward as the cause of meandering by many authors, including Penck in 1894, Davies in 1903, Hedin in 1904 and Vujevic in 1906. Jefferson (24) in 1902 and Schoklitsch in 1930 attributed the origin of meanders to excessive energy in a stream, and in 1936 Russel discussed the effect of changing river stage

as a possible cause. The existence of secondary currents in a stream has often been suggested as the cause of meanders, by Thomson in 1876[48], Hamburg in 1903, de Geer in 1911, Stanley in 1881, Exner in 1919 and Tanner [47] in 1960. Fujiyoshi in 1950[18] put forward an explanation based on the development of convection currents, and Scheijegger[40] has suggested a thermodynamic analogy. Werner[53] in 1951 suggested that a transverse seiche in an initially straight channel could initiate meandering. The concept of transverse oscillations was also the basis of Anderson's[5] work reported in 1967.

Another concept that has come to the fore in recent years recognised the dependence of stream geometry on sediment transport: Callander[9], for example, has very recently considered the stability of the resistance and sediment transport function. Schumm has suggested a dependence also on the percentage of fine sediments in the bed material.

In spite of the various hypotheses, however, the true cause is still obscure. There exists no generally accepted means of predicting, under given conditions of fluid and sediment flux, whether meanders will occur, although there is considerable empirical information which is of value in predicting aspects of the geometrical pattern which might result. There remain considerable differences of opinion about the types of streams which meander. Werner, for example,[53], states that meandering streams are notoriously shallow and that meandering does not occur if the water depth exceeds a certain value. He also states that a certain bed load is necessary to initiate meandering and that superfluous silt content may prevent meandering. Schumm[42], on the other hand, believes that relatively wide shallow channels tend to be straight whereas relatively narrow, deep channels tend to meander, but he also concluded that silt content affects the

geometry [41,43].

Leopold and Wolman[30] agree with Werner, suggesting that in channels deep in relation to width the velocity is nearly constant with depth, hence helical circulation becomes negligible and in curved channels potential flow approximates. This results in a variation of downstream velocity inversely with the radius of the stream lines; the thread of high velocity hugs the convex bank and does not cross the channel. Thus a mechanism for the formation of shoals is absent.

Matthes[33], however, states that helicoidal flow takes place in channels deep in relation to width and he believes that meandering channels are shallow and devoid of helicoidal flow. There was no evidence of helicoidal flow in the Mississippi or other river examined by him.

Bagnold[6] has shown that the resistance to flow in a rigid channel of uniform cross section falls sharply to a defined minimum within a narrow range of $2 < \frac{r}{b} < 3$. In a naturally occurring meandering channel the cross section is not uniform but it has been found by Leopold and Wolman[30] that the majority of river curves have a ratio of curvature radius to channel width between 2 and 3. On the other hand, Leopold et al[32] have concluded that there is no doubt that braided or meandering channels offer larger resistance to flow than otherwise comparable straight channels. This is supported by Blench and Qureshy[7] who have stated that meandering channels of low sinuosity require about 1.5 times the slope of comparable straight channels and very convoluted ones require about 2.5 times this value.

Inglis[21,22] suggested that there were two classes of meandering. In what he called "primary meandering" the excess charge from upstream was said to cross to the convex bank, whereas in secondary meandering the excess charge, being developed in the meander embayment by bank erosion, would deposit as a shoal along the same bank. Friedkin[17]

investigated the movement of material eroded from the bank and concluded that it was deposited further downstream on the same side of the stream as that from which it was eroded, when the banks eroded rapidly, but that when the banks did not erode the sediment crossed from one bank to the other. This deposition of eroded material on the same side was confirmed by White[55].

It was suggested in a previous paper by one of the present Authors[12] that the sediment charge is an important factor in the development and maintenance of a regular sinuous channel. From the previous investigation of the geometry of straight channels it appeared that a low sediment content inhibited the process of meandering in an initially straight channel, but it nevertheless appeared that the mechanism leading to the development of a meander pattern existed in the initially straight channel. Both Friedkin[17] and Matthes[33] believe that bank cutting and the transfer of sediment to a point of deposition on a shoal are principal requirements in the formation of meanders. While agreeing with these very necessary stages in the development of meanders, it is suggested that these are effects rather than original causes of a sinusoidal component of motion, and that until a certain amount of sediment is in movement in shoals on alternate banks erosion of embayments does not occur. This was demonstrated in this series of investigations by the formation of shoals and deeps on alternate sides along a generally straight channel, followed by their steady growth until, almost simultaneously along the length of the channel, bank erosion began and the meander loops developed.

If bank erosion were the initial cause of regular meandering then an obstruction in a straight channel causing erosion of one bank would be expected to produce a pattern of regular sinuous curves. The second author in experiments some years ago with obstructions and low sediment discharges

observed that a series of curves of gradually diminishing amplitude resulted, suggesting a tendency to revert to a straight channel. In addition, such a series of eroded embayments beyond an obstruction developed progressively downstream and not simultaneously along the length of channel.

Some of the confusion which occurs in this field may be due to the lack of any agreed method of classifying meanders. From the results of the present and previous investigations, therefore, it would appear necessary to differentiate between:

(i) conditions which may lead to the erosion of a bank at one or more points resulting in an irregular channel for a limited distance (e.g. non-uniformity of channel bank, obstruction, etc.), and

(ii) conditions necessary for the development and maintenance of a regular sinuous channel in a homogeneous alluvium.

It is the second condition only which is considered in this report.

Definitions and terminology

Nearly all channels in erodible material exhibit some tendency to develop curves, and attempts have been made to define a meandering channel on the basis of symmetry of the meander loops, degree of sinuosity and so on. However, due to the effect of the lack of homogenity of bed and bank materials of natural streams, irregular patterns frequently occur. Schumm[42] classified channels into five groups: (i) tortuous, (ii) irregular (iii) regular (iv) transitional and (v) straight. The experimental laboratory channels established in homogeneous, cohesionless sand in the present investigation have been grouped as follows:

(a) straight or slightly irregular channels;
(b) regular sinuous or meandering channels, and
(c) braided channels (i.e. multi-thread systems divided by islands or shallows).

Channel width b is the width of a straight or meandering channel, which in the present context develops by bank erosion from a channel initially too narrow.

Cross-over is that part of the meandering channel where the flow crosses from one bank where the flow is of one sense (clockwise or counter-clockwise) to the other bank where the sense of rotation is reversed.

Freeboard is the height between the water surface and the top of the channel bank.

Meander belt width β is the width of a belt between parallel lines which contain the meandering channel.

Meander length λ is the distance measured along the axis of the valley between two corresponding points on successive meanders. A meander therefore consists of two embayments or loops in one of which the flow is clockwise and in the other counter-clockwise.

Sinuosity is a measure of the deviation of a meandering channel from a straight line and is the ratio of the length of the thalweg to the straight line distance along the axis of the valley. The definition is usually reserved for the more symmetrical meandering channels.

Straight line water surface slope and straight line bed slope are the average gradients of the water surface and the stream bed respectively along a straight line between two points. They both have the same value as the valley slope if the channel is neither aggrading nor degrading.

Thalweg is the line followed by the deepest part of the main waterway of a non-prismatic channel.

Thalweg water surface slope is the average gradient of the water surface between two points following the thalweg, i.e. the curved path of the deep water channel.

Threshold sediment charge is, in the present context, the minimum sediment transport rate for a given sand and discharge necessary for the development of meanders.

Tortuosity describes the degree of sinuosity of a river channel and is most frequently used for the channel with a large sinuosity, wide meander belt and the less regular meander patterns of natural rivers.

Valley slope is the slope of the surface of the sand in the general direction of flow.

SCOPE OF INVESTIGATION

Much of the work available at the moment on meandering streams is either the analysis of natural rivers, whose discharges range from a few tens to millions of cusecs (13, 21, 31, 43) or the observation of very small streams in laboratory flumes (9, 17, 38 et al) having a discharge rarely as large as one cusec. The present laboratory experiments were designed to bridge a part of the gap between these two fields of investigation by establishing, in effect, small natural streams in a large area of sand under closely controlled conditions.
A previous series of experiments in this area of sand has already been reported upon (1, 2), and the results will be referred to again in the present report where appropriate, although they related mostly to straight channels. Also, a limited series of experiments was conducted in an adjacent strip of coarser sand some years ago[20].

Of the many factors that may be considered to influence meanders, the discharge and the initial sediment transport rate were experimentally variable in these tests, but the

material of bed and banks could not be varied conveniently because of the very large quantities required. The density and viscosity of the water varied with seasonal temperature changes, but were not controlled: allowance was made for these changes in assessing certain of the results. The valley slope, of some influence in the development of meanders, was variable within confined limits, but the same value was used at the start of most of the tests.

The discharge was generally maintained constant, one of the aims of the investigation being to determine the relation between this parameter and the resultant meander shape, but some tests were made with the discharge continuously varied throughout to establish the effect of such a cyclical discharge compared with a steady flow.

After a first run for familiarisation with the equipment a series of steady discharges of 0.4, 0.7, 1.2 and 2.0 cusecs was used, combined with sand injection rates of 100, 150, 300 and 600 mg/litre (equivalent to parts per million by weight) to give sixteen tests. Three variable-discharge tests were also made, each having a hydrograph of 8 hour period and nearly sine wave form, the mean discharge being 0.7 cusec. In two of the variable discharge experiments this was modulated from 0.05 to 1.95 times the mean, combined with sand injection rates of 150 and 300 mg/litre in turn. In the third experiment, at 300 mg/litre the flow modulation was from 0.5 to 1.5 of the mean.

It is to be expected that the initial conditions will affect the stages of development of meandering in an initially straight channel in alluvium. Several initial and operating conditions are possible as follows:

(a) Sediment charge
  (i) below threshold for meandering;
  (ii) above threshold.

(b) Initial channel width
  (i) less than dynamically stable width of a meandered channel;
  (ii) more than this but less than the meander belt width;
  (iii) more than meander belt width.

(c) Valley slope
  (i) initially greater than the straight-line water surface slope for a stable meandered stream and maintained constant;
  (ii) as above but varied throughout the run by bank level adjustment to maintain a constant freeboard, i.e. valley slope adjusted to remain parallel to straight line water surface slope;
  (iii) initially less than the straight line water surface slope for a stable meandered stream and maintained constant;
  (iv) as above but varied throughout the run to maintain a constant freeboard.

A comprehensive coverage of the above initial and operating conditions would require twenty four different tests for every chosen discharge and the series would clearly be unreasonably prolonged. Thus, in the investigations described in this paper only four of the possible combinations of conditions were used for the various discharges, as follows:

	(a) Sediment charge	(b) Initial width	(c) Valley slope
Condition A	i	i	iv
Condition B	ii	i	iv
Condition C	ii	i	iii
Condition D	ii	ii	iii

In each case both the development and the final channel pattern were different. Under Condition A the channel

remained straight although it increased its width and shoals developed on alternate sides. Under Condition B a meander pattern developed as the bank levels were adjusted throughout to maintain a constant freeboard. When the valley slope was kept constant as in Condition C, the water soon overtopped the banks in the upper reaches as the gradient increased and a wide braided channel resulted. In the last case, the stream developed a meander pattern within the sinuous channel previously formed by a larger discharge but as the meander migrated and increased its slope it cut through the previously formed shoals and developed a wide braided pattern.

RESEARCH FACILITIES

The main series of experiments was made at the Hydraulics Research Station of the Ministry of Technology at Wallingford over a period of about two years. An open-air area 300 ft long by 34 ft wide filled with uniformly graded sand was used, this being in fact one of three sections of the experimental area. Plate 1 and Fig. 1 give a general picture.

Water supply was by shallow well from the local ground water, stored below sand surface level in a reservoir tank of about 16 000 cu ft capacity at one end of the area. A 5 cusec capacity horizontal axial flow pump moved water from this storage tank through a 15 in pipe to the head of the sand strip, where it was discharged into an expansion chamber with low velocity. The discharge to the stream was controlled firstly by altering pump delivery and secondly by means of a broad-crested weir discharging from the expansion chamber. A streamlined splitter block could be

moved across the weir crest, dividing the flow into the required fraction which passed to the experimental channel, the remainder of the near-constant pump delivery being taken in a by-pass channel directly back to the storage sump. For the majority of the tests at steady discharge, a convenient breadth was selected and the head over the weir adjusted to give the required flow. For the tests with continuously variable discharge, an electric drive was provided which traversed the flow splitter across the weir crest under the control of a cam representing the hydrograph shape, so varying the breadth of weir delivering flow to the test channel while the head remained constant.

The sand in which the experiments took place † was found by sieve analysis to have a median grain diameter of 0.15 mm, with 90 per cent lying between 0.1 and 0.23 mm diameter, see Fig. 2. Samples were taken from several locations early in the period of experiments to confirm this close grading, and at intervals during the experimental period to ensure that no sorting effect was present as the sand was re-worked. The sand bed was 1½ ft deep, underlain by 2 ft of coarser sand containing ground drains to prevent waterlogging; below this the existing sub soil had been compacted by sheep's foot rollers.

The ground drain flow was continuously metered over a V-notch weir back into the main supply well, but some overall loss of water from the sand area into the underlying ground water was suspected. Hence the outlet discharge from the experimental sand strip was not necessarily the difference between the metered inflow and ground water return.

A sharp crested rectangular weir (of multi-unit design in order to obtain the necessary head requirements of B.S.

† The same sand as used in the experiments of ref. 1, but after sieving to remove isolated gravel particles and elutriation to eliminate almost entirely particles below 0.06 mm.

3680: Pt 4A: 1965) was provided to measure outflow at the lower end of the sand strip. Immediately upstream of this weir was a large settling basin which effectively removed transported sand from the water before it was returned over the sharp crested weir to the main pump for re-circulation. The settling basin was preceded by a tilting tail gate for control of water surface level at the stream outlet.

A sand injector was provided at the upstream end of the area, feeding dry sand the same as that of the bed and banks, by free fall into a lined section of channel immediately downstream of the variable-breadth weir at the inlet. This ensured good mixing of the dry sand with the flow. The injector was a commercial screw-feed machine, which could provide any chosen rate of sand injection by a combination of variable hydraulic gear box and a range of feed screw pitches and diameters. During tests with variable discharge the injector gearbox was linked to the flow splitter control in such a way as to maintain a chosen constant proportion of sand in the varying water discharge.

The amount of sand in transport at the downstream end of the stream was estimated by passing the entire flow of water and associated sediment through a trough of 6 in square section extending across the whole width of the settling basin, just beyond the tilting tail-gate. The trough was designed to give a good roller action within it, and had multiple withdrawal tappings into a common manifold which was connected to a small centrifugal pump. In this way a large representative sample of the water-sand complex could be taken into a settling tank of known capacity, and hence the concentration could be obtained after settling and weighing. In practice, the withdrawal tappings commanding inactive parts of the stream cross section were temporarily closed off, and the pumping speed through the tappings in use was adjusted to match the flow velocity in the trough in an attempt to take an undisturbed sample.

Water surface levels were measured at nine points an average distance of 30 ft apart by a series of pitot-static probes carried on beams spanning the channel. Each was connected by siphon tube to a stilling pot provided with a vernier hook gauge reading to 0.001 ft. As these open-air experiments were continued throughout two winters, methyl alcohol was added to the water in the stilling pots as an "anti-freeze"; the resultant density change of the liquid column was calculated to be within the accuracy of measurement.

Cross sections of the channels, whether during a test or when drained at completion, were usually taken with a simple beam and probe. This was quicker to set up at any point of the extensive sand area than the automatic bed plotting device available, though such an instrument would have been suitable if a large number of plots at one section had been needed.

Vertical photographs were taken during the tests (Plates 2-12 by mounting a camera on the specially lengthened boom of a mobile crane, the camera shutter then being released by an electric solenoid from the ground. Three exposures covered the whole length of a channel, and when slightly enlarged produced overlaps of a scale about 1 in to 10 ft. By using some ground control in the form of objects of known position, many results were obtained from these photographs by scaling.

DESCRIPTION OF EXPERIMENTS

Each experiment was started by preparing a straight channel down the centre of the sand strip from the restricted section under the sand injector to the tilting tail-weir. This initial channel was of trapezoidal cross-section sufficient to run full at the start of the test. Depending on the discharge to be used its depth varied between 5 and 6½ in and its top width between 1 ft 8 in and 4 ft 6 in (side slopes 45°). Allowing for a freeboard of about 0.75 in the initial width/depth ratio of the channel varied from 4.7 for the smallest discharge channels to 9.4 for the largest. The bed and flood plain flanks of the channel were carefully adjusted by surveyor's level to a constant slope, usually 0.0020 except for the final test when the initial valley slope was 0.0026.

The flow was started slowly, entering from the head of the channel (it was not possible to back flood the area) on an axial line, and not artificially inclined against either bank. As soon as the full required discharge had been obtained, the water surface slope was adjusted by means of the tilting tail gate to be parallel with the initial bed gradient, usually 0.002. After this, no adjustment was made to the tail water control during a test and the channel gradient was free to vary. Sand injection was started when the discharge was at its required value, and the experiment then ran continuously for a period, varying between about 5 and 35 days in the different tests, until it was judged that a stable channel pattern had formed in at least part of the length of the stream. This was not always a regular meander pattern, but such straight or braided channels as developed were highly informative in themselves. As the channel widened or meandered during a test at constant freeboard, sand was moved on the flood plain as necessary to ensure a uniform bank height with

respect to water surface level, in order not to introduce variations in the amount of sand available for transport when bank cutting took place.

Observations were taken twice each day of inlet and outlet discharge, water surface levels at the nine measuring points, sand injection rate and water temperature. Sand transport rate at outlet was of necessity measured rather less frequently, due to the time needed to obtain and process large numbers of samples. Photographs were taken from the mobile crane and cross-sections surveyed at representative stages of development of the channels, as frequently as daily in the later tests. For most of the photographs the flow was temporarily reduced to about one third of the chosen value, and a dye injected at the head of the channel in order to show the shoal pattern clearly. At the conclusion of each experiment the channel was drained, and meander length and breadth and thalweg length were measured, together with final cross-sections and ripple profiles.

In the subsequent analysis, account was taken of the reduction in discharge along the channels due to exfiltration. For want of a better basis, a linear rate of loss between inlet and outlet was assumed; the resultant flow at the centre point of compartments near the centre and towards the downstream end of the channel was then calculated. The exact location of the compartments varied from test to test, and was chosen by inspection to include only stable channel forms. By this means two results could usually be obtained for the relations between discharge and other dependent variables in any one test of a given nominal discharge.

EXPERIMENTAL RESULTS

Development of channel

Immediately after the introduction of water and sediment at the upstream end of the prepared trapezoidal channel, the bed developed ripples which were symmetrical about a longitudinal line. After a period of between about 6 hours for the higher discharges and about 24 hours for the lower values, the symmetry deteriorated and shoals began to form near the sides of the channel along its length. These were at fairly regular intervals, the shoals forming on alternate sides of the channel with deeps near the opposite bank. The shoals continued to increase in length and height while they migrated fairly rapidly downstream, increasing their pitch as the channel became wider through general bank erosion even though it remained straight. Then, almost simultaneously along the channel, embayments were eroded in the banks opposite each shoal. These expanded, producing a sinuous channel pattern in plan, and also migrated downstream, although more slowly than the shoals had done. The pitch of these features then remained constant, or tended to decrease slightly.

At the upstream end of the initially straight channel the pattern of development differed somewhat. A shoal formed in the centre of the channel near the inlet, and scour occurred on both sides. The shoal increased in length, width and height as the channel widened, until the banks on both sides of the channel became markedly concave. The resulting bulb-like shape at the entry to the channel was a familiar feature in almost every test of the series; (see Plate 12). It expanded fairly evenly, but flow was disposed to concentrate more to one side or the other where more marked bank erosion would occur. The deposition of sediment to form the central shoal at the inlet and the flanking shoals along the length of the

channel usually began almost simultaneously.

In the later stages of development of the central shoal, when flow tended to concentrate towards one side, the predominant stream was observed to change from one side to the other on more than one occasion without apparently seriously affecting the downstream pattern of erosion points. This may have been due perhaps to the distance between the "bulb" and the first erosion point which was about twice the distance between erosion points downstream, i.e. equal to about one meander length. The predominating flow from the bulb did not attack the opposite bank downstream; it scoured the bed near the centre but slightly towards the opposite bank. The current then swung back again to erode the bank further downstream, from which point onwards erosion occurred at intervals on alternate sides. The failure of the main current emerging from the bulb to attack the opposite bank was presumed to be due to the effect of the stream from the opposite side of the bulb. Sufficient disturbance was produced to cause bed scour at their junction, but the stronger flow was deflected to an angle insufficient to cause bank erosion.

With variable flow, the sequence of stages leading up to the formation of a sinuous channel was similar to those for a channel with a steady discharge except for two main differences:

(i) the channel with a variable flow took longer to develop meanders than one with a steady discharge equal to the mean of the variable flow, and the same sediment injection rate, and

(ii) the pattern of the final channel was similar to that of channels conveying a steady discharge greater than the mean value of the variable flow.

The slower rate of development of the channel conveying the variable flow may be due to the scouring of

the shallow cross-over points at low flow and the deposition of sand in the deep pools. Thus, as the discharge increases, work has to be done to scour the pools before any further development of the sinuous pattern can occur.

One special test was conducted to examine the influence of an increase in sediment injection rate on the development of the channel. This was test 20, in which initially no sediment was added at the upstream end and meanders did not develop. Later, when sediment was introduced, the channel progressed from a shoaled pattern to full meandering. The implications of this experiment are discussed later.

Development of channel within previously formed wider channel

A channel was allowed to develop as described above using a discharge of 2.0 cusecs and a sediment injection rate of 600 mg per litre. The channel gradually increased in width but remained straight for 20 hours, but by 28 hours after the start of the test embayments had been eroded at intervals along the channel. These gradually grew larger until by about 15 hours later a marked sinuous channel pattern had developed with a wave length of about 52 feet in the upper reaches.

The operating flow was then reduced to 0.5 cusec with a sediment charge of 300 mg per litre. Under these conditions the water surface was below the top of the shoals formed previously and the channel followed a sinuous course.

About three days later the water surface slope had increased and a channel had been cut through the most upstream of the original shoals and the stream was adopting a sinuous path along the original thalweg with a meander length of about 27 feet. From this stage onwards the sinuous pattern developed. When some of the original shoals were cut through, being on the eroding bank, an island developed in the middle of the channel. On each side of the island

where the discharge was about half the total flow in the channel there was evidence of the development of a secondary sinuous pattern with a wavelength of the order of about 20 feet. Finally the channel developed a braided pattern as more of the original shoals were cut and small islands were formed within the wide channel formed by the 2.0 cusec flow.

The first part of the test followed the usual pattern of development of a channel with a sediment charge in excess of the threshold value, initial channel and valley slopes less than the final straight line water slope, an initial channel width which was too narrow and a constant freeboard. The second part of the test followed the pattern for a channel with a sediment charge in excess of the threshold value, initial channel and valley slopes less than the straight line water surface slope, a constant valley slope (not constant freeboard) and an initial channel width intermediate between the width which would develop under constant freeboard from a channel initially too narrow and the width of the meander belt which would develop under the same circumstances. Thus the flow was able to attack the channel banks causing embayments and a sinuous pattern but as the water surface slope increased and the shoals were eroded the whole valley was flooded and a braided channel developed.

Geometry as function of discharge

Intuitively, it is anticipated that the geometrical dimensions of a meandered channel may be dependent on:

(i) the discharge, and its variation with time if unsteady;
(ii) the slope of the valley containing the stream;
(iii) the rate of sediment transport
(iv) the resistance of the bed and banks to movement, which in turn is dependent on

grain size, specific gravity and cohesive strength;

(v) the viscosity and density of the liquid.

However, the slope of the stream and the rate of sediment transport are not independent variables. It was clear from previous experiments, and confirmed in the new experiments, that local slopes and rates of sediment transport are interdependent. If, for example, the channel slope is less than that required to carry forward the injected sediment load under the particular conditions of flow and stream geometry, aggradation will occur at the head of the stream. Thereby the slope increases, until it is adequate, with the discharge and geometry obtaining, to convey the full injected charge. The slope of the channel therefore cannot be controlled in this type of experiment: it is self-adjusting once the operating conditions are determined.

Previous work, in both field and laboratory, has indicated that the meander length is strongly dependent on discharge, and hence this will be examined first. The equation suggested by Inglis in 1938 [21] is well known:

$$\lambda = \alpha\, Q^{0.50} \qquad \ldots\ldots (1)$$

where α is of the order of 25 to 30 (in ft. sec units), when Q is the maximum discharge, or of the order of 36 when Q is the dominant discharge.

The results of the present experiments are shown on Fig. 3, in terms of discharge against meander length. Using the method of least squares, the equation best fitting the results at a median grain size of 0.15 mm and with discharges ranging from 0.238 to 2.74 cusecs is:

$$\lambda = 37.8\, Q^{0.506} \qquad \ldots\ldots (2)$$

The correlation coefficient is 0.958 and the standard error is 0.0426 log units. This equation agrees closely with the form of equation 1 given above, and confirms a strong

correlation between meander length and discharge. Some of the early tests were affected by algal growth on the banks or by other difficulties which prevented full meander development. These runs are shown by a different symbol in Fig. 3. Excluding these doubtful results the equation becomes

$$\lambda = 38.0\, Q^{0.467} \qquad \ldots\ldots (3)$$

It is also possible that at low discharge the loss of flow through exfiltration has a greater effect on the channel pattern because it was proportionately higher. A further correlation was therefore made excluding all runs with discharges less than 0.4 cusec. This yielded

$$\lambda = 38.0\, Q^{0.454} \qquad \ldots\ldots (4)$$

It is only by examining data covering a wider discharge range that the uncertainty in the value of exponent of Q can be resolved, and this point will be returned to later. The value of the exponent is of some significance, as in Ref. 1 an exponent of 0.42 in the width equation was deduced from straight channel data obtained at Wallingford, compared with the usually accepted regime figure of 0.50.

Fig. 4a shows the relation between channel width and discharge for the present results together with the points obtained for the investigations on straight channels (1) undertaken previously at Wallingford using the same sand. It will be seen that in all cases the width of the meandered channels is greater than that of the straight channels at the same discharge. The best-fit equation to the straight channel data was:

$$b = 3.6\, Q^{0.42} \qquad \ldots\ldots (5)$$

and it appears from Fig. 4a that the freely-developed meandered channels had widths, almost without exception, of at least twice the width of a straight channel for the same discharge. The relationship:

$$b = 7.2\ Q^{0.42} \qquad \ldots\ldots (6)$$

forms a lower limit to the width data for meandering channels. Streams that remained straight but had prominent alternate shoals lay in the band:

$$5.0 < \frac{b}{Q^{0.42}} < 7.0 \qquad \ldots\ldots (7)$$

There has been diversity of opinion about the width to depth ratio of meandering channels. From these investigations, it is apparent that it is only streams of low sediment charge that remain straight; streams that develop meanders are dependent on a higher sediment charge, and also have a greater width to depth ratio. On the other hand, bank erosion will presumably only occur when channels are not excessively wide, and thus there may be an upper limit to the width of the channel which will meander in this fashion.

Turning next to the effect of discharge on the width of the meander belt, Fig. 4b shows considerable scatter, far greater than that in Fig. 3, where meander length is similarly plotted. This is not unexpected, as it appeared from the pattern of development that the ultimate stable wave length might be achieved before the width of the meander belt had reached a maximum. Given more time - in some tests at least - wider meander belts might have resulted, but even more important is the fact that the belt width may depend appreciably on factors other than discharge. This will be referred to again later in this report.

From a least squares analysis of the data from the same tests used for the derivation of equation (3) the regression of meander belt width on discharge is:

$$\beta = 18.50\ Q^{0.505} \qquad \ldots\ldots (8)$$

The correlation coefficient is 0.838 and the standard error is 0.082 log units.

The meander length being strongly dependent on discharge, and the belt-width and channel width also being dependent on discharge (amongst other factors), it may be instructive to consider how the ratios between these three linear measurements vary with discharge. Fig. 5 shows the ratios λ/b, β/b, and λ/β: scatter is appreciable in each case. The best-fit equations, again using data from the same tests used in the derivation of equation (3), are:

$$\frac{\lambda}{b} = 4.47\ Q^{0.148} \qquad \ldots\ldots (9)$$

$$\frac{\beta}{b} = 2.17\ Q^{0.186} \qquad \ldots\ldots (10)$$

$$\frac{\lambda}{\beta} = 2.06\ Q^{-0.038} \qquad \ldots\ldots (11)$$

The correlation coefficients are 0.399, 0.461 and 0.136 respectively. The standard errors are 0.085, 0.090, and 0.070 in log units respectively. Thus, although the ratio of meander length to belt width is not very dependent on discharge, the two geometric ratios involving channel width show a variation with discharge in the present range of experiments, though this is of doubtful significance.

Inter-relationship of channel dimensions

When plotting meander length against discharge the scatter is small provided that, as in these tests, a single grade of material is used. If the different grades of material occurring in nature some slightly cohesive, are considered however, it is to be expected that scatter would occur because of differing mobility of bed sediments and erodibility of banks. A further factor that has influenced previous authors in analysing prototype data is that bank-full channel widths and meander lengths are measurable even if somewhat ill-defined, whereas the discharge that generated the pattern may be difficult to

measure or define. Hence, the use of purely geometric plots, for example meander length against channel width. Fig. 6a shows the results of the present tests from which the following relation has been deduced using data from the same tests used in the derivation of equation (3):

$$\lambda = 6.84\ b^{0.760} \qquad \ldots\ldots\ (12)$$

The coefficient of correlation is 0.680 and the standard error is 0.089 in log units.

The scatter of points in this graph is more pronounced than in Fig. 3, but in our case this may in part be due to the difficulty in defining the channel width in sinuous channels including shallow water zones.

The suggestion of Hjulstrom (reported in Ref. 30) that meander length is dependent on the meander breadth was also investigated. Fig. 6b shows the relation between these two variables using data from the same tests as previously, which lead to the equation.

$$\beta = 0.335\ \lambda^{1.103} \qquad \ldots\ldots\ (13)$$

The coefficient of correlation is 0.887 and the standard error is 0.070 log units. The scatter in this figure is not quite as great as that in Fig. 6a but it is doubtful whether this means that λ and β are variables dependent on the same independent variables. Leopold and Wolman[30] also concluded that there is no confirmation that λ is dependent on β.

To complete the examination of the inter-relationship of the dimensions of a meandered channel, Fig. 6c shows meander belt-width in terms of channel width. This demonstrated that

$$\beta > 1.5b \qquad \ldots\ldots\ (14)$$

but in very few cases did belt-width exceed three times the channel width in these experiments.

Influence of sediment charge on meander geometry

In these investigations shoals developed on alternate sides of the initially straight channels during the early phases of all twenty-one tests. From observation of the stages in the development of the channels it appeared that the mechanism of bank erosion may be governed by the growth of these shoals. Unless the shoals become large enough in relation to the strength of the channel bank to deflect the flow sufficiently to cause the local erosion of an embayment on the opposite bank downstream, a natural sinuous pattern will not occur. Once the erosion of embayments has begun there is a further contribution to the sediment available for shoal building on the same side and downstream of the embayment. The expansion of the meander once begun thus progresses rapidly. This would suggest that for any discharge and sediment size once a channel has changed from a straight to meander pattern there is a lower limit of meander belt width and meanders of gradually increasing amplitude do not occur. This is in agreement with experience.

For a given discharge and particle size, the dimensions of the shoals are governed by the sediment transport rate. This suggests that there may be a threshold value of sediment transport rate (and hydraulic gradient) below which meandering will not occur and above which regular meanders will naturally form and be maintained provided that the water surface slope required to transport the sediment is not restricted by the valley slope. When the valley slope is insufficient to allow the stream to transport the liquid and solid discharges, a wide braided pattern develops.

The view that sediment is necessary in the formation of a meandering pattern has been put forward by Werner[53] who suggested also that superfluous sediment would prevent meandering, and by Inglis[22] in his definitions of

primary and secondary meandering. On the other hand, however, Schumm (41) has declared that sinuosity is not conducive to bed load transport, hence as the bed load decreases the channel becomes narrower and steeper and tends to meander.

Plates 2, 3, 6(a) and 6(b) show the development of shoals in a straight channel and the failure of these channels to meander. These cases all occurred with low sediment transport rates. Plates 4(a), 4(b), 7(a) and 7(b) show the development of meanders. Plates 7(a) and 7(b) are of particular interest because by comparison with plates 6(a) and 6(b) they demonstrate the development of meanders after the introduction of sediment.

Fig. 7 shows the sediment transport rates and discharges for these investigations to which have been added the results of laboratory studies of several other observers. In general those channels which were classified as being "straight" have lower sediment changes than channels of nearly similar discharges but which were termed "meandered". From the information available, however, it is not possible to deduce a statistical relation for the threshold value of sediment change above which meandering occurs in the narrow range of discharges examined although a line has been drawn which separates the data into two groups.

Test 20 demonstrated generally the effect of sediment charge on the development of meanders. In the first phase of this experiment, no sediment was injected at the head of the channel although bank erosion as the channel widened initially provided a temporary source of sediment supply along the channel. In this phase, shoals developed on alternate sides of the channel but only at the downstream end where the sediment charge due to bank erosion was temporarily high did any meanders develop (see Plate 6). After some time the channel widening process stopped, and the meanders in the downstream section

developed cut-offs, apparently tending to straighten the channel. A temporary spur was inserted to prevent cut-off developing, which was effective while in position, but as soon as it was removed the stream reverted to cutting through the shoal, i.e. confirmed its tendency to straighten. When sediment was introduced into the channel in the second phase of the experiment the shoals began to increase in size, the water surface slope increased and meandering occurred almost simultaneously along the channel.

This offers a partial explanation of the immediate cause of meandering channels and provides a method of predicting whether a small channel in alluvium of median grain size 0.15 mm will remain straight or develop a meandered or braided pattern. It does not, however explain the development of the shoals which may be caused by secondary currents, although attempts to measure the magnitude and direction of these currents in both straight and meandered channels were inconclusive.

The first author in an earlier paper put forward the view that a stream's tendency to meander increased with increasing solids transport and slope, and it will be of interest to examine the extent to which a solids concentration in excess of the threshold may affect the geometry. The plot of meander length against discharge given in Fig. 3 shows some scatter, and it is possible that this arises from the variation in X, the solids transport rate expressed as a concentration by weight of the fluid flow.

Since it has been concluded that the principal factor affecting meander length is the discharge and that the effect of other factors must necessarily be smaller, it is to be expected that large variations in sediment charge will produce only small changes in meander length. A multiple correlation of λ with X and Q gave the relationship:

$$\lambda = 42.9 \; X^{-0.021} \; Q^{0.480} \qquad \ldots\ldots (15)$$

for $52 < X < 902$ mg/litre

$0.245 < Q < 1.94$ cusecs

The correlation coefficient with respect to Q was 0.70 indicating a significant dependence, whilst that with respect to X was 0.21 which cannot be said to confirm a dependence of λ upon X. The coefficient of correlation with respect to λ and Q was 0.975 and the standard error was 0.023 log units. The poor correlation with X may have arisen from the inadequacy of the selected form of equation above in respect of this variable, but this point was examined by plotting $\lambda/Q^{0.48}$ against X in Fig. 8. There appeared to be no dependence on X, the average value of $\lambda/Q^{0.48}$ being 38.0*.

Sediment movement and migration of meanders

When the rate of erosion at any position is larger than the local transport capacity of the flow, there must be deposition just downstream from the point where erosion takes place. This raises the bed level near the banks creating an increasing obstacle to the current, which will then be shifted towards the opposite bank increasing the rate of erosion there. The same process is repeated from bank to bank over the total length of the channel (Fig. 9) so that the development of shoals into meanders may be

* By equation (2), it might have been expected that $\lambda/Q^{0.506}$ was the appropriate parameter to plot against X. The change of exponent from 0.506 in the single correlation to 0.480 in the multiple correlation arises from the necessary omission of some discharge data in the latter case, because the associated sediment concentrations were not known. As these data happen to lie at the extremes of the plot, their omission has a noticeable effect on the exponent of Q. A plot of $\lambda/Q^{0.506}$ may be seen in Ref. 11.

practically simultaneous over the 300 ft length of the channel. The zones of erosion become the concave banks; the shoals grow in width and height, forcing the flow more and more against the banks, thus increasing the meander amplitude until a condition of equilibrium is reached when the shoals stop growing laterally.

Simultaneous with the vertical and lateral growth there is a longitudinal downstream movement of the shoals which, in advanced stages of development, present a definite front. This front moves forward as the material is carried over the shallow back of the shoal and deposits in the stagnant zone downstream of it. This movement of the front is not symmetrical because the transport of material is greater where the velocity of the water is greater, that is towards the deeper thread of the channel. This asymmetry of movement and the pattern of flow are responsible for the formation of the typical deeps of stagnant water (Fig. 9) in the lee of shoals.

Fig. 10a shows the development of the pitch of bed features with time, for run 14. The shoals in the still-straight channel increased in pitch from about 25 ft when the stream was 20 hours old, up to a maximum of 27½ ft soon after meanders began, thereafter reducing over a period of a few days to about 25 ft again. The 10 per cent increase in pitch of bed feature as meanders developed and the channel widened was small, but definite.

The shoals were also observed to migrate downstream. Obviously if the shoals both migrate and increase their pitch simultaneously, those downstream would have to travel faster than those upstream, if those upstream were not to advance on to and ultimately destroy those immediately downstream. Fig. 10b shows the development of shoals in terms of location and migration with time during test 20. It may be seen that the rate of downstream

migration increased up to a limiting value of about 5.4 ft /day after ten days. Where the limiting rate was reached, for example at shoal 5, the shoal immediately downstream was ultimately destroyed. This may be observed to have occurred about hour 350 at a point 170 ft along the channel. Shoal 6 no longer existed at hour 400.

An investigation was undertaken during test 11 of the movement of bed material. The discharge at inlet was 1.2 cusecs and the sediment injection rate was 300 p.p.m. This value of sediment injection rate was thought to be below the threshold value for the development of meanders at this discharge, but due to the loss of water by seepage along the channel and the gain in sediment charge from bank erosion, conditions developed downstream which were conducive to the formation of meanders.

In order to confirm that eroded material does not cross to the opposite side of the channel, an attempt was made to trace the movement by using sand grains, of the same size range as the sand in the experimental area, coated with a fluorescent resin. A small amount of fluorescent sand replaced a zone of the bank material where erosion was in process. Later, to determine where the material had been transported, a series of underwater samples of the surface of the bed were taken. For this purpose metal plates 6" x 7" were covered with a layer of silicone grease (not fluorescent) and then pressed against the surface of the bed taking samples as the grains stuck to the grease. The number of fluorescent particles was then counted under ultraviolet light.

The direct examination of greased plate samples was preferred to a photographic recording method, because the sand in the model had a fluorescent background (organic filaments, shells) which in the photographic plate could not be distinguished from the tracer.

Initially a small amount of tracer was placed in two of the bends, 30 gms in a bend with small rate of erosion and 10 gms in one with greater movement, at 159 and 195 ft respectively from the upstream end of the channel. The distribution of the transported tracer over the bed after 6 and 23 hours is shown in Fig. 11b. Sampling was only made over the bend immediately downstream from the source.

It can be seen that a great part of the material is found over the shoal, whereas in the main channel or thalweg only very near the source is the concentration of particles considerable. The sample taken in the deep stagnant water shows also that sand is continuously depositing here, as the lee-side of the shoal advances.

The release of fluorescent material from the banks is rapid so that the change of surface concentration of particles with time results from the combined effects of erosion and transport of the fluorescent particles, their deposition in zones of low velocity and their covering by new sand deposited over them. The low bed concentration along the thalweg tends to show that the transport there is more intense, not allowing the settlement there of tracer grains in transport.

When the traces of this first test had disappeared, a new source of 100 grammes of fluorescent sand was placed at the concave bank in section 175 with the purpose of sampling over a longer length of channel. Samples were taken at 1, 19 and 43 hours after erosion of tracer with results shown in Fig. 11a. During this period tracer was observed only in the first shoal beyond the source until the last measurement when a few particles appeared in the third shoal beyond the source. This suggests that the sand eroded from the right bank remained on the right bank.

Flow resistance and slope

In seeking an explanation of meandering it is sometimes assumed that a stream which meanders does so in order to decrease its gradient which would otherwise be too steep.

Consider first a stream whose gradient is initially too flat to convey its discharge and associated sediment load. It would seem reasonable that it should maintain a straight channel while increasing its gradient. Any decrease in potential thalweg water surface slope through the development of meanders would be contrary to the reasons for the increasing gradient. Thus it would seem necessary during valley aggradation for a channel to remain straight and on the contrary, meanders might be expected to occur only when a channel was initially too steep and was tending to reduce its gradient. But in all those tests in which meanders developed, they did so after the water surface slope had increased from the initial value (of 0.0020 or 0.0026) and after the start of meandering the straight line water surface slope continued to increase. An example of the increase in water surface slope is shown in Fig. 12 demonstrating conclusively that meandering does not result from a natural tendency for a too-steep channel to reduce its effective gradient: meandering in these experiments brought with it an increasing straight line gradient.

In the first author's 1964 paper on alluvial streams, the rough-turbulent equation:

$$\frac{v}{v*} = 2\sqrt{8}\log 12.3\frac{m_s}{k_s} \qquad \ldots\ldots (16)$$

was used in analysing resistance data from straight channels, and k_s, the equivalent sand roughness, was found to equal or exceed the measured mean ripple height. With shoaled and meandered channels, k_s lay between 0.1 ft and 0.5 ft, i.e. considerably in excess of the ripple height (see Fig. 15, ref. 2) A plot of the dimensionless Chezy

coefficient against relative roughness, m/k, where k is the mean ripple height, showed that the small straight alluvial channels followed very approximately the rough-turbulent equation, but that the value of C' dropped by between 10 and 30 percent when the stream meandered. It is well established, therefore, that meandered channels impose a greater resistance to flow than otherwise similar straight ones.

Two methods of evaluating slope have been used, S denoting the straight-line slope, and i_m denoting the slope along the thalweg. These give rise to two values of the dimensionless Chezy coefficient,

$$C'_s = \frac{Q_m}{A_m\sqrt{gm_m S}} \qquad \ldots\ldots (17)$$

and

$$C'_m = \frac{Q_m}{A_m\sqrt{gm_m i_m}} \qquad \ldots\ldots (18)$$

Clearly $C_m' > C_S'$, as $i_m < S$. Furthermore,

$$\frac{C_m'}{C_S'} = \sqrt{\frac{S}{i_m}} = \sqrt{\sigma} \qquad \ldots\ldots (19)$$

In a meandered channel, it is not easy to obtain an accurate measure of A and m. The area and shape of the flow section vary considerably along the channel. There are zones of exceedingly shallow water that contribute very little to the flow, yet, taking the usual definition of m = A/P, have an appreciable effect on the computed hydraulic mean depth. This is particularly the case at skewed sections taken at the bends, and there is also a possibility that sections taken may include dead water zones in the lee of the advancing shoals. The cross-over sections are rather better defined, and hence in applying equations 16, 17 and 18 in the present analysis A and m relate to a typical cross-over.

C'_s and C'_m have been plotted in Fig. 13 against m. The average values are 5.50 and 6.43 respectively, the ratio between them agreeing with the mean sinuosity of about 1.37. Values of C'_m between 4.3 and 8.5 were found, with a considerable degree of scatter. An explanation of this was sought in the variation of the sediment transport rate from test to test, without success, nor is it explained by the variation of sinuosity.

It is not valid to extrapolate the rough-turbulent equation to the case of tortuous channels, and it is doubtful if extrapolation to such low values of m/k_m could be supported even in straight channels. Nevertheless, to permit comparison with results previously obtained for straight channels, equ. 16 has been shown on Fig. 13 for

k_m values of 0.05 ft (0.015m) to 0.5 ft (0.15m)

In terms of straight line slope, the roughness on this basis lies between about 0.1 and 0.5 ft in these small meandered streams. In terms of thalweg slope, it lies between about 0.07 and 0.3 ft. Increasing sinuosity must increase the apparent roughness whichever basis is used, but a further explanation offered for the variation is that scatter arises from the difficulty of defining a representative cross-section. Very many more sections, carefully excluding dead water zones, would have to be taken to improve the correlation.

The overall drag in a non-prismatic free-boundary channel consists of the following:

(i) The component of the boundary shear stress against the direction of flow;

(ii) The net upstream component of the pressures exerted by the bed forms on the fluid;

(iii) The head losses due to changes in cross-section (contractions and expansions with distance) and changes in alignment (horizontal bends).

The first of the above-named contributions is the minor part of the overall drag in a rippled fine-sand stream, but nevertheless this direct shear stress is instrumental in causing movement of the bed sediment. The second part arises from the pressure difference between the exposed and lee-faces of bed roughness features and is the major source of resistance in a straight alluvial channel, the dimensions of the bed-forms providing some indication of the roughness (e.g. k_s) at low transport rates. However, the third type of head loss will presumably become important in tortuous channels with alternating skewed deeps and shallow cross-overs.

With a bed of fine sand a turbulent-transition equation would be appropriate for the boundary shear contribution; for the bed form losses, the rough turbulent equation applies, with the proviso mentioned above regarding extrapolation to low values of m/k_m; and for the form losses due to changes in section and alignment, the Chezy equation would apply, with C' dependent on geometry. It is clear from the scatter in Fig. 13 however that elaborate analysis of the data available would be profitless. For <u>small</u> meandered channels in fine sand, the broad conclusion to be drawn is that

$$C'_s = \frac{Q_m}{A_m\sqrt{gm_mS}} \simeq \frac{2\sqrt{8}}{\sqrt{\sigma}} \log 12.3 \frac{m_m}{k_m} \quad \ldots\ldots (20)$$

where k_s lies between 0.07 ft and 0.3 ft. Acknowledging that k_s will probably be greater the larger the channel, and that much of the resistance is due to form losses, a more generally applicable relationship is: C'_m = approx 6.5 in channels of low tortuosity.

Because of the increase in resistance with meandering, it may be anticipated that the slopes of stable meandered channels will exceed those of straight ones at the same discharge in the same sediment, and hence slope may provide a method of classifying the channel type that will exist.

Two methods of evaluating slope have been utilised in Fig. 14. In the upper diagram, the average water surface slope on a straight axial line is plotted against discharge, for both straight and shoaled channels (2), and meandered ones from the current test series. In the lower figure, the slope has been taken with reference to the length of the deep-water channel, the thalweg. There is obviously less extra slope compared with a straight channel when expressed in the latter way. A division between straight, shoaled and meandered channels can be drawn: in terms of straight line slope, the boundaries for discharges of the order of 1 cusec (0.03 cumec) are roughly as follows:

Straight: $S < 0.0015\ Q^{-0.12}$ ..... (21)

Shoaled: $0.0015\ Q^{-0.12} < S < 0.0021\ Q^{-0.12}$ ..... (22)

Meandered: $S > 0.0021\ Q^{-0.12}$ ..... (23)

The latter condition is consistent with the onset of meandering according to Fig. 12. If the slopes of the meandered channels are measured along the thalweg, they reduce to below $0.002\ Q^{-0.12}$, i.e. fall back into the shoaled region of slopes. Nevertheless, they remain up to twice as steep as straight channels at the same discharges in the same material. No great significance should be attached to the exponent of Q in the above inequalities; the information does not permit us to draw a firm conclusion regarding the variation of limiting slope with discharge, but the tendency for slopes to reduce with increasing flow is as expected.

Cross-sectional shape and resistance compared with a straight channel

Cross-sections were measured at a typical cross-over and a typical bend at the conclusion of each run. The results are shown by distinctive symbols in Fig. 15.

In the previous research on straight channels cross-sections were measured at fixed locations and averaged, and the greater scatter in the present results, in the absence of any averaging, presumably indicates the rather random variation in a tortuous natural channel from section to section.

Areas and hydraulic mean depths were also expressed as ratios of the corresponding quantities in straight channels in the same sediment at the same discharge. Areas were found to be greater by a factor of 1.71, on average, the same figure applying to both cross-over and bends, indicating that the area at a bend in a meandered stream is the same as the area at a cross-over.

$$A_m = 1.71\ A_s \quad \text{..... (24)}$$

Hydraulic mean depths at the bends were 0.60 times those in a straight channel, and at the cross-overs they were 0.67 times the values in a straight channel

$$m_m = 0.60\ m_s \text{ (at bends)} \quad \text{..... (25)}$$

$$m_m = 0.67\ m_s \text{ (at cross-overs)} \quad \text{..... (26)}$$

Flow in straight and meandering channels may be described in terms of the Chezy equation as follows:

$$Q_s = C'\ A_s\ \sqrt{g\ m_s\ i_s} \quad \text{..... (27)}$$

$$Q_m = C'_m\ A_m\ \sqrt{g\ m_m\ i_m} \quad \text{..... (28)}$$

and;

$$C'_m = \zeta_r\ \zeta_f\ C' \quad \text{..... (29)}$$

where ζ_r and ζ_f are factors by which the coefficient C' for a straight channel must be multiplied to take into account any change in the roughness of the channel and form losses due to bends and changes of section, respectively.

In Fig. 13 a group of points cluster round

$$m_m = 0.125 \text{ ft} \qquad \ldots\ldots (30)$$

$$C'_m = 5.9$$

Referring to Fig. 14 the ratio of the average slope of channels which meandered to the average slope of channels which remained straight is given by

$$i_m = 1.9\, i_s \qquad \ldots\ldots (31)$$

Assuming that

k_m = 0.145 ft (the geometric mean of the values 0.07 and 0.3 ft derived on page)

m_m = 0.635 m_s (the mean of equations 25 and 26)

$Q_m = Q_s$

from eqns. 27 and 28

$$C'_m = 0.53\, C' \qquad \ldots\ldots (33)$$

$$\therefore \quad C' = 11.1 \qquad \ldots\ldots (34)$$

which value is consistant with earlier tests (see Fig. 14 ref. 1).

Is this change in value of C' with meandering due mainly to an increase in the relative roughness arising from a reduction in the hydraulic mean depth and/or increase in absolute roughness, or is it on account of the form losses due to bends and changes of section? So far as absolute roughness (k) is concerned, measurements of ripple geometry show that these bed features remain about the same size, if anything rather smaller, after meandering.*

$$k_m = k_s \qquad \ldots\ldots (35)$$

* See Ref. 1, Fig. 15a. More recent information confirms this trend (H.K. Mahgoub, unpublished H.R.S. report, August 1966).

The ratio of the relative roughness $(\frac{m}{k})$ of meandering and straight channels therefore does not exceed the ratio of the hydraulic mean depth and the reduction in the value of C' from this cause may be computed from

$$\frac{1}{\sqrt{\lambda}} = 2 \log 12.3 \frac{m}{k} \qquad \ldots\ldots (36)$$

$$C' = \sqrt{\frac{8}{\lambda}} \qquad \ldots\ldots (37)$$

by substituting for m_m, m_s, k_m and k_s, whence

$$\zeta_r = 0.84 \qquad \ldots\ldots (38)$$

i.e. the effect of the increase in relative roughness due to a decrease in the hydraulic mean radius (m) is to reduce C' by a factor of 0.84 or by 16%. Thus the factor reducing the value of C' which arises from the major features (alternating shoals and deeps with cross-overs) and the bend losses may be deduced from equations (29), (31) and (38).

$$\zeta_f = \frac{0.53}{0.84} = 0.63$$

Equations (27), (28) and (29) may be expressed in the form

$$\frac{i_m}{i_s} = \left(\frac{1}{\zeta_r}\ \frac{1}{\zeta_f}\ \frac{A_s}{A_m}\right)^2 \frac{m_s}{m_m} \qquad \ldots\ldots (39)$$

or

$$\frac{i_m}{i_s} = \left(\frac{i_m}{i_s}\right)_r \left(\frac{i_m}{i_s}\right)_f \left(\frac{i_m}{i_s}\right)_A \left(\frac{i_m}{i_s}\right)_m \qquad \ldots\ldots (40\tfrac{1}{4})$$

From this a "balance sheet" may be drawn up, explaining the enhanced slope in a meandered channel as compared with a straight one in the same alluvium at the same flow.

(i) Increased relative roughness due to change in m

$$\left(\frac{i_m}{i_s}\right)_r = \left(\frac{1}{0.84}\right)^2 = 1.42$$

(ii) Form losses (shoals, deeps, bends etc.)

$$\left(\frac{i_m}{i_s}\right)_f = \left(\frac{1}{0.63}\right)^2 = 2.52$$

(iii) Increase in average area of cross section and reduction in velocity

$$\left(\frac{i_m}{i_s}\right)_A = \left(\frac{1}{1.71}\right)^2 = 0.34$$

(iv) Reduction in hydraulic mean depth

$$\left(\frac{i_m}{i_s}\right)_m = \left(\frac{1}{0.635}\right) = 1.57$$

(v) Overall slope ratio given by equation 41

$$\frac{i_m}{i_s} = 1.91$$

The above figures are average values for the current range of experiments, and would not be generally applicable to all meander geometries. It is nevertheless significant that, even with the slope defined by the length of the thalweg, "form" losses arising from the non-prismatic geometry accounted for almost 60% of the fall, the roughness of the lesser bed features (ripple texture) accounting for a little over 40%.

This might appear surprising in these meanders of fairly low tortuosity, but Allen and Shahwan (4) concluded from model experiments and field results on the River Irwell that the boundary roughness accounted for between one-third and one-half of the total resistance (depending on the tortuosity of the reach in question), the remaining head loss being due to naturally occurring changes in alignment and cross-section. Thus the present result under free-

boundary conditions is in accord with Allen and Shawan's result from a specific rigid boundary model.

Although the hydraulic mean depth of a meandered channel proved to be considerably less than that of a straight channel carrying the same flow in the same alluvium, the maximum depths at both bends and cross-overs exceeded the average depth of a straight channel. This is illustrated in Fig. 16, in which maximum depths are compared with mean depths in a straight channel (see ref. 2, Fig. 8(b)):

At cross-overs in the meandered channels the excess averaged 45%

$$\text{i.e. } d_{mc} = 1.45\ d_s \qquad \ldots\ldots (42)$$

and at bends the excess averaged 63%

$$\text{i.e. } d_{mb} = 1.63\ d_s \qquad \ldots\ldots (43)$$

where d_{mc} and d_{mb} are the maximum depths at the cross-overs and the bends in a meandered channel and d_s is the average depth in a straight channel.

If the slope necessary to overcome bed rougnness is i_k then from (i), (iii) and (iv) above

$$i_k = 1.42 \times 0.34 \times 1.57\ i_s = 0.76\ i_s \qquad \ldots\ldots (44)$$

$$d_{mc}\ i_k = 1.45\ d_s \times 0.76\ i_s$$

$$= 1.10\ d_s\ i_s \qquad \ldots\ldots (45)$$

and

$$d_{mb}\ i_k = 1.63\ d_s \times 0.76\ i_s$$

$$= 1.24\ d_s\ i_s \qquad \ldots\ldots (46)$$

Thus the combination of maximum depth and reduced bed roughness yield a maximum drag on the bed at the bends and cross-overs of a meandering channel about 24% and 10% greater than the average shear stress on the bed of a straight channel. The maximum drag on the bed of a meander

bend also provides a measure of the drag near the base of the adjacent concave bank. The shear on the bank is of course of importance in determining the extent to which the bend will develop; slow erosion at the bend implies a slightly higher shear than when erosion of the banks ceases in a stabilizing straight channel.

The above conclusions have assumed that the slope at bends and cross-overs is the same as the average slope along the thalweg line. There is no information to show the degree to which this assumption departs from the truth but there is reason to believe that the slope does change and this will affect the computed values.

The deductions are also based on the value of the ratio of i_m to i_s, quoted in equation (32) above and obtained from Fig. 14. Reference to this figure also shows that the straight channels are separated from the meandered channels by a line given by

$$i = 0.0021\ Q^{-0.12}$$

In the vicinity of this line where channels are on the point of developing a meander pattern or have just meandered equation (32) becomes

$$i_m = i_s \qquad \ldots\ldots (32)$$

Adopting the reasoning and methods described above the following comparison of results may be made.

Equation	Item	Value assuming $\frac{i_m}{i_s} = 1.9$	Value assuming $\frac{i_m}{i_s} = 1.0$
33	$C'_m : C'$	0.53	0.73
34	C'	11.1	8.1
38	ζ_r	0.84	0.84
39	ζ_f	0.63	0.87
	$(i_m/i_s)_r$	1.42	1.42
	$(i_m/i_s)_f$	2.52	1.32
	$(i_m/i_s)_A$	0.34	0.34
	$(i_m/i_s)_m$	1.57	1.57
44	$i_k : i_s$	0.76	0.76
	$d_{mc}\ i_k : d_s\ i_s$	1.10	1.10
	$d_{ms}\ i_k : d_s\ i_s$	1.24	1.24

Thus at the threshold value of slope there is no change in the value of the ratio of the maximum shear stresses in meandering and straight channels at the bends and cross-overs as deduced previously. The roughness of the bed features accounts for about 75% of the losses compared to 40% previously and the form losses arising from the non-prismatic boundary geometry account for only about 25%.

Dimensional analysis of meander geometry

Before extending the analysis to include data from other sources, it is desirable to approach the problem from dimensional and similarity considerations.

The dependent quantities that describe an alluvial channel are functions of:

(a) The imposed flow and force field. This is represented by the variables Q and gS, the latter being the effective gravitational component generating the driving force on the fluid, which is also a measure of the influence of the fluid flow on the boundary.† If flow were non-steady, or if valley slope varying with distance prevented development of a quasi-uniform channel, or if a non-regime sediment transport rate were imposed causing accretion or erosion, additional variables would be needed to define the imposed flow.

(b) The sediment properties. D and γ_s are the main parameters, where D is a representative diameter (taken as D_{50}), and γ_s is the immersed weight per unit volume of the solid particles. Again, a full treatment of sediment properties would include a measure of uniformity of grading, particle shape factors, cohesive strength, angle of repose, and would make allowance for different properties of bed and bank materials if the channel is cut in alluvium different from that which forms its bed load.

(c) The fluid properties. The fluid is adequately described by μ and ρ

Any dependent quantity = ϕ [Q, gS, D, γ_s, μ, ρ]

i.e. meander length, = ϕ_1[Q, gS, D, γ_s, μ, ρ]... (33)

† It is implicity assumed at this stage that the gravitational acceleration g is not directly relevant, i.e. that the Froude number is sufficiently small for free surface effects to be neglected. This will be referred to again later.

and sediment concentration,

$$X = \phi_2[Q, gS, D, \gamma_s, \mu, \rho] \quad \ldots\ldots (34)$$

This equation may be re-written

$$gS = \phi_3[Q, X, D, \gamma_s, \mu, \rho] \quad \ldots\ldots (35)$$

and hence we may also write an alternative functional equation for meander length

$$\lambda = \phi_4[Q, X, D, \gamma_s, \mu, \rho] \quad \ldots\ldots (36)$$

This is more suited to the present set of experiments, because the sediment concentration was imposed whereas the channel gradient was not.

Before attempting to deduce dimensionless groups, consider the likely relative importance of the independent variables. Clearly Q is of considerable importance: the dimensions of a channel must depend strongly on the quantity of water to be conveyed. The evidence is that X does not have much effect on geometry. This was the conclusion for straight streams (2) and has already emerged from the correlation of λ with Q and X mentioned earlier in this report. This series of experiments with a single sediment provides no basis for assessing the importance of D and γ_s, so these variables must be retained. Also ρ, the mass density of the fluid, cannot be eliminated because fluid inertial stresses are dependent upon density.

The viscosity of the fluid may affect the problem either in association with the fluid flow or in association with the performance of the sediment grains. The former association can probably be discounted in the present case, because the fluid phase is of fully developed rough turbulent flow, with turbulence on a major scale added by form losses. If the sediment were coarse (>1 or 2mm for s = 2.65), the latter association could also be discounted, but as much data on meandered channels relates to fine-medium sand, viscosity must also remain as a variable at this stage. It may be of less importance than

Q, D and γ_s, but is unlikely to be insignificant.

It is concluded therefore that

$$\lambda = \phi_5\left[Q,\ D,\ \gamma_s,\ \rho,\ (\mu),\ (X)\right] \quad \ldots\ldots (37)$$

where the variables in parenthesis are secondary.

Regarding sediment diameter as the main linear scaling factor, a legitimate dimensionless form is:

$$\frac{\lambda}{D} = \phi_6\left[\frac{Q}{D^{5/2}}\sqrt{\frac{\rho}{\gamma_s}},\ \frac{Q}{D\nu},\ (X)\right] \quad \ldots\ldots (38)$$

Preferably, the Reynolds number term should not mix bulk flow variables with boundary variables, and so may be replaced by one which relates sediment to fluid without bringing in the imposed flow as follows, noting that $\gamma_s/\rho = g\ (s-1)$

$$\frac{\lambda}{D} = \phi_7\left[\frac{Q}{D^2\sqrt{gD(s-1)}},\ \left(\frac{D\sqrt{gD(s-1)}}{\nu}\right),\ (X)\right] \ldots (39)$$

The term including ν takes account of the influence of the fluid on the individual grain, and an analogy with the familiar initial motion function (Shields, ref. 45; Yalin, ref. 58) is apparent. For initial motion,

$$\frac{\tau_c}{\gamma_s D} = \phi_8\left[\frac{D}{\nu}\sqrt{\frac{\tau_c}{\rho}}\right] \quad \ldots\ldots (40)$$

which may be written

$$\frac{\tau_c}{\gamma_s D} = \phi_9\left[\frac{D\sqrt{gD(s-1)}}{\nu}\right] \quad \ldots\ldots (41)$$

Thus the second parameter in ϕ_7 could legitimately be replaced by $\tau_c/\gamma_s D$, as follows:

$$\frac{\lambda}{D} = \phi_{10}\left[\frac{Q}{D^2\sqrt{gD(s-1)}},\ \left(\frac{\tau_c}{\gamma_s D}\right),\ (X)\right] \ldots (42)$$

This substitution introduces the possibility of reducing the number of dimensionless groups to be considered. It is

probable that the sediment diameter's influence is primarily via its effect on the stability of the boundary, and possible that this is its sole influence. If this is so, τ_c may take precedence over D in the function, which would then appear as

$$\frac{\lambda g(s-1)}{(\tau_c/\rho)} = \phi_{11}\left[\frac{Q\,g^2\,(s-1)^2}{(\tau_c/\rho)^{5/2}},\ \left(\frac{\tau_c}{\gamma_s D}\right),\ (X)\right] \ldots (43)$$

ϕ_7 and ϕ_{11} lead to two similar forms of plotting meander length data, namely

$$\frac{\lambda}{D} \text{ versus } \frac{Q}{D^2\sqrt{gD(s-1)}}$$

$$\text{and } \frac{\lambda g(s-1)}{(\tau_c/\rho)} \text{ versus } \frac{Q\,g^2\,(s-1)^2}{(\tau_c/\rho)^{5/2}}$$

where τ_c is determined for the particular sediment from the initial motion function. These plots would be physically indistinguishable if all the sediments considered were coarse. It will also be realised that the second plotting system is equivalent to

$$\frac{\lambda}{D_e} \text{ versus } \frac{Q}{D_e^{\,2}\sqrt{gD_e(s-1)}}$$

where

$$\frac{D_e}{D} = \frac{\dfrac{\tau_c/\rho}{g(s-1)D}}{\dfrac{\tau_c/\rho}{g(s-1)D}} \text{ for coarse sediment} \quad \ldots (44)$$

In Fig. 18 both methods of plotting are shown, the D_e/D factors applied to the lower diagram being estimated from the inset initial motion curve.† Incorporated in the plots are laboratory data from a number of sources with sand sizes in the transition range.

† There has been considerable discussion about the concept and definition of initial motion in recent years (Ref. 35). The traditional curve ascribed to Shields (Ref. 45 for example) is subject to re-scrutiny. The relationship used herein is based on evidence from several sources.

The plot of laboratory data based on actual D_{50} size yields, as a best fit equation

$$\frac{\lambda}{D} = 123 \left(\frac{Q}{D^2 \sqrt{gD(s-1)}} \right)^{0.376} \quad \ldots\ldots (45)$$

i.e. $$\lambda \propto D^{0.060} Q^{0.376} \quad \ldots\ldots (46)$$

for material of given specific gravity.

The corresponding equations in terms of D_e are

$$\frac{\lambda}{D_e} = 130 \left(\frac{Q}{D_e^2 \sqrt{gD_e(s-1)}} \right)^{0.371} \quad \ldots\ldots (47)$$

i.e. $$\lambda \propto D_e^{0.0725} Q^{0.371} \quad \ldots\ldots (48)$$

Because there is rather more scatter in the latter plot, it might be thought that equation 46 above correctly expresses the influence of sediment size on the relationship of meander length to discharge, and that its influence is therefore as a characteristic dimension rather than through its mobility. Further reflection tends to disprove such a conclusion, and moreover suggests that the appearance of D as the main linear scaling factor is spurious.

Consider the data plotted in Fig. 18 in groups of experiments with similarly sized sand:

(a) 0.15 mm, (H.R.S. experiments)
(b) 0.21 mm to 0.26 mm,
(c) 0.45 mm to 0.70.

It so happens that the experiments with large sand sizes were conducted at much lower discharges than those with the finer materials. Thus the groups tend to be separated, groups (a) and (b) being to the top right, and (c) being to the bottom left. This separation enables a reasonable average line at a slope of about 0.375 to be drawn through all the data, but inspection shows that the slope of the average line does not accord with the trend of data for any single sediment

size. Conversion to D_e as in Fig. 17b merely reduces the horizontal separation. In reality, however, these plots demonstrate the inadequacy of these two parameter plots in taking full account of sediment size.

The best-fit equations to the groups of data considered separately are given in Table 1 below. The omission of gravity itself from the original list of variables might reasonably be questioned in view of the theories put forward for meandering based on the transverse propagation of a surface disturbance, a recent example being published by Anderson (5). Clearly if gravitational surface waves are a material factor, their propagation will depend on depth and width as well as gravity. However, just as meander length is a dependent variable, so also are the depth and width of a channel in non-cohesive alluvium dependent on the same variables, so that only gravity need be added to ϕ_5 above, yielding

$$\lambda, \text{ d, or b} = \phi_{12} \left[Q, D, \gamma_s, \rho, g\ (\mu)\ (x) \right] \ldots\ldots (57)$$

Analogous with ϕ_7, we might express this in dimensionless form:

$$\frac{\lambda}{D}, \frac{d}{D}, \text{ or } \frac{b}{D} = \phi_{13} \left[\frac{Q}{D^2\sqrt{gD(s-1)}}, \frac{Q}{D^2\sqrt{gD}}, \left(\frac{D\sqrt{gD(s-1)}}{\nu} \right), (x) \right] \quad \ldots\ldots (58)$$

The additional group occurs because of the inclusion of g as a separate variable, in addition to its implicit appearance as $\gamma_s/\rho(s-1)$. If attention is confined to a single value of the relative weight density of the solids, then the Froude-like parameter including (s-1) may be dropped. Moreover, as both λ/D and d/D are functions of the same dimensionless groups, for a single value of s we may re-write ϕ_{13} as:

$$\frac{\lambda}{d} = \phi_{14} \left[\frac{Q}{d^2\sqrt{gd}}, \left(\frac{D\sqrt{gd(s-1)}}{\nu} \right), (x) \right] \quad \ldots\ldots (59)$$

Anderson's analysis (5), on the basis of a transverse seiche produced the equation

$$\frac{\lambda}{\sqrt{A}} = \left(\frac{2\pi^3}{c}\right)^{\frac{1}{2}} F^{\frac{1}{2}} \qquad \ldots\ldots (60)$$

where c is a numerical constant, i.e.

$$\frac{\lambda}{d} = \text{constant}\ \frac{\sqrt{AV}}{d(gd)^{\frac{1}{2}}} = \text{constant}\left(\frac{Q}{d^2\sqrt{gd}}\right)^{\frac{1}{2}} \quad \ldots (61)$$

This is, not surprisingly, in line with ϕ_{14}, and in his paper Anderson plotted data - including the present Authors' - in support of his equation.

He went on to eliminate the depth d by utilising the empirical relationship of depth to discharge deduced in Ref. 2, concluding that

$$\lambda = 39\ Q^{0.39} \qquad \ldots\ldots (62)$$

By reference to data from other sources, Anderson apparently confirmed that this relationship was applicable to laboratory-scale channels in alluvium in which bed and banks were uniform and equally erodible.

This equation is analogous to those deduced in the present study for all data taken together (see eqns. 46 and 48, and Table 1), which also gave an exponent close to 0.4. However, on detailed examination of Anderson's Fig. 8, the same doubts must be expressed about the correctness of the overall picture as were voiced about Fig. 18 of the present report: a slightly different exponent would emerge if experiments with different sediments were treated separately, and the relationship is thus not truly independent of sediment size.

It is frustrating that the correlations so far referred to are good when looked at broadly, but are deficient when looked at in detail. The overall picture suggests that λ is dependent on Q raised to a power of

approximately 0.4, i.e. a Froude-scale relationship, although could enter in association with the sediment property (s-1) rather than alone. A detailed look suggests that the exponent of the λ-Q relationship is above 0.4, probably about 0.45 and perhaps as great as the conventional value of 0.5, and that a three parameter correlation is needed to take proper account of sediment size.

It was suggested above that sediment diameter should not be used as the primary linear scale factor, because its effect is secondary. A far more searching form of plotting is provided if the dimensionless meander length parameter is selected to illustrate departure from a Froudian function, associating Q and λ, and rearranging ϕ_{13} to read:

$$\frac{Q}{\lambda^2\sqrt{g\lambda}} = \phi_{14}\left[\left(\frac{\lambda}{D}\right), \left(\frac{D\sqrt{gD(s-1)}}{\nu}\right), (x), (s-1)\right] \quad \ldots (63)$$

Apart from suspecting a fairly weak dependence on all the variables on the left, we have no indication about their relative significance. In Fig. 19a, the available data from laboratory experiments has been plotted in this form. Tests in a given sand at a uniform temperature are run at a constant value of $D\sqrt{gD(s-1)}/\nu$; X, the sediment concentration is not known for all tests but is shown within the plotted points where known for the main H.R.S. data; the plot is confined to sands with much the same value of s.

In Fig. 19b, possible fitting lines are shown for the data in the upper part of the figure, as follows:

I. An overall fit to the data, discounting trends in results from individual sources.
This yields:

$$\frac{Q}{\lambda^2\sqrt{g\lambda}} \propto \left(\frac{\lambda}{D}\right)^{\frac{1}{2}} \qquad \ldots\ldots (64)$$

i.e. for a given sediment one would deduce

$$\lambda \propto Q^{0.36} \qquad \ldots\ldots (65)$$

in line with the overall fit in Fig. 18, and not very far removed from Anderson's exponent.

II. A Froudian relationship independent of λ/D and $D\sqrt{gD(s-1)}/\nu$ (alternatively referred to as $D_g^{3/2}$). This corresponds to

$$\lambda \propto Q^{0.4} \qquad \ldots\ldots (66)$$

i.e. very close to Anderson's relationship, but ignoring both the broad trend and the variation within individual sets of data.

III. The equation that fits best (by eye) the data from three sources for sands of 0.21, 0.23 and 0.26 mm

$$\frac{Q}{\lambda^2\sqrt{g\lambda}} = \eta\left(\frac{\lambda}{D}\right)^{-1} \text{ where } \eta = 0.90 \qquad \ldots\ldots (67)$$

if parallel lines are drawn through the centre of data for other values of $D\sqrt{gD(s-1)}/\nu$, the corresponding values of η required are given on the figure. Such lines would yield:

$$\lambda \propto Q^{0.67}$$

but this exponent value does not find confirmation at other values of $D_g^{3/2}$.

IV. Lines have also been shown through the centre of the individual groups of data for the equation

$$\frac{Q}{\lambda^2\sqrt{g\lambda}} = \eta'\left(\frac{\lambda}{D}\right)^{-\frac{1}{2}} \qquad \ldots\ldots (69)$$

where η' = function $(D_g^{3/2} = D\sqrt{gD(s-1)}/\nu)$

Values of η' are tabulated on Fig. 22.

It will be realised that this gives the Inglis exponent

$$\lambda \propto Q^{0.50} \qquad \ldots\ldots (70)$$

but is a reasonable fit only for the finest material, $D_g^{3/2} = 5.6$, i.e. $D_g = 3.15$

V. Also added for comparison is the equation deduced by multiple correlation earlier, for the finer of the H.R.S. sands.

This searching method of plotting shows how easy it is to oversimplify the correlation, either by looking broadly at the collection of data whilst plotting in such a way that the departures of individual groups from the overall trend are obscured or minimised, or by assuming that experiments with only one size of sediment will provide an exponential equation that is generally applicable and extrapolatable. In fact, setting aside any preconception that a log-log plot of the data must be fitted by a set of parallel straight lines, the bold curves in Fig. 19b, would fit the data well, within the framework of ϕ_{14} above. (What evidence there is suggests that X has a minor effect, with high concentrations tending to be above the trend line and low concentrations below).

It should be remembered that the data in Fig. 19 are for sands, s ≈ 2.65. Only one experimental result for material of different specific gravity is known to the authors, namely a run by Friedkin (15) in a coarse material called "Haydite", s = 1.85. This gives a value of $D_g^{3/2} = 105$ or $D_g = 22.3$ which lies outside the range of the sands considered, and so this one point does not permit us to judge the relevance of Fig. 19b to other specific gravities. The "Haydite" point has been added to the figure.

The influence of a periodic variation in discharge.

Seasonal variations of discharge occur in natural channels and there has been considerable discussion over the years about the influence of such variable flows on

meander geometry, centred around the question "What discharge within the range of observed flows dominates the resultant stream geometry?" Carlston (10) has recently reviewed field evidence, seeking a definition for discharge that would give the best correlation with measured geometry. If optimum correlation were a suitable definition of "dominant discharge", the conclusion from Carlston's results would be that mean annual discharge is the appropriate value, although Schumm has expressed reservations (43). However, other opinions point to a definition in terms of the discharge, or discharge range, that does the greatest amount of work in sediment transportation, with the mean flow in the mouth of maximum discharge as a strong contender. Carlston, however, does not support the Inglis suggestion (22) that a flow slightly in excess of bank-full discharge is the dominant condition.

Fortunately, when examining laboratory data, an un-equivocal definition is possible, namely:

> "Dominant discharge is the steady flow, probably lying within the range of imposed periodically varying flow, that would yield the observed meander length in an otherwise similar experiments."

Three types of discharge variation have been studied at Wallingford:

(i) A discharge varying sinusoidally about its mean value, with extremes of approximately zero and double the average

(ii) A discharge varying sinusoidally about its mean value, with extremes of approximately 0.5 and 1.5 times the average

(iii) A repeating hydrograph similar to the typical annual discharge pattern of a large river in Nigeria (R. Kaduna).

The first two types of hydrograph were run as part of the present fundamental research, with an eight hour period, and the third type was used with a seven day period some years previously in experiments that preceded an ad hoc investigation of river training (3). All three hydrographs are shown in Fig. 20 (exfiltration has been assessed and allowed for, and this accounts for differences between the nominal pattern and actual variation).

The definition of dominant discharge given above, coupled with equation 3, which relates meander length to steady discharge in the range of Wallingford experiments in 0.15 mm sand, yields:

$$Q_d = \lambda^{2.14}/2400 \quad \text{(ft sec units)} \quad \ldots\ldots (71)$$

The results are summarised in Table III, including a comparison of the dominant discharge calculated as above with maximum and mean discharges. It will be seen that the average value of the ratio Q_d/Q_{max} for the runs with a sinusoidal pattern is 0.95, with no significant difference between the runs including zero flow and those which varied over a smaller range. In contrast, the ratio Q_d/Q_{max} for the natural hydrograph is 0.42, which would refute any conclusion that in general $Q_d \approx Q_{max}$. A factor that may be of importance is that the natural hydrograph involved over-bank flows (in model as in prototype) whereas the sinusoidal pattern was entirely within banks. The prototype figure attached to bank-full flow when scaled to model conditions approximates to 0.55 cusec, so that $Q_d/Q_b = 1.13$, which clearly supports Inglis proposition for natural circumstances including an over-bank period of discharge.

Also in Table III, the results have been expressed as the percentage of time for which the flow exceeded the calculated dominant discharge. Because the sinusoidal patterns are relatively flat topped and the dominant value

is towards the peak, the percentage exceedances show scatter - indeed in one case Q_d marginally exceeds Q_{max} and gives zero exceedance - but the average is about 17 per cent. In the case of the natural hydrograph, the flow exceeds the dominant for about 18 per cent of the cycle time (just over two months per annum in prototype terms), but a little reflection will show that this is not very meaningful. For about 50 per cent of the time the flow in the river is a mere trickle, so much so that in the model it was lost in exfiltration and did not reach the outlet from the model area. Now it is hardly conceivable that the meander geometry would be influenced by the duration of this drought period, so that if the cycle time had been increased from 7 days to 14 days by trebling the duration of drought, the percentage of time for which the dominant discharge was exceeded would have reduced to about 10 per cent. Percentage exceedance is not likely to be a significant parameter if the flow cycle includes a drought period.

Comparison with Field Data.

It was hoped that with the above analysis it would be possible to define a suitable measure of discharge to permit the correlation of variable discharge data in nature with steady discharge tests in laboratory. The evidence from our tests with sinusoidal and natural hydrographs is that for such a correlation to be valid the discharge for a variable flow run is significantly above the mean. In fact, if flow is entirely contained within the channel banks and varies sinusoidally, the discharge effective in generating the meander pattern is about 95% of its maximum value. When overbank flow occurs with a natural hydrograph, then the effective discharge is still much above the mean yet significantly less than 95% of maximum - whether this is because some of the flow is conveyed over

the flood-plain or because of the peaky form of a typical natural hydrograph is not known. However, definition of a formative discharge linked to bank-full conditions is consistent with traditional views of Inglis and his colleagues, and also with any theory that is based on transverse surface waves. By reference to the Kaduna river model, 'dominant' seems to be 10 to 20 per cent above bank-full discharge when the discharge pattern entails seasonal conveyance over flood-plains.

Despite many published data from the field, incorporation of these field data is in some cases suspect as the following brief review indicates:

Rivers in Orissa, India. (27)

These provide traditional data, much used and repeated, and upon which Inglis relied to a considerable extent. Carlston says that Inglis originally mistook the definition of meander length, and he also considers that the selection of figures for λ were subjective, and biased. He suggests there are serious doubts as to whether the associated discharge figures are really bank-full values, or somewhat above bank-full, and he questions how they were estimated anyway. The Orissa data have to be viewed with reserve.

Leopold and Wolman's data on American rivers. (27)

Discharges were determined in several different ways, but in only one case was bank-full flow deduced from an established stage-discharge curve. In others, the flood with a 1.15 year return period was assumed to equate to bank-full; some were interpolated from a regional function with drainage area. Their listed meander lengths are very rounded figures, suggesting they are not statistical averages over a number of meanders, but subjective assessments of typical values, so again they must be treated with reserve.

Carlston's data on American rivers. (10)

Carlston adopted a critical attitude to previously published data, but unfortunately his values for bank-full discharge were not deduced directly, the 1.5 year return period flood was used on the basis of an unpublished study by D.M. Thomas of the U.S.G.S. without proof that in all catchments and all parts of a catchment bank-full conditions were achieved with equal frequency. Carlston preferred other discharge measurements which he concluded gave better correlation ("better" in the sense of reduced scatter). His collection of data is from a limited geographic region, and he did not consider whether other climatological conditions would fit in with this. Meander lengths are, however, statistical averages over a reach and appear reliable.

Zeller's data for Swiss rivers. (52)

His analysis of 50 river reaches was on the basis of the 50 year flood -- such a rare event that it seems unlikely that it would be sufficiently confined to the river channel as to dominate the channel geometry. The data is not given in detail but his plots do permit average meander lengths to be extracted for river reaches, and data is given on mean discharge and relative flows for different return periods that enables one to deduce, for example, the mean annual flood.

Speight's data for River Angabunga. (41)

A spectral analysis of the plan form of this river in central Papua was carried out by Speight in respect of the channels as they existed in 1938, 1957, and 1963. He used thalweg wavelengths which may nevertheless be converted to the more usual definition from a knowledge of tortuosity. The longest wavelength emerging from the analysis he concluded to be less significant than

the second and third ones, referred to as B and C peaks "Because the C-peak is characteristically the shortest wavelength dominant peak the subjectivity assessed typical meander wavelength often corresponds to it....." "The B-peak is the most stable and persistent peak of all, dominating nearly all plans-course spectra ....." In addition to the elegant analysis of plan form, a flood frequency study was included, unfortunately related to an incomplete catchment (the gauging station was upstream of the meandered reaches). Bank-full flows were deduced from channel roughness, taking the section to tree-line and grass-line; and mean annual flow was also quoted.

Schumm's data on American and Australian Rivers (44)

He recorded in the appendix to his paper the meander length, the mean annual discharge and the mean annual flood discharge of 33 rivers in the United States and one river in Australia at three sites. All the river channels were located in the semi-arid to sub-humid regions of the Great Plains of the United States and in the riverine plain of New South Wales, Australia. All are alluvial channels in that their beds and banks are composed of sediment that is being transported by the river.

H.R.S. data on R. Kaduna, Nigeria. (3)

The meander length here was deduced from aerial photographs and averaged over a stretch of many miles. The bank-full discharge was deduced from an established stage- discharge relationship within this reach. This river in Nigeria seems to have very different characteristics from American rivers cited by Carlston because it is over-bank at a discharge of 40 per cent of mean annual flood, and remains over-bank for almost 20 per cent of the year, demonstrating that a regional relationship of bank-full discharge to

return period cannot be applied to other different geographic regions. For example, flows of bank-full and above occur in Britain for approximately 0.6 per cent of the time (36), and the bank-full discharge has a return period of under a year compared with 1.5 years assumed by Carlston.

These varied data from different sources, with the several authors' differing bases of evaluating the parameters, are not strictly compatible. However, the present Authors have endeavoured to compare these field observations with their own model scale results for the following definitions of discharge:

(a) mean flow
(b) the mean flow of the highest twelfth of the cycle period
(c) bank-full discharges.

Not all sources of data permit plotting on all three systems, and from the remarks above it will be realised that bank-full discharges are in some cases inferred from an assumed frequency of occurrence or otherwise estimated rather than deduced directly from a stage-discharge relationship. Despite the deficiencies, the comparisons in Fig. 21 are of considerable interest.

The mean flow plot in Fig. 21b includes Carlston's best-fit equation to his own field data:

$$\lambda = 106.1\ Q_a^{0.46} \qquad \ldots\ldots (72)$$

and the H.R.S. laboratory result (eqn. 3) for the steady discharge runs

$$\lambda = 38.0\ Q^{0.467}$$

The close similarity of the exponents is remarkable but the large difference in the coefficients reinforces the conclusion already drawn that the average discharge is not the dominant condition, in the sense of giving meander

lengths equal to the corresponding steady flow. The Swiss rivers agree well on average, although if treated separately would have produced a slightly higher exponent. The variation of meander length from reach to reach of the Angabunga river is not explained, as all carry an equal mean annual discharge, but it is the B-peak that is closest to Carlston's equation. The one Nigerian river also lies close to, but below this equation.

The difference in coefficients is reduced when the mean of the highest twelfth of the flow is taken, the equations then being

$$\text{Carlston,} \quad = 80.0\ Q^{0.46} \qquad \ldots\ldots (73)$$

$$\text{H.R.S.} \quad = 38.0\ Q^{0.467}$$

The tests with sinusoidal variation conducted at Wallingford plot on the same line as the steady flow runs on this basis, but the model R. Kaduna is low. The prototype R. Kaduna lies right on the extrapolation of the laboratory equation however.

Although Carlston's analysis led him to conclude that the exponent of a meander length to discharge relationship based on mean discharge is 0.46, he deduced that the corresponding relationship based on a flood discharge - and he naturally included his own use of a 1.5 year flood as a bank-full flow in this category - must have an exponent above 0.46. He supported this theoretically by reasoning that the long-term mean discharge is directly proportional to catchment area, whereas flood flows are proportional to some power of area less than one, a usual empirical value being 0.75. Thus if the "mean discharge" exponent is 0.46, the "flood discharge" exponent must be about 0.61. His data for the 1.5 year flood, taken from a limited American geographic region, appeared to support this contention, his best-fit equation being

$$\lambda = 8.2\ Q_{1.5}^{0.62} \qquad \ldots\ldots (74)$$

This is shown on Fig. 21a, together with the H.R.S. laboratory data and equation, and field information from the other sources referred to in this report.

When these data from other regions are brought in and compared with laboratory scale data, the case for an exponent of 0.62 seems inadequately established. The extrapolated laboratory equation is close to five of the six Swiss points, the Orissa data which Carlston has cast doubts on, and the C-peak values for the Angabunga. The last, together with the prototype Kaduna are close to Carlston's equation also, as the two equations intercept at a flow of approximately 15,000 cusecs. The bank-full basis may also explain the variation in meander length in the three reaches of the Papuan river.

We are left wondering, on inspecting Fig. 21a whether the information on some of the smaller American rivers is suspect, either because the estimate of bank-full conditions in those cases was inaccurate or inappropriate, or because an incompatible spectral length has inadvertently been selected - a sub-meander pattern generated by flows much below the channel capacity.

Unfortunately few of those researchers reporting field data on meander length have given corresponding sediment data. Thus few of the prototype results can be added to the type of non-dimensional plot which highlighted the influence of sediment size in laboratory scale channels. Figure 22 includes the available field data from the Angabunga (involving a guess at the sediment sizes described as "gravel" and "medium sand"), the Kaduna, and Leopold and Wolman's information on American rivers. The tentative family of curves, depending on D_g, have been taken from the laboratory plot in Fig. 19 and are shown by broken lines. The field data have indicated alongside them their approximate D_g values, where

$$D_g = \sqrt[3]{\frac{g(s-1)}{\nu^2}}\, D \qquad \ldots\ldots (75)$$

Looking first at the B-peak of the spectral analysis of the Angabunga (upright A symbol), the Kaduna (triangle) and the Mississippi (extreme right-hand point), the sparse field data nevertheless extends the laboratory information adequately. The gravel reach of the Angabunga (D_g = 280) together with the Leopold and Wolman data at D_g values of 990 and 920, appear to define a limiting curve for coarse grained sediments*.

The Angabunga C-peak values (inverted A symbol) lie well above the curves appropriate to their sediment size, because of course this spectral value for meander length is some 1.4 to 1.8 times less than the B-peak. (Meander length appears to the power -2.5 in the ordinate). Leopold and Wolman's** points marked 88, 250, 530, 540 and 580 also lie above or to the right of the D_g > 30 line, which could be due to incompatible values of either Q, λ or D. These data relate to catchment areas of 4 to 24 square miles, with bank-full discharges yielded by run-offs of 30 to 60 cusecs/sq mile, compared with Nixon's data (see Fig. 5 ref 36) which would suggest no more than 10 cusecs/sq. mile from British rivers of this order of magnitude with an annual rainfall of some 30 inches. So it seems that if the American bank-full discharge estimates in these small gravelly rivers are realistic, they may nevertheless be such infrequent occurrences as not to define the formative flow. This suggests a qualification to the use of bank-full discharge as the dominant condition for meander formation: If the river does not run bank-full

* It is thought that when D_g > 30, viscous effects no longer influence sediment motion, and hence it would follow that any functional relations implying sediment transport would cease to depend on D_g at values above 30, e.g. sands coarser than 1.3 mm. The Haydite point on Fig. 19 is close to this limit.

** Respectively referenced 67, 25, 65, 68 and 30 in Appendix G, ref. 27.

at least annually, then a flow related to the mean annual flood may be more appropriate. More reliable field data with associated sediment analysis are highly desirable before drawing firm conclusions about the validity of the extrapolation of the analysis based on the three non-dimensional parameters, but indications are that the field results are reasonably consistent. This confirms White's opinion (54 discussion to ref. 19) "that particle size influenced the meander length to some extent".

SUMMARY AND CONCLUSIONS

Many reports on meandering channels have been written, some being based on small-scale laboratory experiments and some on the analysis of field-scale data, and almost as many hypothesis have been put forward to explain why rivers meander. The purpose of the present series of experiments was to add to the fund of data, in the hope that by conducting research under the controlled conditions available in a laboratory, yet at a much larger scale than in most previous experimental investigations, greater confidence could be placed in the relevance of the results to practical conditions in nature.

The study was made in a closely graded sand of 0.15 mm median diameter at discharges between a quarter and two cusecs. Tests with time-varying flows provided evidence about the discharge that could be said to dominate the processes defining channel geometry and size, adding weight to the opinions expressed by some previous authors and discounting others. By incorporating laboratory data from other sources, suggestions are made as to the influence of sediment size on meander length -

a fairly small but quite significant effect. A new dimensional analysis of the factors affecting regime geometry in alluvial channels provides suitable parameters for the analysis of both laboratory and field data, and what limited field data is available is consistent with the extrapolation of the laboratory predictions. This analysis also provides possible explanations for the many different exponents and coefficients put forward from time to time in equations relating meander length to discharge.

Consideration has also been given to the conditions leading to meandering, the conclusion being that meandering streams are steeper than straight ones, and provide a much greater resistance to flow. Clearly, there is a potential instability in straight streams.

The conclusions may be summarised as follows:

1. Meandering will begin in an initially straight channel in alluvium if instability in bed sediment transport leads to a series of relatively large shoals on alternative sides of the stream. Only after these shoals become prominent does bank erosion produce the sinuosity characteristic of meandering.

2. There are two dynamically stable channel patterns viz. straight or slightly irregular, and regular sinuous channels.

3. Excluding certain results which are suspect because of the growth of algae, the meander length under steady conditions at flows between 0.25 and 2.0 cusecs in 0.15 mm sand correlates well with discharge, the best-fit equation being

$$\lambda = 38.0\, Q^{0.467}$$

4. The width of a meandered channel is at least twice that of a straight channel in the same sediment at the same discharge: channels with prominent shoals

that nevertheless remain straight are under twice as wide.

5. No conclusion is drawn from these tests about the relationship of meander breadth to discharge: the correlation is poor.

6. The geometrical ratios λ/b, β/b and λ/β have been plotted but the only conclusion of significance is that meander length may well be proportional to channel breadth as has previously been reported.

7. A multiple correlation of meander length with discharge and sediment concentration (i.e. sediment flux as proportion of fluid flux) does not prove any appreciable dependence on sediment concentration, the exponent of X being small and ill confirmed:

$$\lambda = 42.9\ X^{-0.021}\ Q^{0.480}$$

8. The pitch of shoals increases by some 10 per cent as the channel widens and as the shoals develop, but the wavelength of meanders finally observed agrees closely with that of the initial shoals.

9. If the sediment injection rate is above the threshold value after the formation of meanders on an aggrading channel the meander length tends to decrease as the sediment load increases.

10. The migration of meanders downstream was confirmed: in these experiments rates up to 5 ft/day were observed.

11. Tests with fluorescent sand showed that material eroded from one bank did not cross over to the other bank.

12. There is a threshold value of sediment charge below which channels of small discharge tend to remain straight and above which they meander, provided the available valley slope does not prevent the development of the hydraulic gradient necessary to

transport the discharge and sediment.

13. When the valley slope is insufficient for the development of the hydraulic gradient necessary to transport the discharge and sediment as a straight or meandering channel depending on the sediment charge then a braided channel pattern develops.

14. Channel resistance has been analysed both in terms of straight-line slope and slope along the thalweg: the dimensionless Chezy C values that resulted average about 5.5 and 6.5 respectively, compared with about 10 in a straight channel in the same sediment.

15. These small streams can be categorised as straight, straight but exhibiting well developed shoals and meandered depending on the straight-line slope as follows:

$$\text{Straight:} \quad S < 0.0015\ Q^{-0.12}$$

$$\text{Meandered:} \quad S > 0.0021\ Q^{-0.12}$$

Straight but shoaled: S between above limits

16. The minimum slope of a meandered channel is about 1.5 times the maximum slope of a straight channel without marked shoal development in the same sediment at the same discharge.

17. Comparing the energy loss of a meandered channel with that of a straight channel without well developed shoals 60% may be due to its non prismatic boundary i.e. arising from bends and major variations in cross section. Comparing the energy losses at the threshold of meandering, however, the losses due to shoaling are about 25% of the total.

18. Maximum depths at bends and at cross-overs, are about the same as those occurring in a straight channel in the same alluvium at the same discharge.

19. The maximum bed shear in a meandered stream is about 25% greater than the average bed shear in the corresponding straight channel.

20. Dimensionless parameters may be derived incorporating either the actual D_{50} size of the sediment or an adjusted diameter deduced from the initial motion function, but no preference for the latter emerges.

21. The use of sediment size as the linear quantity to convert a λ, Q plot to dimensionless terms is apt to provide spurious relationships: such a two parameter plot does not adequately account for the influence of sediment size.

22. It has not been possible on the available evidence to say whether free-surface effects are important in the meandering process. Gravity influences sediment mobility, but whether it directly controls geometry via surface waves has not been established or refuted.

23. A three parameter plot elucidates the influence of sediment size, the preferred dimensionless variables being λ/D, $Q/\lambda^2\sqrt{(g\lambda)}$, and $D\sqrt{gD(s-1)}/\nu$.

24. On the above basis, various relationships of meander length to discharge may be compared with laboratory data, but it is concluded that no simple exponential relationship is completely satisfactory.

25. It follows that neither the Inglis relationship of λ to $Q^{0.5}$, nor the Froude-type one of λ to $Q^{0.4}$, nor any of the many empirical alternatives put forward have special significance. Almost any exponent from 0.375 to 0.6 could be deduced depending on the range and grouping of data and the parameters used.

26. In the almost complete absence of data for sediments of other densities, the influence of relative specific gravity has not been established, but one result for

light-weight material is consistent with the three parameter plot mentioned above.

27. Dominant discharge has been defined herein as the steady flow that would yield the observed meander length in an otherwise similar experiment.

28. With sinusoidal variation, the dominant discharge was 95 per cent of the maximum of the cyclic variation whereas with a modelled natural hydrograph involving over-bank flow the proportion was 42 per cent.

29. Inglis' recommendation that bank-full flow or a little more was the "dominant" condition is supported by this investigation, with the definition in 28 above.

30. A comparison of field scale data on meander geometry with laboratory tests at steady discharge suggests that bank-full discharge, or a flood with a recurrence interval of about a year, (whichever is the lesser in a given reach of river) generates the plan geometry.

31. Carlston's preference for mean annual discharge does not accord with the available evidence with the above definition of dominant discharge.

32. With some uncertainty regarding gravel rivers, prototype data confirm and extend the results of laboratory tests when plotted on the basis of the preferred three dimension-less parameters.

33. It is desirable that field studies of meandering should include adequate analysis of the sediment character, and a full study of the discharge pattern based on direct measurement, as well as the plan geometry.

34. Laboratory scale experiments using sediments having a wide range of specific gravities would help clarify the roles of sediment mobility and gravity waves in the initiation of meanders.

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LIST OF SYMBOLS

			Dimensions
A	=	Area of cross section of a channel	L^2
A_m	=	Area of cross section of a meandering channel	L^2
A_s	=	Area of cross section of a straight channel	L^2
b	=	Width of channel	L
C'	=	Dimensionless Chezy Coefficient for straight channels in equation $v = C' \sqrt{g\, m_s\, i_s}$	-
C'_m	=	Dimensionless Chezy Coefficient for meandering channels using thalweg water surface slope in $v = C'_m \sqrt{g\, m_m\, i_m}$	-
C'_s	=	Dimensionless Chezy Coefficient for meandering channels using a straight line water surface slope in equation $v = C'_s \sqrt{g\, m_m\, S}$	
D	=	Median grain diameter	L
D_e	=	Adjusted grain diameter to allow for effect of Reynolds number on initial motion $\tau_c/\rho = \overline{0.C4\ g(s-1)}$	L
D_g	=	Dimensionless parameter defining the grain size $D \sqrt[3]{g(s-1)/\nu^2}$	-
d	=	Depth of flow	L
d_{mb}	=	Maximum depth of meandering channel at a bend	L
d_{mc}	=	Maximum depth of meandering channel at a cross-over	L
d_s	=	Mean depth of straight channel	L
F	=	Froude number	-
g	=	Acceleration due to gravity	LT^{-2}
i_k	=	Slope necessary to overcome bed roughness in a meandered channel	-
i_m	=	Water surface slope in a meandered channel measured along the thalweg	-
k	=	Mean ripple height	L

k_m	=	Equivalent sand roughness in a meandered channel	L
k_s	=	Equivalent sand roughness in a straight channel	L
l	=	Length of thalweg corresponding to one meander length	L
m_m	=	Hydraulic mean depth of a meandering channel	L
m_s	=	Hydraulic mean depth of a straight channel	L
P	=	Wetted perimeter of channel	L
Q	=	Liquid discharge, volume of flow per unit time	L^3T^{-1}
Q_a	=	Mean annual discharge	L^3T^{-1}
Q_b	=	Discharge at bank-full stage	L^3T^{-1}
Q_d	=	Dominant discharge	L^3T^{-1}
Q_f	=	Mean annual flood discharge	L^3T^{-1}
Q_m	=	Discharge in a meandering channel	L^3T^{-1}
Q_{max}	=	Maximum discharge occurring during a hydrographic cycle	L^3T^{-1}
Q_s	=	Discharge in a straight channel	L^3T^{-1}
Q_t	=	Mean discharge during the period of the highest twelfth discharges each year	
r	=	Radius of meander bend	L
S	=	Valley slope or water surface slope of a meandering channel measured along a straight line parallel to the axis of the valley	-
s	=	Specific gravity of solids	-
v	=	Mean velocity of flow = $\frac{Q}{A}$	LT^{-1}
v^*	=	Shear velocity $\sqrt{g\ m\ i}$	LT^{-1}
X	=	Ratio by mass of the sediment discharge to the liquid discharge.	-
α	=	Coefficient in the empirical equation relating λ and Q	-
β	=	Width of the meander belt	L
γ	=	Immersed weight per unit volume of solids $\rho g(s-1)$	MLT^{-2}
δ	=	Standard deviation from the mean	-
ζ_r	=	Factor by which C' must be multiplied to account for change in roughness occurring in meandering channels	-

ζ_f	=	Factor by which C' must be multiplied to account for increased resistance due to form losses (bends, non prismatic section etc.)	-
η	=	A coefficient used in Fig. 17 and equations 67 and 69	-
η'	=	A function of $D_g^{3/2}$	-
λ	=	Wave length of meander or twice the pitch of shoals	L
μ	=	Viscosity of water	$ML^{-1}T^{-1}$
ν	=	Kinematic viscosity of water = $\frac{\mu}{\rho}$	L^2T^{-1}
ρ	=	Mass density of water	ML^{-3}
σ	=	Sinuosity of meander = $\frac{l}{\lambda} = \frac{S}{i_m}$	-
τ	=	Overall boundary shear stress	$ML^{-1}T^{-2}$
τ_c	=	Boundary shear stress to initiate sediment motion	$ML^{-1}T^{-2}$
ϕ	=	A mathematical function of :	-
ψ	=	A mathematical function of :	-

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TABLES

TABLE 1

Dimensionless correlation

Best-fit equations

	Sediment size	Equations	
(a)	0.15 mm	$\frac{\lambda}{D} = 26.9\left(\frac{Q}{D^2\sqrt{gD(s-1)}}\right)^{0.467}$	.... (49)
		$\frac{\lambda}{D_e} = 41.5\left(\frac{Q}{D_e^2\sqrt{gD_e(s-1)}}\right)^{0.445}$	.... (50)
(b)	0.21 - 0.26 mm	$\frac{\lambda}{D} = 7.58\left(\frac{Q}{D^2\sqrt{gD(s-1)}}\right)^{0.552}$	.... (51)
		$\frac{\lambda}{D_e} = 7.53\left(\frac{Q}{D_e^2\sqrt{gD_e(s-1)}}\right)^{0.557}$	.... (52)
(c)	0.45 - 0.70 mm	$\frac{\lambda}{D} = 59.7\left(\frac{Q}{D^2\sqrt{gD(s-1)}}\right)^{0.453}$	.... (53)
		$\frac{\lambda}{D_e} = 56.8\left(\frac{Q}{D_e^2\sqrt{gD_e(s-1)}}\right)^{0.454}$	.... (54)
	All together	$\frac{\lambda}{D} = 123\left(\frac{Q}{D^2\sqrt{gD(s-1)}}\right)^{0.376}$	.... (55)
		$\frac{\lambda}{D_e} = 130\left(\frac{Q}{D_e^2\sqrt{gD_e(s-1)}}\right)^{0.371}$	.... (56)

The exponents in the above are not far removed from 0.4, the value that would eliminate D and D_e from the relationship. It is clear therefore that the use of sediment diameter as a basic scaling dimension is unsound, and the interpretation of the dimensional analysis given above requires modification.

TABLE II

STEADY DISCHARGE EXPERIMENTS

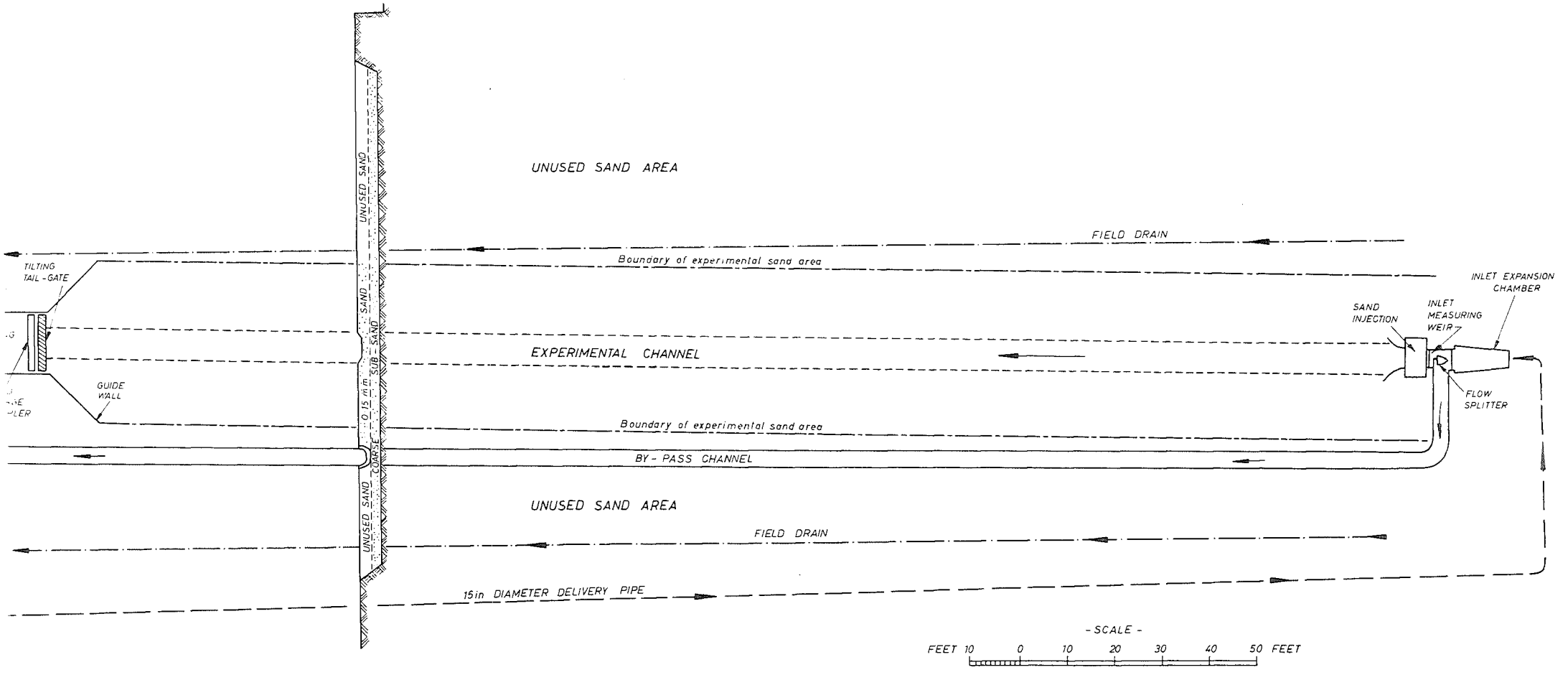
Run No.		Discharge Cusecs	Sed. Trans. Rate mg/litre	Straight line Water Surface slope	Thalweg Water-Surface slope	Meander length feet	Meander breadth feet	Channel width -feet Mean	Temperature °C Mean	Equiv. Grain size at 15°C feet x 10^{-4}	Velocity of fall ft/sec	Sinuosity	Remarks
1	M	3.26						9.53	13.0	4.76	0.0452		Run stopped early. Not fully developed.
1	D	2.74	-	-	-	59.6	16.7	12.77					
2	M	0.332	-	-	-	20.4	6.8	3.25	11.7	4.68	0.0442		Not fully developed.
2	D	0.239	-	-	-	17.5	3.7	2.80					
3	M	0.251	-	-	-	16.5	5.9	3.61	11.5	4.67	0.0440	1.23	Growth of algae on banks.
3	D	0.206	-	-	-	-	-	3.23					
4	M	0.373	-	0.00220	0.00207	23.7	6.7	3.98	10.0	4.57	0.0430	1.06	Not fully developed
4	D	0.337	-	0.00283	0.00210	15.5	7.4	4.77				1.35	
5	M	0.328	-	0.00240	0.00202	22.8	8.0	4.61	9.0	4.50	0.0419	1.18	
5	D	0.301	-	0.00291	0.00191	18.2	9.3	6.29				1.53	
6	M	0.655	-	0.00196	0.00141	33.9	14.1	6.68	3.4	4.14	0.0364	1.38	
6	D	0.621	-	0.00279	0.00196	37.9	19.7	6.05				1.42	
7	M	0.628	125	0.00214	0.00146	31.4	15.4	5.98	5.0	4.24	0.0380	1.47	
7	D	0.579	108	0.00292	0.00189	32.3	17.6	5.89				1.55	
8	M	1.132	325	0.00142	0.00113	40.8	16.6	8.35	5.4	4.26	0.0383	1.26	
8	D	1.099	451	0.00310	0.00212	39.9	21.8	8.75				1.46	
9	M	1.935	405	0.00207	0.00155	48.7	19.4	10.32	6.4	4.33	0.0394	1.33	
9	D	1.872	487	0.00308	0.00232	51.4	28.2	10.1				1.33	
10	M	1.091	311	0.00237	0.00175	38.1	16.1	8.20	9.8	4.56	0.0425	1.35	
10	D	1.051	421	0.00237	0.00175	42.7	21.9	9.03				1.35	
11	M	1.038	442	0.00203	0.00176	35.5	15.2	9.16	13.0	4.76	0.0452	1.15	
11	D	0.996	490	0.00304	0.00179	36.8	21.9	10.00				1.70	
12	M	1.067	565	0.00330	0.00236	38.8	18.2	7.63	14.5	4.86	0.0466	1.39	
12	D	1.037	537	0.00261	0.00199	40.5	17.7	7.39				1.31	
13	M	0.559	390	0.00221	0.00149	28.4	14.7	6.82	16.7	5.01	0.0487	1.49	
13	D	0.494	439	0.00340	0.00192	27.2	17.8	7.10				1.76	
14	M	0.568	612	0.00265	0.00197	27.4	12.2	6.51	17.8	5.08	0.0498	1.34	
14	D	0.503	617	0.00256	0.00193	26.9	12.5	5.42				1.33	
15	M	1.82	568	0.00203	0.00184	49.5	15.1	9.42	19.6	5.20	0.0514	1.11	Not fully developed
15	D	1.71	902	0.00305	0.00220	48.2	33.2	15.00				1.38	
19	M	0.353	84	0.00320	0.00229	21.5	10.8	7.08	9.7	4.55	0.0423	1.40	
19	D	0.245	52	0.00247	0.00193	20.1	8.3	5.44				1.28	
20/I	M	0.649	73	0.00191	0.00172	-	-	6.03	5.0	4.24	0.0380	1.11	Straight channel
20/I	D	0.612	137	0.00220	0.00174	27.8	9.2	6.40				1.30	Meandered at high sediment load. Reverting to straight channel
20/II	M	0.602	125	0.00238	0.00176	28.6	12.9	9.44	5.0	4.24	0.0380	1.35	
20/II	D	0.556	107	0.00263	0.00181	32.0	16.2	10.16				1.45	
21	M	0.304	108	0.00283	0.00199	22.1	11.1	6.84	7.5	4.40	0.0404	1.42	
21	D	0.246	116	0.00325	0.00276	21.2	10.5	6.55				1.18	

TABLE III

VARIABLE DISCHARGE EXPERIMENTS

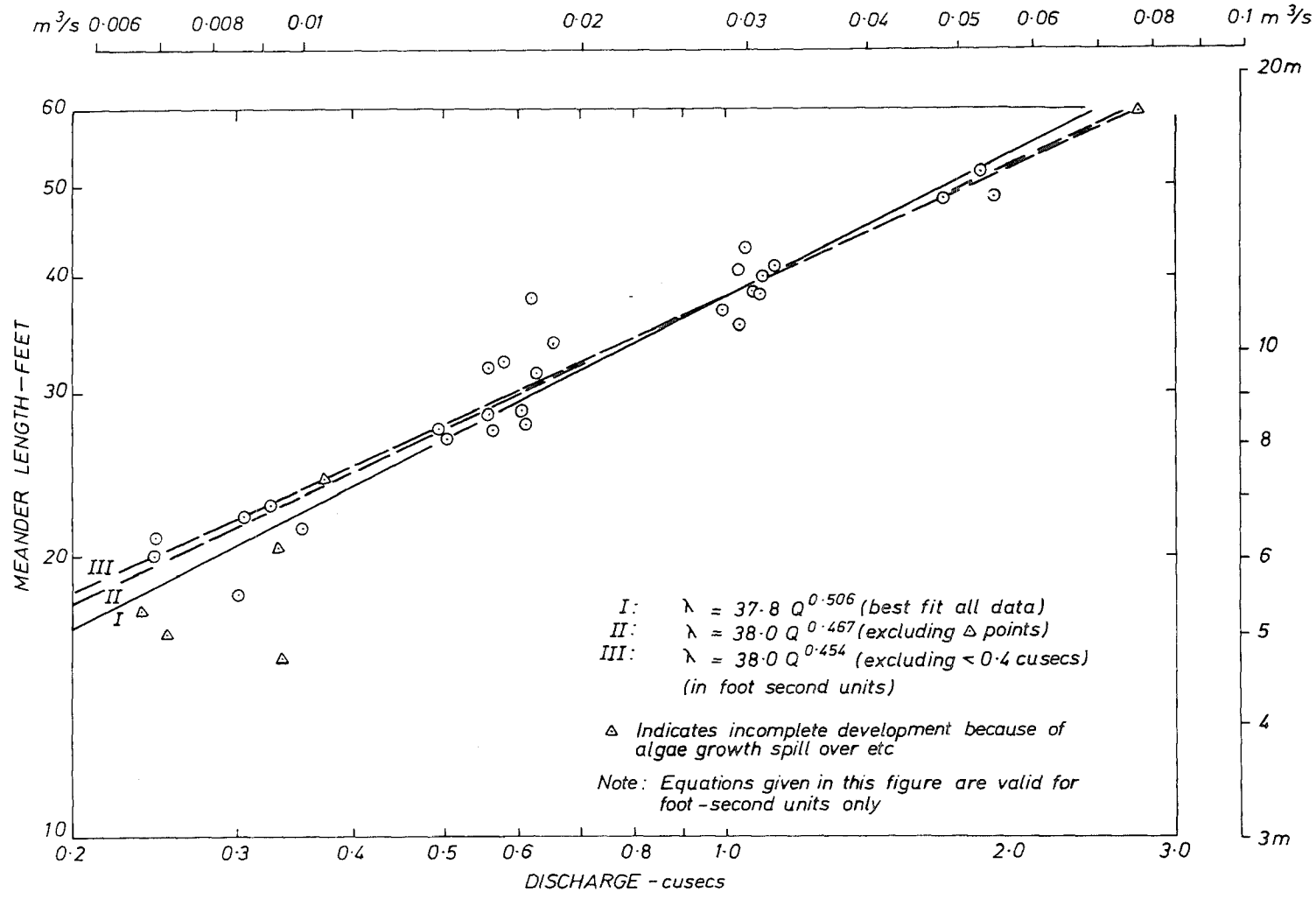
Run No.	Duration hrs	Discharge - cusecs			Flow pattern	Observed Meander Length ft	Dominant Discharge Q_d Cusecs	Ratio Dominant to max Discharge	Ratio Dominant to mean Discharge	$\frac{Q_d - Q_{mean}}{Q_{max} - Q_{mean}} = \alpha$	arc Sin α degrees	Percent of cycle Q exceeds Q_d	$\lambda_{Q_{max}}$	$\frac{\lambda_{obs}}{\lambda_{Q_{max}}}$
		Max Q_{max}	Mean Q_{mean}	Min Q_{min}										
16M	365	1.168	0.563	0	sin (i) (8 hr)	37.9	0.996	0.853	1.769				40.9	0.927
16D	"	1.103	0.506	0	"	38.4	0.975	0.884	1.927				39.8	0.965
16(a)	"	1.135	0.534	0	"	38.1	0.985	0.868	1.845	0.750	48.6	23.0	40.4	0.944
17IM	193	1.164	0.579	0	"	40.9	1.172	1.007	2.024				40.9	1.000
17ID	"	1.111	0.533	0	"	39.0	1.058	0.952	1.985				40.0	0.975
17I(a)	"	1.137	0.556	0	"	39.9	1.115	0.978	2.004	0.963	74.4	8.7	40.5	0.987
17IIM	384	1.164	0.509	0	"	39.0	1.058	0.909	2.079				40.9	0.953
17IID	"	1.101	0.453	0	"	39.6	1.094	0.994	2.415				39.8	0.996
17II(a)	"	1.132	0.481	0	"	39.3	1.076	0.951	2.240	0.913	65.9	13.4	40.4	0.973
18IM	295	0.890	0.578	0.267	sin(ii) (8 hr)	32.0	0.693	0.779	1.199				36.0	0.389
18ID	"	0.831	0.528	0.228	"	36.0	0.892	1.073	1.689				34.7	1.037
18I(a)	"	0.860	0.553	0.247	"	34.0	0.788	0.917	1.426	0.766	50.0	22.2	35.3	0.964
18IIM	393	0.877	0.587	0.249	"	32.6	0.721	0.823	1.228				35.7	0.913
18IID	393	0.821	0.538	0.206	"	38.0	1.000	1.218	1.859				34.4	1.103
18II(a)	"	0.849	0.562	0.227	"	35.3	0.854	1.007	1.520	1.018	-	nil	35.1	1.033
Kaduna	1010	1.50	0.250(b)	0	Hyd(iii) (7 day)	30.6	0.630(c)	0.42	2.52			17.8(d)	45.9	0.667

(a) Average of M & D Sections
(b) Over 7 days
(c) Corresponds to average if all flow were discharged in 2.78 days (8.11 days = 1 proto yr; 2.78 days = 0.343 yrs ≃ 4 months)
(d) Corresponds to 65 days in prototype ≃ 2 months
(i) Discharge varying sinusoidally about mean value with extremes approximately equal to 5% and 195% of the mean. Period of oscillation 8 hours.
(ii) Discharge varying sinusoidally about mean value with extremes approximately equal to 50% and 150% of the mean. Period of oscillation 8 hours.
(iii) Repeating hydrograph similar to typical annual discharge pattern of the Kaduna River in Nigeria.



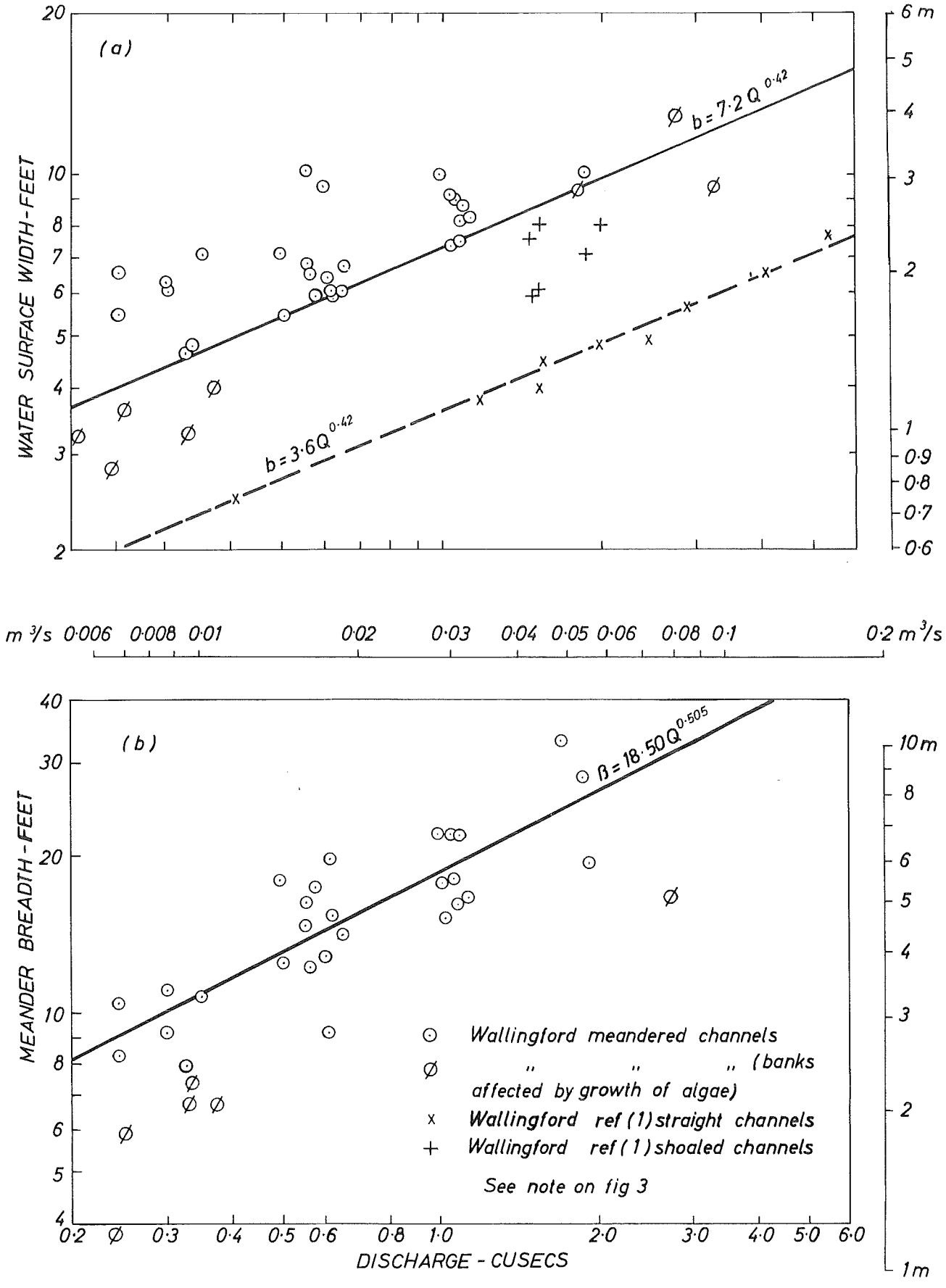
PLAN OF EXPERIMENTAL AREA

FIG 1



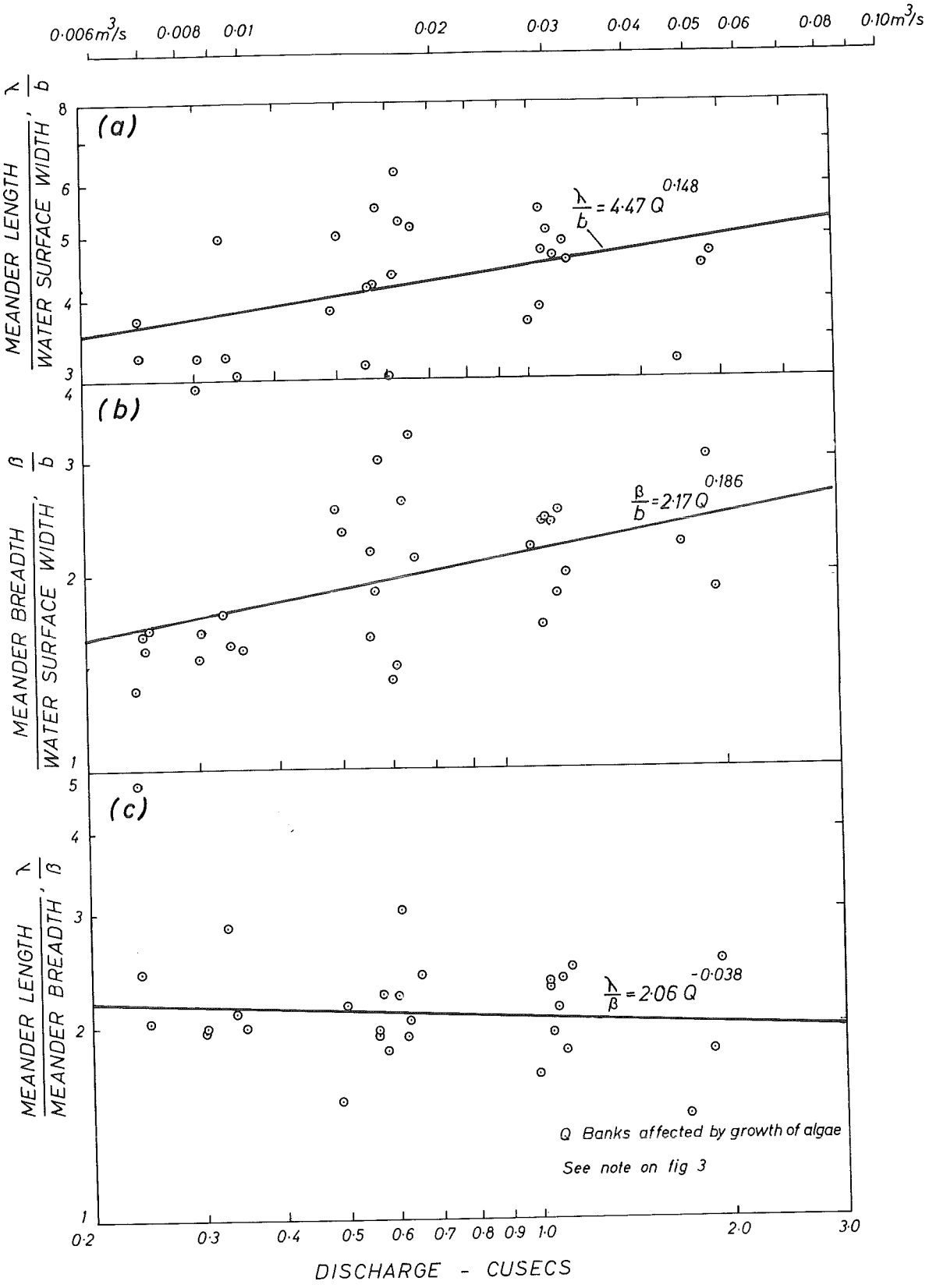
VARIATION OF MEANDER LENGTH WITH DISCHARGE

FIG 3



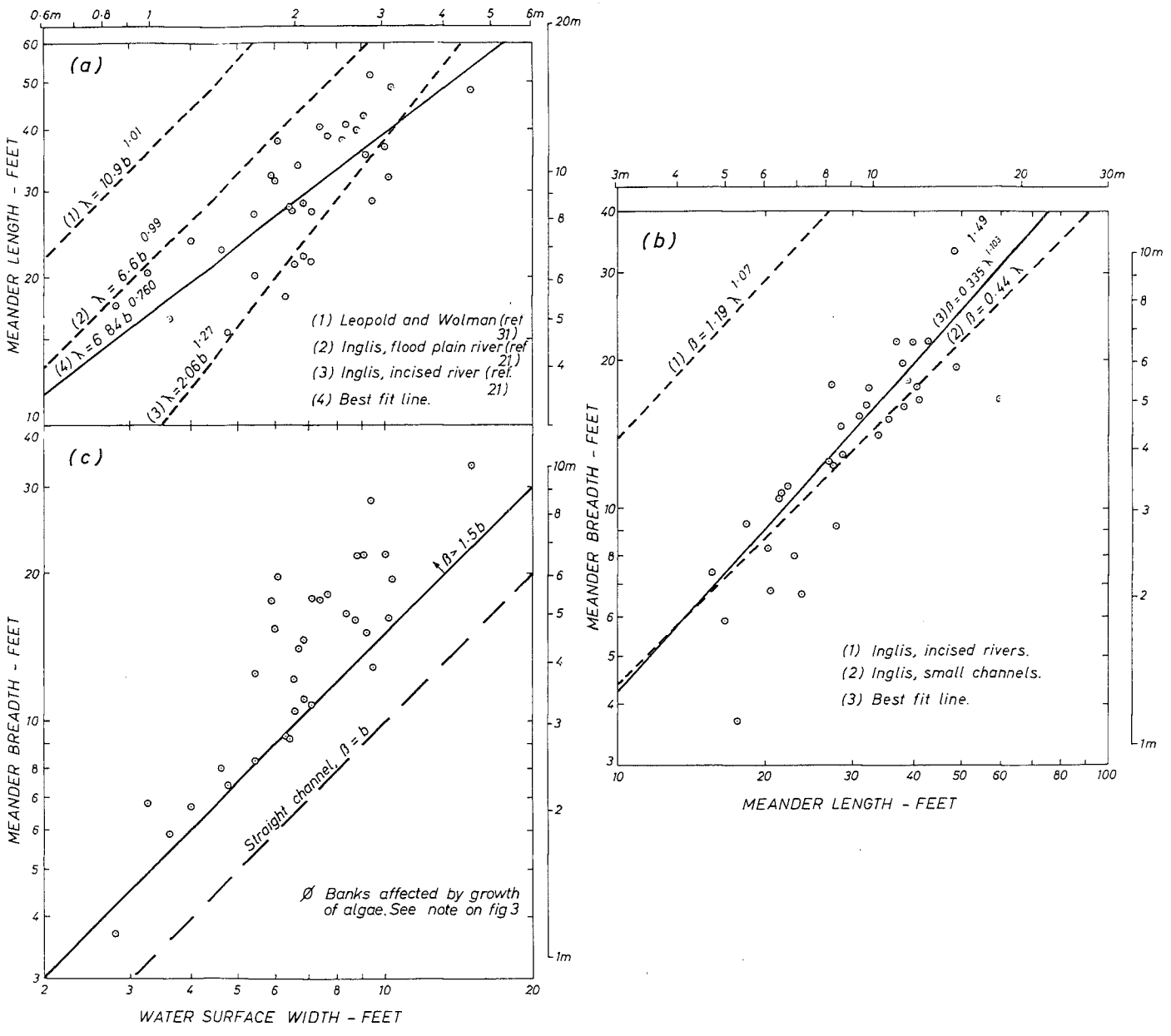
RELATION BETWEEN WATER SURFACE WIDTH MEANDER BREADTH AND DISCHARGE

FIG 4



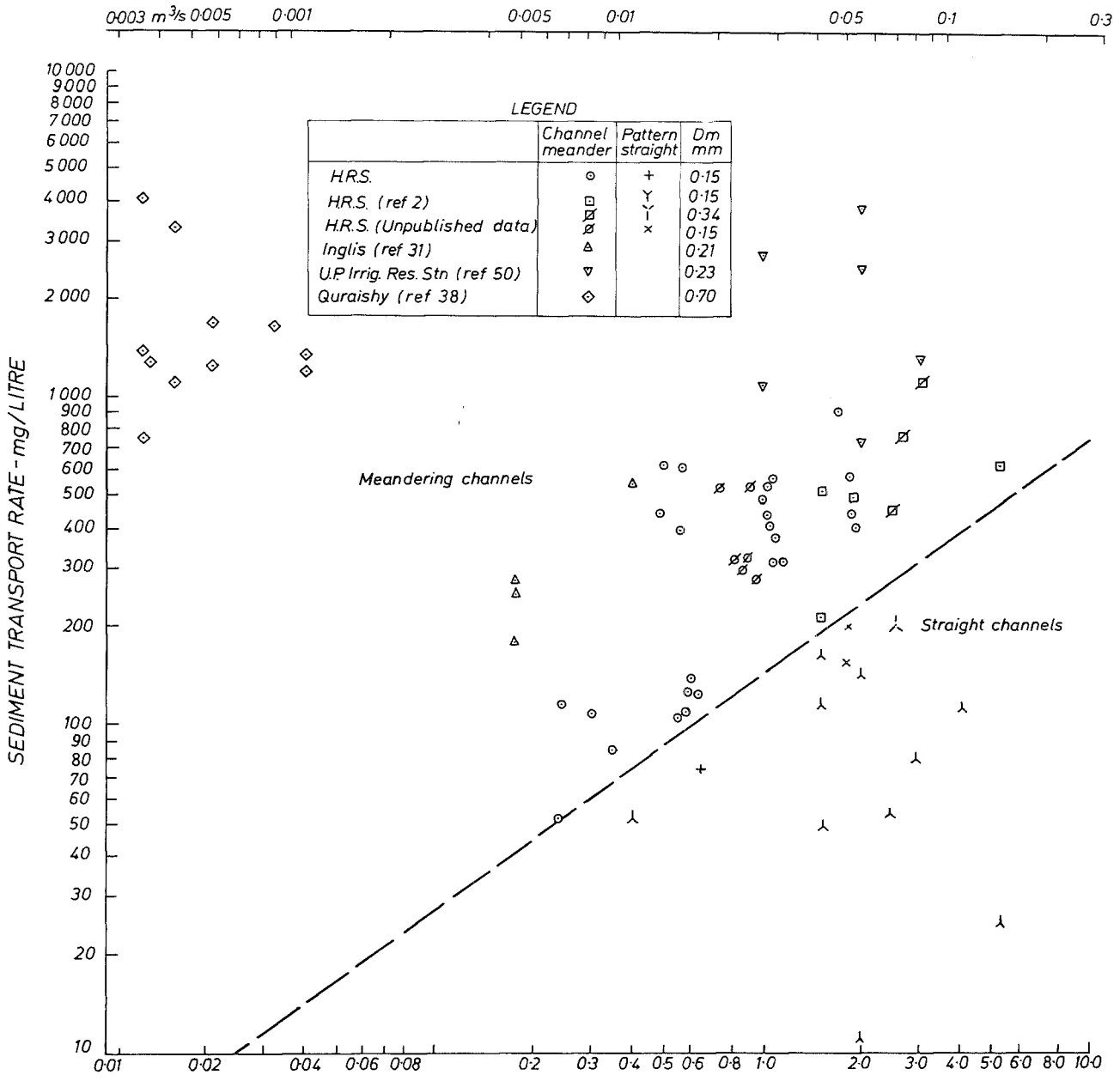
EFFECT OF DISCHARGE ON RATIOS OF PLAN DIMENSIONS

FIG 5



COMPARISON OF PLAN DIMENSIONS

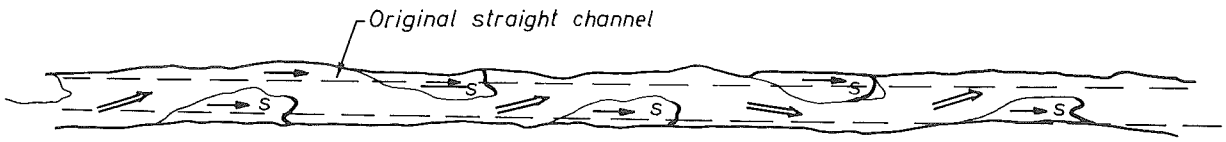
FIG 6



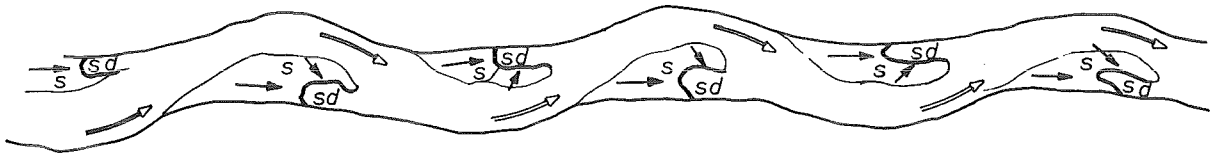
DISCHARGE - CUSECS

LIMITING SEDIMENT LOADS

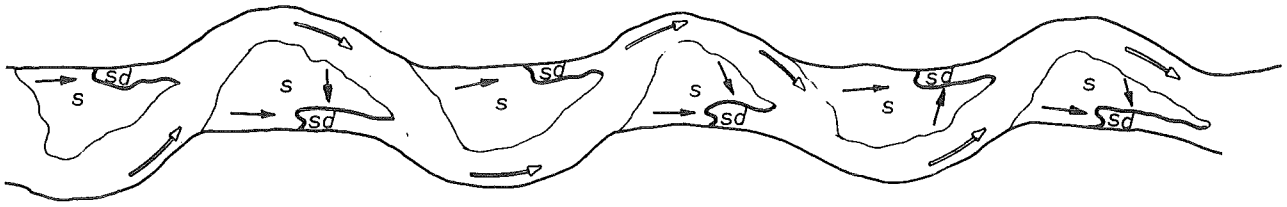
FIG 7



(a)- ENLARGING AND SHOAL FORMATION



(b)- BANK EROSION



(c)- MEANDERING

SYMBOLS

S - Shoal

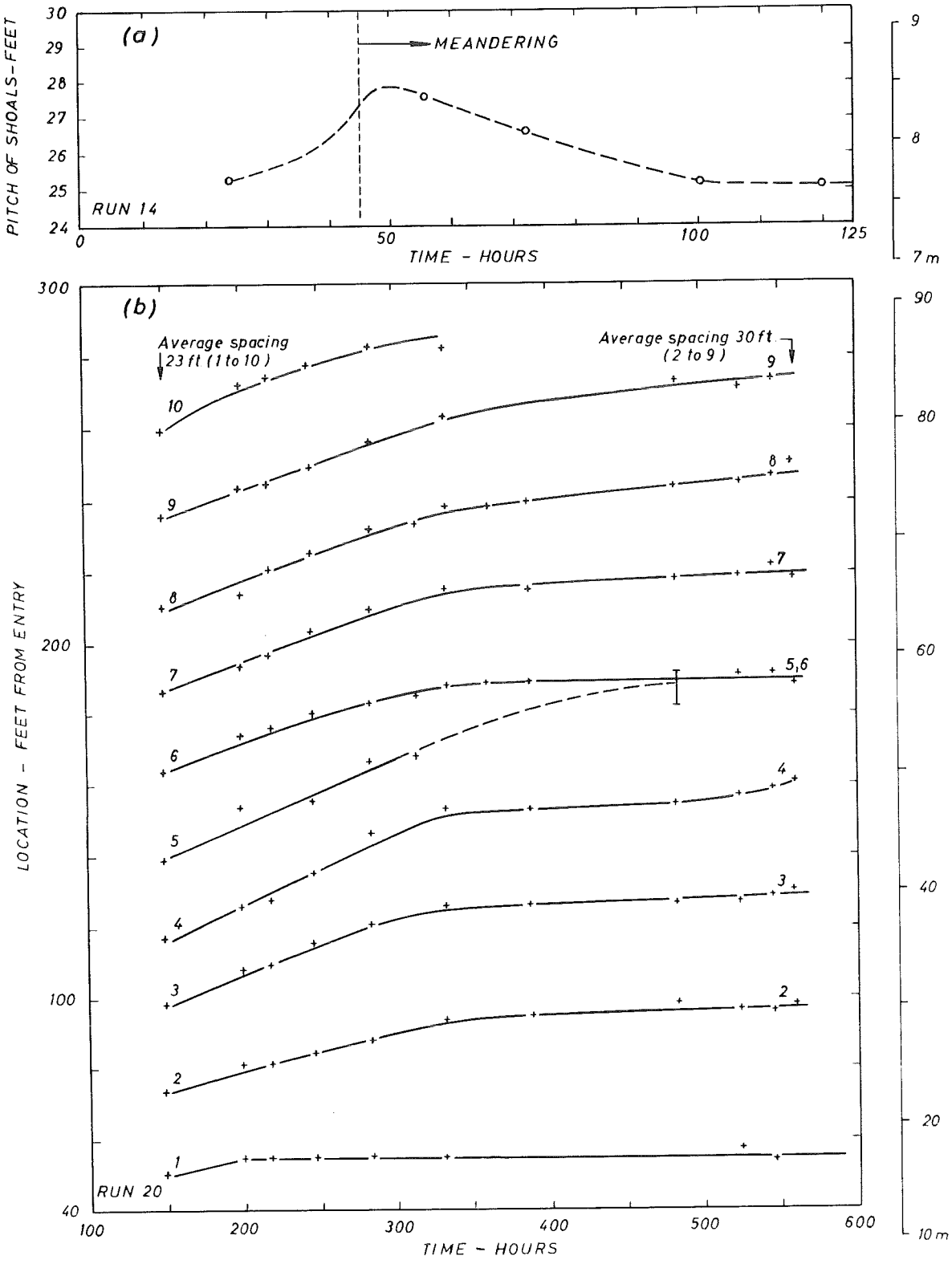
Sd - Stagnant. water, deep

⟹ Main water flow

⟶ Subsidiary flow due to difference in head between main flow and the stagnant deeps.

DEVELOPMENT OF MEANDERING

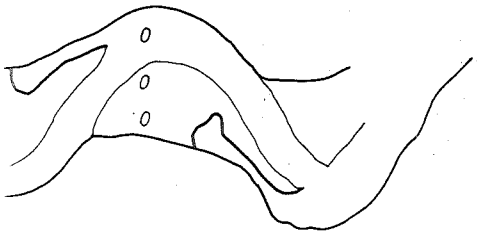
FIG 9



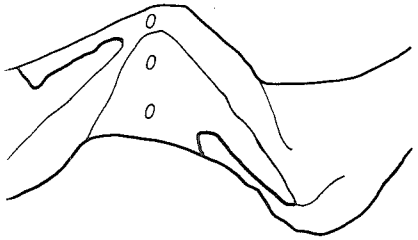
DEVELOPMENT OF SHOALS AND MEANDERS

FIG 10

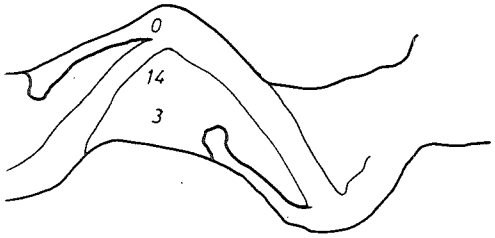
)



SION OF TRACER



EROSION OF TRACER



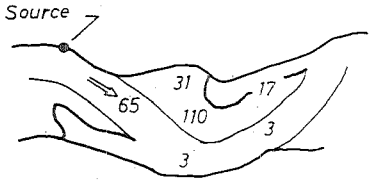
EROSION OF TRACER

SYMBOLS

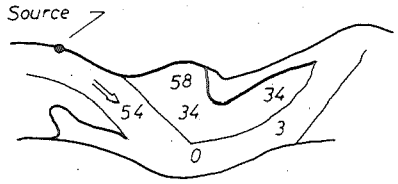
540 - Number of particles per ft^2

X - Not sampled

(b)

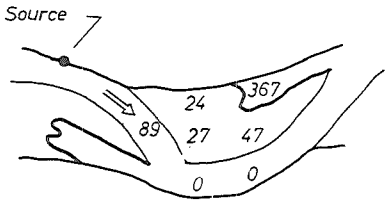


6 hrs after erosion of tracer

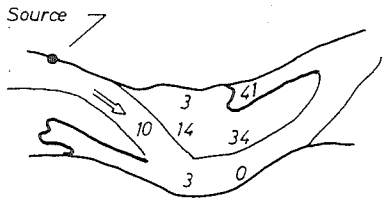


23 hrs after erosion of tracer

(i) SOURCE AT SECTION 159



6 hrs after erosion of tracer

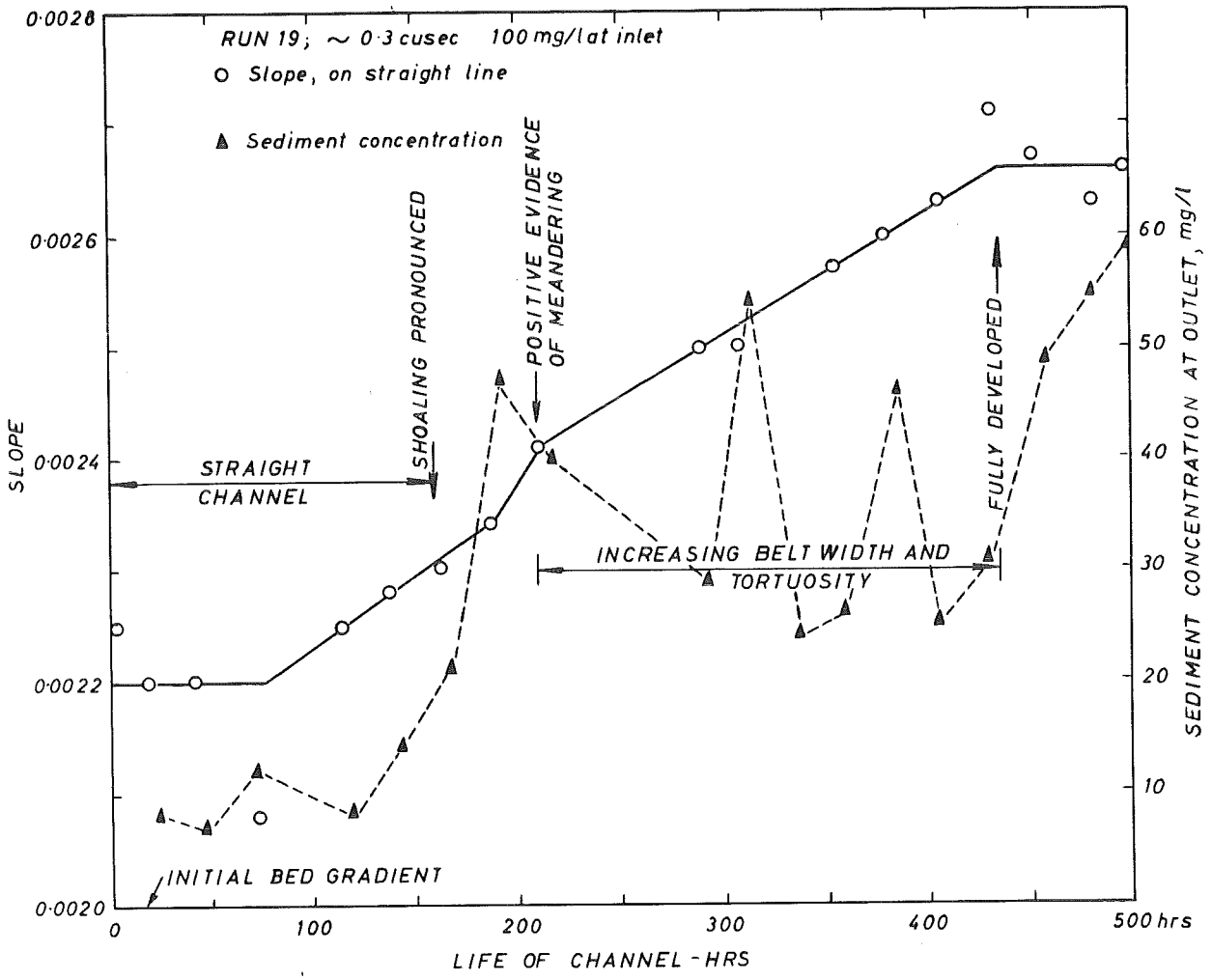


23 hrs after erosion of tracer

(ii) SOURCE AT SECTION 195

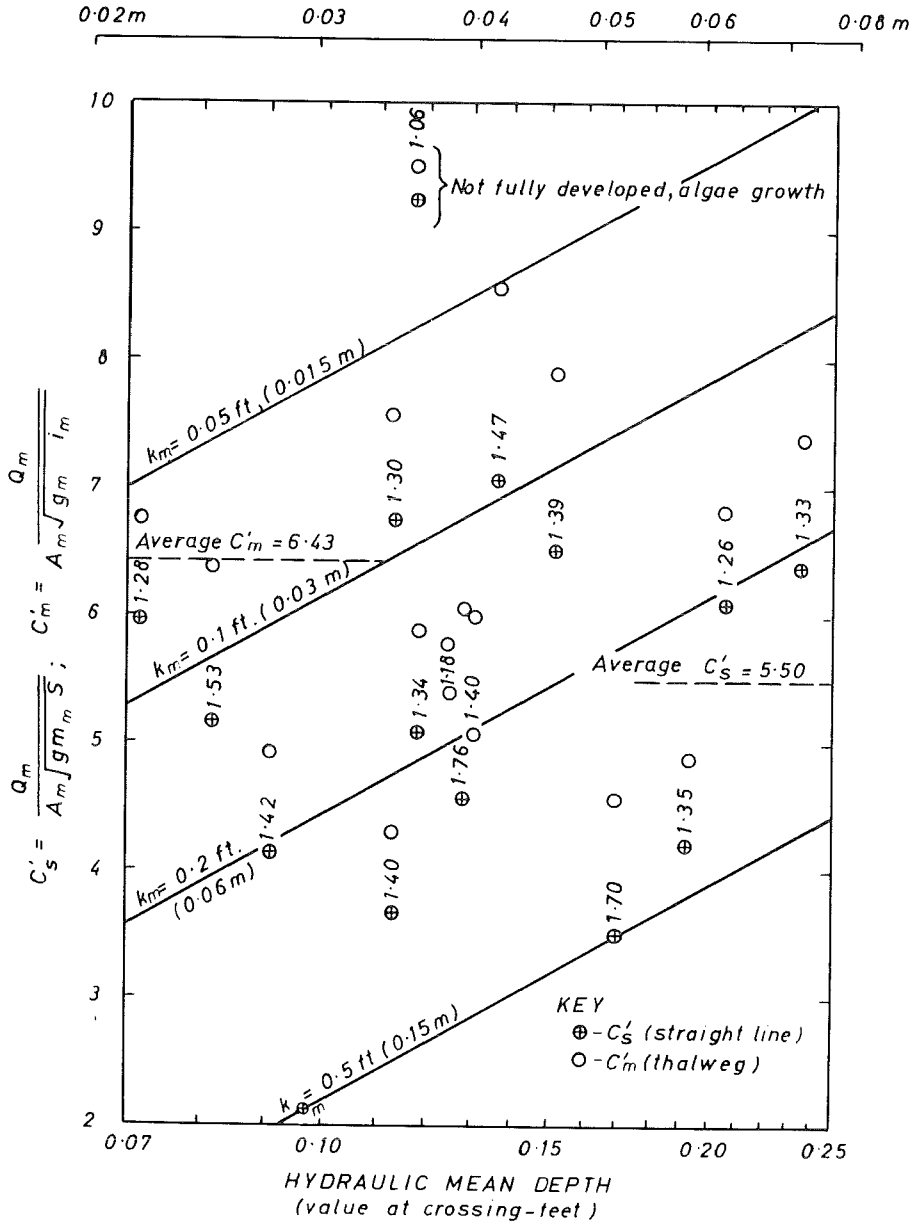
'LUORESCENT TRACER STUDY OF MOVEMENT OF ERODED BANK MATERIAL

FIG 11



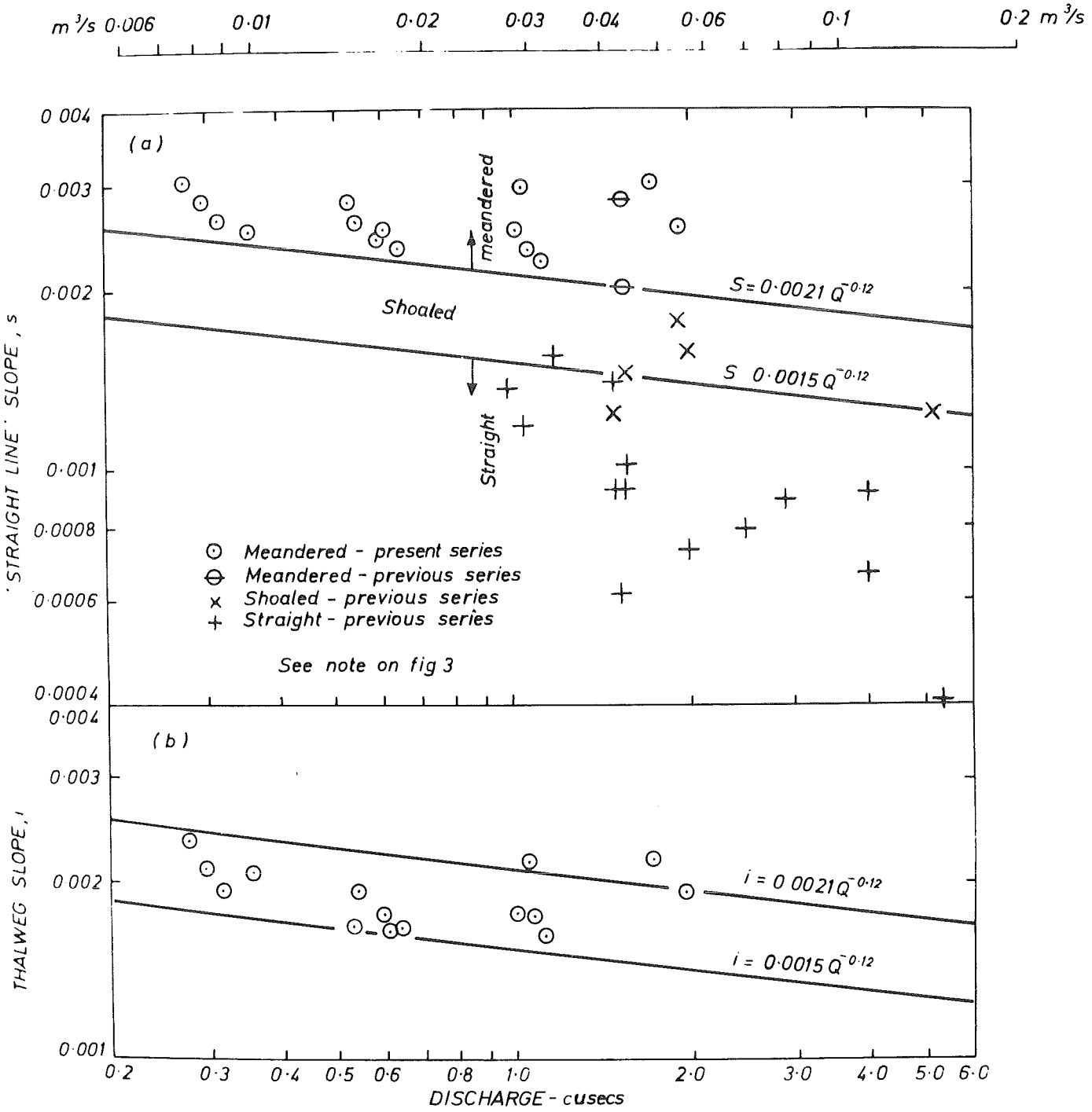
TYPICAL VARIATION OF SLOPE WITH TIME

FIG 12



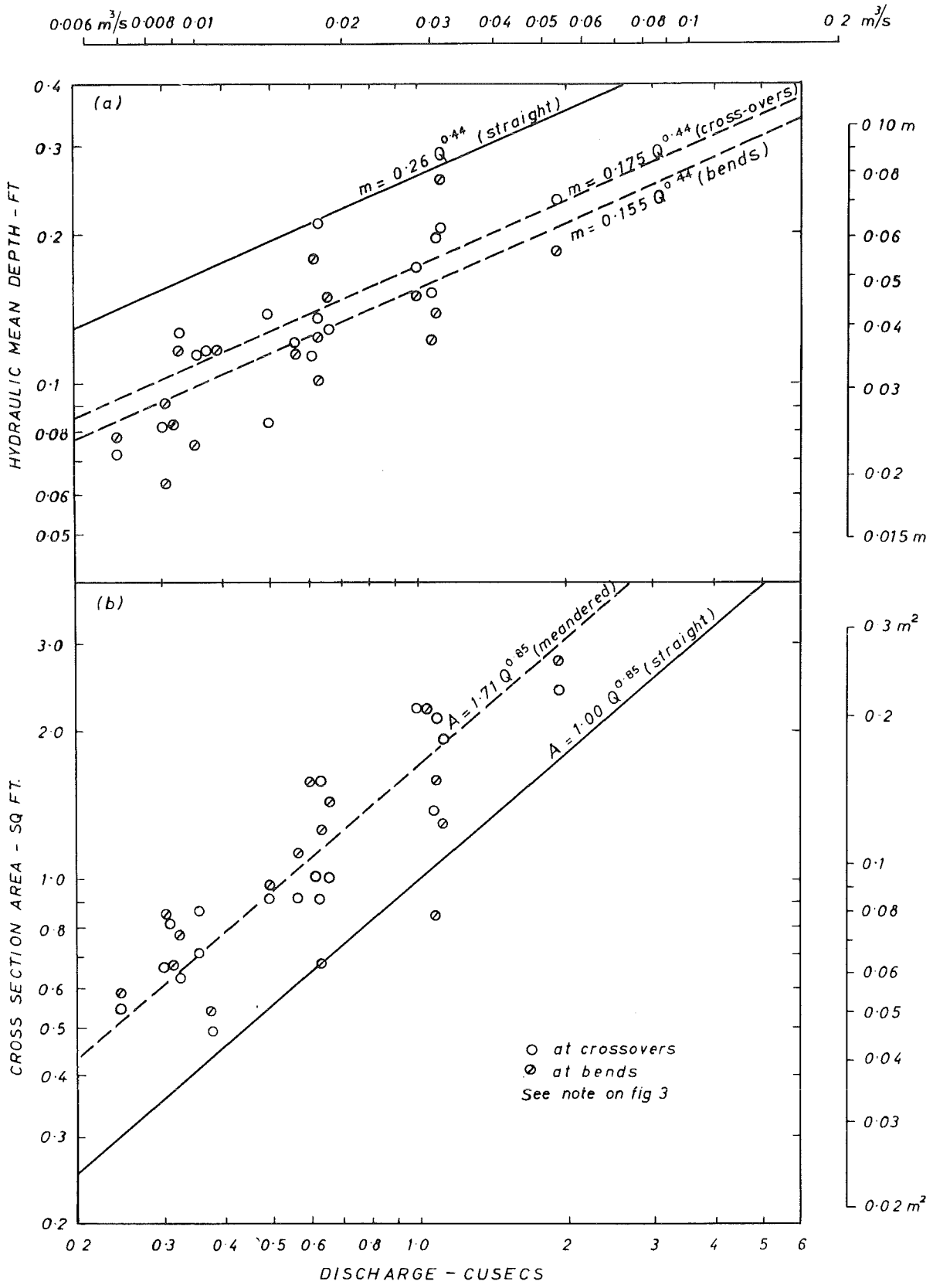
RESISTANCE ANALYSIS

FIG 13



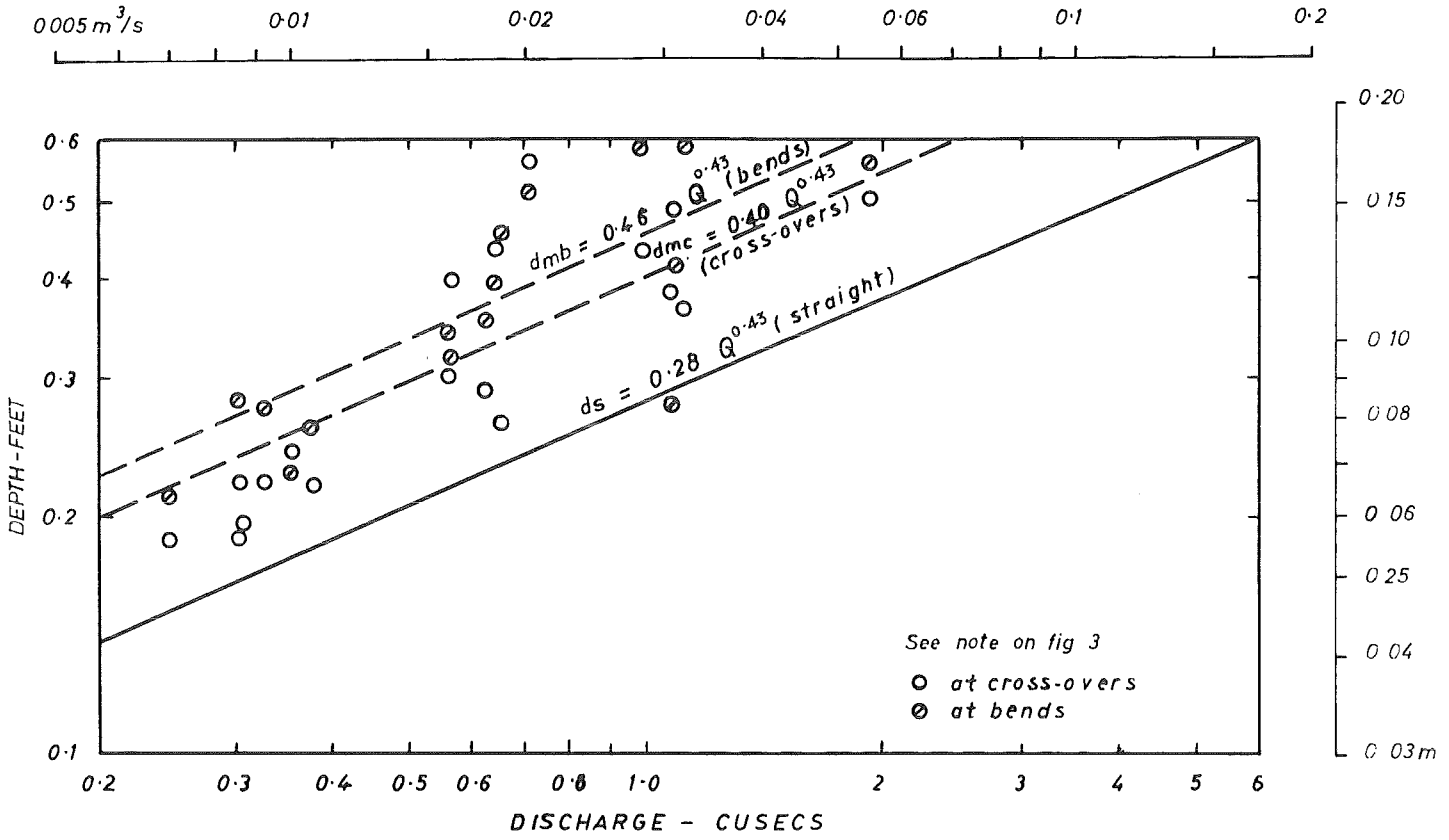
LIMITING SLOPES

FIG 14



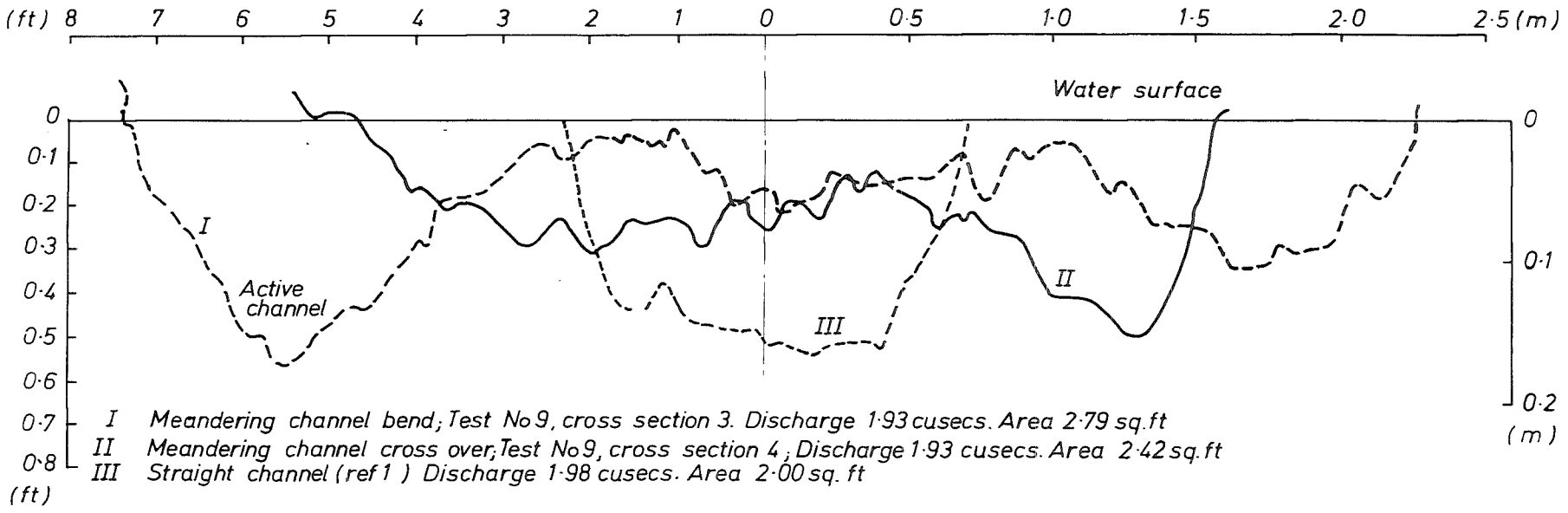
AREA AND HYDRAULIC MEAN DEPTH

FIG 15



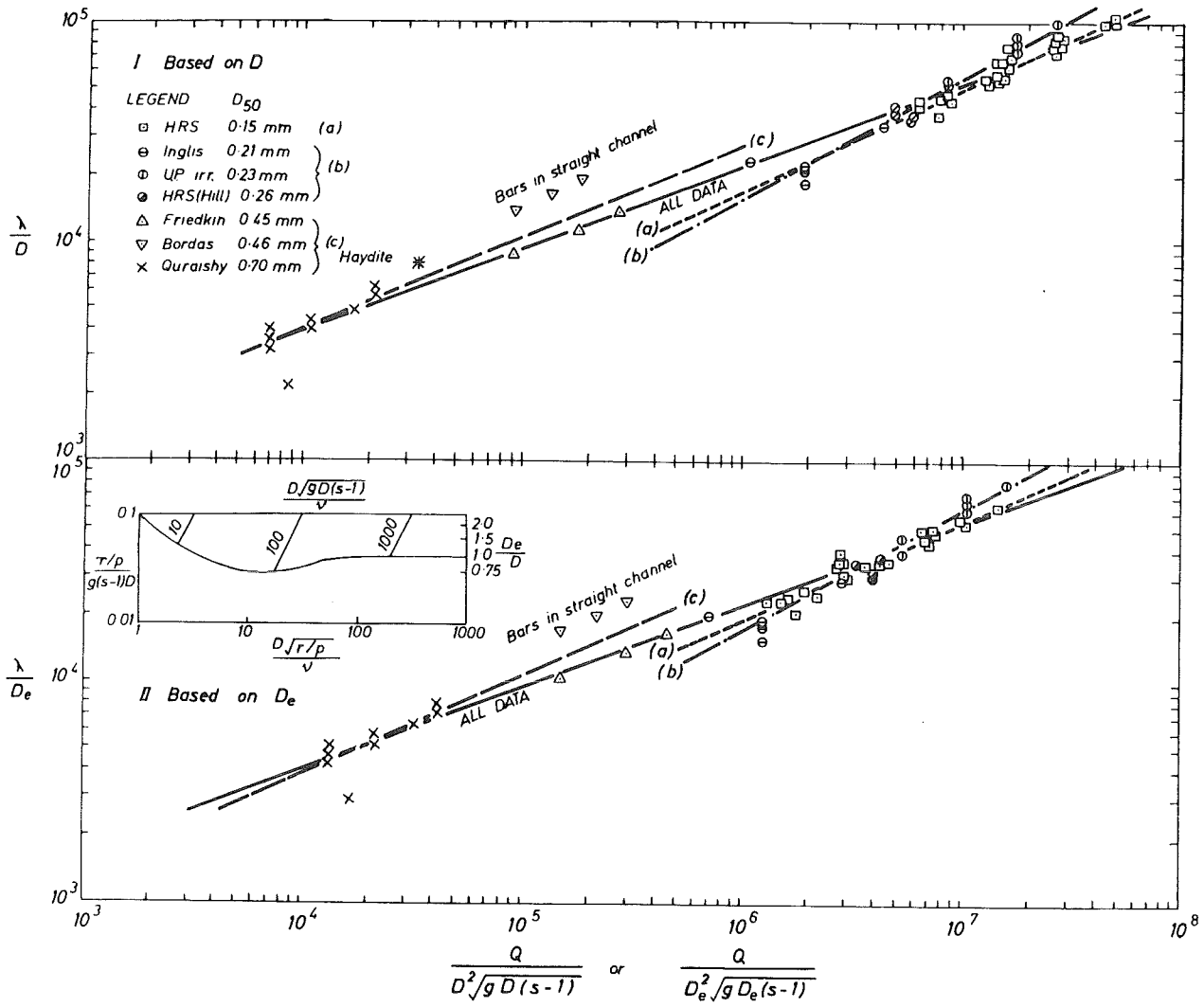
MAXIMUM DEPTHS AT BENDS AND CROSS-OVERS

FIG 16



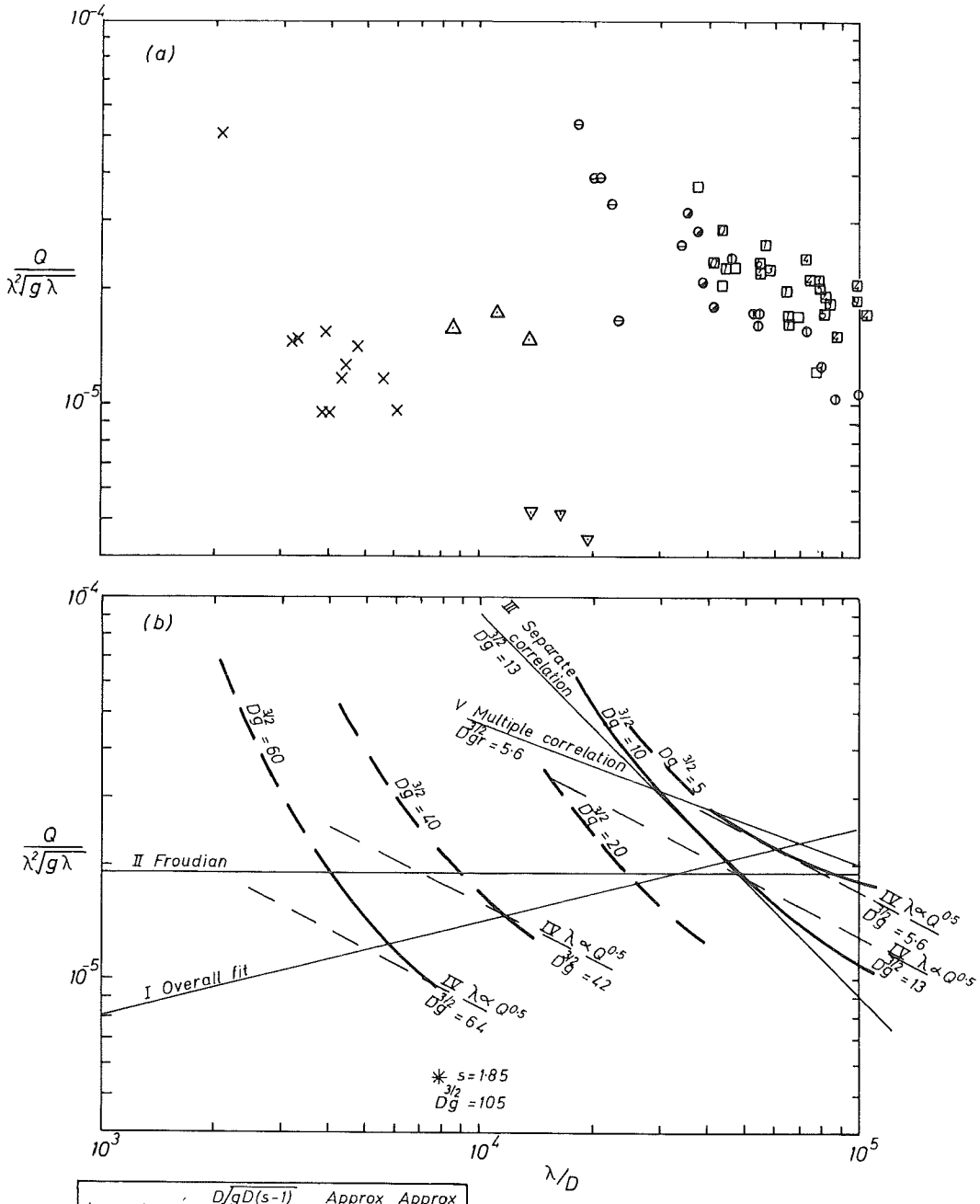
CROSS - SECTIONS OF STRAIGHT AND MEANDERED CHANNELS

FIG 17



DIMENSIONLESS MEANDER LENGTH VERSUS DIMENSIONLESS DISCHARGE: LABORATORY DATA FROM SEVERAL SOURCES

FIG 18



Legend	$\frac{D\sqrt{gD(s-1)}}{v}$ mean	Approx η	Approx $10^3\eta'$
□ HRS	5·6	1·37	5·45
(inset figures, x ~ 10^2 x ppm)			
⊖ Inglis	13·2	0·90	4·22
⊕ U.P. Irr.	12·3		
◒ HRS (Hill)	14·6		
△ Friedkin	42·0	0·17	1·60
▽ Bordas	39·0		
× Quraishy	63·9	0·06	0·85

Equations

I $\frac{Q}{\lambda^2\sqrt{g\lambda}} \propto \left(\frac{\lambda}{D}\right)^{1/4}$; $\lambda \propto Q^{0.36}$

II $\frac{Q}{\lambda^2\sqrt{g\lambda}}$ = Constant, median value 1.9×10^{-5}

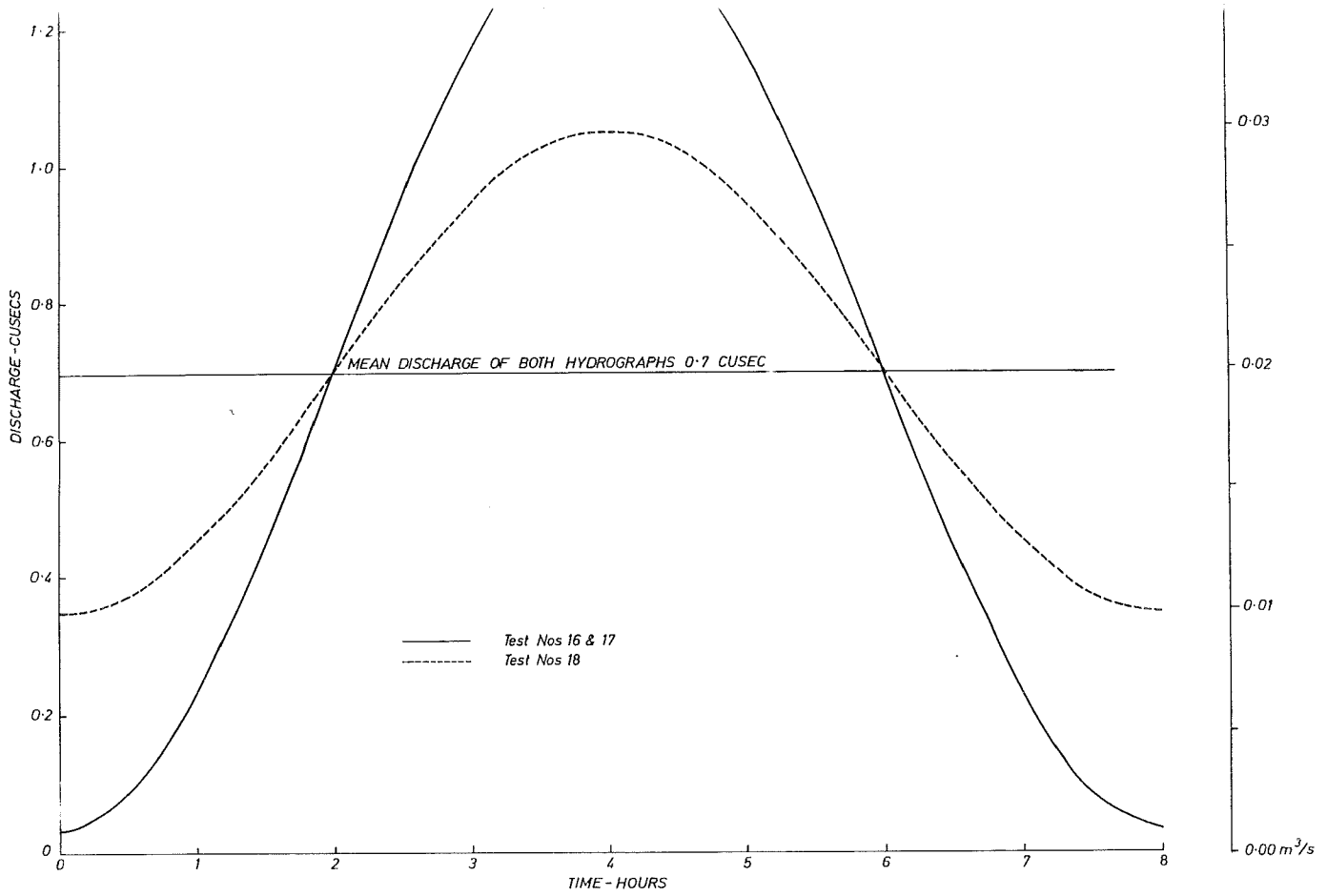
III $\frac{Q}{\lambda^2\sqrt{g\lambda}} = \eta\left(\frac{\lambda}{D}\right)^{-1}$, where $\vartheta = \phi(Dg)$

IV $\frac{Q}{\lambda^2\sqrt{g\lambda}} = \eta'\left(\frac{\lambda}{D}\right)^{-1/2}$; $\lambda \propto Q^{0.5}$

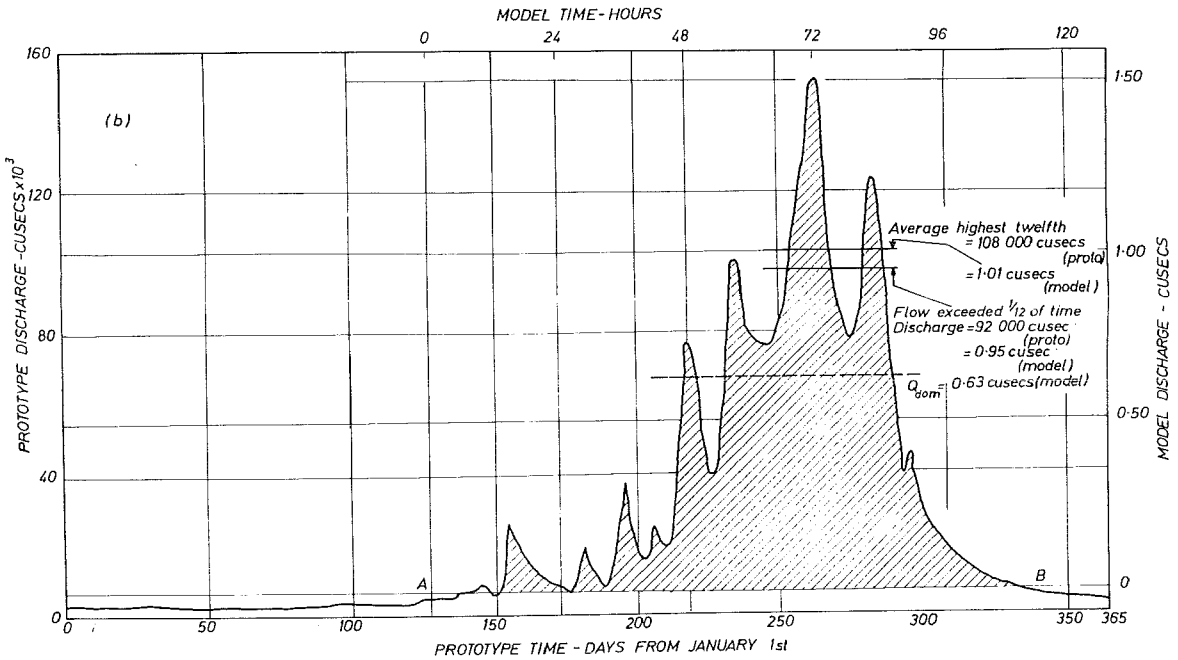
V $\lambda = 42.9\, x^{-0.021} Q^{0.480}$ for $X = 400$

DIMENSIONLESS CORRELATION OF MEANDER GEOMETRY: LABORATORY DATA FROM SEVERAL SOURCES

FIG 19



HYDROGRAPHS FOR VARIABLE DISCHARGES



TYPICAL ANNUAL HYDROGRAPH
KADUNA RIVER

VARIABLE FLOW HYDROGRAPHS

FIG 20

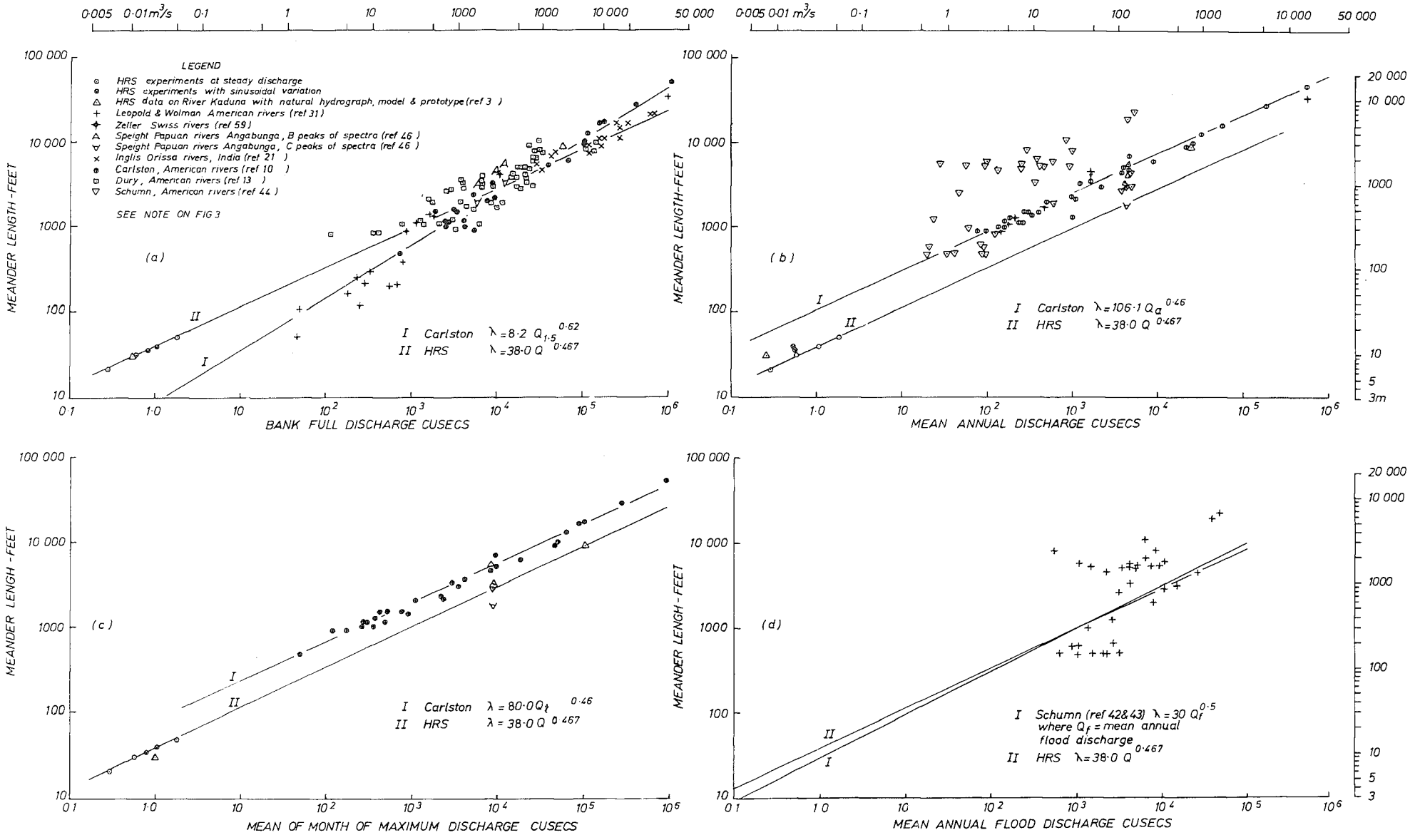


FIG 21

FIELD & HRS DATA ON MEANDER LENGTH AND DISCHARGE

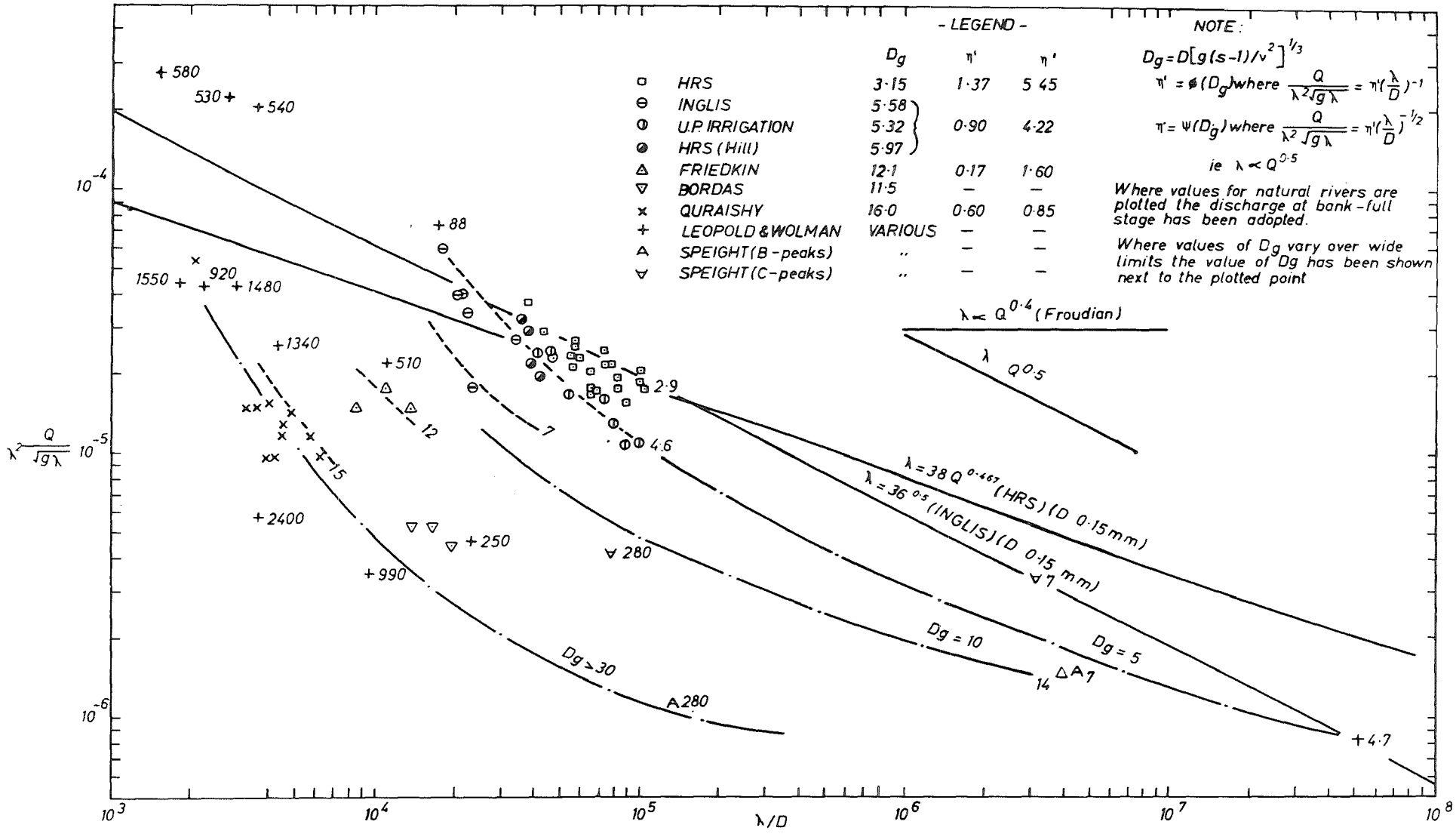


FIG 22

DIMENSIONLESS CORRELATION OF MEANDER GEOMETRY: INCORPORATION OF FIELD DATA

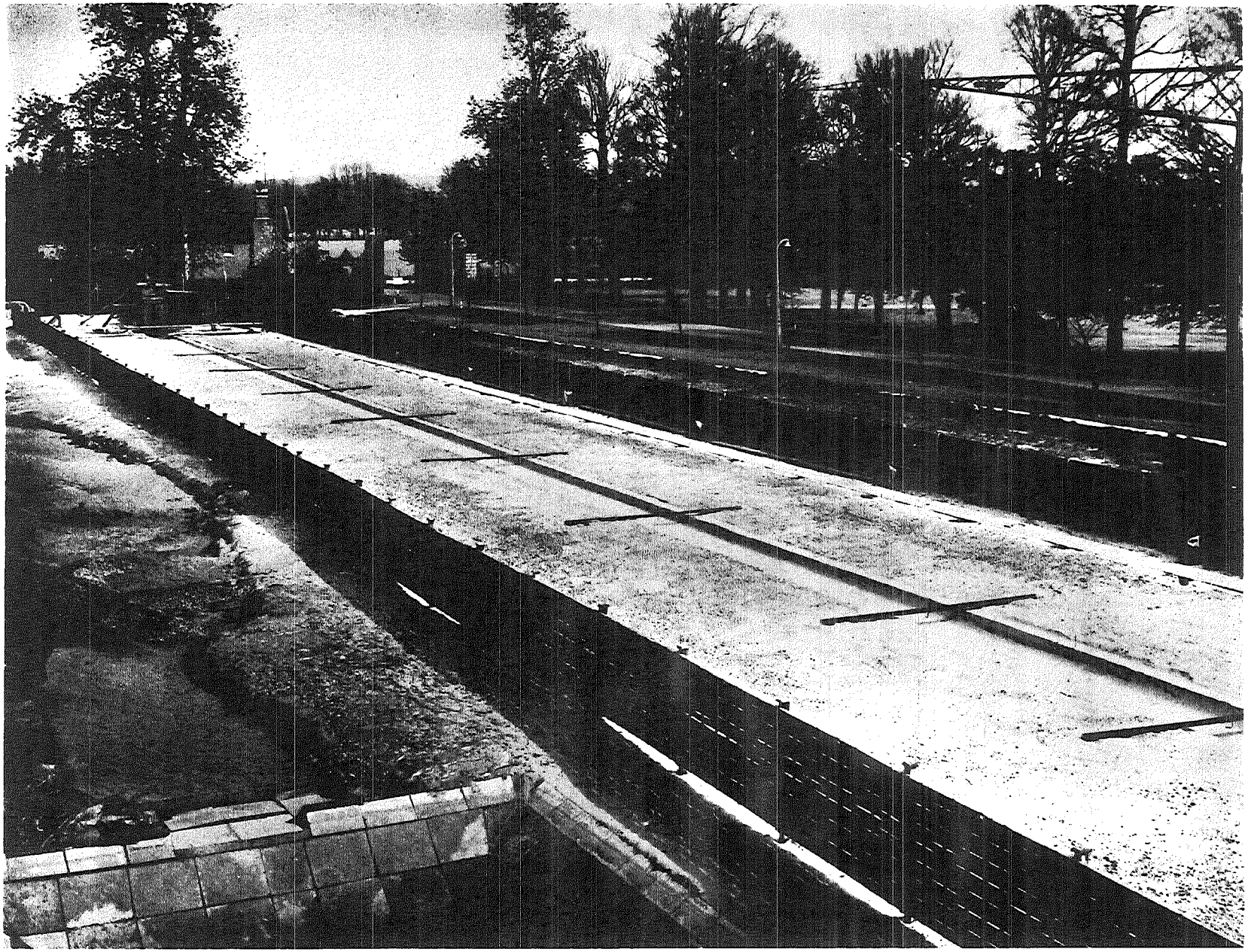


PLATE 1. General view of area showing initial straight channel.

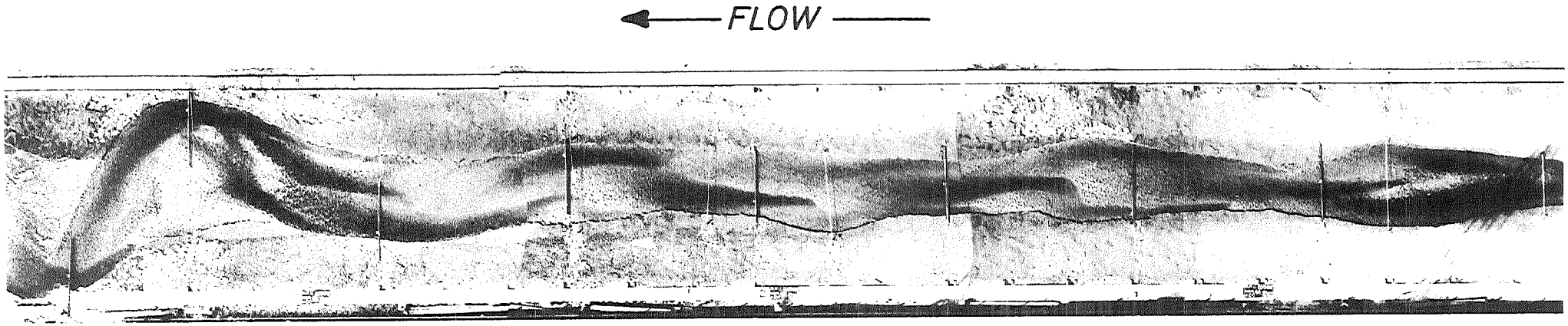


PLATE 2. Channel with very low upstream sediment injection rate (Run 15, hour 145, low flow)

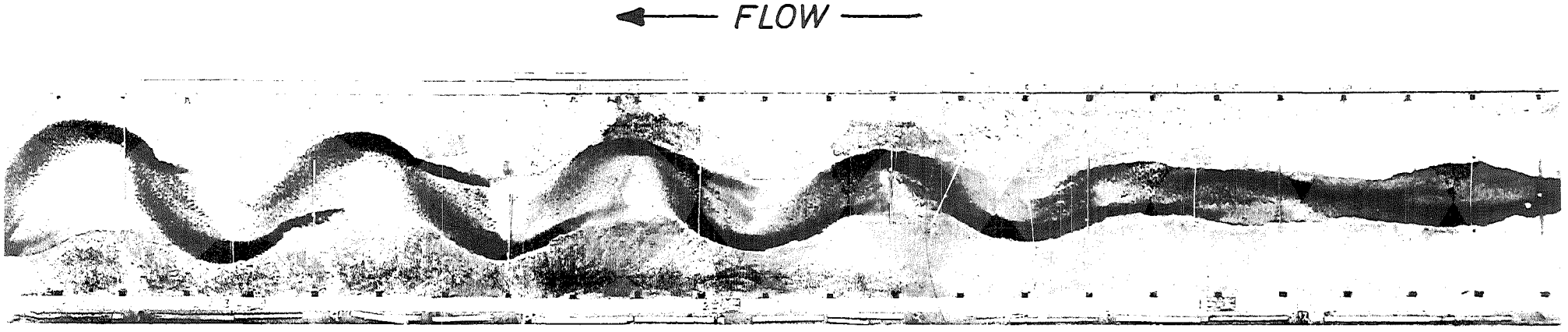


PLATE 3. Channel with slightly inadequate upstream sediment injection rate (Run 8, hour 312, low flow)

FLOW

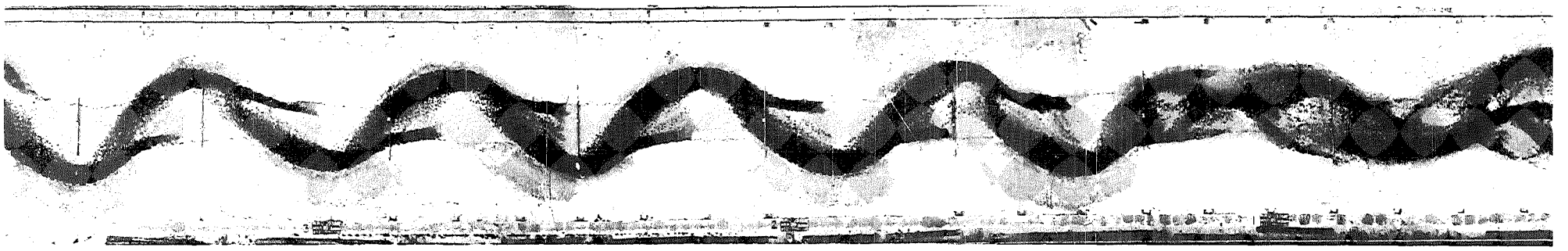


PLATE 4(a). Channel showing well developed meanders at low flow (Run 12, hour 99 low flow)

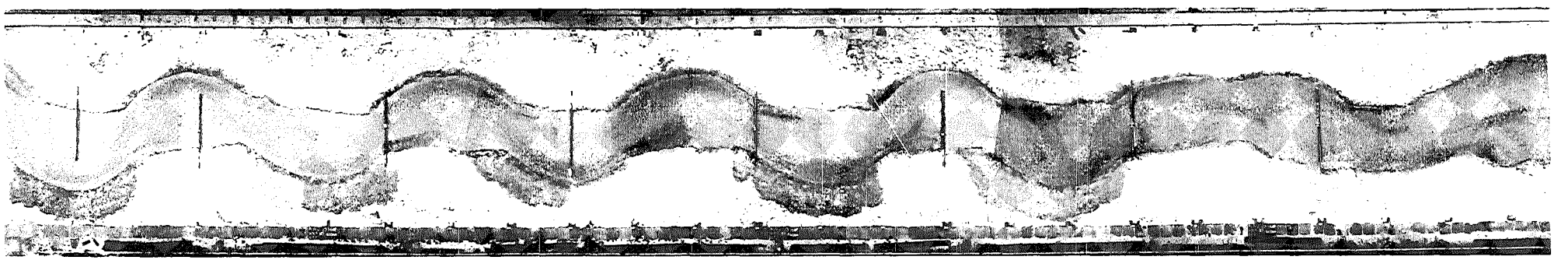


PLATE 4(b) Channel showing well developed meanders at high flow (Run 12, hour 99, normal flow)

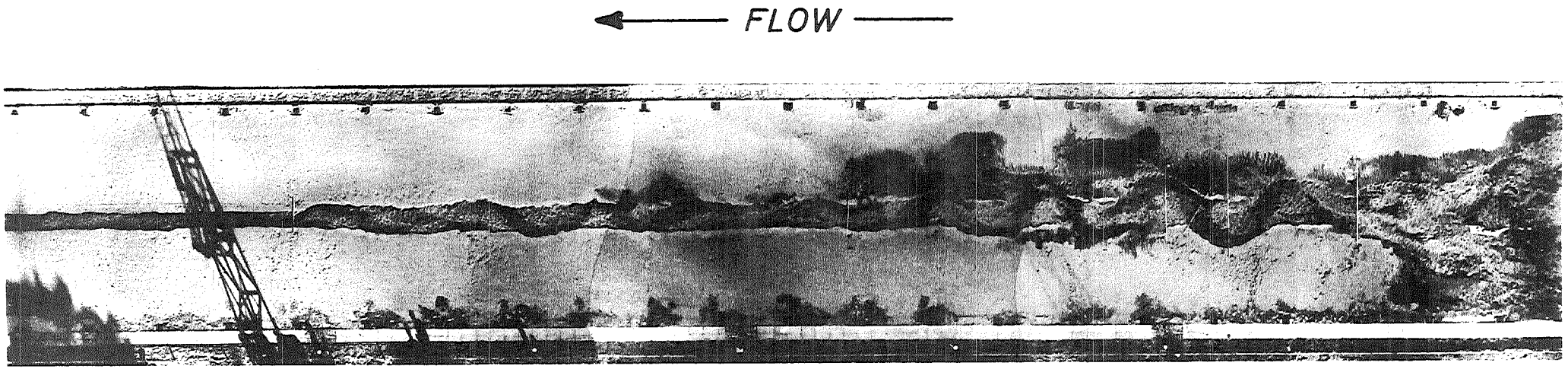


PLATE 5. Channel with high upstream sediment injection rate showing braiding. (Run 3, hour 236, low flow)

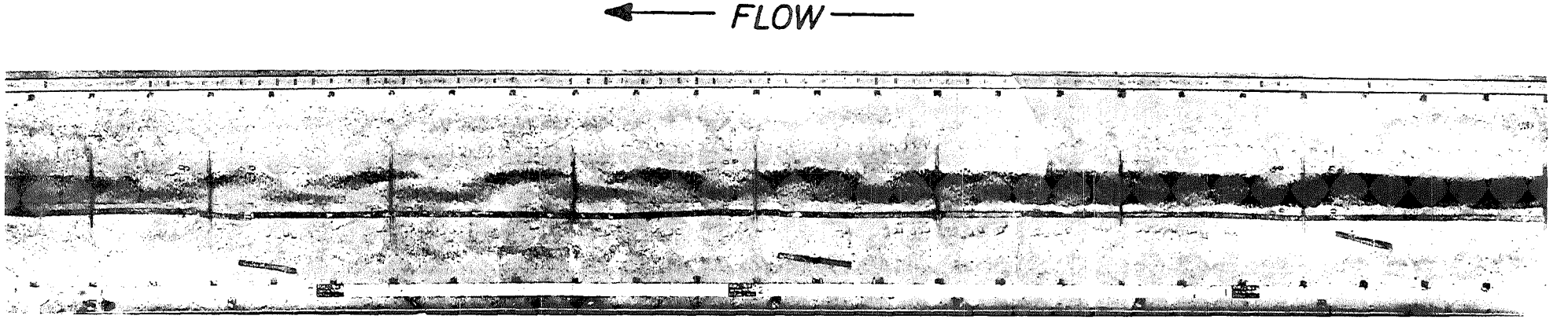


PLATE 6(a). Channel with no injection of sediment upstream showing well developed shoals at 145 hours (Run 20/I, hour 145 low flow)

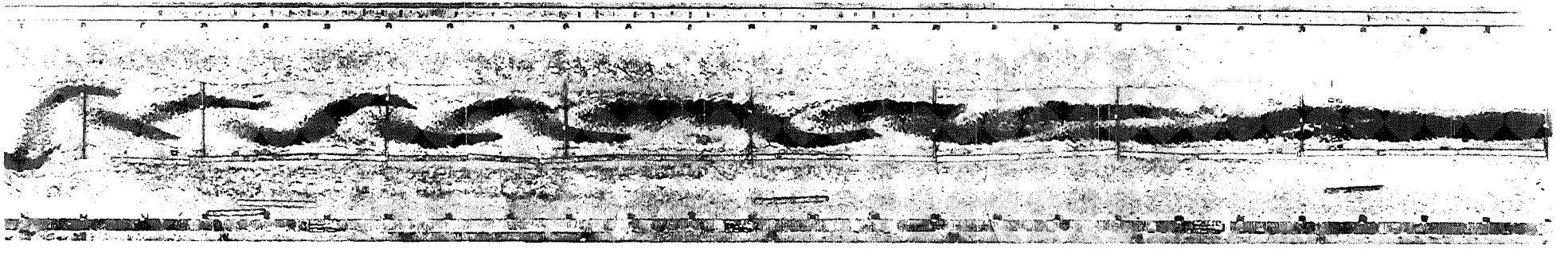


PLATE 6(b) Channel with no injection of sediment upstream showing well developed shoals at 362 hours and the maintenance of a straight channel. (Run 20/I, hour 362, low flow)

FLOW

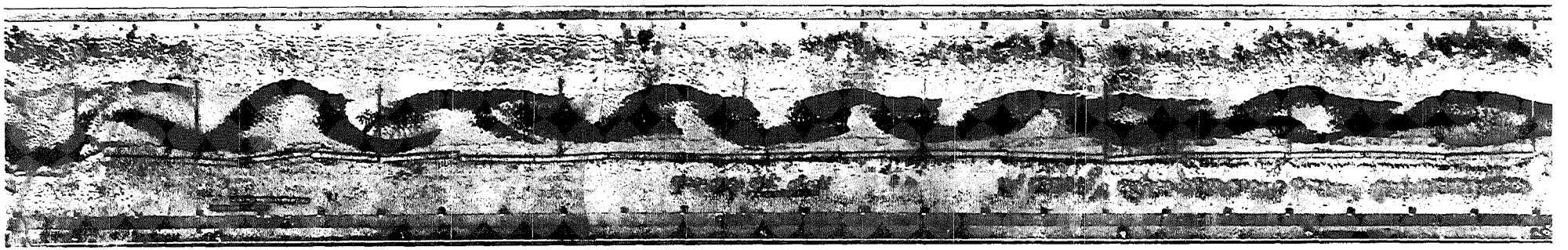


PLATE 7(a) Channel showing the development of meanders after the injection of sufficient sediment (Run 20/II hour 559, low flow)

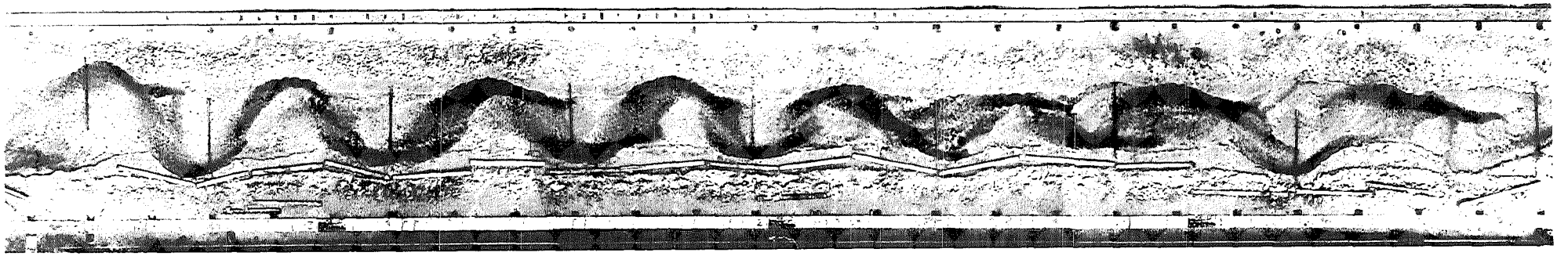


PLATE 7(b) Channel showing increase in meander belt width (Run 20/II, hour 794, low flow)

FLOW

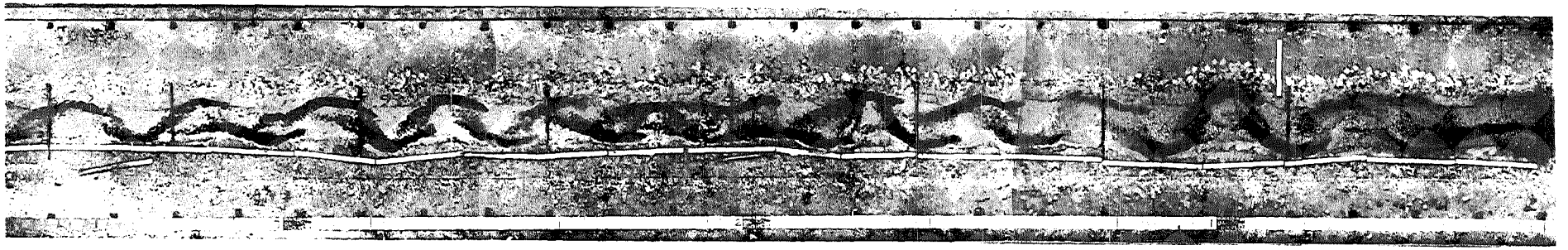


PLATE 8(a) Channel with adequate rate of sediment injection upstream (Run 19, hour 452, low flow)

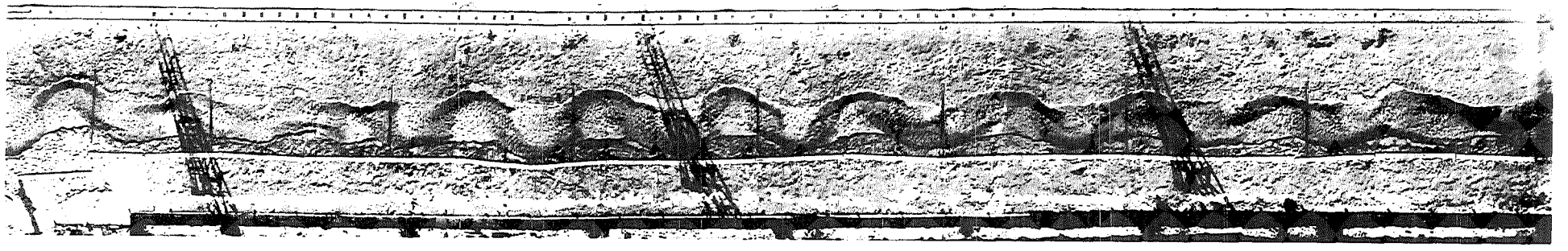


PLATE 8(b) Channel width same discharge and sediment injection rate, showing change in belt width due to initial slope (Run 21, hour 480, low flow)

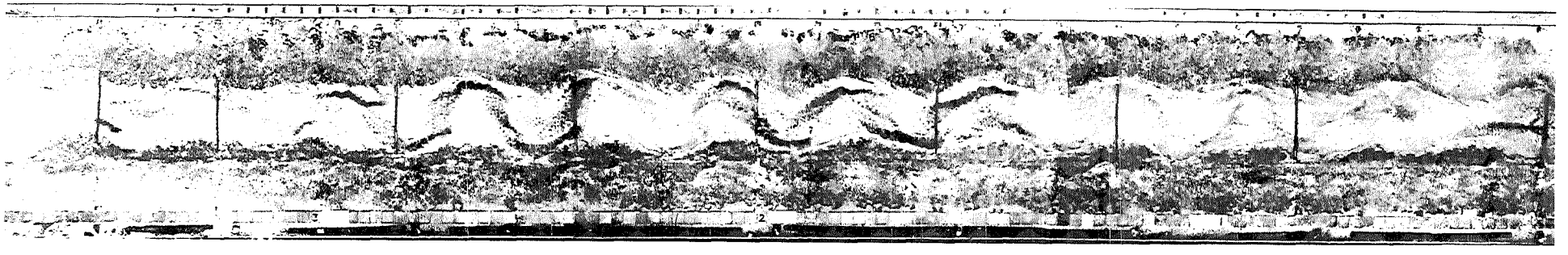


PLATE 8(c) Channel showing increase in belt width with time (Run 21, hour 651, low flow)

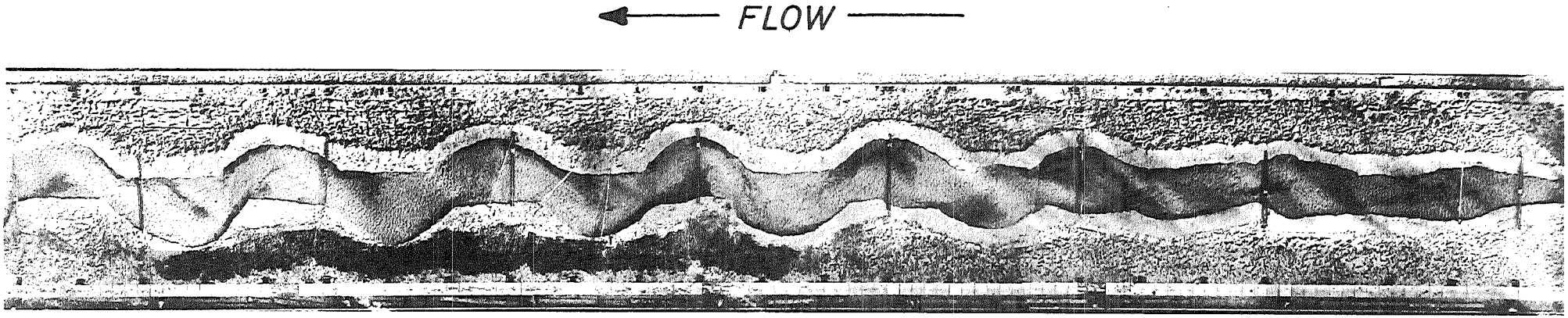


PLATE 9. Channel with sufficient sediment showing a change of phase (Run 7, hour 379, low flow)

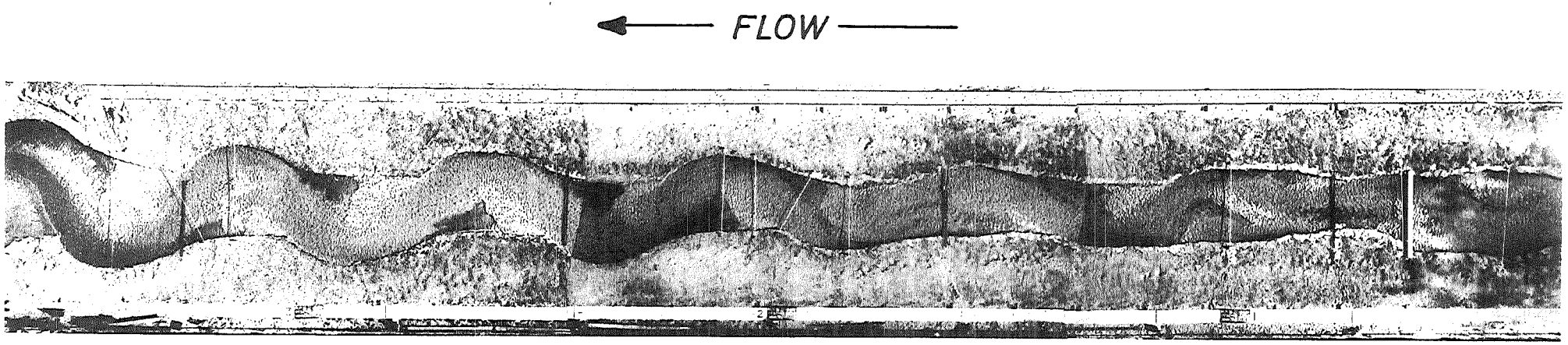


PLATE 10. Channel with variable flow and low rate of sediment injection upstream (Run 16, hour 360, low flow)

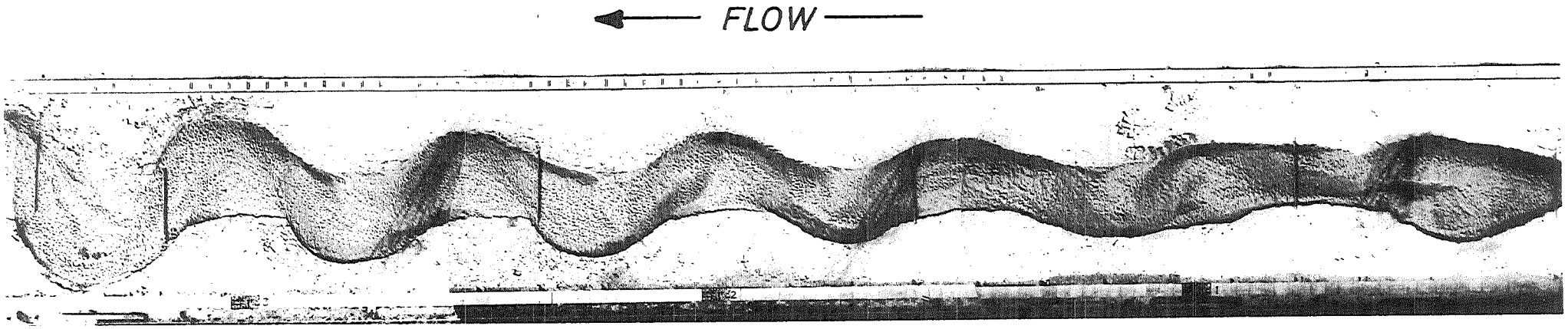


PLATE 11. Channel with variable flow and low rate of sediment injection upstream (Run 17, hour 293, low flow)

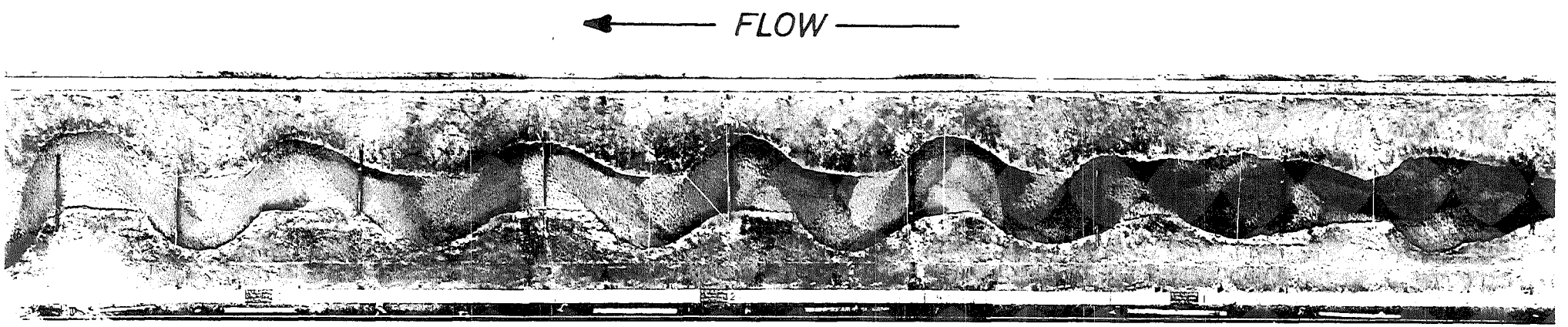


PLATE 12. Channel with variable flow and adequate rate of sediment injection (Run 18, hour 295, low flow)