Journal of the
ENGINEERING MECHANICS DIVISION
Proceedings of the American Society of Civil Engineers

ARCHIEF
20 MEI 1980
Lab. v. Scheepsbouwkunde
Technische Hogeschool
Delft

EARTHQUAKE RESPONSE STATISTICS OF NONLINEAR SYSTEMS

By Shih-Chi Liu,[1] A. M., ASCE

INTRODUCTION

The need for making ductility a specific requirement in the design of earthquake resistant building has been recognized for many years. This need has led to a number of theoretical and experimental investigations being made into the behavior of structures having various nonlinear yielding force-deformation mechanisms. Hanson and Connor (6,7),[2] in a series of tests of reinforced concrete frames conducted by the Portland Cement Association (PCA) have shown that properly designed frame members and connections could develop significant ductile deformation and resist severe earthquake without loss of strength. Adequate energy dissipation is provided by ductility of the reinforcing steel. Based upon their results, Clough (4) has defined a stiffness degrading model to reflect the characteristic behavior of reinforced concrete frames under cyclic loadings. Four different earthquake ground motion records are used as input mechanisms to study the relative earthquake resistance of structures exhibiting a stiffness degrading property as compared with the performance of equivalent ordinary elasto-plastic structures. He has shown that the response frequency of the stiffness degrading model is deduced by the loss of stiffness, and that this change in frequency tends to eliminate the resonant effect of earthquake input and thus reduces the response. For long period structures, the reduction of stiffness in the degrading model developed during earthquakes does not cause any significant change in the maximum responses or the ductility factor.

Clough's most significant results were obtained by deterministic means and are highly dependent on the specific input excitation. Since an earthquake is recognized as a random phenomenon, a nondeterministic or probabilistic method of analysis will permit a full understanding of structural behavior.

Note.—Discussion open until September 1, 1969. To extend the closing date one month, a written request must be filed with the Executive Secretary, ASCE. This paper is part of the copyrighted Journal of the Engineering Mechanics Division, Proceedings of the American Society of Civil Engineers, Vol. 95, No. EM2, April, 1969. Manuscript was submitted for review for possible publication on August 7, 1968.

[1] Member of Technical Staff, Bell Telephone Labs., Whippany, N.J.

[2] Numerals in parentheses refer to corresponding items in Appendix I.—References.

This paper is concerned with such analysis and considers the complex yielding mechanisms of elasto-plastic and stiffness degrading models.

The objectives of this study are: (1) To establish the probabilistic maximum response of nonlinear elasto-plastic and stiffness degrading models, using a digitally generated stochastic process to represent strong ground motion caused by earthquake; (2) to compare and correlate the response statistics of these nonlinear systems with their corresponding linear elastic systems, i.e., systems having the same initial stiffness and viscous damping ratios; and (3) to determine the response accumulation of these nonlinear systems when subjected to consecutive earthquake excitations. The last objective is motivated by the consideration that many small quantities of permanent deformation caused by a sequence of moderate-intensity earthquakes may be accumulative and cause the eventual failure of the structure.

The probabilistic maximum response is established by using Monte Carlo computations or computer experiments, following a separate deterministic analysis which generates the response ensemble. In the response accumulation study the importance of existing permanent set induced in structural members by earthquake excitations is evaluated. The input or exciting mechanisms for the response accumulation study were provided by repetitive segments of the primary phase of the El Centro 1940 N-S earthquake accelerogram and five short duration acceleration bursts.

NONLINEAR STRUCTURAL MODELS

The structural system considered in this investigation is shown in Fig. 1. It is a single-mode oscillator consisting of a rigid girder of mass M, a viscous damper having a damping constant, λ, supported by weightless columns having a total lateral displacement stiffness, k. When this system is subjected to an earthquake excitation, its response is characterized by the displacement relative to the ground, u. During motions which exceed the elastic limit of the system it is assumed that the stiffness, k, of this system varies nonlinearly in accordance with the following two types of material behavior.

Elasto-plastic Model.—The elasto-plastic system represented by Fig. 2 is characterized by two factors; (1) The yield strength or resistance, V_y, i.e., the load at which yielding occurs, and (2) the initial elastic stiffness $k_e = V_y/X_y$, in which X_y is the yield displacement corresponding to V_y. This system is the simplest representation of idealized structures and has been used in many previous nonlinear earthquake response studies (11,12). It should be noted that the system is assumed to be symmetrical so that the motion is unbiased. Displacement of this system beyond the elastic limit takes place with no increase of load and the unloading stiffness is identical with the initial elastic value.

Stiffness Degrading Model.—This model was suggested by Clough for approximating the behavior of concrete frame structures as reported in the recent PCA test (6). The initial behavior of this system, Fig. 3 is identical with the elasto-plastic system, Fig. 2, and is characterized by the same yield strength and initial elastic stiffness factors. After loading, yielding, and unloading, however, the negative loading stiffness is assumed to be defined by two points on the force-deflection diagram: (1) The point at which the positive unloading terminated, and (2) the current negative yield point, CYP_n. For the

initial negative loading, CYP_n is called the initial negative yield point, IYP_n, and is defined by the initial negative yield condition. However, for the negative loading thereafter, the CYP_n is defined by the maximum negative displacement which occurred at any previous time, and the corresponding negative yield force.

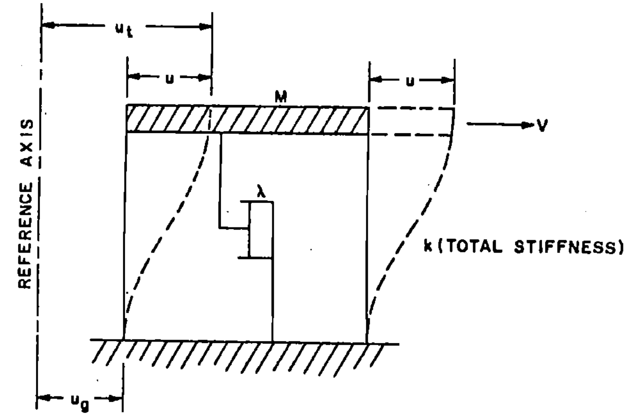

FIG. 1.—BASIC SINGLE-MODE DYNAMIC MODEL WITH EARTHQUAKE EXCITATION

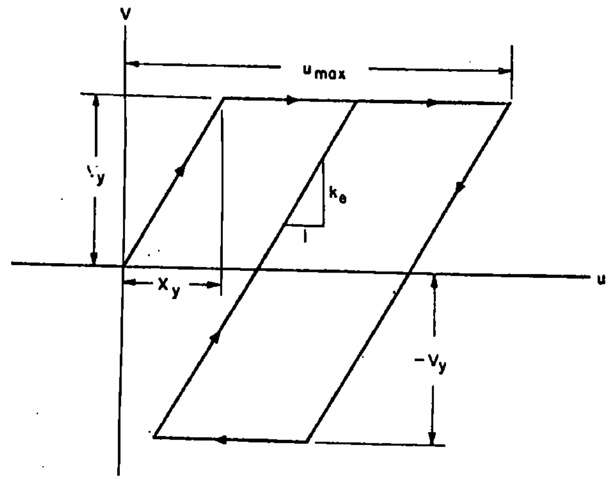

FIG. 2.—ORDINARY ELASTO-PLASTIC BEHAVIOR

Unloading from the negative loading zone is identical with the elasto-plastic system, but the subsequent positive loading is controlled by the degrading stiffness property. Similar to the negative loading case, the reduced positive stiffness is defined by two points on the force-deflection diagram: (1) The point at which the negative unloading terminated, and (2) the positive yield point, CYP_p. The CYP_p is defined analogous to the CYP_n, corresponding to the previous maximum positive displacement and the corresponding positive yield force.

To cover the wide range of structural properties defined by the variables T, λ, X_y, in which $T = 2\pi\,(M/k_e)^{1/2}$ is the period of the system, the following systems, each with both ordinary elasto-plastic and stiffness degrading force-deformation relationships, are considered.

1. System with T = 0.3 sec, λ = 0.02, X_y = 0.088 in.
2. System with T = 0.3 sec, λ = 0.10, X_y = 0.088 in.
3. System with T = 2.7 sec, λ = 0.02, X_y = 3.42 in.
4. System with T = 2.7 sec, λ = 0.10, X_y = 3.42 in.

These systems, covering short period and long period structures, each with two different nonlinear mechanisms and two viscous damping ratios, are

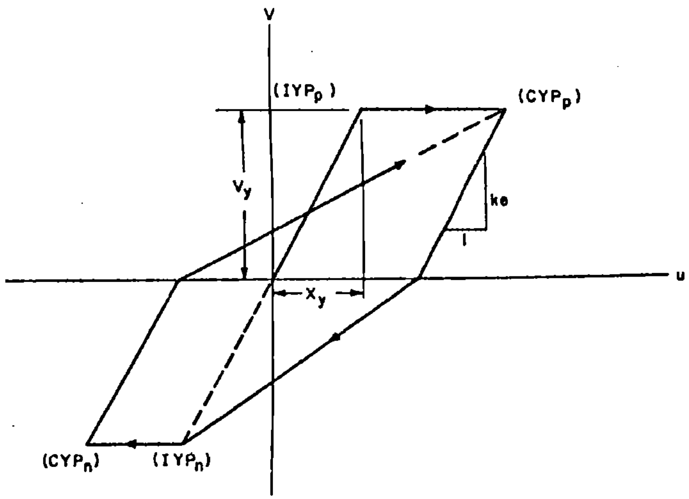

FIG. 3.—STIFFNESS DEGRADING BEHAVIOR

chosen to demonstrate how the relative dynamic behavior is influenced by each structural parameter. The yield displacement, X_y, is defined by the strength ratio B in accordance with the relation $B = V_y/W$, in which $W = Mg$ is the total weight of the system. The values for B are selected for these systems on the basis that the yield resistance V_y would equal twice the design load as specified in Section 2312(*d*) of the 1967 Uniform Building Code (13) for moment resisting frames, i.e., $B = 2KC = (2)(0.67)(0.05)(T)^{1/3}$.

METHOD OF ANALYSIS

The equation of motion of the system as shown in Fig. 1 within the elastic range is

$$\ddot{u} + \frac{4\pi}{T}\lambda\dot{u} + \frac{4\pi^2}{T^2}u = -\ddot{x}_g(t) \quad (1)$$

in which $\ddot{x}_g(t)$ is the input function to the system, u_g represents the relative displacement of the support and u_t is the total displacement of the girder.

It is obvious that the parameters T and λ completely define the linear characteristics of a structure in the elastic range. However, when the structure displaces beyond the yield level, the nonlinear property controls the motion, and the stiffness varies in accordance with the response regime, following the law defined for each of the two different models. In this case, the equation of motion may be written

$$\ddot{u} + \frac{4\pi}{T}\lambda\dot{u} + \frac{V(u)}{M} = -\ddot{x}_g(t) \quad (2)$$

in which $V(u)$, the spring force, is a function of the current displacement according to the specified nonlinear behavior. Due to the complexities imposed by the nonlinearity, Eq. 2 can only be solved by a step-by-step numerical integration procedure. By this procedure, the input function is divided into very short equal time intervals $\Delta\tau$ and the output acceleration is assumed to vary linearly over each interval (5). In this case, the response quantities at steps n and $n + 1$ can be expressed by the following simple relations

$$\left.\begin{aligned} \dot{u}_{n+1} &= \dot{u}_n + \frac{\ddot{u}_n}{2}\Delta\tau + \frac{\ddot{u}_{n+1}}{2}\Delta\tau \\ u_{n+1} &= u_n + \dot{u}_n\,\Delta\tau + \frac{\ddot{u}_n}{3}\Delta\tau^2 + \frac{\ddot{u}_{n+1}}{6}\Delta\tau^2 \end{aligned}\right\} \quad (3)$$

where the dot represents the time derivative.

The response history of a structure is obtained by programming using Eqs. 1, 2 and 3 in a CDC 6600 digital computer.

RESPONSE STATISTICS

The relative behavior of the elasto-plastic and stiffness degrading systems under random-type ground motions can be better demonstrated by the comparison of the response statistics obtained from a large ensemble of input functions than the comparison of deterministic responses from specifically given accelerograms. If a stochastic representation of earthquake is possible, then we encounter a typical nonlinear random transformation problem. The exact solution of nonlinear random transformation problems governed by the associated Fokker-Planck equation generally requires that the input process be Gaussian, stationary with constant power spectrum, and that the nonlinearity existing in the stiffness term may be derived from a potential. For more general cases in which the input process is nonstationary, or the system's nonlinearity is characterized by a function of velocity as well as displacement, the general Fokker-Planck equation has not been resolved at the present stage. Although approximate methods such as pertubation and equivalent linearization techniques are applicable for certain special cases, unfortunately, those methods are all restricted to cases of small nonlinearities.

For the general random input process, and for yielding structures whose nonlinear characteristics can be defined as being piece-wise linear such as elasto-plastic systems, bilinearly elastic systems or systems exhibiting the stiffness degrading property, closed form solutions for the response statistics are practically impossible to obtain. Analytic methods applied to such nonlinear random vibration problems do not necessarily reduce the amount of work when compared with numerical methods which can always be performed by a modern analog or digital computer. Therefore, an efficient way to solve this problem is to use the Monte Carlo technique, i.e., to establish an input ensemble of known characteristics, determine each member of the output ensemble by a separate deterministic analysis, and then evaluate the output ensemble statistically.

Stochastic Model of Ground Motion.—The stochastic model of ground motion used in this investigation is a Gaussian stationary process with nonuniform power spectral density. The development of this model is based upon the theory of spectral simulation (9). As shown in Fig. 4, consider the point re-

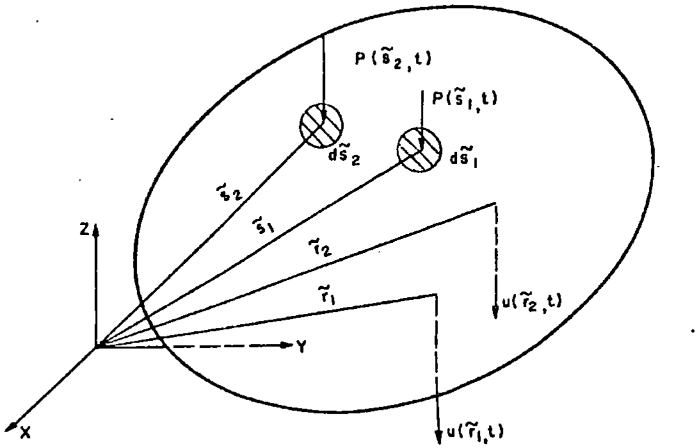

FIG. 4.—MULTIPLE LOADS AND RESPONSES OF THE STRUCTURE

sponse $u(\tilde{r})$ of a continuous body subjected to a point random load $p(\tilde{s})$, in which $\tilde{r}$ and $\tilde{s}$ are position vectors. It can be shown (9) that the response power spectrum $S_u(\tilde{r}, \omega)$ is related to the load spectrum $S_p(\tilde{s}, \omega)$ by an infinite combination of various modes

$$S_u(\tilde{r}, \omega) = S_p(\tilde{s}, \omega) \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} H_n^*(\omega) H_m(\omega) v_n(\tilde{r}) v_m(\tilde{r}) v_n(\tilde{s}) v_m(\tilde{s}) \quad (4)$$

in which $v_n(\tilde{r})$ represents the undamped normal modes, $H_m(\omega)$ is the frequency transfer function in the m^{th} mode, and $H_n^*(\omega)$ is the complex conjugate of $H_n(\omega)$.

The power spectral density function of many existing earthquake accelerograms are found to have single peaks only (10). This indicates for such cases only one mode predominates the entire motion and a single-mode spectral

simulation will be satisfactory to model the random ground motion. For such situations, Eq. 4 reduces to

$$S_u(\tilde{r}, \omega) = S_p(\tilde{s}, \omega) \, |H(\omega)|^2 \quad \ldots \quad (5)$$

which is the familiar expression for a one-dimensional case.

The problem of spectral simulation is to impose on a given body random motions whose spectral densities are equal to those observed. In earthquake engineering we are concerned with simulating random ground motions by matching the real power spectral density $S_u'(\omega)$ with the idealized, mathematically realizable spectral density $S_u(\omega)$, as given by Eq. 4 or Eq. 5. Such spectral simulation procedure is justified on the basis that strong motion earthquake accelerograms are generally Gaussian (2,3) and the power spectral density is sufficient to provide a complete statistical description for a Gaussian process.

A simple algebraic formula which gives smooth spectral density $S_u(\tilde{r}, \omega)$ may be obtained by assuming the input spectral density $S_p(\tilde{r}, \omega)$ = constant, and letting the modal transfer function $H_n(\omega)$ be that of a single-mode linear oscillator when considering that the base acceleration is input and the total acceleration of the mass is the output; more specifically

$$H_n(\omega) = \frac{1 + 2i\lambda_n\left(\dfrac{\omega}{\omega_n}\right)}{1 - \left(\dfrac{\omega}{\omega_n}\right)^2 + 2i\lambda_n\left(\dfrac{\omega}{\omega_n}\right)} \quad \ldots \quad (6)$$

in which λ_n and ω_n represent the modal damping and frequency respectively, and $i = \sqrt{-1}$ is the imaginary unit.

The spectral comparison or equivalence procedure can be performed with the guidance of a root-mean-square error criterion

$$\bar{\epsilon} = \sqrt{\sum_{i=1}^{l} (S_u' - S_u)^2} = \text{minimum} \quad \ldots \quad (7)$$

in which l is the total number of power spectral density in the significant frequency range.

Synthetic or artificial ground-motion acceleration may now be modelled by filtering a Gaussian stationary white noise through a single- (or multi-) degree-of-freedom linear system whose properties are determined by the spectral simulation procedure using Eqs. 4 through 7 as just described. Such procedure has merit in its ability to realize the local site geology. The generation of synthetic acceleration is obtained by employing the methods of Housner and Jennings (8).

A total of 50 artificial accelerograms each with a duration of 30 sec are generated to simulate El Centro 1940 N-S earthquake and used as input process to the nonlinear systems. The linear filter used in generating artificial earthquakes is a single-mode oscillator with a natural frequency ω_1 = 15.5 rads/s and viscous damping λ_1 = 0.42.

Statistics of the Maximum Displacement Response.—Since the maximum or peak, or single highest displacement (SHD) is used to measure the damage of nonlinear structures produced by dynamic loads, its statistics will be the ultimate concern of this study. For each of the eight structures investigated, the maximum values of displacement response are obtained by Monte Carlo computations of the output samples, using a digital computer. The mean, $\bar{u}$, and standard deviation, σ, of the maximum displacement (SHD) sequence for all eight cases and for three different durations of earthquake excitation are listed in Table 1. Both $\bar{u}$ and σ increase with the duration of excitation for a fixed structural system, indicating that time will allow the structure to "phase in" with the input and reach a larger response. This effect of extended duration of the input stationary process applies to the linear system as well as to the nonlinear yield type systems.

TABLE 1.—RESPONSE STATISTICS OF STRUCTURES SUBJECTED TO AN ENSEMBLE OF ARTIFICIAL EARTHQUAKES SIMULATING EL CENTRO 1940 N-S EARTHQUAKE

Structural Properties					Maximum Response Within Input Duration Range					
					First 10 Seconds		First 20 Seconds		30 Seconds	
Period (seconds)	Strength Ratio	Absolute Yield Displacement (inch)	Damping Ratio	Nonlinear Model	$\bar{u}$ (inch)	σ (inch)	$\bar{u}$ (inch)	σ (inch)	$\bar{u}$ (inch)	σ (inch)
0.3	0.10	0.088	0.02	Linear	—	—	—	—	0.768	0.115
				S-D[a]	1.670	0.542	2.166	0.764	2.480	0.711
				E-P[b]	1.739	0.805	2.509	1.312	3.214	1.613
			0.10	Linear	—	—	—	—	0.354	0.050
				S-D	0.932	0.273	1.179	0.338	1.327	0.360
				E-P	1.089	0.440	1.536	0.727	1.947	0.910
2.7	0.048	3.42	0.02	Linear	—	—	—	—	14.145	3.067
				S-D	9.234	4.382	12.187	5.693	14.325	5.831
				E-P	9.691	4.669	13.457	6.482	16.846	7.430
			0.10	Linear	—	—	—	—	8.767	1.312
				S-D	6.755	2.578	8.826	3.388	9.985	3.256
				E-P	7.079	2.950	9.573	4.067	11.567	4.565

[a] *S-D* = Stiffness Degrading System.
[b] *E-P* = Elasto-plastic System.

For a given damping, the mean and standard deviation both increase with an increase in initial period of the structure. This behavior, as is expected from the shape of earthquake response spectra for linear structures, also holds for the nonlinear yielding structures considered in this study.

The results in Table 1 show that all of these nonlinear structures are displaced beyond their yield limits. For comparison purposes, $\bar{u}$ and σ of the

corresponding linear systems with same stiffness and damping ratio subjected to the entire 30-sec input are also obtained and presented in the same table. It can be clearly seen from the values of $\bar{u}$ that, for short period structures, a strong-motion earthquake like El Centro 1940 N-S component would produce a much higher response for nonlinear yield type models than for linear models. For long period structures, the difference in $\bar{u}$ between linear and corresponding nonlinear models are relatively small.

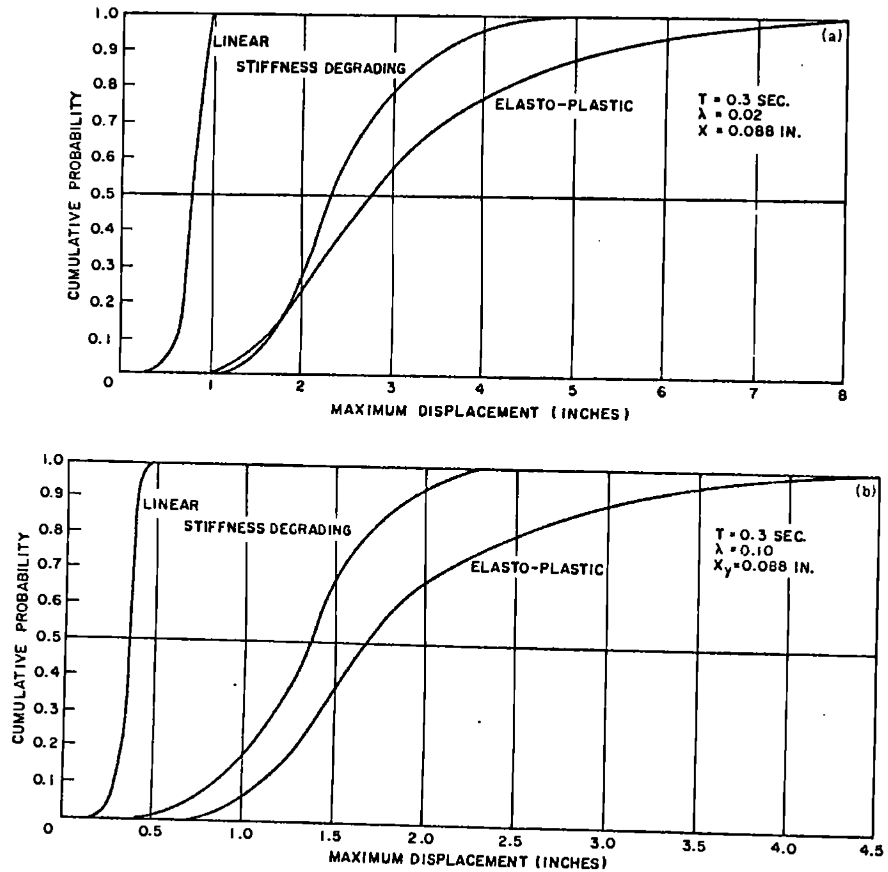

FIG. 5.—COMPARISON OF PEAK DISPLACEMENT DISTRIBUTIONS OF LINEAR AND NONLINEAR STRUCTURES

With regard to the variance of the mean peak structural displacement, the corresponding standard deviation σ for a linear system is smaller than that for either of the two corresponding nonlinear systems considered. Therefore the maximum displacement distribution for the nonlinear system is expected to spread over a relatively wider response range than that for the corresponding linear system having the same initial properties. This result suggests that one can design with more confidence for linear and stiff systems than for any other systems, is using the mean peak displacement response as the sole earthquake resistant design parameter.

To compare and correlate the responses of linear and nonlinear systems, the cumulative probability distribution of maximum displacement is used. The distribution diagrams for each of the four cases classified by the period and damping ratio of the systems are presented in Figs. 5(*a*) to 5(*d*). For a short period structure, either low damped or highly damped, the probability distri-

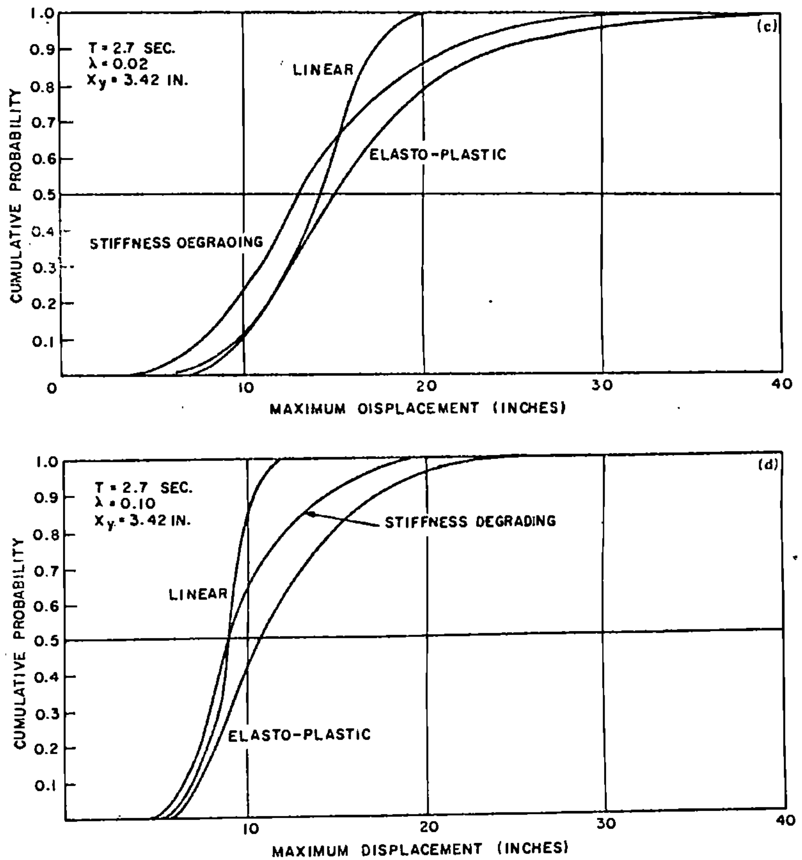

FIG. 5.—*CONTINUED*

butions of the maximum displacement for both nonlinear structural models are spread over a considerably wider range than those for the corresponding linear models [Fig. 5(*a*) and 5(*b*)]. Between the two nonlinear models, the elasto-plastic system apparently has larger $\bar{u}$ and σ than the corresponding stiffness degrading system. Relatively lower $\bar{u}$ and smaller σ are observed for the linear short period systems than for the corresponding nonlinear systems. However, when the period of the structure is increased the apparent differences in peak responses between linear and nonlinear models become

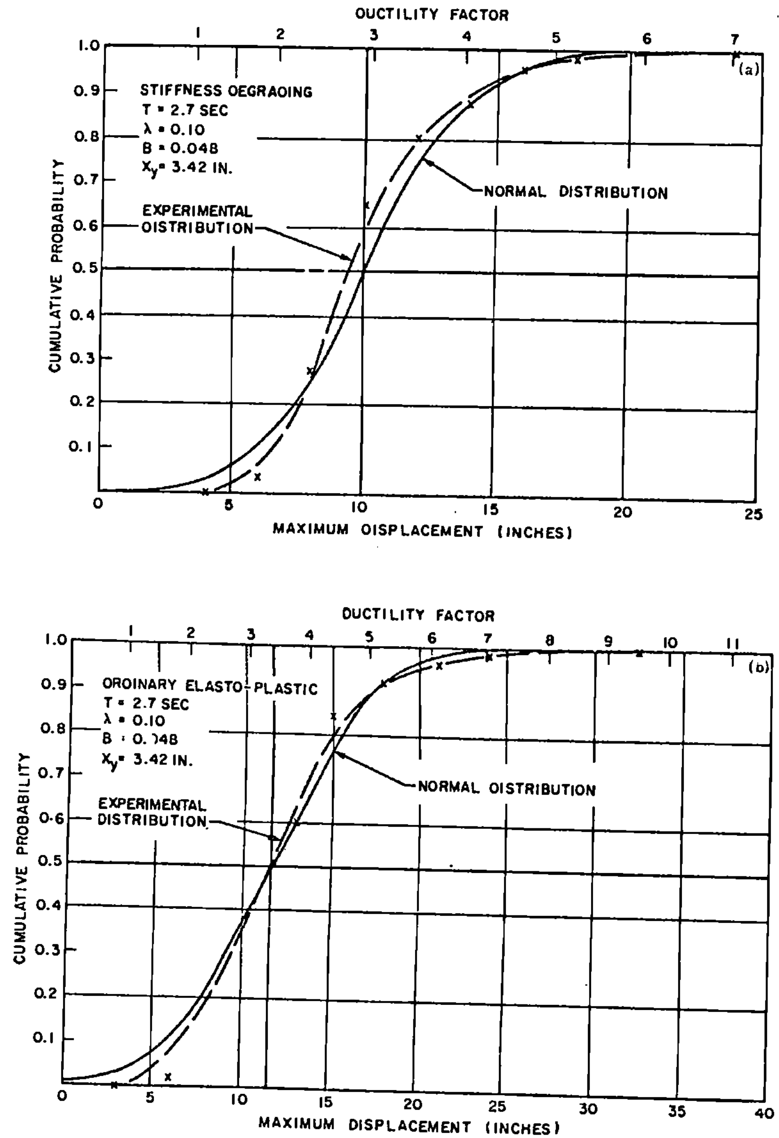

FIG. 6.—COMPARISON OF NORMAL AND EXPERIMENTAL DISTRIBUTIONS

smaller. For long-period structures the linear models still have smaller standard deviations than the corresponding nonlinear models, however, their mean peak displacement responses are of approximately the same order of magnitude as those of the corresponding nonlinear systems [Figs. 5(*c*) and 5(*d*)]. This result is valuable when dealing with high-rise buildings whose fundamental periods are generally long. In such cases, the nonlinear behavior of the structure may be disregarded without significant loss of accuracy when evaluating the peak displacement response produced by the random-type earthquake excitations.

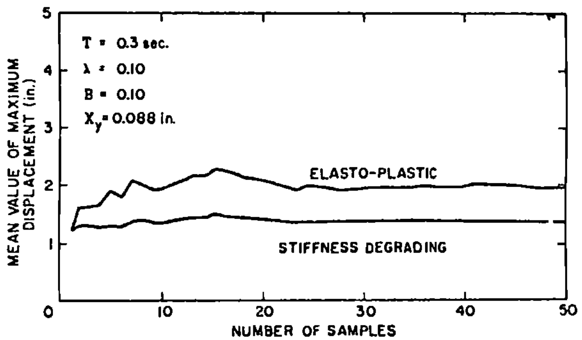

FIG. 7(*a*).—CONVERGENCE OF MEAN VALUE OF MAXIMUM STRUCTURAL RESPONSE TO ARTIFICIAL EARTHQUAKE

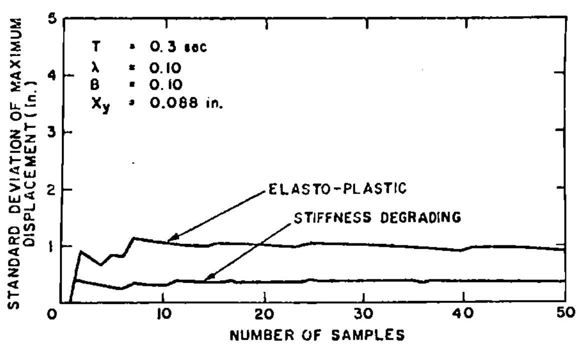

FIG. 7(*b*).—CONVERGENCE OF STANDARD DEVIATION OF MAXIMUM STRUCTURAL RESPONSE TO ARTIFICIAL EARTHQUAKE

The distribution of peak displacement response of nonlinear structures as presented in Figs. 5(*a*) to 5(*d*) are also compared with theoretical normal distributions constructed with the experimental $\bar{u}$ and σ taken from Table 1. The results are presented in Figs. 6(*a*) and 6(*b*). Note that the peak response of the nonlinear models can be measured also in terms of the ductility factor μ, defined as $\mu = u_{max}/X_y$. The theoretical normal distributions are symmetrical about the mean. For all cases, both the experimental curve and the theoretical normal curve reach the unity probability before or at about the 3σ level be-

yond the mean. It is therefore evident that the 3σ level above the mean may serve as the ultimate design limit for both types of nonlinear single-mode systems considered in this study. Earthquake-induced damage beyond that level, which corresponds to a failure probability of only one out of thousands of earthquakes, would be improbable and any consideration of setting the design requirement beyond the 3σ limit would be impractical.

Fig. 6 shows that in general, over the range of one standard deviation above and below the mean, the experimentally cumulative probabilities are slightly larger than the theoretical normal values. Over this range, the exceeding

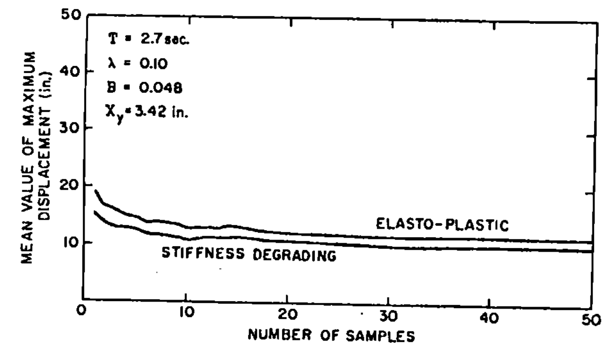

FIG. 7(*c*).—CONVERGENCE OF MEAN VALUE OF MAXIMUM STRUCTURAL RESPONSE TO ARTIFICIAL EARTHQUAKE

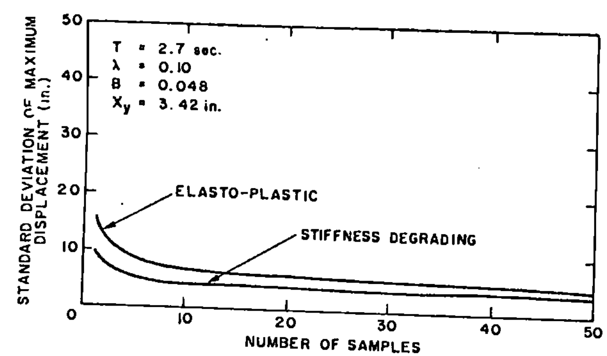

FIG. 7(*d*).—CONVERGENCE OF STANDARD DEVIATION OF MAXIMUM STRUCTURAL RESPONSE TO ARTIFICIAL EARTHQUAKE

probabilities from normal distribution are higher than the experiments and would be safe to use for design. Beyond this range, i.e., the low or high response ranges, the experimental cumulative probabilities are smaller than the theoretical normal values.

Curves showing the variation of mean and standard deviation with respect to the sample size are plotted in Figs. 7(*a*) to 7(*d*). These curves demonstrate the convergence property of the peak response and are necessary to justify the sufficiency of using a finite number of samples. Although the standard deviation takes a few more samples than the mean to converge, both of them stabilize to constant values after taking approximately 25 samples into account. Therefore, the 50 artificial earthquakes used in this analysis are sufficient to derive the statistics of the nonlinear systems.

Although a single input member earthquake might cause higher maximum response to either of the two nonlinear models considered in this investigation, it is evident from the mean maximum response diagrams [Figs. 7(*a*) and 7(*c*)] that the earthquake will induce higher response for the ordinary elasto-plastic model than the corresponding stiffness degrading model. This relatively lower response behavior for the stiffness degrading model is attributed to its ability in reducing the resonance with the earthquake and, more importantly, to its higher internal energy dissipation capability. A stiffness degrading system gives rise to hysteresis loops for all cycles of vibration after any amount of initial yielding while the elasto-plastic system only develops hysteresis loops during the cycles of vibration which exceed the yield limit.

DAMAGE ACCUMULATION BY CONSECUTIVE EARTHQUAKES

A problem of interest in earthquake and structural engineering is the determination of the probable maximum accumulated damage in a structure

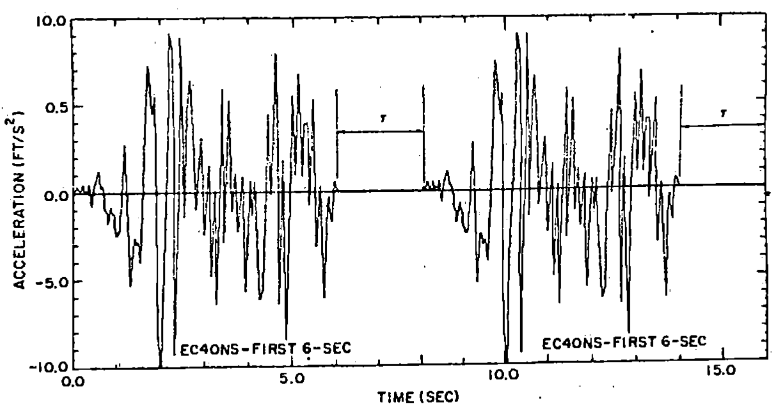

FIG. 8.—CONSECUTIVE GROUND EXCITATION FORMED BY SEGMENTS OF NATURAL EARTHQUAKE

when it is subjected to consecutive ground motions and when each single shock is strong enough to cause permanent deformation to that structure. This problem, often disregarded by engineers, arises from the practical viewpoint that a structure, having tolerated permanent deformation by earlier earthquakes, will also survive future excitations. Future forces may not be strong enough to seriously damage the structure if the structure retains its elastic property through all previous excitations. However, in case the many small quantities of permanent set of a structure produced by previous loadings are accumulated, any further ground disturbances to the structure, even a light-intensity after-shock, may cause the complete failure of that structure. A typical example of consecutive ground shocks can be provided by the current Parkfield, California earthquakes (1). On June 28, 1966 two large earthquakes occurred at 0409:56.5 (magnitude M = 5.3) and 0426:13.8 (magnitude M = 5.5) in the Parkfield-Cholame area. A third major shock occurred on the next day at 1953:26.2, with a comparable magnitude M = 5.0.

In the following section the effect and importance of existing permanent deformations of nonlinear yield-type structures on their future earthquake responses are analyzed. It is intended that this investigation will direct engineers' attention to this possible but serious mechanism of structural failure due to earthquakes. This damage accumulation phenomenon may possibly explain the failure of some structures in an active, moderate-intensity seismic area, which were designed under careful seismic resistant consideration, but without having had the deformation accumulation effect taken into account.

TABLE 2.—RESPONSES OF NONLINEAR STRUCTURES DUE TO CONSECUTIVE GROUND MOTIONS

Structural Properties (1)			First Excitation (6-second) (2)		Transition Zone (3)	Second Excitation (6-second) (4)	
Period (second)	Damping Ratio	Nonlinear Model	Maximum Displacement (inch)	Time of Occurrence (second)	Permanent Set (inch)	Maximum Displacement (inch)	Time of Occurrence (second)
0.3	0.02	*E-P*	-1.76	5.52	-1.30	-3.06	13.53
		S-D	-3.54	5.64	-1.24	-4.58	11.01
	0.10	*E-P*	-1.01	5.49	-0.77	-1.78	13.50
		S-D	-1.63	5.52	-0.41	-1.84	10.89
2.7	0.02	*E-P*	-10.83	3.78	-7.67	-18.50	18.72
		S-D	-11.74	3.81	-3.17	-13.49	18.90
	0.10	*E-P*	-8.06	3.27	-6.04	-14.10	18.30
		S-D	-8.44	3.30	-2.23	-9.56	18.72

It should be pointed out that in what follows, no attempt has been made to statistically analyze this damage accumulation problem, although such analysis clearly provides a new research area of practical importance in earthquake engineering.

Accumulative Response Due to Natural Earthquake.—All nonlinear yielding type systems considered in the previous section are now subjected to loading represented by two segments of the same earthquake accelerogram. The loading diagram as shown in Fig. 8 was formed by repeating the first six-second position of El Centro 1940 N-S earthquake, each followed by a null-amplitude portion with duration τ which induces free vibration of the structure. The τ values are so determined that they are long enough to pick up the permanent deformation of the structure due to the first shock.

The response of structures is solved numerically in accordance with Eqs. 1 to 3. Basic results included the response history and the corresponding force-displacement diagram. Some typical response histories are shown in Figs. 9(a) to 9(d) with the first two corresponding force-displacement diagrams shown in Figs. 10(a) to 10(b). The maximum displacement responses and the corresponding times of occurrence for each nonlinear system are summarized in Table 2. For all cases the maximum response during the second excitation is substantially larger than that procured by the first excitation. For all elasto-plastic systems, the maximum displacement due to the consecutive ground motions is identical to the sum of the maximum displacement and the

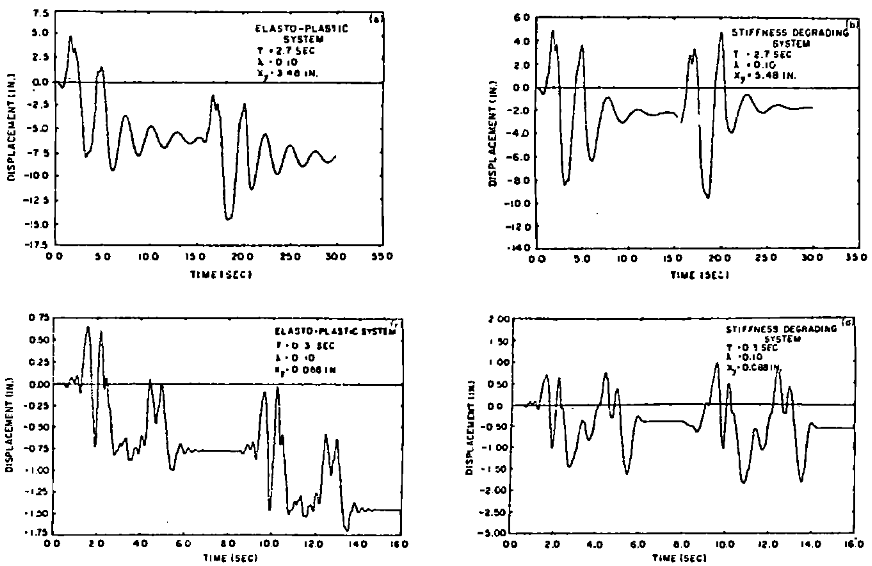

FIG. 9.—DISPLACEMENT RESPONSE HISTORY OF NONLINEAR STRUCTURES SUBJECTED TO CONSECUTIVE GROUND MOTIONS

permanent set produced by the first excitation [see Figs. 9(a) and 9(c)]—a result which directly follows from the equation of motion (Eq. 2). Although this relation is only approximately true, it will give conservative results for stiffness degrading models. From the response histories it is evident that for all elasto-plastic systems the motion during the second excitation is identical to that during the first excitation, except for a shift in the initial displacement due to the previously developed permanent deformation of the structure [Figs. 9(a) and 9(c). For the stiffness degrading cases, however, the system's motion during the second excitation is obviously distorted by the permanent set [Figs. 9(b) and 9(d). This phenomenon can be explained by the basic difference between the two nonlinear models; the elasto-plastic model is history-independent, while the effective stiffness of the stiffness degrading model is governed by

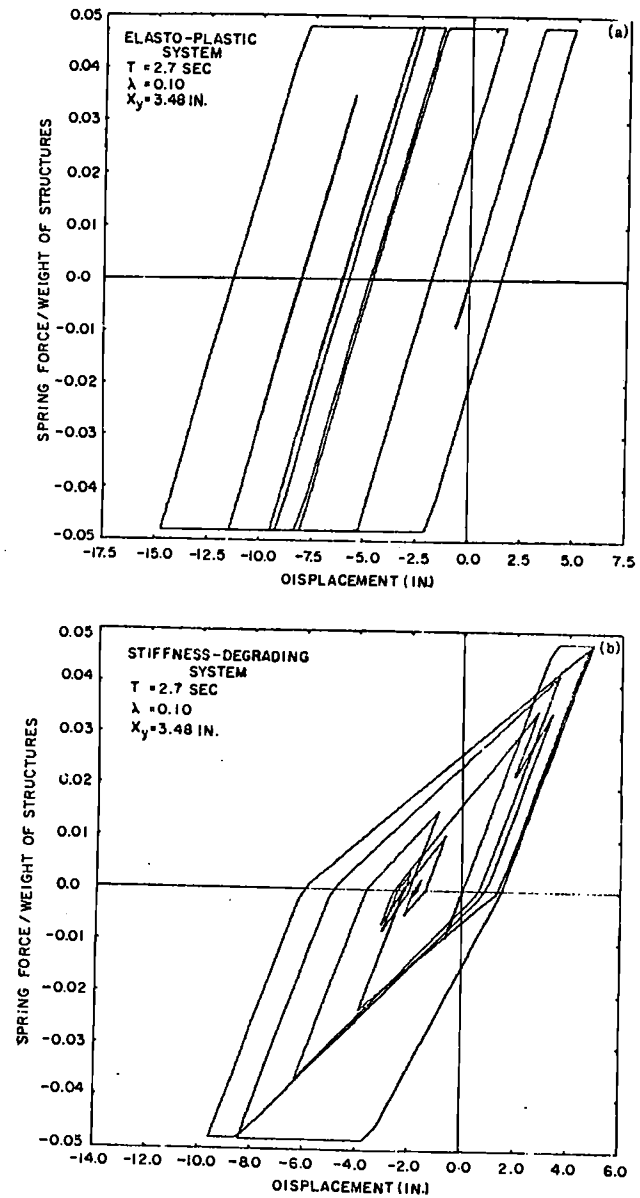

FIG. 10.—FORCE-DISPLACEMENT RESPONSE DIAGRAM OF NONLINEAR STRUCTURES SUBJECTED TO CONSECUTIVE GROUND MOTIONS

by its past motion, i.e., the stiffness degrading model is history-dependent [see Figs. 10(*a*) and 10(*b*)].

A close study of the numerical results shows that the damage accumulation for stiffness degrading systems is not so severe as the corresponding elasto-plastic systems. The accumulated maximum displacement response for the former type of nonlinear structures is generally less than the direct sum of the previous residual value and the undistorted peak response value (i.e., peak response with zero initial conditions) produced by the current excitation. The less severe damage accumulation in the stiffness degrading system is again attributed to its higher internal energy dissipation capability, as previously explained.

Accumulative Response Due to Random Impulsive Loadings.—Following the same procedure of analysis as in the previous section, the stiffness degrad-

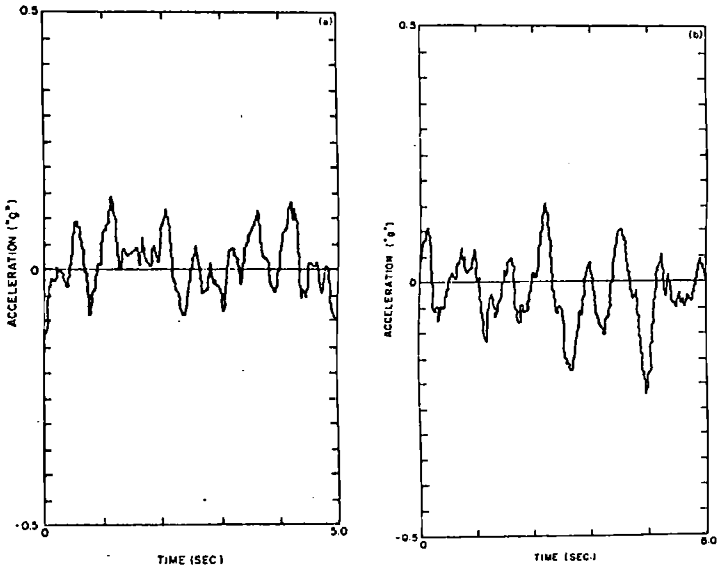

FIG. 11.—SHORT DURATION BURST, NOS. 1 AND 2

ing system with τ = 0.3 sec, λ = 0.02 and B = 0.1 is further subjected to consecutive impulsive loadings of 21-sec duration formed by any three of a total of five short-duration acceleration bursts [Figs. 11(*a*) and 11(*b*)], which are segments arbitrarily taken from the artificial earthquake ensemble previously used in simulating the El Centro 1940 N-S earthquake. The duration of each individual burst is chosen to be 5 sec which corresponds to the approximate time of the primary phase of strong-motion earthquake accelerograms.

The maximum and permanent displacement responses of the structure produced by each individual burst as well as the corresponding occurrence time of the response are listed in Table 3.

A total of seven different loading combinations were investigated. The results during the first 14-sec excitation period and the entire 21-sec period are presented in Table 4. A typical response history is shown in Fig. 12. It

is noted in Table 4 that the second load case resulted in an absolute maximum displacement of 4.129 in. Comparing this value with the corresponding single burst-induced maximum displacement of 3.498 in. (see Table 3), there is a substantial increase (18%) in the absolute maximum response.

TABLE 3.—NONLINEAR SINGLE BURST RESPONSE, STIFFNESS DEGRADING SYSTEM, T = 0.3 SECONDS, λ = 0.02, B = 0.1, x_y = 0.088 INCH

(1)	Maximum Response (2)		Permanent Response (3)	
Burst No.	Displacement (inch)	Time of Occurrence (second)	Displacement (inch)	Time of Occurrence (second)
1	-1.896	4.44	-1.167	6.21
2	3.498	4.35	2.687	6.09
3	-1.643	4.15	-0.576	6.27
4	-2.302	4.68	-1.830	6.09
5	1.803	2.94	-0.029	6.09

TABLE 4.—NONLINEAR RESPONSE TO CONSECUTIVE BURSTS STIFFNESS DEGRADING SYSTEM, T = 0.3 SECONDS, λ = 0.02, B = 0.10, X_y = 0.088 INCH

Input \ Output		Maximum Response				Permanent Response	
		14-Second		21-Second		14-Second	21-Second
Load Case	Order of Combination of 5-Second Bursts	Displacement (inch)	Time of Occurrence (second)	Displacement (inch)	Time of Occurrence (second)	Displacement (inch)	Displacement (inch)
1	(1) - (2) - (3)	2.598	11.37	2.626	15.06	1.774	0.205
2	(2) - (5) - (1)	4.129	9.66	4.129	9.66	-0.230	-0.100
3	(4) - (5) - (3)	-3.28	11.43	-4.036	18.15	0.000	-1.984
4	(3) - (5) - (2)	-2.393	11.94	3.133	18.36	0.000	2.243
5	(5) - (2) - (4)	3.778	11.37	4.060	15.63	2.940	-0.888
6	(3) - (2) - (1)	2.919	11.37	2.919	11.37	2.098	-0.683
7	(3) - (4) - (5)	-2.748	11.70	-3.199	18.96	0.000	-2.250

Fig. 12 illustrates another important result: the permanent set produced by previous loadings may be either positively or negatively added to the current response. This feature is not obvious when using segments of the same earthquake to model the consecutive ground excitation. Considering the most severe situation, i.e., the positively additive case based upon the numerical results obtained, the following formula may be used to estimate the maximum accumulative displacement response U_{max} of structures produced by a sequence of earthquake excitations, $x_{g,j}(t)$, $j = 1, \ldots, n$,

$$U_{max} = U_{max,j} + u_p \quad (8)$$

where $U_{max,j}$ and u_p indicate respectively the maximum, undistorted displacement response of the structure due to the current loading $x_{g,j}(t)$ and the permanent set produced by all previous earthquake loadings.

Effects of Duration and Intensity.—It has been shown that the permanent set u_p existing in a structural element may substantially increase the earthquake response. It is therefore desirable to investigate the effects of duration and intensity of the earthquake excitation on u_p. Physically, u_p is the displacement at a time t_p when the structure comes to rest. More specifically, it can be defined as

$$\left.\begin{array}{l} u_p = u(t_p) \\ \dot{u}(t_p) = 0, \; t_p > T_o \end{array}\right\} \quad (9)$$

where T_o is the duration of the excitation.

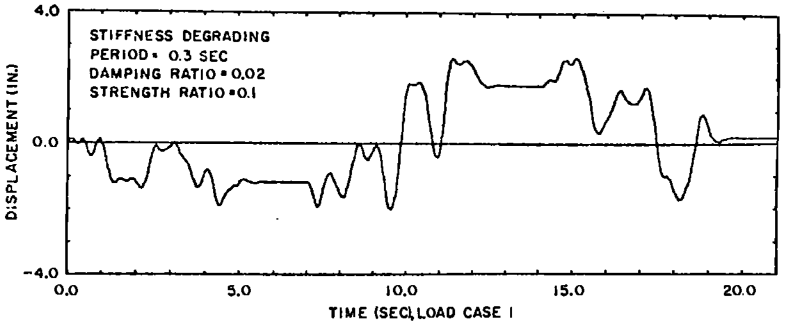

FIG. 12.—DISPLACEMENT RESPONSE DUE TO CONSECUTIVE BURSTS (1) - (2) - (3)

In Eq. 9, u_p is a function of many variables including the characteristics of the excitation (deterministic or nondeterministic), the structural properties such as λ, T, V_y or X_y, and the type of yielding mechanism. A preliminary analysis using an arbitrary sample member of artificial earthquake shows irregular variation of u_p with respect to T_o (varying in the range of 5 to 30 sec) and the rms amplitude of the excitation (varying in the range of 0.5 to 2.5 ft per sq sec) for nonlinear structures considered in this study. There is no evidence in the results that u_p would increase by any definite manner with the increase of the duration or intensity of the excitation.

CONCLUSIONS

This investigation demonstrates that response statistics of nonlinear yielding structures as well as linear structures can be easily obtained by treating the earthquake as a random process and using a nondeterministic method of

analysis. The mean value, variance, and probability distribution of the maximum response provide more reliable criteria for seismic design of structures than the traditionally used response spectra. Based upon results from a large number of samples, the stiffness degrading system shows more energy dissipation in the hysteresis loops during the cycles of motion beyond the yield limit as compared with the corresponding elasto-plastic system. In general, the response distribution for the nonlinear models has larger mean and variance than the distribution of the corresponding linear models. The difference between the peak earthquake response distributions of linear and nonlinear structures becomes smaller when the natural or initial period of the structure is increased.

It is also shown that the response of elasto-plastic or stiffness degrading structures produced by consecutive earthquake excitations are accumulative and may be approximately determined by a simple superposition rule. The effect of the permanent deformation existing in a structure on its future earthquake response is important. In designing a structure in an active, moderate-intensity seismic area, the damage accumulation should be considered when estimating the ultimate earthquake resistance capacity of that structure.

ACKNOWLEDGMENTS

The work was partially done during the course of the writers graduate research at the University of California, Berkeley, California. The writer wishes to thank J. Penzien and R. W. Clough of the University of California for their valuable discussions and suggestions.

APPENDIX I.—REFERENCES

1. Algermissen, S. T., et al., "Studies on the Parkfield, California Earthquakes of June, 1966," *Bulletin of the Seismological Society of America*, Vol. 57, No. 6, December, 1967, pp. 1131–1266.
2. Caughey, T. K., and Stumpf, H. J., "Transient Response of a Dynamic System Under Random Excitation," *Transactions*, American Society of Mechanical Engineers, Vol. 83, Series E, 1961, pp. 563–566.
3. Caughey, T. K., and Stumpf, H. J., author's closure to discussion by Goldberg, J. E., and Bogdanoff, J. L., of "Transient Response of a Dynamic System Under Random Excitation," *Transactions*, American Society of Mechanical Engineers, September, 1962, pp. 594–595.
4. Clough, R. W., "Effect of Stiffness Degradation on Earthquake Ductility Requirements," *Report No. 66-14*, Structures and Materials Research, University of California, Berkeley, California, 1966.
5. Clough, R. W., and Wilson, E. L., "Dynamic Response by Step-by Step Matrix Analysis," *Symposium on Use of Computers in Civil Engineering*, Lisbon, Portugal, 1962.
6. Hanson, N. W., and Conner, H. W., "Reinforced Concrete Beam-Column Connections for Earthquakes," Portland Cement Association, 1965.
7. Hanson, N. W., and Conner, H. W., "Seismic Resistance of Reinforced Concrete Beam Column Joint," *Proceedings*, Journal of the Structural Division, ASCE, Vol. 93, No. ST5, Proc. Paper 5537, October, 1967, pp. 533–560.

8. Housner, G. W., and Jennings, P. C., "Generation of Artificial Earthquakes," *Proceedings*, American Society of Civil Engineers, Vol. 90, No. EM1, Proc. Paper 3806, February, 1964, pp. 113–150.
9. Liu, S. C., and Jhaveri, D. P., "Spectral Simulation and Earthquake Site Properties," to be presented at ASCE National Meeting at Louisville, Kentucky, Apr., 1969.
10. Liu, S. C., and Jhaveri, D. P., "Power Spectral and Correlation Analysis of Ground-Motion Accelerograms," (to be published).
11. Penzien, J., "Dynamic Response of Elasto-Plastic Frames," *Transactions*, American Society of Civil Engineers, Vol. 127, 1962, p. 1.
12. Penzien, J., "Elasto-Plastic Response of Idealized Multi-Story Structures Subjected to Strong Motion Earthquakes," *Proceedings*, Second World Conference on Earthquake Engineering, Japan, 1960.
13. Uniform Building Code, *Section 2313(d)*, 1967 Edition.

APPENDIX II.—NOTATION

The following symbols are used in this paper:

B = V_y/w;
g = acceleration due to gravity;
$H(\omega)$ = frequency transfer function or complex frequency response of single-degree-of-freedom system;
$H_n(\omega)$ = frequency transfer function in generalized coordinates;
$H_n^*(\omega)$ = complex conjugate of $H_n(\omega)$;
i = $\sqrt{-1}$;
k = total stiffness of structure;
k_e = initial or elastic stiffness of nonlinear structure;
l = total number of discrete power spectrum values;
M = total mass of structure;
$p(\tilde{s})$ = distributed input process;
$\tilde{r}$ = response position vector;
S_p, S_u = power spectral density functions;
S_u' = actual power spectral densities calculated from given accelerogram;
$\tilde{s}$ = load position vector;
T = period of structure;
T_o = duration of earthquake excitation;
t_p = time when structure comes to rest;
U_{max} = maximum accumulative displacement response;
$U_{max,j}$ = maximum, undistorted displacement with zero initial conditions;
$u(\tilde{r})$ = response process;
$u(t)$ = relative displacement;
u_n = displacement at time $n\Delta\tau$;
$\dot{u}_n$ = velocity at time $n\Delta\tau$;
$\ddot{u}_n$ = acceleration at time $n\Delta\tau$;
u_p = permanent displacement;
$\bar{u}$ = mean value of maximum displacement responses;

$V(u)$ = spring force;
V_y = yielding force or resistance;
$v_n(\tilde{r}), v_n(\tilde{s})$ = undamped normal modes;
W = total weight of structure;
X_y = yield displacement;
$\ddot{x}_g(t)$ = earthquake ground acceleration;
$\bar{\epsilon}$ = mean square error of spectral comparison;
λ = fraction of critical coefficient of viscous damping;
λ_n = modal ground damping;
σ = standard deviation;
t = time;
μ = ductility factor;
ω = frequency;
ω_n = modal ground frequency.