

# On a Hysteresis Model for Transient Analysis

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**Abstract:** The letter describes an approach to a representation of transient behavior due to switching off complex nonlinear circuits. For this purpose the widely known Jiles model (JM) was implemented into alternative-transient-program (ATP). Calculations have been made on simplified transformer single-phase and three-phase circuits.

**Keywords:** Jiles model, hysteresis, ATP-EMTP, overvoltage.

**Introduction:** So far a great amount of research has been conducted on the representation of hysteresis behavior of ferromagnetic materials. Among all models, Jiles model (JM) [1] takes a significant place in the representation of dynamic hysteresis in ferromagnetic materials. Since the early 1980s many publications have been written about the application of this model for different kinds of materials. Later it was shown that this model could also be used for ferroresonance studies [2].

When switching off transformer magnetizing currents the only load that is seen by the circuit breaker is the transformer core, which is a nonlinear inductance. In order to represent the behavior of the transformer core during switching it is important that a dynamic hysteresis model be included into the transformer circuit. After the transformer is de-energized, the flux no longer follows the 50 Hz hysteresis curve and ends with a residual flux. The consideration of this effect strongly influences the released magnetic energy from the core, which further influences the generated overvoltages due to transformer switching. The approach used in this work provides the possibility for studying the transients in complex nonlinear circuits. A concrete example of the dynamic flux decay and generated overvoltages is presented. It is shown that the parameters of the hysteresis model strongly affect the peak magnetizing current, which is responsible for the maximum overvoltage level. We consider a special case when an ideal chopping (without re-ignition) in the circuit breaker takes place.

**The Theory of the JM:** The hysteresis effect of any ferromagnetic material can be described through the JM, given by a first-order nonlinear differential equation, which can be solved numerically to give the magnetization  $M$  or magnetic field  $H$  if one of these parameters is known. We use the JM in the following form:

$$\frac{dH}{dM} = \frac{k \cdot \delta - \alpha \cdot \left( M_{an} - M + k \cdot \delta \cdot c \cdot \frac{dM_{an}}{dH_c} \right)}{M_{an} - M + k \cdot \delta \cdot c \cdot \frac{dM_{an}}{dH_c}} \quad (1)$$

The effective field is given by

$$H_c = H + \alpha M(H). \quad (2)$$

The parameter  $\delta$  in (1) is:

$$\delta = \begin{cases} 0 & \text{if } \frac{dH}{dt} < 0 \text{ and } M_{an}(H_c) - M(H) \geq 0 \\ 0 & \text{if } \frac{dH}{dt} > 0 \text{ and } M_{an}(H_c) - M(H) \leq 0 \\ 1 & \text{otherwise.} \end{cases}$$

There are three parameters appearing in (1),  $c$  (dimensionless) is a ratio of the initial normal to the initial anhysteretic susceptibility,  $\alpha$  (dimensionless) is a mean field parameter representing the coupling between the domains, and  $k$  (A/m) is a measure of hysteresis. An increase or decrease of the parameter  $k$  leads to hard or soft hysteresis, respectively. If  $k = 0$  then no hysteresis exists. Furthermore, the shape of the

hysteresis is determined by an anhysteretic magnetization  $M_{an}(H)$ . It is given by the expression

$$M_{an} = M_s \left[ \coth\left(\frac{H_c}{a}\right) - \frac{a}{H_c} \right] \quad (3)$$

where  $M_s$  (A/m) is a saturation magnetization and  $a$  (A/m) is a parameter that shows how the anhysteretic magnetization scales with  $H_c$ . Equation (1) is solved by means of Runge-Kutta method of fourth order in order to calculate the magnetization. Numerical computation of (3) and its derivative at small values of  $H_c/a$  ( $H_c/a < 0.5$ ) was obtained by developing (3) in Taylor series in order to avoid the problem of calculating its value around  $H_c = 0$ . Figure 1 presents the variation of the steady-state hysteresis for different values of the parameter  $k$ . One could see that an increase of this parameter leads to an increase of the hysteresis losses. Thus, different shapes of steady-state hysteresis loops are available. The determination of all parameters in the JM is possible if we have a measured hysteresis curve. The complete procedure of parameter extraction from experimental data and parameters for some magnetic materials is given in [1].

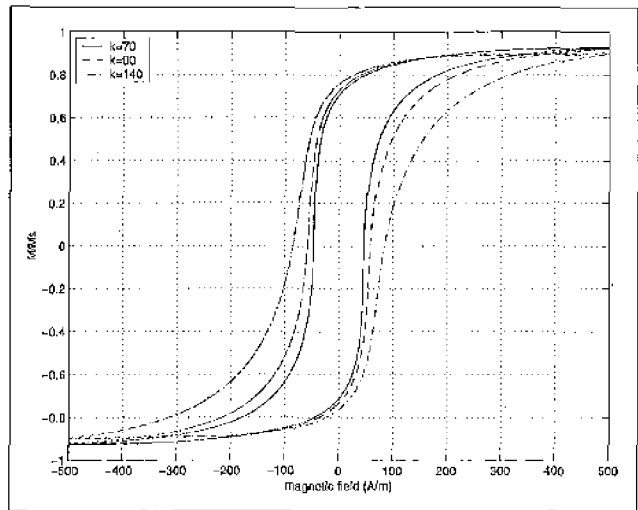


Figure 1. Calculated steady-state magnetization versus applied magnetic field for different parameter  $k$

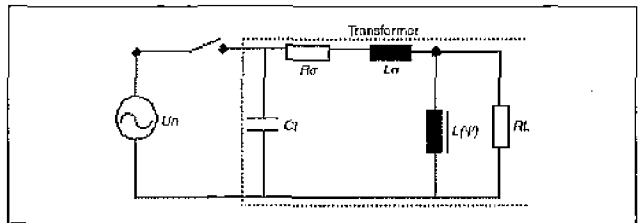


Figure 2. Single-phase transformer circuit

Hysteresis Model Data		Circuit Data		Transformer Core Data	
$k$	60 A/m	$U/N$	15000 V	$N$	792 turns
$\alpha$	8.13-5	$C_L$	1 nF	$S$	388 cm <sup>2</sup>
$c$	0.2	$R_G$	0.1 $\Omega$	$l$	3.2 m
$M_s$	$1.34 \cdot 10^6$ A/m	$L_G$	5 mH		
		$R_L$	$8.E+5 \Omega$		

**Application of the Model:** The implementation of the model in ATP is done by means of nonlinear inductor TYPF-94 [5]. By means of an input voltage at each time step, the magnetic flux density is calculated as

$$B = \frac{1}{NS} \int (U_A - U_B) dt \quad (4)$$

where  $N$  is the number of turns,  $S$  is the cross section of the magnetic core, and  $U_A$  and  $U_B$  are the voltages of the inductance nodes. Magnetization is expressed as

$$M = \frac{B}{\mu_0} - H \quad (5)$$

where  $H$  is the calculated magnetic field from the previous time step. Having the value of  $M$ , (1) can be solved to give the magnetic field at each time step and the nonlinear inductance. The current through the nonlinear inductance is calculated by the magnetic field as

$$i = \frac{Hl}{N} \quad (6)$$

where  $l$  is the length of the magnetic flux path. This method can be repeated as many times as the number of nonlinear inductances in the circuit. The calculation was made for the simplified transformer circuit given in Figure 2. The parameters of (1) and the circuit in Figure 2 are given in Table 1. The calculation is done for different values of parameter  $a$ . For the steady-state case, calculated results by ATP were also checked numerically. Neglecting the transformer losses we can write

$$\frac{dB}{dt} = \frac{1}{NS} \left( U_N \cos(\omega t) - R_\sigma i - L_\sigma \frac{di}{dt} \right) \quad (7)$$

Taking into account (5) and (6), (1) and (7) are solved numerically. Calculated results are compared with the ATP simulation and are shown in Figure 3. Having the model implemented into ATP we will show two examples of switching simplified single-phase and three-phase unloaded transformer. Results of the simulation for the single-phase circuit are shown in Figures 4 and 5. The circuit breaker is switched at peak magnetizing current in order to get the maximum value of the voltage. It can be seen that the parameter  $a$  influences the peak magnetizing current. This causes a different peak voltage after switching. Figure 5 shows the residual flux in the transformer core after the magnetizing current has disappeared. For a three-phase case, the same circuit from Figure 2 was used and the transformer is considered as three single-phase transformers. The windings are star connected.

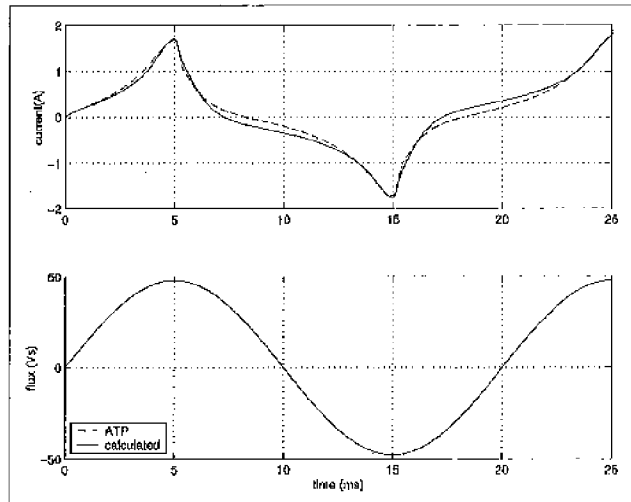


Figure 3. Calculated and simulated magnetizing current and flux

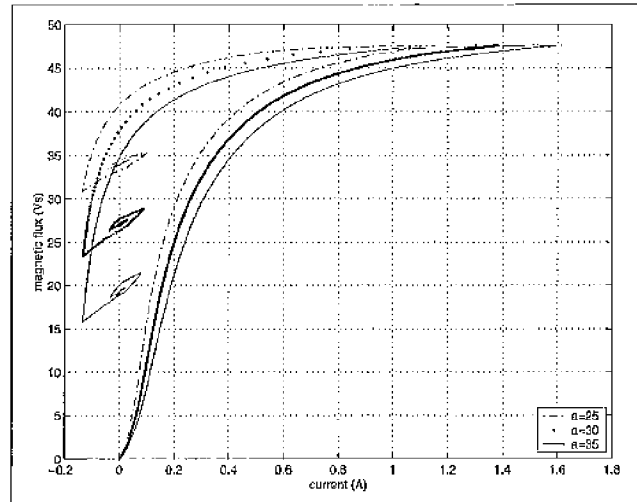


Figure 5. Transformer flux after de-energizing

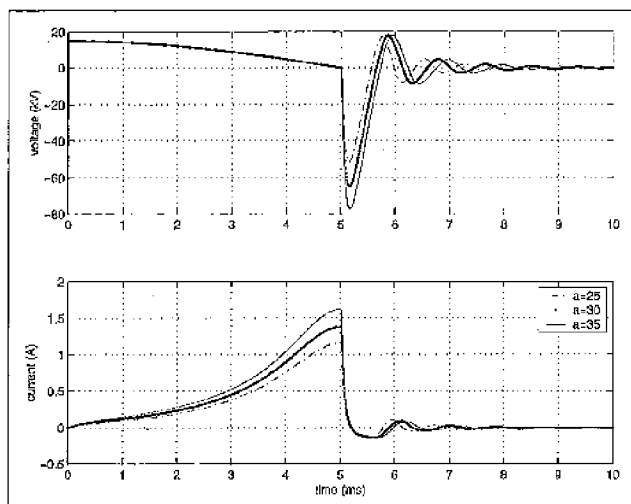


Figure 4. The magnetizing current and terminal voltage after de-energizing

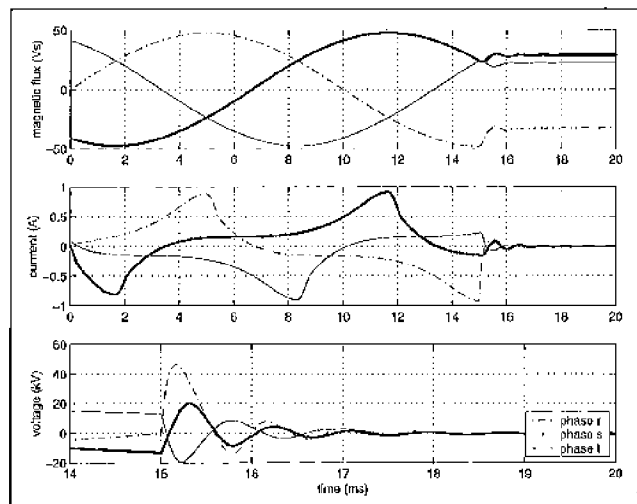


Figure 6. Transformer winding fluxes, magnetizing currents, and terminal voltages

The circuit breaker CB is set to open at maximal magnetizing current in phase R. The results of the simulated transformer fluxes, magnetizing currents, and the overvoltages after de-energizing are shown in Figure 6.

**Conclusion:** Due to lack of experimental results and field tests, we compared our results with previous experimental and analytical analyses. In [4] an experimental measurement of a dynamic flux decay and switching overvoltage of a laboratory single-phase transformer was done, whereas [3] deals with an implementation of the Preisach model for transformer transient studies.

The results presented in this letter are in good agreement with the results given in these references. We believe that this approach can find a further interest in the calculation of transient phenomena in networks with a greater number of nonlinear elements.

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## Novel Energy-Based Lyapunov Function for Controlled Power Systems

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**Abstract:** This letter first derives Hamiltonian equation for a power system with excitation control. A modified energy-based Lyapunov function is then presented for fast transient stability analysis of controlled power systems.

**Introduction:** It is widely known that excitation control plays an important role in enhancing small-signal and transient stability of power systems. The traditional method for transient stability analysis of the effects of excitation system on transient stability improvement is based on step-by-step numerical integration techniques. With the increased pressure to maximize power transfers, electric utilities are being pushed to operate their systems much closer to their stability limits. Thus fast transient stability analysis method based on transient energy function (i.e., direct method) has been a significant tool in power system planning, operation, and control. During the past two decades, significant progress has been made in practical applications of direct method to transient stability analysis of power systems. Several major direct methods have been proposed [1], [2]. There have also been several attempts to extend the direct method to include excitation control system [3], but these techniques have only carried out an approximate estimation of the effectiveness of excitation control on transient stability. Therefore, in this paper, first one machine connected to an infinite bus system with excitation control has been converted into a Hamiltonian system. Then a modified energy-based Lyapunov function is presented for fast transient stability analysis of controlled power systems.

**Review of the Main Results of Hamiltonian System [4]:** A forced Hamiltonian system can be described as

$$\begin{aligned} \dot{x} &= [J(x) - R(x)] \frac{\partial H(x)}{\partial x} + g(x)u \\ \dot{y} &= g^T(x) \frac{\partial H(x)}{\partial x} \end{aligned} \quad (1)$$

where  $x \in M$ , a manifold,  $u, y \in R^m$ ,  $J(x)$  is a skew-symmetric matrix and defines a generalized Poisson bracket on  $M$ .  $R(x)$  is a nonnegative symmetric matrix and defines a symmetric bracket on the state manifold  $M$ . In general, the function  $H(x)$  represents the total stored energy. Its energy balance can be written as

$$\frac{dH(x)}{dt} = - \frac{\partial^T H(x)}{\partial x} R(x) \frac{\partial H(x)}{\partial x} + u^T y \quad (2)$$

where the first term on the right hand represents the energy dissipation due to the resistive elements in the system and the second term is the power externally supplied to the system.

According to Lyapunov stability definition, the function  $H(x)$  is a Lyapunov function for the equilibrium  $x_s$  if the function  $H(x)$  has a local minimum at  $x_s$  and  $dH/dt < 0$  is satisfied. Thus for the unforced system, that is  $u = 0$ , the total stored energy  $H(x)$  is a Lyapunov function for investigating the stability of the equilibrium  $x_s$ . For the forced Hamiltonian system, the forced equilibria  $\bar{x}_s$  are the solutions of

$$[J(\bar{x}_s) - R(\bar{x}_s)] \frac{\partial H}{\partial x}(\bar{x}_s) + g(\bar{x}_s)u = 0. \quad (3)$$

In general,  $\bar{x}_s$  will not be a minimum (or extremum) of  $H(x)$ . Furthermore, inserting  $u = \bar{u}$  in (2) yields

$$\frac{dH(x)}{dt} = - \frac{\partial^T H(x)}{\partial x} R(x) \frac{\partial H(x)}{\partial x} + \bar{u}^T g^T(x) \frac{\partial H(x)}{\partial x} \quad (4)$$