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An operational simulation framework for modelling the multi-interaction of two-wheelers on mixed-traffic road segments

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ABSTRACT

In recent years, the interest in riding in cities using the two-wheeler (e.g., bicycles, electric bicycles, electric mopeds, etc.) increases. Mixed-traffic road segments are one of the most common traffic scenes where the mixed two-wheeler flows exist. Because the movements are often not restricted by lanes, the two-wheeler uses lateral road space more freely and shows obvious multilateral interactions (i.e. multi-interaction) with others, bringing issues that endanger traffic safety. A precise estimation of its impacts on traffic operation and safety is necessary, while the microscopic simulation model can satisfy the need as a helpful tool. However, most existing simulation models of these three types of two-wheelers are essentially focusing on handling the one-on-one interaction. The capability to deal with the two-wheeler multi-interaction in mixed traffic is still rare, and the description of what endogenous tasks are contained by the multi-interaction has also not given by literature. To this end, this paper first defines what the multi-interaction entails on the operational behaviour level, claiming that it contains three intertwined processes, namely a (mental) perception, a (mental) decision, and a physical process. The (mental) perception and decision processes represent the recognition of interactions and the response to traffic conditions, while the physical process refers to the execution of these mental activities. A three-layer simulation framework has then been developed, where each layer sequentially corresponds to one of the operational behaviour tasks. Integrated component models are also proposed in each layer to cover these operational tasks. A Comfort Zone model is hence put forward to dynamically perceive the multiple interactive road users, while a Bayesian network model is developed to deal with the decision-making process under multi-interaction situations. Meanwhile, a behaviour force model is also proposed to capture the non-lane based movements following the selected behaviour and current interaction states. Finally, we face validate the proposed models by the comparison between simulation results and observations obtained from trajectory dataset. Results indicate the model performance matches the observed interaction and motion well.

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1. Introduction

Road segment separated by traffic markings is a widely adopted traffic design on urban roads [1]. In most cases, such a road segment is the mixed-traffic context where motorized vehicles (e.g., cars) and types of two-wheelers (e.g., bicycles, electric bicycles (e-bikes), electric mopeds (e-mopeds), etc.) commonly exist [2]. The renaissance of active modes [3] and the obviously increasing number of e-mopeds [4] and e-bikes [5] further normalizes the mixed two-wheeler traffic on urban roads. However, due to the non-strict using rights of road space and non-compliance with traffic regulations [6], different kinds of road users in fact move and interact on the single-surface area with the same priority, leading to many issues that restrict traffic safety [7]. Therefore, realistically representing of complex interactions for different types of road users arising in mixed-traffic road segments is necessary for the basic measurement of traffic operations [8]. For this purpose, microscopic simulation models are helpful [9]. In previous studies, most models focus on reproducing behaviours of a single traffic mode like vehicles [10], pedestrians [11], and bicycles [12]. However, the discussion of how two-wheelers of various sorts interact with different road users (both inter- and intra-modal interactions) is by far not as mature as that of other traffic modes. Therefore, a precise microscopic simulation model of mixed two-wheelers is still needed to support the management of mixed traffic under different conditions on urban road segments. In this paper, we put attention to the mixed two-wheeler flow that contains bicycles, e-bikes, and e-mopeds due to their extensive existence in many areas, like China [13], West Africa [14], and some European countries like Spain [15]. Herein, both e-bikes and e-mopeds are special bicycles that are powered via electric engines (with similar dynamics performance), while the major difference between these two types of electric two-wheeler is their structure and size [16].

On realistic mixed-traffic road segments, we often encounter the situation where implicit interactions simultaneously occur between a two-wheeler and several road users [17], as shown in Fig. 1. This phenomenon is named in this paper as the multi-interaction of two-wheelers. In such scenes, multiple interactive road users would jointly affect the current two-wheeler's operation. It means, for any two-wheelers at any point in time, they need to simultaneously handle multilateral interactions with several road users during the whole operation process, which is from perceptions, decisions, to actions based on the traffic environment [18]. Two-wheelers interact on the two-dimensional space, performing on adjusting trajectories by executing control dynamics based on the complex human decision-making process. Therefore, describing the multi-interaction of two-wheelers is far from trivial. Furthermore, multiple road users who interact with the two-wheeler are not always set in stone, but constantly change following the variations of the traffic context. This is mainly due to each individual's dynamics (e.g. moving direction, speed, etc.) altering while moving, which leads to the extent of impacts of other road users on the two-wheeler's operations constantly changing. Hence, such a dynamic feature makes the multi-interaction become more challenging to be represented. Moreover, the multi-interaction sometimes occurs between a two-wheeler and several different types of road users. The differences in aspects like acceleration ability and shape/dimension between them would also make interactions become more complex. For that, the ability of a simulation model to capture the two-wheeler dynamic multi-interaction is crucial [19,20], which determines whether it can reproduce the realistic behaviour of two-wheelers in mixed traffic to improve simulation modelling performance and understand how two-wheeler multi-interaction affects traffic safety and operation.

Advances in current bicycle traffic simulation models can guide the development of simulation models for two-wheelers. This is because the bicycle is one of the typical two-wheelers. According to the definition of behaviour for bicycles made by Gavriilidou et al. [21], tasks of interaction during riding belong to the same operational level and should be simultaneously modelled from both mental (decisions) and physical (actions) processes. For decades, several research efforts have been made to capture the two-wheeler (mainly related to cyclists) interactions, however, they cannot well represent the two-wheeler behaviour to some extent. Early microscopic models such as lane-based models and most of the commercial simulation software such as VISSIM, Transmodeller, etc. pay more attention to handling the physical activities of agents based on virtual lanes [22], as shown in Fig. 2(a). Meanwhile, most mainstream models, namely Cellular Automata (CA), deal with the movements and interactions using the principle of spatial-discrete assumptions [23]. The one-on-one interaction between cells and lanes has not been adjusted to fit the operations of two-wheelers in reality [21,24]. Even recent simulations using force-based models [25] can reflect the contact with several road users on the two-dimensional space, they are still regarded essentially as the pure dynamics control of the operational physical process that is influenced by the interactive forces from adjacent agents [26]. Some attempts based on the social force model are also made for operational mental process modelling. However, these models generally default the nearest front road users within a certain lateral space as the interaction object to simplify the interaction into the one-on-one pattern, and influences of other road users are still regarded as effects exerted by repulsive forces. Behaviour choice during the mental progress is hence generally considered as a result that is only influenced by the specified agent [27,28]. This kind of assumption restricts the ability of existing models to deal with the two-wheeler multi-interaction that is accompanied by decision-making tasks on the two-dimensional space (Fig. 2(b)). Furthermore, several agent-based models and hybrid models are beneficial for capturing the whole interaction process of two-wheelers from decisions to actions [29–31]. These models have the ability to describe the detail of behaviours of agent and imitates the real-world heterogeneity [32,33]. In the decade, research of Lee et al. [33] and Lee and Wong [34] have provided a good reference to modelling the multi-interaction on the two-dimensional space for motorcycles in mixed traffic consisting of vehicles and motorcycles. Despite they gave good guidance of the further research, due to the different dynamics and behaviour features [3,24], models for such a mixed bicycle flow in our study may need to be revisited and discussed to represent the multi-interaction

Fig. 1. Multi-interactions of a two-wheeler in a real-life scenario.

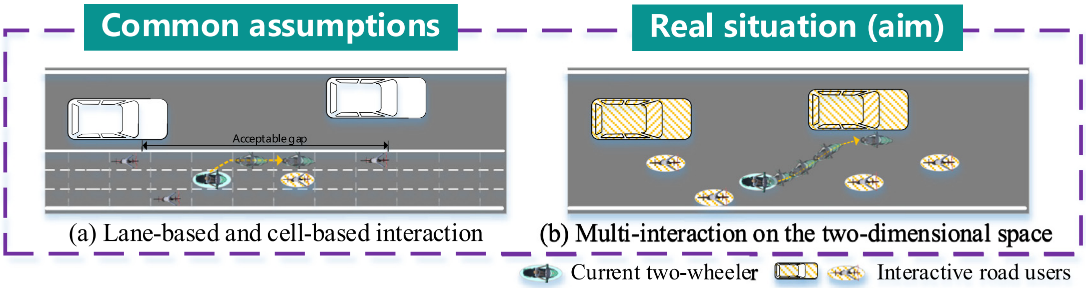


Fig. 2. Comparison of general assumptions of the two-wheeler interaction and the reality.

among bicyclists, especially for their operational mental activities. In all, most existing simulation models of two-wheelers (only including bicycles, e-bikes, and e-mopeds) roughly provide the option to simulate their interactions and behaviours. Two-dimensional models, such as the social force model and its modified variations, bring the ability to represent the movement on the continuous space and contact with multiple road users on the operational physical level, however, they still need additional components to deal with simultaneous multi-interactions on the operational mental level. This is likely because there are rare concepts in related literature that well represent what endogenous tasks are contained on the operational level (especially the mental layer) when the multi-interaction happens. Models for capturing the corresponding task of the operational mental process are also still lacking in the real two-dimensional conditions without lane/cell-based hypotheses. When we face the need for multi-interaction modelling of such a non-motorized two-wheeler flow, the description of these endogenous tasks and their connections between the mental activities, and with the physical process would also be crucial.

To this end, we first give a proper definition of what the two-wheeler multi-interaction entails on the operational behaviour level. We argue that three major tasks for multi-interactions are owned by the operational behaviour level, namely, interaction perception and behaviour decision during the mental process, as well as the dynamic control for the physical process. At the same time, we correspondingly design a three-layer simulation framework to capture the dynamic multi-interactions throughout these operational tasks. In each layer of the framework, we develop integrated components to describe connotations for each task. First, a "Comfort Zone (CZ)" model is introduced to specially capture the multiple interactive road users that dynamically vary following the traffic context. Secondly, this paper develops for the first time the use of the Bayesian network (BN) model to represent the decision-making procedure for two-wheelers under multi-interaction situations. Thirdly, a behaviour force model is designed to support the non-lane based movements of two-wheelers under the guidance of the behaviour choice and multi-interaction. Finally, the proposed models are face validated and tested using a trajectory dataset collected on a mixed-traffic road segment in Shanghai, China, demonstrating their satisfactory performance.

The paper is outlined as follows. In Section 2, we review the relevant state-of-the-art and summary the research gaps. Section 3 describes the definition of the two-wheelers operational level and then explains the proposed simulation framework for representing two-wheeler multi-interaction. Section 4 introduces the specific methods proposed in each layer of the framework. In Section 5, the model calibration results are presented, along with the face validation of the model using simulation. Finally, Section 6 concludes the study and suggests possible directions for future research.

2. Related works

Over the last decade, some microscopic models dedicated to modelling operational behaviours of the two-wheeler have been developed (mainly about the bicycle traffic). Twaddle et al. [35] and Paulsen et al. [36] have summarized the mainstream modelling approaches that describe the state of the art. Asaithambi et al. [37] also mentioned some related studies in their work. However, this section is not intended to be an additional review of methodologies, but as an underpinning for the proposed model we develop next. Therefore, we would discuss how these related works characterize and model the operational behaviours of two-wheelers in this section.

At the outset, some studies have made investigations to understand behaviour differences among distinct types of two-wheelers, because the two-wheeler flows generally consist of different types of road users [38,39]. Operational performance is the most popular topic in literature. Jin et al. [13] and Paulsen et al. [36] compiled surveys on the dynamics of two-wheelers. The result showed the most explicit distinction between different types of two-wheelers is their speed and acceleration. Correspondingly, some other studies also claimed that some behaviour features of bicycles are distinct from that of e-mopeds and e-bikes, such as more moderate interactions [40,41], lower overtaking frequency [42], and more conservative yielding behaviour [43]. But essentially speaking, the main discrepancy of these results is that they are powered in different ways, i.e. electric engines for e-mopeds and e-bikes and human physical strength for cyclists, respectively. Hence, these features are still the manifestation of dynamics differences between these two-wheelers on their operational physical level. Hence, considering the mixed two-wheeler flow comprised of e-mopeds, e-bikes, and bicycles is widespread, it is beneficial to provide the measurement of their operations together by a generic simulation model to improve the model suitability and capability.

Actually speaking, several studies have attempted to simulate two-wheeler flows. In respect of microscopic models, longitudinally continuous models [44,45], Cellular automata (CA) [46,47], and force-based models [48,49] are the most utilized underlying methods. As for the former two kinds of models, virtual lanes and cells are hypothesized as the basic unit to simulate road users' movements and interactions [50,51]. However, its discrete nature in space hinders the ability to represent realistic behaviours and interactions of road users [52]. From the perspective of two-wheeler motion, longitudinally continuous models can only capture the state change in the longitudinal direction, while CA can only mechanically express movements as jumps between discrete cells. More adjustments need to be made to represent the two-dimensional movements brought by interactions with other road users. On the other hand, the force-based models govern the movements of road users through forces exerted by desired destinations and other road users. As a result, two-dimensional movements of agents can be interpreted [53] and interaction with several road users can be considered. However, the interaction forces naturally and indiscriminately describe collisions/conflicts with other road users (or rather physical contacts) [3]. Therefore, they are still limited to reflecting the multi-interaction during the operational physical process, and cannot consider the impact of mental activities, especially the behaviour decision-making result on the simulated agent operations.

Based on these underlying models, previous studies also developed some agent-based models [54,55] and hybrid models [56,57] to capture the two-wheeler's interactions. As for the agent-based model, a good review has been made by Bazzan and Klügl [58], and it is considered as a powerful method that can be used to capture the complex movements and interactions of agents from the microscale to the macroscale [31]. The key task in developing an agent-based model is the identification of agent goals from the strategies, tactical, and operations [59]. However, most of the current agent-based models that aim to describe the two-wheeler operational behaviours are still developed on the basis of the CA, such as Vasic and Ruskin's work [60], and Zhao and Sadek's work [61]. Therefore, they still cannot depict the two-wheeler's interaction and movements on a refined operational behaviour level. Despite this, Lee and Wong [34] developed an agent-based model for powered motorcycles to attempt to depict their interaction on the two-dimensional space. In this model, headway models in both longitudinal and lateral directions are developed to first describe the motorcycle's safety margins and then the path choice model is utilized to depict spacing where the motorcycle can move safely, which jointly cooperate to deal with the motorcycle's multi-interaction. They really provided a good reference and a feasible way for modelling the road user's motion on the two-dimensional space. In recent, Mohammed et al. [31] presented the solution of the agent-based model for investigating the continuous trajectories of two-wheelers. The generative adversarial imitation learning approach in this model requires high-quality and large-amount trajectory data [62], though there is still a challenge of collecting enough two-wheeler trajectory data [21]. Meanwhile, the nature of this black box model also leads to its lack of ability to explain the process of two-wheeler interactions. Another category is the hybrid model that combining several different models for achieving the simulation of road users. For example, Yang et al. [48] captured the dispersion of bicycles and the corresponding movements by the integration of a quadratic curve model and a force-based model. Li et al. [28] developed a hybrid model to describe the movement of through bicycles from the entrance of the simulation roads to their destinations. In this model, the interaction of agents are captured by several different force-based models, and a dynamic boundary model is utilized to determine the available riding space. Generally, the advantage of this type of model is that it can realize different simulation functions by integrated models, so as to describe the interaction process more realistically. However, for the mixed two-wheeler flow that consisting of bicycles and electric bicycles, rare discussions have been made to attempt to represent the two-wheeler operational mental activities in multi-interaction situations.

Several additional mathematical methods have also been embedded into these simulation models to supplementally discuss the mental activities of two-wheelers while interacting, such as the heuristics constraints [25], the decision

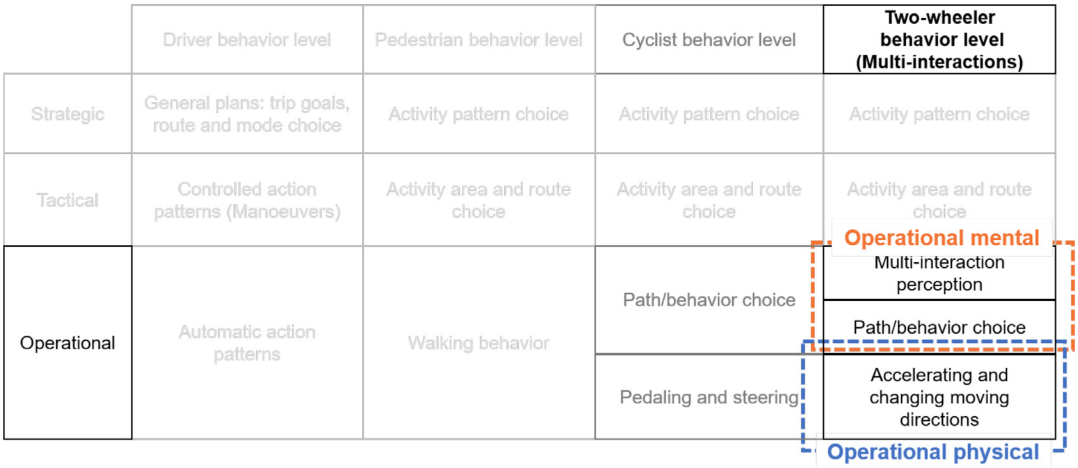


Fig. 3. Conceptual definitions of behaviour levels for different traffic modes [21] and for the two-wheeler multi-interaction proposed in this paper.

tree method [28], game theories [3], collision avoidance methods [63,64], and discrete choice theory [21]. For instance, heuristics constraints are widely adopted for practices that perform the decision-making process due to their convenience and efficiency. In its implementations, the nearest front road user in spacing usually defaults to be the interactive road user who impacts two-wheeler behaviour choices [19]. Behaviour is selected based on the speed and relative distance of the above specific object, while the influences of other road users are mainly considered as elements of collision avoidance [35]. As for other approaches, they are generally integrated with the force-based model to fulfill the needs. However, even though these models provide insight into different interaction strategies, they are still essentially limited to solving the one-on-one interaction [19,20]. This means that, agents of these models can only handle a single interaction with another road user in predefined situations during their mental activities, which fail to deal with interactions with two or more objects. Hence, there is still a lack of models to represent the nature of the two-wheeler multi-interaction governing the operational mental behaviour level. The core shortage is likely that previous models have not well defined for two-wheelers who they perceive have impacts (interactive road users) and how they select (behaviour decisions) based on the multi-interaction traffic context.

In summary, the primary findings of the related works are as follows: (1) Due to the universality of mixed two-wheeler flows in urban roads, a simulation model that can simultaneously represent interactions of two-wheelers with several types is necessary and possible for the traffic management of urban roads. (2) The force-based model is adept at depicting the multi-interaction on the operational physical level, but additional components are still needed to represent the movements under the guidance of behaviour decisions from mental activities. (3) Existing models for describing mental activities can only handle the one-on-one interaction, no matter whether interacting with the predefined leader or specific road users in simple situations. There is a lack of conceptions and definitions to sufficiently represent and perceive the multilateral interactions of two-wheelers on the two-dimensional space. Meanwhile, efforts are also required to deal with the complex decision-making process while multi-interactions. In all, models concerning two-wheelers should have the ability to capture the multi-interaction in terms of both operational mental and physical behaviours to obtain a more reasonable formulation.

3. Conceptual framework for simulation modelling

In this section, we first discuss the overall definition of what the two-wheeler operational behaviour includes, followed by the description of the proposed simulation framework in the rest of this section.

Fig. 3 aggregates the definitions of the behavioural level for different traffic modes proposed by Michon [65], Hoogendoorn and Bovy [66], and Gavriilidou et al. [21]. Referring to the definition of cycling operational behaviour in Gavriilidou et al. [21], the performance of behaviour choices and automatic actions are the major processes for cyclists on the operational level (Fig. 3). Therefore, they define the operational mental process and the operational physical process to capture the cyclists' decisions and actions, respectively. As mentioned in Section 2, the major operation difference between different types of two-wheelers is their sources of engine power. For cyclists, they execute their dynamics through pedalling and steering, and for e-moped riders and e-bike riders, they are in the form of braking and steering. However, no matter what form they act in moving, it can still be considered as some changes in acceleration magnitudes and moving directions. Meanwhile, the decision-making process is the indispensable link for all of these riders/cyclists before the execution of movements. Therefore, we believe the definition of the operational process for cyclists can be generalized to represent different types of two-wheelers to some extent.

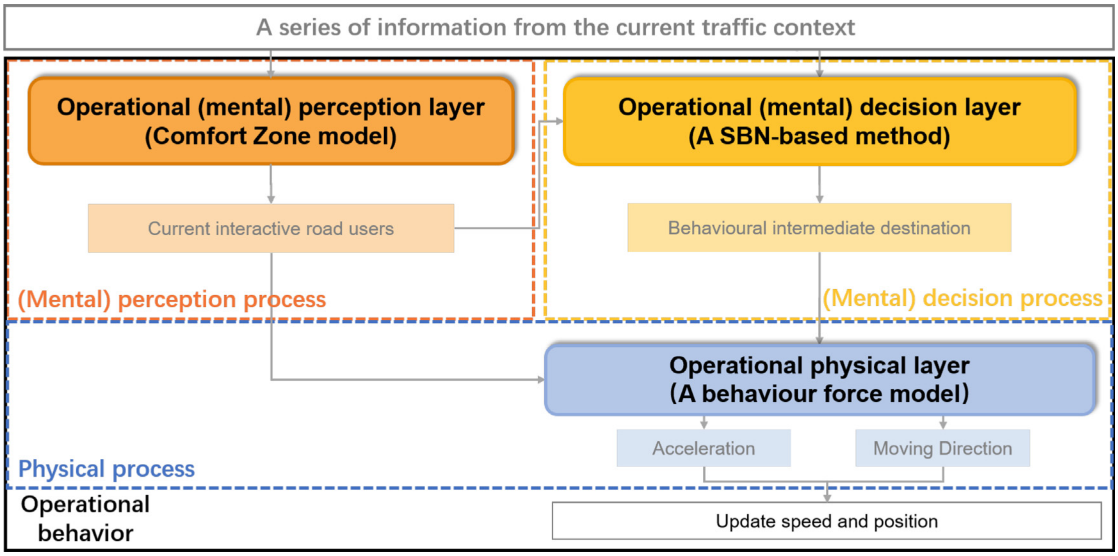


Fig. 4. The structure of the proposed simulation framework based on the operational behaviour definition.

However, the definition of cyclist operational behaviour still needs improvements to represent the two-wheeler's operations when we consider the extension of the multi-interaction situation. To the authors' best knowledge, the main restriction is reflected on the operational mental level. This is because the mental process is a combination of tasks to perform manoeuvers based on what road users perceive from the surroundings and how they context-specially respond to the traffic context [67]. Meanwhile, human behaviour is regarded as a cognitive process with the serial tasks of perception, decision, and action [68], of which the former two tasks belongs to the mental level. Hence, these procedures, i.e. perception and decision, should be modelled together if we want to well describe the mental activities of human traffic participants like two-wheelers.

Therefore, we extend the operational mental level by distinguishing two new processes to describe the two-wheeler's multi-interaction, as visualized and compared in Fig. 3. We sequentially call these two processes the "operational (mental) perception" process and the "operational (mental) decision" process. They jointly describe the inner assignments to resolve the multi-interaction of two-wheelers while conducting their mental activities. On the mental level, the two-wheeler perceives interactive road users during the mental perception process, and then he/she selects behaviours to interact with these captured objects. Behavioural choices refer, among other things, to freely moving, following, overtaking, yielding, stopping, etc. After that, one of these possible behaviours would be executed. At this moment, control dynamics of two-wheelers on the two-dimensional space are necessary, which is based on the current behaviour choice result and interaction state. This is the task handled in the physical layer mentioned in the definitions.

Based on the above definition, we put forward the conceptual simulation framework for representing the multi-interaction of two-wheelers, as shown in Fig. 4. We fit these three layers within the framework under the guidance of the operational behaviour definition we have given above. The associations between these three layers are described as follows.

- ***Operational (mental) perception layer:*** This layer is the upper mental layer that provides guidance to both mental and physical activities. In this layer, the two-wheeler captures the current multiple interactive road users based on the current traffic context. Potential interactive road users include bicycles, e-bikes, e-mopeds, motorized vehicles, etc., which commonly exist on mixed-traffic road segments. Subsequently, the captured interactive road users are communicated to both the operational (mental) decision layer and operational physical layer, representing the impacts from surroundings that need to be taken into account for decisions and actions.
- ***Operational (mental) decision layer:*** This layer focuses on describing the mental activities by considering the behavioural decision-making process under the multi-interaction situation. A suitable behaviour choice result would be first given here. This is described by a single decision-making model covering behaviour alternatives. After that, the intermediate destination that corresponds to the selected behaviour would be chosen. For each selected behaviour, the corresponding intermediate destination points the way and direction of moving. Herein, alternative behaviours in this study include freely moving, following, and overtaking. This is because they are the most common behaviours for two-wheelers on road segments [24,28]. One intermediate destination is the mapping of a specific behaviour choice. As a result, the intermediate destination would be input into the operational physical layer for execution.

- ***Operational physical layer:*** This layer aims to capture the dynamics control of each individual to realize the two-dimensional movements. Under the guidance of the behaviour decision-making result and the current interaction state, the state of the current two-wheeler would alter in the forms of acceleration and moving direction to determine the velocity and position of each individual two-wheeler.

The above three layers jointly express the whole mental and physical tasks for the multi-interaction. On an aggregated scale, the two mental layers have an interaction with the physical layer. After applying the selection obtained in the two mental layers, the new state of the current object would be updated. Then, the change in the physical layer would also influence the new decision to be made during the mental activities. They together determine the behaviour and interaction of two-wheelers on the operational level. In the next Section 4, specific methods used in each layer would be successively presented in detail.

4. Mathematical modelling

In this section, we first discuss the behavioural hypothesis regarding our proposed models in Section 4.1, followed by the development of corresponding model for each behaviour process (Sections 4.2 and 4.3 for the operational mental process and operational physical process, respectively).

4.1. Model assumptions

The assumptions related to behaviours and interactions of two-wheelers on the operational level are made as follows.

1. Three types of two-wheelers, i.e., e-moped riders, cyclists, and e-bike riders, are considered.

2. The two-wheeler perceives the surrounding traffic context and captures interaction through their sensory organs in the same way, however, perception is more sensitive to the front than the rear [69].

3. The two-wheeler would assess influential attributes from the current traffic context, and then select a suitable behaviour from a set of alternatives while making a decision.

4. Influential attributes are extracted from all the interactive road users at each time step, and these surrounding interactive road users are equally treated by the two-wheeler when he/she makes behaviour decisions.

5. The two-wheeler aims to optimize his/her physical status based on the behavioural choice produced from the operational mental process.

6. The decision made in the operational mental layer is a real-time transition with no anticipations.

7. The movement during the physical process are open-space based with no lane discipline.

These above hypotheses form the foundation of the models we utilized to specialize each layer of the proposed framework. In the rest of this section, we will present the specific mathematical modelling methods in detail for each layer we define in the framework.

4.2. Modelling the operational mental process

This process aims to realize the operational mental activities of two-wheelers while interacting with multiple road users. As mentioned in Section 3, two parts consist of this process in the simulation framework, i.e. the operational (mental) perception layer, and the operational (mental) decision layer.

For the upper layer, the proposed Comfort Zone (CZ) model would be implemented here to identify the dynamic multi-interaction for two-wheelers on the two-dimensional space, while a decision-making model would also be developed in the lower layer to represent the complex human decision-making process while multi-interactions. The details are presented in the rest of this section.

4.2.1. Operational (mental) perception layer

In this layer, the proposed CZ model focuses on capturing multiple interactive road users to consider their influence on the two-wheeler's operations. It essentially characterizes the two-wheeler demand for riding space that varies with the traffic context, so as to explain how other road users influence and interact with two-wheelers.

The model assumes that each two-wheeler is surrounded by an invisible comfort zone. This view is inspired by the "Behavioural Adaptation Concept" in the psychological area outlined by Näätänen and Summala [70] and Summala [71]. They claim that each person is surrounded by an invisible social space, and invasions by others into this space can arouse discomfort and trigger responses. By analogy, we can reasonably consider that two-wheelers' interactions are constructed by others' invasion of their social space. Note that, an invasion means there is one road user itself located inside the comfort zone of the rider. As a result, by capturing others that are inside the social area, all the interactive road users can be found and the multi-interaction of the two-wheeler can therefore be attempted to be well represented.

In this model, we claim that "comfort" is the feeling that the two-wheeler obtains by maintaining space and time with other traffic participants while riding, which determines the size of the comfort zone. This concept is similar to the key hypothesis of the famous "task difficulty homeostasis" theory, which considers road users maintain driving task difficulty within their acceptable limit by adjusting speed and distance to other road users [72]. As for a two-wheeler, he/she continuously makes decisions to maintain its comfort feelings of riding task by adjusting the manoeuvers and

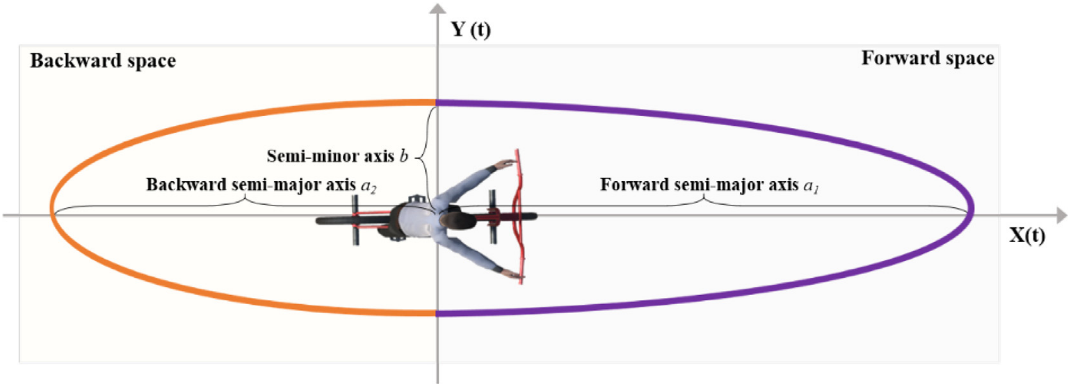


Fig. 5. Schematic diagram of a two-wheeler's comfort zone.

dynamics. Thus, if the threat of another road user exceeds the acceptable limit of the comfort, the two-wheeler is likely to make a response to change the relative status with this road user who locates inside his/her comfort zone. At that time, the interaction between the current two-wheeler and the road user would be established. Hence, the core of the model is how to quantify the size of the comfort zone for each two-wheeler to capture road users that impact its movements.

In this paper, we define the two-wheeler's comfort zone as a bubble coupled in two-dimensional space to capture its multiple interactive road users. We consider the ellipse is more suitable for shaping the comfort zone. This is because traffic participants' attention to stimuli/threats is regarded to be an anisotropic reaction [73], which explains why they tend to react on road users that locate at orientations parallel with or orthogonal to the moving direction. Meanwhile, because temporal indicators are considered as more suitable to reflect the influences/threats of other road users [74], the comfort zone would be measured temporally in this paper. Furthermore, since traffic participants obtain information from the surrounding traffic context mainly relying on their vision, they would naturally pay more attention to the front compared with that to the rear. As such, the comfort zone is shaped as a bubble consisting of two half ellipses with different semi-major axes, as shown in Fig. 5. The shape of the comfort zone can be described by the formula:

$$\begin{cases} x = a \cdot \cos\theta \\ y = b \cdot \sin\theta \end{cases}, \text{where } a = \begin{cases} a_1, & \text{if} \quad \theta \in (-\pi/2, \pi/2) \\ a_2, & \text{if} \quad \theta \in (\pi/2, 3\pi/2) \end{cases} \tag{1}$$

where a denotes the semi-major axis (including a_1 and a_2, representing the forward and backward semi-major axes, respectively); b denotes the semi-minor axis; θ denotes the angle between the line connecting the origin of each half ellipse to any point on it and the moving direction of the current object. Generally, two-wheelers' cognitive way of the threat from surroundings is likely to be similar, though the attention degree is not the same [75]. Therefore, a_1 and a_2 can be reasonably regarded to be determined following the same law of nature, and a_2 is likely to be shorter than a_1. Therefore, we assume in this paper the semi-major axis a_1 of the front is proportional to the semi-major axis a_2 of the rear, i.e., $a_1 = \lambda \times a_2$. Herein, λ is a parameter that is used to capture relations between a_1 and a_2.

The size of a two-wheeler's comfort zone is not fixed but varies. Since a two-wheeler's perception of comfort is generally not unchanging, its comfort zone is also an attribute of the rider, which will also vary to adapt to variations of the traffic context and the two-wheeler's status. Besides, the comfort zone can also be expressed as a constitutional characteristic that is affected by the two-wheeler's dynamics. Therefore, the current traffic environmental conditions and the two-wheeler's motion status are two uppermost elements that influence the size of the comfort zone. Furthermore, the heterogeneity between different types of two-wheelers has also been found to have significant impacts on interactions and behaviours [8,76]. As such, variables that determine the size of the comfort zone in this paper are discussed from the traffic environmental conditions, the two-wheeler's motion status, as well as the different types of two-wheelers. The definition of the selected variables in each aspect is presented below.

- ***Traffic environmental conditions*** we select traffic density k as the influential variable. It is assumed in this paper that the traffic density k is inversely proportional to the size of the comfort zone (because we measure the zone temporally). This is because Guo et al. [77] found that two-wheelers' responses to interactions differed according to traffic density conditions. Generally, the lower the traffic density around, the larger the time spacing with others the two-wheeler would pursue, and vice versa. Meanwhile, Saifuzzaman et al. [78] found when other conditions being equal, the higher the spacing (spatial distance that can reflect traffic density to some extent), the more time (temporal distance) there is available for road users to keep with others. Hence, we can infer that the traffic density k is inversely proportional to the temporal distance keeping to other road users.
- ***Two-wheeler's riding status*** we choose the speed v as the influential variable. We hypothesize that the temporal size of the comfort zone varies inversely with the individual speed v. Several studies have claimed that the two-wheeler's

interaction would be affected by its own speed [77,79]. When other conditions being equal, the higher the speed, the less time (temporal distance) road users maintain due to the limited travelling space, which reflects their expectation of travel efficiency [28]. Wang et al. [80] also found that the faster the speed of a road user, the lower the road user's perceived safety. That is to say, with the increase of a road user's speed, the experienced risk would increase. At this time, road users could accept a decreasing time distance because they would invest more mental effort to cope with it [78,81] Therefore, we can consider that the temporal distances keeping with other traffic participants are inversely proportional with the two-wheeler's speed v.

- ***Type of the two-wheeler:*** referred from a previous study [82], we use the coefficient S_t of the type to represent the perceptual differences caused by the discrepancy of the dynamic performance of two-wheelers. As mentioned in Section 1, the e-moped riders, e-bike riders, and cyclists are the three main types of two-wheelers we considered in this study.

These above variables are combined in a linear summation to reflect their joint impacts. Therefore, the size of a two-wheeler's comfort zone can then be calculated by the following formulas:

$$a = f_a(k, v, S_t, C_d) = S_t \cdot C_d \cdot (\alpha_1/v^{\beta_1} + \alpha_2/k^{\beta_2} + \delta_1) \tag{2}$$

$$b = f_b(k, v, S_t) = S_t \cdot (\alpha_3/v^{\beta_3} + \alpha_4/k^{\beta_4} + \delta_2) \tag{3}$$

where $f_a(k, v, S_t, C_d)$ represents the function of the semi-major axis a and $f_b(k, v, S_t)$ is the function of the semi-minor axis b. C_d denotes the direction coefficient of the semi-major axis to distinguish a_1 and a_2, and it has no direct value and only helps us describe the difference between the front and the rear of the comfort zone; α_1, α_2, α_3, and α_4 are correlation coefficients; β_1, β_2, β_3, and β_4 are also correlation coefficients that need to be calibrated; and δ_1 and δ_2 are constant terms. The detailed calibration process of these coefficients would be discussed later in Section 5.2.1.

Finally, the comfort zone can be quantitatively measured to capture interactive road users. Saifuzzaman et al. [78] explained that the estimation of distance (mainly the temporal distance because it reflects the influence of a combination of a road user's speed and position) is a basic skill of a traffic participant. This is also the safe margin in any situation that the traffic participant (including two-wheelers) determines to maintain [83]. Therefore, when the temporal distance with a road user exceeds the time the two-wheeler wants to keep with others, we can consider that the road user brings risks and discomfort felt to the two-wheeler. As a result, if the road user insides the comfort zone, the corresponding interaction would be established. When extending such a case to more relations between the current two-wheeler and all of the other road users, multiple interactive road users can be captured. This can be formulaic described as follows:

$$\Omega = \left\{ m | b_n^2 x_m^2 + a_n^2 y_m^2 < a_n^2 b_n^2 \right\} \tag{4}$$

where Ω denotes the collection of current interactive road users; a_n and b_n represent the semi-major axis and semi-minor axis of the current two-wheeler n, respectively; x_m and y_m are the longitudinal and lateral positions of a road user m, respectively.

4.2.2. Operational (mental) decision layer

Based on the interactive road users captured by the CZ model, this layer describes the decision-making modelling of two-wheelers at the current time step. As mentioned in Section 3, the two-wheeler would first choose a suitable behaviour from alternatives and then build up an intermediate destination that corresponds to the selected behaviour. Therefore, two steps jointly describe the function of this layer, i.e. behaviour decision-making and intermediate destination selection for behaviour execution. These two steps would be sequentially introduced in detail.

(1) Behaviour decision-making

A behaviour decision-making model is developed here to choose a suitable behaviour based on the current multi-interaction state. As mentioned in Section 3, three behaviour decisions are in the alternatives, i.e. freely moving, following, and overtaking, in this paper. Herein, the overtaking behaviour means the process two-wheelers approach from behind and pass another road user travelling in the same direction. Two-wheelers consider overtaking to be crossing to one side of the road so as to pass another road user in front. Meanwhile, the following behaviour describes how a two-wheeler in the rear follows another road user and adjusts its position, speed or acceleration to avoid rear-end collision. Besides, the freely moving behaviour captures the trend of moving towards the final destination if there is no need to follow or overtake other road users.

Honestly speaking, it is hard to obtain modelling variables of the model for decision-making process under the multi-interaction scenarios. This is because the multiple interactive road users scatter on the two-dimensional space. The interaction relationships constantly change and the number of interactive road users is not the same at each moment. Therefore, a qualitative discussion is needed to deal with this challenge before building up the specific decision-making model in this layer.

Benefitted from the comfort zone characterized by the CZ model, the rough position relationship between interactive road users and the current two-wheeler can be intuitively understood to some extent. This provides us with a modelling viewpoint: First, we group all of the interactive road users into several specific areas based on their position. Then, the influence of interactive road users in the same area would be considered as a whole to act on the two-wheeler

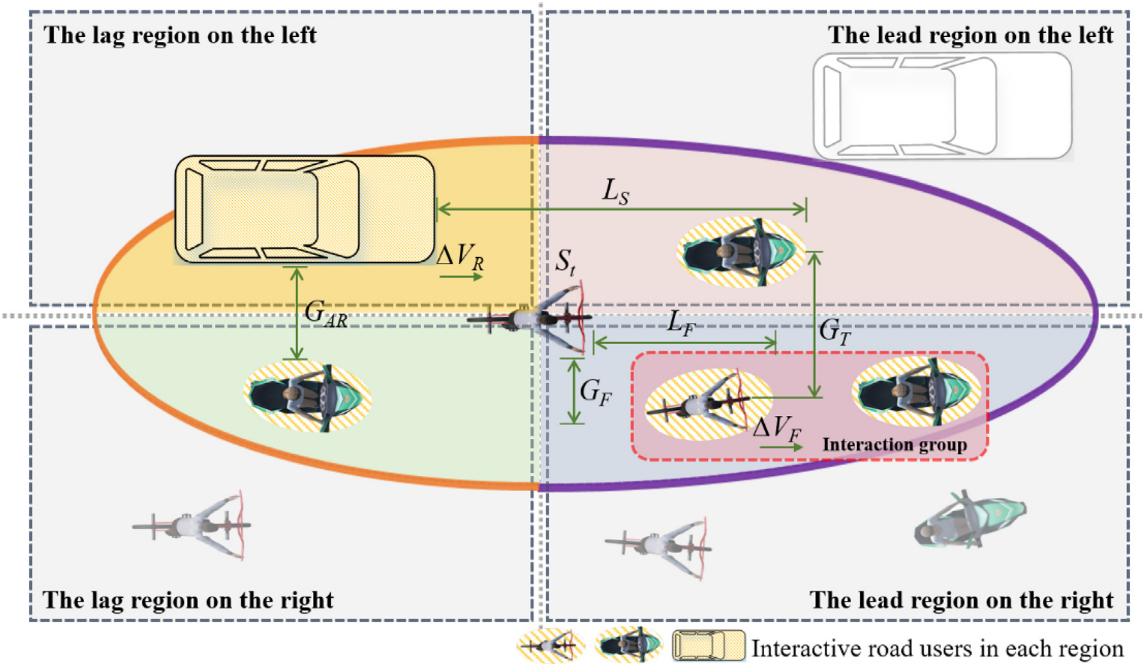


Fig. 6. Schematic of the influential regions, interaction groups, and variables for behaviour decisions.

mental activities. We consider such a hypothesis have the potential to handle the decision-making process with multi-interactions. This is because it depicts the two-wheeler's overall assessment of the influence from a specific area to some extent. Meanwhile, the similar solution for explaining the driver attention has been successfully implemented in car-following behaviour modelling [84]. Therefore, we attempt to handle the decision-making process in multi-interaction situations by this view, i.e. dividing interactive road users into different interaction groups and considering the overall impact of road users from each specific group while modelling the decision-making process.

As such, we hypothesize that the two-wheeler make decisions based on four interaction groups from four certain areas around, i.e., the lead and lag regions on its left- and right-hand sides, as shown in Fig. 6. The saddle is taken as the reference for subdividing. Interactive road users located inside each region are regarded as an interaction group. On this basis, influential variables for the decision-making process are extracted in units of the four interactive groups. In our approach, values of any variables would be the weighted average of corresponding attributes of all interactive road users in the corresponding area.

Then, the variables for capturing the behavioural decisions can be determined. We discuss in this paper from the following four aspects:

- One perspective is the speed difference and longitudinal distance to the lead interaction groups, as they represent the impact ahead and whether two-wheelers can efficiently move with no block to their destination.
- Another perspective is the speed difference with the lag interaction groups, which reflects the threat of the lag object to the safety of the current two-wheeler.
- The third perspective considers the travelling space on the continuous surface, i.e. available spacing in both the lateral and longitudinal direction.
- The last one is the type of the current two-wheeler. It represents the heterogeneity in decision-making process.

Several attributes from these above three aspects are considered in the decision-making model via the random forest model. Finally, a total of eight variables are selected. A full list of these influential variables' notations and definitions is provided in Table 1 and also shown in Fig. 6.

In this layer, a Bayesian network (BN) method is adopted here to specialize the two-wheeler's decision-making process with the consideration of the comprehensive influences from multi-interactions. The method is selected for two main reasons. First, the decision-making process with multi-interactions reflects the two-wheeler's trade-offs of influences from multiple road users. Therefore, the decision model should have the ability to represent the internal connections between multiple road users or model attributes and explain how they collectively affect the decision results. Second, the two-wheeler behaviour decision is flexible, and they commonly do not obey fixed rules to make decisions [85]. We, therefore, consider the two-wheeler's behaviour decision as an uncertainty issue with probabilistic semantics. Meanwhile, the use of the BN method is demonstrated by its application in describing behaviour choices of other human traffic participants [86].

Table 1
Variables for decision-making modelling.

Notation	Definition (unit)
ΔV_F	Speed difference with the nearest interaction group in the front (m/s).
L_F	Longitudinal distance from the nearest interaction group in the front (m).
L_S	Longitudinal gap between the lead and the lag interaction groups on the left/right[a] (m).
G_F	Lateral gap with the nearest interaction group in the front (m).
G_T	Lateral distance between the two interaction groups in the front (m).
ΔV_R	Speed difference from the nearest interaction group behind (m/s).
G_{AR}	Lateral gap between the two interaction groups behind (m).
T_t	Type of the current two-wheeler.

[a]Note: if the current two-wheeler is on the left side of the space, G_S means the longitudinal gap between the lead and the lag interaction groups on the right. If the current two-wheeler is on the right side of the space, G_S means the longitudinal gap between the lead and lag interaction groups on the left.

Therefore, we use a BN method to model the decision-making process of two-wheelers. Besides, to the author's knowledge, this paper also develops for the first time the use of the BN model to represent two-wheeler operational behaviour. More specifically, a static BN is utilized here.

In Section 5.2.2, we would further discuss the modelling steps and results of the BN model after we introduce the dataset utilized in this study.

(2) Intermediate destination choice for behaviour execution

To execute the selected behaviour, we should first determine the intermediate destination for each alternative behaviour. For this purpose, we filter the dominant interactive object from the current multiple interactive road users to capture the intermediate destination. In this paper, the interactive road user with the highest influence intensity would be regarded as the dominant interactive object for intermediate destination choice. The dominant interactive object expresses the most urgent interaction that needs to be dealt with. A defined concept "influence intensity" is proposed to capture the dominant interactive object here. For the calculation of the influence intensity, the speed and position relationship are the primarily considered attributes. Meanwhile, the type of interactive road user is also considered. Speed and position relationships indicate whether the corresponding road user has influences on the two-wheeler's behaviour. From Tao et al. [87]'s work it can be seen that the farther the distance from the current road user to another individual, the smaller the influence brought by the individual on the current road user, and vice versa. Meanwhile, the speed of other individuals is often significant, which commonly shows proportional to their impact on the current road user [82]. However, it is worth noting that when the road user locates behind the current object, it is best to consider their speed difference [88] because this index can represent whether the road user is approaching. Furthermore, the type of road user decides to some extent the importance that the two-wheeler attaches to it, and can also represent the influence of different road users in mixed traffic. For example, keeping the same speed and distance, the two-wheeler generally pays more attention to the impact of a motor vehicle, rather than a bicycle. Hence, the influence intensity of a road user m is calculated by the following formula:

$$E_m = \begin{cases} v_m \cdot S_m / D_m, & if \quad x_n < x_m \\ (v_m - v_n) \cdot S_m / D_m, & if \quad x_n \geq x_m \end{cases} \tag{5}$$

where E_m denotes the influence of the interactive road user m; S_m and D_m are the type of m and the distance to the current two-wheeler n, respectively; v_n and v_m represent the speed of n and m, respectively; and x_n and x_m are the longitudinal positions of n and m, respectively. In Eq. (5), $x_n < x_m$ means the road user m is ahead of the two-wheeler n, while $x_n \geq x_m$ means m is to the rear of or next to the two-wheeler n. Herein, the values of S_m in the simulation are 1.2, 1.6, 3.6 for the bicycle (including e-bikes), electric moped, and motorized vehicles, respectively, which is determined by each type of the road user's cross-sectional area that can reflect the human first-time judgment on the impact the road user can exert on he/she.

After that, the intermediate destination can be determined based on the selected behaviour and the dominant interactive object at each time step. Since the decision-making model selects a suitable behaviour from three alternative behaviours, i.e. freely moving, following, and overtaking, each kind of behaviour is also designed to correspond to a specific intermediate destination. As presented in Fig. 7, the intermediate destination is set up as follows: (1) the intermediate destination of the freely moving behaviour is put on the two-wheeler's final destination; (2) the intermediate destination of the following behaviour is set at the tail of the dominant interactive object; (3) the intermediate destination of the overtaking behaviour is located alongside the head of the dominant interactive object with a fixed distance w_n, which consists of the width of the overtaking object and an additional shy-away distance of 0.5 m [89]. Note that, the overtaking behaviour can be executed from both the right- and left-hand sides. As such, the specific position of the intermediate destination would also be determined by which side has more overtaking space in this model.

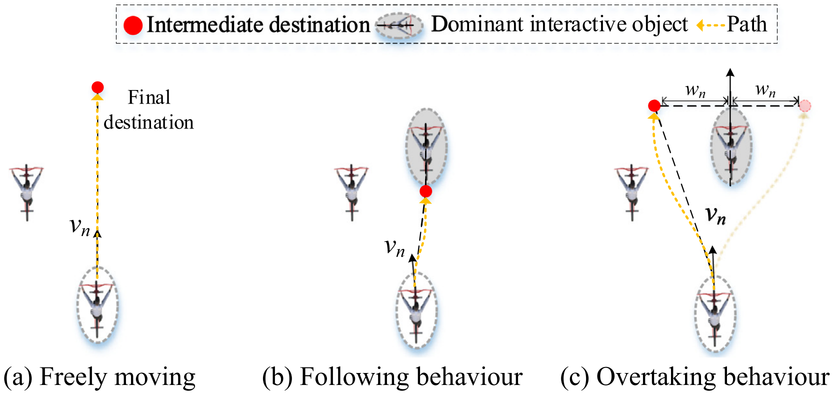


Fig. 7. Schematic of intermediate destinations of different selected behaviours.

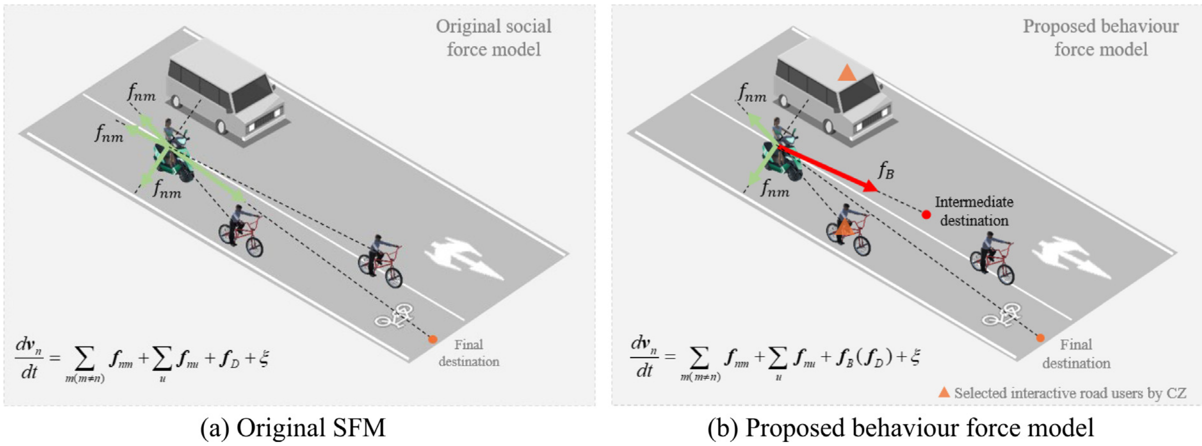


Fig. 8. Schematic of intermediate destinations of different selected behaviours.

4.3. Modelling the operational physical process

After selecting the intermediate destinations and behaviour at the current time, the behaviour path would be built up from the current position to the intermediate destination to achieve the two-dimensional movement in this physical process. The path would be specifically executed by the control dynamics of two-wheelers in the form of acceleration and moving direction.

In this section, we develop a behaviour force model to capture the continuous movements of two-wheelers by considering the behaviour choice and the state of multi-interactions. Of this model, the original social force model (SFM) [90] is selected as the foundation to represent the two-wheeler's operational physical process. This is due to the nature of the SFM can help to depict continuous movements of agents and avoid physical contact with other road users, which would also provide the ability to consider multi-interactions during the physical activities. Herein, the original SFM controls the dynamics of agents by the driving force and repulsive forces exerted by final destinations and other road users, respectively (Fig. 8(a)).

However, the implementations of social force-based models often require additional elements to consider the impact of behaviour decisions on the simulated object's movements. Therefore, in this paper, the behaviour force model introduces the behaviour force $\boldsymbol{f}_B$ into the original SFM to realize the dynamics control of the two-wheeler under the guidance of the decision-making results, as shown in Fig. 8(b). Because the freely moving behaviour captures the trend of moving towards the final destination, we can hence use the driving force to describe the freely moving behaviour and the driving force model can also be regarded as one of the force models for alternative behaviours. Therefore, the resultant force for this behaviour force model can be presented as follows:

$$\frac{d\boldsymbol{v}_n}{dt} = \sum_{m(m\neq n)} \boldsymbol{f}_{nm} + \sum_u \boldsymbol{f}_{nu} + \boldsymbol{f}_B + \boldsymbol{f}_D + \xi \tag{6}$$

where $\boldsymbol{f}_{nm}$ denotes the repulsive force exerted by a road user m; $\boldsymbol{f}_{nu}$ represents the repulsive force of an obstacle u for the current two-wheeler n; $\boldsymbol{f}_D$ denotes the driving force. When the two-wheeler selects the freely moving behaviour, $\boldsymbol{f}_D$ would exist as the behaviour force $\boldsymbol{f}_B$ of the freely moving behaviour and guides the two-wheeler move towards the final destination; ξ is the stochastic component. Herein, $\boldsymbol{f}_{nm}$, $\boldsymbol{f}_{nu}$, as well as $\boldsymbol{f}_D$ are identical to the original SFM.

In the behaviour force model, the behaviour decision result would be exerted at each time step by calculating the corresponding behaviour force $\boldsymbol{f}_B$. The direction of a specific behaviour force directs to the corresponding intermediate destination of the current path. The dynamics control is achieved by iterating the resultant force of the behaviour force and repulsive forces exerted by others. Herein, the behaviour force $\boldsymbol{f}_B$ brings the change of the speed by acceleration and also the adjustment of the moving direction, while the repulsive force $\boldsymbol{f}_{nm}$ ensures no collision will happen. Finally, the behaviour force $\boldsymbol{f}_B$, as well as repulsive forces $\boldsymbol{f}_{nm}$ exerted by interaction road users would jointly determine the state update of each simulation two-wheeler at every time step. Herein, because the traffic marking between the bicycle lane and the adjacent vehicle lane would not strictly limit the two-wheeler's movement to the vehicle lane, the force of the traffic marking would be regarded as a repulsive force exerted by a virtual object (the same type as a two-wheeler) in the simulation [64].

Herein, one behaviour corresponds to a specific force model to calculate the related force. As mentioned above, three alternative behaviours are involved in this study, i.e. freely moving, following, and overtaking. If an alternative behaviour is selected on the operational mental process, the corresponding behaviour force model would be utilized to specialize the behaviour force. The given force models of these three alternatives are listed as follows.

(1) Freely moving force model

The force $\boldsymbol{f}_B(t)$ for the freely moving behaviour at time t is provided by the forward driving force model referred to the original SFM, which is presented as follows:

$$\boldsymbol{f}_B(t) = \frac{v_d \boldsymbol{e}_d - v_n}{\tau_d}, \text{ where } \boldsymbol{e}_d(t) = \frac{\boldsymbol{r}_n^d - \boldsymbol{r}_n}{\left|\boldsymbol{r}_n^d - \boldsymbol{r}_n\right|} \tag{7}$$

where v_d is the desired speed, and τ_d denotes the relaxation time; $\boldsymbol{e}_d(t)$ represents the desired direction; $\boldsymbol{r}_n^d$ denotes the intermediate destination and $\boldsymbol{r}_n$ represents the momentary position of the current simulation two-wheeler. For the subsequent formulas of other behaviour forces, the calculation of $\boldsymbol{e}_d(t)$ is consistent, pointing to the corresponding intermediate destination.

(2) Following force model

Since the effectiveness of simulating the following behaviour of two-wheelers has been proved [91], the Intelligent Driver Model (IDM) developed by Treiber et al. [92] is used to represent the force $\boldsymbol{f}_B(t)$ at time t for the following behaviour, which is calculated as follows:

$$\boldsymbol{f}_B(t) = a_m \cdot (1 - (\frac{v_n}{v_d})^{\delta_e} - (\frac{S_d}{\Delta S})^2) \cdot \boldsymbol{e}_d(t) \tag{8a}$$

$$S_d = s_0 + s_1 \cdot \sqrt{\frac{v_n}{v_d}} + T_d \cdot v_n + \frac{v_n \cdot \Delta v}{2\sqrt{a_m \cdot b_f}} \tag{8b}$$

where a_m is the maximum acceleration of the simulation two-wheeler; S_d is the desired minimum spacing; ΔS is the spacing from the front edge of the follower to the rear end of the leader; s_0, s_1 are the jam distance; T_d is the desired time headway; Δv is the velocity difference to the leader; δ_e is the acceleration index; b_f is the comfortable deceleration.

(3) Overtaking force model

In this study, the overtaking behaviour force is calculated based on Ni's work [41] that decouples the force in the lateral and longitudinal directions. The longitudinal component acceleration $\boldsymbol{f}_B^x$ and the lateral component acceleration $\boldsymbol{f}_B^y$ is obtained by the following Eq. (9a) and (9b), respectively.

$$\boldsymbol{f}_B^x(t) = (-0.12 \cdot \Delta s + 0.72) \cdot \boldsymbol{e}_x \tag{9a}$$

$$\boldsymbol{f}_B^y(t) = \frac{y_d}{2} \cdot \frac{\pi}{(t_o)^2} \cdot \cos(\frac{t_o - t'}{t_o} \cdot \pi) \cdot \boldsymbol{e}_y \tag{9b}$$

where Δs is the longitudinal distance from the current two-wheeler to the overtaken object; y_d is the desired lateral offset (it is determined by the position of the intermediate destination) for the overtaking behaviour; t_o, t' are the duration time and start time of the lateral offset, respectively.

Last but not the least, the implementers of the repulsive force $\boldsymbol{f}_{nm}$ in this layer would only include the multiple interactive road users that are captured by the CZ model (Fig. 8(b)). This can help us represent the multi-interaction on the operational physical level. Besides, the anisotropic interaction feature [93] would be considered based on the different calibration results of $\boldsymbol{f}_{nm}$ for different types of road users. Furthermore, the desired speed, maximum acceleration, and maximum turning angle would also have corresponding constraints that are determined based on the empirical dataset to reflect different two-wheelers' power performance in the operational physical layer.

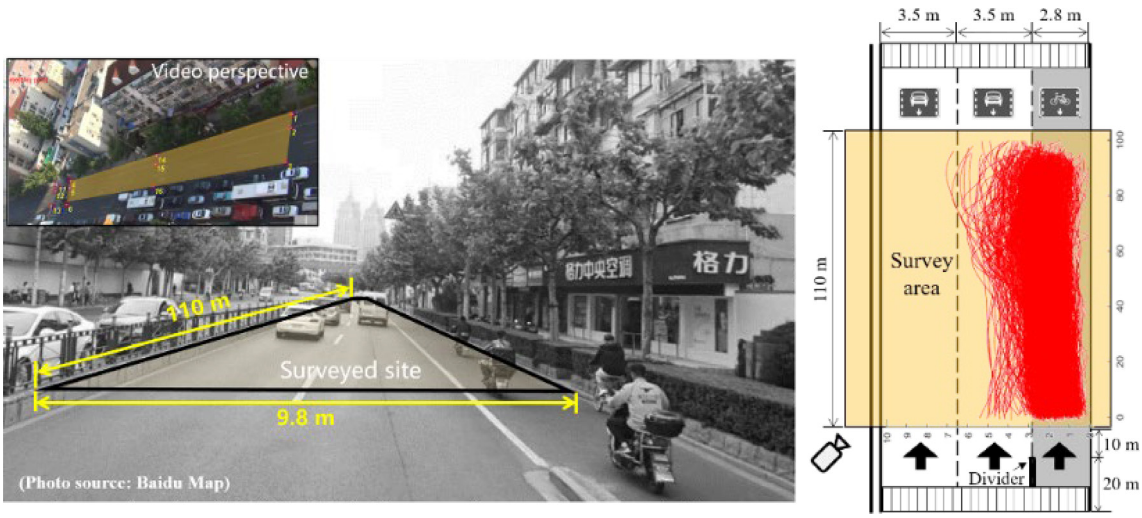


Fig. 9. Layout of the surveyed site and obtained two-wheeler trajectories.
Source: photo source [94]

5. Calibration and validation

This section presents the calibration and validation of the proposed components in the simulation framework. First, the dataset used in this study is introduced in Section 5.1. Then, we calibrate the parameters of the CZ model and estimate the BN model for behaviour decision-making in Sections 5.2.1 and 5.2.2, respectively, followed by the calibration of the parameters involved in the behaviour force model in Section 5.2.3. Finally, a face validation will be given to prove the model performance in Section 5.3.

5.1. Site and dataset description

The trajectory dataset used in this study was collected on a 110 m × 9.8 m mixed-traffic segment in urban road in Shanghai, China (Fig. 9). The site is a shared road segment where motorized vehicles and two-wheelers (mainly consist of e-mopeds riders, e-bike riders and cyclists) use the same road space. The speed limitation of motorized vehicles is 50 km/h, while no limitation for two-wheelers. Meanwhile, it is 40 m away from intersections, which can reduce the influence of the traffic signal. During the investigation, the flow of the two-wheeler traffic is around 700–900 road user/h, while that of the motorized vehicle traffic is around 400–500 veh/h. Besides, the widths of bike lane and adjacent vehicle lane are 2.8 m and 3.5 m, respectively. At the place that is 10 m away from the beginning of the survey area, there is a lane divider (i.e. curbstone) exists, which completely separates the bicycle lane and the adjacent vehicle lane. Subsequently, the curbstone is replaced by a traffic marking, which makes it possible for two-wheelers to enter the adjacent vehicle lane. Furthermore, because the curbstone is far enough from the survey area, it also ensures that interactions would not be influenced by the curbstone. In this surveyed site, the two-wheeler moves freely and frequently interact with surrounding road users, and the movement of road users has no obvious priority. Hence, trajectories of two-wheelers are of significant variation. As can be seen in Fig. 9, a lot of trajectories cover on the adjacent vehicle lane. Such a phenomenon happens all the way from the beginning to the end of the survey area. Trajectories finally locate at both the bicycle lane and the adjacent vehicle lane, and show a leftward trend. Based on our survey, one of the reasons is that several two-wheelers enter the adjacent vehicle lane to overtake the front road users. Meanwhile, some of them would not come back to the bicycle lane until they leave the surveyed site. Furthermore, a number of two-wheelers ride inside the vehicle lane since they enter the surveyed site. These truths would make trajectories distribute leftward. Several studies have reported such a phenomenon [8,17] and this is very common in such a mixed-traffic road segment in China.

The trajectory dataset was extracted from video recordings using a high-accuracy video processing assistant software developed by Suzuki and Nakamura [95], which has been widely used in previous studies [8,26,41,53]. Video cameras were mounted on a nearby tall building to record the movements of two-wheelers and other road users at the surveyed site. The video recorded traffic during the rush hour (from around 4:00 p.m. to 5:00 p.m.) on April 20, 2017. The weather during the investigation period was fine. Furthermore, the resolution of the video is 1080p. Herein, the spatial resolution is 1920 × 1080. Besides, the trajectories were extracted at a time resolution of 0.12 s. The near-end and far-end resolution of the video is 0.065 and 0.125 m/pixel, respectively.

Finally, 1,256 two-wheeler's trajectories (1,050 for e-mopeds and e-bikes, and 206 for bicycles) and 548 interactive motorized vehicle trajectories were obtained. Note that, a few e-bikes were observed in this area. Because the number is

Table 2
Descriptive statistics of major characteristics parameters of interaction behaviours.

	Speed (m/s)				Following behaviour (following headway (s))				Overtaking behaviour (overtaking lateral clearance)			
	Mean	S. D.	15% quantile	85% quantile	Mean	S. D.	15% quantile	85% quantile	Mean	S. D.	15% quantile	85% quantile
Bicycles	4.5220	1.4567	3.0713	6.0887	1.6229	0.6044	1.0223	2.2058	0.2773	0.1089	0.1517	0.3886
E-bikes /e-mopeds	7.2010	1.8797	5.4672	9.0751	1.1002	0.4192	0.6755	1.4549	0.1942	0.0721	0.1237	0.2671

not too many, we would treat these e-bikes as e-mopeds for the first step towards extensive model calibration. Besides, the extracted trajectories contain 29,591 frames of trajectories with an interval of 0.12 s. Parameters for each frame of trajectories include coordinates, speed, acceleration, etc.

We further extract the two-wheeler's behaviour periods from the trajectory dataset. As already mentioned, three typical behaviours of two-wheelers are included in this study, i.e. freely moving, following, and overtaking. A rule-based filter is applied in this study to collect the trajectories of each behaviours. For the following behaviour, flitting rules are in line with the criterion in Hoogendoorn and Daamen [89]. Besides, overtaking interactions are found preliminarily by plotting space–time diagrams to find where the trajectories crossed, and then rules in Mohammed et al. [24] are utilized to identify whether these potential periods are exactly overtaking periods. Last but not the least, frames not considered as following or overtaking interactions by the filter are recognized as the freely moving behaviour. This is also because no other behaviours are obviously found in our surveyed site. Finally, each frame of trajectories is labelled as freely moving", "following", or "overtaking" to indicate the two-wheeler's behaviour. Herein, 407 following periods and 170 overtaking periods are collected. Table 2 gives the descriptive statistics of the typical behavioural characteristics parameters of each interaction behaviours of two-wheelers, as well as that of the speed of different types of two-wheelers. Headway for the following behaviour and the overtaking lateral clearance are involved in Table 2, which can characterize the main features of these two behaviour. Herein, the above overtaking lateral clearance is defined in this paper as the quotient of the overtaking lateral distance to the two-wheeler's desired speed. Because the definition of the above term is related to the desired speed, therefore, it is also a psychological variable that reflects the two-wheeler's expectation of spacing in the lateral direction while overtaking. More specifically, two-wheelers often expect to overtake the front road user in a safe condition (no collision), therefore, they may allow ample (even redundant) lateral spacing. Since the desired speed describes the possible speed the two-wheeler can reach and the lateral distance is always speed-related, the definition of the overtaking lateral clearance using the desired speed ensures a sufficient distance while interacting. At this moment, two-wheelers usually pursue their desired speed to make sure to overtake smoothly [96]. In this study, the desired speed of each two-wheeler is defined as the 85th quantile of its own speed distribution. Please note, such a setting of the desired speed is feasible in the free flow condition, adjustment may be needed for the high-density condition.

5.2. Model calibration

5.2.1. Calibration approach of the CZ model

In this section, several coefficients of the CZ model need to be calibrated first. Herein, a generic procedure for calibration needs to be provided first. We provide the calibration procedure of the CZ model in this study, and the specific operation in each step is also discussed. The procedure is presented as follows:

Step 1 Define characteristic indicators of interactions that can represent the need for riding space of two-wheelers based on applied scenarios.

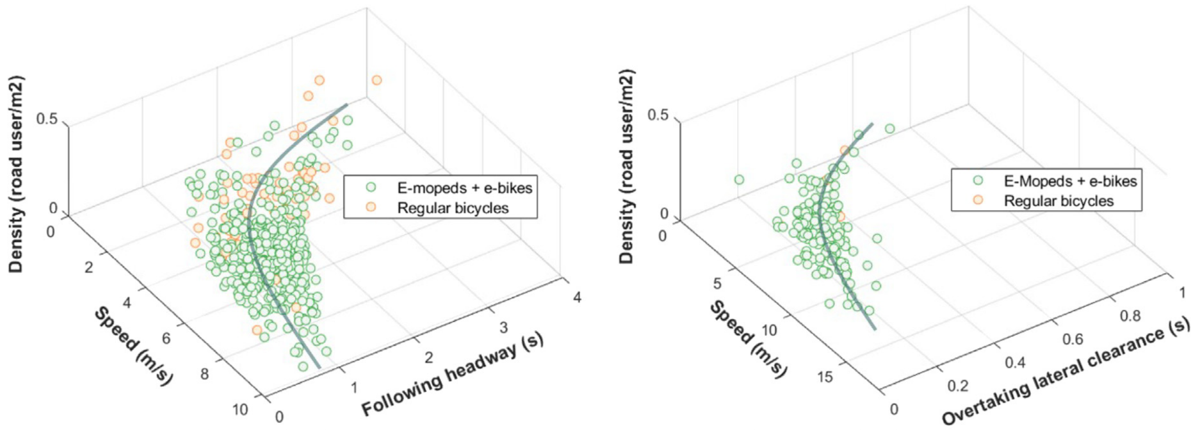
Step 2 Extract values of selected parameters and modelling variables from the dataset.

Step 3: Obtain model coefficients using mathematical methods and check goodness of fit.

Step 4: Face verify through numerical examples under different context, and optimize parameters.

Note that, Step 1 is the manifestation for considering interaction characteristics of two-wheelers in specific application scenarios. This is because the comfort zone of the two-wheeler essentially describes features of interactions with others. It represents the riding space that the two-wheeler tries to keep if he/she wants to feel comfortable while riding. Therefore, choosing suitable indicators of interactions in a specific scenario to characterize the comfort zone of two-wheelers is reasonable.

In the application of the CZ model in this study, we first select that the headway of the following behaviour and the lateral clearance of the overtaking behaviour are surrogate measures for a and b. As mentioned above, the above overtaking lateral clearance is the quotient of the overtaking lateral distance to the two-wheeler's desired speed. This approximation allows the interaction features and the comfort zone of two-wheelers to be expressed reasonably. This is because the following headway represents the acceptable distance in time from the front road user that two-wheelers try to maintain [89]. Meanwhile, the overtaking lateral clearance depicts the lateral headway needed for two-wheelers to keep safe while executing an overtaking manoeuver [8]. Furthermore, these above selected measurable parameters are also reported in previous methods used to quantify the comfort/safety boundary of a road user [97,98]. However, they have



(a) Following headway (b) Overtaking lateral clearance

Fig. 10. Relationship between speed v, density k, and two related interactive feature parameters.

Table 3
Calibration results of coefficients of variables in the CZ model.

Formula	Coefficient	Variable	Value	P value	Goodness of fit
$f_a(k, v, S_t, C_d)$	α_1	v	5.4190	0.0079*	$SSE = 3224$
	α_2	k	−0.3064	0.0377**	$R^2 = 0.4145$
	β_1	v	0.2914	0.0218**	$RMSE = 0.3929$
	β_2	k	−1.0000	–	F-value = 414.5
	δ_1	–	−2.0870	–	
$f_b(k, v, S_t)$	α_3	v	4.3510	*0.0052**	$SSE = 0.4698$
	α_4	k	−0.0038	0.0438**	$R^2 = 0.4541$
	β_3	v	1.9300	0.0191**	$RMSE = 0.0556$
	β_4	k	−1.0000	–	F-value = 77.3
	δ_2	–	0.1027	–	

Note: (1) According to the calibrated results, the parameter S_t for the two-wheeler's type is set as 2.5 for e-mopeds (including e-bikes) and 3.1 for bicycles; (2) Significance codes: < 0.01* and 0.05**; (3) Values of β_2 and β_4 are assigned due to the final model form.

not modelled for two-wheelers and have also not considered relationships between interactive feature parameters and other environmental attributes. We attempt to serve as a bridge between the two-wheeler's comfort zone and potential influencing attributes that can represent the traffic context.

Secondly, we extract the values for the corresponding variables from the trajectory dataset, i.e. speed v, density k, following headway, and overtaking lateral clearance. Fig. 10 visualizes the scatter diagram to describe the relationships between v, k, and other above two interactive feature parameters. It can be seen that a linear combination of speed v and density k in observations is inversely proportional to both the following headway and overtaking lateral clearance. It shows that the order of nature in the empirical data is consistent with our model assumptions and corresponding underpinning in literature [77,80]. Meanwhile, the performances of interactive features of different types of two-wheelers are also inconsistent, especially of their following headway (no obvious differences between these different types of two-wheelers because bicycles seldom execute overtaking while our observation).

Thirdly, the model coefficients needs to be determined and checked. In this study, we use the Fitting Toolbox in MATLAB® by the curve-fitting method to achieve the aim. The regression results are listed in Table 3. We used the F-test to verify whether the regression results could be confirmed. The F-value shows that the regression results are significant, as visualized in Table 3. Therefore, the calibrated results can be considered to explain the relationships between model variables. It also indicates the model is capable of representing the two-wheeler's interactive features.

Finally, we qualitatively measure the effect by intuitively visualizing several numerical examples with different traffic context. In Fig. 11(a), when the traffic density is fixed, the current two-wheeler (herein, the object is an e-moped) focuses on road users with larger temporal distances as its speed decreases. Fig. 11(b) visualizes another scenario in which the traffic density increases but the two-wheeler's speed does not change. When the density is higher, the two-wheeler's comfort zone becomes smaller and the two-wheeler will be more concerned about interactions with temporally closer road users. The findings of previous study [99] confirm that road users pay more attention to others at smaller temporal distances when their speeds are faster and the traffic density is lower because these objects pose a greater threat to safe movement, which is consistent with the performance of the CZ model. As such, we can consider the calibration results of the CZ model is reasonable.

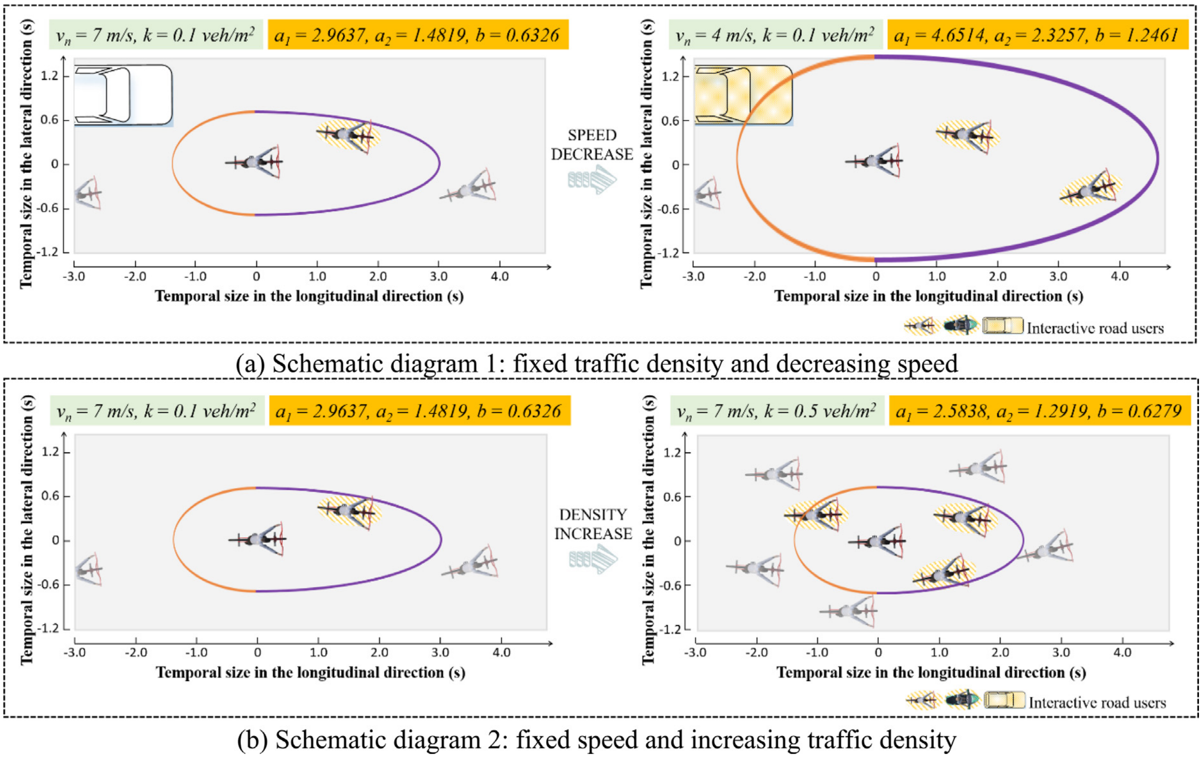


Fig. 11. Schematic diagrams of the variation of the comfort zone in different scenarios.

Table 4
Results of the decision-making model (SBN) for training and testing datasets.

Dataset	Observed	Predicted			
		Freely moving	Following	Overtaking	Correct prediction (%)
Training	Freely moving	10,957	34	161	98.25%
	Following	77	469	13	83.90%
	Overtaking	458	61	2,566	83.18%
	Overall percentage (%)				94.57%
Testing	Freely moving	10,946	30	179	98.13%
	Following	79	467	9	84.14%
	Overtaking	422	81	2,582	83.70%
	Overall percentage (%)				94.59%

5.2.2. Estimation of the decision-making model

The proposed BN-based behaviour decision-making model is calibrated and validated based on the trajectory dataset mentioned in Section 4.1. Three steps are required to construct a SBN model: attribute selection, structure learning, and parameter estimation [100].

Attributes in SBN consist of two parts: the hidden state and the observations in the graph. The hidden state stands for the estimation results, i.e., the two-wheeler's decision results in this study, represented as a single random attribute. Each observation, i.e., model variables (as shown in Table 1) that have influences on the two-wheeler's behaviour decisions, corresponds to a unique node in the graph and stores the joint probability distribution of this node for all its direct parent nodes. Values of these attributes are obtained from the trajectory dataset mentioned above. As mentioned in Section 5, a total of 29.591 frames of trajectories are contained in the dataset, and each of them has the corresponding behaviour label. Here, we utilized all of them to train and test the decision model. Besides, to guarantee the data requirements for SBN learning, we further use the zero-mean normalization method to eliminate the dimensional differences between variables. Meanwhile, the equal depth method is also adopted to discretize the original continuous values [101]. Meanwhile, we also need to consider that pieces of different behaviours should be evenly distributed in the training and testing dataset. Finally, the dataset was then divided into training and testing datasets in a ratio of approximately 1:1.

As for the further two steps required to build a SBN, we adopt the K2 algorithm presented by Cooper and Herskovits [102] to determine the structure and utilize the expectation–maximization (EM) algorithm based on maximum

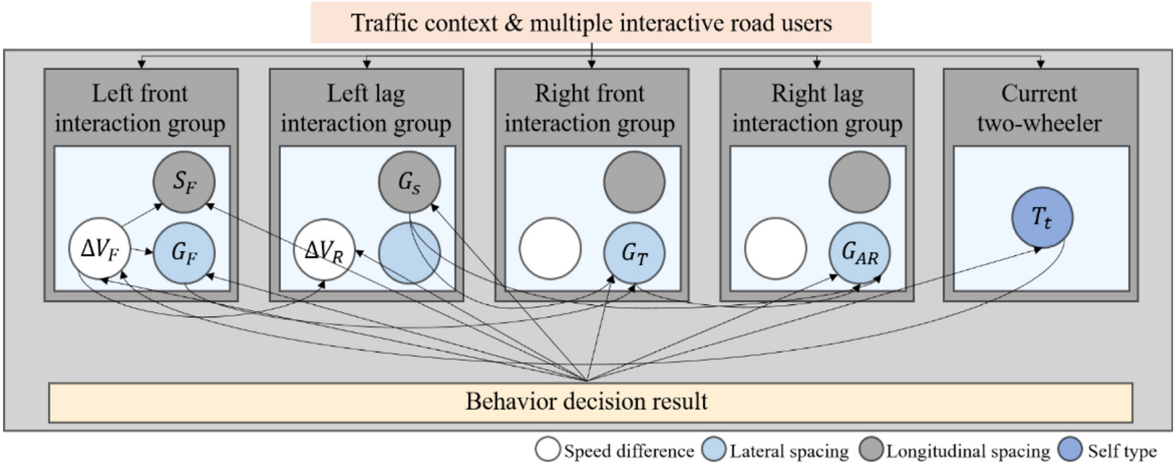


Fig. 12. The BN model structure.

Table 5
Variables calibration of force-related models.

Type of parameter	Parameter	Value	Description	Corresponding Model
Measurable parameters	v_d	9.08 6.09	Mean value of e-bikes/e-mopeds' desired speed (m/s) Mean value of bicycles' desired speed (m/s)	-
Calibrated parameters	A_{mn}^{rb} B_{mn}^{rb}	0.42 6.43	Repulsive force of bicycles	Repulsive force model
	A_{mn}^{mb} B_{mn}^{mb}	0.76 7.11	Repulsive force of e-bikes/e-mopeds	
	A_{mn}^{car} B_{mn}^{car}	1.63 9.31	Repulsive force of motorized vehicles	
	τ_d	5.06 3.41	Buffer time of e-bikes/e-mopeds Buffer time of bicycles	Freely moving force model
	a_m	1.17 0.55	Maximum acceleration/deceleration of e-bikes/mopeds (m/s^2) Maximum acceleration/deceleration of bicycles (m/s^2)	Following force model
	δ_e	4.00 4.00	Acceleration exponent of e-bikes/e-mopeds Acceleration exponent of bicycles	
	b_f	0.94 0.43	Comfortable deceleration of e-bikes/e-mopeds (m/s^2) Comfortable deceleration of bicycles (m/s^2)	
	s_0	1.14 0.72	Gap at standstill of e-bikes/e-mopeds (m) Gap at standstill of bicycles (m)	
	T_d	1.50 1.96	Desired time headway of e-bikes/e-mopeds (s) Desired time headway of bicycles (s)	

likelihood estimation [103] for parameter estimation. These works are finished through the FullBNT 1.0.4 Toolbox in MATLAB®. The estimated decision-making model based on the training dataset is visualized in the following Fig. 12.

We also used the dataset to assess the accuracy of the decision-making model. The prediction results for the training and testing datasets are shown in Table 4. Finally, the overall accuracy is 94.58%, indicating that the model can reflect most two-wheeler real-life behaviours and can respond to context variations.

5.2.3. Calibration of the behaviour force model

The genetic algorithm (GA) is adopted to find the optimum values of parameters of the behaviour force model used in the physical process. As a result, the calibrated parameters are given in Table 5.

Herein, the Toolbox of GA in MATLAB® is utilized to achieve this objective. The root mean square percentage errors (RMSPE) of the average speed is set as the objective function [104]. The population size, maximum number of generations, and number of stall generations of GA are set as 200, 500, and 100, respectively. If the change is less than the function tolerance, i.e. 1×10^{-6} in this study, the algorithm would stop. Besides, the calibration process is repeated 10 times and the set of parameters with the minimum RMSPE error are selected. Furthermore, v_d is set as the 85% quantile of the two-wheeler's speed distribution extracted from the trajectory dataset, which is regarded as measurable parameters.

5.3. Face validation using simulation

Based on the calibration results, simulations are implemented to validate the proposed model. Simulations are performed through coding in MATLAB®. The simulation area is a road segment, of which the size is 110 m × 9.8 m (consistent with the surveyed site mentioned in Section 4.1). Meanwhile, the interval of the simulation step is 0.12 s, which is also consistent with the interval between adjacent frames of the trajectories. In this study, two types of validation are performed. On the one hand, the face validation [105] is utilized to prove the model effectiveness from the perspective of individual behaviour level (Sections 5.3.1–5.3.4). On the other hand, the validation via replicated simulation is implemented to discuss the stability and robustness of the proposed model (Section 5.3.5).

As for the simulation of the face validation, prediction results performed by the proposed model would be compared with the observations in the dataset. Herein, the face validation has been proven to be possible for verifying the model performance from a more individual behaviour level in previous studies [21,26,28,76]. The simulation utilized the estimated models to compute the velocity and position update of simulated objects at each time step. Meanwhile, each simulated object would be simulated in turn until it leaves the road segment. This means that the traffic context information provided for each simulated individual would always be the same as that contained by the observed trajectories. Then, we can intuitively verify whether the model can accurately represent the realistic interaction and movement of each individual. The performance of simulated two-wheelers is compared with the empirical data in terms of decision process analysis (see Section 5.3.1), overtaking interaction results (see Section 5.3.2), trajectory distribution (see Section 5.3.3), distributions of travel time, and anticipated collision time (see Section 5.3.4). Herein, the decision process analysis can visualize the two-dimensional behaviour and interaction of two-wheelers on the individual level [28]. Overtaking interaction results can describe the accuracy of the decision and action process while interacting [96]. Trajectory distribution can overall represent the two-dimensional behaviour of two-wheelers [27]. Travel time measures the accuracy in mobility [26] and anticipated collision time reflects the safety performance when interacting with multiple road users.

As for the replicated simulation, the control variable is the density of the two-wheeler flow. By varying the density of two-wheeler flow, the flow rate and average speed can be measured at different density levels, and the fundamental diagram can be obtained. Herein, the density is calculated by dividing the occupied area of two-wheelers by the area of the simulated road segment, which is consistent with the previous literature [8,77]. At each density level, the simulation would be repeated 20 times and the length of the simulation period would be set as 3600 s. Besides, to decrease the influence of the warm-up period and the initial state, the simulation data would be obtained after discarding the first 2000 time steps and last 2000 time steps (i.e. 240 s). The average value of the rest time steps would be used for discussion.

5.3.1. Operation mental process analysis

To verify the rationality of the proposed model in representing the operational mental and physical processes while interacting, we randomly choose a two-wheeler with obvious interactions as an example. We visualize its behaviour choice results in the simulation, along with the simulated and empirical trajectories in both lateral and longitudinal positions over time, as shown in Fig. 13.

Initially, the selected two-wheeler moves freely for approximately 4.5 s. Then, when approaching a slower cyclist in front, the simulated object executes a lateral swerving movement and an overtaking interaction occurs during the next 5.8 s, finishing at around 10.3 s. The simulated two-wheeler then follows another traffic participant and leaves the road segment. From Fig. 13, we can find that the overtaking and following occasions in the simulation are close to the observations. Furthermore, the trajectory generated by the model has a similar locational distribution to the empirical trajectory. During the first freely moving phase (between 0 s and 4.5 s) in Fig. 13, we can obviously see that the empirical two-wheeler moves rightward slightly. This is because two-wheelers commonly show a randomness feature of their movement by turning the handlebar. Under the freely moving condition on the two-dimensional space, such a feature would be more obvious and is hard to be fully captured by models. At the same time, the simulated two-wheeler would move forward to its destination directly based on the current behaviour decision result that is consistent with the reality. This may be the reason for the simulated two-wheelers moving leftward but the empirical one moving rightward slightly. At around 4.5 s, the overtaking behaviour would be triggered because the decision model on the operational mental (decision) layer accurately makes such a behaviour choice. Then, it outputs the result into the operational physical layer to control the simulated two-wheeler to execute the selected behaviour. As shown in Fig. 13, our model successfully captures the moment for the overtaking interaction as well as the dynamics during the interaction. When the overtaking interaction ends around 10 s, the simulated two-wheeler decides to follow another front road user until it leaves the road segment. The reason for the slightly leftward deviation of the empirical two-wheeler may still be brought by the extremely high degree of movement freedom of two-wheelers on the two-dimensional space. However, we can still indicate that the proposed model can precisely capture the interaction with the decision-making process of a single two-wheeler, and can represent the real movement progress on the whole. Furthermore, the control dynamics of the two-wheelers can also be well characterized. Finally, we also visualize several other cases to prove the model performance, as shown in Fig. 14.

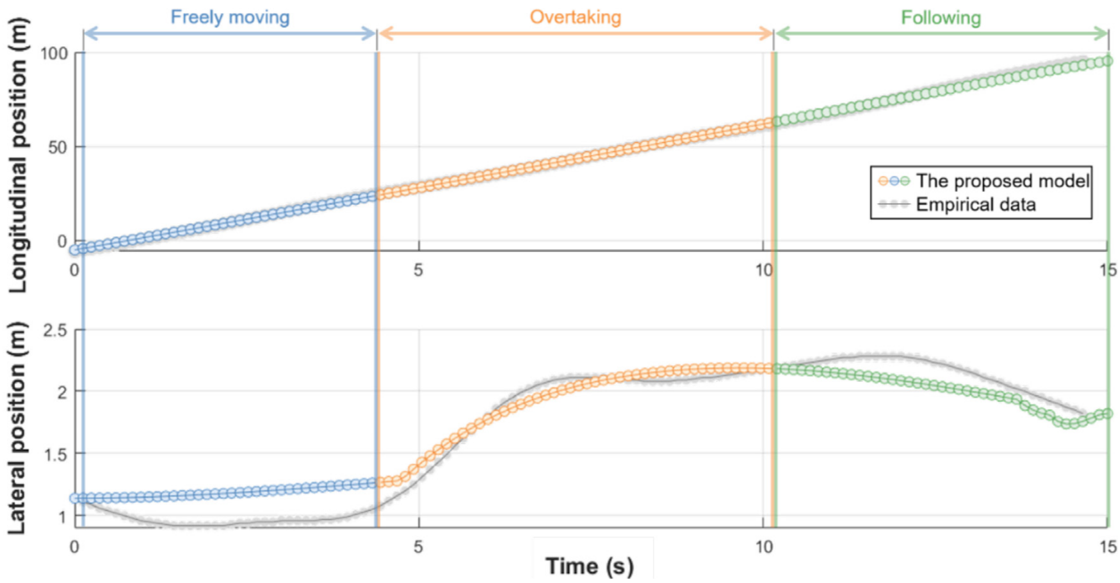


Fig. 13. Comparison of a single two-wheeler operation process between simulation and observation.

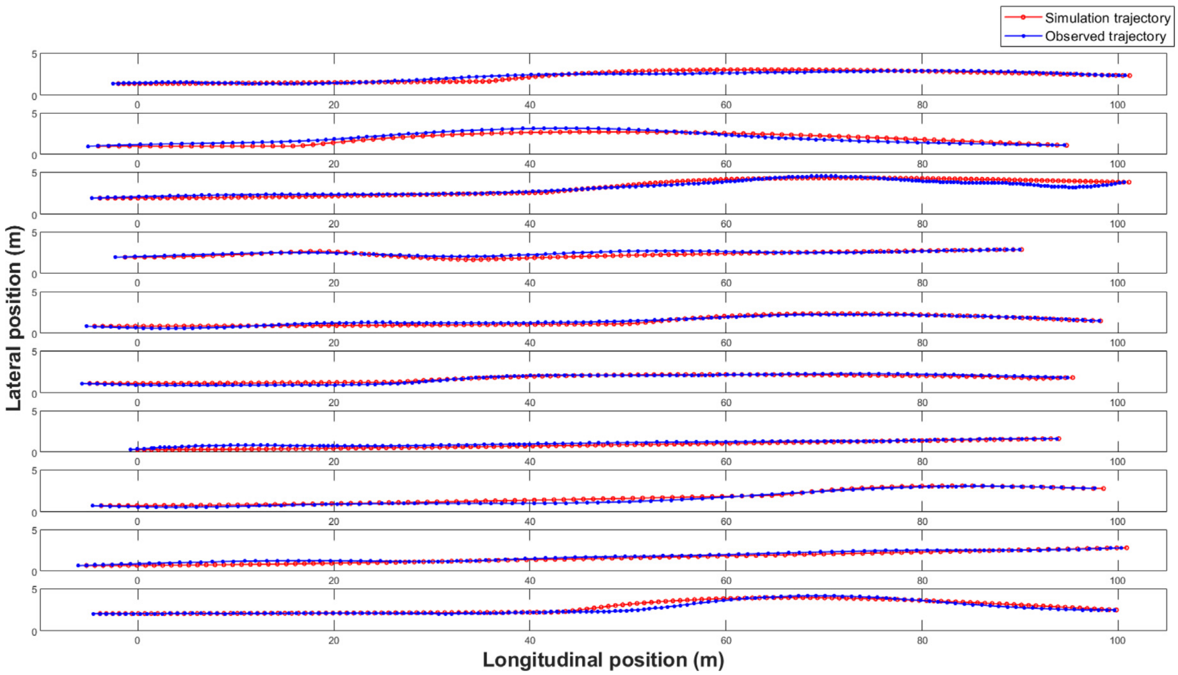


Fig. 14. Comparison of other cases for the simulated and observed trajectories.

5.3.2. Overtaking interaction results

Because overtaking is one of the most common riding interactions [24], we also face verify the model's performance in terms of two aspects: the overtaking prediction accuracy, and the aggregated distributions of the overtaking lateral distance. These two indexes can help us test the performance of the proposed model from the mental and physical levels, respectively.

We first calculate the overtaking prediction accuracy. Of the 170 overtaking interactions recorded in observations, a total of 156 times are captured by the proposed model in simulation. Therefore, the prediction accuracy of the overtaking interaction at the overall behaviour level is 91.76%, which means the model can reproduce most of the two-wheeler's overtaking manoeuvres. Besides, we calculate the accuracy of the decision choice at each moment during overtaking

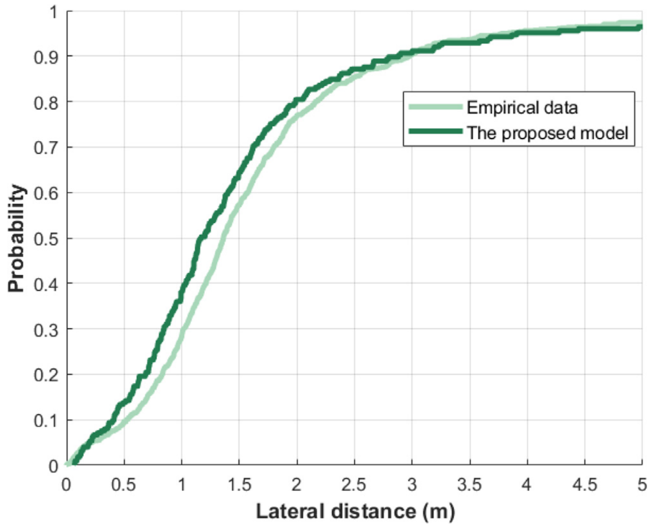


Fig. 15. Comparison of observed and predicted overtaking lateral distance interactions.

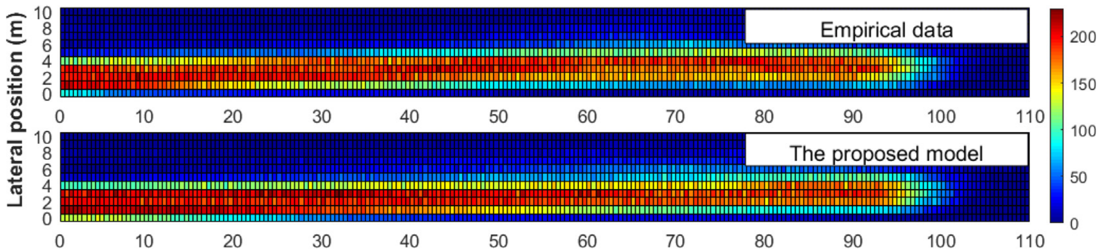


Fig. 16. Location distributions of two-wheelers' trajectories.

interactions. A total of 6170 time step are included in overtaking interactions, while 5148 of them are predicted by the decision model as the moment for the overtaking interaction. Hence, the prediction accuracy is around 83.44%. Furthermore, Fig. 15 shows the cumulative distributions of the overtaking lateral distance. It can be seen that the predicted (simulated) result is similar to the true (observed) result. These simulation results are owing to the model's reasonable behaviour choices on the operational mental level and the dynamics control considering decision-making results on the operational physical level. Therefore, the model can be considered a great representation of reality.

5.3.3. Trajectory distribution

We also compare the distributions of the empirical and simulated trajectories to check whether the locations of two-wheelers are similar on the whole. For this comparison, the whole surveyed site (110 m × 9.8 m) is divided into 0.5 m × 0.5 m cells and the occupancy frequencies of the simulations and observations for each cell are counted. The location distributions are presented as a heat map in Fig. 16.

It is clear from Fig. 16 that the location distributions generated by the proposed model are similar to the observations. Meanwhile, the observed distributions of trajectories show a trend for moving on the left side of the bicycle lane, which is well captured in the simulation. The results displayed in the two heat maps have less variance. The reason for the small difference between observations and simulation results may because the motion of the two-wheeler on the two-dimensional plane is highly free, and the freedom of movement in reality is certainly higher than that of the simulation. Furthermore, we can also observe that the leftward trend of the trajectory distribution is also captured by the proposed simulation model. It indicates that our model can represent the two-wheeler to make a behaviour decision close to the reality, and our model can also show an accurate dynamics control of the simulated two-wheeler. When the trajectory of the two-wheeler can be accurately reproduced in the simulation, the leftward trend of the whole trajectory distribution would be naturally captured. Such a result is jointly realized by the cooperation of every component model in our proposed simulation framework, and is also benefitted from our framework. First, the CZ model can capture interactions of two-wheelers so as to provide realistic information for the next two tasks, namely, operational mental decision and operational physical process. Based on that, our proposed SBN model can output a reasonable behaviour choice for the simulated two-wheelers, which has been proven in Table 4. Finally, the behaviour force model can produce the smooth trajectory on the two-dimensional space to represent the coupled movement in longitudinal and lateral directions.

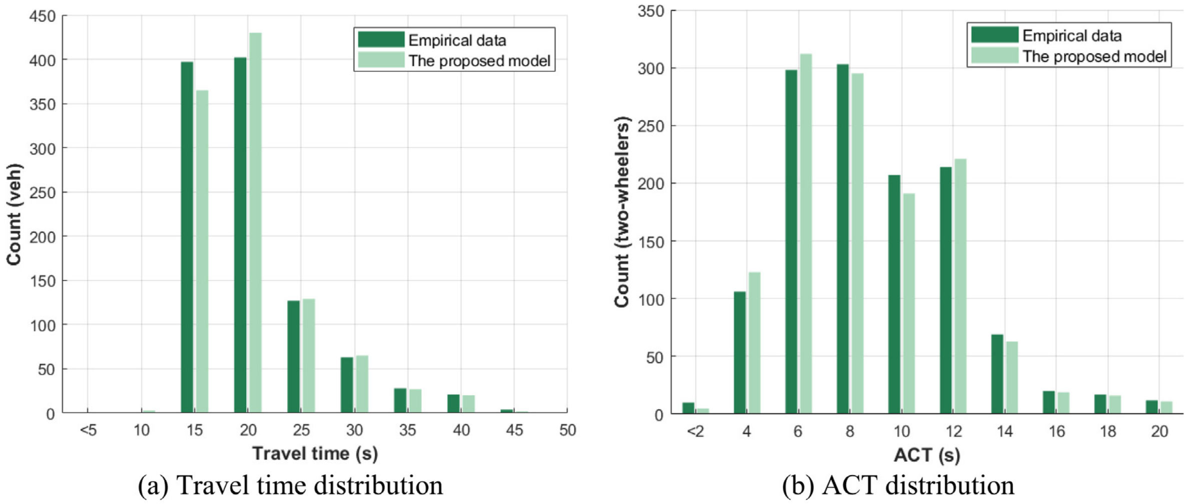


Fig. 17. Comparison of the travel time and ACT distributions.

Table 6
Descriptive statistics for travel time and ACT distributions.

	Travel time (s)		ACT (s)	
	Mean	S. D.	Mean	S. D.
Empirical data	17.6336	5.7727	7.5765	3.0129
Proposed model	17.7526	5.6105	7.9031	3.2314

5.3.4. Travel time and anticipated collision time (ACT) performance

The travel time distribution and the ACT distribution are compared in this subsection to verify the performance of the proposed model in terms of efficiency and safety aspects, respectively. Herein, the travel time is the most common surrogate indicator used to evaluate efficiency in the traffic simulation. Meanwhile, the ACT is a surrogate safety indicator for trajectory-based safety assessment and can consider the simultaneous interaction of multiple road users on the two-dimensional space [106], which can be calculated by the following Eq. (10):

$$ACT = \begin{cases} \dfrac{\delta}{\left(\frac{\partial \delta}{\partial t}\right)}, & if\ \dfrac{\partial \delta}{\partial t} > 0 \\ \infty, & otherwise \end{cases} \tag{10}$$

where δ denotes the shortest distance between two road users; $\partial\delta/\partial t$ is the closing-in rate of the two-wheeler to another interactive object; Herein, The factors determining the closing-in rate $\partial\delta/\partial t$ of two road users are their speed, acceleration, heading angle, and yaw rate. The components of the closing-in rate can be described by operators that takes the vector sum of speed, acceleration, heading angle, and yaw rate, jointly.

Fig. 17 shows that both the travel time and ACT distributions generated by the simulation are closer to the empirical data. Table 6 also shows the means and standard deviations of the travel time and ACT distributions. Hence, these results indicate that the distributions of travel time and ACT of simulation and observations do not show noticeable differences, and the proposed model can well represent the two-wheeler dynamics. Furthermore, the ACT accuracy also indicates that the proposed model has the potential to support the traffic safety assessment as a simulation tool.

5.3.5. Model robustness performance

Fig. 18 shows the fundamental diagrams of two-wheelers under different density values in the replicated simulation. From Fig. 18 we can see that the relationship between the density and the speed, as well as the relationship between the density and the flow rate, is similar to the empirical findings in the literature [77]. Therefore, the results can first indicate that the proposed model can generally reflect the basic law of nature of the two-wheeler traffic flow. At the same time, it can be also seen from Fig. 18 that the average speed and flow rate are distributed within an acceptable range at each density level. Therefore, it also shows that the model has good robustness to reproduce the two-wheeler flow characteristics, that is, stably describing the operation of two-wheeler traffic flow.

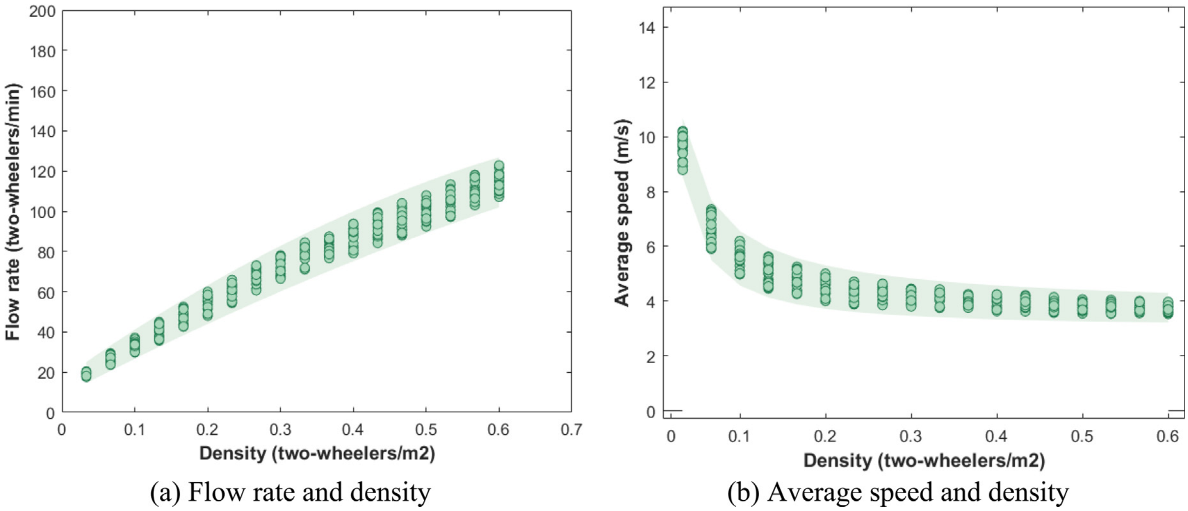


Fig. 18. Flow rate and average speed as functions of density.

6. Conclusions and outlook

Multi-interaction is one of the critical characteristics for two-wheelers on mixed-traffic road segments. Its negative impacts lead to influences in traffic operation and bring potential safety problems. Therefore, the description of the multi-interaction is crucial if microscopic simulation models are designed to reproduce the two-wheeler's operations with high accuracy to satisfy the demand of traffic planning, facility designs, and traffic management. However, existing simulation models lack the ability to handle and describe the multi-interaction, no matter in terms of operational mental or physical activities while moving. More attempts should be made to reach the goal of high accuracy representing of two-wheeler operations.

In this paper, we first define the tasks that are contained by the multi-interaction on the operational behaviour level, and correspondingly put forward a simulation framework that supports representing the two-wheeler multi-interaction in simulation. The framework is designed as a three-layer structure to capture the inner tasks of the multi-interaction in terms of operational (mental) perception, operational (mental) decision, and operational physical processes. At the same time, we also develop corresponding components in each layer of the framework to specifically model the framework. Herein, the "Comfort Zone" model is developed to capture multiple interactive road users. The dynamic variation of the interaction is captured by the size-variable comfort zone defined for each two-wheeler. Besides, the BN model is specifically applied for the first time to describe the two-wheeler's decision-making behaviour considering multi-interactions. Furthermore, a behaviour force model is developed to represent the non-lane based movement that is guided by the behaviour decision results and current interaction states.

Using the trajectory dataset collected on a mixed-traffic road segment in Shanghai, China, proposed models are estimated and face validated. The simulation results indicate that the two-wheeler's operations and interactions can both be accurately replicated. For instance, compared with the empirical data, 91.76% of the overtaking behaviours within the field-measured results can be reproduced in simulations, while trajectories, travel time, and ACT distributions also show no significant difference with the reality. Overall, the proposed framework addresses the current shortcomings in modelling the two-wheeler's multi-interactions. It can also provide a foundation for microscopic simulation tools to evaluate traffic efficiency, make safety assessments, and design infrastructure for mixed-traffic flows. The developed framework and the models of tasks of two-wheeler multi-interactions in simulation is the major contribution of this study, which can potentially enable the accurate performance of future simulation tools.

This study also has some room for improvement. First, it is still necessary to make step-by-step developments, which may be achieved by including more attributes in the CZ model, such as weather conditions, types of facilities, and human factors such as age or gender. The improvement process could be accelerated by collecting more empirical evidence and conducting virtual experiments. Furthermore, more data would also be collected to validate the developed model. The hypothesis of the elliptical shape still needs to be further proven and the calibration method also needs to be improved, especially calibrating the model at not only longitudinal and lateral directions but also other oblique directions. In addition to improvements aimed at higher accuracy, another research direction is the adjustment of the decision model to consider the effect of interactive road users' motion trends, because the two-wheeler's anticipation of interactive objects' movement may affect its behavioural decisions. Furthermore, the applicability of the framework could be verified by extending it to more scenarios such as some spaces shared with pedestrians, mixed-traffic scenarios with high density, and even for scenarios with different flow directions. Besides, the discussions of the using experience of the SBN model and how the SFM presents the two-wheeler movement features are also worth to be further pushed forward. We aim to collect more pieces of evidence and verify our observations through controlled experiments to achieve these potential works. The present study paves the way for this future research.

CRediT authorship contribution statement

Ying Ni: Conceptualization, Methodology, Writing – original draft, Writing – review & editing, Supervision. **Yixin Li:** Conceptualization, Methodology, Software, Data analysis, Calibration, Validation, Writing – original draft, Writing – review & editing. **Yufei Yuan:** Writing – original draft, Writing – review & editing. **Jian Sun:** Supervision, Project administration.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

The authors do not have permission to share data.

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Appendix A. Supplementary material

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.physa.2022.128441.

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