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TECHNICAL NOTE 1

EXPERIMENTAL EVIDENCE OF THREE-DIMENSIONAL  
PERTURBATIONS IN THE REATTACHMENT OF  
A TWO-DIMENSIONAL LAMINAR BOUNDARY LAYER AT  $M=2.05$

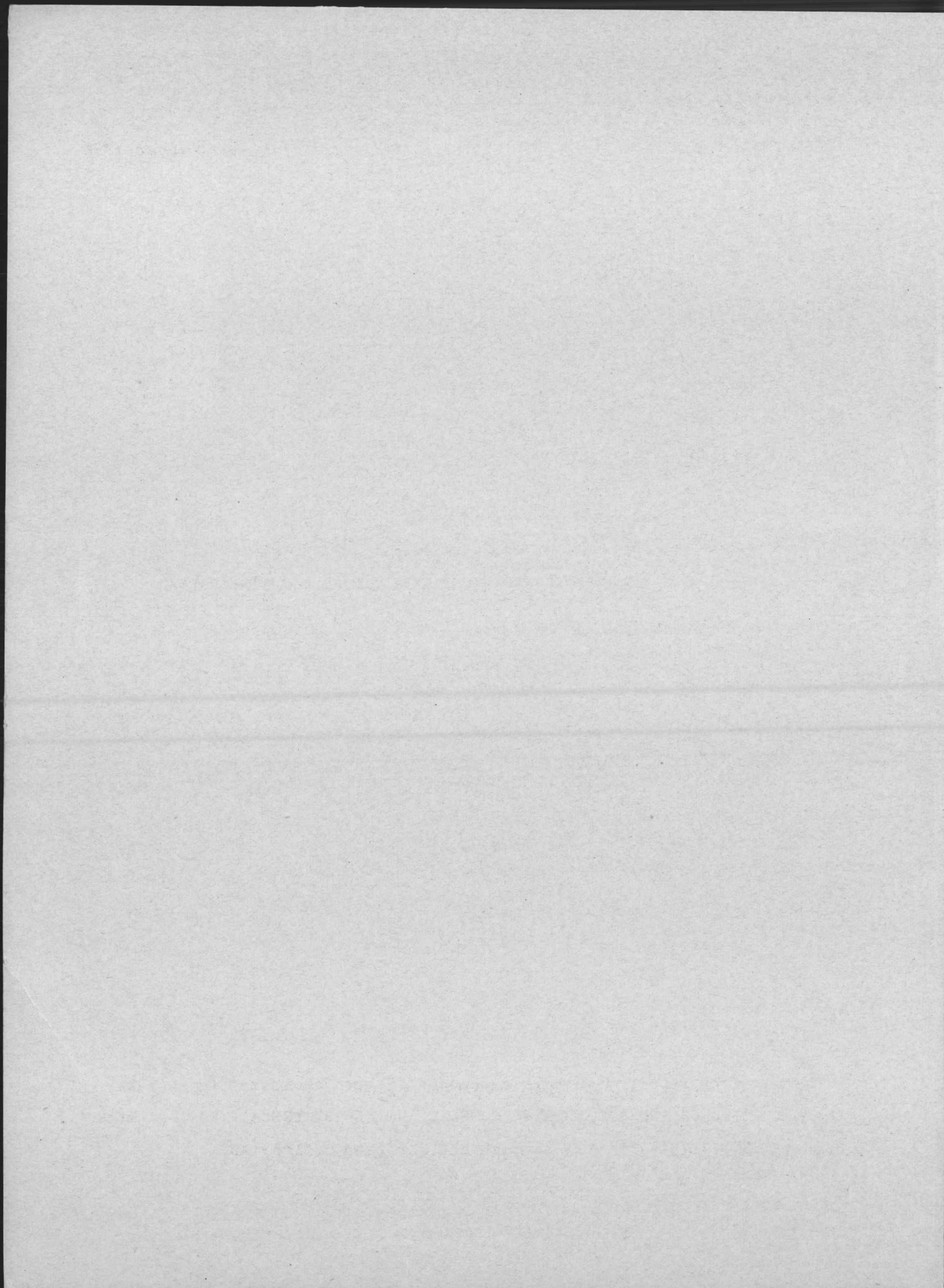
By Jean Ginoux



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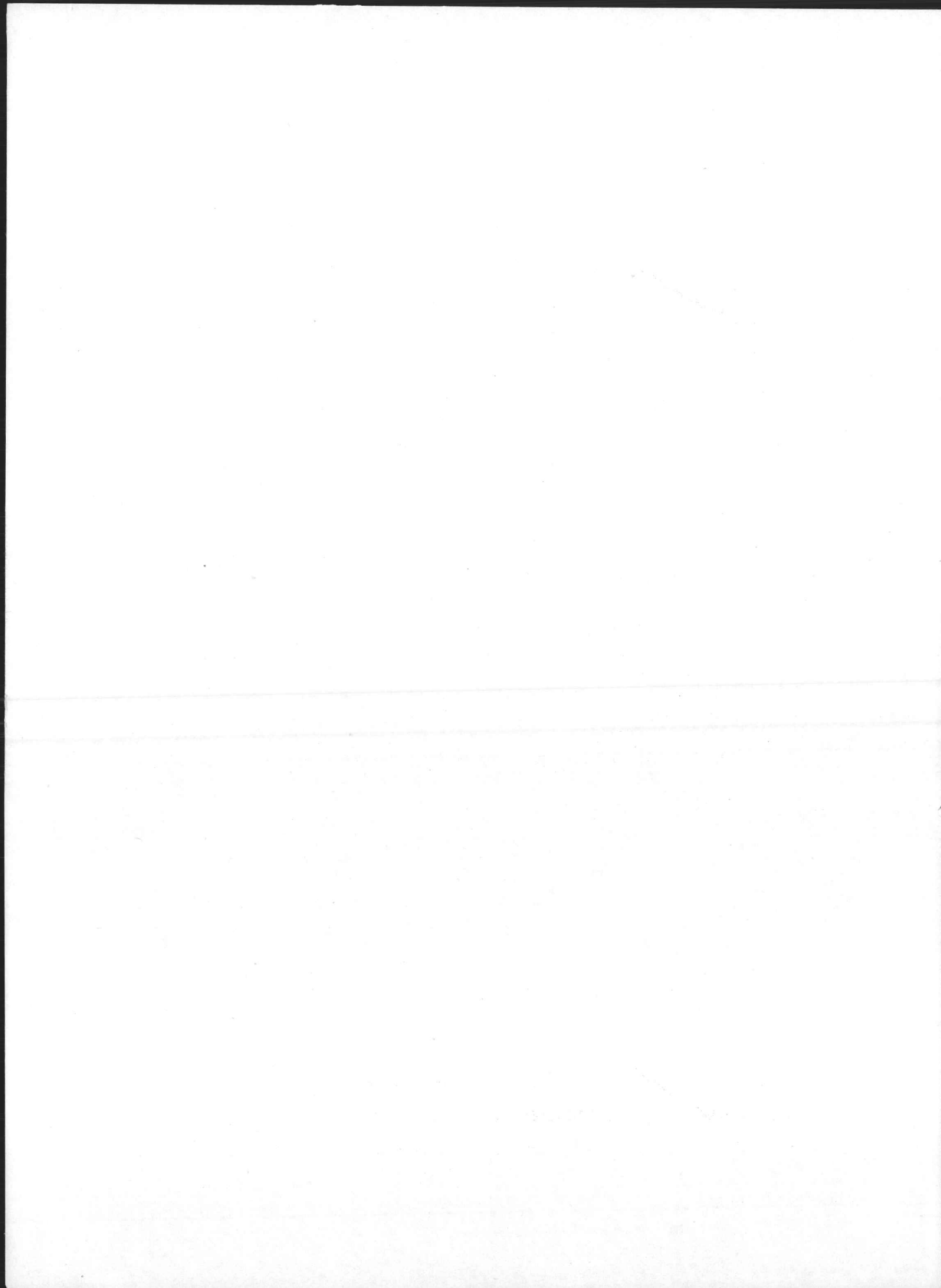


EXPERIMENTAL EVIDENCE OF THREE-DIMENSIONAL PERTURBATIONS  
IN THE REATTACHMENT OF A TWO-DIMENSIONAL LAMINAR  
BOUNDARY-LAYER AT  $M = 2.05^*$

by

Jean J. GINOUX

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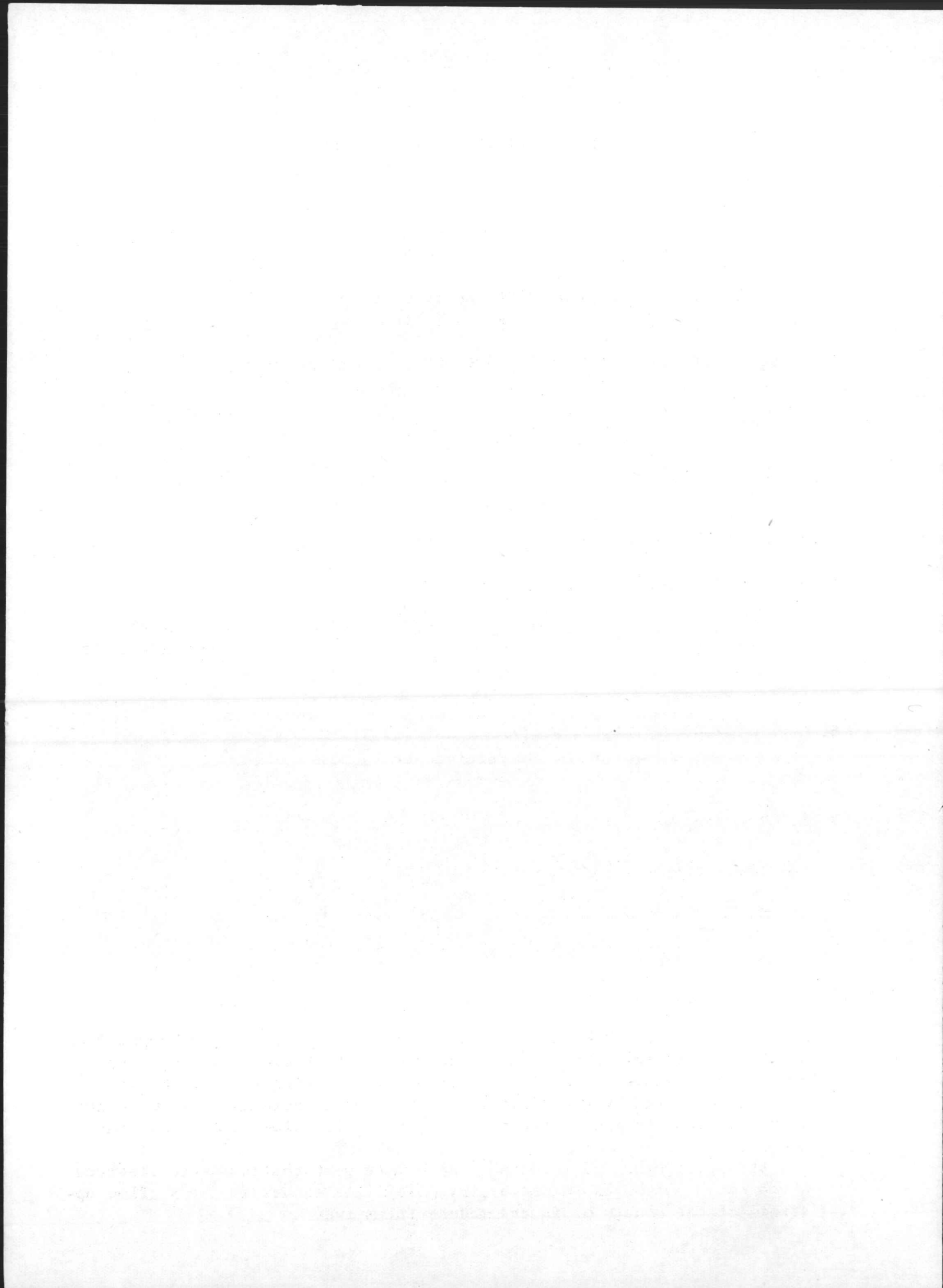


## FOREWORD

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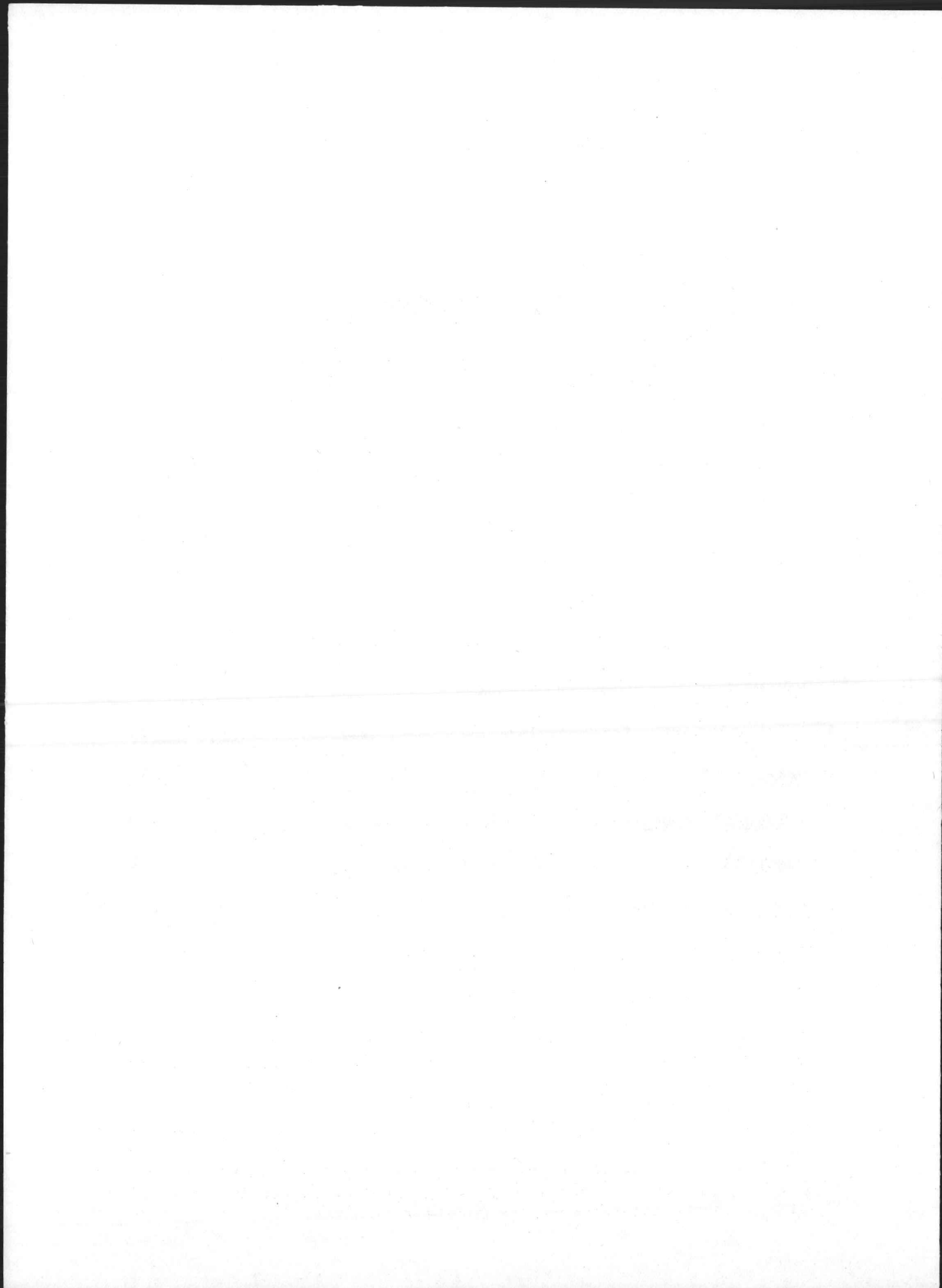
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Strong, regular and repeatable span-wise perturbations were observed which could not be explained by irregularities either in the airflow upstream of the models or in the models themselves.

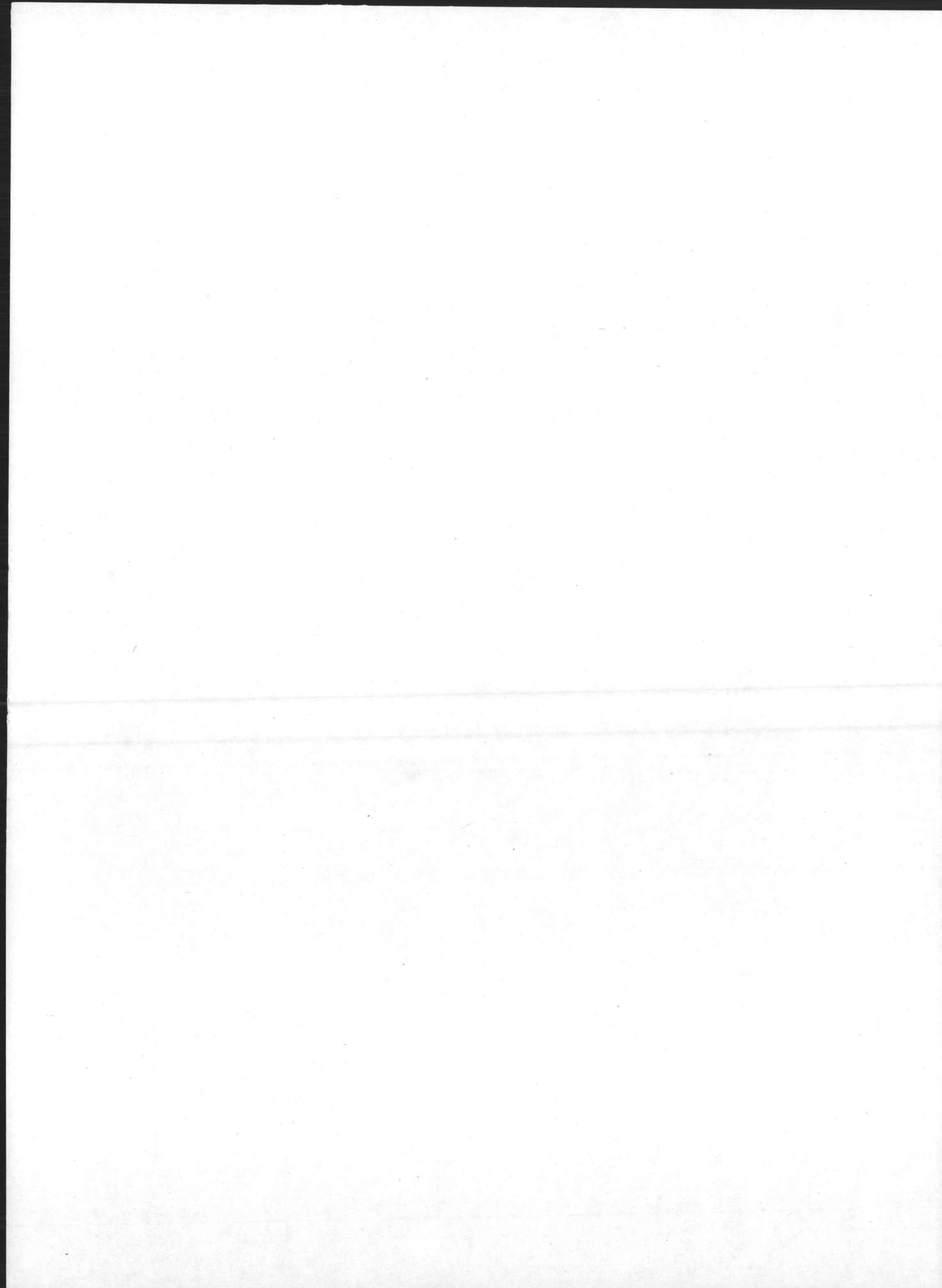


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IN THE REATTACHMENT OF A TWO-DIMENSIONAL LAMINAR  
BOUNDARY-LAYER AT  $M = 2.05$

b y

Jean J. GINOUX

T.C.E.A. and C.N.E.R.A.

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ABSTRACT.

The reattachment of a laminar boundary-layer after separation has been investigated at a Mach number of 2.05. Backward facing step models were used that completely spanned the working section. Surface flow was observed by a sublimation technique and detailed span-wise surveys were made in the reattachment region of the flow with total head probes.

Strong, regular and repeatable span-wise perturbations were observed which could not be explained by irregularities either in the airflow upstream of the models or in the models themselves.

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## INTRODUCTION

The wind-tunnel tests which are described in this note arose incidentally from a research programme undertaken by the author to study the reattachment of a laminar boundary-layer after separation at supersonic speeds.

In the course of this programme, a detailed survey was begun to check that the flow was effectively two-dimensional with the particular configuration used. This survey produced the somewhat surprising result that there were strong, regular and repeatable span-wise perturbations in pressure, which could not be explained by irregularities either in the air-flow upstream of the model or in the model itself. It thus appears possible that these perturbations are fundamental to the type of flow under consideration.

Other published work describing tests in which a similar result occurred, or might have been expected to occur, is considered at the end of this note.

The work here described is as yet incomplete and the author hopes to continue further with this particular line of investigation.

## DESCRIPTION OF THE TUNNEL

The tests were made at a Mach number of 2.05 in the 40cms x 40cms (16 inches x 16 inches) supersonic wind-tunnel at Rhode-Saint-Genève. This is a continuous closed-circuit tunnel, driven by an axial compressor with a maximum power of about 800 kW. The tunnel is operated at a stagnation pressure which can be varied between 0.1 and 0.35 at absolute, giving a free-stream Reynolds number, at  $M=2.05$ , of  $1.2 \times 10^6$  and  $4.5 \times 10^6$  per metre respectively. A silica-gel dryer, located in a by-pass, maintains the absolute humidity of the air below  $0.1 \times 10^{-3}$ .

Conventional shadow and schlieren systems are available, using spherical mirrors and a parallel beam of light. A spark light-source is used for pictures. Special measuring devices or techniques used in the present research are described where appropriate in the body of this report.

## MODEL CONFIGURATIONS

The geometry, dimensions and designations of the various models are given in figure 1.

The models were formed by assembling together a certain number of interchangeable pieces. The same leading edge flat plate was used in models S-1, S-3, W-1 and W-3. A second one was mounted on models S-2, S-4 and on the two-step model. The leading edge thickness was in both cases equal to 0.1 mm (0.004 inch).

Wedge models were formed by fastening three different wedges against the step base of model S-1.

Models completely spanned the 40 cms test section and were mounted on a double support allowing for the adjustment of the angle of attack. Figure 9 shows one of the models in the test chamber.

## FLOW AROUND THE STEP MODELS

An earlier series of tests (reference 9) was made to determine the conditions under which a purely laminar reattachment could be obtained on a solid surface behind a backward facing step, using Chapman's definition (reference 1), namely that the static pressure distribution on the wall is not affected by changes in Reynolds number.

The free stream Reynolds number of the wind tunnel can be varied between  $1.2 \times 10^6$  and  $4.5 \times 10^6$  per metre, by changing the stagnation pressure. It was found that such a range was too small to obtain, on a single model, all the possible flow configurations; that is, turbulent, transitional and purely laminar types of reattachment. It was thus decided to design various models having different step heights to cover the complete range of flow.

It was concluded from the shadow and schlieren pictures of the flow, together with pressure distributions, that transitional reattachments were obtained on the single-step models and on the wedge models. A purely laminar reattachment existed behind the forward step (a) of the two-step model, while a turbulent one was found downstream of the rearward step (b).

For the purpose of the present discussion, only a few typical flow pictures are shown (figure 2- a to f).

#### FLOW ON THE MODEL SURFACE

In the initial phase of the experiments, some doubt arose about the possibility of getting a perfectly two-dimensional flow over a model with a backward facing step which completely spanned the tunnel. It was therefore decided to investigate carefully the flow on the model surface, using a sublimation technique.

Azobenzene was used as an indicator. Surfaces of the models were thoroughly degreased and a 5 % solution of azobenzene in 100/120 petroleum ether was sprayed on the surface to form a thin film. In order to get it adhering firmly to the surface, the models were sprayed from a distance of about half a metre, using a standard spray gun with a slow rate of feed. When the gun was held too close to the surface, the azobenzene was still in solution on reaching the model resulting in the formation of large crystals which did not adhere to the surface. After spraying, the bigger crystals were removed by rubbing the surface with a cotton wool cloth, in order to avoid an increase in the surface roughness.

The spectrum of azobenzene was measured by spraying samples on transparent pieces of glass and was found to be very flat except in the blue region of the spectrum. It was thus found very convenient to spray some tracing blue colour on the models before spraying the azobenzene. A good contrast could thus be obtained between regions of high and low sublimation rates.

An average of ten successive applications of azobenzene were found to be necessary to build up a film of the right thickness. It appeared difficult always to get a perfectly uniform film of azobenzene owing to the step in the model. Tests were therefore repeated several times under the same conditions to eliminate this effect. Running times of the tunnel varied between half-an-hour and two hours to give an adequate picture of the surface flow, depending upon the thickness of the azobenzene layer and the tunnel pressure.

Using this sublimation technique, span wise (i.e. three-dimensional) perturbations were found to be present in the reattachment region in all the

transitional types of flow, that is on the single-step and wedge models and also in the case of a purely laminar reattachment on the two-step model. No such perturbations were noticed in the turbulent flow over step (b) of the two-step model.

Three-dimensional perturbations which appeared in the form of striations, where the sublimation rate was high, are visible on the various pictures of figure 3 (a to g). From these, it can be seen that selective transverse wave lengths seemed to exist.

For a few models, the tests were re-made several times and consistently similar striation patterns were found on each occasion.

One of the tests was made with a longer running time, to find out if the azobenzene striations were extending downstream of the reattachment region of the flow. Although the contrast was very poor, an extension was observed in the turbulent region of the flow, as can be seen on figure 4a.

Tests made on the two-step model, where a purely laminar reattachment was obtained, showed (figure 2f and 3g) that the appearance of three-dimensional perturbations did not indicate incipient turbulence in the flow. It was only after they had travelled a certain distance that transition appeared, as shown by a higher sublimation rate of the azobenzene layer.

#### EFFECT OF POSSIBLE FREE-STREAM PERTURBATIONS

The next step in the research was to eliminate possible experimental sources of errors which might be responsible for the existence of three-dimensional perturbations in the reattachment region of the flow, in order to determine whether they were effectively inherent in the particular types of flow under consideration.

Such perturbations could possibly be introduced by irregularities either in the free-stream or in the models. The two possibilities are considered in the following discussion.

When the first tests, made on model S-1, revealed the existence of spanwise perturbations, the quality of the supersonic stream was examined with particular attention.

At that time, the drying equipment was not working satisfactorily

and it was found better to run the tunnel with atmospheric humidity. The effect of free-stream irregularities owing to the condensation shock was thus examined.

With the existing equipment, the absolute humidity could be reduced down to  $0.6 \times 10^{-3}$ . Although it was not possible, under these conditions, to suppress completely the condensation shock, its intensity could be modified. Doing this, no change was observed in the flow over the model. This was further confirmed by changing the position of the model in the test chamber, as will be seen below, and by subsequent runs made with dry air, using the new and very efficient drying equipment.

Other irregularities in the free-stream could be possibly formed in the settling chamber of the tunnel where the main cooler is located. A series of parallel vertical water-cooled tubes is used to maintain a constant stagnation temperature.

It so happened that the number of irregularities on the S-1 model surface was nearly equal to twice the number of cooling tubes, that is to the number of vortex sheets shed by them. A theoretical analysis was made, which showed that, if any transverse total pressure gradient were present in the settling chamber, axial vortices could be formed in the converging part of the nozzle and be present in the test section (see appendix). The vortex strength is given by

$$dk = 1/q \times dh/dN' \times d\theta$$

where  $k \rho \vec{V}$  is the axial component of curl  $\vec{V}$   
 $q$  is the dynamic pressure  
 $h$  is the total head  
 $N'$  is the directional normal to the side walls  
 $\theta$  is the slope of the stream lines.

To check this influence, three 13-mesh screens (0.5 mm diameter) followed by two 55-mesh screens (0.2 mm diameter) were placed in the settling chamber downstream of the cooler. However, the same pattern of flow was still observed in the flow around the step model.

A last check was made by mounting the model vertically, instead of horizontally. This was possible in the square test section - although not easy because of the wall divergence. Again the same irregularities were revealed by the sublimation technique (see figure 4c - a five mm scale was placed on the model surface for the picture).

It was concluded from these tests that possible free-stream perturbations ahead of the model were not responsible for the three-dimensional perturbations associated with the boundary-layer reattachment.

\* i. e. 13 per inch.

Perturbations introduced by the model itself were also considered : i.e. by irregularities in the leading edge and the step edge and surface roughness.

A test with a longer running time, was made using the S-1 model coated with azobenzene, to see if any perturbations could be detected on the model surface upstream of the step. Although they could hardly be seen, very thin and irregularly spaced striations appeared, as is shown on figure 4b.

Further attention was thus given to irregularities in the leading edge. A new flat plate was machined in a different material, but with the same leading edge thickness. However, perturbations with the same wave length were observed in the reattachment region of the flow. An optical examination of the leading edge showed no particular waviness.

Thus, it was concluded that, if irregularities in the leading edge were responsible for the flow perturbations :

- 1° these were extremely small and non observable
- 2° such irregularities, as did exist, were necessarily different in the two edges tested, although the observed perturbations were of the same wave length.

The same conclusion could be stated for any irregularities in the edge of the step itself.

The increased surface roughness owing to the azobenzene crystals was found to be without any influence on the phenomenon. With a clean model, the same flow perturbations were detected by transverse total head surveys made in the reattachment region, as will be seen in the following section.

The effect of incidence of the model was also checked. The same flow perturbations were observed at angles of attack equal to  $+ 1^{\circ}30'$  and  $- 1^{\circ}30'$ .

#### DETAILED SURVEY OF THE FLOW ON MODEL S-1

A transitional type of reattachment existed on model S-1. Although transition was located close to the reattachment (see figure 3b), the flow was affected by changes in free-stream Reynolds number. This is shown on figure 5, where the total head measured on the model axis is plotted against the distance from the step base, for two different values of the stagnation pressure. In these surveys, the probe was moved in contact with the model surface.

In order to check the validity of the results given by the sublimation technique, a detailed survey of the flow around model S-1 was made with a total head probe.

Surveys were made in a spanwise direction (y-axis) using a total-head probe located against the model surface. The probe was supported through a metallic plate replacing the test chamber window and was moved with a micrometric device. Special care was taken to avoid the nose of the probe moving away from the model surface during the displacements; otherwise, the reading would have been affected by the significant total pressure gradient normal to the surface.

A fixed total-head probe was located on the centre line of the surface, at the same distance from the step base as the moving probe. Pressure differences were measured with a Staham strain-gauge pick-up and a Brown recorder.

Transverse surveys were made at a given stagnation pressure ( $p_0$ ) at various distances (x) from the step, in the reattachment region of the flow, where three-dimensional phenomena had been previously observed with the sublimation technique.

For a few surveys, pressures were recorded for a probe moving both away and towards the side wall, in order to check the validity of the results. No differences were observed within the accuracy of the measurement which was evaluated to be  $\pm 3$  mm of water.

The results obtained are shown on figure 6 for different values of the distance (x) from the step base at a free-stream stagnation pressure of 220 mm Hg  $\pm 6$  mm.

It was concluded that important pressure changes were present in a spanwise direction. The maximum total-head pressure gradient was found to be equal to about 20 mm of water per millimetre, which has the same order of magnitude as the maximum axial total-head pressure gradient given on figure 5.

Maximum and minimum pressure differences were found to be very irregular. However, the distances between the peaks on the pressure curves were about equal. The number of these peaks was found to be in agreement with the number of striations detected on the model surface with the sublimation technique.

Thus, there seemed to exist a definite wave length for these spanwise perturbations.

It was also concluded as shown by figure 6, that pressure changes were present in the whole reattachment region, from the reattachment line (located at about  $x = 65$  mm, as indicated by the oil-film technique) to the maximum downward position of the probe obtained with the measuring device. This position was in the region of constant static pressure, as measured on the model axis.

In the reversed part of the flow, the total-head probe was not correctly set, being directed against the free-stream; no attempt was made to detect any pressure changes.

Figure 6 shows that there was no noticeable change in the pattern of the span-wise perturbations as the flow moved towards the trailing edge of the model; however, some attenuation of the pressure peaks was obtained. Figure 7 is a schematic representation of the complete set of measurements, where this result can be better observed. Also shown on figure 7 are the azobenzene striations taken from the picture of figure 8 which is a full-scale enlargement of a portion of figure 3a (included in the picture is a 5 mm strip scale).

A transverse survey was also made at 8 mm upstream of the step, in order to check if any pressure changes were present in the boundary-layer before it passed over the step. The total head was found to be constant within the accuracy of the measurements.

For comparison, a transverse total head survey was made at a given distance from the step base ( $x = 60$  mm) on model S-3, having the same leading edge as model S-1, but a smaller step.

Pressure changes, shown on figure 10, were found having about the same wave length as for the 15 mm step. As is shown on figure 10, the same type of irregularities were present for two-different free-stream stagnation pressures.

This seems to indicate that the wave length of the flow perturbations is not affected appreciably by the step height.

From an approximate count of the number of span-wise striations on the various models coated with azobenzene, it was found that the wave length was larger for  $L = 225$  mm (see figure 1) than for  $L = 120$  mm. To the extent that any conclusion can be formed from the results obtained with only two leading-edge lengths, it appears that the wave length of the flow perturbations increases with increasing thickness of the boundary-layer immediately upstream of the step.

## OPTICAL ANALYSIS OF THE FLOW PERTURBATIONS

An attempt was made to visualize the three-dimensional perturbations of the flow by using a conventional shadow system. A transparent plexiglass surface was used instead of the metallic plate which formed the wall downstream of the step and the model was mounted vertically in the supersonic test section. With such an arrangement, it was possible to use the shadow system.

No indication of three-dimensional perturbations in the flow was obtained. It was possible that the density of the flow and the strength of the perturbations were too small to see anything on the screen. No picture was taken.

The optical quality of the plexiglass was not good enough to make use of the schlieren system.

## RESULTS OBTAINED BY OTHER INVESTIGATORS

As far as the author is aware the three-dimensional perturbations observed in these tests have not previously been observed under the same flow conditions. However, in a few cases, three-dimensional perturbations have been reported, specially in incompressible flow.

It is the purpose of this section to summarize experiments where similar perturbations might possibly have occurred.

Dean Chapman et al. have carried out a very interesting and detailed investigation of separated flows in supersonic streams, using backward facing step models with a leading edge thickness of 0.13 mm (0.005 inch), (reference 1).

The flow over the centre portion of the models was judged by the authors to be essentially two-dimensional because :

1° several pressure orifices located spanwise 2 inches off-centre revealed only small variations of static pressure;

2° the pattern formed by oil film on a model surface was normal to the flow direction over a sizable centre portion of span;

3° modifications of the end plates of the models had no effect on midspan pressure distributions.

However, according to the results of the present investigation, three-dimensional perturbations could have been present in Chapman's work, although not detectable by any of the above three tests.

As is shown above, the wave length of the flow perturbations decreases as the length (L) of the leading edge plate (or the boundary-layer thickness) is reduced from 225 mm down to 120 mm. Chapman used a smaller value of L (equal to 51 mm), and the resulting wave length would be correspondingly smaller - possibly about 1.5 mm.

Such perturbations, if they existed, would be hardly detectable, except possibly by surveying the flow with a very small total-head probe and this Chapman did not do.

An investigation of the base pressure behind wedges at Mach numbers 2 and 3, in the laminar and transitional regimes, was reported by Charwat and Yakura (reference 2).

Trailing vortices with axes parallel to the main flow were observed by the authors, over a portion of the span. They were explained in terms of non-uniform mixing rate in the wake, due to early transition. The model base was coated with kerosene-lampblack suspension and 4 pairs of trailing vortices were seen to be shed from the base. The authors were able to observe the circulation for a short time before the kerosene evaporated. A pair of vortices was seen at either end of the span and two other pairs were located symmetrically on each side of mid-span. The phenomenon was consistently reproducible at  $M = 2$ . The base pressure was found to be uniform across the span. No transverse total-head survey was made in the wake.

During a research, carried out by the author, on turbulent boundary-layer reattachments, three-dimensional perturbations of the flow were observed. The research (not published) was made in a blowdown supersonic wind tunnel at the Forrestal Research Center - Princeton University - under U.S. Air Force Contract.

The phenomenon was observed by spreading an oil film on the surface downstream of a backward facing step located in the nozzle wall. Figure 11 shows schematically the oil pattern, visually seen on both sides of the reattachment line. Alternate regions of high and low speeds of the oil were present. No explanation was given to the phenomenon which might be different from the present one, revealed only for the laminar and transitional types

of reattachment.

The same technique was used in the present investigation to detect the position of reattachment. Although, no detailed oil pattern could be seen, alternate zones of high and low speeds of the oil were very roughly observed locally.

Three-dimensional flow perturbations have been considered, in the case of an incompressible flow, both theoretically and experimentally. Görtler has evolved a theory on three-dimensional perturbations and Hama et al. have observed three-dimensional vortices in a water tank.

Görtler has made a theoretical investigation of the stability of incompressible boundary-layers on concave walls, relative to small disturbances, in the shape of vortices, whose axes are parallel to the main flow (references 10 and 11). Friction was duly allowed for in the calculations and linearized equations were derived for a wall curvature large compared to the boundary-layer thickness and for disturbance velocities small with respect to the basic flow velocity. The Reynolds number at which vortices of the particular type can exist for the first time without decaying again, was found to be :

$$\frac{V_0 \delta}{\nu} = 16 \sqrt{\frac{R}{\delta}}$$

It involved vortices whose wave length ( $\lambda$ ) was given by :

$$\lambda = 5\delta$$

where  $\delta$  was the boundary-layer thickness  
 $R$  was the radius of curvature of the surface.

A very interesting investigation has been made on transition from laminar to turbulent flow in a water tunnel by Hama et al. at the University of Maryland (reference 12). A flat plate was moved at constant velocity in a water tunnel and two-dimensional vortices were obtained either by a trip wire located on the surface or by a stepdown. A special device was used to visualize the vortices.

It was found that a two-dimensional discrete vortex line has a strong tendency, in shear flow, to form a three-dimensional vortex loop, with a marked transverse wave length. The wavy vortex pattern occurred consistently along almost the same geometrical location. Tests were made in order to determine whether the three-dimensional waviness of the vortices was

possible in the absence of a probable leading edge effect. It was also shown that concave streamline curvature, obtained upstream of the trip wire was not responsible for the three-dimensional perturbations. This was proved when a similar phenomenon was observed with a stepdown for which no such curvature existed.

A comparison was made between the measured wave length and the theoretical wave length of Görtler vortices. It was concluded that the Görtler instability theory could not explain the wave length observed.

The authors reached the conclusion that undetectable randomly distributed imperfections in the model, such as in the wire diameter, could modulate a three-dimensionality of the flow, having a selective wave length, depending upon the mean flow velocity distribution.

The formation and development of vortex loops were found to be essential features preceding a turbulence spot.

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A P P E N D I X

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AXIAL VORTICITY IN STEADY IDEAL FLUID FLOW

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by Jean SMOLDEREN, TCEA.

The equation of motion for steady isentropic flow of an ideal fluid,

$$(\vec{V} \text{ grad})\vec{V} = - \text{grad } i \quad (1)$$

can be written :  $\vec{V} \times \text{rot } \vec{V} = \text{grad } (V^2/2 + i) = \text{grad } h \quad (2)$

taking into account a well known vector identity; h represents the total energy per unit mass of fluid.

Equation (1) yields the component of vorticity normal to the stream lines :

$$(\text{rot } \vec{V})_n = - (\vec{V} \times \text{grad } h)/V^2 \quad (3)$$

We therefore write :

$$\text{rot } \vec{V} = k \rho \vec{V} - (\vec{V} \times \text{grad } h)/V^2 \quad (4)$$

k being a scalar.

Our aim is to obtain an expression for this quantity which is clearly the ratio of the axial vorticity (component of vorticity in the direction of flow) to the mass flow.

Taking the divergence of both members of (4), we obtain :

$$0 = \text{div } (k \rho \vec{V}) - \text{div } \left[ (\vec{V} \times \text{grad } h)/V^2 \right] \quad (5)$$

We have, by the condition of continuity :

$$\text{div } (k \rho \vec{V}) = \rho \vec{V} \cdot \text{grad } k + k \text{ div } (\rho \vec{V}) = \rho \vec{V} \cdot \text{grad } k \quad (6)$$

Furthermore, according to well known formulas of vector analysis :

$$\begin{aligned} \text{div } \left[ (\vec{V} \times \text{grad } h)/V^2 \right] &= (\vec{V} \times \text{grad } h) \cdot \text{grad } \frac{1}{V^2} + \frac{1}{V^2} (\text{grad } h \cdot \text{rot } \vec{V} \\ &\quad - \vec{V} \cdot \text{rot } \text{grad } h) \end{aligned}$$

The terms between brackets vanish as a consequence of equation (2) and the irrotationnality of gradients.

Equation (5) then reduces to :

$$\rho \vec{V} \cdot \text{grad } k = (\vec{V} \times \text{grad } h) \cdot \text{grad}(1/V^2) = (-2/V^4) (\vec{V} \times \text{grad } h) \cdot \text{grad}(V^2/2) \quad (7)$$

But as  $\text{grad}(V^2/2) = \text{grad } h - \text{grad } i = \text{grad } h + (\vec{V} \cdot \text{grad}) \vec{V}$

Equation (7) is equivalent to

$$\rho \vec{V} \cdot \text{grad } k = (-2/V^4) (\vec{V} \times \text{grad } h) \cdot (\vec{V} \cdot \text{grad}) \vec{V} = (2/V^4) \text{grad } h \cdot \vec{V} \times (\vec{V} \cdot \text{grad}) \vec{V} \quad (8)$$

The factor between brackets is the vector product of velocity and acceleration and will thus involve only the normal acceleration (Centripetal acceleration), i.e. :

$$\vec{V} \times \vec{N}(V^2/R)$$

$\vec{N}$  being the unit vector of the principal normal to the streamline, oriented towards its centre of curvature, R its radius of curvature. We may write :

$$\vec{V} \times \vec{N} = |\vec{V}| \vec{N}'$$

$\vec{N}'$  being the unit vector of the binormal to the streamline. Equation (8) becomes :

$$\rho \vec{V} \cdot \text{grad } k = (2/V)(1/R) \vec{N}' \cdot \text{grad } h \quad (9)$$

or finally, introducing the symbols  $(d/dl)$  and  $(d/dN')$  to describe differentiation along a streamline and along its binormal respectively,

$$dk/dl = (2/\rho V^2)(1/R)(dh/dN') = (1/q)(1/R)(dh/dN') \quad (10)$$

q being the dynamic pressure.

Equation (10) is a differential equation for the evolution, along a streamline, of the (reduced) axial vorticity. A useful differential form of this equation is :

$$dk = (1/q)(dh/dN') d\theta \quad (11)$$

$d\theta$  being the infinitesimal angle between the velocity vectors at neighboring points on the streamline.

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## R E F E R E N C E S

1. Dean R. CHAPMAN, Donald M. KUEHN and Howard K. LARSON  
"Investigation of separated flows in supersonic and subsonic streams with emphasis on the effect of transition".  
NACA TN 3869, March 1957.
2. A.F. CHARWAT and J.K. YAKURA  
"An investigation of two-dimensional supersonic base pressures".  
J.A.S., February 1958.
3. H.H. KORST, R.H. PAGE and M.E. CHILDS  
"A theory for base pressures in transonic and supersonic flow".  
University of Illinois TN 392-2, March 1955.
4. Dean R. CHAPMAN, William R. WINBROW and Robert H. KESTER  
"Experimental investigation of base pressure on blunt-trailing-edge wings at supersonic velocities".  
NACA TR 1109, 1952.
5. Eugene S. LOVE  
"Base pressure at supersonic speeds on two-dimensional airfoils and on bodies of revolution with and without fins having turbulent boundary-layers".  
NACA TN 3819, January 1957.
6. Luigi CROCCO and Lester LEES  
"A mixing theory for the interaction between dissipative flows and nearly isentropic streams".  
J.A.S., October 1952.
7. G.E. GADD  
"Interaction between wholly laminar or wholly turbulent boundary-layers and shock waves strong enough to cause separation".  
J.A.S., November 1953.
8. Raymond L. CHUAN  
"An investigation of shock wave-boundary-layer interaction".  
USC Report 40-201, August 1955.
9. Jean J. GINOUX  
"An investigation of supersonic laminar boundary-layer reattachments".  
US Air Force - ARDC Contract; to be published.

## 10. H. GOERTLER

"Über eine dreidimensionale instabilität laminarer Grensschichten on konkaven wänden".

Ges. d. Wiss. - Göttingen, 1940.

## 11. H. GOERTLER

"On the three-dimensional instability of laminar boundary-layers on concave walls".

NACA TM 1375, June 1954.

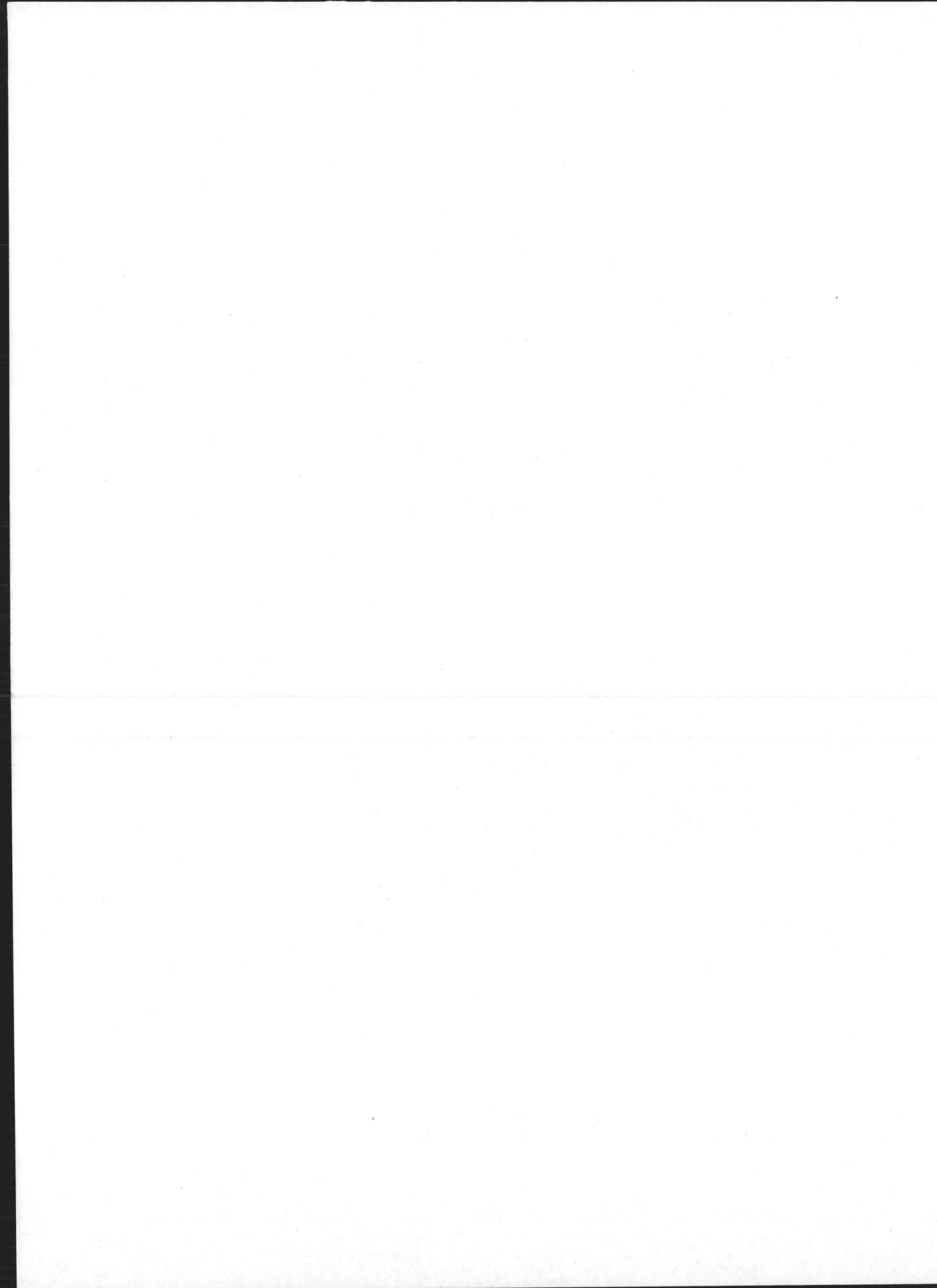
## 12. Francis R. HAMA, James D. LONG and John C. HEGARTY

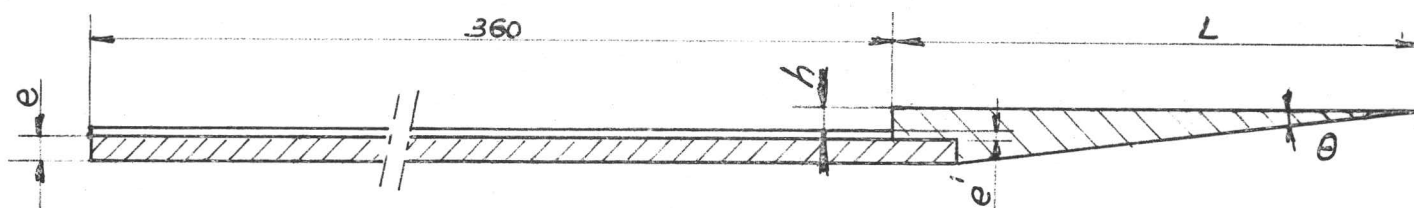
"A transition from laminar to turbulent flow"

University of Maryland - Technical Note BN-81, August 1956.

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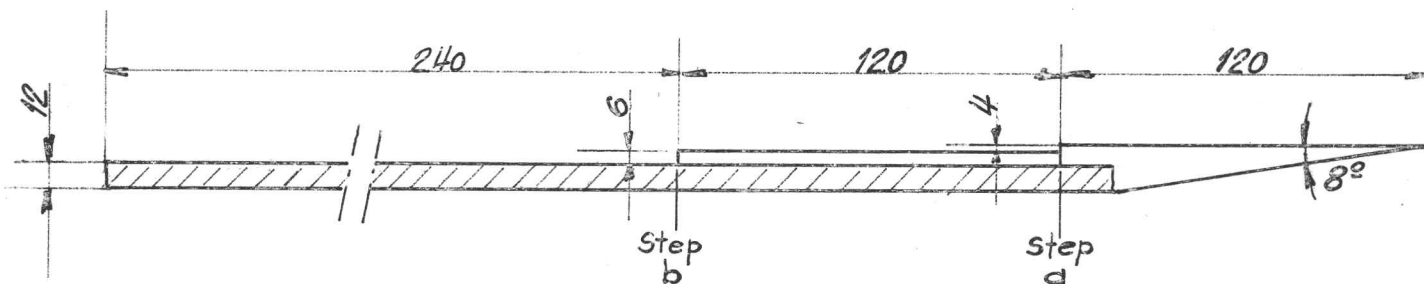
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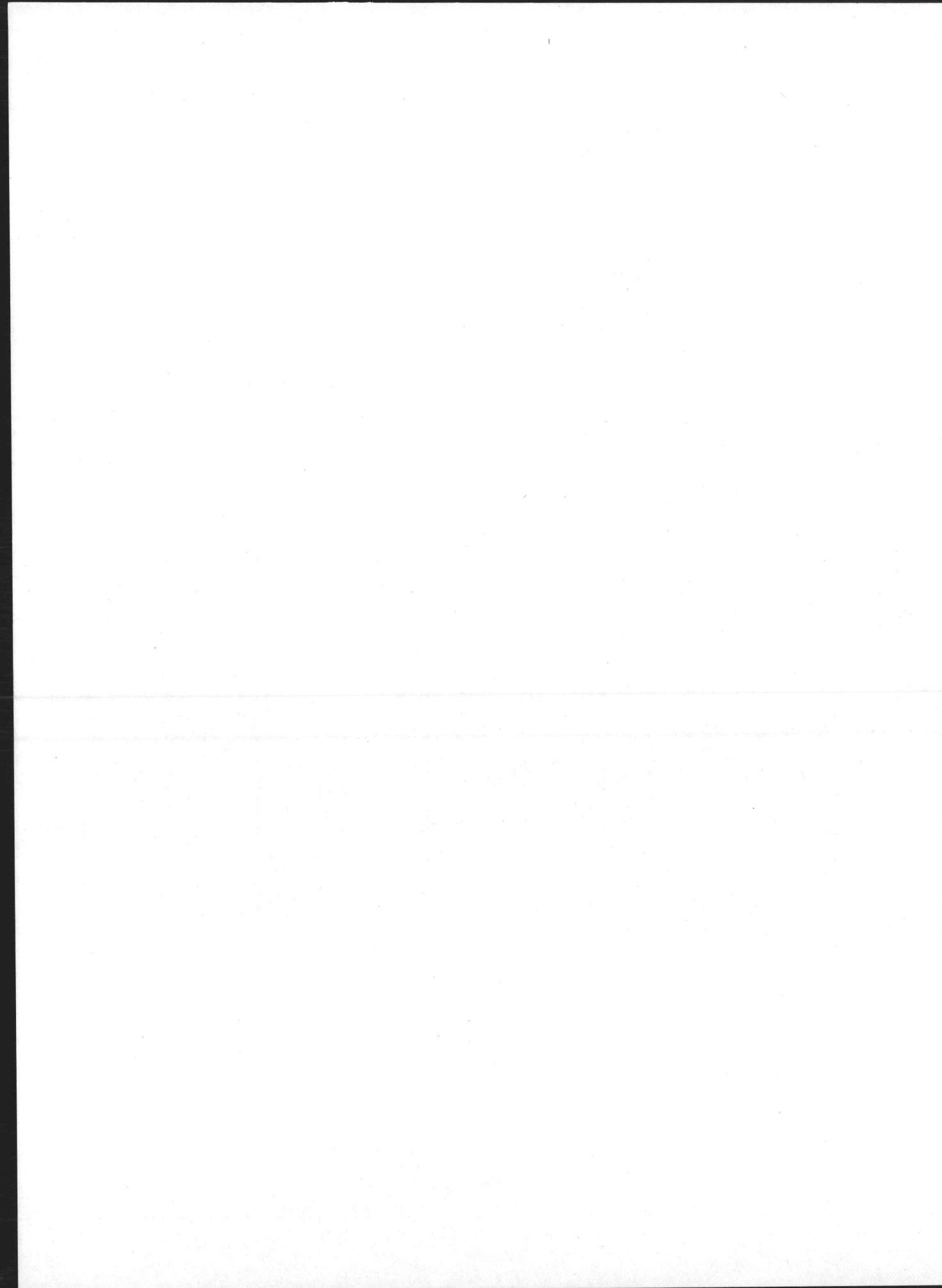
Model designation	L (mm)	h (mm)	$\theta$ (degrees)	e (mm)	e' (mm)
S-1	225	15	5°40'	7	0
S-2	120	15	8°	7	0
S-3	225	10	5°40'	12	5
S-4	120	10	8°	12	5

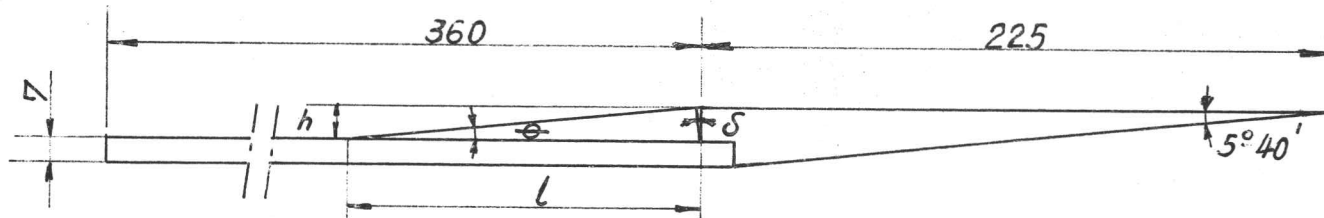
Single Step Models



Double Step Model (D S-1)

FIGURE 1 - Model Configurations and Dimensions.

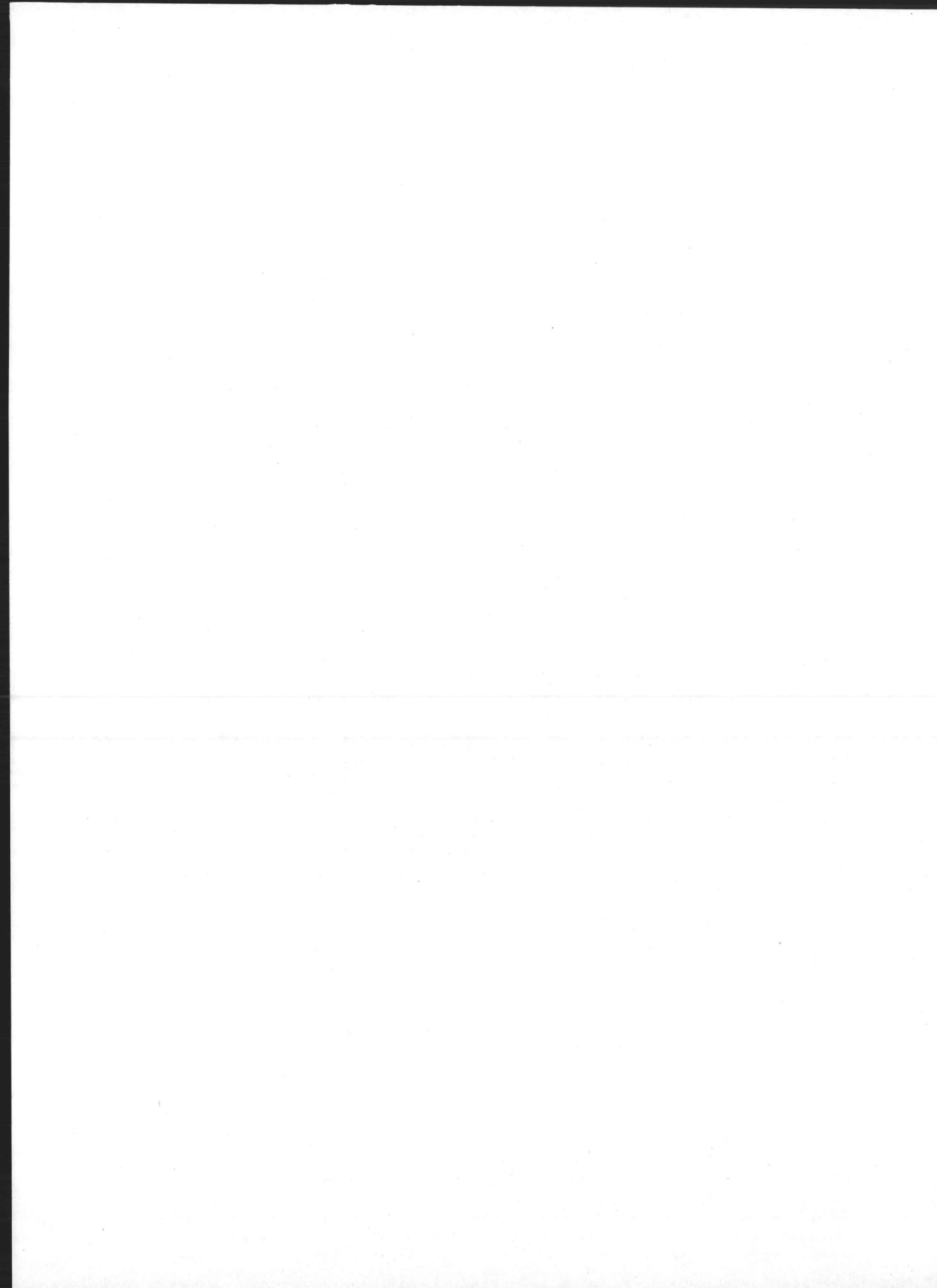




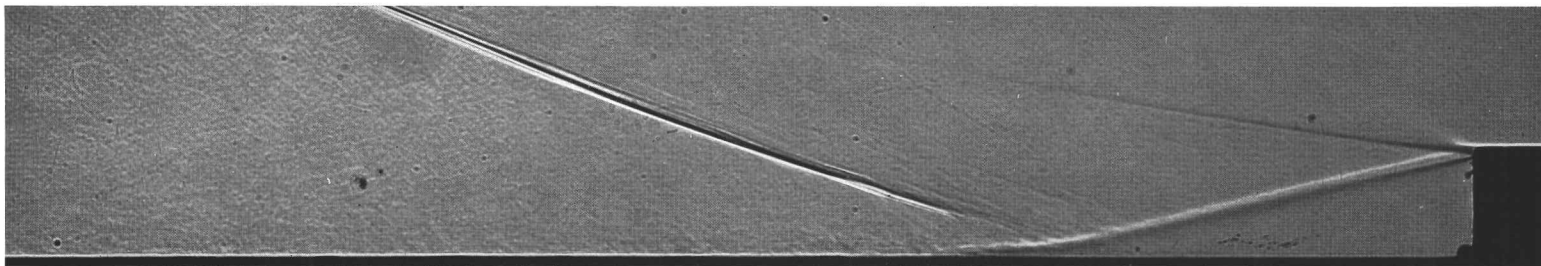
Model designation	$h$ (mm)	$l$ (mm)	$\phi$ (degrees)	$\delta$ (degrees)
W-1	15	60	14	0
W-2	15	57	15.5	15.5
W-3	15	155	5.5	0

Wedge Models

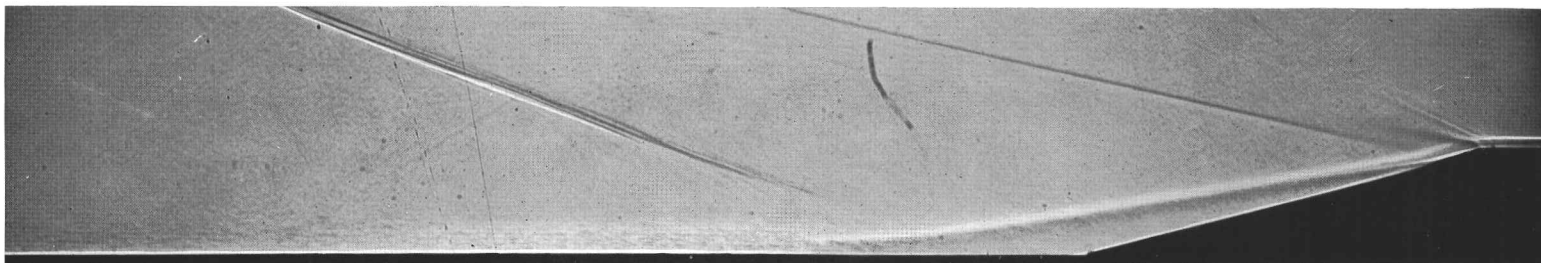
FIGURE 1 - Concluded.



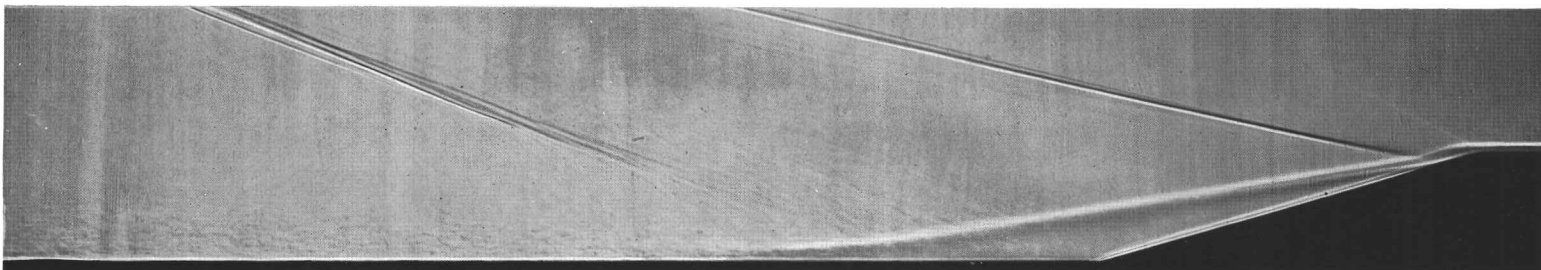
a. Model S-1



b. Model W-1



c. Model W-2



d. Model W-3

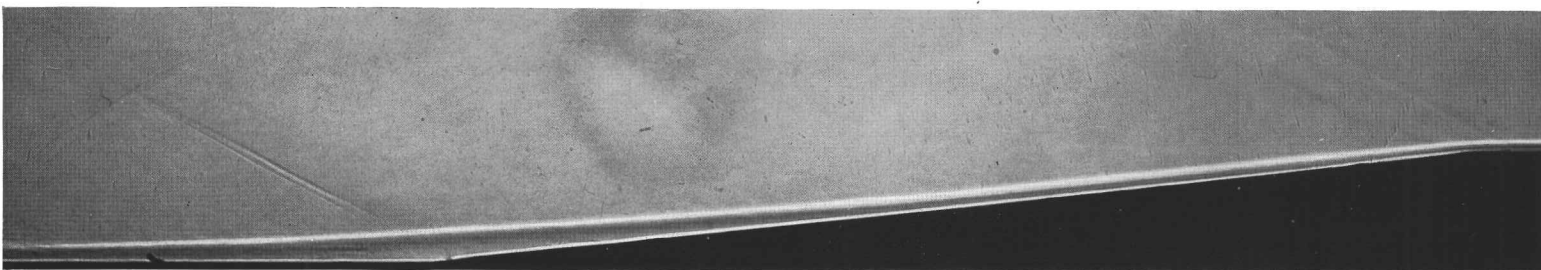
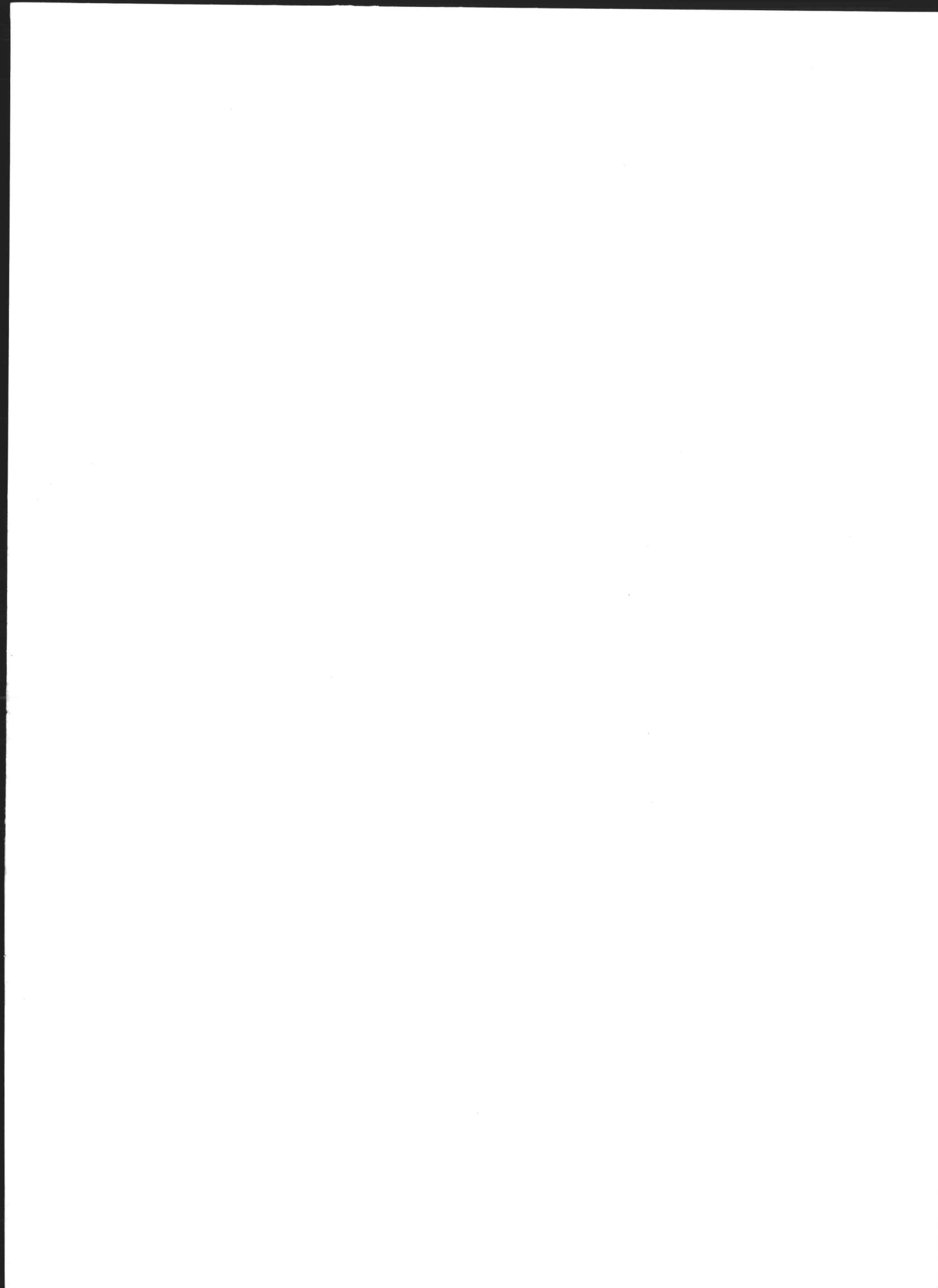
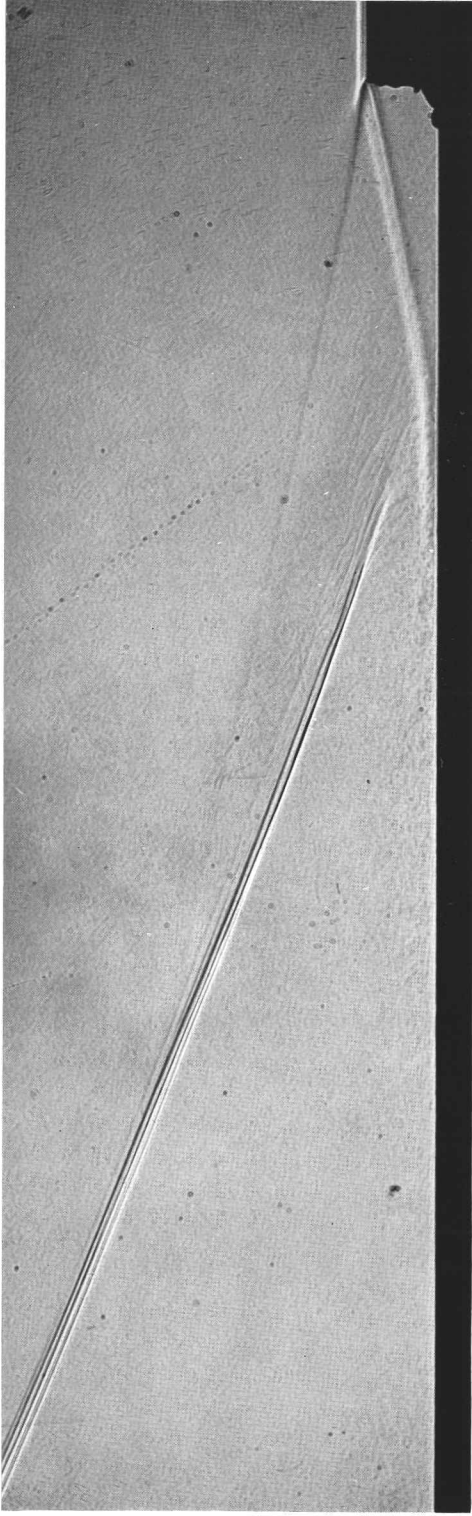


Figure 2. Shadowgraphs of the flow around different model configurations at  $M = 2.05$ . ( $p_o = 220$  mm Hg)



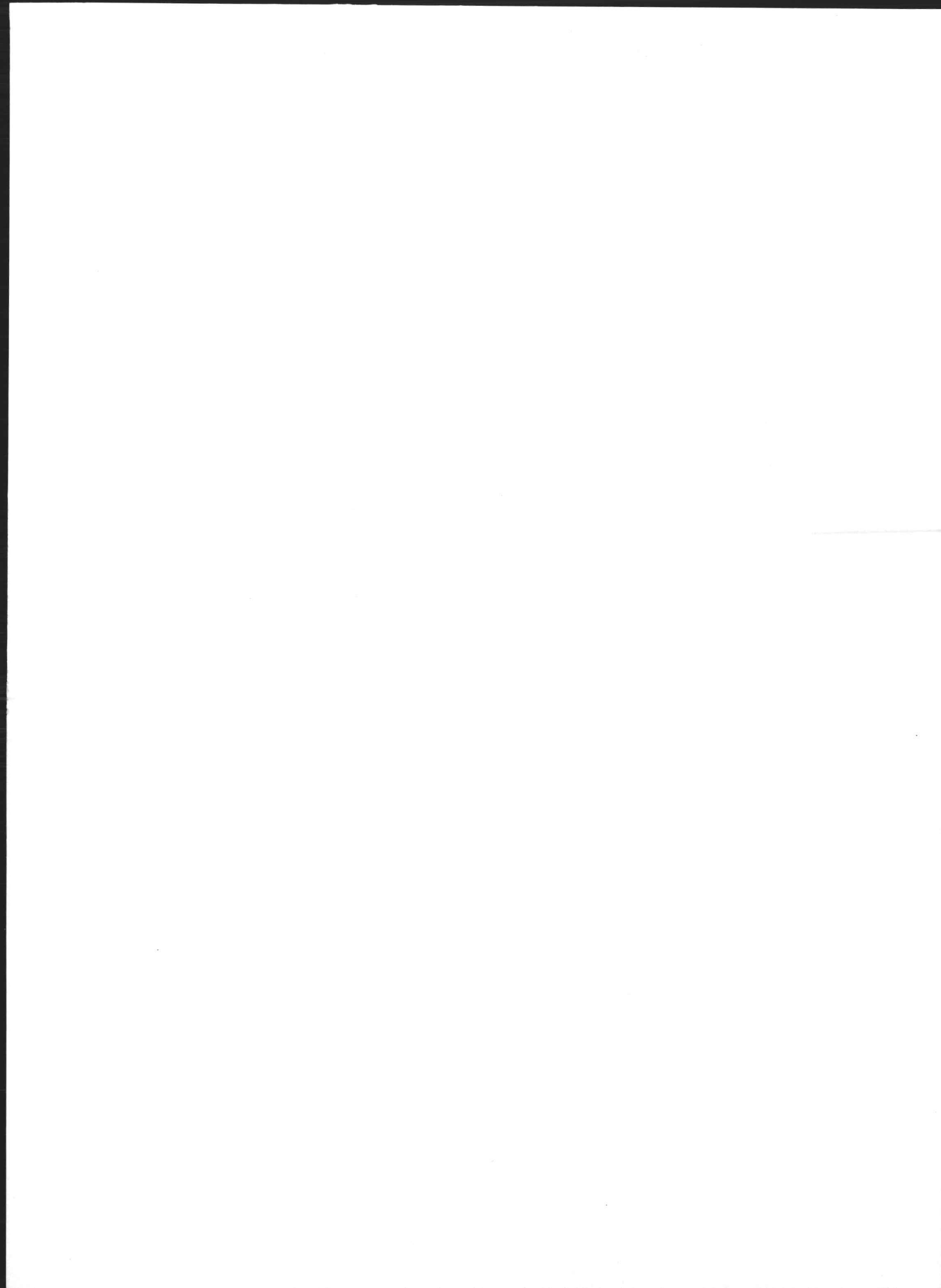
e. Model S-3

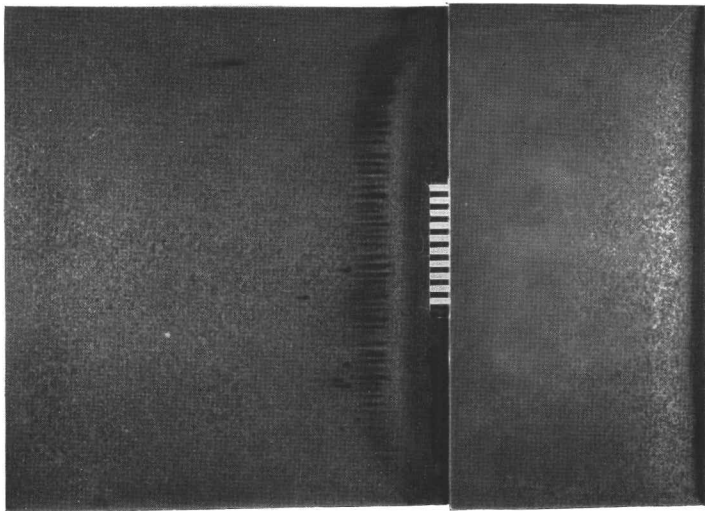


f. Two-step model



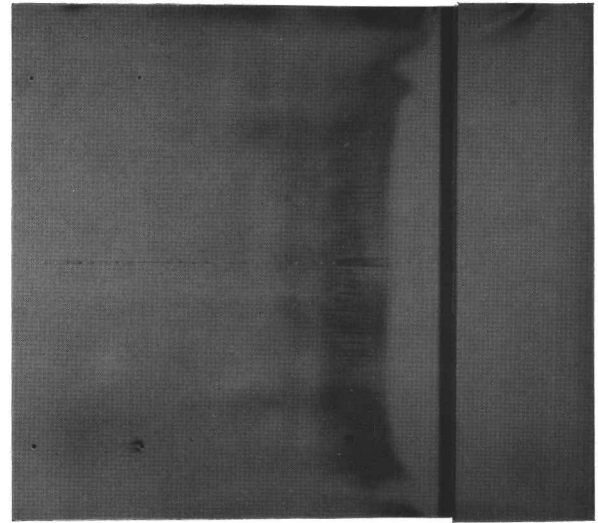
Figure 2. Concluded



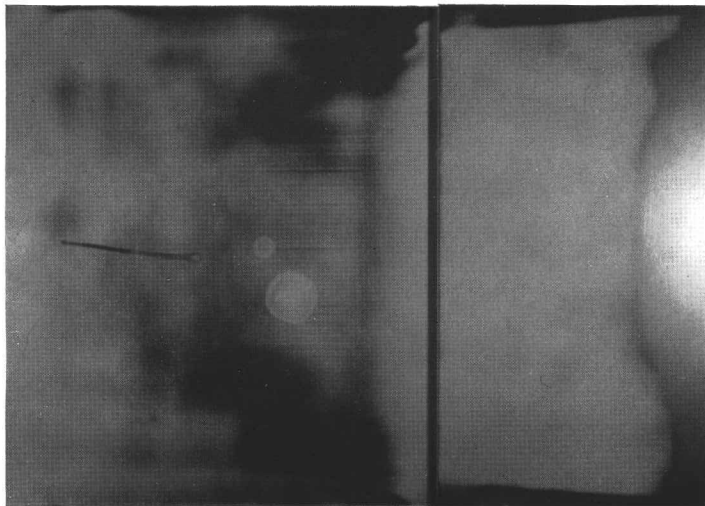


a. Model S-1

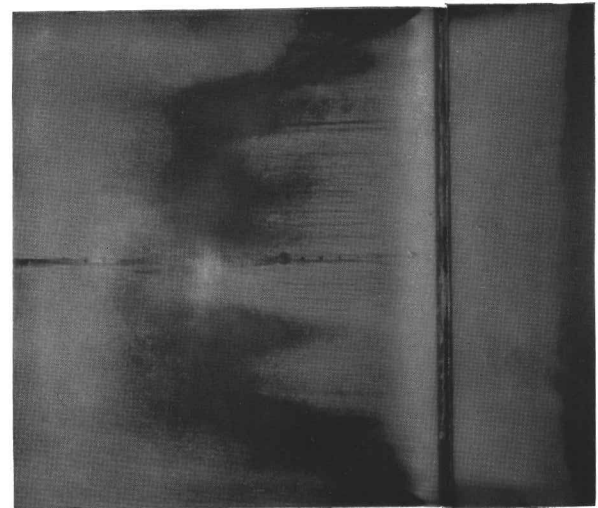
The width of each white band of the scale is 5 mm



b. Model S-2

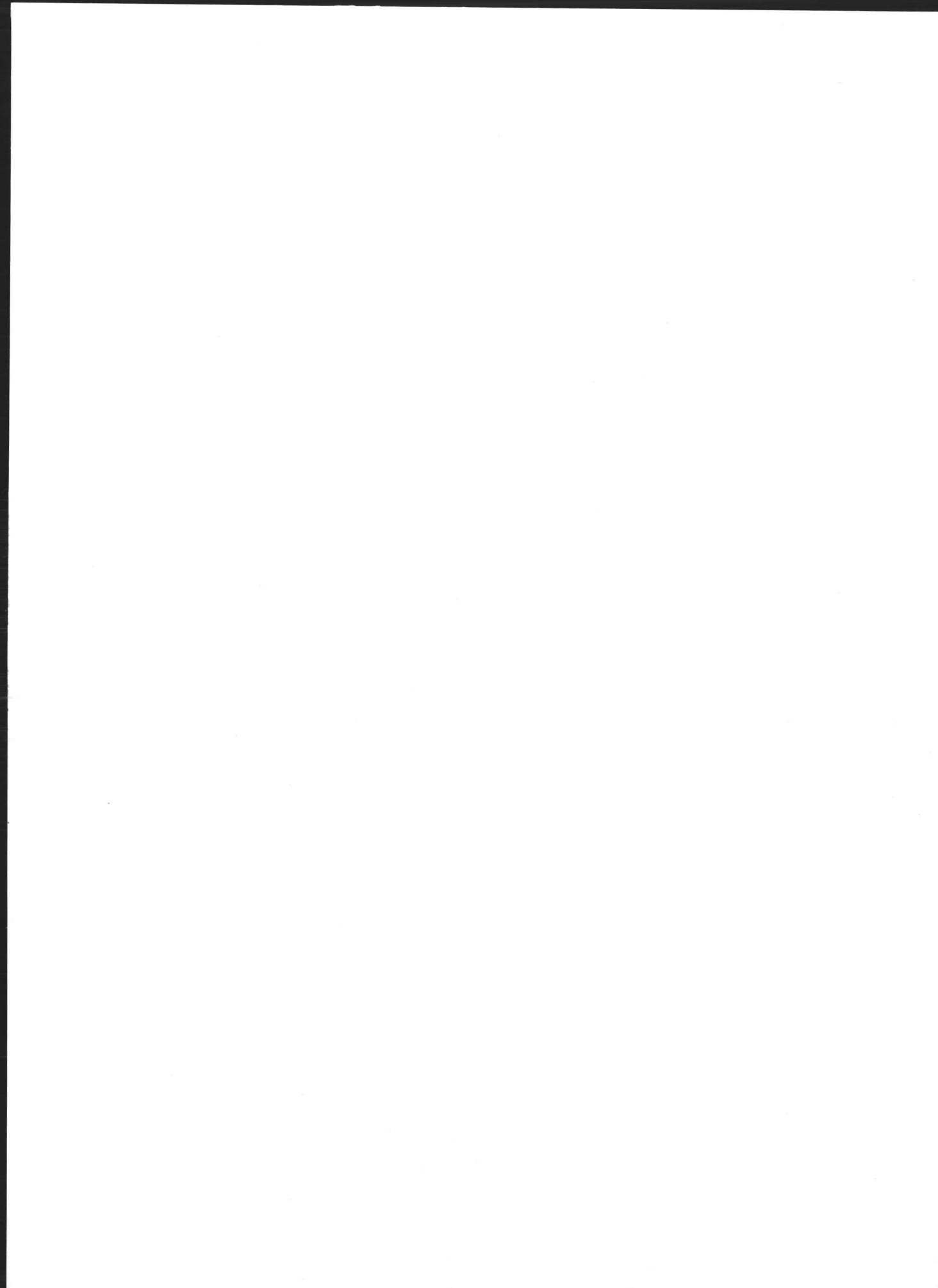


c. Model S-3

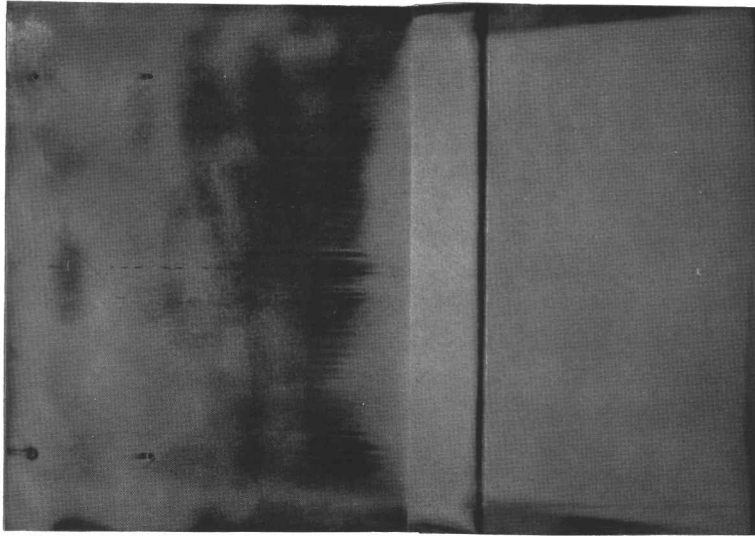


d. Model S-4

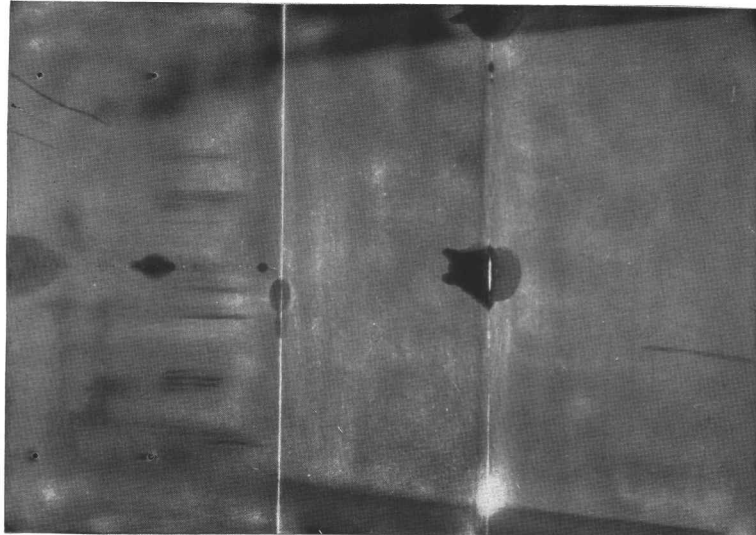
Figure 3. Flow on the model surface shown by the azobenzene.



e. Model W-2



f. Model W-3



g. Two-step Model

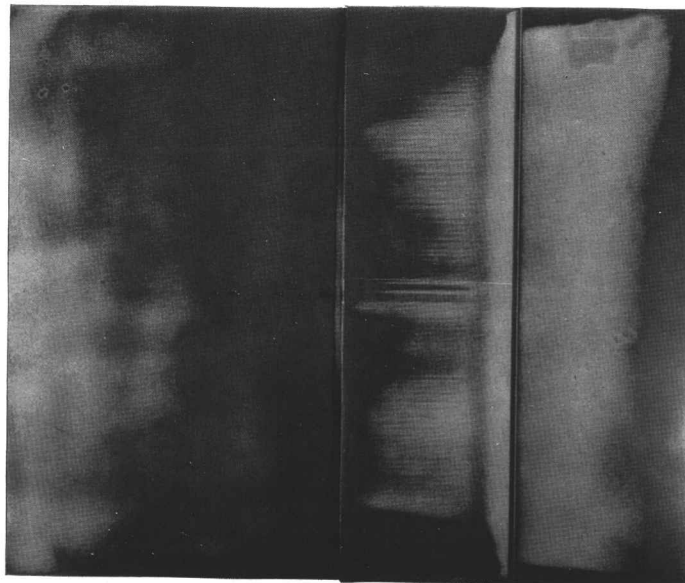
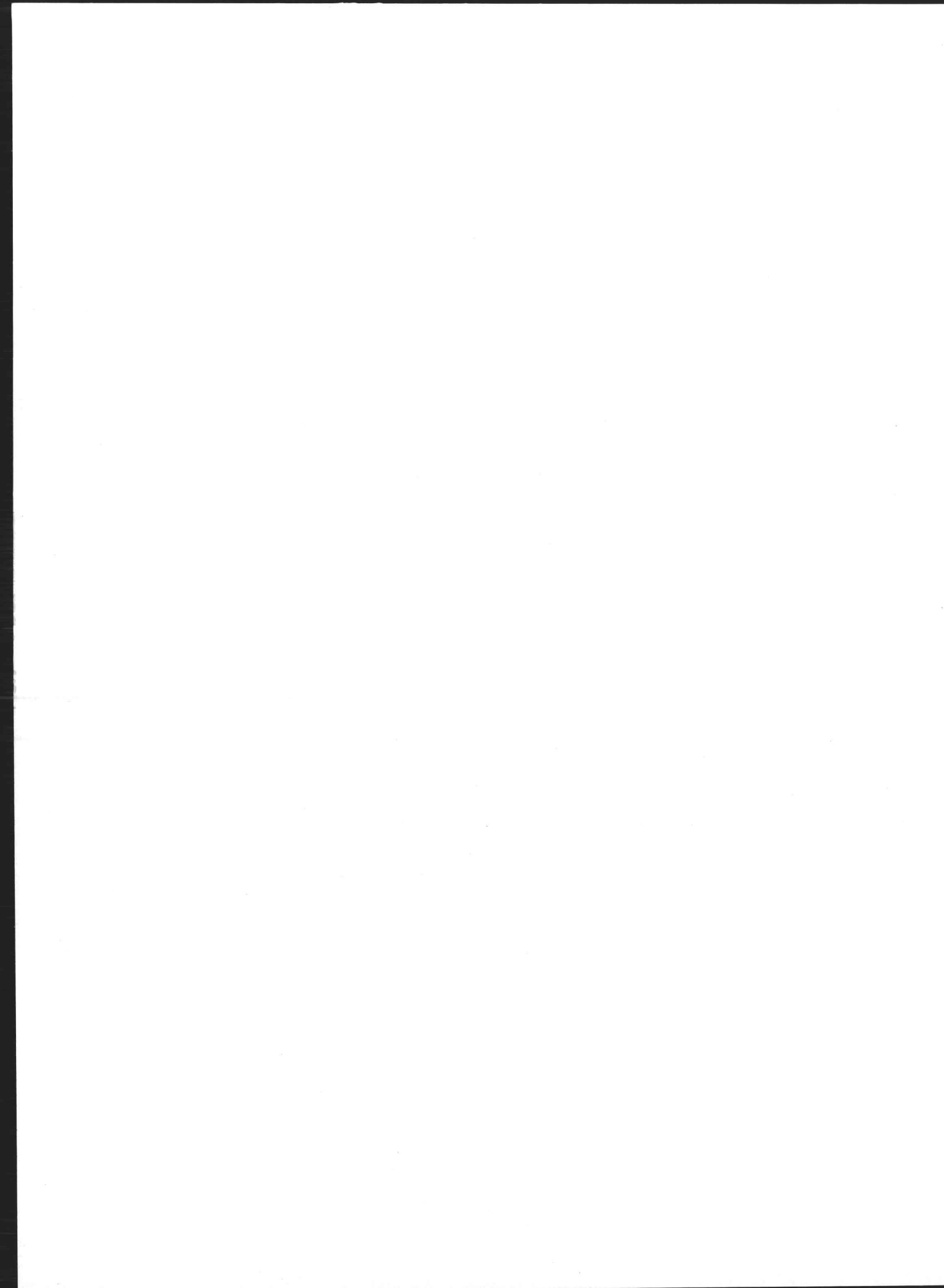
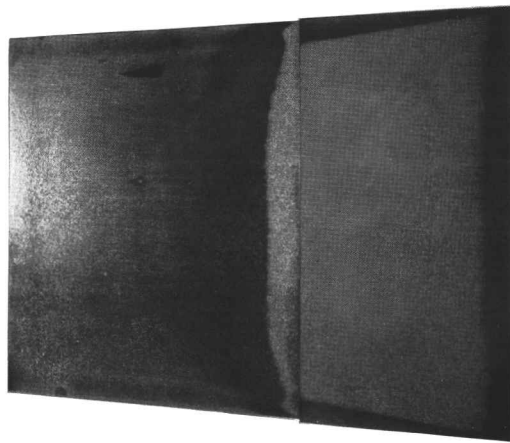
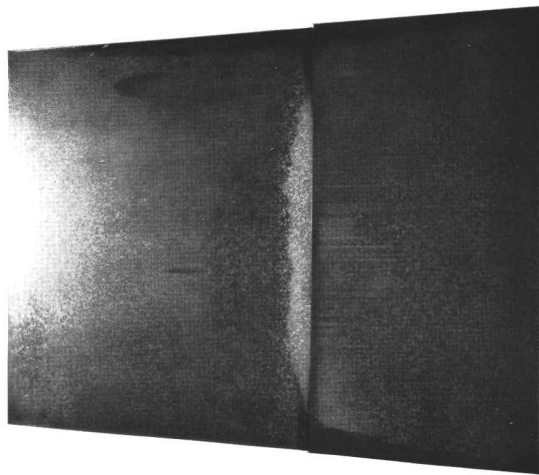


Figure 3. Concluded

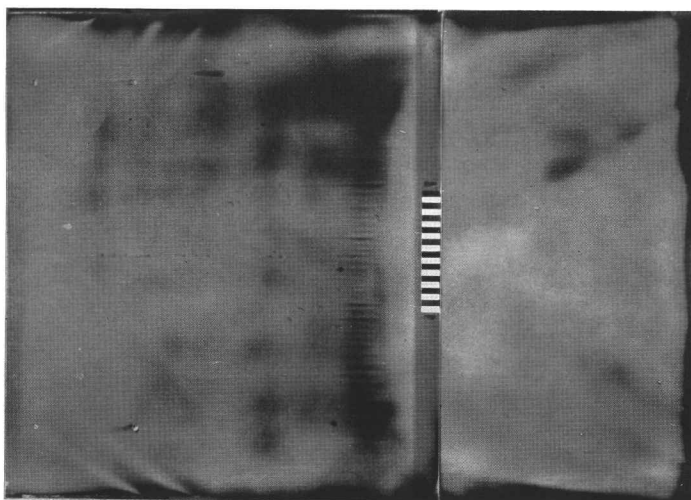




a. Striations downstream of the step



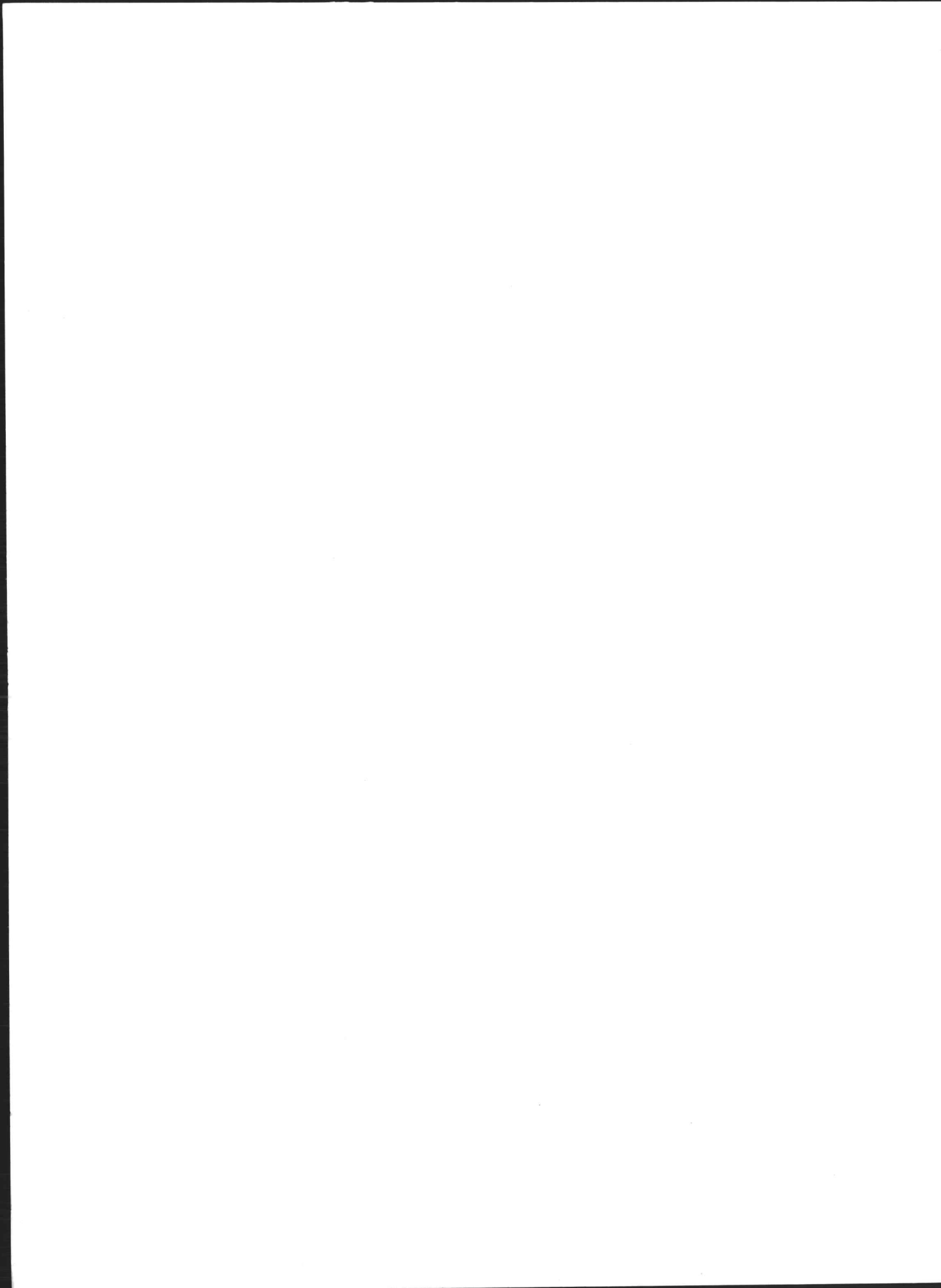
b. Striations on leading-edge flat plate

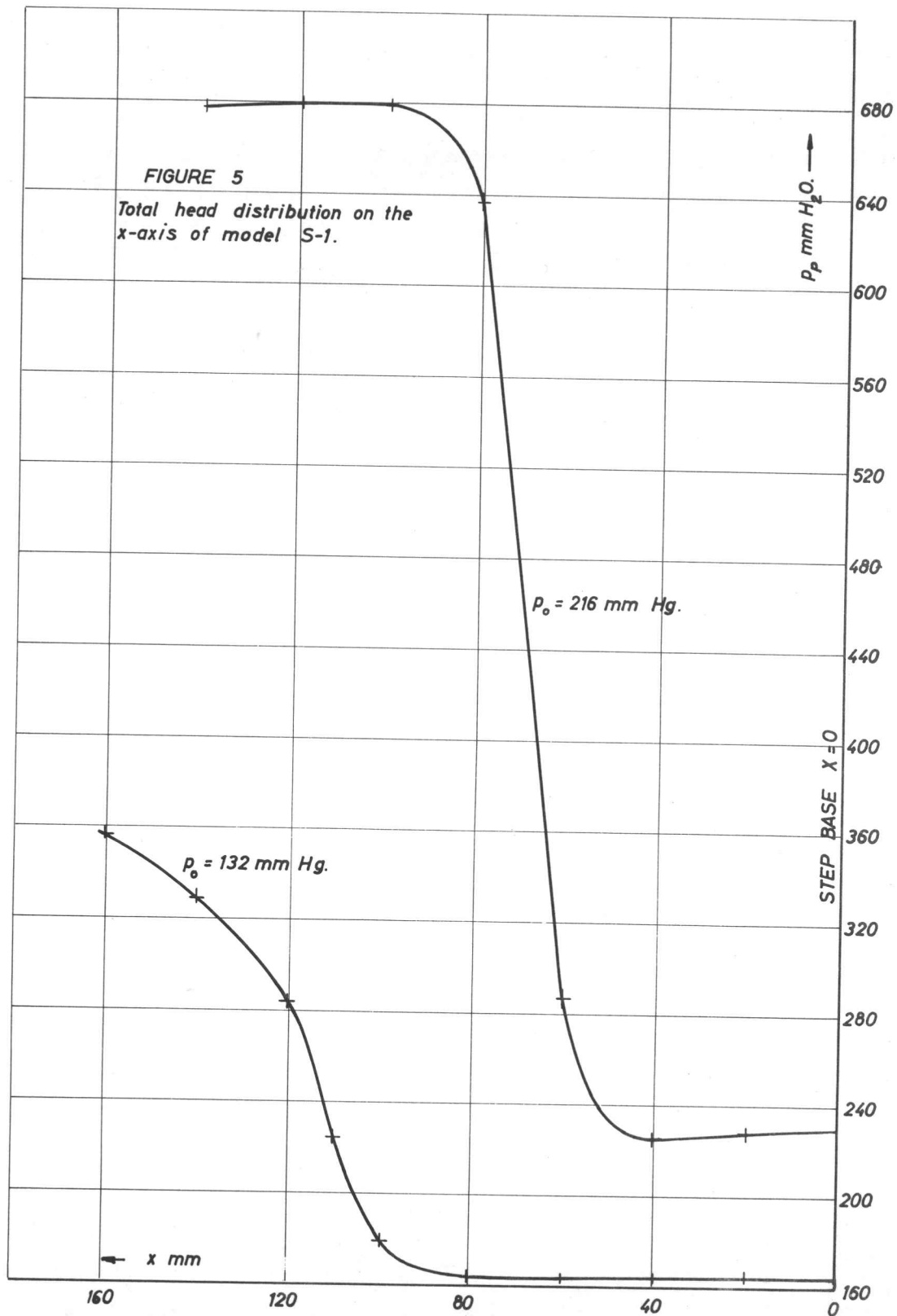


c. Vertically supported model.

The width of each white band of the scale is 5 mm.

Figure 4. Surface flow on model S-1







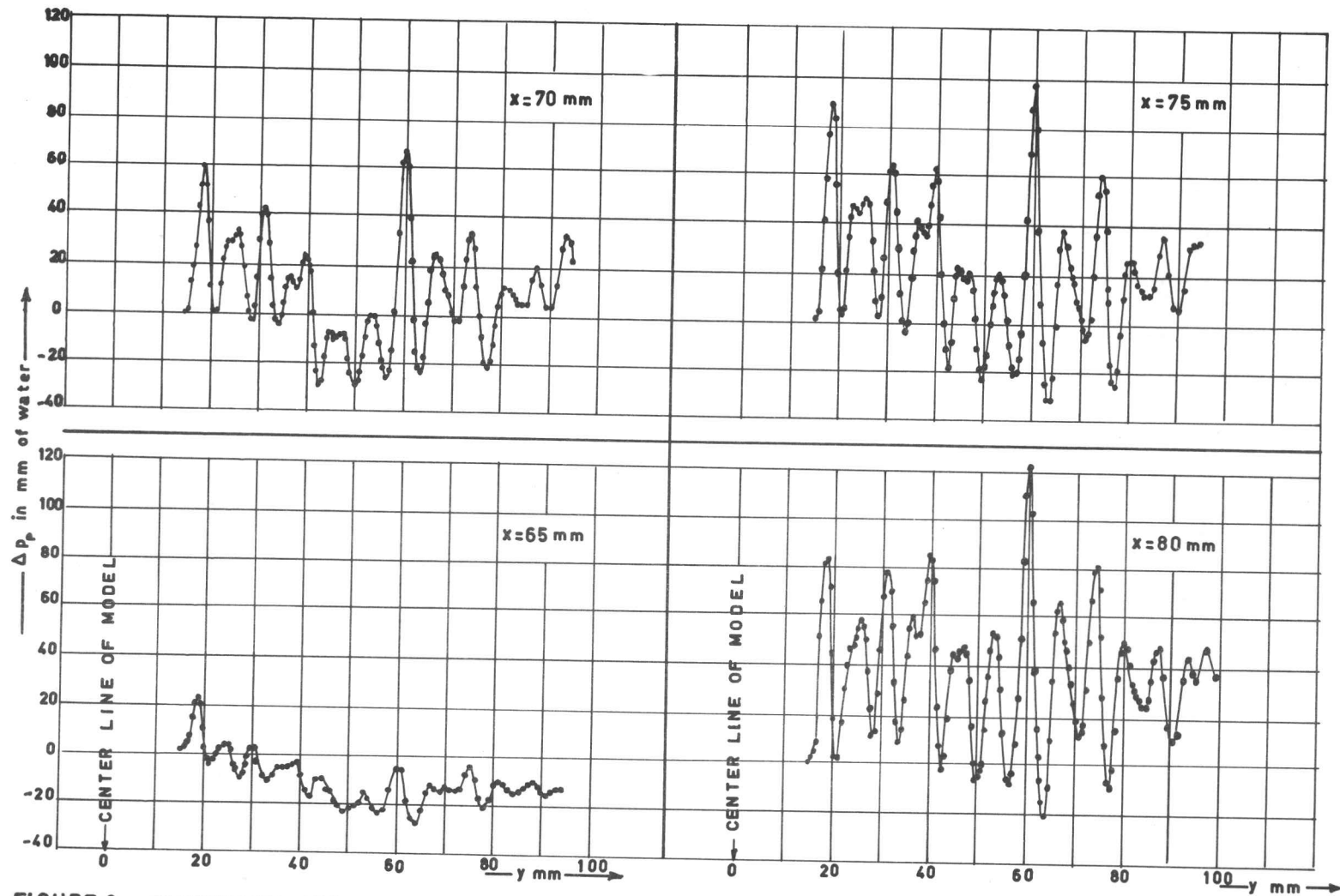
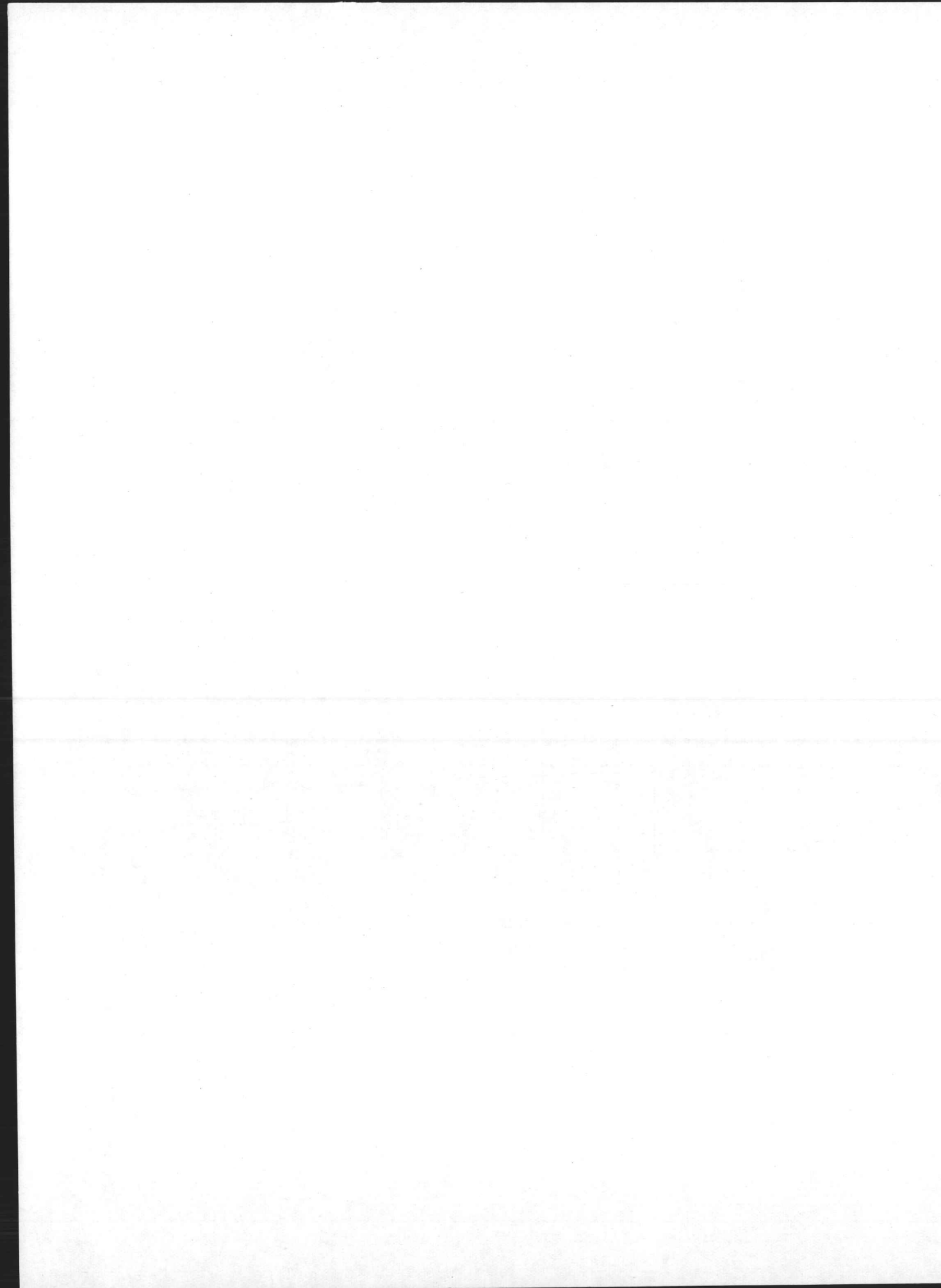


FIGURE 6 - TRANSVERSE TOTAL HEAD DISTRIBUTIONS ON MODEL H15BA1 AT VARIOUS DISTANCES FROM THE STEP BASE.



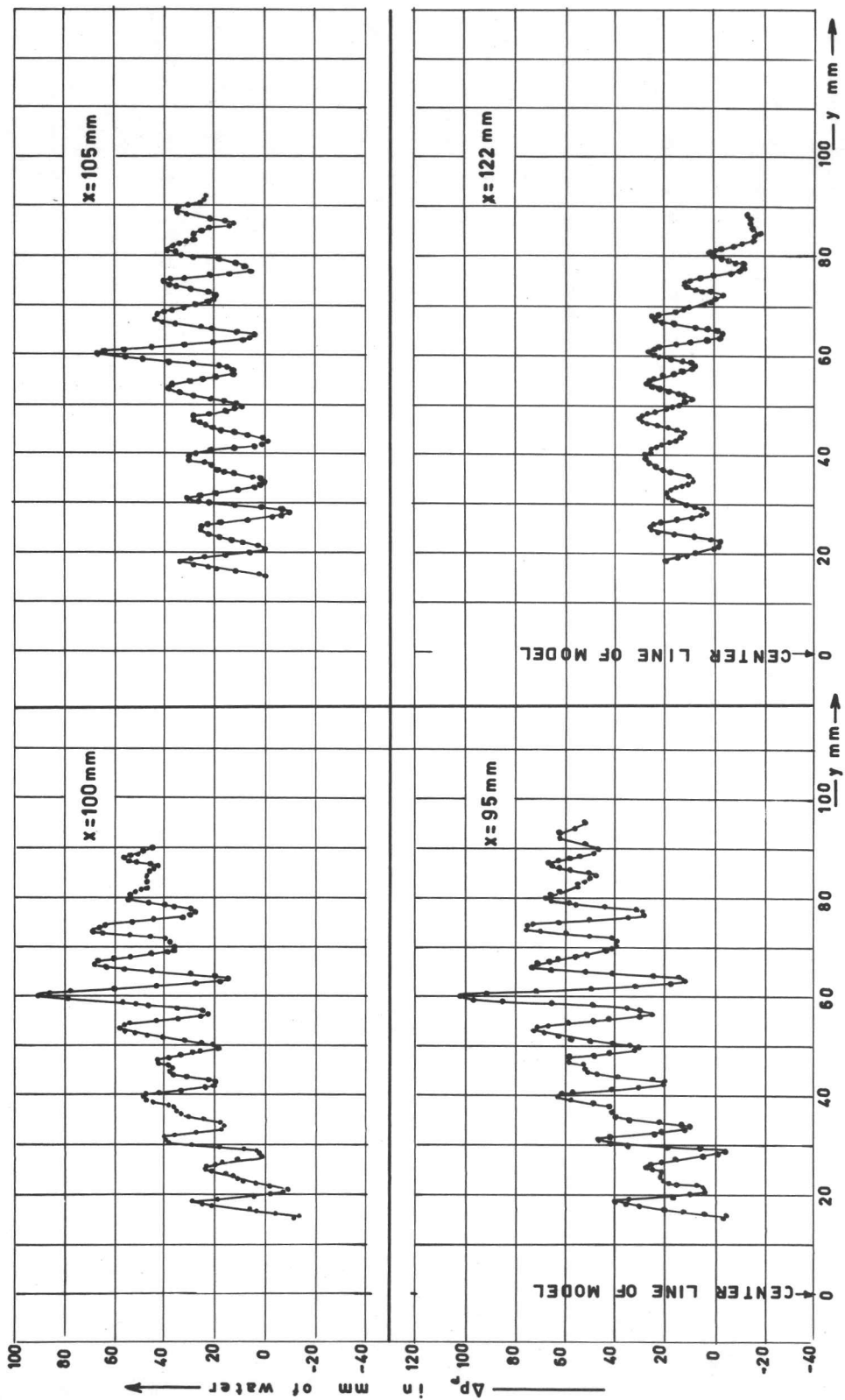
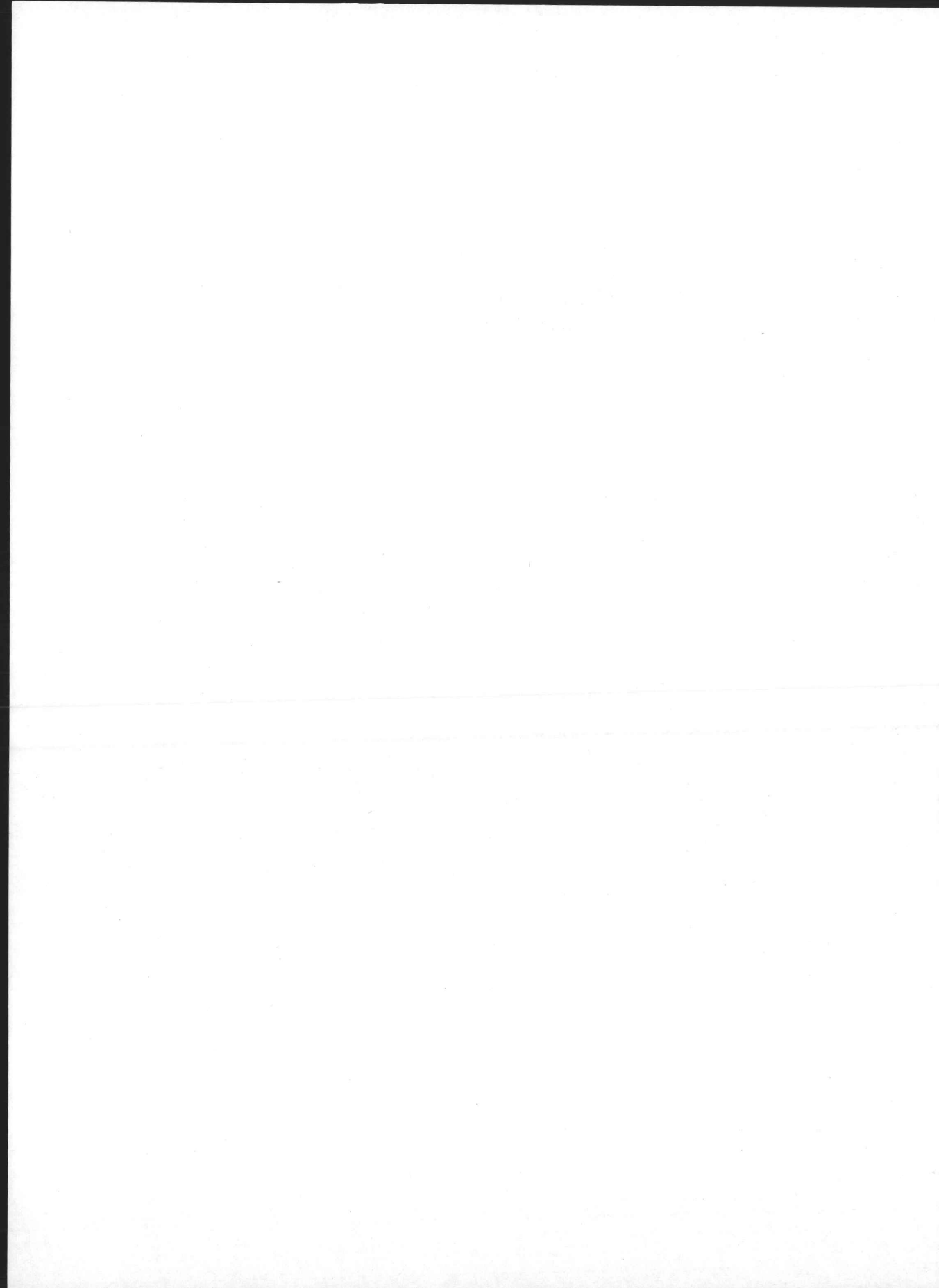


FIGURE 6 - CONCLUDED



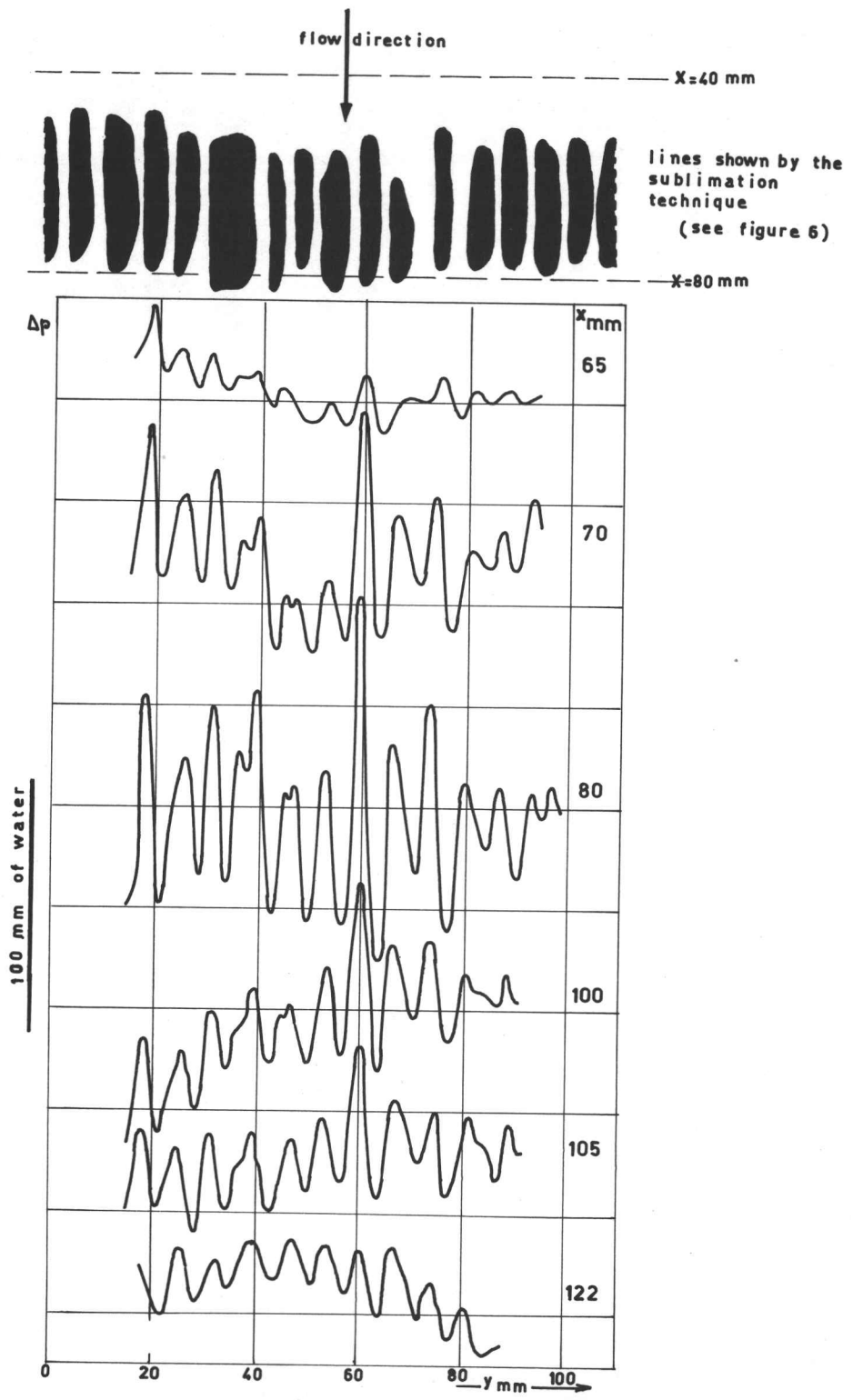
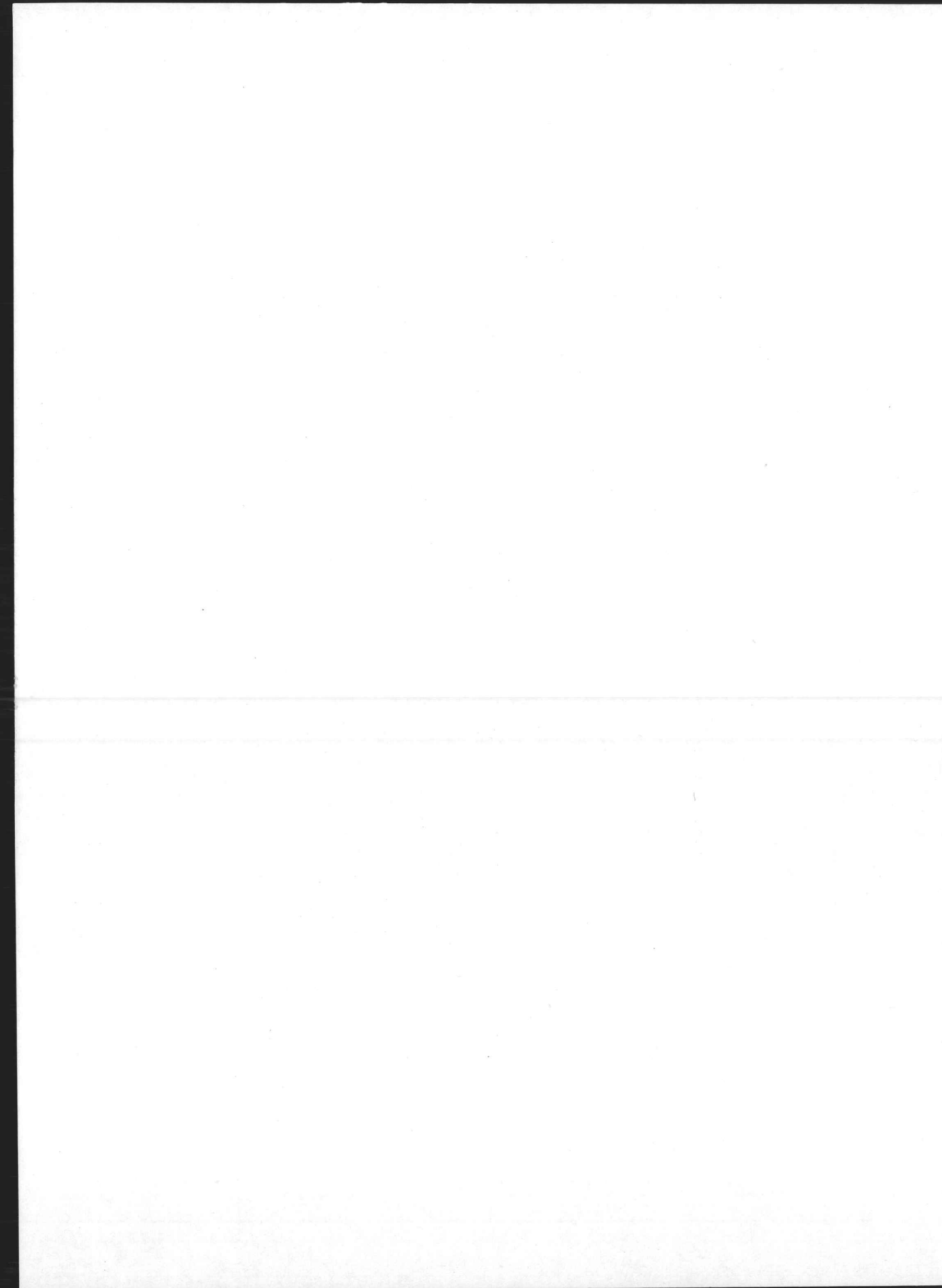


FIGURE 7 - SUMMARY OF TRANSVERSE SURVEYS



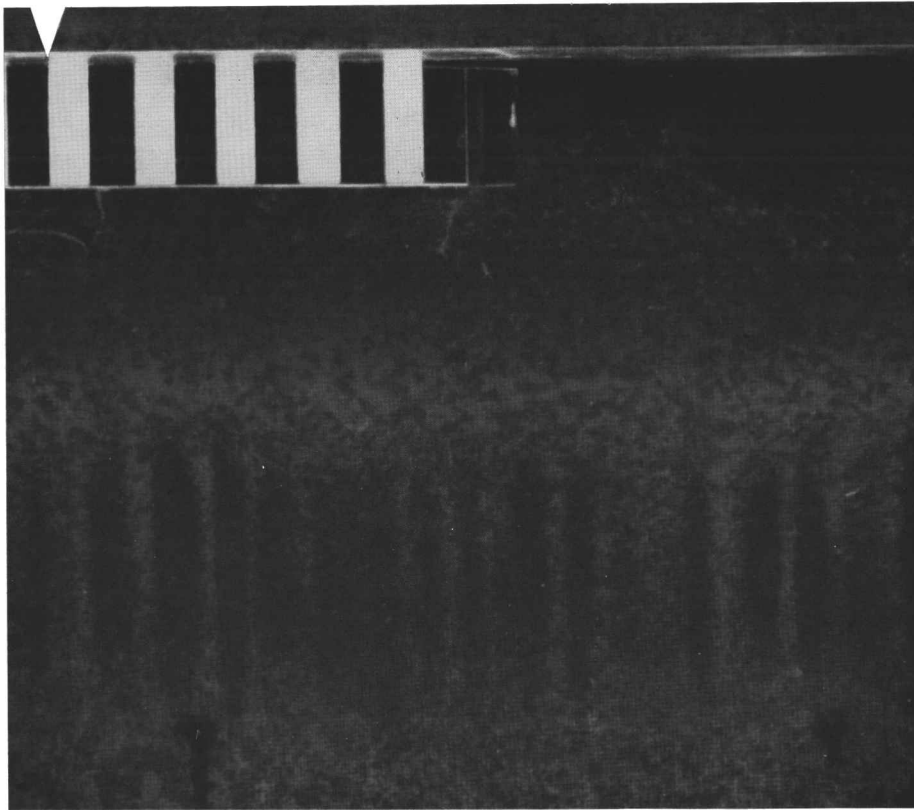


Figure 8. Full scale enlargement of a portion of figure 3a

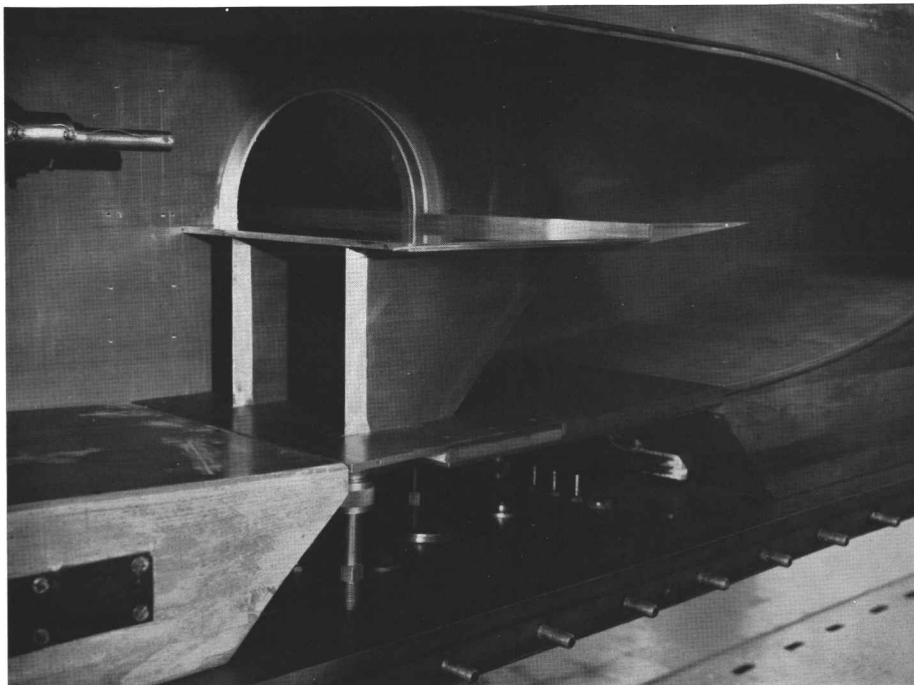
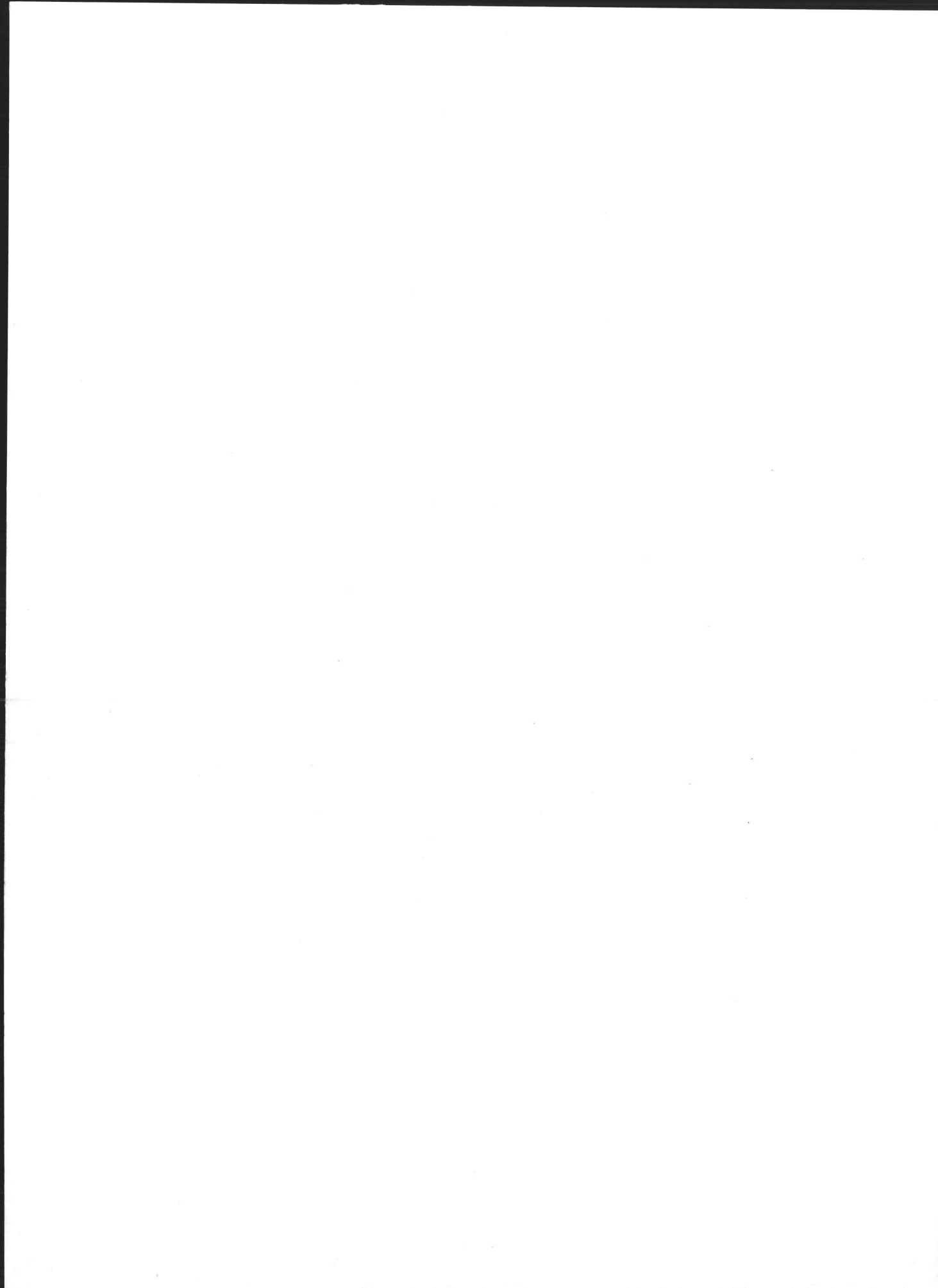


Figure 9. Model S-1 in the test section



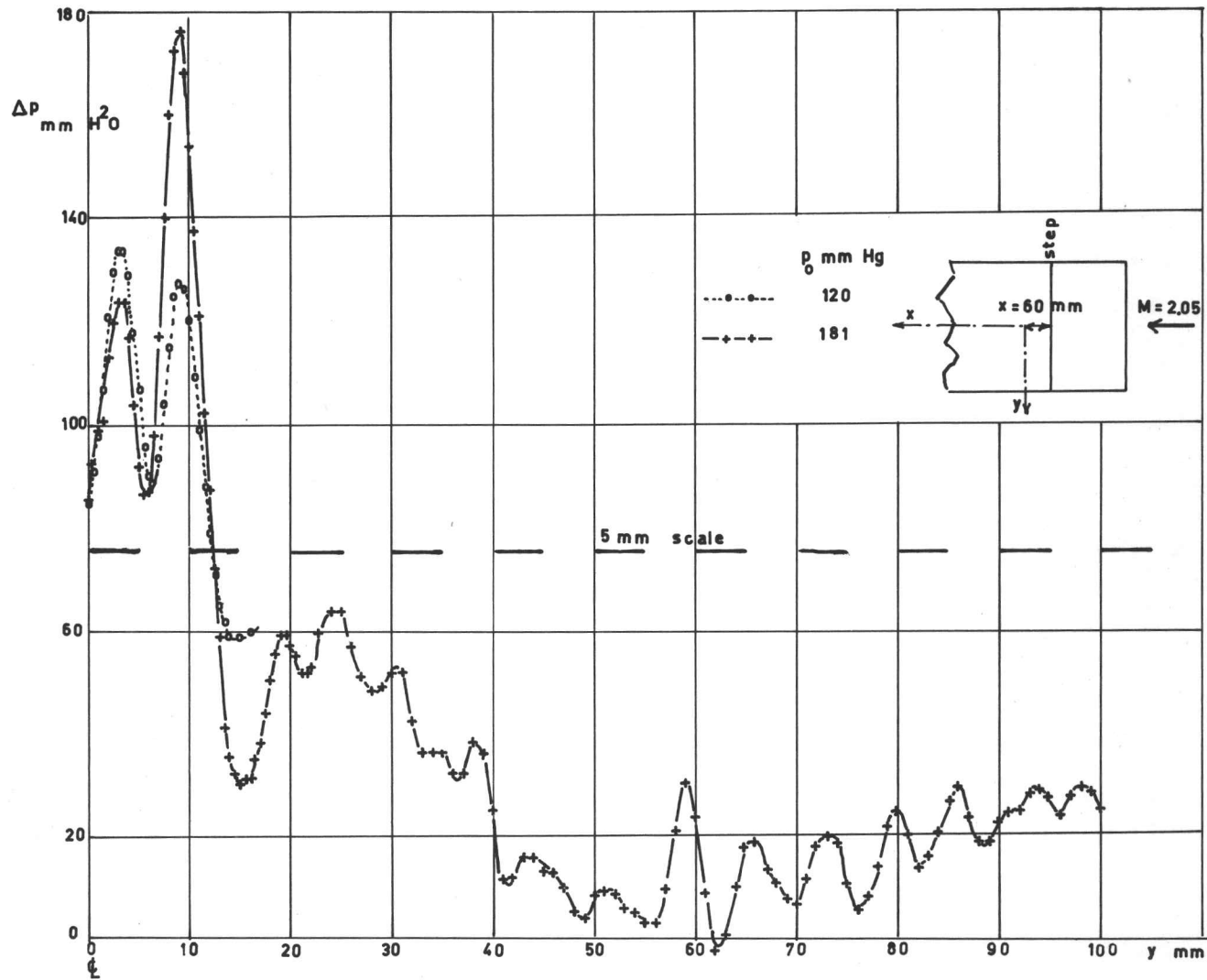
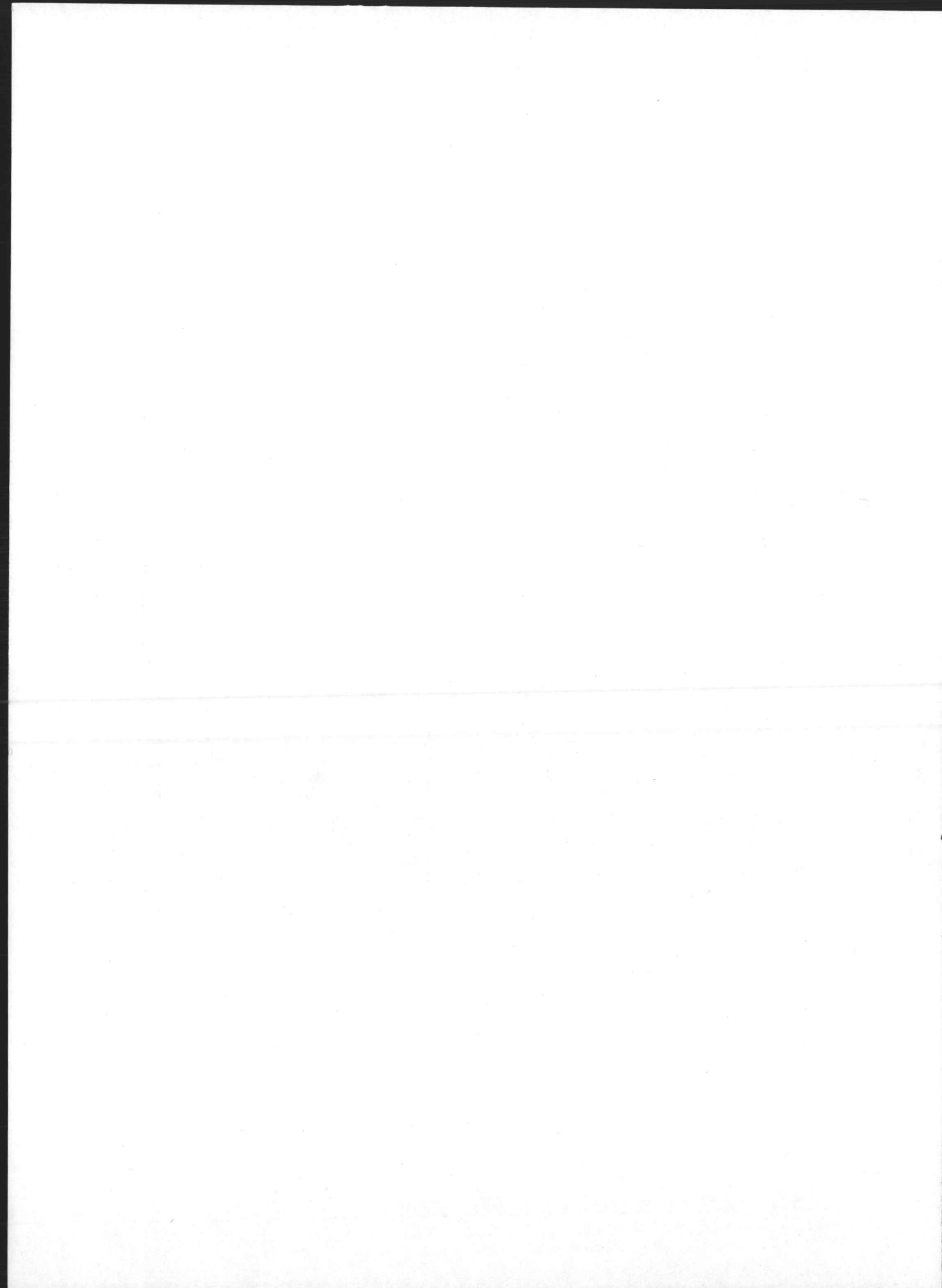


FIGURE 10 - TRANSVERSE SURVEY ON MODEL h10 BA1



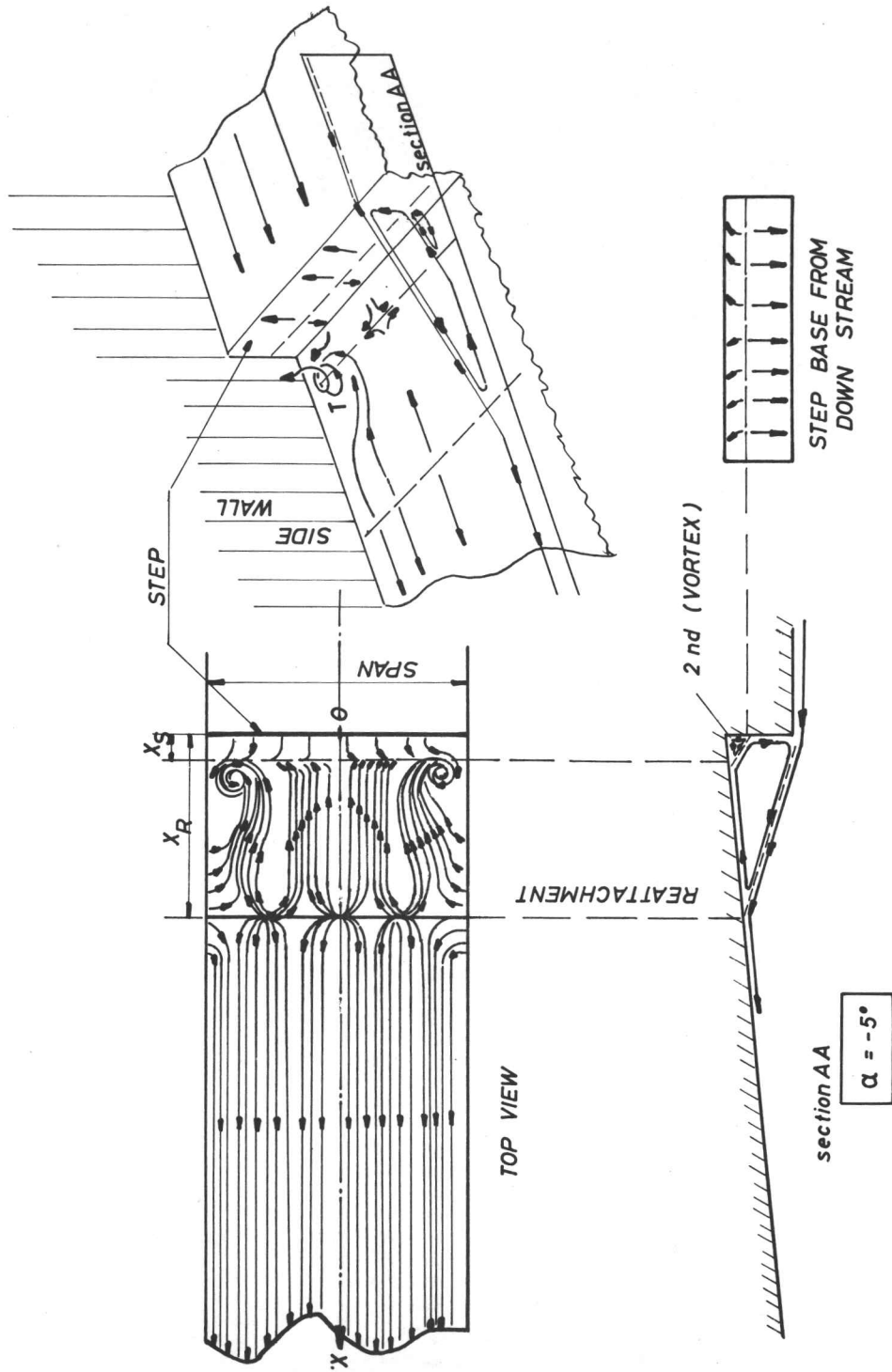
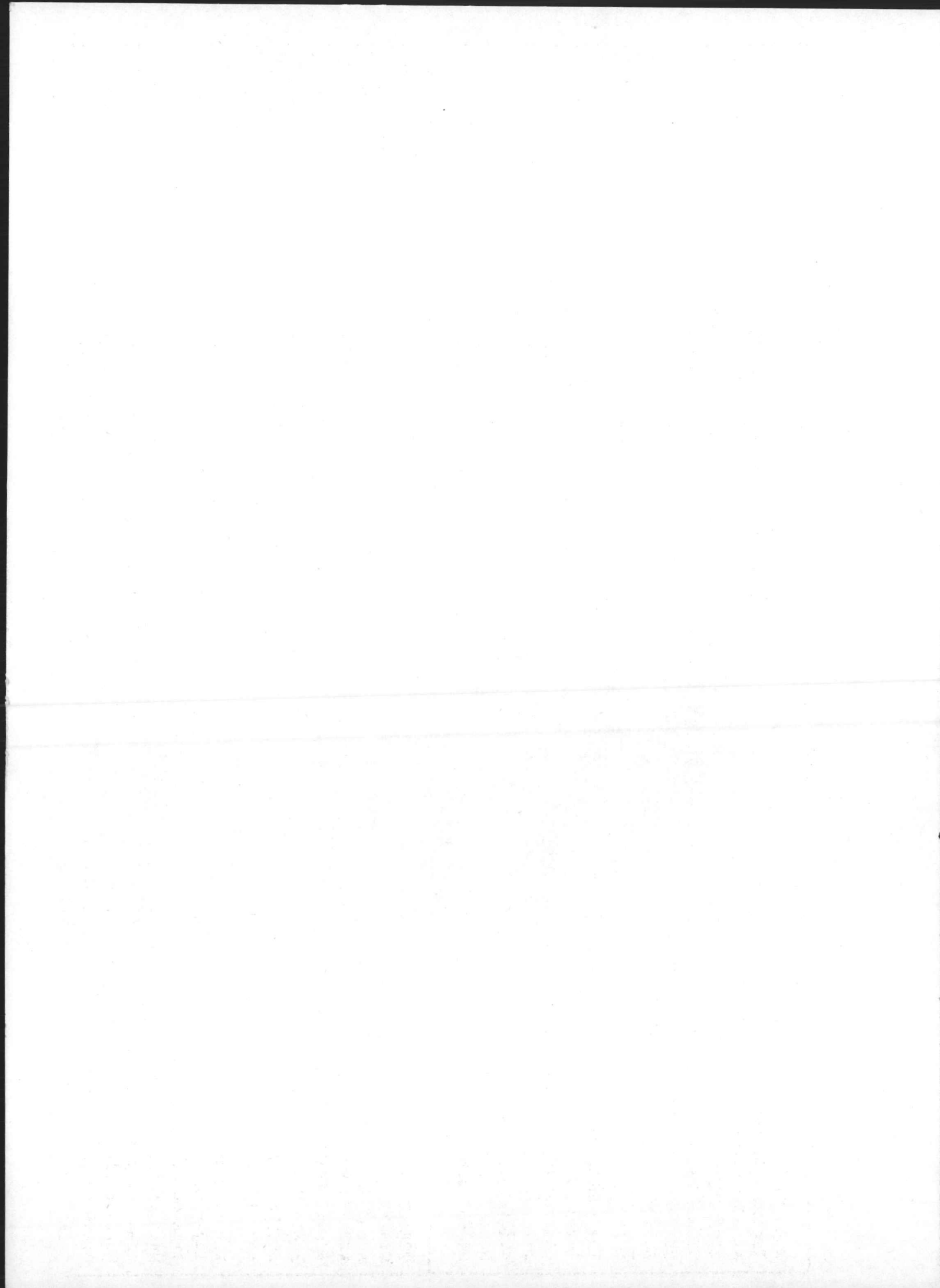


FIGURE 11 OIL FLOW ON MODEL



<p>TCEA TN 1 Training Center for Experimental Aerodynamics. EXPERIMENTAL EVIDENCE OF THREE-DIMENSIONAL PERTURBATIONS IN THE REATTACHMENT OF A TWO-DIMENSIONAL LAMINAR BOUNDARY LAYER AT M=2.05. Nov. 1958. Jean J. Ginoux</p> <p>The reattachment of a laminar boundary layer after separation has been investigated at a Mach number of 2.05. Backward facing step models were used that completely spanned the working section. Surface flow was observed by a sublimation technique and detailed span-wise (over)</p>	<ol style="list-style-type: none"> <li>1. Flow, Compressible (1.1.2)</li> <li>2. Flow, Viscous (1.1.3)</li> <li>3. Flow, Laminar (1.1.3.1)</li> <li>4. Flow, Turbulent (1.1.3.2)</li> </ol> <p>I. GINOUX, Jean J. II. TCEA TN 1.</p>	<p>TCEA TN 1 Centre de Formation en Aérodynamique Expérimentale. DE L'EVIDENCE DE PERTURBATIONS TRI-DIMENSIONNELLES DANS LE RECOLLEMENT D'UNE COUCHE LIMITE LAMINAIRE SUPERSONIQUE A UN NOMBRE DE MACH DE 2,05. Nov. 1958. Jean J. Ginoux.</p> <p>Une étude expérimentale a été faite du recollement d'une couche limite laminaire après séparation, à un nombre de Mach de 2,05. Les modèles formés par des plaques planes couvrant toute l'envergure de la chambre d'essai comportaient une marche descendante provo- (voir au verso)</p>	<ol style="list-style-type: none"> <li>1. Ecoulement, Compressible(1.1.2)</li> <li>2. Ecoulement, Viscueux (1.1.3)</li> <li>3. Ecoulement, Laminaire (1.1.3.1)</li> <li>4. Ecoulement, Turbulent(1.1.3.2)</li> </ol> <p>I. GINOUX, Jean J. II. TCEA TN 1.</p>
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TCEA TN 1

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D'importantes perturbations tri-dimensionnelles, régulièrement espacées en envergure, ont été systématiquement observées. Il a été montré que d'éventuelles irrégularités dans l'écoulement en amont des modèles ou dans les modèles eux-mêmes ne pouvaient pas être la cause des perturbations observées.

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Strong, regular and repeatable span-wise perturbations were observed which could not be explained by irregularities either in the airflow upstream of the models or in the models themselves.

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