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# Multichannel wavefield reconstruction using smooth slope information from multicomponent data

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## Summary

Local slopes carry useful information about the directionality of the predominant events in a seismic dataset and therefore can be used to steer the reconstruction process of sparsely sampled data. However, in the presence of spatial aliasing (for example, in the crossline direction of streamer data), conventional algorithms fail to provide a reliable estimate of such slopes and only low-frequency, smooth versions of the slope field can be produced. We show that provided the availability of multi-component data, and more precisely the pressure wavefield and its first-order gradient, such slopes are naturally embedded in the data and can be easily obtained by smoothed division of those wavefields. We further show that the estimated slopes can be used as regularization in a multi-channel sparse interpolation problem, providing additional guidance to the reconstruction process compared just using the pressure data and its gradient at the available traces. Numerical examples on 2D and 3D datasets confirm the effectiveness of the proposed two-stage process for multi-channel seismic data reconstruction.

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## Introduction

The transition to a low carbon future calls for the acquisition of highly sparse seismic data. Multi-component recording systems, whereby the pressure wavefield and its spatial derivative(s) are recorded simultaneously, represent a key enabler in that they allow relaxing the spatial sampling. The benefit of such systems was first presented by Roberstoon et al. (2008), who adapted the well-known Papoulis' generalized sampling theorem to geophysical applications. Later, Vassallo et al. (2010) and Ruan and Vasconcelos (2019) embedded gradient measurements in seismic reconstruction algorithms, relaxing the spatial Nyquist criterion by a factor of two or three (depending on the order of gradients available alongside the pressure data), or even more when suitable sparse bases are used to represent the sought seismic wavefield. In both cases, gradients are treated as extra measurements in the solution of a sparse reconstruction inverse problem and their modelled counterpart is obtained by simply applying one (or more orders of) spatial derivative in the frequency-wavenumber domain as part of the modelling operator.

In this work, we realize the potential of estimating local slopes from multi-component seismic data. First, we devise a robust approach to estimate such slopes by means of smoothed division of the pressure and gradient data. Second, we propose to use them to regularize the multi-channel sparsity-promoting inversion process, which we efficiently solve by means of the Fast Iterative Shrinkage-Thresholding Algorithm (FISTA). Our methodology is validated on both 2D and 3D datasets.

## Theory

Let's begin by recalling the equation of motion,

$$a_y = \frac{\partial v_y}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial y}, \tag{1}$$

where $p(y,x,t)$ is the pressure wavefield, $v_y(y,x,t)$ and $a_y(y,x,t)$ are the particle velocity and acceleration wavefields, respectively, $t$ is the time coordinate, and $\rho$ is the density of the medium. Throughout this work, $y$ represents the poorly sampled spatial direction (e.g., crossline in marine seismic) and $x$ is the finely sampled spatial direction (which is dropped in the 2D scenario). Equation 1 lies at the foundations of the previously mentioned multi-channel reconstruction algorithms as well as the one proposed in the work. More precisely two key observations can be made based on such equation:

1. provided knowledge of the density of the medium at receiver locations, recordings of particle velocity (or particle acceleration) in an acoustic medium (e.g., water column) give us direct access to first-order gradient measurements of the pressure wavefield;
2. under the assumption that seismic data can be represented as a superposition of local plane waves, first-order gradient measurements can be used to compute local slopes. Such slopes are in fact linked to the time and spatial gradients of the pressure field via the local plane wave equation

$$\frac{\partial p}{\partial y} + \sigma\frac{\partial p}{\partial t} = 0, \tag{2}$$

which, after defining $\hat{a}_y = -\rho a_y$, we can be written as function of time derivatives only as

$$\hat{a}_y + \sigma\frac{\partial p}{\partial t} = 0, \tag{3}$$

*Multi-channel local slope estimation*

Following Vassallo et al. (2009), local slopes can be estimated directly from multi-channel data by simply inverting equation 3 for $\sigma$. This approach is particularly appealing in the case of sparsely sampled data as the time derivatives can be always easily evaluated. However, in order to constrain the reconstruction process with local slopes, we aim to obtain a densely sampled slope field (i.e., at every spatial location of the sought after reconstruction grid). Leveraging the fact that local slopes are smoothly varying in time and space, we define the following inverse problem:

$$\begin{bmatrix} -\hat{\mathbf{a}}_y \\ \mathbf{0} \end{bmatrix} = \begin{bmatrix} \mathbf{P}_t\mathbf{R} \\ \varepsilon\nabla \end{bmatrix} \sigma \quad (4)$$

where $\hat{\mathbf{a}}_y$ is the scaled acceleration at the available traces, $\mathbf{P}_t$ is a diagonal operator with the time derivatives of the pressure data at the available traces, $\mathbf{R}$ is a restriction operator and $\nabla$ is the Laplacian operator.

*Multi-channel sparsity promoting inversion*

Once a densely sampled local slope field has been estimated, the process of reconstructing missing seismic traces is formulated in a similar fashion to Ruan and Vasconcelos (2019). More specifically:

$$\begin{bmatrix} \mathbf{p} \\ s_y\mathbf{p}_y \\ \mathbf{0} \end{bmatrix} = \begin{bmatrix} \mathbf{R}\mathbf{F}^H \\ s_y\mathbf{R}\mathbf{F}^H\mathbf{D}_{\mathbf{k_y}} \\ \varepsilon(\mathbf{D_y} + \Sigma\mathbf{D_t})\mathbf{F}^H \end{bmatrix} \mathbf{z} \rightarrow \mathbf{d} = \mathbf{G}\mathbf{z} \quad (5)$$

where $\mathbf{p}$ and $\mathbf{p}_y$ contain the available traces for the pressure and first-order spatial gradient wavefields, respectively, $s_y$ is a scaling factor introduced to balance the contributions of the two datasets, $\mathbf{F}$ is a multi-dimensional Fourier operator that transforms a seismic wavefield from time-space to frequency-wavenumber, $\mathbf{D}_{\mathbf{k_y}}$ is a diagonal matrix containing $jk_y$ that applies the first-order spatial derivative in the frequency-wavenumber domain. Finally, the third row of equation 5 is the discretised version of equation 2 that regularizes the solution $\mathbf{u} = \mathbf{F}^H\mathbf{z}$ to be consistent with the estimated local slopes $\sigma$ arranged along the diagonal of the operator $\Sigma$. Here $\mathbf{D_t}$ and $\mathbf{D_y}$ apply time and spatial derivatives directly in the time-space domain via a firth order, centered finite-difference stencils.

In this work, we follow the commonly used approach of representing seismic data in the frequency-wavenumber domain via the operator $\mathbf{F}$ and seeking for a sparse representation that fits the data:

$$\mathbf{z} = \arg\min_{\mathbf{z}} \|\mathbf{d} - \mathbf{G}\mathbf{z}\|_2^2 + \lambda\|\mathbf{z}\|_1 \quad (6)$$

This optimization problem is solved via the FISTA algorithm. Note that the proposed method is generic and other preconditioners such as windowed frequency-wavenumber decomposition (Abma and Kabir, 2006), rank-reduction (Trickett et al., 2010) or Neural Networks (Ravasi, 2022) could be used instead.

## Numerical Examples

The proposed method is first tested on a 2D line of the Mississippi Canyon streamer data. We consider a low-pass filtered version of a single shot gather with cut-off frequency $f_c = 35Hz$ (Figure 1a). The low-pass filter is required to remove any spatial aliasing such that we can synthetically generate the particle acceleration data (which is not physically recorded) by applying a spatial derivative in the frequency-wavenumber domain as in equation 1. The original data in Figure 1a is sampled every $25m$ for a total of 177 receivers. The data is subsampled by factor of 5 (i.e., new spatial sampling equals to $125m$) as shown in Figure 1b: the frequency-wavenumber spectrum, displayed in the bottom row, is severely aliased from around $15Hz$. Prior to interpolation, a time

gain equal to $t/2$ is applied to the available traces and normal moveout correction with constant velocity of $v = 1500m/s$ is carried out to reduce aliasing in the data. First, we attempt to reconstruct the missing traces by considering only the pressure and gradient data (i.e., the first two rows in equation 5): whilst the addition of the gradient data provides an improvement over single-channel interpolation (approximately $3dB$ more than in the single-channel case, not shown here for lack of space), some of the complex events in the near offset are still partially affected by aliasing. On the other hand, when inversion is constrained with local slopes estimated by solving equation 3 upfront, the reconstruction is significantly improved (with a SNR of $13.55dB$, approximately $7dB$ more than in the former case). We also show the slopes estimated on both the fully sampled and subsampled data (Figures 1e and f): their trace-by-trace Pearson correlation coefficient shows the high degree of similarity between the two despite the later was estimated from a fifth of the traces of the former. Finally, we note that the cost of estimating the

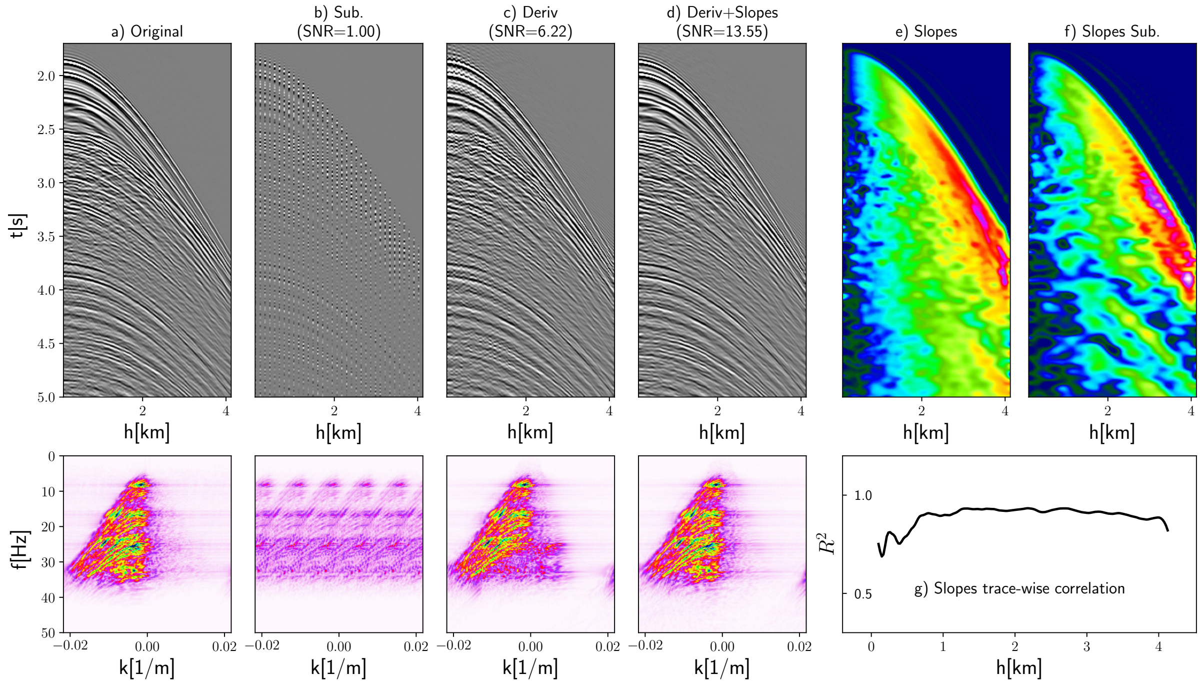


Figure 1: Mississippi Canyon data reconstruction. a) Original, full sampled data, b) Subsampled data, c) Reconstructed data using only pressure data and its spatial gradient, and d) Reconstructed data using both the spatial gradient data and local slopes. e-f) Local slopes estimated by solving equation 4 with full data and subsampled data, and g) their trace-by-trace Pearson correlation coefficient.

slopes and including them in the inversion process is negligible, and no additional data is required as the information is completely contained in the pressure and gradient data as shown in equation 3.

An example of 3D reconstruction is now presented for a synthetic ocean-bottom cable dataset modelled from the SEG/EAGE Overthrust model (Figure 2a). In this case, the spatial sampling over the inline direction is equal to $20m$, whilst the crossline direction is sampled every $100m$ (Figure 2b). This gives rise to aliasing effects from a frequency of about $10Hz$. A similar flow to the one described for the 2D dataset is applied, with the only difference that normal moveout is applied with a velocity of $v = 2000m/s$ and a $t^2$ gain is used: the crossline local slopes are displayed in Figure 2c and the pressure data reconstructed on a $20m \times 20m$ grid (for a total size of $90 \times 177 \times 801$) is shown in Figure 2d. Although not shown here, the proposed reconstruction method provides an improvement of $14dB$ over single-channel reconstruction and of $12dB$ over multi-channel reconstruction with pressure and gradient data (i.e., without local slopes). Both numerical examples have been implemented using the GPU backend of the PyLops framework for matrix-free inverse problems (Ravasi and Vasconcelos, 2020). Slope estimation and data reconstruction for the 3D example run under 2 minutes on a Intel(R) Xeon(R) CPU @ 2.10GHz equipped with a single NVIDIA GEForce RTX 3090 GPU.

## Conclusions

We have presented a two-stage process to reconstruct aliased, multi-component seismic data: first, local slopes are estimated directly from the time derivatives of the pressure and crossline particle acceleration wavefields by means of a spatially regularized inverse process. Next, such slopes are used in a sparsity-promoting inversion process to reconstruct missing traces in the pressure wavefield. Under the same sampling conditions and same choice of the sparsifying transform, local slopes are shown to act as a strong additional constraint to the inverse process and significantly improve the reconstructed wavefield.

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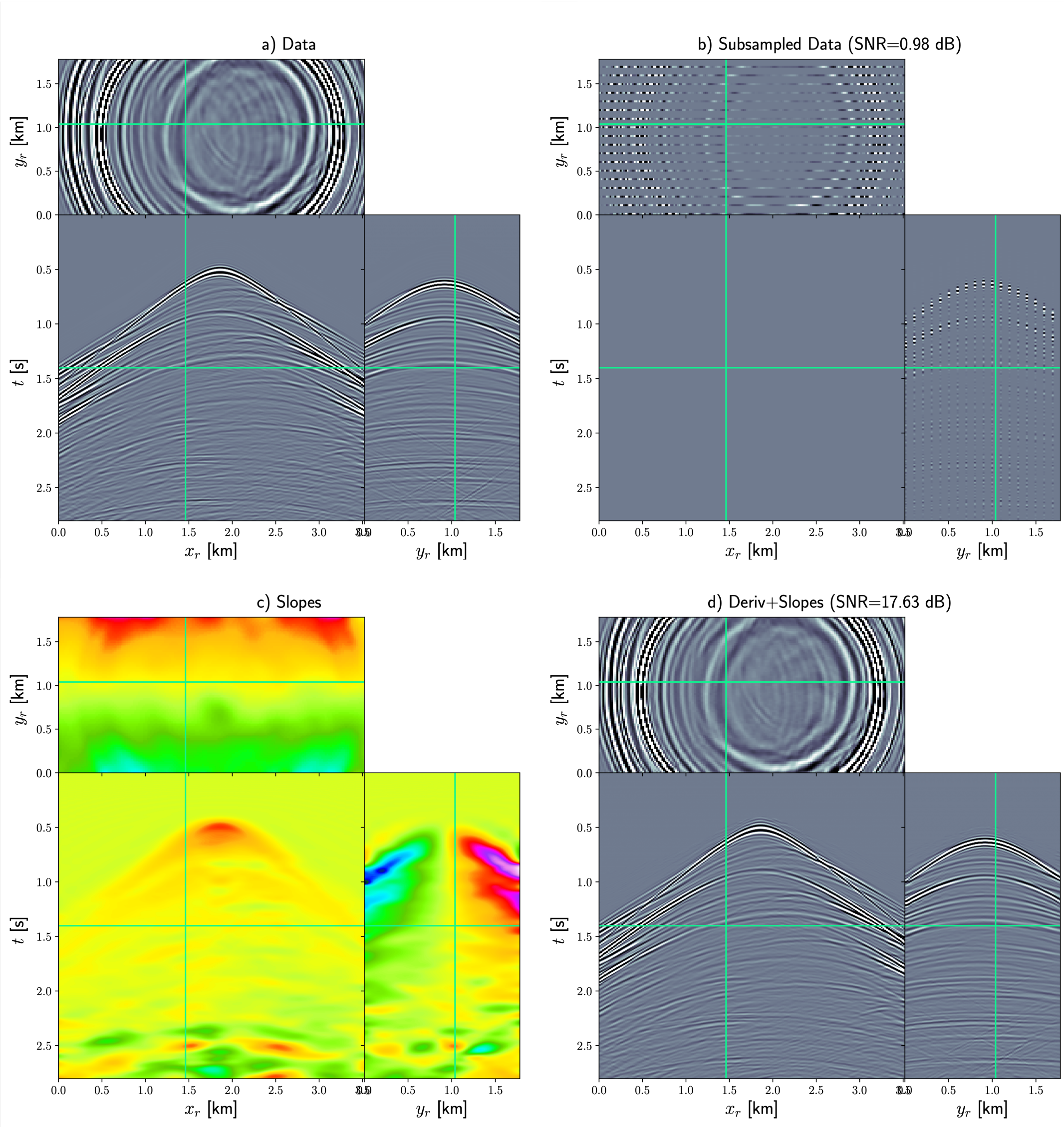


Figure 2: SEG/EAGE Overthrust data reconstruction a) Original, full sampled data, b) Subsampled data, c) Reconstructed data using both spatial derivative data and local slopes, d) Local slopes.

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