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Stochastic analysis of aircraft composite structures under space-dependent environmental and material uncertainties

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Abstract

The application of uncertainty quantification (UQ) methods to realistic industrial-scale structures remains a challenging task, due to the typical geometric complexity, the existence of various sources of uncertainties and the high dimension of structural models. These challenges are even bigger when dealing with composite material structures, which are knowingly prone to uncertainties arising from manufacturing processes and environmental influences. In this context, the present paper intends to contribute to increase the maturity level of UQ techniques to composite aeronautic structures, under the combined effects of space-dependent material and environmental fluctuations. Variations in temperature, laminate thickness and fiber volume fraction are jointly considered, being represented as random fields discretized using the Karhunen-Loève Expansion (KLE), while fiber angles are treated as random variables. Aiming at expanding the range of situations possibly found in practice, both Gaussian and non-Gaussian random fields are considered within a methodology combining the Iterative Translation Approximation Method (ITAM) and KLE. A micromechanical model is used to represent temperature- and moisture-dependent material properties, capturing the coupled effects of environmental degradation of material properties and hygrothermally-induced stresses. Monte Carlo Simulation (MCS) is employed to perform uncertainty quantification for buckling loads and vibration natural frequencies of a regional aircraft composite wing structure modeled with a relatively high-dimension finite element model. Additionally, global sensitivity analysis based on Sobol' indices is conducted to identify the most influential random parameters, where structural responses are approximated using artificial neural network (ANN)-based surrogate models. From the simulation scenarios analyzed, accounting for different values of standard deviations attributed to random variables and correlation lengths assigned to random fields, the statistics of structural responses are assessed. The significant spread of structural responses highlights the importance of incorporating the considered types of uncertainty in analysis and design procedures for achieving robust and reliable aerospace composite structures.

Keywords Uncertainty quantification · Hygrothermal loads · Non-Gaussian random field · Micromechanical modeling · Karhunen-Loève expansion

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1 Introduction

It is broadly recognized that, as compared to traditional materials used in variety of engineering areas, polymer matrix-based composite materials present a number of advantages, such as high strength-to-weight ratios, impact and corrosion resistance, larger design space, lower thermal conductivity, among others. Although clear advantages favor their widespread use, composite materials are highly affected by variabilities arising from a range of sources, which can potentially compromise their structural performance [1]. The need to account for such uncertainties in the analysis of engineering composite systems has been particularly prominent in the aerospace industry, where lighter and fuel-efficient aircraft are pursued, while guaranteeing critical reliability requirements. In this context, the field of Uncertainty Quantification (UQ), which has been continuously gaining maturity, provides tools for the management of uncertainty in engineering systems and, more recently, has enabled to devise techniques aiming to reduce the amount of testing required for aircraft certification [2].

Significant sources of variability influencing composite materials arise from the intrinsic complexity of their manufacturing processes, which can only be partially mitigated by automated techniques, such as Automated Tape Laying (ATL) and Automated Fiber Placement (AFP) [3]. The unavoidable imperfections commonly include geometric deviations, voids and variations in volume fractions of fibers and matrix, inadequate bonding between components, fiber misalignment, residual stresses, and incomplete curing, which can propagate and increase the uncertainty in the overall stiffness and strength of the structure with respect to the original design [4].

Aerospace applications commonly experience severe fluctuations in temperature and humidity, which makes the environmental effects also critical in the design of composite aircraft structures. According to [5], the typical external temperature for a commercial jet aircraft can vary from -80°C to 50°C across the different flight phases. In addition, humidity in the atmosphere can range from practically dry to the saturated condition.

Temperature and moisture variations can have significant effects on the structural behavior of composite structures, following two different mechanisms, as stated by Gibson [6]: softening of the polymer matrix is induced by elevated temperature, which is aggravated by the presence of moisture as the matrix glass transition temperature is reduced; and increase in temperature and/or moisture content leads to expansion/swelling of the polymer matrix which, under the practically insensitivity of the fibers to hygrothermal variations, induces stresses within the material.

Several studies reported in the literature investigate the effects of environmental conditions on the mechanical behavior of composites. Deflections, buckling and natural frequencies of thick composite laminates are analyzed by Patel et al. [7], considering the degradation of the lamina material properties at elevated temperature and moisture. In the work of Upadhyay et al. [8], considerable deterioration in the flexural response of a laminated composite plate was observed with the elastic and hygrothermal material properties considered dependent on temperature and moisture concentration. Li and Nie [9] also demonstrated the strong influence of temperature-dependent material properties on the buckling response of variable angle tow (VAT) laminates.

Regarding the effects of stresses induced by hygrothermal changes, Whitney and Ashton [10] included the effect of thermal expansion and swelling strains in the analysis of bending, vibration, and buckling of laminates with symmetric and asymmetric layups. Considering also the degradation in material properties due to the elevation of temperature and moisture concentration, Amoushahi and Goodarzi [11] investigated the natural frequencies and buckling capacity of composite laminated plates, including pre-buckling in-plane analysis under hygrothermal effects. A benchmark review on the analysis of laminated composite and sandwich structures subjected to hygrothermal conditions was conducted by Garg and Chalak [12], where an extensive bulk of literature is presented and categorized based on the type of analysis and theory employed.

The previously cited studies have been conducted under the assumption of deterministic hygrothermal effects, neglecting the complexity of transfer mechanisms involving uncontrolled environmental factors and the influence of intrinsic variability in these processes. This assumption cannot be adequately applicable in many problems of practical interest, for which variations in material properties of composite laminates due to environmental influences and limitations of manufacturing processes may be more appropriately modeled as random quantities.

Many research works have been devoted to the consideration of uncertainties in composite structures. In [13], the variation of fundamental natural frequencies of cantilever composite beams is analyzed considering uncertainties in material properties, thickness and fiber orientation of individual laminae as random variables. Randomness in loads, geometry and material properties are considered in [14] when analyzing the buckling resistance of simply-supported laminated composite plates. Variability in the buckling load of composite laminates is also investigated in [15] within a multi-scale uncertainty quantification framework, where uncertainties of material properties, strength and geometry parameters at micro- and macro-scales are accounted for. In

[16], hygrothermal buckling loads of straight-fiber laminate plates, subjected to uniform temperature/moisture changes and random variations of lamina thickness and material properties, are analyzed. Also concerning environmental uncertainties, stochastic eigenfrequency and dynamic response of the graphite-epoxy laminated composite plates are evaluated in [17], considering the uniform temperature increment as a normally distributed random variable.

Very often, probabilistic structural analyses may require the representation of spatially-distributed random quantities. In these cases, the concept of random field has to be employed, frequently in combination with discretization techniques to make numerical computations amenable [18]. In the literature, different methods intended for the approximate representation of random fields can be found. Among them, the most popular and frequently preferred method, due to its advantageous characteristics, is the Karhunen-Loève Expansion (KLE) [19].

Some studies have been conducted employing KLE to incorporate spatially varying variables in structural analysis involving composite materials. In [20], the impact of spatial randomness in fiber orientations on the vibrational behavior of a rectangular fiber-reinforced composite plate was analyzed, where uncertainty in each ply orientation was described by KLE as an independent Gaussian random field. Ply orientations with spatial variability discretized by KLE were also accounted for in [21], where an approach using lamination parameters to model random fields was proposed. The lamina fiber volume was likewise modeled as random fields with KLE discretization in [22] and [23], both analyzing variable stiffness composites. The first study investigated, assuming fiber misalignment random fields, the stochastic characterization of the buckling response, while the second was dedicated to aeroelastic stability analysis under subsonic and supersonic flow. In the work of Naskar et al. [24], the probabilistic descriptions of stochastic dynamics and stability of composite laminates were characterized by quantifying the effect of spatial randomness of micro- and macromechanical material properties, also discretized by KLE.

Regarding environmental spatial randomness in composites, Parviz et al. [25] analyzed the stochastic thermal buckling of composite plates under space-distributed random temperature, with KLE employed to discretize the Gaussian temperature random field, which led to spatially varying mechanical properties, as a consequence of assuming them as temperature-dependent. Similarly, in [26], temperature was assumed as a Gaussian random field and material properties considered dependent on it while conducting stochastic dynamic analyses of composite plates. In [27], the moisture variation over laminated composite plates was modeled as a random field, with the addition of temperature

stochastic field, to investigate the effect of Gaussian space-dependent hygrothermal variations on the buckling response of composite plates.

In the probabilistic structural analysis, UQ computations have been extensively performed with the Monte Carlo Simulation (MCS) [28]. This is certainly the most popular simulation-based method, due to its power and simplicity, with the inconvenience of involving high computational effort when complex systems are dealt with. When it comes to real-world structures, which often involve a large number of random variables, such disadvantage may lead to the infeasibility of the stochastic analyses (the so-called *curse of dimensionality*). Aiming to avoid prohibitive computational costs, methods for constructing surrogate models have been proposed. Among them, in light of recent advances in the field of Artificial Intelligence (AI), Artificial Neural Networks (ANNs) have emerged as computationally effective techniques to approximate mathematical models [29].

Zhou et al. [30] present an overview of techniques for direct and inverse uncertainty problems in the field of structural mechanics, highlighting the successful use of different variants of ANNs applied to a range of relevant problems. A methodology based on deep neural networks, intended for the construction of surrogate models to deal with stochastic systems featuring large numbers of random quantities, is proposed by Tripathy and Bilonis [31] and demonstrated with a problem on uncertainty propagation in a stochastic elliptic partial differential equation with an uncertain diffusion coefficient. In [32], the focus is directed to the use of ANNs in combination with MCS. The underlying idea is to achieve dimensionality reduction of the training space of the ANN by using the random phase angles of the spectral representation as inputs, instead of the random variables describing the uncertain parameters.

Dealing specifically with structural reliability analysis, in the work conducted by Elhewy et al. [33], the ANN-based response surface method demonstrated to be more efficient and accurate than the polynomial-based response surface method. Structural reliability problems are also addressed in [34], where a new artificial neural network-based response surface method is proposed in conjunction with the uniform design method (UDM) for predicting the failure probability. The method involves the selection of training datasets for constructing an ANN model by the UDM, approximation of the limit state function by the trained ANN model, and estimation of the failure probability using the first-order reliability method (FORM). The two companion papers [35] and [36] investigate the use of ANNs in the reliability analysis of composite structures. In the first study, considering randomness in loads, lamina thickness and ply angles, finite element (FE) models are substituted by trained ANNs for reliability assessment based on Tsai-Wu failure criterion,

while in the second, a combination of trained ANNs with a genetic algorithm was used for reliability-based design optimizations taking the reliability index as constraint. The ANN approach was also employed in the stochastic natural frequency analysis of composite plates in [37], accounting for randomness in the elastic properties, density and orientation angles, all of them modeled as random variables. In the review paper [38], where an extensive literature assessment comprising methods, concepts and applications of reliability in composites is presented, ANN has been recognized as an advantageous technique to deal with large composite structural systems.

The association of FE method with uncertainty modeling and quantification of composite materials has been made in numerous previous investigations, mainly due to the capacity of FE models to provide sufficient accurate prediction capability of complex structures in a multiphysical framework. Stefanou et al. [39] propose a method to link microstructure uncertainty with the random variation of material properties at the macro-scale, capable of accounting for arbitrary probability distributions and correlation structure of the macroscopic mechanical properties. The approach is based on the use of the Extended Finite Element Method (XFEM) and MCS for the computation of the effective properties of two-phase composites. In [40], a numerical investigation of the vibrational behavior composites under uncertainty in fiber orientations is performed. The spectral stochastic FE method is used in combination with a truncated KLE for the random fiber orientations, and the generalized polynomial chaos is employed to represent the structural responses. The coefficients are estimated using probabilistic collocation points, where commercial FE codes can be used in a non-intrusive way. Considering damping parameters as random variables, Sepahvand [41] characterizes the uncertainty in damping and frequency response functions of composite plates. Generalized polynomial chaos expansions constructed for the uncertain parameters are used non-intrusively as inputs to a deterministic FE model.

A very relevant discussion is conducted in [42], raised by the limiting assumptions of the Stochastic Finite Element (SFE) Method when the spatial variation of uncertain material properties is described by random fields, due to the lack of measured data to support the assumptions. Thus, accounting for the fact that information at the macroscopic material randomness can be deduced by observing the micromechanical characteristics of materials, the authors attempt to stimulate a systematic and multidisciplinary effort towards establishing a link between microstructural material models and stochastic material properties at the macro-scale. This latter can, then, be inputted to SFE models.

In the literature, research works investigating the effect of random variabilities in industrial-scale composite

structures are scarce. A reliability-based design and optimization methodology is developed for self-adaptive composite marine propellers in the work of Young et al. [43], where the FORM is employed for probabilistic optimization under the influence of material and load uncertainties. Regarding composite parts of aircraft, uncertainties affecting material properties and laminate stacking sequence in a multi-scale modeling are taken into account in [44] to evaluate the stochastic buckling mode of an aircraft fuselage airframe, with the sparse polynomial chaos approach being used in uncertainty and sensitivity analyses to handle the high number of random quantities. For real-scale composite aircraft wings, two works, [45] and [46], have incorporated variations in material properties into stochastic analyses, where the latter is dedicated to multi-level aeroelastic tailoring optimization encompassing a combination of robust and reliability-based design methods, also including uncertainties in ply thickness, and the former is intended to simulate random fields over curved surfaces and perform uncertainty quantification concerning vibration and buckling responses.

In the context delineated above, the present paper proposes a strategy associating different UQ techniques for the stochastic assessment of real-world industrial composite structures, under the combined effects of multiple sources of uncertainty likely to occur in practice. Uncertainties in temperature, laminate thickness, and fiber volume fraction over the structure are modeled as random fields discretized by KLE, while the variability in fiber-direction angles is modeled as random variables. Both Gaussian and non-Gaussian random fields are considered by using an efficient methodology combining the Iterative Translation Approximation Method (ITAM) and the KLE, denoted by KL-ITAM [47]. A micromechanical approach is used for the stochastic prediction of the elastic and hygrothermal properties of the fiber-reinforced composites, where these properties are considered dependent on temperature and moisture concentration. Humidity absorption is also considered to vary spatially due to fiber volume variation according to the rule of mixtures. Hence, the stochastic analysis simultaneously accounts for the dual impact of environmental uncertainties: the degradation of material properties and hygrothermally-induced stresses.

Considering a regional aircraft composite wing represented by a large FE model, uncertainty quantification is performed with respect to the buckling loads and natural frequencies, taking into consideration different levels of variability of the random hygrothermal conditions and material characteristics, as well as distinct correlation lengths assigned to the random fields. The stochastic analyses are conducted using MCS in conjunction with Latin Hypercube Sampling (LHS). Then, Sobol' indices are computed for global sensitivity analysis, where structural responses are

predicted using ANN models trained based on the MCS-generated data.

To the best of the authors' knowledge, the simultaneous consideration of spatial randomness in hygrothermal conditions and material properties of industrial-scale composite structures, in conjunction with the inclusion of non-Gaussian random fields in the modeling of these uncertainties, have not been addressed so far.

The paper is organized as follows: the main theoretical background supporting the methods and numerical procedures used in this work are summarized in Sect. 2. In Sect. 3, the structural model of the regional jet wing is introduced, together with preliminary analyses carried out to support the definition of the set of parameters and conditions required by the numerical procedures employed for UQ. Subsequently, the numerical simulations for uncertainty propagation and sensitivity analysis of the structural responses of the composite wing, along with a detailed discussion of the results, are presented. Finally, in Sect. 4, concluding remarks on the research are provided.

2 Theory

This section is dedicated to reviewing the theoretical formulation underlying the characterization of spatially varying random quantities and to deriving the formulations related to the construction of a stochastic structural model intended for buckling and vibration analyses, encompassing both micro- and macromechanical modeling. In addition, the formulation related to sensitivity analysis based on Sobol' indices, with a brief description of surrogate modeling using ANNs, is presented.

2.1 Random fields discretized by KLE

In the Karhunen-Loève Expansion (KLE) approach, a continuous random field $W(\mathbf{x}, \Theta)$, which is function of the position vector $\mathbf{x}$, defined in the global coordinates (X, Y, Z) over a domain D , can be approximated using the following expansion truncated to M terms [19]:

$$W(\mathbf{x}, \Theta) = \bar{W}(\mathbf{x}) + \Delta W(\mathbf{x}, \Theta) = \bar{W}(\mathbf{x}) + \sum_{n=1}^M \sqrt{\lambda_n} \xi_n(\Theta) f_n(\mathbf{x}), \quad (1)$$

where $\bar{W}(\mathbf{x})$ is the mean field and $\xi_n(\Theta)$ is a set of zero-mean, unit-variance Gaussian uncorrelated random variables, in which Θ belongs to an appropriate sample space. In addition, λ_n and $f_n(\mathbf{x})$ are, respectively, the eigenvalues and eigenfunctions of the following integral eigenvalue

problem associated with an adopted covariance function $C(\mathbf{x}_1, \mathbf{x}_2)$:

$$\int_D C(\mathbf{x}_1, \mathbf{x}_2) f_n(\mathbf{x}_2) d\mathbf{x}_2 = \lambda_n f_n(\mathbf{x}_1), \quad n = 1, \dots, M. \quad (2)$$

This eigenvalue problem has analytical solutions for certain types of covariance functions defined on rectangular domains [19]. Numerical solutions are required for other types of covariance functions and/or domains whose geometrical shapes are complex [48], which is the case considered herein. In the present work, the Nyström method is chosen to numerically solve the eigenproblem by using a numerical integration scheme to approximate Eq. (2), as follows:

$$\sum_{j=1}^{N_e} a_j C(\mathbf{x}_i, \mathbf{x}_j) \hat{f}_n(\mathbf{x}_j) = \hat{\lambda}_n \hat{f}_n(\mathbf{x}_i), \quad i = 1, \dots, N_e, \quad (3)$$

where $\mathbf{x}_i$ and $\mathbf{x}_j$ refer to the centroid positions of the i -th and j -th element of the integration mesh, respectively, N_e is the total number of subdomains used for space-discretizations, and a_j are the areas of each subdomain. In addition, $C(\mathbf{x}_i, \mathbf{x}_j)$ is one element of a symmetric $N_e \times N_e$ square matrix composed by the values of covariance between random variables at positions $\mathbf{x}_i$ and $\mathbf{x}_j$. More details of the Nyström method can be found in [48].

With the solutions of Eq. (2), the covariance function can be approximated using M terms. By definition, such function is bounded, symmetric, positive definite, and admits the following spectral decomposition:

$$C(\mathbf{x}_i, \mathbf{x}_j) = \sum_{n=1}^M \hat{\lambda}_n \hat{f}_n(\mathbf{x}_i) \hat{f}_n(\mathbf{x}_j), \quad n = 1, \dots, M. \quad (4)$$

2.2 The KL-ITAM method

Gaussian probability distributions can be preferably associated with random fields owing to the Central Limit Theorem and the convenient use of well established simulating methods, besides the fact that a complete description is enabled by second-order statistical moments. Nevertheless, many physical phenomena are likely to follow non-Gaussian distributions, which requires the use of specific methods that frequently make use of translation processes [49]. In such methods, the nonlinear transformation of an underlying Gaussian random field to the specified non-Gaussian counterpart demands a compatible relationship between the target covariance function and target non-Gaussian marginal

distribution, which is not guaranteed when both are independently specified [50].

In the work of Kim and Shields [47], an efficient methodology is proposed to overcome the aforementioned limitation of translation-based methods. Based on the Iterative Translation Approximation Method (ITAM) and using the KLE, the KL-ITAM method identifies a Gaussian random field $W_G(\mathbf{x}, \Theta)$ with its respective underlying Gaussian autocorrelation function (ACF) that, when mapped using the translation process theory, matches the prescribed non-Gaussian distribution and the target ACF.

The KL-ITAM method can be implemented in the following steps:

Step 1) Initialization of the underlying Gaussian ACF. An initial guess for the underlying Gaussian ACF, denoted by $R_G(\mathbf{x}_i, \mathbf{x}_j)$, is chosen. Assuming a Gaussian random field to be translated with zero mean and unit variance, the target non-Gaussian covariance function $C(\mathbf{x}_i, \mathbf{x}_j)$ can be utilized to obtain the initial ACF with such characteristics by employing the following relation:

$$R_G(\mathbf{x}_i, \mathbf{x}_j) = \frac{1}{\sigma_N^2} C(\mathbf{x}_i, \mathbf{x}_j), \quad (5)$$

where σ_N is the standard deviation associated with the non-Gaussian random field.

Step 2) Computation of the non-Gaussian ACF. The iterative process starts with the computation of the normalized Gaussian correlation function as below:

$$\rho_G(\mathbf{x}_i, \mathbf{x}_j) = \frac{R_G(\mathbf{x}_i, \mathbf{x}_j)}{\sqrt{R_G(\mathbf{x}_i, \mathbf{x}_i) R_G(\mathbf{x}_j, \mathbf{x}_j)}}. \quad (6)$$

Then, the translation process is performed according to:

$$R_N(\mathbf{x}_i, \mathbf{x}_j) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F_N^{-1}\{\Phi[w_i], \mathbf{x}_i\} F_N^{-1}\{\Phi[w_j], \mathbf{x}_j\} \phi\{w_i, w_j; \rho_G(\mathbf{x}_i, \mathbf{x}_j)\} dw_i dw_j, \quad (7)$$

where $w_i = W_G(\mathbf{x}_i)$, $w_j = W_G(\mathbf{x}_j)$, $F_N^{-1}\{\cdot, \mathbf{x}_k\}$ is the inverse marginal non-Gaussian cumulative distribution function (CDF) at point $\mathbf{x}_k$, and Φ is the standard Gaussian CDF (i.e., zero mean and unit variance). The joint Gaussian probability density function is given as:

$$\phi\{w_i, w_j; \rho_G(\mathbf{x}_i, \mathbf{x}_j)\} = \frac{1}{2\pi\sqrt{1 - \rho_G(\mathbf{x}_i, \mathbf{x}_j)^2}} \exp\left(-\frac{w_i^2 + w_j^2 - 2\rho_G(\mathbf{x}_i, \mathbf{x}_j)w_iw_j}{2(1 - \rho_G(\mathbf{x}_i, \mathbf{x}_j)^2)}\right). \quad (8)$$

Step 3) Check of convergence. The relative difference between the estimated and the target non-Gaussian ACFs is verified at each iteration t according to:

$$\text{diff}^{(t)} = 100 \sqrt{\frac{\sum_{i=1}^{N_e} \sum_{j=1}^{N_e} [R_N^{(t)}(\mathbf{x}_i, \mathbf{x}_j) - R_N^T(\mathbf{x}_i, \mathbf{x}_j)]^2}{\sum_{i=1}^{N_e} \sum_{j=1}^{N_e} [R_N^T(\mathbf{x}_i, \mathbf{x}_j)]^2}}, \quad (9)$$

where, considering $\mu_N(\mathbf{x}_i)$ and $\mu_N(\mathbf{x}_j)$ as the means of the random field at positions $\mathbf{x}_i$ and $\mathbf{x}_j$, respectively, the target non-Gaussian ACF is determined as:

$$R_N^T(\mathbf{x}_i, \mathbf{x}_j) = \mu_N(\mathbf{x}_i) \mu_N(\mathbf{x}_j) + C(\mathbf{x}_i, \mathbf{x}_j). \quad (10)$$

The iterative process is stopped when the relative difference is smaller than a determined tolerance or when it does not vary significantly in the course of iterations. When convergence is achieved, the process jumps to Step 5, otherwise Step 4 is performed, followed by repetition of steps 2 and 3.

Step 4) Update of the underlying Gaussian ACF. The following equation is used to update the underlying Gaussian ACF:

$$R_G^{(t+1)}(\mathbf{x}_i, \mathbf{x}_j) = \frac{R_N^T(\mathbf{x}_i, \mathbf{x}_j)}{R_N^{(t)}(\mathbf{x}_i, \mathbf{x}_j)} R_G^{(t)}(\mathbf{x}_i, \mathbf{x}_j). \quad (11)$$

The updated ACF has to be positive semi-definite to be admissible. Such property is maintained by finding the nearest positive semi-definite ACF to the updated ACF. In this work, the *nearcorr* function available in MATLAB[®] [51] is used for this purpose.

Step 5) Simulation of the random field using KLE. Using the Gaussian ACF generated by the iterative process, KLE is performed as described in Sect. 2.1. The discretized Gaussian random field is then mapped to the non-Gaussian one using the following equation:

$$W_N(\mathbf{x}) = F_N^{-1}\{\Phi[W_G(\mathbf{x})], \mathbf{x}\}. \quad (12)$$

It should be mentioned that, although originally developed for handling non-Gaussian distributions, the KL-ITAM method can be applied to Gaussian random fields as well. In this case, the methodology is useful for translating a zero-mean and unit variance Gaussian random field to another one with arbitrary values of mean and standard deviation.

2.3 Micromechanical modeling of hygrothermal and material properties

In the present work, the most relevant sources of uncertainties related to environmental conditions and material properties are considered to vary spatially throughout the structure. The temperature, fiber volume fraction, and laminate thickness, denoted as $T(\mathbf{x}, \Theta)$, $V_f(\mathbf{x}, \Theta)$, and $h(\mathbf{x}, \Theta)$, respectively, are modeled as random fields.

The variation in temperature and moisture content has a predominant effect on polymer matrix materials. According to [52, 53], the degradation of matrix properties can be estimated using the following empirical retention ratios, duly adapted to account for randomness in temperature, applicable to mechanical and hygrothermal properties, respectively:

$$F_m(\mathbf{x}, \Theta) = \frac{P}{P_o} = \left[\frac{T_{gw} - T(\mathbf{x}, \Theta)}{T_{go} - T_o} \right]^{0.5}, \quad (13)$$

$$F_h(\mathbf{x}, \Theta) = \frac{1}{F_m(\mathbf{x}, \Theta)}, \quad (14)$$

where P_o and P refer to the matrix strength or stiffness parameters without and with hygrothermal degradation, respectively. In addition, T_o denotes the test temperature at which P_o is measured (reference condition), and T_{go} and T_{gw} are, respectively, the glass transition temperatures for dry (reference) and wet (with degradation) conditions. In Eq. (13), all temperatures are in $^{\circ}F$.

The relation between T_{go} and T_{gw} can be experimentally obtained, with the following empirical relation suggested in [53]:

$$T_{gw} = (0.005M_r^2 - 0.10M_r + 1.0) T_{go}, \quad (15)$$

where M_r denotes the weight fraction of moisture in the matrix.

As a consequence of including randomness in fiber volume fraction and considering the retention ratios defined above, the mechanical and hygrothermal properties of a single lamina can be stochastically represented using a micro-mechanical approach. Based on [6], the spatially stochastic tensile moduli along and perpendicular to the fiber direction ($E_1(\mathbf{x}, \Theta)$ and $E_2(\mathbf{x}, \Theta)$, respectively), shear modulus ($G_{12}(\mathbf{x}, \Theta)$), longitudinal and transverse coefficients of thermal ($\alpha_1(\mathbf{x}, \Theta)$ and $\alpha_2(\mathbf{x}, \Theta)$, respectively), and hygroscopic expansion ($\beta_1(\mathbf{x}, \Theta)$ and $\beta_2(\mathbf{x}, \Theta)$, respectively) can be expressed as:

$$E_1(\mathbf{x}, \Theta) = E_{f1}V_f(\mathbf{x}, \Theta) + F_m(\mathbf{x}, \Theta)E_m[1 - V_f(\mathbf{x}, \Theta)], \quad (16)$$

$$E_2(\mathbf{x}, \Theta) = F_m(\mathbf{x}, \Theta)E_m \left[1 - \sqrt{V_f(\mathbf{x}, \Theta)} + \frac{\sqrt{V_f(\mathbf{x}, \Theta)}}{1 - \sqrt{V_f(\mathbf{x}, \Theta)} \left(1 - \frac{F_m(\mathbf{x}, \Theta)E_m}{E_{f2}} \right)} \right], \quad (17)$$

$$G_{12}(\mathbf{x}, \Theta) = F_m(\mathbf{x}, \Theta)G_m \left[1 - \sqrt{V_f(\mathbf{x}, \Theta)} + \frac{\sqrt{V_f(\mathbf{x}, \Theta)}}{1 - \sqrt{V_f(\mathbf{x}, \Theta)} \left(1 - \frac{F_m(\mathbf{x}, \Theta)G_m}{G_{f12}} \right)} \right], \quad (18)$$

$$\alpha_1(\mathbf{x}, \Theta) = \frac{E_f\alpha_fV_f(\mathbf{x}, \Theta) + F_m(\mathbf{x}, \Theta)E_m\alpha_m[1 - V_f(\mathbf{x}, \Theta)]}{E_fV_f(\mathbf{x}, \Theta) + F_m(\mathbf{x}, \Theta)E_m[1 - V_f(\mathbf{x}, \Theta)]}, \quad (19)$$

$$\alpha_2(\mathbf{x}, \Theta) = (1 + \nu_{12m})F_h(\mathbf{x}, \Theta)\alpha_m[1 - V_f(\mathbf{x}, \Theta)] + (1 + \nu_{12f})\alpha_fV_f(\mathbf{x}, \Theta) - \alpha_1(\mathbf{x}, \Theta)\nu_{12}, \quad (20)$$

$$\beta_1(\mathbf{x}, \Theta) = \frac{E_f\beta_fV_f(\mathbf{x}, \Theta) + F_m(\mathbf{x}, \Theta)E_m\beta_m[1 - V_f(\mathbf{x}, \Theta)]}{E_fV_f(\mathbf{x}, \Theta) + F_m(\mathbf{x}, \Theta)E_m[1 - V_f(\mathbf{x}, \Theta)]}, \quad (21)$$

$$\beta_2(\mathbf{x}, \Theta) = (1 + \nu_{12m})F_h(\mathbf{x}, \Theta)\beta_m[1 - V_f(\mathbf{x}, \Theta)] + (1 + \nu_{12f})\beta_fV_f(\mathbf{x}, \Theta) - \beta_1(\mathbf{x}, \Theta)\nu_{12}, \quad (22)$$

where the subscripts f and m refer to fiber and matrix, respectively, and 1 and 2 denote the quantities along the longitudinal and transverse directions of the fibers on a lamina, respectively.

The presence of voids are neglected and, therefore, the sum of volume fractions of fiber and matrix (V_m) equals 1. In addition, the matrix Poisson's ratio is considered not degraded by hygrothermal influences (according to [53]); the same assumption is made for the matrix mass density. Consequently, using the rule of mixture, the Poisson's ratio and the density of a fiber-reinforced composite lamina can be obtained, respectively, as follows:

$$\nu_{12}(\mathbf{x}, \Theta) = \nu_{12f}V_f(\mathbf{x}, \Theta) + \nu_{12m}[1 - V_f(\mathbf{x}, \Theta)], \quad (23)$$

$$\rho(\mathbf{x}, \Theta) = \rho_fV_f(\mathbf{x}, \Theta) + \rho_m[1 - V_f(\mathbf{x}, \Theta)], \quad (24)$$

where ν_{12f} and ν_{12m} are the Poisson's ratios of fiber and matrix, respectively, and ρ_f and ρ_m are the fiber and matrix mass densities, respectively.

The moisture content in a single lamina is affected by the stochastic space-dependent fiber volume, becoming also a space-distributed random quantity. In the present research, the condition of fully saturated equilibrium (i.e., maximum absorbed moisture) is assumed, although the moisture absorption is likely to vary with time when the material is

exposed to ambient conditions. Based on [54], and using the rule of mixture, the random field representing the moisture content in the composite is expressed as follows:

$$M_c(\mathbf{x}, \Theta) = M_r \left(\frac{[1 - V_f(\mathbf{x}, \Theta)]\rho_m}{[1 - V_f(\mathbf{x}, \Theta)]\rho_m + V_f(\mathbf{x}, \Theta)\rho_f} \right). \quad (25)$$

It should be recalled that the micromechanics models used to estimate the material properties are key to ensure adequate representativeness. In this sense, Shan et al. [55] investigate the rule of mixture as one of the four commonly used micromechanics theoretical models used for calculating the elastic parameters of unidirectional carbon fiber reinforced polymer (CFRP) composites. The authors noticed that the purely deterministic rule of mixture showed a large difference in the calculated values compared to reference results, while the developed probabilistic analysis method based on the Bridging Model and the MCS proved to be fast and more accurate.

Another possible approach to deal with uncertainties affecting the intrinsic properties of multiphase materials, including laminate composites, is the use of probabilistic homogenization methods based on Representative Volume Element (RVE) or Statistical Volume Element (SVE), some of which have proved to be superior to the rule of mixture in previous studies [56]. In [57], some homogenization methods are implemented and confronted to determine the basic probabilistic characteristics of the CFRP composite in the presence of Gaussian uncertainty in its material components. More recently, machine learning techniques have been used for probabilistic FEM-based homogenization of fiber-reinforced composite structures [58]. Despite the advantages provided by more evolved homogenization techniques, the stochastic version of the rule of mixture, adopted herein, is considered sufficient for the established purpose.

2.4 Stochastic structural model

The stochastic fields described in the previous section are employed for the construction of stochastic structural models of laminated composite structures, under the assumptions

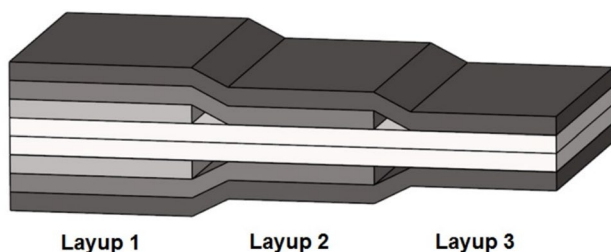


Fig. 1 Illustration of a composite part constituted of three different layups. In this example, each of the 8 laminae has its fiber angle represented by a single random variable

of the Classical Lamination Theory (CLT). All random fields are assumed not to vary across the laminate thickness. Hence, the thicknesses of individual laminae are also treated as random fields, derived by dividing the total laminate thickness at a given location by the number of laminae.

According to the CLT and considering the local coordinates x , y , and z , where x and y are in the lamina plane, with x being originated from the projection of the global X material direction, and z is the out-of-plane direction, the constitutive stress–strain relations in a state of plane stress for a k -th orthotropic lamina are written as [59]:

$$\sigma_k(\mathbf{x}, y, \Theta) = \begin{Bmatrix} \sigma_x(\mathbf{x}, y, \Theta) \\ \sigma_y(\mathbf{x}, y, \Theta) \\ \tau_{xy}(\mathbf{x}, y, \Theta) \end{Bmatrix}_k = \bar{Q}_k(\mathbf{x}, y, \Theta) [\epsilon^0 + z_k(\mathbf{x}, y, \Theta)\kappa - \epsilon^H(\mathbf{x}, y, \Theta)], \quad (26)$$

where $\bar{Q}_k(\mathbf{x}, y, \Theta)$ is the stochastic transformed stiffness matrix, $z_k(\mathbf{x}, y, \Theta)$ is the distance from the neutral surface of the laminate to the center plane of the k -th lamina in the direction of plate thickness - which is stochastic as the ply thicknesses are random -, and the vectors of mid-plane strains, curvatures and stochastic hygrothermal strains are denoted by ϵ^0 , κ , and $\epsilon^H(\mathbf{x}, y, \Theta)$, respectively.

An additional random quantity is introduced at the macromechanical level, namely the fiber-direction angle $\theta(\Theta)$, which is modeled as a random variable. This aims to account for potential deviations in the orientation of individual laminae, which is likely to occur during the manufacturing of large-scale composite structures. In such a manufacturing scheme, laminae of varying sizes are assembled with the largest of them typically spanning the entire component to ensure ply continuity and adequate ply-drops between adjacent layups, as represented in Fig. 1. Therefore, the assumption of uniform variability of the fiber-direction angle over each lamina, attributable to uncertainties arising from its placement as a whole in the layup assembly, is considered reasonable.

As a consequence of including the fiber angles as random variables, the stochastic transformed stiffness matrix $\bar{Q}_k(\mathbf{x}, y, \Theta)$ in Eq. (26) can be written as:

$$\bar{Q}_k(\mathbf{x}, y, \Theta) = T_\sigma^{-1}(\Theta) \begin{bmatrix} Q_{11}(\mathbf{x}, y, \Theta) & Q_{12}(\mathbf{x}, y, \Theta) & 0 \\ Q_{12}(\mathbf{x}, y, \Theta) & Q_{22}(\mathbf{x}, y, \Theta) & 0 \\ 0 & 0 & Q_{66}(\mathbf{x}, y, \Theta) \end{bmatrix} T_\sigma^{-T}(\Theta), \quad (27)$$

where

$$T_\sigma(\Theta) = \begin{bmatrix} \cos^2 \theta(\Theta) & \sin^2 \theta(\Theta) & 2 \sin \theta(\Theta) \cos \theta(\Theta) \\ \sin^2 \theta(\Theta) & \cos^2 \theta(\Theta) & -2 \sin \theta(\Theta) \cos \theta(\Theta) \\ -\sin \theta(\Theta) \cos \theta(\Theta) & \sin \theta(\Theta) \cos \theta(\Theta) & \cos^2 \theta(\Theta) - \sin^2 \theta(\Theta) \end{bmatrix},$$

$$\begin{aligned}
 Q_{11}(x, y, \Theta) &= \frac{E_1(x, y, \Theta)}{1 - \nu_{12}^2(x, y, \Theta) \frac{E_2(x, y, \Theta)}{E_1(x, y, \Theta)}}, \\
 Q_{22}(x, y, \Theta) &= \frac{E_2(x, y, \Theta)}{1 - \nu_{12}^2(x, y, \Theta) \frac{E_2(x, y, \Theta)}{E_1(x, y, \Theta)}}, \\
 Q_{12}(x, y, \Theta) &= \frac{\nu_{12}(x, y, \Theta) E_2(x, y, \Theta)}{1 - \nu_{12}^2(x, y, \Theta) \frac{E_2(x, y, \Theta)}{E_1(x, y, \Theta)}}, \\
 Q_{66}(x, y, \Theta) &= G_{12}(x, y, \Theta).
 \end{aligned}$$

With the displacements along the direction x , y , and z being denoted as u , v , and w , respectively, and ψ_x and ψ_y denoting the cross-section rotations about the y -axis and x -axis, respectively, the vectors of mid-plane strains, curvatures and stochastic hygrothermal strains are given, respectively, as follows:

$$\begin{aligned}
 \epsilon^0 &= \begin{Bmatrix} \epsilon_x^0 \\ \epsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} = \begin{bmatrix} \partial/\partial x & 0 & 0 \\ 0 & \partial/\partial y & 0 \\ \partial/\partial y & \partial/\partial x & 0 \end{bmatrix} \begin{Bmatrix} u \\ v \\ w \end{Bmatrix}, \\
 \kappa &= \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} = \begin{bmatrix} \partial/\partial x & 0 \\ 0 & \partial/\partial y \\ \partial/\partial y & \partial/\partial x \end{bmatrix} \begin{Bmatrix} \psi_x \\ \psi_y \end{Bmatrix}, \\
 \epsilon^H(x, y, \Theta) &= \mathbf{T}_\epsilon(\Theta) \begin{Bmatrix} \alpha_1(x, y, \Theta)[T(x, y, \Theta) - T_o] + \beta_1(x, y, \Theta)[M_c(x, y, \Theta) - M_o] \\ \alpha_2(x, y, \Theta)[T(x, y, \Theta) - T_o] + \beta_2(x, y, \Theta)[M_c(x, y, \Theta) - M_o] \\ 0 \end{Bmatrix},
 \end{aligned} \quad (28)$$

where T_o and M_o refer to the temperature and moisture content for the measured properties P_o (reference condition), respectively, and the matrix $\mathbf{T}_\epsilon(\Theta)$ is given as:

$$\mathbf{T}_\epsilon(\Theta) = \begin{bmatrix} \cos^2 \theta(\Theta) & \sin^2 \theta(\Theta) & -\sin \theta(\Theta) \cos \theta(\Theta) \\ \sin^2 \theta(\Theta) & \cos^2 \theta(\Theta) & \sin \theta(\Theta) \cos \theta(\Theta) \\ 2 \sin \theta(\Theta) \cos \theta(\Theta) & -2 \sin \theta(\Theta) \cos \theta(\Theta) & \cos^2 \theta(\Theta) - \sin^2 \theta(\Theta) \end{bmatrix}.$$

In a composite laminate formed by N laminae, the stochastic resultant forces ($N_x(x, y, \Theta)$, $N_y(x, y, \Theta)$, $N_{xy}(x, y, \Theta)$) and moments ($M_x(x, y, \Theta)$, $M_y(x, y, \Theta)$, $M_{xy}(x, y, \Theta)$) are then given as:

$$\begin{aligned}
 \begin{Bmatrix} N_x(x, y, \Theta) \\ N_y(x, y, \Theta) \\ N_{xy}(x, y, \Theta) \\ \vdots \\ M_x(x, y, \Theta) \\ M_y(x, y, \Theta) \\ M_{xy}(x, y, \Theta) \end{Bmatrix} &= \begin{bmatrix} \mathbf{A}(x, y, \Theta) & \mathbf{B}(x, y, \Theta) \\ \mathbf{B}(x, y, \Theta) & \mathbf{D}(x, y, \Theta) \end{bmatrix} \\
 \begin{Bmatrix} \epsilon_x^0 \\ \epsilon_y^0 \\ \gamma_{xy}^0 \\ \vdots \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} &= \begin{Bmatrix} N_x^H(x, y, \Theta) \\ N_y^H(x, y, \Theta) \\ N_{xy}^H(x, y, \Theta) \\ \vdots \\ M_x^H(x, y, \Theta) \\ M_y^H(x, y, \Theta) \\ M_{xy}^H(x, y, \Theta) \end{Bmatrix},
 \end{aligned} \quad (29)$$

where the matrix partitions $\mathbf{A}(x, y, \Theta)$, $\mathbf{B}(x, y, \Theta)$, and $\mathbf{D}(x, y, \Theta)$ are the stochastic extensional, bending-extensional coupling, and bending stiffness matrices, given, respectively, as:

$$\begin{aligned}
 &(\mathbf{A}(x, y, \Theta), \mathbf{B}(x, y, \Theta), \mathbf{D}(x, y, \Theta)) \\
 &= \sum_{k=1}^N \bar{\mathbf{Q}}_k(x, y, \Theta) \int_{z_{k-1}(x, y, \Theta)}^{z_k(x, y, \Theta)} (1, z, z^2) dz,
 \end{aligned} \quad (30)$$

and the stochastic hygrothermal forces, $N_x^H(x, y, \Theta)$, $N_y^H(x, y, \Theta)$, and $N_{xy}^H(x, y, \Theta)$, and moments, $M_x^H(x, y, \Theta)$, $M_y^H(x, y, \Theta)$, and $M_{xy}^H(x, y, \Theta)$, are given as:

$$\begin{aligned}
 &\left(\begin{Bmatrix} N_x^H(x, y, \Theta) \\ N_y^H(x, y, \Theta) \\ N_{xy}^H(x, y, \Theta) \end{Bmatrix}, \begin{Bmatrix} M_x^H(x, y, \Theta) \\ M_y^H(x, y, \Theta) \\ M_{xy}^H(x, y, \Theta) \end{Bmatrix} \right) \\
 &= \sum_{k=1}^N \bar{\mathbf{Q}}_k(x, y, \Theta) \epsilon^H(x, y, \Theta) \int_{z_{k-1}(x, y, \Theta)}^{z_k(x, y, \Theta)} (1, z) dz.
 \end{aligned} \quad (31)$$

Based on finite element discretization, and disregarding damping effects, the formulation summarized above leads to the following stochastic equations of motion:

$$\mathbf{M}(\Theta) \ddot{\mathbf{d}}(t) + [\mathbf{K}(\Theta) + \mathbf{K}_G + \mathbf{K}_H(\Theta)] \mathbf{d}(t) = \mathbf{Q}(t), \quad (32)$$

where $\mathbf{d}(t)$ is the vector of nodal displacements and rotations, and $\mathbf{Q}(t)$ is the vector of applied forces and moments. The deterministic geometric stiffness matrix $\mathbf{K}_G$ associated to mechanical loads (N_x^M , N_y^M , and N_{xy}^M) and the stochastic mass, structural, and hygrothermal stiffness matrices, denoted as $\mathbf{M}(\Theta)$, $\mathbf{K}(\Theta)$, and $\mathbf{K}_H(\Theta)$, respectively, are given as:

$$\mathbf{K}_G = \sum_{e=1}^{N_e} \int_{-1}^1 \int_{-1}^1 \mathbf{B}_G^T \begin{bmatrix} N_x^M & N_{xy}^M \\ N_y^M & N_{xy}^M \end{bmatrix}^{(e)} \mathbf{B}_G |\mathbf{J}| dy' dx', \quad (33)$$

$$\mathbf{M}(\Theta) = \sum_{e=1}^{N_e} \int_{-1}^1 \int_{-1}^1 \rho(\mathbf{x}^{(e)}, \Theta) h(\mathbf{x}^{(e)}, \Theta) \mathbf{B}_M^T \mathbf{B}_M |\mathbf{J}| dy' dx', \quad (34)$$

$$\begin{aligned}
 \mathbf{K}(\Theta) &= \\
 &\sum_{e=1}^{N_e} \int_{-1}^1 \int_{-1}^1 \mathbf{B}_K^T \begin{bmatrix} \mathbf{A}(\mathbf{x}^{(e)}, \Theta) & \mathbf{B}(\mathbf{x}^{(e)}, \Theta) \\ \mathbf{B}(\mathbf{x}^{(e)}, \Theta) & \mathbf{D}(\mathbf{x}^{(e)}, \Theta) \end{bmatrix}^{(e)} \mathbf{B}_K |\mathbf{J}| dy' dx',
 \end{aligned} \quad (35)$$

$$\begin{aligned}
 \mathbf{K}_H(\Theta) &= \\
 &\sum_{e=1}^{N_e} \int_{-1}^1 \int_{-1}^1 \mathbf{B}_G^T \begin{bmatrix} N_x^H(\mathbf{x}^{(e)}, \Theta) & N_{xy}^H(\mathbf{x}^{(e)}, \Theta) \\ N_y^H(\mathbf{x}^{(e)}, \Theta) & N_{xy}^H(\mathbf{x}^{(e)}, \Theta) \end{bmatrix}^{(e)} \mathbf{B}_G |\mathbf{J}| dy' dx',
 \end{aligned} \quad (36)$$

where N_e is the total number of elements, $\mathbf{J}$ is the Jacobian matrix of transformation relating the element local normalized coordinates (x', y') and the global coordinates (X, Y) , and the strain-displacement matrices $\mathbf{B}_G$, $\mathbf{B}_K$, and $\mathbf{B}_M$ are formed from derivatives of the interpolation functions.

In the present investigation, the commercial software MSC Nastran is used to build the FE model and solve the equations related to buckling [60] and modal analyses [61]. As will be further clarified later on, the introduction of the

stochastic variations is performed in a non-intrusive manner, by interfacing MATLAB scripts, which are used as pre- and post-processing for FE models, with the FE solver MSC Nastran. In FE approach, the space-distributed characterization of stochastic properties is represented by assigning unique values to each FE considering its centroidal position.

The linear buckling problem, derived from Eq. (32), is governed by the following stochastic eigenvalue problem:

$$[K(\Theta) + K_H(\Theta) + \lambda_M(\Theta) K_G] d(\Theta) = 0, \quad (37)$$

where $\lambda_M(\Theta)$ are the eigenvalues corresponding to the buckling load factors and $d(\Theta)$ are the eigenvectors corresponding to the buckling mode shapes. The lowest buckling load factor is named herein the *critical load factor*.

For solving numerically the linear buckling problem of Eq. (37), the SOL 105 of MSC Nastran is employed. The Lanczos method is chosen to solve the real eigenvalue problem. As the buckling behavior depends on the stress state in the structure caused by applied loads, these initial stresses, also known as pre-buckling stresses, are obtained from linear static analysis. Therefore, disregarding the time-dependent terms, the following linear static problems for the mechanical (Q^M) and thermal loads (Q^T), respectively, are first solved:

$$K(\Theta) d^M = Q^M, \quad (38)$$

$$K(\Theta) d^T = Q^T. \quad (39)$$

From the displacements d^M and d^T , the initial stresses states are calculated for building the stiffness matrices K_G and K_H , respectively. Then, the linear buckling problem of Eq. (37) can be solved.

Aiming at performing modal analysis, the following stochastic eigenvalue problem is derived from Eq. (32):

$$[K(\Theta) + K_H(\Theta) - \omega^2(\Theta) M(\Theta)] q(\Theta) = 0, \quad (40)$$

where ω are the natural frequencies and $q(\Theta)$ are the associated vibration mode shapes. The SOL 103 of MSC Nastran is used for the numerical resolution of this eigenvalue problem, with the Lanczos method also being used to solve the real eigenvalue problem.

2.5 Sensitivity analysis combined with ANN metamodeling

In stochastic analyses involving multiple sources of uncertainty, a measure of the relative contribution of each random parameter on the variation of responses is of utmost importance [62]. Owing to the ability to handle complex, non-linear and/or non-additive models, the Sobol' sensitivity

indices, proposed by Ilya M. Sobol [63], are widely used in the context of global sensitivity analysis. Based on the decomposition of the total variance of the model output into fractions attributable to individual inputs and their interactions, Sobol' indices are variance-based dimensionless measures of the influence of random parameters.

Monte Carlo-based estimators are frequently employed to compute Sobol' indices due to their straightforwardness, with the disadvantage of requiring high number of model evaluations for adequate convergence. In the work of Azzini and Rosati [64], an innovative algorithm including dynamic adaptive variances was proposed to overcome this drawback, which demonstrated high rate of convergence over traditional estimators. Such an approach, as a variance-based method, also allows to consider groups of inputs, which is particularly appealing in the stochastic analysis presented hereafter.

According to [64], the Sobol' first-order sensitivity index of the i -th input random parameter, which indicates its individual contribution to the output (response) variance, is estimated as follows:

$$S_i = \frac{\sum_{j=1}^N \{[y(a_j) - y(c_j^i)][y(d_j^i) - y(b_j)]\}}{\frac{1}{2} \sum_{j=1}^N \{[y(a_j) - y(b_j)]^2 + [y(c_j^i) - y(d_j^i)]^2\}}, \quad (41)$$

where $y(\cdot)$ denotes the response to the j -th input sample, which is represented by a row in a matrix of size N total samples by k random variables. In total, four sample matrices of size $N \times k$ are required for the computation of S_i : two random independent matrices, denoted as a and b ; one matrix c derived from a , where all columns of a are maintained with the exception of the i -th column, which is replaced by the i -th column of b ; and one matrix d derived from b , formed by all columns of b except from the i -th column, which is replaced by the i -th column of a .

In accordance with [65], the Sobol' total-order sensitivity index, which accounts for both the direct effect of a given input parameter i and all its interactions with other input factors, can be estimated as follows:

$$ST_i = \frac{\sum_{j=1}^N \{[y(b_j) - y(d_j^i)]^2 + [y(a_j) - y(c_j^i)]^2\}}{\sum_{j=1}^N \{[y(a_j) - y(b_j)]^2 + [y(c_j^i) - y(d_j^i)]^2\}}. \quad (42)$$

The use of surrogate models for model evaluations is very common in conjunction with Monte Carlo-based estimators for the computation of Sobol' indices, as it sometimes guarantees the feasibility of global sensitivity analysis involving complex and large-scale structural models. In the present paper, Artificial Neural Networks (ANNs) are used as a technique to approximate the structural responses (linear buckling analyses, Eq. (37), and modal analyses, Eq. (40)), within the MCS framework.

ANNs are computational models where a network of interconnected processing elements, known as artificial neurons, processes the input signals and has each of its connection associated with a weight. The weights are adjusted during the training phase to minimize the difference between predicted and actual outputs, commonly using optimization algorithms such as stochastic gradient descent and back-propagation [66].

A layered architecture is typically the arrangement for the network of nodes from an ANN, consisting of an input layer, one or more hidden layers, and an output layer. In addition, the neurons processing involves an activation function, whose choice, together with the definition of the number of training samples, layers and neurons per layer, determine the general capability of the ANN model.

The accuracy of the response predictions using ANN models is generally assessed with the Mean Squared Error (MSE) between the actual responses from MCS and the ANN predicted ones. MSE is a widely used metric for evaluating ANN performance and is sensitive to outliers, as it penalizes larger deviations more heavily due to the squaring of the differences. In addition, MSE also serves as an essential loss function during the training process, guiding optimization algorithms in iteratively adjusting the weights.

In this paper, the function *fitrnet* from MATLAB® [51] is used to train a fully connected network, where a sigmoid function is chosen as the activation function. The determination of numbers of training sample sets, hidden layers and nodes inside each layer is discussed in Sect. 3.4. A unique ANN model is used for each structural response considered, i.e., the output layer has only one node and the input layer is composed by the corresponding MC sample sets of random variables. For assessing the accuracy of ANN models (validation), the MSE between the responses obtained from MCS, not included in the training process, and the ones predicted with the trained ANN, is utilized as an indicator.

3 Test-case description and results

This section begins with a description of the FE model representing the regional aircraft composite wing of interest in this work, followed by the choices made to characterize the simulation scenarios to be considered in the numerical computations. Next, the probabilistic descriptions of the random fields and random variables involved in those simulations and the sampling-based results from MCS-LHS are presented, aiming to characterize the stochastic behavior of the critical buckling load, the first five buckling modes, and the first five natural frequencies of the composite wing. Different values of standard deviations of the input random parameters and correlation lengths associated to random

fields are considered in the numerical simulations. The stochastic responses are discussed in detail.

Thereafter, since the structural responses are predicted with ANN models for the computation of Sobol' sensitivity indices, the architecture of such models, namely, the number of layers, neurons per layer, and training sample sets, is defined. The ANN models are then trained with data derived from MCS-LHS results. Finally, a global sensitivity analysis based on Sobol' indices is performed for each of the stochastic responses, enabling the identification of the respective most influential random parameters.

3.1 The composite wing model

The FE model of the composite wing structure from a regional transport aircraft, referred to as the Embraer Benchmark Wing (EBW), is presented in Fig. 2. This model is part of a collaborative research and development partnership between Embraer and leading international institutions, serving as a benchmark model for testing and validation of new methodologies.

The EBW consists in a right wing, fully clamped at its root. Quadrilateral and triangular shell elements are used to represent skins, spars and rib covers, whereas beam elements are used to model stringers, spar caps, rib caps, aileron hinges, actuators, pylon frames, and links.

The pylon and the ribs are assumed to be made of aluminum alloy, while all other components are made of IM7/8552 carbon fiber reinforced polymer composite, which is commonly used in aerospace applications, and whose nominal properties, measured at $T_o = 75^\circ\text{F}$ and $M_o = 0\%$, are indicated in Table 1. Only symmetric laminates are considered, leading to a null matrix $\mathbf{B}$ in Eq. (29), and stiffness matrices $\mathbf{A}$ and $\mathbf{D}$ are constructed for components represented by shell elements. Composite beam elements require property homogenization and, therefore, are modeled with equivalent isotropic materials by using effective modulus.

Interpolation constraint elements are used in the model to include lumped masses, which account for the masses of components attached to the wing (e.g., fuel, actuators,

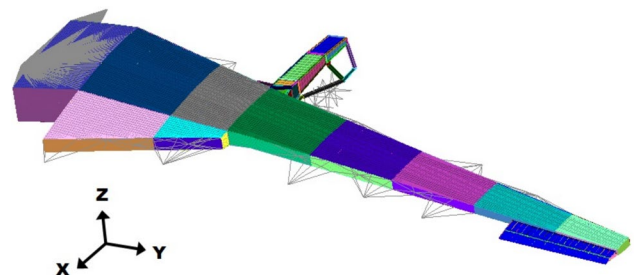
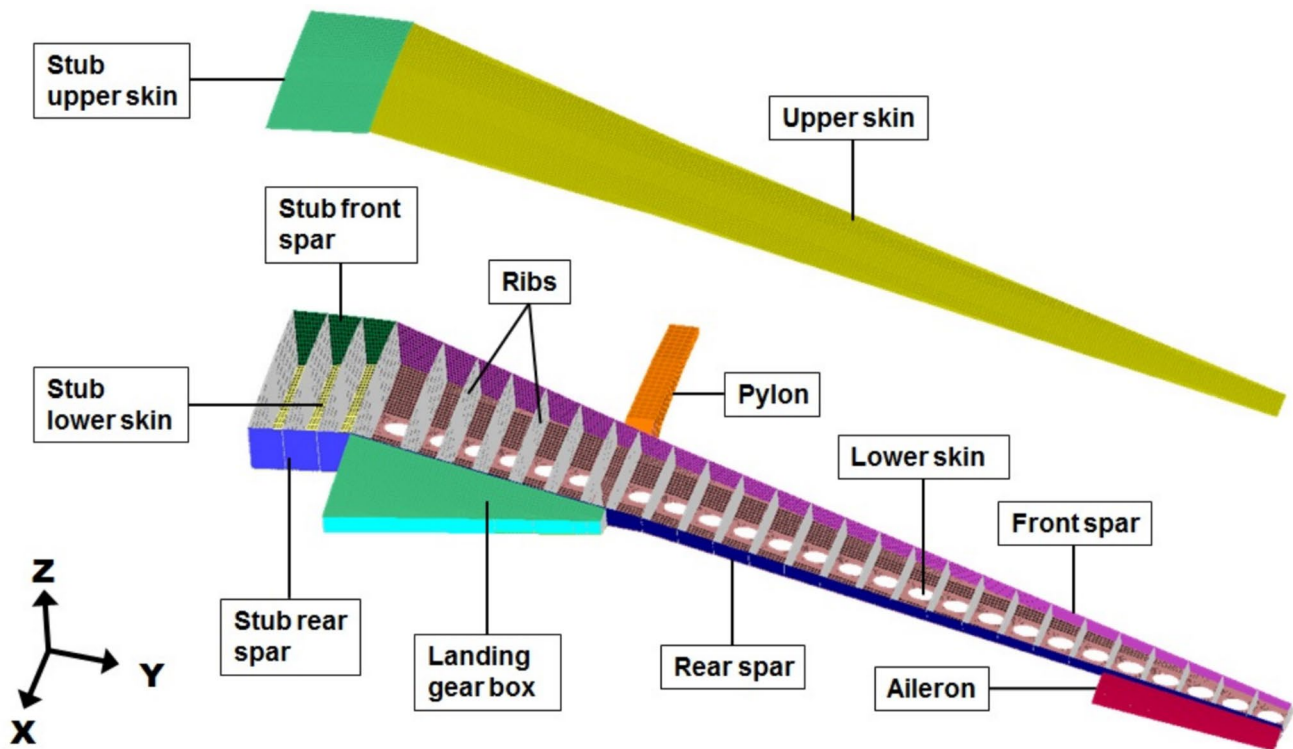


Fig. 2 The FE composite wing model, where different colors indicate groups of elements of unique geometry, layups and/or material properties

Table 1 Elastic and hygrothermal properties of IM7 carbon fiber and 8552 epoxy matrix, at $T_o = 75^\circ\text{F}$ and $M_o = 0\%$ [67–70]

Mat.	E_1 (GPa)	E_2 (GPa)	G_{12} (GPa)	ν_{12}	α ($10^{-6}/^\circ\text{F}$)	β	T_{go} ($^\circ\text{F}$)	ρ (kg/m^3)	V
IM7	276	19.5	70	0.28	− 0.36	0	—	1780	0.55
8552	4.67	4.67	1.70	0.37	28.89	0.33	392	1301	0.45

**Fig. 3** Main components of the EBW

landing gear). Regarding the applied loads, a load case consistent with a wing upward bending under maximum take-off weight (MTOW) is considered. Also, loads are multiplied by a factor of 1.5 to be in accordance with ultimate load conditions, as required for aircraft certification.

The main components of the EBW are presented in Fig. 3. Each of the four components of the stub is constituted by a unique stacking sequence, while other components of the model feature different layups, which are indicated by distinct colors in Fig. 2.

3.2 Simulation conditions

The discretization of a random field using KLE requires the appropriate choice of the number of terms M kept in the truncated series expansion, as per Eq. (1). In addition, the mesh used for space discretization of a random field has not to be the same as the one used for the structural FE model. Hence, the adoption of a coarser grid for the discretization of a random field can be explored to achieve reduction of computational cost. Both aspects are addressed in this subsection to ensure satisfactory accuracy of stochastic results,

where the resulting covariance matrices of a random field obtained after different choices for series truncation and space discretization are analyzed. For this assessment, a standard Gaussian random field $W(\mathbf{x})$ is considered and the upper skin component of the EBW is selected as the applicable domain.

In accordance with many investigations reported in the literature [20–24], an exponential covariance function is assigned to the random fields in all subsequent analyses, i.e.:

$$C(\mathbf{x}_1, \mathbf{x}_2) = \sigma_N^2 e^{-d(\mathbf{x}_1, \mathbf{x}_2)/l_c}, \quad (43)$$

where l_c is the correlation length (CL) and $d(\mathbf{x}_1, \mathbf{x}_2)$ is the geodesic distance between the positions $\mathbf{x}_1$ and $\mathbf{x}_2$. This distance metric is defined as the shortest distance between points on a curved surface and is more adequate for arbitrary geometry surfaces than Euclidean distances, in particular when dealing with curved panels. It should be pointed-out that one of the distinguished features of the present case study is that part of the domains over which the random fields are represented exhibit irregular shape, presenting significant curvature along the chord direction.

Numerical algorithms are frequently employed to determine the geodesic distance between points in a general surface. The *Fast Marching* algorithm developed by Sethian [71], which solves a discretized version of the Eikonal equation, available in the Fast Marching MATLAB toolbox [72], is used in the present work.

3.2.1 Space-discretization of the random fields

The pairwise geodesic distances between all FE mesh centroids of the EBW upper skin are computed and, using Eq. (43), a reference analytical covariance matrix C_a of dimensions $N_T \times N_T$ is obtained, where N_T is the total number of elements of the upper skin FE mesh. A coarse grid of N_C elements is obtained by grouping elements from the original grid and a similar procedure is performed to obtain a covariance matrix C_b of dimensions $N_C \times N_C$, with $N_T > N_C$.

Following the procedure described in Sect. 2.1 and using C_b , the KLE is employed to discretize $W(\mathbf{x})$ over the coarse grid, according to Eq. (1), with $M = N_C$. Subsequently, MCS is combined with LHS to generate 10,000 realizations of the stochastic field. To map the field values from the coarse mesh onto the EBW upper skin mesh, an interpolation technique is applied, where a radial basis function (RBF) of the type *multiquadric*, available in MATLAB [73], is used.

A covariance matrix of the interpolated random field is then computed from the samples using the following formula:

$$C_s(\mathbf{x}_n, \mathbf{x}_m) = \frac{1}{S-1} \sum_{i=1}^S [W(\mathbf{x}_n)_i - \bar{W}(\mathbf{x}_n)][W(\mathbf{x}_m)_i - \bar{W}(\mathbf{x}_m)], \quad (44)$$

where $\mathbf{x}_n$ and $\mathbf{x}_m$ refer to the centroid position of the n -th and m -th element of the mesh, respectively, and S is the total number of samples.

To compare the covariance matrix from sample realizations and the reference analytical C_a , the difference between those matrices is computed as follows:

$$\text{diff}(\%) = 100 \sqrt{\frac{\sum_{n=1}^{N_T} \sum_{m=1}^{N_T} [C_s(\mathbf{x}_n, \mathbf{x}_m) - C_a(\mathbf{x}_n, \mathbf{x}_m)]^2}{\sum_{n=1}^{N_T} \sum_{m=1}^{N_T} [C_a(\mathbf{x}_n, \mathbf{x}_m)]^2}}. \quad (45)$$

Three different values of correlation length (CL) for Eq. (43) are accounted for in the investigation: $L_w/15$, $L_w/3$ and L_w , where L_w refers to the total length of the wing. Also, considering s_e as the average size of the N_T finite elements of the upper skin mesh, where s_e is approximately $L_w/325$, three levels of random field mesh refinement are considered, with larger element average sizes $2s_e$, $3s_e$ and $4s_e$. Figure 4 illustrates the difference between the mesh refinement sizes.

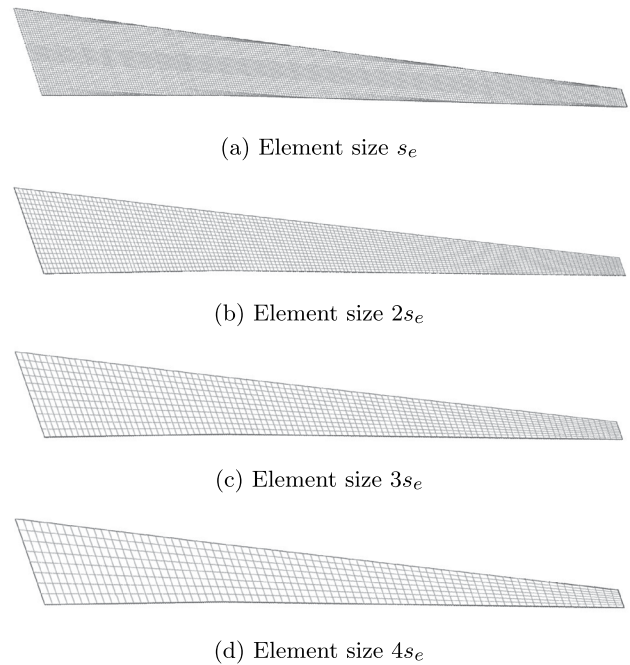


Fig. 4 Original upper skin mesh (a) and coarse upper skin meshes: $2s_e$ (b), $3s_e$ (c), and $4s_e$ (d)

Table 2 Differences between the covariance matrix computed from sample realizations and the reference analytical one for different values of coarse mesh size and correlation length

FE size	CL		
	$L_w/15$	$L_w/3$	L_w
$2s_e$	4.24%	1.27%	0.81%
$3s_e$	4.67%	1.37%	0.95%
$4s_e$	5.16%	1.57%	1.03%

The differences between the theoretical and the sampling-based covariance matrices, as indicated by Eq. (45), are presented for the nine possible scenarios in Table 2. Larger differences are observed when the element average size increases, as expected, and/or the CL decreases, due to the increased difficulty in accurately representing high-frequency spatial variability, typical of low-CL random fields.

Based on the results, the coarse mesh with FEs of size $3s_e$ is selected to discretize the random fields in all further analyses, as it brings a reduction of the number of random variables, while maintaining a satisfactory accuracy. Furthermore, such coarse mesh size is adopted for all other components involved in the subsequent uncertainty quantification.

3.2.2 Truncation criterion for KLE

The computations described in the previous Sect. 3.2.1 accounted for N_C terms in the KLE discretization over the coarse grid. As the reduction of the number of random

variables is advantageous, the number of terms used in such an expansion must also be analyzed. Considering the coarse mesh with average element size $3s_e$ applied to the upper skin, the analytical covariance matrix C_b , previously derived using geodesic distances, is adopted as a reference.

Subsequently, KLE is performed to discretize the standard Gaussian random field using C_b , as described in Sect. 2.1. In accordance with [74], the eigenvalues from Eq. (2) are sorted in decreasing order and a truncation criterion is used, according to which M is the minimum number of eigensolutions for which the ratio of the sum of the neglected eigenvalues ($\sum \lambda_{NEG}$) to the sum of the retained ones ($\sum \lambda_{RET}$) is lower than a threshold value. Then, a covariance matrix, denoted by C_k , is built only with the M terms according to the approximation indicated by Eq (4).

Using three different values for the truncation threshold and the same three values of CL previously considered, the number of retained eigensolutions and the difference between C_k and C_b are obtained for each of the nine scenarios. A formula similar to Eq. (45) is used, where C_s and C_a are replaced by C_k and C_b , respectively.

The results are found in Table 3, where the numbers of terms in KLE are presented, followed by the differences between covariance matrices in parentheses. Lower values of CL lead to slower decay of eigenvalues, as depicted in Fig. 5, which leads to the requirement of more terms in KLE. Despite the higher values of required terms for discretizing the random field, the threshold of 0.05 is selected to be applied in the KLE discretization associated with all components in further analyses. Such threshold value corresponds to the lowest covariance differences for the three values of CL considered.

3.3 Uncertainty quantification

Uncertainty quantification analyses are conducted in this section addressing the stochastic characterization of the critical buckling load, along with the first five buckling modes and natural frequencies of the EBW composite wing.

The uncertainties are assumed to affect the following eight components of the EBW, which are indicated in Fig. 3: upper skin; lower skin; front spar; rear spar; stub upper skin; stub lower skin; stub front spar; and stub rear spar. As described in Sects. 2.3 and 2.4, the stochastic input parameters considered in the analysis include temperature, fiber volume fraction, and laminate thickness, which are modeled as random fields for each component. Additionally, the fiber direction angle of each lamina constituting the layups of each respective component is treated as a random variable (see Fig. 1).

A coarse mesh with FEs of average element size $3s_e$ is built for the above-mentioned eight components to discretize the random fields. Particularly for the lower skin

Table 3 The number of required eigensolutions in KLE according to the criteria threshold value and the correlation length

$\frac{\sum \lambda_{NEG}}{\sum \lambda_{RET}}$	CL		
	$L_w/15$	$L_w/3$	L_w
< 0.15	101 (2.96%)	8 (3.90%)	2 (7.82%)
< 0.10	181 (1.90%)	13 (2.14%)	3 (3.93%)
< 0.05	378 (1.16%)	37 (0.68%)	7 (1.17%)

In parentheses, the differences between the covariance matrices built with all terms and with only the required terms

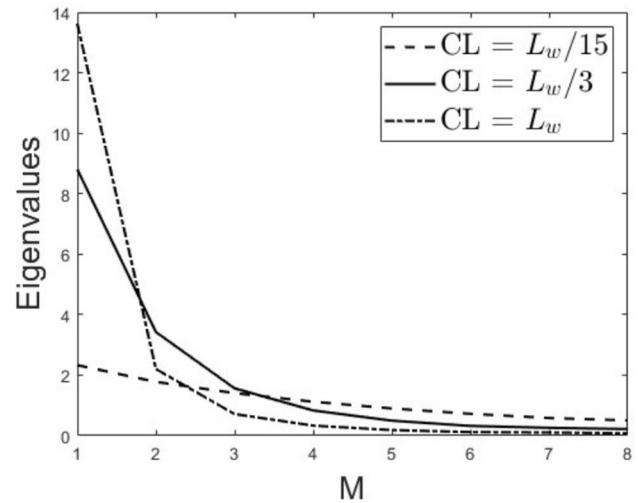


Fig. 5 Decay of KLE eigenvalues for the three values of correlation length considered

and stub lower skin, which have circular cut-outs, a specific procedure is adopted. Although the algorithm employed to determine the geodesic distances is capable of taking into account any path around a circumference in a curved surface domain, the holes need not be accounted for in the construction of the mesh to represent the random fields of interest here. In fact, for the random fields of fiber volume and laminate thickness, as the cut-outs are made after the curing process of the components, they do not alter the existing correlation between different points over the domain, which is determined by the previous manufacturing phases. In the case of the temperature stochastic field, spatial variations in environmental conditions are assumed to be unaffected by the geometric features of the structure.

In Table 4, the probability distributions assigned to each random field and random variable are presented along with the corresponding expected values and standard deviations. A uniform distribution is assumed for the laminate thickness and fiber angle, consistently with the fact that acceptable deviations related to these parameters are commonly defined in terms of absolute manufacturing tolerances enforced in a specific range. In the case of temperature, the assumption of normal distribution is justified by the possibility of positive and negative values, while for the fiber volume, as negative

Table 4 Probabilistic distributions, mean values and standard deviations assumed for each random field (RF) and random variable (RV)

Parameter	Type	Distrib.	Mean	Std. deviation (σ_N)	
				$\Delta=3$	$\Delta=10$
Laminate thickness	RF	Uniform	$\mu_t(\mathbf{x})$	$0.017\mu_k$	$0.058\mu_k$
Fiber volume	RF	Lognormal	$\mu_f = 0.55$	0.0165	0.055
Temperature (°F)	RF	Normal	$\mu_T = 122$	3.66	12.2
Fiber angle (°)	RV	Uniform	μ_a	1.73	5.77

values are not admitted, the lognormal distribution is found more appropriate.

It is worth mentioning that Gaussian and non-Gaussian random fields can be handled thanks to the use of the KL-ITAM method, presented in Sect. 2.2.

Aiming at assessing the influence of the dispersion levels of the input random quantities, two different sets of standard deviation values are assigned to them in the stochastic analyses. They are determined by the introduction of the parameter Δ , which takes two different values $\Delta = 3$ and $\Delta = 10$. For the laminate thickness, using $\mu_t(\mathbf{x})$ to denote the space-dependent mean thickness of the field and μ_k as the average thickness of all layups of a component, the adopted uniform probability density distributions have lower and upper bounds $\mu_t(\mathbf{x}) - \mu_k\Delta/100$ and $\mu_t(\mathbf{x}) + \mu_k\Delta/100$, respectively, and standard deviations $(\mu_k\Delta/50)/\sqrt{12}$. Similarly, for the fiber angles, represented by uniform probability density functions with mean value μ_a , and lower and upper bounds $\mu_a - \Delta$ and $\mu_a + \Delta$, respectively, the standard deviations are $2\Delta/\sqrt{12}$. Finally, for both temperature and fiber volume, the values of the standard deviations are adopted as $\mu_T\Delta/100$ and $\mu_f\Delta/100$, respectively, with μ_T and μ_f as the mean values of each respective random field.

It is worth mentioning that, for the random field representing the laminate thickness, different mean values are admitted throughout the field (as represented by $\bar{W}(\mathbf{x})$ in Eq. (1)) to account for different layup thicknesses, as indicated in Fig. 1. On the other hand, a unique standard deviation is considered for the entire stochastic field over each component. For the other random variables and random fields, one mean value and one standard deviation is attributed to each of them.

Among the values of CL for the exponential covariance function considered in Sect. 3.2.1, two of them, $L_w/15$ and L_w , are accounted for in the subsequent stochastic analyses. Therefore, a total of four cases are considered in the uncertainty analysis, namely: **Case 1:** $\Delta = 3$, $l_c = L_w/15$; **Case 2:** $\Delta = 3$, $l_c = L_w$; **Case 3:** $\Delta = 10$, $l_c = L_w/15$; **Case 4:** $\Delta = 10$, $l_c = L_w$.

In each study case, the KL-ITAM method, presented in Sect. 2.2, is employed to generate the Gaussian and non-Gaussian random fields over the discretization meshes, and KLE is performed using the truncation criterion previously described in Sect. 3.2.2. Table 5 summarizes the resulting

Table 5 Total number of random variables per input parameter for each study case

Parameter	Case			
	1	2	3	4
Laminate thickness	1026	28	1026	28
Fiber volume	1115	30	1115	30
Temperature	1115	30	1115	30
Fiber angle	476	476	476	476
Total	3732	564	3732	564

number of random variables for each study case, considering the eight wing components altogether.

Subsequently, the MCS is combined with LHS to generate 5000 sample sets of random variables, for each study case. Once the random quantities are spatially discretized based on the field discretization grid, the RBF interpolation is performed for distributing those quantities over the FE structural mesh. For exemplification, Fig. 6a–c illustrates realizations of the interpolated random fields over the eight components of EBW, corresponding to **Case 3** and **Case 4**. It can be noticed that high-frequency spatial variations are observed for the random fields with low CL value, while smoother space-variations are seen for the larger value of CL.

In order to verify the adequate characterization of the input random fields using KL-ITAM, the upper and lower skin components are selected for validation based on the obtained covariance functions and marginal distributions. Selecting arbitrarily one element as a reference in the space discretization mesh of the upper skin, whose centroid position is denoted as $\mathbf{x}_a$, Fig. 7 depicts the sample-based covariance of each random field, normalized by the square of the respective standard deviation, for **Case 3** and **Case 4**. Similarly, Fig. 8 depicts the normalized sample-based covariance of each random field, for **Case 3** and **Case 4**, considering a reference point $\mathbf{x}_b$ in the lower skin. Also depicted are the target unit variance covariance, which uses the geodesic distances, and the histograms of the input random quantities at positions $\mathbf{x}_a$ and $\mathbf{x}_b$, indicating the resulting mean and standard deviation.

In addition, Tables 6 and 7 show, for the reference points $\mathbf{x}_a$ in the upper skin and $\mathbf{x}_b$ in the lower skin, respectively, the difference between the normalized sample-based covariance of each random field and the target unit variance covariance. The difference is computed based on a formula

Fig. 6 Realizations of the input stochastic fields (a–c) and random variable (d), corresponding to **Case 3** and **Case 4** ($\Delta = 10$). The values of laminate thickness are normalized by the maximum value

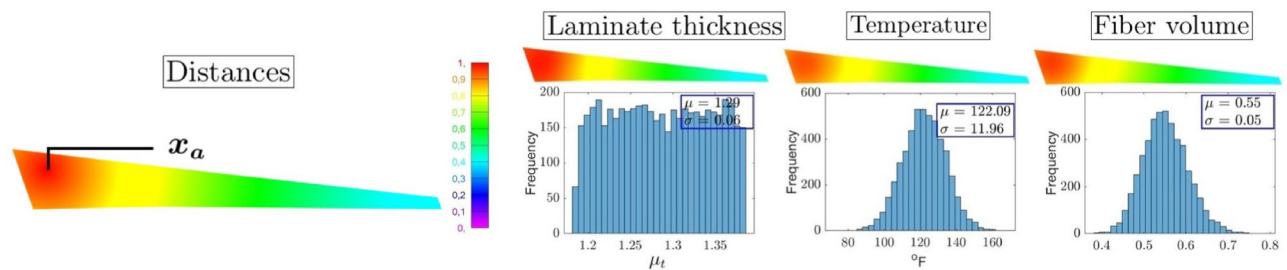
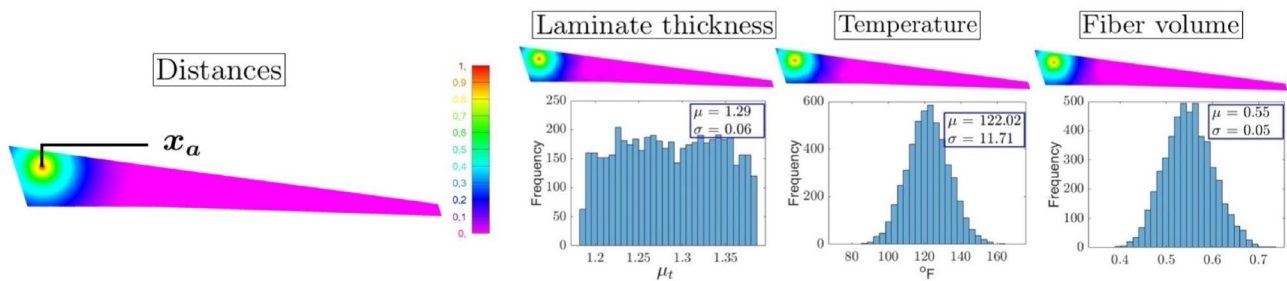
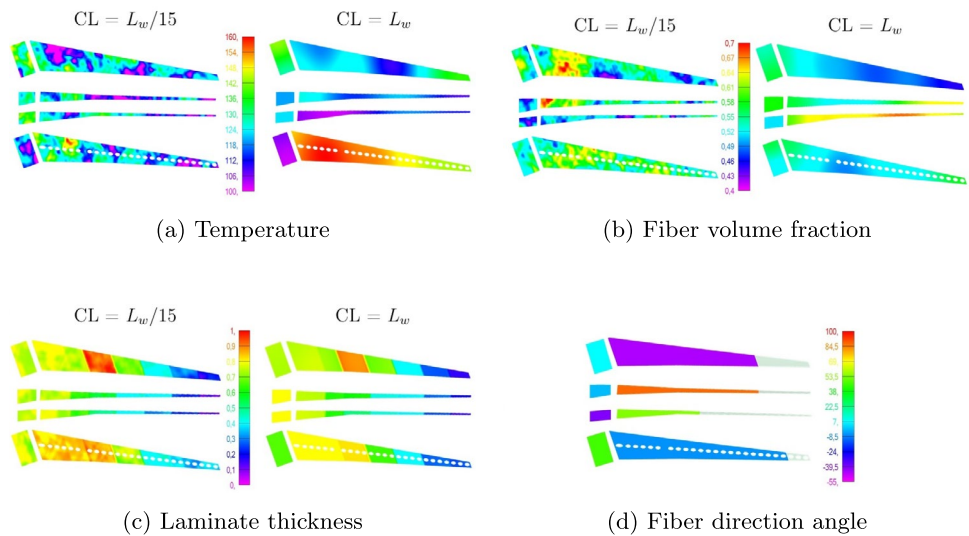


Fig. 7 Covariance functions and PDFs of the input random fields related to position x_a of the upper skin, for **Case 3** and **Case 4**

similar to that in Eq. (45), where x_n assumes a fixed value of x_a or x_b according to the reference point. The values obtained demonstrate the efficiency of the methodology.

In general, a good agreement between the specified and obtained covariance functions and probabilistic distributions can be observed for both values of CL, for all input stochastic fields. The differences indicated in Tables 6 and 7 are compatible with errors arising from the choices in the

KLE truncation criterion and mesh refinement for space-discretization of random fields, as per Sects. 3.2.1 and 3.2.2.

3.3.1 Stochastic buckling analysis

After generating the samples of the random quantities over the structural meshes of the eight EBW components, the stochastic property values are assigned to each element

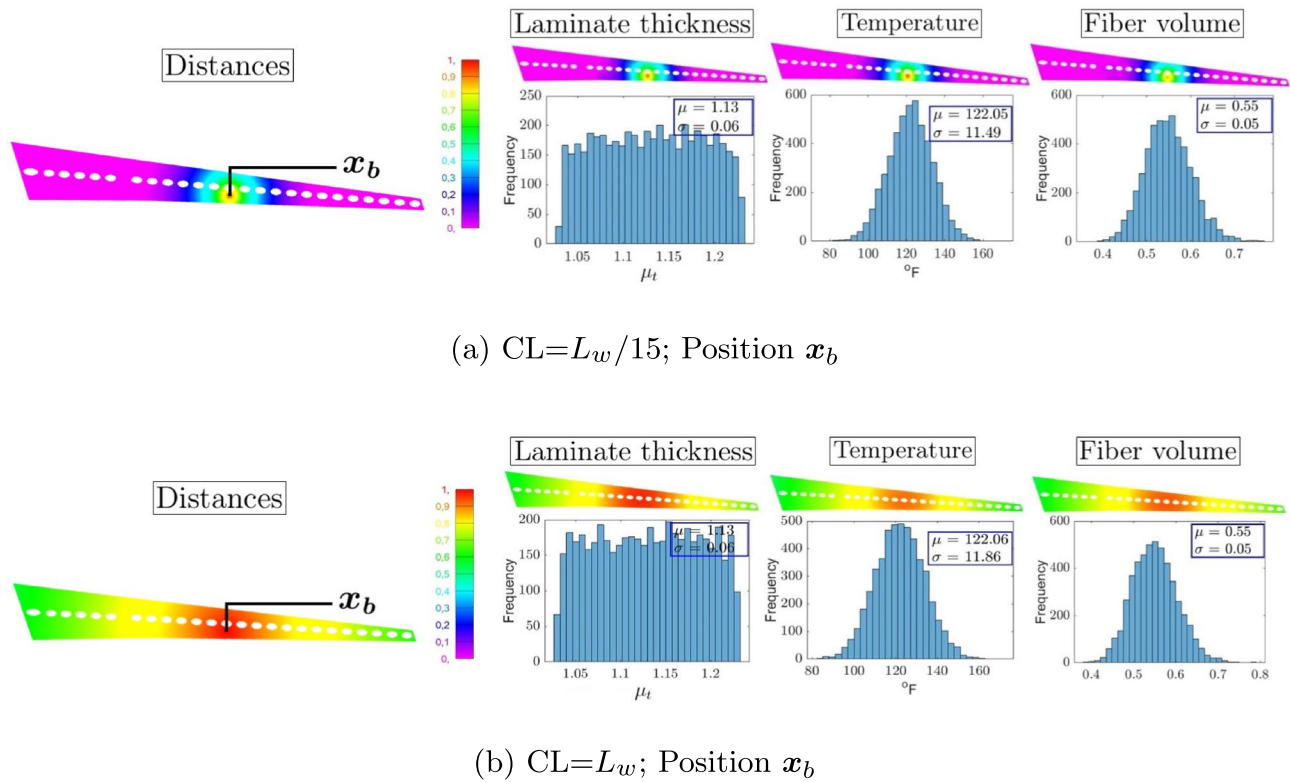


Fig. 8 Covariance functions and PDFs of the input random fields related to position x_b of the lower skin, for **Case 3** and **Case 4**

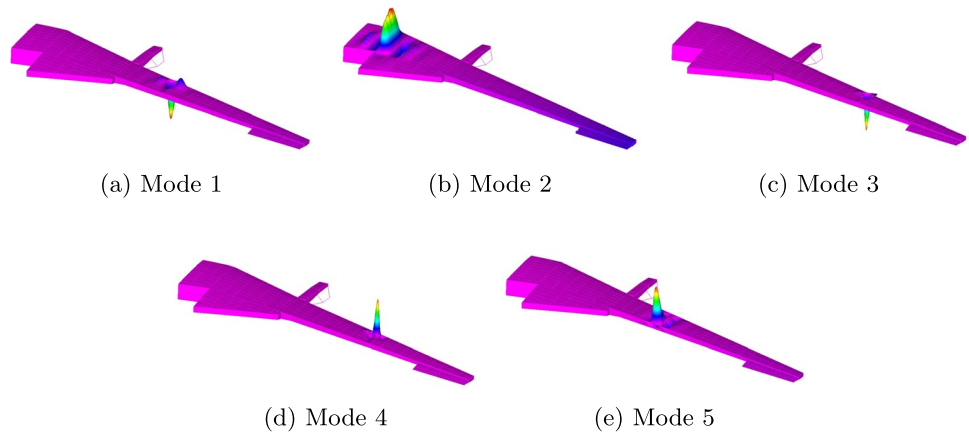
Table 6 Differences between the normalized sample-based covariance of each random field and the target unit variance covariance considering the reference point x_a in the upper skin

Parameter	Case			
	1	2	3	4
Laminate thickness	6.58%	5.56%	6.32%	2.12%
Fiber Volume	5.05%	0.97%	5.64%	1.28%
Temperature	4.49%	1.06%	4.80%	1.09%

Table 7 Differences between the normalized sample-based covariance of each random field and the target unit variance covariance considering the reference point x_b in the lower skin

Parameter	Case			
	1	2	3	4
Laminate thickness	9.91%	5.49%	8.11%	2.40%
Fiber Volume	5.56%	1.88%	5.43%	3.34%
Temperature	7.09%	3.01%	7.40%	1.48%

Fig. 9 The first five deterministic buckling modes of the EBW composite wing

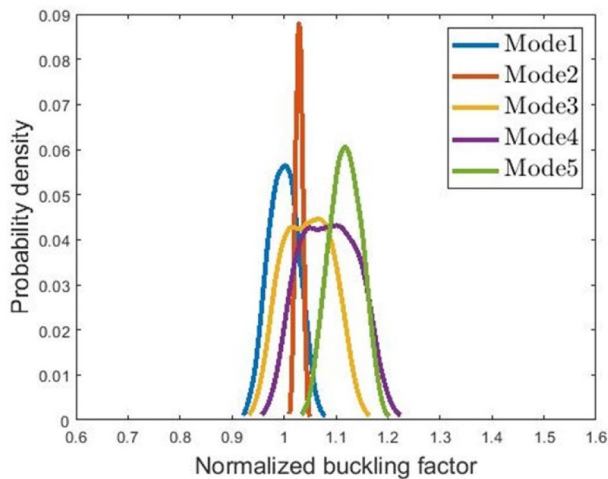


based on the value of each random input parameter at the centroid of the element, thereby characterizing the spatial distribution of these properties. The relations indicated by Eqs. (16)–(24) are employed to degrade lamina properties, whose mean values are found in Table 1, to account for environmental influences and material variations. For computing the consequent moisture random field due to the spatially varying fiber volume, as per Eq. (25), the weight fraction of moisture in the epoxy matrix is assumed as $M_r = 2.407\%$ under fully saturated equilibrium at 85% relative humidity. Such value is determined by using the deterministic version of Eq. (25) and the mean values of V_f , ρ_m , and ρ_f , aiming at achieving a moisture content in the composite of $M_c =$

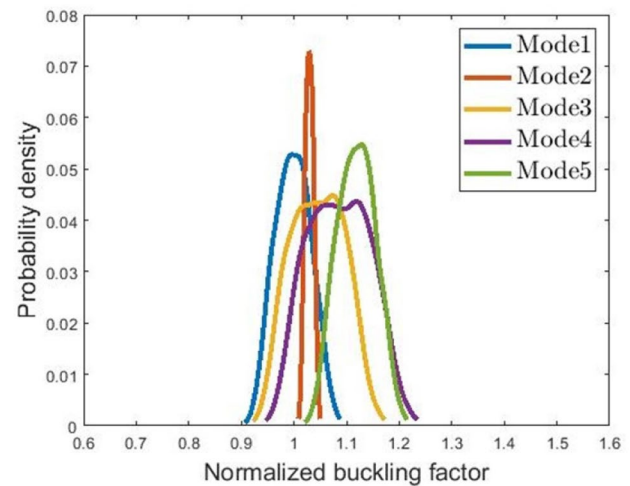
0.90% [75]. Furthermore, the sampled values of the fiber direction angles are assigned to each lamina.

Therefore, for each study case, 5000 linear buckling analyses are performed based on the EBW model, using the SOL 105 of MSC Nastran, as described in Sect. 2.4. It is worth noting that the use of MSC Nastran as the solver after external characterization of space-dependent uncertainties constitutes a non-intrusive approach to conduct stochastic buckling analysis using a commercial FE software.

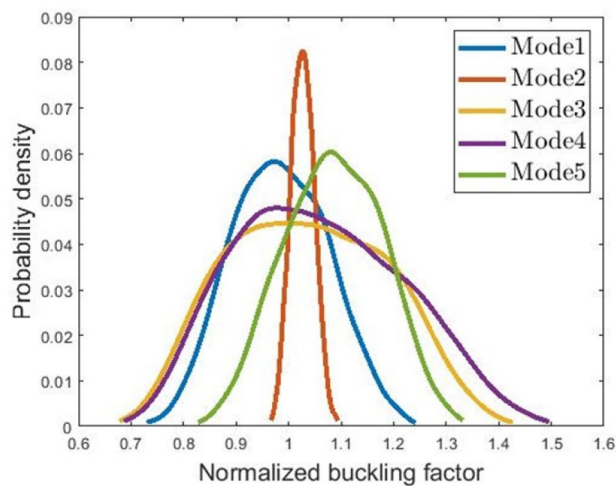
In scenarios where the input random quantities feature high dispersion levels, strong variations of the buckling response are likely to occur. In this context, a procedure based on Modal Assurance Criterion (MAC) was employed



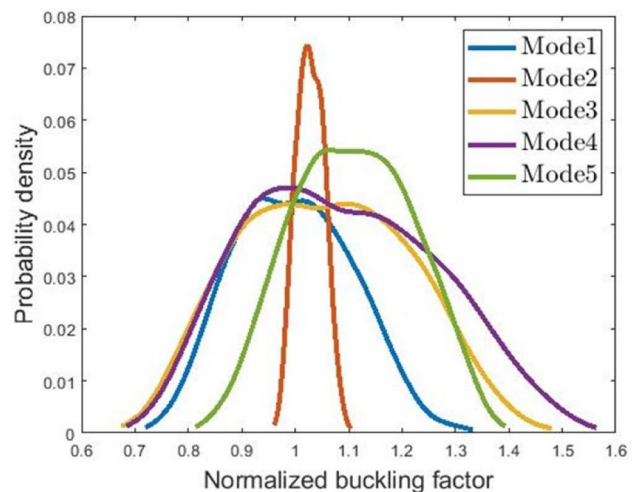
(a) **Case 1** ($\Delta = 3$, $l_c = L_w/15$)



(b) **Case 2** ($\Delta = 3$, $l_c = L_w$)



(c) **Case 3** ($\Delta = 10$, $l_c = L_w/15$)



(d) **Case 4** ($\Delta = 10$, $l_c = L_w$)

Fig. 10 PDFs for normalized buckling factors of the five reference buckling modes for all study cases

Table 8 Characteristics of the PDFs for normalized buckling factors of the five reference buckling modes for all study cases

		Case 1	Case 2	Case 3	Case 4
Mode 1	Mean	0.9988	0.9920	0.9841	0.9970
	Std.Dev.	0.0286	0.0338	0.0919	0.1112
	Skewness	0.0048	0.0135	0.0705	0.1625
	Kurtosis	2.516	2.4525	2.5828	2.5259
Mode 2	Mean	1.0292	1.0293	1.0267	1.0282
	Std.Dev.	0.0067	0.0078	0.0221	0.0261
	Skewness	0.0650	0.0295	0.0957	0.0903
	Kurtosis	2.7529	2.4410	2.7616	2.5030
Mode 3	Mean	1.0457	1.0456	1.0356	1.0544
	Std.Dev.	0.0461	0.0497	0.1483	0.1578
	Skewness	0.0214	0.0050	0.0863	0.0888
	Kurtosis	2.1811	2.1736	2.1966	2.2584
Mode 4	Mean	1.0864	1.0860	1.0578	1.0872
	Std.Dev.	0.0534	0.0578	0.1583	0.1762
	Skewness	0.0532	0.0465	0.2279	0.2199
	Kurtosis	2.1495	2.1629	2.3264	2.2632
Mode 5	Mean	1.1179	1.1187	1.0853	1.1079
	Std.Dev.	0.0307	0.0360	0.09322	0.1129
	Skewness	-0.0270	0.0032	-0.0491	0.0045
	Kurtosis	2.5473	2.4646	2.4927	2.3224

to pair the samples of stochastic responses to the corresponding deterministic mean values, thus accounting for the possible occurrence of mode switching across the samples. MAC is utilized to identify the buckling mode shapes computed from MCS corresponding to the first five mode shapes from the deterministic reference, which are depicted in Fig. 9. The MAC is computed as follows:

$$\text{MAC}_{m,n} = \frac{|\mathbf{d}_{s,m}^T \mathbf{d}_{r,n}|^2}{(\mathbf{d}_{s,m}^T \mathbf{d}_{s,m}) (\mathbf{d}_{r,n}^T \mathbf{d}_{r,n})}, \quad n = 1, 2, \dots, 5, \quad (46)$$

where $\mathbf{d}_{s,m}$ and $\mathbf{d}_{r,n}$ denote the m -th and n -th eigenvectors corresponding to the buckling mode shapes, as indicated by Eq. (37), for the MC sample under analysis and the reference (mean) model, respectively. The criterion $\text{MAC} \geq 0.8$ is adopted to indicate matching between two buckling mode shapes.

The probability density functions (PDFs) for the five buckling factors corresponding to the above-mentioned modes are shown in Fig. 10 for the four cases considered. For each case, the buckling factors are normalized by the eigenvalue corresponding to the first mode of the reference model. As can be observed, the buckling factors are quite close to each other and their PDFs overlap, confirming the possibility of the occurrence of mode-switching across MC samples. Such a tendency is amplified for higher values of CL, and even more notably with the increase of Δ , as greater dispersion is obtained for the corresponding distributions.

The characteristics of the PDFs depicted in Fig. 10, namely mean, standard deviation, skewness and kurtosis, are presented in Table 8. The mean values of the five buckling factors are found to be practically insensitive to the value of CL and Δ employed in each study case. However, the increase of CL and Δ leads to a rise of the standard deviation of all five buckling factors in different proportions: small increment of dispersion is verified with higher value of CL, whereas an increase of practically the same magnitude with which Δ is augmented can be observed for the standard deviation of all buckling factors. Furthermore, the low values of skewness indicate approximate symmetry of the distributions, and the kurtosis values less than 3 denote distributions with flatter peaks and lighter tails compared to a normal distribution.

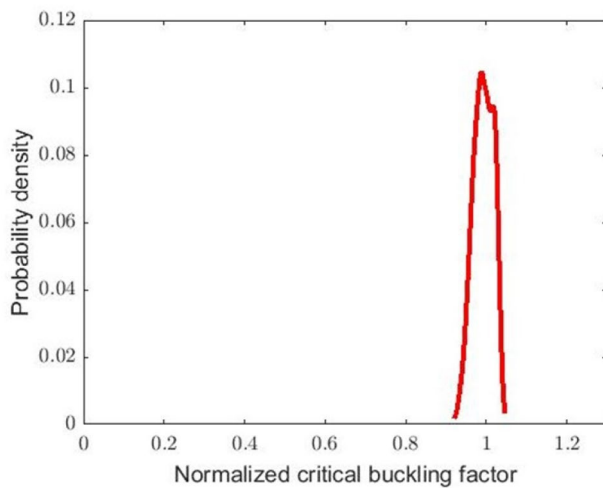
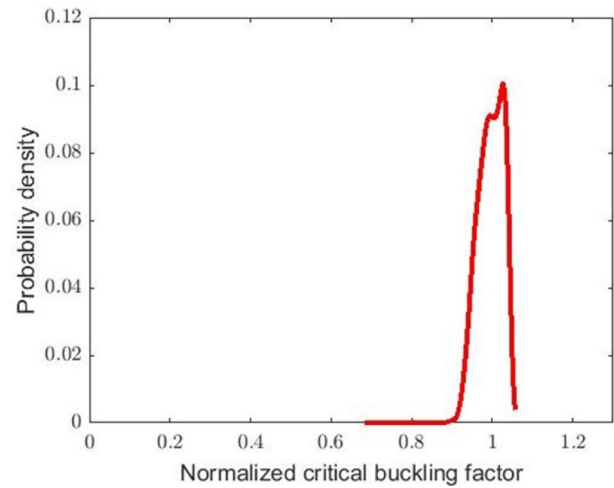
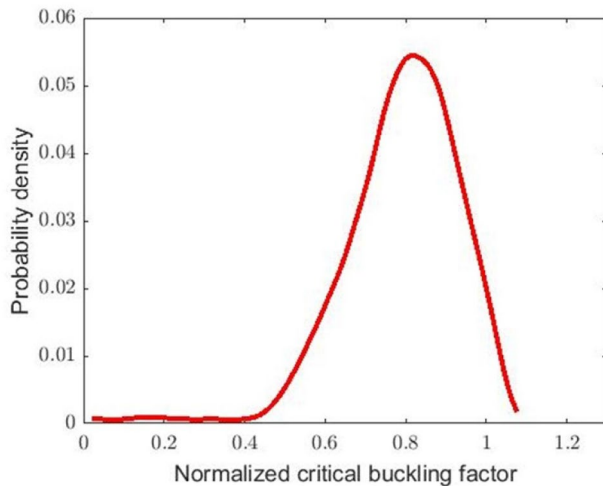
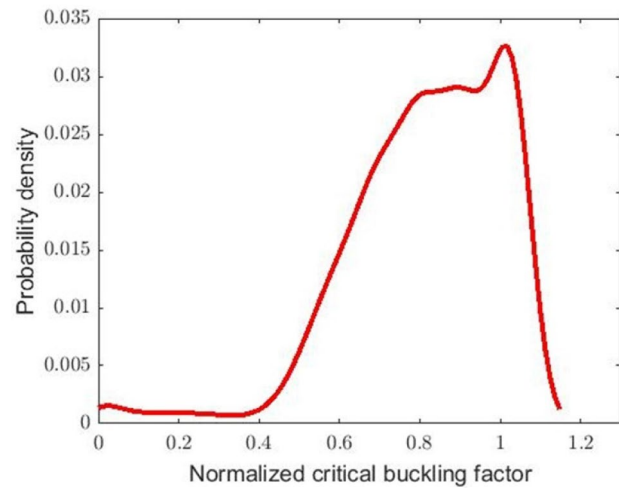
In Fig. 11, the PDFs for the critical buckling factors, which are also normalized by the first buckling factor of reference model, are shown for all study cases. The minimum buckling factor associated with each sample realization can correspond to buckling mode different of all first five from reference model, which explains the appearance of distinct factors compared to the ones already depicted in Fig. 10, in particular for the study cases with $\Delta = 10$. It should also be mentioned that some MC samples of **Case 4** resulted in buckling under only hygrothermal loads. Such samples have been disregarded.

The left-skewed characteristic combined with the higher probability of extreme values observed for PDFs of **Case 2**, **3** and **4** in Fig. 11 is corroborated by the respective negative skewness and high kurtosis presented in Table 9. Also in the same table, the mean of critical buckling factor decreases with the increase of Δ , while the standard deviation presents higher values when Δ and/or CL are augmented.

It is worth mentioning that, when it comes to buckling analysis of wing structures, localized buckling of panels can occur. For cases involving high degree of variability of stochastic input parameters, such regions can present a high sensitivity to local variations of properties. This can explain the occurrence of mode-switching and the significant level of dispersion obtained for the critical buckling factors when $\Delta = 10$.

3.3.2 Stochastic modal analysis

Numerical modal analysis was performed to generate 5,000 samples of the natural frequencies of the EBW model, for each study case. Similarly to buckling analysis, MAC was employed to pair the vibration modes from the samples with those from the mean EBW model, which are depicted in Fig. 12. The same pairing criterion of $\text{MAC} \geq 0.8$ was adopted.

(a) **Case 1** ($\Delta = 3$, $l_c = L_w/15$)(b) **Case 2** ($\Delta = 3$, $l_c = L_w$)(c) **Case 3** ($\Delta = 10$, $l_c = L_w/15$)(d) **Case 4** ($\Delta = 10$, $l_c = L_w$)**Fig. 11** PDFs for normalized critical buckling factors for all study cases**Table 9** Characteristics of the PDFs for the first five normalized buckling factors for all study cases

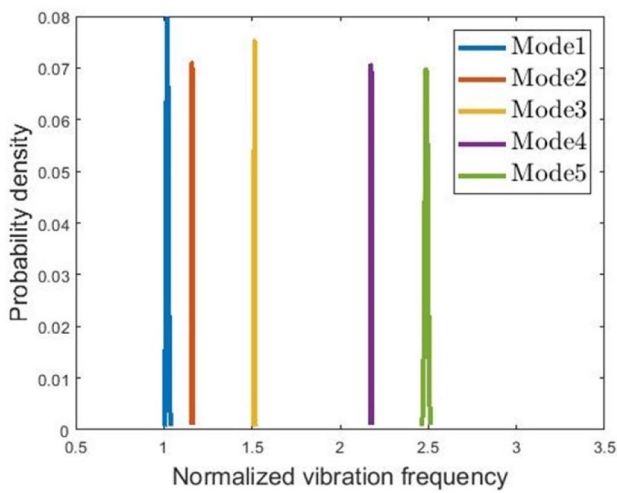
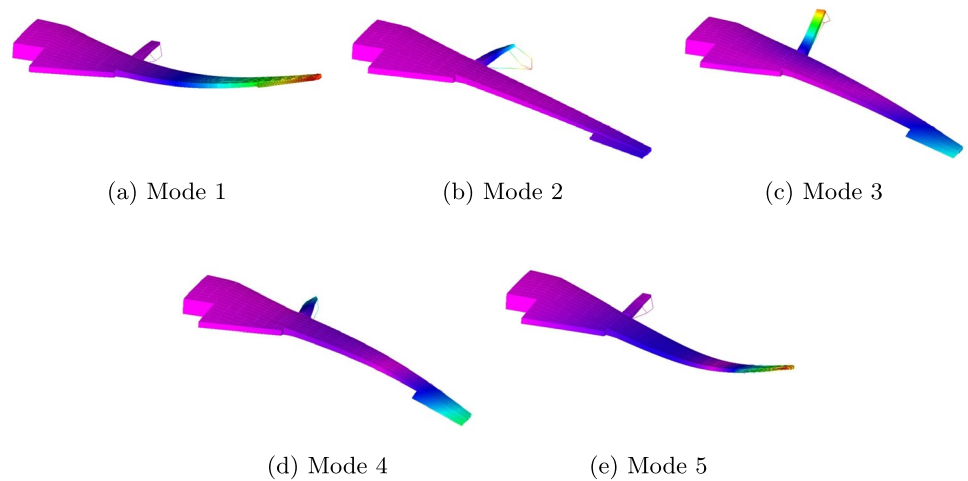
		Case 1	Case 2	Case 3	Case 4
Critical	Mean	0.9923	0.9948	0.7895	0.8143
	Std.Dev.	0.0246	0.0397	0.1456	0.1955
	Skewness	-0.2446	-8.1044	-1.2979	-1.2504
	Kurtosis	2.3553	166.6479	6.9398	5.5758

Figure 13 exhibits the PDFs obtained for the natural frequencies corresponding to the first five vibration modes, the values of which are normalized by the value of the first

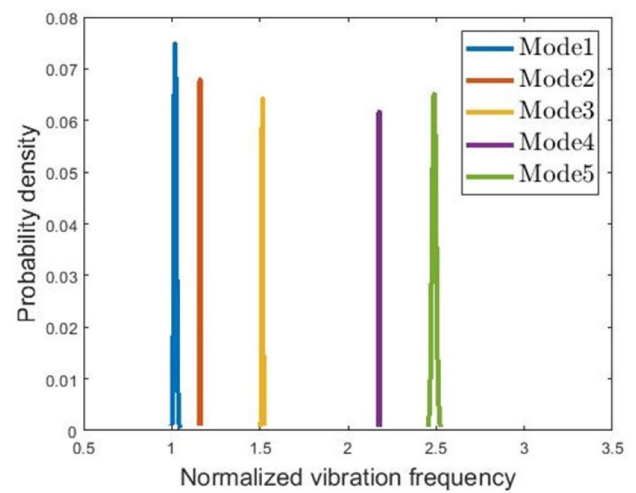
deterministic natural frequency. Mode switching across MC samples has been observed for the study cases with $\Delta=10$.

By analyzing the descriptive statistics presented in Table 10, obtained from the PDFs of Fig. 13, little sensitivity of the mean values of the natural frequencies w.r.t. CL and Δ is observed, similarly to what has been noticed in the previous buckling analysis. On the other hand, the standard deviations of the random natural frequencies are shown to increase with the increase of both CL and Δ . However, different sensitivity trends are observed across the five natural frequencies: while for the 2nd, 3rd and 5th the rate of increase of standard deviation is practically the same as the

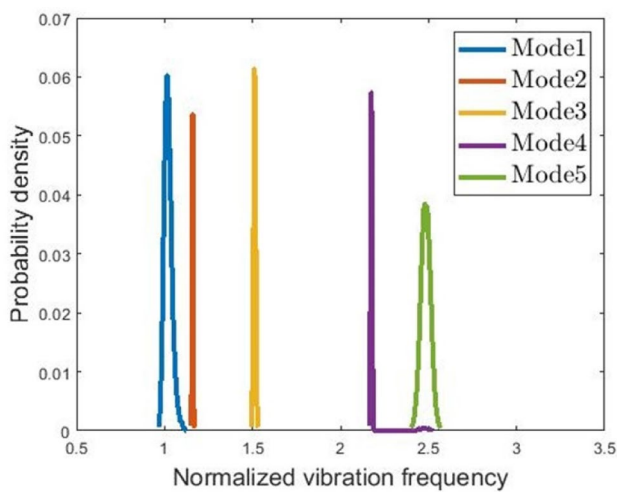
Fig. 12 The first five deterministic vibration modes of the EBW composite wing



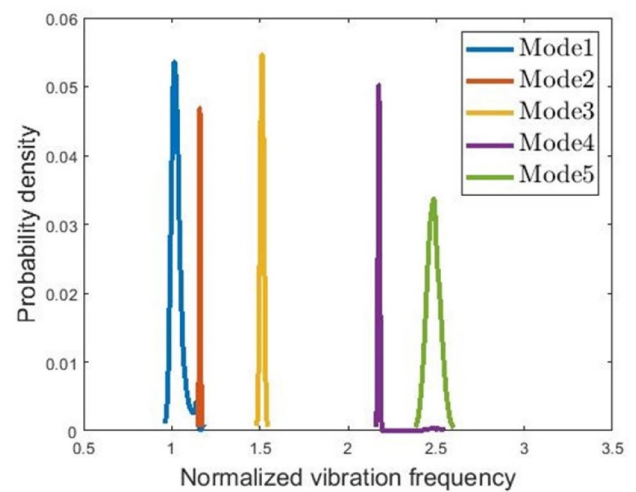
(a) **Case 1** ($\Delta = 3$, $l_c = L_w/15$)



(b) **Case 2** ($\Delta = 3$, $l_c = L_w$)



(c) **Case 3** ($\Delta = 10$, $l_c = L_w/15$)



(d) **Case 4** ($\Delta = 10$, $l_c = L_w$)

Fig. 13 Probability density functions for normalized natural frequencies of the five reference vibration modes for all study cases

Table 10 Characteristics of the PDFs for normalized natural frequencies of the five reference vibration modes for all study cases

		Case 1	Case 2	Case 3	Case 4
Mode 1	Mean	1.0191	1.0197	1.0211	1.0304
	Std.Dev.	0.0061	0.0071	0.0230	0.0350
	Skewness	0.2169	0.3742	0.9810	1.5033
	Kurtosis	3.0736	3.5345	5.9318	5.9509
Mode 2	Mean	1.1600	1.1601	1.1597	1.1600
	Std.Dev.	0.0010	0.0010	0.0031	0.0034
	Skewness	0.0092	− 0.0613	− 0.0367	− 0.0208
	Kurtosis	3.0174	3.0370	3.0595	2.9879
Mode 3	Mean	1.5141	1.5141	1.5121	1.5124
	Std.Dev.	0.0018	0.0031	0.0061	0.0102
	Skewness	− 0.0171	− 0.0615	− 0.0455	0.0067
	Kurtosis	3.0926	2.9814	3.0931	2.9535
Mode 4	Mean	2.1760	2.1760	2.1886	2.1904
	Std.Dev.	0.0012	0.0015	0.0619	0.0674
	Skewness	− 0.0612	− 0.0488	4.4693	4.2874
	Kurtosis	2.9760	2.9267	21.2222	19.7402
Mode 5	Mean	2.4885	2.4885	2.4838	2.4847
	Std.Dev.	0.0081	0.0112	0.0271	0.0370
	Skewness	− 0.0016	0.0330	0.0331	0.1513
	Kurtosis	2.9953	3.0295	2.9929	2.9168

rate at which the Δ is amplified, a higher rate of increase is observed for the standard deviations of the 1st and 4th natural frequencies. Regarding the symmetry and the tailedness of the distributions, the PDFs of all natural frequencies are practically symmetrical with tails and peaks similar to normal distributions for the cases with $\Delta=3$. For the cases with $\Delta=10$, the high values of skewness and kurtosis presented by 1st and 4th natural frequencies indicate right-skewed distributions with heavier tails.

Although the first natural frequencies are associated to mode shapes describing the global motion of the entire structure (as opposed to higher frequencies associated to local modes), considerable dispersions of the first five natural frequencies are observed from the results, confirming the sensitivity of these natural frequencies w.r.t. the random inputs considered. In particular for the 1st, 4th and 5th natural frequencies, the high degree of variability of the input parameters ($\Delta=10$) has led to elevated standard deviation values.

3.4 ANN metamodeling and sensitivity analysis

Sensitivity analysis is performed by computing the first-order and total-order Sobol' sensitivity indices associated to each study case, as described in Sect. 2.5. In each case, every structural response (buckling factors of the critical and five reference buckling modes, and natural frequencies of the five reference vibration modes) is predicted by an unique ANN model constructed for computation of Sobol'

indices. The main interest is the reduction of the high computational cost typically involved in sensitivity analyses, since MC-based estimators require a high number of model evaluations, even when techniques for alleviating the computational effort are used [64]. In the next paragraphs, the ANN architecture is first defined, followed by the estimation of the sensitivity indices.

3.4.1 Definition of ANN architectures

For the definition of the ANN architecture for each study case, i.e. number of training sample sets (N_S), number of layers (N_L), and number of neurons per layer (N_N), the buckling factors and natural frequencies associated to the first reference buckling and vibration modes, respectively, are chosen as outputs. The corresponding MC sample sets of random variables are used as inputs to the ANN model. The same architectures are later used for predicting the other buckling factors and natural frequencies of interest. For a given set of inputs, not included in the training samples, the validation of the ANNs constructed was performed by computing the MSE between the responses obtained from MCS and the ones predicted by the trained ANN.

The evolution of the MSE values as the number of training samples is increased, also considering two different numbers of layers and three distinct numbers of neurons per layer, is depicted in Figs. 14 and 15 for the buckling and vibration responses, respectively. In general, with the increase of the number of training samples, lower values of MSE are obtained. In addition, the 2-layer architectures present overall better performance in all cases examined.

Furthermore, in both Figs. 14 and 15, the trend of better performance is observed for the cases featuring small values of Δ and high values of CL. It should be reminded that, according to Tab. 5, larger values of CL imply smaller numbers of random variables. Thus, the most accurate ANN models in both buckling and vibration analyses, presenting the lowest values of MSE, are those for **Case 2**, with $\Delta=3$ and $CL=L_w$. Conversely, **Case 3**, with $\Delta=10$ and $CL=L_w/15$, exhibits the highest values of MSE.

Regarding the number of neurons per layer, it is observed that the performance is not improved monotonically with the increase of this number, which is accompanied by a significant increase of the computational effort required for training. In addition, increasing unnecessarily the number of neurons in the ANN layers can sometimes cause overfitting, which is to be avoided [76].

Therefore, the values adopted for the parameters of the ANNs used to compute Sobol' sensitivity indices are $N_L = 2$ and $N_N = 100$ for all study cases, and the number of training sample sets are chosen to vary according to the

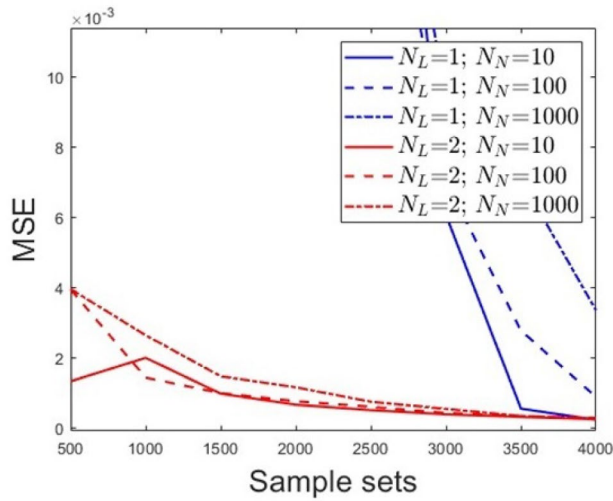
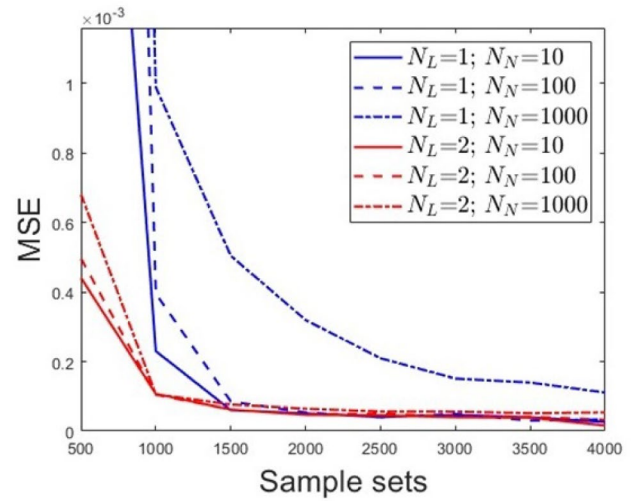
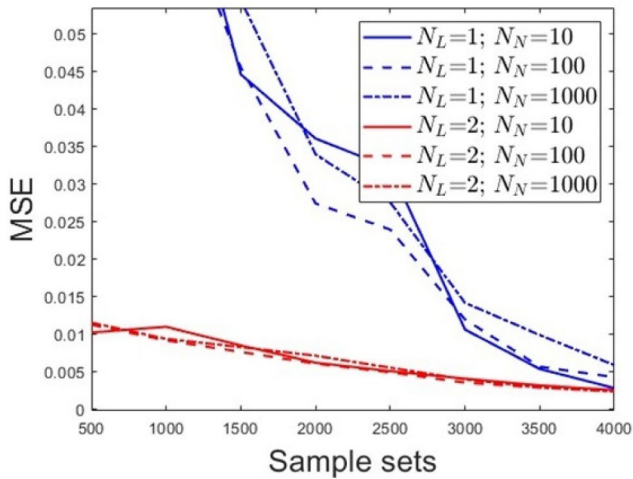
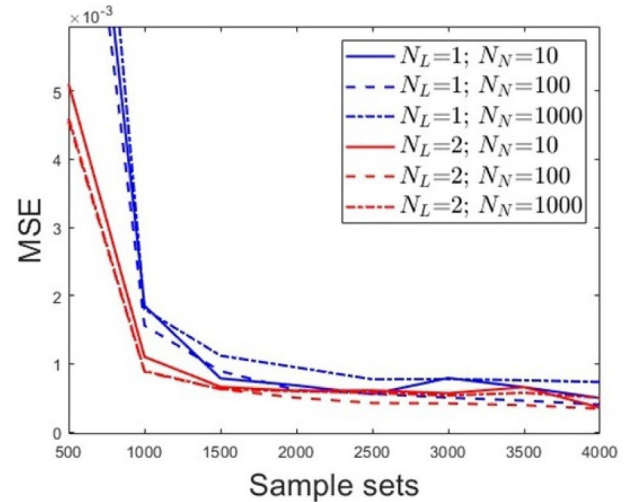

 (a) **Case 1** ($\Delta = 3$, $l_c = L_w/15$)

 (b) **Case 2** ($\Delta = 3$, $l_c = L_w$)

 (c) **Case 3** ($\Delta = 10$, $l_c = L_w/15$)

 (d) **Case 4** ($\Delta = 10$, $l_c = L_w$)

Fig. 14 Performance of ANN models in predicting the first buckling factor, in terms of MSE values as functions of the number of training data sets, for different numbers of layers and neurons per layer

case. Tables 11 and 12 indicate, for buckling and vibration analyses, respectively, the adopted ANN architecture and the resultant MSE values for each study case and response. It is noteworthy that Tables 11 and 12 also report the MSE values corresponding to the ANN predictions, with the adopted ANN architecture, of buckling factors and natural frequencies of other modes that were not accounted for in the definition of the ANN architecture. The low values of MSE obtained for all cases indicate satisfactory performance of the ANNs to approximate outputs to be used in the sensitivity analyses that follow.

3.4.2 Sobol' sensitivity indices

With the purpose of quantifying the relative influence of the variability of each stochastic input parameter - temperature, fiber volume fraction, laminate thickness, and fiber direction angle—on the variation of the output responses of the EBW (buckling factors and natural frequencies), the concept of *groups* for the computation of Sobol' indices is found more appropriate, as each input parameter is represented by a set of random variables, as indicated in Table 5. According to [64], each group can be treated as an individual random

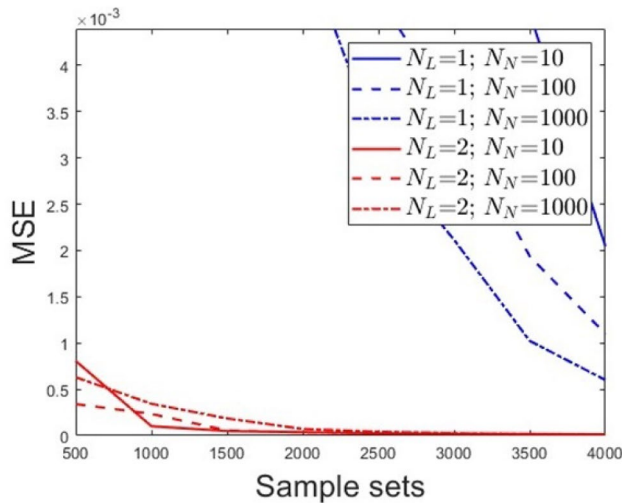
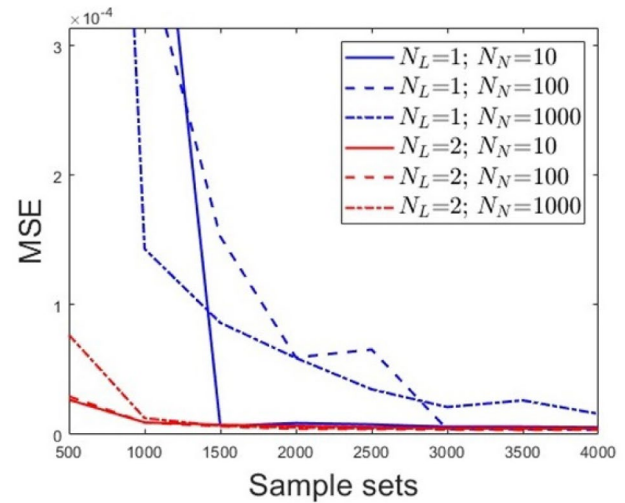
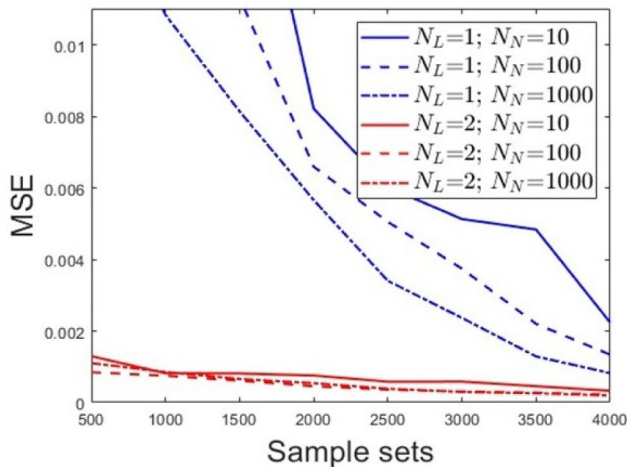
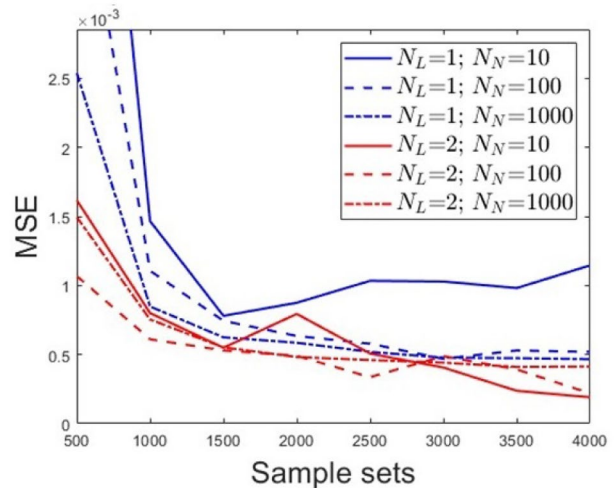
(a) **Case 1** ($\Delta = 3$, $l_c = L_w/15$)(b) **Case 2** ($\Delta = 3$, $l_c = L_w$)(c) **Case 3** ($\Delta = 10$, $l_c = L_w/15$)(d) **Case 4** ($\Delta = 10$, $l_c = L_w$)

Fig. 15 Performance of ANN models in predicting the first natural frequency, in terms of MSE values as functions of the number of training data sets, for different numbers of layers and neurons per layer

quantity as per Eq. (41) while computing the related Sobol' first-order index. In this case, the required matrices are derived by changing simultaneously the columns associated to each random variable included in the group. Analogously, the Sobol' total-order indices of the group can be computed using a similar procedure with Eq. (42).

The sensitivity indices are computed based on the responses predicted by the ANN models, whose parameters are given in Tables 11 and 12. A total of 3000 sample sets, for each response and study case, were found sufficient to achieve satisfactory convergence in the estimation of the sensitivity indices in all analyses.

In Fig. 16, the similar values obtained for the first and total-order indices of the input parameters in all study cases, for the buckling factors of all five modes and the critical buckling factor, indicate that the influence of each random parameter on the variation of the buckling responses is primarily due to its direct effect, with rather small contributions resulting from the interactions with the other parameters. In addition, the laminate thickness is shown to be the most influential factor affecting the buckling responses in nearly all study cases. This can be explained by the obvious importance of thickness on static stability of structural components.

Table 11 MSE obtained with the adopted ANN architecture (in parenthesis) to approximate the critical and the buckling factors of the five modes per study case

	Case			
	1	2	3	4
ANN arch. ($N_S; N_L; N_N$)	(2000;2;100)	(1500;2;100)	(3000;2;100)	(1500;2;100)
Mode 1	7.47×10^{-4}	7.74×10^{-5}	3.62×10^{-3}	6.92×10^{-4}
Mode 2	4.58×10^{-5}	5.94×10^{-6}	2.32×10^{-4}	5.68×10^{-5}
Mode 3	1.61×10^{-3}	1.33×10^{-4}	8.59×10^{-3}	1.27×10^{-3}
Mode 4	2.10×10^{-3}	1.59×10^{-4}	1.62×10^{-2}	1.80×10^{-3}
Mode 5	7.73×10^{-4}	6.73×10^{-5}	4.48×10^{-3}	6.03×10^{-4}
Critical	7.20×10^{-4}	7.85×10^{-4}	3.24×10^{-2}	3.65×10^{-2}

Table 12 MSE obtained with the adopted ANN architecture (in parenthesis) to approximate the vibration frequencies of the five modes per study case

	Case			
	1	2	3	4
ANN arch. ($N_S; N_L; N_N$)	(1500;2;100)	(1000;2;100)	(3000;2;100)	(1500;2;100)
Mode 1	4.96×10^{-4}	5.13×10^{-5}	2.12×10^{-3}	3.10×10^{-3}
Mode 2	1.53×10^{-5}	7.25×10^{-6}	4.72×10^{-5}	8.51×10^{-6}
Mode 3	4.37×10^{-5}	2.10×10^{-5}	1.32×10^{-4}	3.16×10^{-5}
Mode 4	1.98×10^{-5}	1.18×10^{-5}	4.72×10^{-2}	4.05×10^{-2}
Mode 5	8.06×10^{-4}	8.16×10^{-5}	1.82×10^{-3}	4.83×10^{-4}

For **Case 2** and **Case 4**, where the highest CL is employed, the temperature parameter presented very low first and total-order Sobol' indices, as shown in Fig. 16, which implies a negligible influence on the buckling responses variance either individually or considering the overall effect. For the smallest value of CL (**Case 1** and **Case 3**), the influence of temperature, and also fiber angle, on the variation of the buckling responses is increased. In general, although no significant influence of Δ is observed, **Case 1** was the study case where the input parameters presented more equally distributed influence on static stability. Furthermore, only for the critical buckling factor of **Case 4**, the fiber angles demonstrated higher influence on buckling variation than the fiber volume.

The Sobol' sensitivity indices for vibration frequencies of the five modes are presented in Fig. 17. Differences between the first and total-order indices are mainly observed in **Case 1**, indicating that the interaction among parameters is significant in the quantification of their influences when standard deviation and CL have lower values. Similar to buckling analysis, the temperature parameter has minor impact on variation of natural frequencies in scenarios with high value of CL (**Case 2** and **Case 4**). In general, although the fiber

volume fraction presented the highest values of Sobol' indices in the majority of responses and study cases, more balanced influences across the parameters are obtained for vibration frequencies.

In Fig. 17, the cases featuring the highest CL value (**Case 2** and **Case 4**) present very small influence of laminate thickness on the variations of natural frequencies of all modes, with the exception of Mode 5. Also, the fiber direction angle parameter demonstrated to be the most influential one for Mode 2 in **Case 2** and Mode 4 in **Case 2** and **Case 4**. With the increase of CL, either fiber volume or fiber angle are found to exert a more prominent influence on natural frequency variances for all vibration modes. When Δ is augmented, different behaviors are observed, with Modes 1, 4, and 5 presenting practically no variation of influence distributions when CL is high, and in the case of the lowest value of CL, the direct effect of parameters becomes more significant, as first-order indices are close to total-order ones.

It is worth mentioning about sensitivity analysis that, besides providing a systematic approach to identify the variables having the most significant influence on the variation of responses, it can also suggest the exclusion

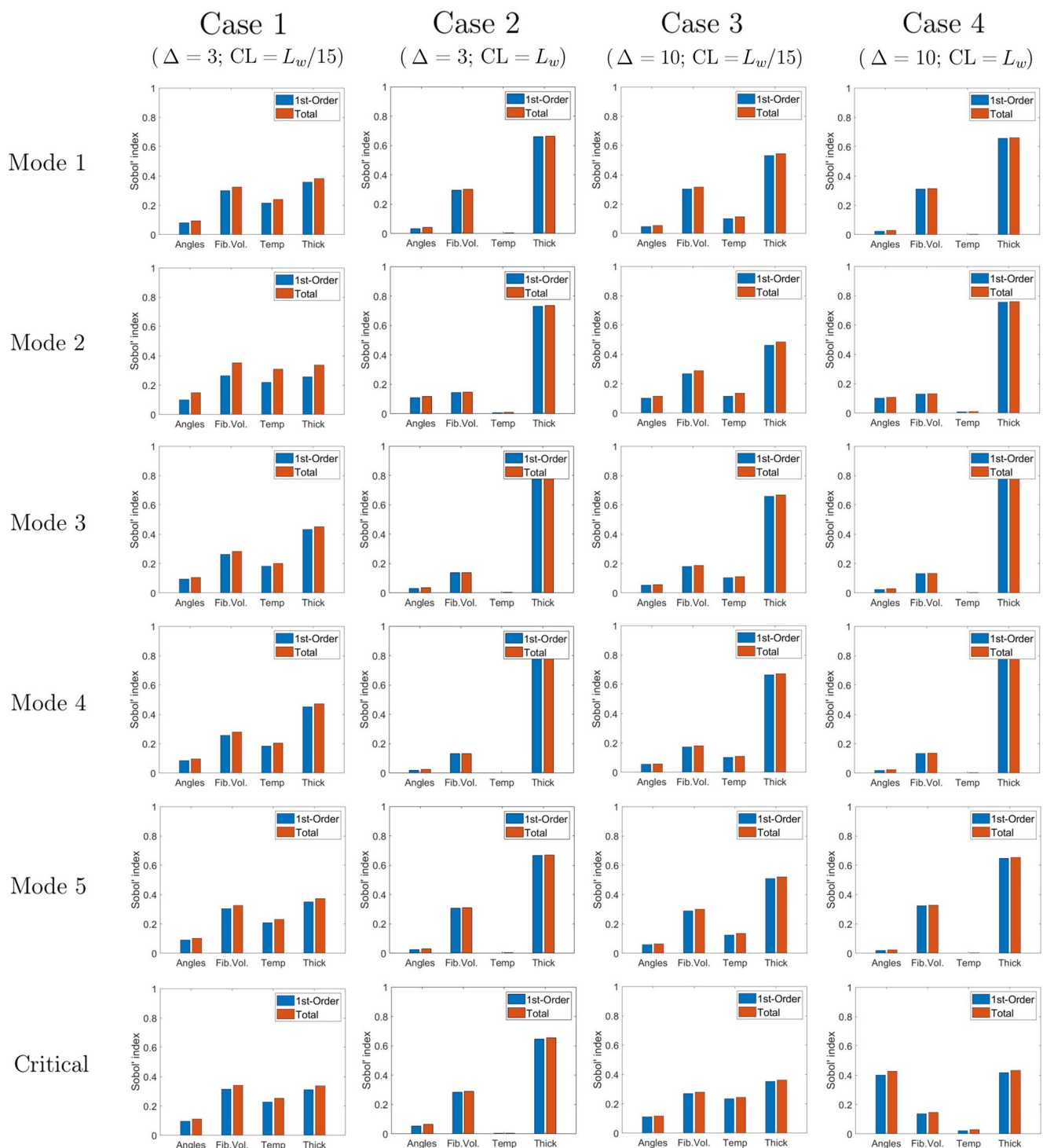


Fig. 16 Sobol' first and total-order of the stochastic input parameters related to the critical and buckling factors of the five modes per study case

of irrelevant random variables from stochastic analysis without significant loss of accuracy in the assessment of system's response variability. This can lead to model simplification and dimensionality reduction of the problem, which is particularly advantageous for complex real-world

structures. In the stochastic analysis of buckling and vibration involving the highest value of CL , with the exception of the first mode of vibration, the Sobol' indices suggest the elimination of the temperature from stochastic input parameters.

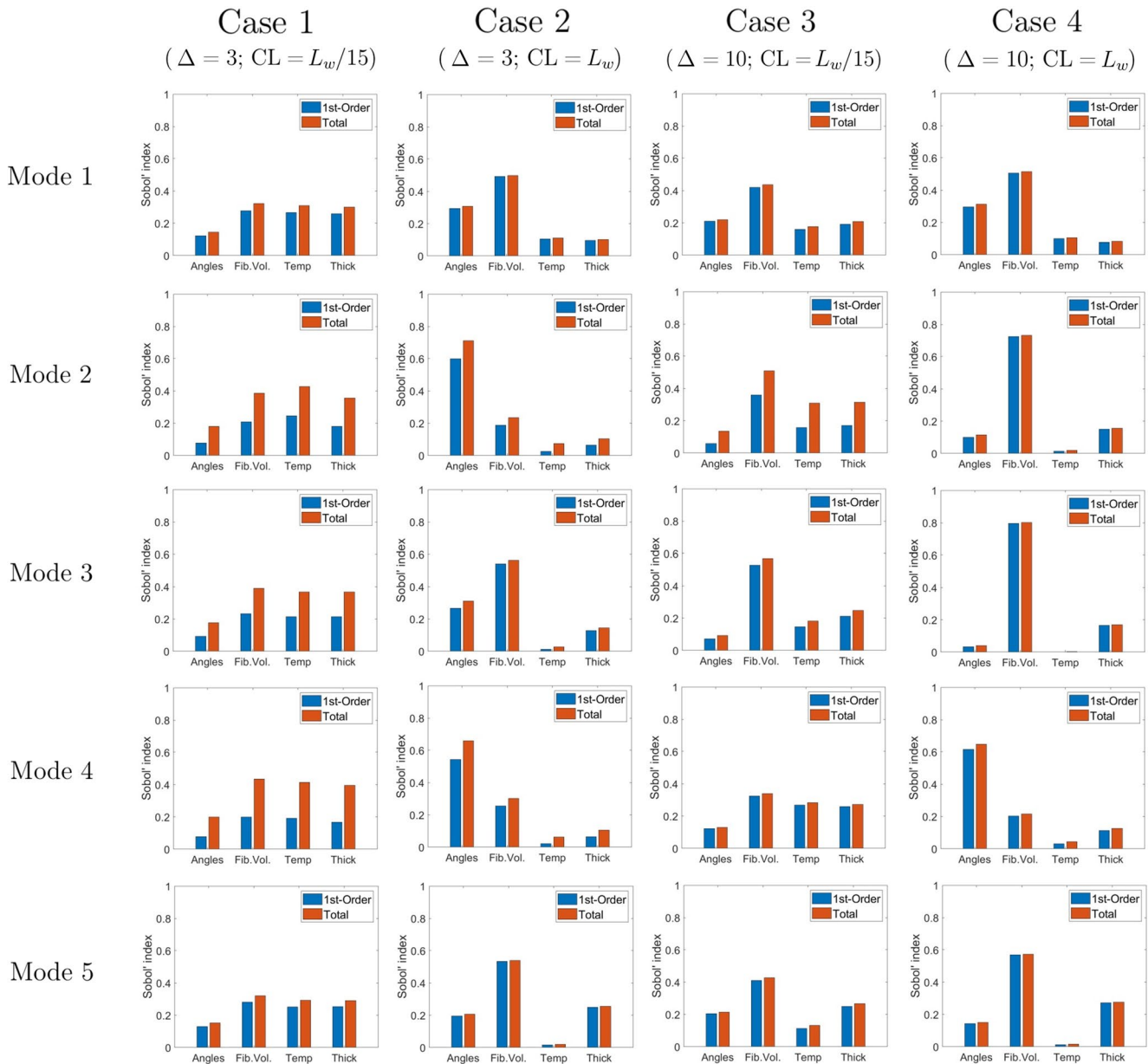


Fig. 17 Sobol' first and total-order of the stochastic input parameters related to the vibration frequencies of the five modes per study case

4 Concluding remarks

The present study investigated the influence of the combined effects of environmental and material uncertainties on the structural responses of rather complex composite structures, taking a regional aircraft composite wing as an application example. According to the methodology, the different sources of random uncertainties, including temperature, fiber volume fraction, laminate thickness, and fiber direction angle, were jointly considered. The formulation underlying the non-intrusive inclusion of those uncertainties into the FE model of the structure was based in both micro- and macromechanical frameworks.

In addition, an ensemble of numerical techniques was employed and evaluated, such as the method combining the ITAM and KLE, denoted as KL-ITAM, to deal with Gaussian and non-Gaussian random fields, and the ANNs to approximate the structural responses for guaranteeing the feasibility of the global sensitivity analysis.

Different scenarios were chosen to be investigated, where the influence of the random input variables on the buckling resistance and natural frequencies of the test structure was assessed. Across these scenarios, different values of relevant parameters of the input random quantities (standard deviation and correlation length) were considered, with the aim of appraising their influences on the

input–output relations and also the accuracy of the predictions by ANN-based surrogate models.

The general conclusion of the investigation is that the methodology proposed was shown to be effective in dealing with the situations considered, which are likely to be found in real-world industrial problems involving high-dimension structural models, frequently characterized by large numbers of random inputs. Although a wing structure has been considered in this study, other types of structures made of composites, in particular those for aeronautical applications, can be investigated using the proposed approach to incorporate uncertainties arising from the fundamental sources. Moreover, as the developed methodology enables the treatment of non-Gaussian random fields over complex domains, it can be readily applied to a general finite element meshed surface of arbitrary geometry, where probabilistic distributions and covariance functions different from those considered in this study may be employed.

Therefore, the major contributions of the present work can be summarized as follows: (1) proposition of a stochastic modeling methodology enabling the simultaneous incorporation of random variables and random fields with different probabilistic properties associated to the key sources of uncertainties — hygrothermal conditions and material properties —, affecting industrial-scale composite structures, where the thermo-mechanical interactions are accounted for; (2) application of the methodology to a set of scenarios that enabled the quantification of the probabilistic characteristics of stochastic structural responses and the assessment of the relative importance of the different types of uncertainty considered; (3) advancement in the maturity of UQ procedures as applied to complex composite aircraft structures.

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Author contributions H.E.A.A.S. performed the writing of the manuscript (original draft), data curation, investigation, software development, and visualization. H.E.A.A.S. and D.A.R. conducted the formal analysis and designed the methodology. D.A.R. supervised the study. All authors contributed to the conceptualization, discussion of results, and writing (review and editing) of the final manuscript. In addition, all authors read and approved the final manuscript.

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Data availability Data sets generated during the study are available from the corresponding author upon request.

Declarations

Conflict of interest D. A. Rade, Associate Editor, was not involved in the peer review or decision-making process of this manuscript. The other authors have no Conflict of interest to declare that are relevant to the content of this article.

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