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A literature review

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Load flow computations for (integrated) Transmission and Distribution systems. A literature review.

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Abstract

Electrical power systems are complex systems and traditionally modeled in two separate systems. Power is generated at the transmission system and at several substations converted to the distribution systems. The increasing amount of generation produced at distribution level can eventually effect the transmission network. An integrated model of both systems can help studying these effects and prevent harmful events on the power system.

Transmission and Distribution systems differ significantly from each other. Where transmission systems are assumed to be balanced and therefore modeled as a single-phase system, the distribution systems are in general unbalanced and should be modeled in three-phase. Furthermore, high R/X ratios of distribution lines, the lower voltage level, the radial structure and the presence of unbalanced loading lead to different solution techniques. Connecting these two systems pose complications for both the solution method and the connection method.

In this report, we present several methods to solve the integrated Transmission-Distribution system. One of them is to omit the simplifications we can make in a transmission system and solve both systems as a three-phase system. Another method is to use a master-slave splitting approach and solve both systems iteratively, using a boundary state. A last method is building an interconnected network which solves the system at once, respecting both the transmission and distribution conditions. Some artificial currents and voltages have to be injected on the boundary.
We compare the different methods on CPU-time, convergence, accuracy and complexity and present the preferable method for the specific network criteria.

Contents

1 Power Flow Analysis

1.1 Introduction

The electrical power system is one of the most important systems in the world. It provides the generation, transmission and distribution of electrical energy. Among most of us, it is regarded as a commodity, while we do not see its complexity.
Most of the power systems work on a frequency of 50 Hz [25], meaning the number of oscillations of Alternating Current (AC). Traditionally, electricity is generated in large power plants whereupon it is transmitted to substations through the transmission network at high voltage AC. The substations are connected to the distribution network, from which electricity is distributed to households on lower voltage AC. Nowadays, the electrical power system is changing from a centralized, passive top-down network to a decentralized, active bottom-up network with the development of Renewable Energy Sources (RES) and its connection to the electrical power system at distribution level. These RES are intermittent and stochastic and make control of the electricity network very hard.

1.2 The Power System in general

The power system consists of a High-Voltage network, a Medium-Voltage network and a Low-Voltage Network. The High-Voltage network is also called the transmission system, it consists of overhead cables which transports the power over large distances, on voltages around $220 - 380$ kV. High voltage leads to lower losses of power. Its operation is the responsibility of the Transmission System Operator (TSO). The Medium and Low-voltage network are operated by the Distribution System Operator (DSO). The Medium-Voltage network consists of overhead cables as well, but power is transported on voltages around $10 - 20$ kV. The Low-Voltage system carries power below 1 kV on underground lines, from substations to end-consumers. The substations connect the three systems. A substation consists of a transformer bank, which transforms the power to lower voltage power. The distribution network used to be entirely radial, meaning that power is generated (or injected) at the substations and then transported in one direction to end-consumers. Nowadays, power is more and more generated at distribution level, causing weakly meshed networks and bidirectional flow. The transmission network is meshed, meaning that nodes are connected in one or more ways to each other.

The TSO is also responsible for control of the frequency balance of the entire system, this is regulated on several trading market and imbalance markets.

1.2.1 Power equations

As said, the power system in Europe is mainly a 50 Hz frequency three-phase AC system [25]. Alternating Current means that voltage and current reverse their direction periodically. The wave form can assumed to be a sinusoidal function:

$$v(t) = \sqrt{2}|V|\sin\omega t \quad \text{and} \quad i(t) = \sqrt{2}|I|\sin(\omega t) \tag{1}$$

ω the angular frequency in [rad/s]

The $|V|$ and $|I|$ are the Root Mean Square (RMS) value or the effective value of the voltage and current. The RMS of the voltage ($|V|$) is found by taking the square root of the mean of $v(t)$:

$$\sqrt{\frac{1}{T}\int_0^T v^2 dt} = \sqrt{2}|V|\sqrt{\frac{1}{T}\int_0^T \sin^2(\omega t)dt} = \sqrt{2}|V|\sqrt{\frac{1}{2}} = |V| \tag{2}$$

T is the period of the sine wave in $[s]$

Power is the product of voltage and current: $P = VI$. As the instantaneous power is continuously varying with time, the average power is more useful for steady-state computations and obtained by taking its integral. The instantaneous power and the average power are written as:

$$p(t) = \frac{v^2(t)}{R} = i^2(t)R \quad \text{and} \quad P = \frac{1}{T}\int_0^T \frac{v^2}{R}dt \tag{3}$$

R is the resistance

Balanced three-phase AC systems Power is transported in three-phase, because of efficiency reasons. The transmission system is in balance while the distribution system is not in balance. Unbalancedness is explained in section 2.2. Balanced means that the sinusoidal voltages and currents are of equal magnitude in all phases, but shifted in phase by $2\pi/3$ rad. It results in solving the three-phase power system with a single-phase equivalent. The other two phases are computed by compensating for the phase-shift.
In a balanced three-phase AC system, the alternating voltage and current are described by:

$$\begin{aligned} v_a &= \sqrt{2}|V|\cos(\omega t) \\ v_b &= \sqrt{2}|V|\cos(\omega t - \frac{2\pi}{3}) \\ v_c &= \sqrt{2}|V|\cos(\omega t - \frac{4\pi}{3}), \end{aligned} \tag{4}$$

$$\begin{aligned} i_a &= \sqrt{2}|I|\cos(\omega t - \phi) \\ i_b &= \sqrt{2}|I|\cos(\omega t - \phi - \frac{2\pi}{3}) \\ i_c &= \sqrt{2}|I|\cos(\omega t - \phi - \frac{4\pi}{3}) \end{aligned} \tag{5}$$

Phasor representation

In power system theory, the effective phasor notation is used [17]. The amplitude (or the peak) of the sinusoidal voltage is $\sqrt{2}$ times the effective phasor or Root Mean Square (RMS): $|V|$. The voltage is rotating counter clockwise with a frequency of 50 Hz. The phasor notation is found with the help of the single-phase equivalent v_s of the voltage representation:

$$v_s = \sqrt{2}|V|\cos(\omega t - \delta_V),$$

δ_V representing the phase-shift (0, $\frac{2\pi}{3}$ and $\frac{4\pi}{3}$ respectively).
Using the Euler identity $e^{\iota\phi} = \cos(\phi) + \iota\sin(\phi)$, the voltage can be written as:

$$v_s = Re\{\sqrt{2}|V|e^{\iota\omega t}e^{\iota\delta_V}\} = Re\{\sqrt{2}Ve^{\iota\omega t}\} \tag{6}$$

Where $V = |V|e^{\iota\delta_V}$. The same can be done for the current, leading to the effective phasor representations:

$$V = |V|\exp(\iota\delta_V), \tag{7}$$
$$I = |I|\exp(\iota\delta_I) \tag{8}$$

Knowing the single-phase values of the voltages and currents, the other quantities can be found by rotating the corresponding phasors with respectively $2/3\pi$ and $4/3\pi$ rad. The voltage, current and power are represented by a per unit normalization, which is the actual value divided by the base value. Later on in this section, the per unit calculations are explained.

Reactive, Active and Complex Power Using the single-phase representation of the balanced three-phase system and the current and voltage relations v_a and i_a in equations (4) and (5), the instantaneous power can be written as:

$$\begin{aligned} p(t) &= v_a(t)i_a(t) \\ &= 2|V||I|\cos(\omega t)\cos(\omega t - \phi) \\ &= |V||I|\cos(\phi)[1 + \cos(2\omega t] + |V||I|\sin(\phi)\sin(2\omega t) \\ &= P[1 + cos(2\omega t)] + Q\sin(2\omega t) \end{aligned} \tag{9}$$

The first term of the last line $P[1 + cos(2\omega t)]$ describes an unidirectional component of the instantaneous power with an average value P, this is called the real or active power in $[W]$. The second term $Q\sin(2\omega t)$ is a bidirectional (or oscillating) component, the amplitude Q is the reactive or imaginary power in $[var]$.
The active power and reactive power are written as:

$$P = |V||I|\cos(\phi) \quad \text{and} \quad Q = |V||I|\sin(\phi) \tag{10}$$

The complex power S can be found by multiplication of the voltage and the complex conjugate of the current ($\overline{I}$). The apparent power $|S|$ is the magnitude of the complex power:

$$S = V\overline{I} = P + \iota Q \quad \text{and} \quad |S| = |V||I|$$

Admittance The relation between voltage and current is described by Ohm's law:

$$I = YV \tag{11}$$

Y is the Admittance, consisting of conductance (G) and susceptance (B) and it is the inverse of impedance (Z) which consists of resistance (R) and reactance (X):

$$Y = \frac{I}{V} = G + \iota B \quad \text{and} \quad Z = \frac{1}{Y} = R + \iota X \tag{12}$$

Power elements (transformers, generators, cables, etc.) exhibit impedance.

Per Unit System Like many engineering calculations, the power system is analyzed in non-dimensional quantities: the per-unit system. The per-unit system has advantages: scaling leads to values within a narrowed range, the voltage is close to unity (as the voltage is chosen as base value) and it eliminates ideal transformers as circuit components [15], which can be seen in section 1.3.
The scaling equation of the per-unit system is:

$$\text{Per-unit value} = \frac{\text{actual value}}{\text{base value}} \tag{13}$$

As the actual value and the base value have the same units, the per-unit value is dimensionless. In order to set up the per-unit system, the base values for Voltage, Current, Impedance and Complex Power are chosen. Two of them are chosen arbitrarily, the third one is deducted from: $V_{base} I_{base} = S_{base}$ and Z_{base} is derived from: $Z_{base} = \frac{V_{base}}{I_{base}}$. As $Z = R + \iota X$ and $S = P + \iota Q$, there is no need for other base values, they can be defined accordingly: $Z_{base} = R_{base} = X_{base}$ and $S_{base} = P_{base} = Q_{base}$, because $Z_{pu} = \frac{R+\iota X}{Z_{base}} = \left(\frac{R}{Z_{base}}\right) + \iota \left(\frac{X}{Z_{base}}\right)$.

1.3 The general three-phase balanced transmission system

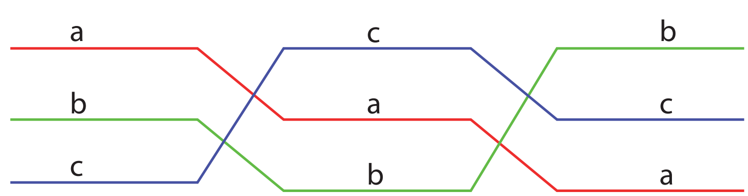

Figure 1: Side-view of transposition of the conductors in a transmission line

As said, the transmission system is balanced, meaning balanced operating conditions and balanced loads. Balanced operating conditions are caused by the general transposition of transmission lines. Transposition is the periodic rotation of positions of conductors in transmission line such that they occupy a different physical position, figure 1. Loads are balanced because of their placement in the system. A full balanced system can be modeled as single-phase instead

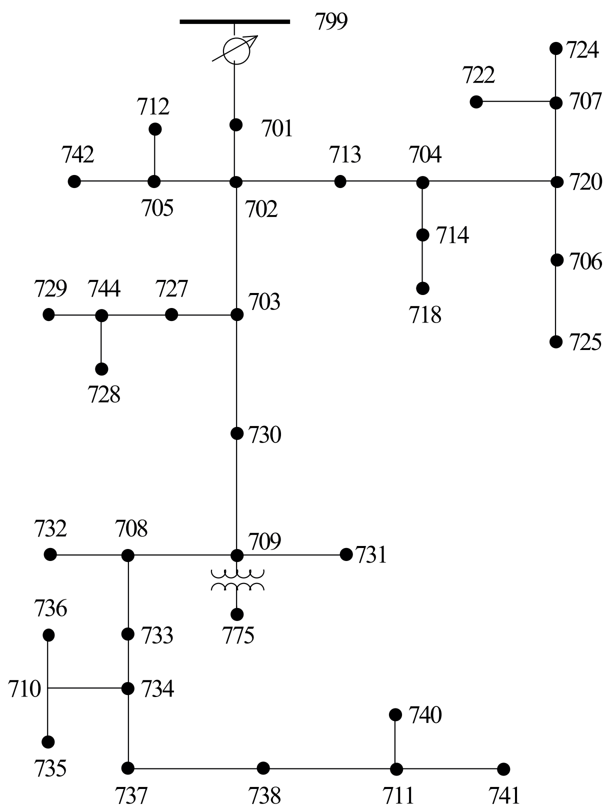

Figure 2: A line diagram of a 37 bus distribution network containing a transformer between bus 709 and 775 [26]

of three-phase. To get a clear idea of what the power system looks like, a one-line diagram is drawn. The one-line diagram shows the necessary information of a power system, figure 2.

1.3.1 Power system model

A power system is modeled as a graph containing branches and nodes which represent the systems' key elements. Branches represent the power cables and transformers, nodes represent the generators, loads and shunts.

Cables carry power over transmission and distribution lines, its most important properties are series impedance ($Z = R + \iota X$) and line charging susceptance (B_c).
Transformers are also represented in the model as branches. Transformers convert voltage and current at high power to lower level [15]. Most common transformers are three-winding transformers, which can be ideal or non-ideal. Ideal transformers do not have resistance while non-ideal transformers have. Transformers are connected in the power system in a Wye or Delta configuration, this is explained later in the section. Transformers do not have to convert power

1 to 1 [14], they can have off-nominal tap ratios, explained in section 2.2.

Generators, loads and capacitors are modeled as nodes. Generators supply a symmetrical load and as a result, current and power is fed into the grid. The bulk of electricity is generated by three-phase synchronous machines, which are modeled as power injectors. Loads convert the electrical energy into a usable form and are modeled as a negative power injection.

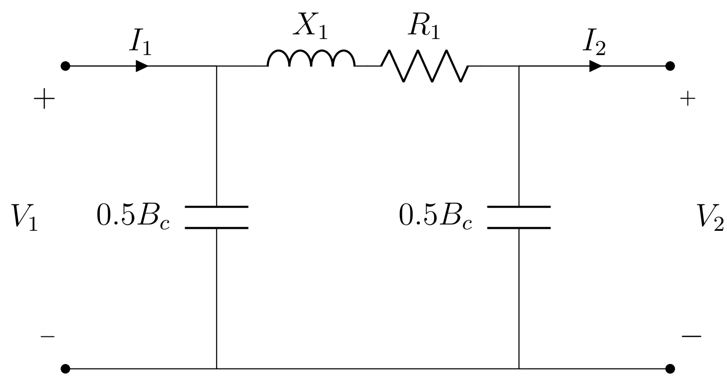

Figure 3: Representation of a long transmission line including shunts

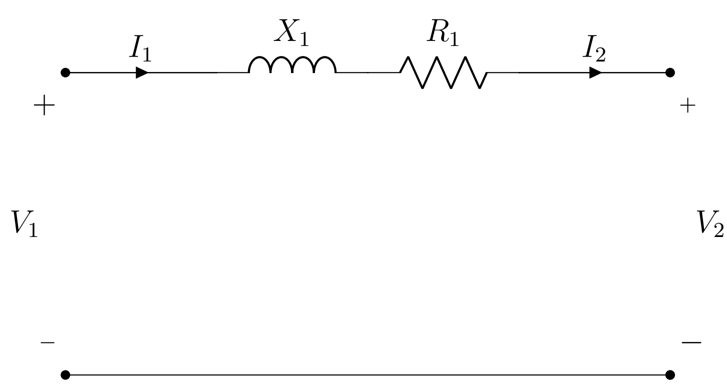

Figure 4: Representation of a distribution line

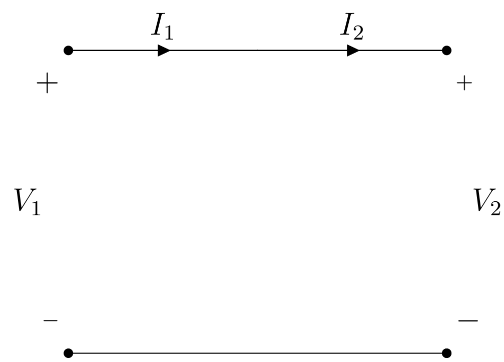

Figure 5: Representation of an ideal transformer

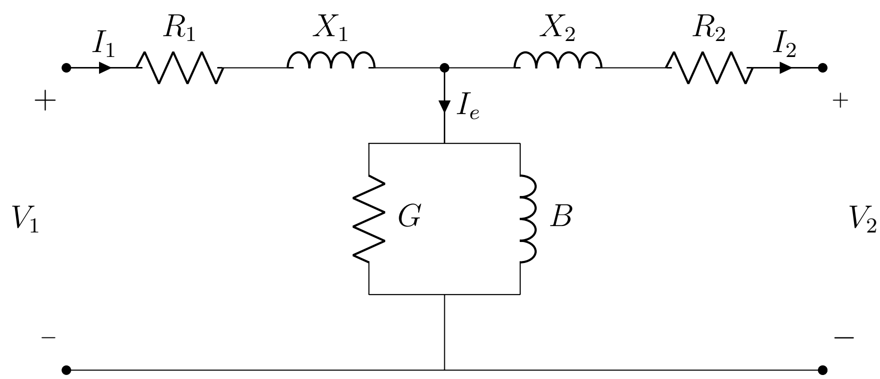

Figure 6: Representation of a practical transformer with external resistance and reactance

1.4 Power Flow model and equations

1.4.1 Topology

Load flow or power flow computations are the computations of power flow in the electricity network and they give insight in the steady-state behaviour of the power system. The network topology, parameters, and the node information are required as input. The four network parameters are:

- the voltage magnitude $|V_i|$
- the voltage phase angle δ_i
- the injected active power P_i
- the injected reactive power Q_i

The network itself is modelled as a system of nodes (buses) and branches (edges). The node information describes the different bus-types. Three bus-types can be distinguished, the load bus (PQ-bus), the generator bus (PV-bus) and the slack bus. Each bus i has its own properties.

They are named after the known information at each bus type.

Table 1: Network Nodes

bus type	known	unknown
PQ-bus	$P_i,\ Q_i$	$\delta_i,\ \|V_i\|$
PV-bus	$P_i,\ \|V_i\|$	$Q_i,\ \delta_i$
slack bus	$\delta_i,\ \|V_i\|$	$P_i,\ Q_i$

The slack bus is a reference bus for the other buses, this is the only node where the phase angle is specified, commonly $\delta = 0$ [24]. The phase angles of the other nodes are measured with reference to the slack bus. The slack bus balances both the difference (the slack) between the total reactive power in- and output plus the total $|I|^2X$ losses and between the total active power in- and output plus the $|I|^2R$. There is always a slack bus in a power system and usually the first generator bus is set as slack bus.
The remaining generator buses are the PV-buses. They have an active power and voltage control. Most of the bus types are generator buses. Load buses (PQ-buses) are modelled as constant power sinks, having a negative injected active and reactive power specified at the node.

The relation between the current injected at the network node and the voltages at the nodes is described by Ohm's law (11) in matrix form:

$$\mathbf{I} = \mathbf{Y}\mathbf{V} \tag{14}$$

Admittance consists of branch admittance (self and mutual) and shunt admittance (per node). The values for I_i and V_i are found by using Kirchhoff's laws. Kirchoff's Current Law (KCL) states that the injected current I_i at each bus i equal is to the the sum from the current flowing from bus i towards bus j, I_{ij}:

$$I_i = \sum_k I_{ik} \tag{15}$$

The same law can be applied for the voltages at each node, which is more often stated as the sum of voltages around a close-loop is zero:

$$\sum_{k=1}^{n} V_k = 0 \tag{16}$$

The admittance is expressed in polar coordinates (center) or in complex coordinates (right-side of the equation).

$$Y_{ij} = |Y_{ij}|(\cos(\theta_{ij}) + \iota\sin(\theta_{ij})) = G_{ij} + \iota B_{ij} \tag{17}$$

The complex power:

$$S_i = V_i\overline{I}_i = P_i + \iota Q_i \tag{18}$$

The last equations form the loadflow equations. They represent the injected active and reactive power as a function of the voltage at node i.

With the use of the admittance matrix and the equivalent circuit models, all the elements can be described by the admittance matrix and Ohm's Law.

The loadflow problem can now be formulated as the determination of the flow of electrical power of a steady-state power system. This is done by computing the voltages (V_i, δ_i) such that the computed active and reactive power injected at each node correspond with the specified values at each node [24]. In mathematical equations:

$$S_i = V_i \overline{I}_i = V_i(\overline{Y\mathbf{V}})_i = V_i \sum_{k=1}^{N} \overline{Y}_{ik} \overline{V}_k \tag{19}$$

The S_i is the complex power injected at node i, I_i is the current flowing through node i, V_i is the voltage at node i, Y is the nodal admittance matrix (generally very sparse and also written as Y_{bus}) and N is the total number of nodes in the power system.

1.5 Transmission and Distribution differences

For modeling transmission systems, assumptions are made that transmission lines are transposed and that the three-phase loading is balanced. But for distribution systems, the loading is in general not balanced, the distribution lines are not transposed and the lines may be in three, two or single phase [18]. It is necessary to perform the calculations in three-phase models to correct for the unbalanced conditions.

The main properties of distribution networks that are distinct from transmission networks are:

- a radial or weakly meshed structure of the distribution network
- high R/X ratio on distribution lines
- unbalanced (dynamic) loads
- Unbalanced operation due to untransposed lines
- inclusion of Distributed Generation (DG)
- multilevel voltage (medium and low voltage distribution cables)

These properties pose challenges for solving the power flow in distribution networks.

Due to these specifications, the transmission system and distribution system are modeled and solved differently. The transmission system is modeled as a single-phase system (phase a), while the distribution system is modeled as a three-phase system (phase a, b, c). This results in changes in the specification of the admittances, the loads and the generators.

1.5.1 Power cables

Single-phase transmission cables are modeled by their admittances: $Y_{ij} = R_{ij} + \iota X_{ij}$, of a cable between nodes i and j. To form the nodal admittance matrix Y_{bus}, the branch admittances are converted to a nodal matrix in blocks.

$$\mathbf{I} = \mathbf{Y}\mathbf{V} \Leftrightarrow \begin{bmatrix} I_f \\ I_t \end{bmatrix} = \begin{bmatrix} Y_{ff} & Y_{ft} \\ Y_{tf} & Y_{tt} \end{bmatrix} \begin{bmatrix} V_f \\ V_t \end{bmatrix} \tag{20}$$

Where Y_{ff} represents admittance at the left node of a branch and Y_{tt} at the right node of a branch. The blocks are placed according the nodal index of a matrix. In case a branch connects node 1 to node 3, the blocks are placed at 11, 13, 31 and 33, this results in a sparse matrix in case of a weakly meshed or radial system.
The block entries are formed as follows: $Y_{ff} = 1/R_{ij} + \iota X_{ij} + 1/2\iota B_{cij}$, $Y_{tt} = Y_{ff}$, $Y_{tf} = Y_{ft} = -1/R_{ij} + \iota X_{ij}$. In case of shunt capacitors, the shunt admittance Y_{sh} is added to the corresponding nodes. The three-phase matrix is more complex.

1.5.2 Distribution Cables

The Dutch distribution system consists mainly of underground lines. Sometimes the medium-voltage network contains overhead lines as well. Like transmission lines, these lines are also modeled by its series impedance and line charging capacitance [21]. Shunt capacitance can usually be neglected for short lines.

Overhead line An overhead distribution line consists usually of four wires, representing phase a, b, c and the neutral phase n. This overhead line, shown in figure 7, is represented by a 4 by 4 impedance matrix Z. The impedance consists of mutual impedances z_{ij} (between conductor i and j or neutral line n) and the self impedances z_{ii} of conductor i. The impedance can be found by using Carson's method. For more explanation we refer to [19].

In order to do calculations in three-phase domain, the impedance matrix need to be reduced to a 3 by 3 matrix [1]. The reduction to a 3 by 3 matrix is done by Kron's reduction:
We start with the 4 by 4 impedance matrix:

$$\begin{bmatrix} z_{aa} & z_{ab} & z_{ac} & z_{an} \\ z_{ba} & z_{bb} & z_{bc} & z_{bn} \\ z_{ca} & z_{cb} & z_{cc} & z_{cn} \\ z_{na} & z_{nb} & z_{nc} & z_{nn} \end{bmatrix} \tag{21}$$

We apply Kron's reduction:

$$Z_{abc} = z_{ij} - \frac{z_{in}}{z_{nn}} \cdot z_{nj} \text{ with } i, j \in \{a, b, c\} \tag{22}$$

[1]Transformers, loads, etc. are represented in three-phase when they are not grounded.

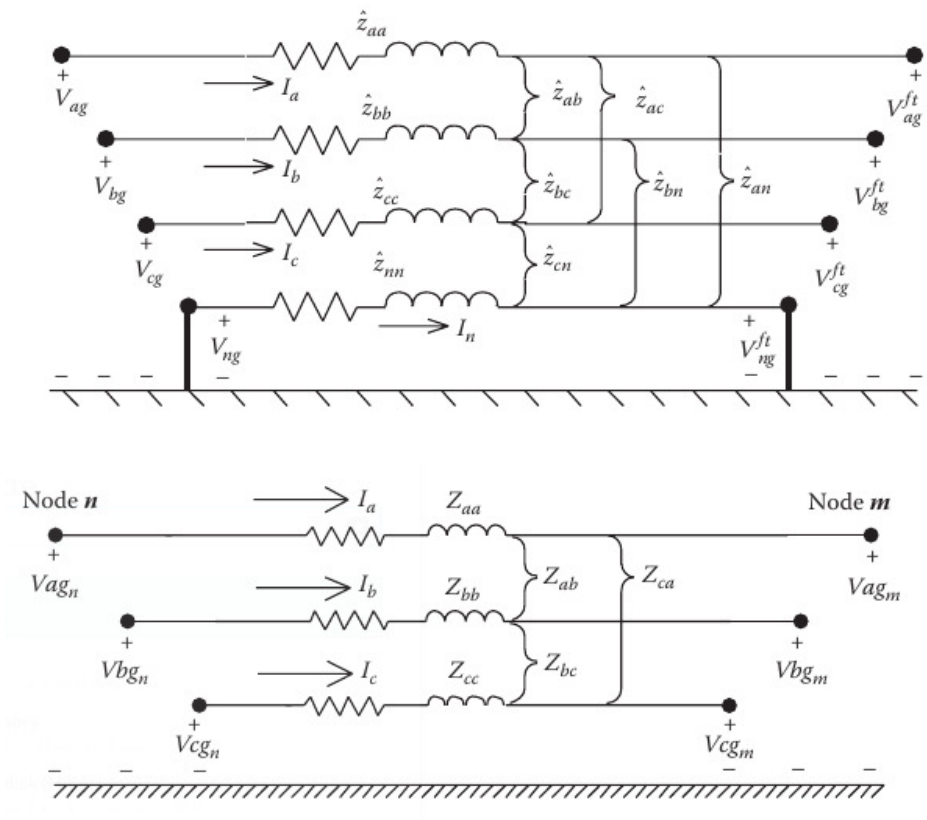

Figure 7: Four-wire grounded Wye line segment (top) and its generalization to a three-phase line segment model (bottom) [19]

The 3 by 3 impedance matrix of a line section l becomes [35]:

$$Z^l = \begin{bmatrix} z^l_{aa} & z^l_{ab} & z^l_{ac} \\ z^l_{ab} & z^l_{bb} & z^l_{bc} \\ z^l_{ac} & z^l_{bc} & z^l_{cc} \end{bmatrix} \tag{23}$$

In case of a balanced (transposed line), the self impedances are equal to each other: $z_{aa} = z_{bb} = z_{cc}$ and the mutual impedances are equal to each other. In case of an untransposed line this is not the case, yet the impedance matrix remains symmetric. Distribution lines are in general not transposed. The voltage equation of this line segment becomes:

$$\begin{bmatrix} V_{ag} \\ V_{bg} \\ V_{cg} \end{bmatrix}_i = \begin{bmatrix} V_{ag} \\ V_{bg} \\ V_{cg} \end{bmatrix}_j + \begin{bmatrix} Z_{aa} & Z_{ab} & Z_{ac} \\ Z_{ab} & Z_{bb} & Z_{bc} \\ Z_{ac} & Z_{bc} & Z_{cc} \end{bmatrix}_{ij} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}, \tag{24}$$

with $Z_{pq} = z_{pq} \cdot length$ [19], $p, q \in \{a, b, c\}$ and i, j the nodes around the branch.

Convert to nodal admittance matrix The three-phase matrix still needs to be converted to the nodal admittance matrix. This is similar to the single-phase nodal admittance matrix, only the block entries $[Y_{ff}\ Y_{ft}\ Y_{tf}\ Y_{tt}]$ contain each a 3 by 3 block matrix themselves, with entries:

$$\mathbf{Y_{ff}} = \begin{bmatrix} Y_{aa} & Y_{ab} & Y_{ac} \\ Y_{ab} & Y_{bb} & Y_{bc} \\ Y_{ac} & Y_{bc} & Y_{cc} \end{bmatrix} + \iota \begin{bmatrix} B_{aa} & B_{ab} & B_{ac} \\ B_{ab} & B_{bb} & B_{bc} \\ B_{ac} & B_{bc} & B_{cc} \end{bmatrix} \tag{25}$$

Distribution lines can contain two-phase or single-phase lines. In the case of a two-phase ab line, the matrix changes into:

$$\mathbf{Y_{ff}} = \begin{bmatrix} Y_{aa} & Y_{ab} & 0 \\ Y_{ab} & Y_{bb} & 0 \\ 0 & 0 & 0 \end{bmatrix} + \iota \begin{bmatrix} B_{aa} & B_{ab} & 0 \\ B_{ab} & B_{bb} & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{26}$$

Shunt admittance of the lines Distribution lines are typically so short that shunt admittance can be ignored [19].

1.5.3 Generators

Generators in transmission system are often big synchronous machines who generate bulk power, while at distribution level, many local sources can be found nowadays. Generation at distribution level is called cogeneration.

Modeling Cogeneration Several papers have profoundly investigated several cogeneration units and classified them amongst three types:

- Synchronous machines (generator)
- Induction Machines (generator)
- Power Electronic Interface (PEI) (static power converter)

Thanks to the relatively fast control of PEI over active power and reactive power of these renewable resources, the units are able to effectively solve or improve many fast phenomena (e.g., fast frequency and voltage deviations) [5]. The cogeneration types (wind, photo-voltaic, hydropower, etc.) can be divided amongst the types as in figure 8.
For modeling purposes it is more convenient to classify cogeneration amongst:

- Constant Power Factor
- Constant Voltage
- Variable Reactive Power

The constant power factor generators are modeled in the system as PQ nodes. The reactive power is expressed in terms of active power and a constant power factor pf:

$$Q = P\tan(\cos^{-1}(pf))$$

The injected currents, necessary for loadflow computations are expressed as:

$$I = \overline{\left(\frac{P + \iota Q}{V}\right)}$$

Synchronous machines can be modeled as PQ node or as PV node, dependent on the voltage control [22]. Most of residential rooftop photovoltaic installations are modeled as PQ-nodes as they do not have reactive power output [9] and therefore are modeled as negative active power loads: PQ-buses with $Q = 0$.
Constant Voltage cogeneration units with voltage control allowance are modeled as PV-nodes. PV-nodes should be treated differently in some solution methods, more in 2.2. Note: a high penetration of PV in distribution networks in terms of conventional generation leads to a decrease of availability of reactive power output. Especially during transients, when the system is disturbed, this can effect the bus voltages. Introduction of distributed PV systems close to the loads, influence the supply of reactive power to the loads. This changes the steady state bus voltage magnitudes, while it is important to maintain steady state voltage stability among the loads. In order to evaluate the effect of PV installations on voltage magnitudes, the load voltage is represented in term of PV generation, showing a quadratic behaviour [9].
Lastly, Variable Reactive Power units are always modeled as PQ node, with Q dependent on V. Induction machines are examples of variable reactive power units, modeled as PQ node but with Q negative [22]. Synchronous machines can also have variable reactive power.

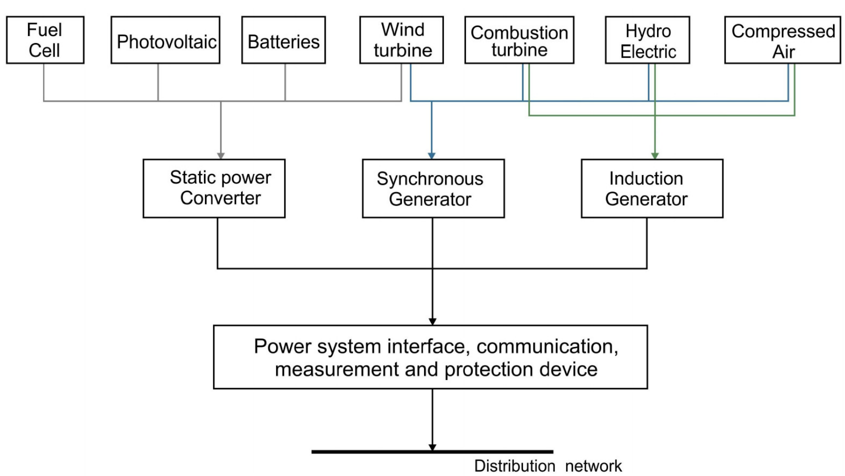

Figure 8: Different modeling types of distributed generation [27]

1.5.4 Loads and Shunts

Loads in single-phase systems are modeled as negative power injectors at nodes, with known active and reactive power (PQ-nodes). They can be modeled as constant Power models, constant Current models or constant Impedance models. Constant power models are just power injections $S = P + \iota Q$, constant current models are modeled as $S = V\overline{I}$, with V unknown and constant impedance models are modeled as $S = YV^2$ with V again as unknown.

Three-phase loads Three-phase loads have the same models, but then represented in three-phase. Three-phase loads can be connected in Wye or Delta connection. Loads are modeled as a Wye-connected load or Delta-connected load and dependent on their type, they can consume three-, two-, or single-phase power.
For each load, the complex power needs to be calculated, which can be done by describing the load by a constant impedance (Z), constant current (I) or constant real and reactive power (P) matrix [19].

Constant Power

$$\begin{aligned} S_a &= P_a + \iota Q_a, \\ S_b &= P_b + \iota Q_b, \\ S_c &= P_c + \iota Q_c \end{aligned} \tag{27}$$

Constant Current The power is represented by the line currents according the relationship:

$$\begin{aligned} I_a^L &= \frac{P_a - \iota Q_a}{\overline{V_a}} \quad \Leftrightarrow \quad P_a + \iota Q_a = \overline{I_a} V_a, \\ I_b^L &= \frac{P_b - \iota Q_b}{\overline{V_b}} \quad \Leftrightarrow \quad P_b + \iota Q_b = \overline{I_b} V_b, \\ I_c^L &= \frac{P_c - \iota Q_c}{\overline{V_c}} \quad \Leftrightarrow \quad P_c + \iota Q_c = \overline{I_c} V_c \end{aligned} \tag{28}$$

During the Newton-Raphson (or other) iterative process, the line-to-neutral voltage will change during each update until convergence is achieved.

Constant Impedance The constant load impedance is determined according $Z^{-1} = Y$ and the relationship:

$$\begin{aligned} Z_a &= \frac{|V_{an}|}{\overline{S_a}} \quad \Leftrightarrow \quad P_a + \iota Q_a = |V_a|\overline{Y_a} \\ Z_b &= \frac{|V_{bn}|}{\overline{S_b}} \quad \Leftrightarrow \quad P_b + \iota Q_b = |V_b|\overline{Y_b} \\ Z_c &= \frac{|V_{cn}|}{\overline{S_c}} \quad \Leftrightarrow \quad P_c + \iota Q_c = |V_c|\overline{Y_c} \end{aligned} \tag{29}$$

Due to voltage-dependence of the loads, constant models are not always suitable [20] and a polynomial load model is introduced. The polynomial load model is a combination of the constant impedance model, the constant current model, the constant power model, and the exponential load model [12].

$$\begin{aligned} &P = P_0(a_0 + a_1 V + a_2 V^2 + a_3 V^{1.38}) \\ &Q = Q_0(b_0 + b_1 V + b_2 V^2 + b_3 V^{3.22}) \\ &a_0 + a_1 + a_2 + a_3 = b_0 + b_1 + b_2 + b_3 = 1 \end{aligned} \tag{30}$$

V	p.u. value of the node voltage
P_0, Q_0	real and reactive power consumed at the specific node under the reference voltage
a_0, b_0	parameters for constant power load component
a_1, b_1	parameters for constant current load component
a_2, b_2	parameters for constant impedance load component
a_3, b_3	parameters for expontential load component

These parameters can be defined by experiments and empiric values.

Load connections Loads are connected in a Wye or in a Delta connection. For a Delta connected load, the load models need to undergo a certain transformation. Loads in the Netherlands are never connected in Delta and therefore the Delta transformation is not considered.

Two- and single-phase loads In case of two- or single-phase loads, the missing phases are set to zero in the matrix operations. The same current, power and impedance models are used.

Shunts Shunt capacitors are necessary for voltage regulation and reactive power support. They are modeled as three-phase connections with, in case of two- or single-phase connections, zeros on the missing phases.
The same models and connections as the loads can be used for the shunt models. The difference is that the shunt capacitance is not represented in the power vector but is accounted for in the nodal admittance matrix.

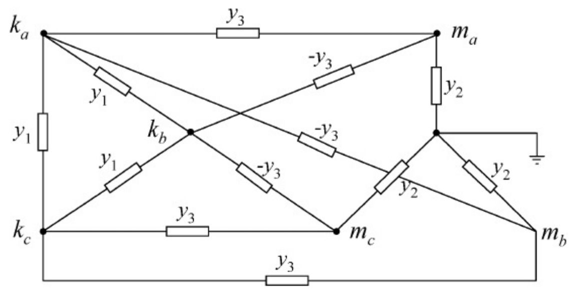

Figure 9: A clear representation of a Delta - Grounded Wye transformer

1.5.5 Transformers

Transformers form the link between the distribution and transmission network. They transform the high voltage to lower level voltage. Single-phase transformers are already explained in section 1.3, the focus here is on three-phase transformers. A transformer consists of a primary and secondary winding. A winding can be presented in a Delta or (grounded) Wye representation. They can be connected to each other in **radial** distribution networks in the following combinations [27]:

- Grounded Wye - Grounded Wye
- Grounded Wye - Wye
- Grounded Wye - Delta
- Wye - Grounded Wye
- Wye - Wye
- Wye - Delta
- Delta - Grounded Wye
- Delta - Wye
- Delta - Delta

In the United States, they use *open* Wye or Delta connections as well [19]. Figure 9 shows a Wye-Delta connected transformer.

The transformer model is represented by a leakage admittance matrix Y_T (the T stands for transformer) and the core losses are represented by primary (f:from) and secondary(t:to) voltages V^f and V^t respectively.

General transformer models Like the standard distribution line model, the transformer matrix Y_T is divided into four blocks Y_T^{ff}, Y_T^{ft}, Y_T^{tf} and Y_T^{tt}:

$$Y_T = \begin{bmatrix} Y_{ff}^T & Y_{ft}^T \\ Y_{tf}^T & Y_{tt}^T \end{bmatrix} \tag{31}$$

The matrix blocks contain different elements, depending on the connection. Table 2 represents these different entries.

Table 2: Leakage Admittance Matrix composition of transformer connection

Connection Type		Self Admittance		Mutual Admittance	
Bus P	**Bus S**	Y_{ff}	Y_{tt}	Y_{ft}	Y_{tf}
Gr. Wye	Gr. Wye	Y_{II}	Y_{II}	$-Y_{II}$	$-Y_{II}$
Gr. Wye	Wye	Y_I	Y_I	$-Y_I$	$-Y_I$
Gr. Wye	Δ	Y_{II}	Y_I	Y_{III}	Y_{III}^T
Wye	Gr. Wye	Y_I	Y_I	$-Y_I$	$-Y_I$
Wye	Wye	Y_I	Y_I	$-Y_{II}$	$-Y_{II}$
Wye	Δ	Y_I	Y_I	Y_{III}	Y_{III}^T
Δ	Gr .Wye	Y_I	Y_{II}	Y_{III}^T	Y_{III}
Δ	Wye	Y_I	Y_I	Y_{III}^T	Y_{III}
Δ	Δ	Y_I	Y_{II}	$-Y_I$	$-Y_I$

Where

$$Y_I = \begin{bmatrix} y_t & 0 & 0 \\ 0 & y_t & 0 \\ 0 & 0 & y_t \end{bmatrix},\ Y_{II} = \frac{1}{3}\begin{bmatrix} 2y_t & -y_t & -y_t \\ -y_t & 2y_t & -y_t \\ -y_t & -y_t & 2y_t \end{bmatrix} \text{ and } Y_{III} = \frac{1}{\sqrt{3}}\begin{bmatrix} -y_t & y_t & 0 \\ 0 & -y_t & y_t \\ y_t & 0 & -y_t \end{bmatrix},$$

and y_t is the leakage admittance matrix between phases [27].

1.6 Symmetrical components and the sequence frame approach

Yet we modeled everything in the phase frame approach which leads to a system of three phases (a, b and c), coupled impedance, and line-to-line voltages. Instead of a phase-frame approach, a sequence frame approach can be used as well. Charles Fortescue discovered in 1913 that three independent components are present in an electrical system: a positive, negative and zero sequence component and any unbalanced system can be expressed in these independent components [29]. He called this methodology: Symmetrical Components.

1.6.1 (Un)balanced systems

Thus far we only considered either totally balanced transmission networks or completely unbalanced distribution networks. In practice, transmission networks are not always completely balanced because not all lines are transposed and distribution networks can have either unbalanced loading or unbalanced operating conditions (untransposed lines) but do not have to be completely unbalanced. This results in different modeling techniques. A system with balanced operating conditions is called a symmetrical system and one can use the symmetrical component methodology which allows you to decouple the system into three independent sequences. It depends on the loading conditions whether you can assume only one sequence or should keep three sequences.

1.6.2 Symmetrical components explained

Generators supply three-phase voltages of which positive voltage sequences are always present. The negative sequence is a set of balanced phasor voltages, equal in magnitude but displaced $2\pi/3$ in a counter-clockwise rotation sequence of $a-c-b$. The zero sequence is also a set of the three phasors equal in magnitude but without rotation.
The sequence and phasor currents can be expressed by each other:

$$\begin{aligned} I_a &= I_1 + I_2 + I_0 \\ I_b &= a^2 I_1 + a I_2 + I_0 \\ I_c &= a I_1 + a^2 I_2 + I_0 \end{aligned} \tag{32}$$

or in matrix-notation:

$$\begin{aligned} I_k^{abc} &= T_1 I_k^{012} \\ I_k^{012} &= T_2 I_k^{abc} \end{aligned}$$

with:

$$\begin{aligned} T_1 &= [1 \ a^2 \ a]^T \\ T_2 &= \frac{1}{3}[1 \ a \ a^2] \end{aligned}$$

shift-operator: $a = e^{\frac{2}{3}\pi\iota}$.
The same relationship holds for the voltages.

A balanced system only has positive sequence voltages and currents as they have equal magnitude and phase angles, resulting in the absence of negative and zero sequences.

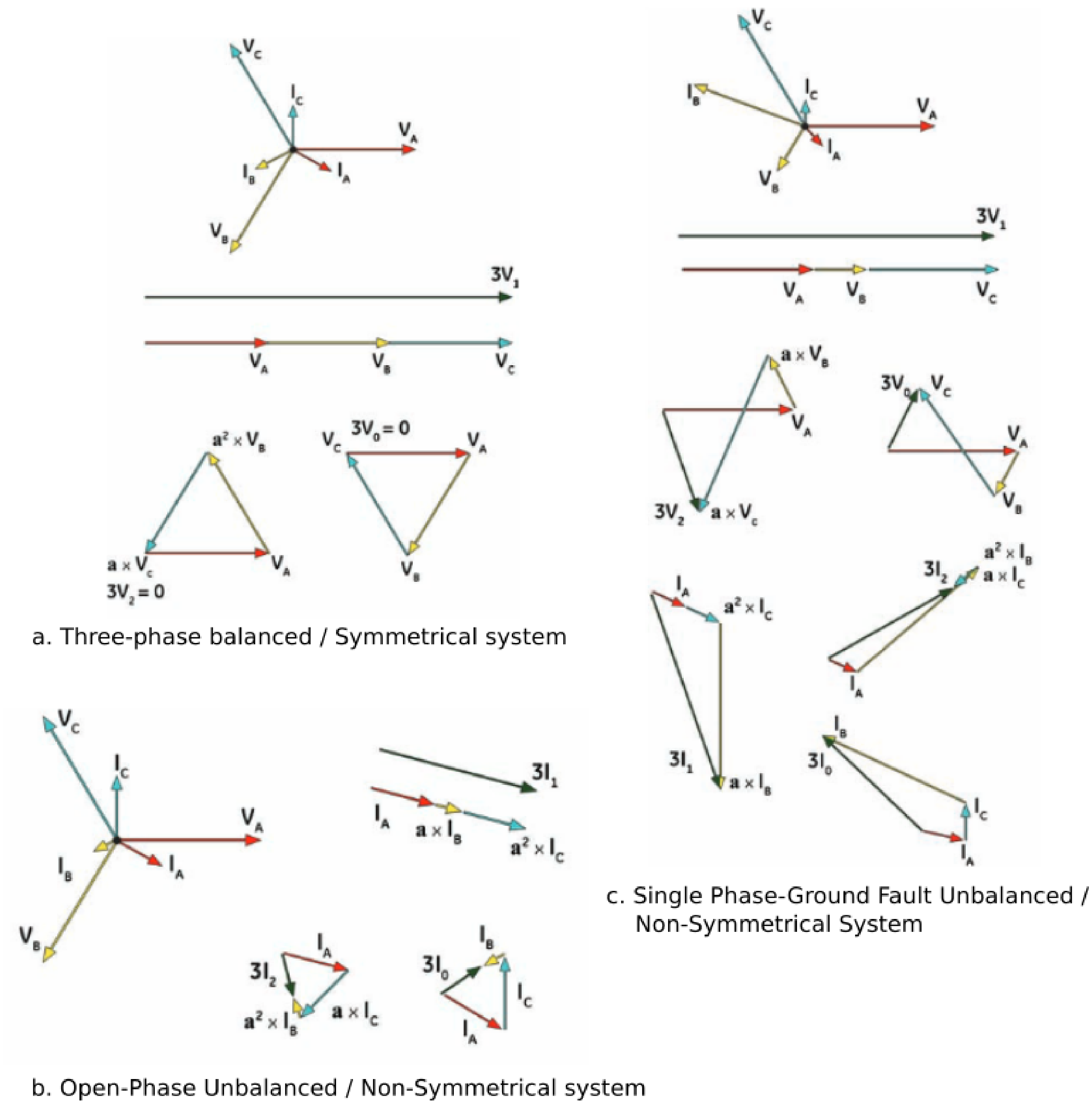

Figure 10: Three (non-) symmetrical systems [29]

Consequence of the zero sequence component in power systems The zero sequence represents the three individual phasor components of an unbalanced systems, which are equal in magnitude and phase. Since they are in phase, the zero sequence currents sum up to n times the magnitude of the individual zero sequence currents. Mostly, the influence of zero sequences are negligible. But in case of irregular large zero-sequence events, such as lightning strikes, the sum of these currents can be larger than the individual phase currents. Conductors are not designed to handle these large currents, which could lead to overheating of neutral conductors.

2 Power system solvers

2.1 General Solvers for Transmission networks

The load flow equations form a system of many non-linear equations, which can be solved by several methods of which Newton-Rapshon and Gauss-Seidel are mostly used.

2.1.1 Newton-Raphson

A general nonlinear system of equations can be written as [2]:

$$\mathbf{f}(\mathbf{x}) = \mathbf{0} \tag{33}$$

where $\mathbf{x}$ is a vector of N unknowns and $\mathbf{f}$ is a vector function of $\mathbf{x}$. Given a starting value x^0, the Newton-Raphson method solves this vector equation with the following method:

$$\begin{aligned} \mathbf{J}(\mathbf{x}^\nu)\boldsymbol{\Delta}\mathbf{x}^\nu &= \mathbf{f}(\mathbf{x}^\nu) \\ \mathbf{x}^{\boldsymbol{\nu+1}} &= \mathbf{x}^\nu + \boldsymbol{\Delta}\mathbf{x}^\nu \end{aligned} \tag{34}$$

The $\mathbf{J}(\mathbf{x}^\nu)$ is the Jacobian with elements $J_{ij} = \frac{\partial f_i}{\partial x_j}$.

Algorithm 1: Newton-Raphson method

1 The Newton-Raphson algorithm is as follows:

1. Set $\nu = 0$ and choose appropriate starting value x^0;
2. Compute $\mathbf{f}(\mathbf{x}^\nu)$;
3. Test convergence:
   If $|f_i(\mathbf{x}^\nu)| \leq \epsilon$ for $i = 1, 2, ..., N$ then x^ν is the solution
   Otherwise go to 4;
4. Compute the Jacobian matrix $\mathbf{J}(\mathbf{x}^\nu)$;
5. Update the solution:
   $$\begin{aligned} \boldsymbol{\Delta}\mathbf{x}^\nu &= -\mathbf{J}^{\mathbf{-1}}(\mathbf{x}^\nu)\mathbf{f}(\mathbf{x}^\nu) \\ \mathbf{x}^{\boldsymbol{\nu+1}} &= \mathbf{x}^\nu + \boldsymbol{\Delta}\mathbf{x}^\nu \end{aligned}$$
6. Update iteration counter $\nu + 1 \rightarrow \nu$, go to step 2.

Newton-Raphson for powerflow To apply the Newton-Raphson method for powerflow computations the entities $\mathbf{x}$, $\mathbf{f}(\mathbf{x})$ and $\mathbf{J}(\mathbf{x})$ have to be determined in terms of power variables. Goal is to determine the complex power S_i at every node, given by equation (19) by computing the voltage magnitude V_i and angle δ_i at each node of the system. As the generator and slack buses are described by $|V_i|$ and not V_i as in eq. (19), it is better to use the quantities $|V_i|,\ \delta_i,\ P_i$ and Q_i. The vector $\mathbf{x}$ of state-variables is then written as:

$$\mathbf{x} = \begin{bmatrix} \delta_2 \\ \vdots \\ \delta_N \\ |V_{N_{g+2}}| \\ \vdots \\ |V_N| \end{bmatrix} \tag{35}$$

Notice the subscript of the unknown quantities: the quantities δ_1 and $|V_1|$ (the phase angle and voltage magnitude of the slack bus) are known and can be left out of the vector. Furthermore, the voltage magnitude $|V_g|$ of all the generator buses can be dropped as we must make sure that the update for the known voltage is equal to 0. The size of the vector $\mathbf{x}$ remains: $2N - N_G - 2 = 2N_L + N_G$ (N_G number of generator buses, N_L number of load buses).
The complex power S_i has to be rewritten to a system of variables $|V_i|,\ \delta_i,\ P_i$ and Q_i. We use the phasor notation of V: $V_i = |V_i|e^{\iota\delta_i}$, the complex conjugate $\overline{V_k} = |V_k|e^{-\iota\delta_k}$, the admittance matrix $Y_i = G_i + \iota B_i$ and the phasor difference $\delta_{ij} = \delta_i - \delta_j$:

$$\begin{aligned} S_i &= V_i\overline{I}_i = V_i(\overline{Y\mathbf{V}})_i = V_i\sum_{k=1}^{N}\overline{Y}_{ik}\overline{V}_k \\ &= \sum_{k=1}^{N}(G_{ik} - \iota B_{ik})|V_k||V_i|(\cos\delta_{ik} + \iota\sin\delta_{ik}) \end{aligned} \tag{36}$$

The active power is derived in similar way, note $\delta_{ii} = \delta_i - \delta_i = 0$:

$$\begin{aligned} P_i(\mathbf{x}) &= \sum_{k=1}^{N}|V_i||V_k|(G_{ik}\cos\delta_{ik} + B_{ik}\sin\delta_{ik}) \\ &= |V_i|^2G_{ii} + \sum_{\substack{k=1 \\ k\neq i}}^{N}|V_iV_k|(G_{ik}\cos\delta_{ik} + B_{ik}\sin\delta_{ik}) \quad \text{[17]} \end{aligned} \tag{37}$$

And the reactive power:

$$Q_i = -|V_i|^2B_{ii} - \sum_{\substack{k=1 \\ k\neq i}}^{N}|V_iV_k|(G_{ik}\sin\delta_{ik} - B_{ik}\cos\delta_{ik}) \quad \text{[17]} \tag{38}$$

Power-mismatch formulation The state-variables $\mathbf{x}$ have to be determined in such a way that the specified and computed power injections are equal:

$$\Delta\mathbf{P}(\mathbf{x}) = \mathbf{P}_s - \mathbf{P}(\mathbf{x}) = \mathbf{0} \quad \text{and} \quad \Delta\mathbf{Q}(\mathbf{x}) = \mathbf{Q}_s - \mathbf{Q}(\mathbf{x}) = \mathbf{0}.$$

Where $\mathbf{P}(\mathbf{x})$ (or $\mathbf{Q}(\mathbf{x})$) is the computed active (reactive) power flow *out* of the node and $\mathbf{P}_s$ (or $\mathbf{Q}_s$) the known active (reactive) *injected* power at the node. These are the power-mismatch equations and are combined in vector $\mathbf{F}(\mathbf{x})$:

$$\mathbf{F}(\mathbf{x}) = \begin{bmatrix} \Delta\mathbf{P}(\mathbf{x}) \\ \Delta\mathbf{Q}(\mathbf{x}) \end{bmatrix} \tag{39}$$

Newton-Raphson works in such a way that the iterations stop as soon as $\mathbf{x}$ leads to equality of the specified and computed injected power, thus when $\mathbf{F}(\mathbf{x}) = \mathbf{0}$. (Actually when $\mathbf{F}(\mathbf{x}) < \epsilon$, a certain tolerance value).
Before convergence has reached, the vector $\mathbf{x}$ is updated with a small difference $\Delta\mathbf{x}$ until $\mathbf{F}(\mathbf{x} + \Delta\mathbf{x}) = \mathbf{0}$.
The iterative formula of Newton-Raphson requires the Jacobian J of the loadflow equations, which is the negative value of the Jacobian H of the power-mismatches: $\mathbf{J} = -\mathbf{H}$.
H looks as follows:

$$H(\mathbf{x}) = \left[\begin{array}{ccc|ccc} \frac{\partial P_2}{\partial \delta_2}(\mathbf{x}) & \dots & \frac{\partial P_2}{\partial \delta_N}(\mathbf{x}) & \frac{\partial P_2}{\partial |V_{N_g+2}|}(\mathbf{x}) & \dots & \frac{\partial P_2}{\partial |V_N|}(\mathbf{x}) \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \frac{\partial P_N}{\partial \delta_2}(\mathbf{x}) & \dots & \frac{\partial P_N}{\partial \delta_N}(\mathbf{x}) & \frac{\partial P_N}{\partial |V_{N_g+2}|}(\mathbf{x}) & \dots & \frac{\partial P_N}{\partial |V_N|}(\mathbf{x}) \\ \hline \frac{\partial Q_{N_g+2}}{\partial \delta_2}(\mathbf{x}) & \dots & \frac{\partial Q_{N_g+2}}{\partial \delta_2}(\mathbf{x}) & \frac{\partial Q_{N_g+2}}{\partial |V_{N_g+2}|}(\mathbf{x}) & \dots & \frac{\partial Q_{N_g+2}}{\partial |V_N|}(\mathbf{x}) \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \frac{\partial Q_N}{\partial \delta_2}(\mathbf{x}) & \dots & \frac{\partial Q_N}{\partial \delta_N}(\mathbf{x}) & \frac{\partial Q_N}{\partial |V_{N_g+2}|}(\mathbf{x}) & \dots & \frac{\partial Q_N}{\partial |V_N|}(\mathbf{x}) \end{array}\right] \tag{40}$$

The four blocks of the Jacobian are all quadatric matrices and so is the entire Jacobian [2]. The values of the Jacobian entries are as follows:

$$\begin{aligned} \frac{\partial P_i}{\partial \delta_i} &= -Q_i - |V_i|^2 B_{ii}, \\ \frac{\partial P_i}{\partial \delta_j} &= |V_i V_j|(G_{ij}\sin\delta_{ij} - B_{ij}\cos\delta_{ij}), \\ \frac{\partial P_i}{\partial |V_i|} &= 2|V_i|G_{ii} + \sum_{\substack{k=1 \\ k\neq i}}^{N} |V_k|(G_{ik}\cos\delta_{ik} + B_{ik}\sin\delta_{ik})), \\ \frac{\partial P_i}{\partial |V_j|} &= |V_i|(G_{ij}\cos\delta_{ij} + B_{ij}\sin\delta_{ij})) \end{aligned} \tag{41}$$

$$
\begin{aligned}
\frac{\partial Q_i}{\partial \delta_i} &= P_i - |V_i|^2 G_{ii}, \\
\frac{\partial Q_i}{\partial \delta_j} &= |V_i V_j|(-B_{ij}\sin\delta_{ij} - G_{ij}\cos\delta_{ij}), \\
\frac{\partial Q_i}{\partial |V_i|} &= -2|V_i|B_{ii} + \sum_{\substack{k=1\\k\neq i}}^{N} |V_k|(-B_{ik}\cos\delta_{ik} + G_{ik}\sin\delta_{ik})), \\
\frac{\partial Q_i}{\partial |V_j|} &= |V_i|(-B_{ij}\cos\delta_{ij} + G_{ij}\sin\delta_{ij}))
\end{aligned}
\tag{42}
$$

With the entries of the Jacobian $H(\mathbf{x})$, the magnitude of the voltages and phase-angles and the algorithm, the Newton-Raphson method is ready to use.

2.1.2 Other methods

Decoupled Loadflow The active and reactive power flow control can be estimated independently from each other, resulting in 'decoupling' the active power and voltage angles from the reactive power and voltage magnitudes. Decoupling is possible when the following operating conditions are valid:

- the resistance of overhead transmission lines is much smaller than the reactance (or in terms of conductance and susceptance: line susceptance is much larger than line conductance):
$$G_{ij}\sin(\delta_j - \delta_i) \ll B_{ij}\cos(\delta_j - \delta_i)$$
- the differences between the voltage angles are small
$$\sin(\delta_j - \delta_i) = \delta_j - \delta_i \quad \text{and} \quad \cos(\delta_j - \delta_i) = 1$$
- the reactive power injected into a node is much smaller than the reactive flow that would result if all lines connected to that bus were short-circuited to reference [24]
$$Q_i \ll |V_i|^2 B_{ii}$$

This results in a decoupled Jacobian matrix $\mathbf{H}$. The off-diagonal blocks in the matrix $\mathbf{H}(\mathbf{x})$ (40), are close to 0. Decoupling starts by setting these offdiagonal blocks equal to $\mathbf{0}$. The matrix-vector equation $\mathbf{H}(\mathbf{x})\mathbf{x} = \mathbf{F}(\mathbf{x})$ changes then in two separate matrix-vector systems:

$$
\begin{aligned}
\mathbf{H}_{11}\mathbf{x}_1 &= \Delta\mathbf{P}, \\
\mathbf{H}_{22}\mathbf{x}_2 &= \Delta\mathbf{Q}
\end{aligned}
\tag{43}
$$

To solve this system, the algorithm is as follows:

Algorithm 2: Decoupled Loadflow

The Decoupled loadflow algorithm is as follows:

1. Calculate matrices $\mathbf{H}_{11}$ and $\mathbf{H}_{22}$
2. Compute the LU factorization of $\mathbf{H}_{11}$ and $\mathbf{H}_{22}$
3. Set iteration counter: $\nu = 0$ and initial iterates δ_i and $|V_i|$
4. Calculate the active power mismatches: $\frac{\Delta P_i(|V_i|^\nu, \delta_i^\nu)}{|V_i|^\nu} = \mathbf{H}_{11}\Delta\delta$
5. Update the voltage angles: $\delta_i^{\nu+1} = \delta_i^\nu + \Delta\delta_i^\nu$
6. Calculate the reactive power mismatch with the updated voltage angle:

$$\frac{\Delta Q_i(|V_i|^\nu, \delta_i^{\nu+1})}{|V_i|^\nu} = \mathbf{H}_{22}\Delta\mathbf{V}$$

7. Update the voltage magnitudes: $|V_i|^{\nu+1} = |V_i|^\nu + \Delta|V_i|^\nu$
8. Repeat steps 4 till 7 until convergence is reached

2.1.3 DC loadflow

DC loadflow is a linearized version of the loadflow problem. Is speeds up the computation time, but also alters the equations and thus effects the final solution. DC loadflow can also be applied to find an initial value for the other iterative techniques [24].
In DC loadflow, the following approximations are made:

- the node voltage magnitudes are 1 pu in active power equations
- voltage magnitudes are approximated by: $|V| = 1 + \Delta|V|$ and $\frac{1}{1+\Delta|V|} \approx 1 - \Delta|V|$ in reactive power equations
- the resistances of the transmission lines are neglected ($G_{ij} = 0$)
- the differences between voltage angles are 0 (see Decoupled loadflow)

2.1.4 Gauss-Seidel iteration

Gauss-Seidel is an other iterative method to solve the loadflow equations [2]. The dependent state variables are the bus voltages V_i, thus in order to derive Gauss-Seidel for the loadflow equations, the complex power equation (19) has to be rewritten to a voltage-current relationship from which the state voltages V_i can be determined:

$$\begin{aligned} S_i &= P_i + \imath Q_i \\ &= V_i \sum_{k=1}^{N} \overline{Y_{ik} V_k} \\ &= V_i \sum_{\substack{k=1 \\ k \neq i}}^{N} \overline{Y_{ik} V_k} + V_i \overline{Y_{ii} V_i} \quad \Leftrightarrow \qquad (44) \\ V_i \overline{Y_{ii} V_i} &= S_i - V_i \sum_{\substack{k=1 \\ k \neq i}}^{N} \overline{Y_{ik} V_k} \quad \Leftrightarrow \\ \overline{V_i} &= \frac{1}{\overline{Y_{ii}}} [\frac{S_i}{V_i} - \sum_{k=1}^{N} \overline{Y_{ik} V_k}] \qquad (45) \end{aligned}$$

Taking the complex conjugate of equation (45), the voltage is expressed as:

$$V_i = \frac{1}{Y_{ii}} [\frac{\overline{S_i}}{\overline{V_i}} - \sum_{k=1}^{N} Y_{ik} V_k] \qquad (46)$$

Gauss-Seidel iterative scheme Gauss-Seidel is an iterative scheme, based on LU-splitting. A matrix-vector equation $\mathbf{Ax} = \mathbf{b}$ is solved iteratively as:

$$\mathbf{L}\mathbf{x}^{\nu+1} = \mathbf{b} - \mathbf{U}\mathbf{x}^{\nu} \qquad (47)$$

Where $\mathbf{A} = \mathbf{L} + \mathbf{U}$, a lower ($\mathbf{L}$) and upper ($\mathbf{U}$) triangular matrix, of which $\mathbf{L}$ contains the diagonal elements of $\mathbf{A}$.
If the sequence of iteration converges ($\mathbf{x}^\nu \to \mathbf{x}^*$), then

$$\mathbf{x}^* = \mathbf{h}(\mathbf{x}^*) \tag{48}$$

With $\mathbf{h}(\mathbf{x}) = \mathbf{L}^{-1}(\mathbf{b} - \mathbf{U}\mathbf{x})$ and $\mathbf{x}^*$ the exact solution.
The $\mathbf{x}^{\nu+1}$ is computed from:

$$\mathbf{x}^{\nu+1} = \mathbf{L}^{-1}(\mathbf{b} - \mathbf{U}\mathbf{x}^\nu) \tag{49}$$

Gauss-Seidel for the loadflow equations To apply the iterative scheme (49) on the voltage equation of (46), the Gauss-Seidel loadflow iterative scheme, becomes:

$$\begin{aligned} \Delta V_i &= \frac{1}{Y_{ii}} \left(\frac{\overline{S_i}}{\overline{V_i}} - \sum_{k=1}^{N} Y_{ik} V_k^\nu \right) \\ V_i^{\nu+1} &= V_i^\nu + \Delta V_i \end{aligned} \tag{50}$$

The Gauss-Seidel is not used in practice as convergence is very slow compared to Newton-Raphson. Sometimes the iteration does not even converge.

2.2 Power Flow Solvers in Distribution Networks

The special properties of the distribution network add complexity to the solvability of distribution power flow. Researchers have been investigating methods that are able to converge for distribution networks. Their starting point were the conventional transmission network methods: Newton-Raphson, Gauss-Seidel and Decoupled Loadflow algorithms, which have been adapted over the years to convergent distribution networks.
This chapter gives an overview of the work done so far: it is an historic overview of the methods and their modifications to different characteristics of the distribution network. Their main (dis-) advantages are given, including a brief description of the algorithm, modifications and convergence properties.
The proposed algorithms can be divided into the following categories:

- Newton-Raphson like methods
- Modified Gauss-Seidel methods
- Direct methods
- Compensation based methods
- Forward and Backward Sweep methods
- Miscellaneous methods

The several categories can be classified as node or branch based methods [11]. The node based methods use the node voltage or current injections as state variables, while branch based methods use branch currents or branch powers as state variables to solve the power flow problem. Newton-Raphson, Modified Newton and Implicit Z_{bus} methods are node-based methods, Sweep based methods are branch based and Direct methods use both branch and bus currents to obtain the power flow solution and fall in a third category.

2.2.1 Possible solvers and their (dis)-advantages

We review NR-methods, modified GS methods, compensation based methods, direct methods and FBS methods.

Newton-Raphson methods The Newton-Raphson method is modified to a three-phase method using Current mismatches instead of Power mismatches resulting in the Newton-Raphson Three-phase Current Injection Method (NR-TCIM). The method is really advanced and robust [27]. Its convergence behaviour does not change in case of different loading conditions or high R/X ratios and also work on unbalanced systems. It can handle Distributed Generation (DG) by taking into account cogeneration as PV-node and solve this system separately. The only disadvantage is that the methods are not tested on weakly meshed systems [13].

Modified Gauss-Seidel Due to LU-factorization of the very big admittance matrix Y, modified Gauss-Seidel (GS) method becomes very slow [6]. Some modifications have been introduced, in which the LU-factorization only takes place on smaller sub-admittance matrices. By decoupling the voltage relations in separate phases, the method is also suitable for unbalanced systems [32][34]. The number of iterations of the proposed method does not increase too much with the increase of system size. Furthermore, the phase-decoupled approach is suitable for parallel processing techniques.
Like NR-TCIM, the method has not been tested on weakly meshed systems, and is probably not suitable for these distribution feeders.

Direct methods The bus-injection to branch-current / branch-current to bus-voltage (BIBC-BCBV) method is based on modified GS, but found a way to avoid LU factorization by making advantage of the radial structure of a distribution network [33]. Also in case of weakly meshed structures, it has found a way to handle loops in the system. The inclusion of distribution generation is not a problem and the method works also on unbalanced systems [11].
But the methods have only been tested on very small systems (33 buses). Probably because it takes too much time to define the BIBC and BCBV matrices, which are based on the systems structure.

Compensation based methods The compensation based methods uses Forward and Backward Sweeps (FBS) to update the voltage relation. It is one of the first proposed methods that knows how to convert weakly meshed systems to radial systems [28]. They appoint breakpoints in the systems, which are nodes that form a closed loop in the system, causing a meshed structure. They open the breakpoints by injecting them with artificial currents. They state that this breakpoint method is also suitable for other algorithms [7][35]. The compensation based methods develop in the FBS methods. These methods are easy to implement, but very sensitive to high R/X ratios and probably not suitable for large-scale systems[3].

General conclusion Based on this review, we can conclude that the use of NR-TCIM, separately handling PV-nodes, will probably be best for large-scale weakly meshed unbalanced systems *with* inclusion of DG. To handle the breakpoints due to weakly meshed systems, the proposed theory of the compensation based methods can be tried [10][3].
It is also an idea to test the direct methods on large-scale systems.

Table 1 Summary of Phase frame power flow

Phase frame					
Algorithm		R/WM/M	UB	DGs	CG
Forward/ Backward sweep		R	✓	✗	Max. Bus voltage mismatch
Compensation Based method		R/WM	✓	✓	Max. Bus Real and reactive Power mismatch
Gauss Implicit Z_{Bus}		R/WM/M	✓	✓	Max. bus voltage mismatch
Modified Newton/Newton like methods		R/WM/M	✓	✓	Max. Bus Real and reactive Power mismatch / current mismatch/ loop voltage mismatch
Miscellaneous power flow methods	BIBC/BCBV matrices method	R/WM	✓	✓	Max. Load current mismatch
	Loop impedence Matrix method	R/WM	✓	✗	Max. bus voltage mismatch

Table 2 Summary of Sequence frame power flow

Sequence frame				
	R/WM/M	UB	DG	CG
Forward /Backward sweep	Radial Part of Mixed network	✓	✗	Max. Bus voltage mismatch
Compensation Based method	Not discussed			
Gauss Implicit Z_{Bus}	Not discussed			
Newton Based methods	Main 3 phase part of mixed network	✓	✓	Positive sequence voltage mismatch or power mismatch

R-Radial network; WM-Weakly meshed network; M- Mesh or Mixed type network
UB-Unbalanced; DG-Distributed Generation; CG-Convergence criteria

Figure 11: Overview of the several proposed algorithms and their applications. The figure is taken from [4]

2.2.2 Newton-Raphson three-phase current injection method

As Newton-Raphson was chosen to be most suitable for these types of power networks, it is reviewed here in more detail.

General idea Instead of using power mismatches as in the standard Newton-Raphson, current mismatches are suggested [13] as a better alternative for unbalanced distribution networks. This method is called the Three-phase Current Injection Method (TCIM). Instead of applying the Newton-Raphson method ($\mathbf{F}(\mathbf{x}) = \mathbf{J}(\mathbf{x})\Delta\mathbf{x}$) to power mismatches:

$$\begin{aligned} S_i &= V_i(\overline{Y\mathbf{V}})_i \\ \Delta P &= P_i - P(\mathbf{x}) \\ \Delta Q &= Q_i - Q(\mathbf{x}) \end{aligned} \tag{51}$$

It is applied to Ohm's Law ($\mathbf{I} = Y\mathbf{V}$) and according current mismatches:

$$
\begin{aligned}
I_i &= (Y\mathbf{V})_i \\
\Delta I^{Re} &= I_i^{Re} - I^{Re}(\mathbf{x}) \\
\Delta I^{Im} &= I_i^{Im} - Q^{Im}(\mathbf{x})
\end{aligned}
\tag{52}
$$

Or in more detail:

$$
\begin{aligned}
\Delta Re(I_i^A) &= \frac{(P^s)^A Re(V_i^A) + (Q^s)^A Im(V_i^A)}{(Re(V_i^A))^2 + (Im(V_i^A))^2} \sum_{k\in\Omega_i} \sum_{S\in\alpha_p} (G_{ik}^{AS} Re(V_i^S) - B_{ik}^{AS} Im(V_i^S)) \\
\Delta Im(I_i^A) &= \frac{(P_i^s)^A Im(V_i^A) - (Q_i^s)^A Re(V_i^A)}{(Re(V_i^A))^2 + (Im(V_i^A))^2} - \sum_{k\in\Omega_i} \sum_{S\in\alpha_p} (G_{ik}^{AS} Im(V_i^S) + B_{ik}^{AS} Re(V_i^S))
\end{aligned}
\tag{53}
$$

where:

$$
\begin{aligned}
A, S &\in \alpha_p \\
\alpha_p &= \{a, b, c\} \text{ three phases} \\
i &= \{1, ..., N\}, N \text{ total number of buses} \\
\Omega_i &- \text{set of buses directly connected to bus } i
\end{aligned}
$$

The three-phase current mismatch equations can be represented in terms of power mismatches:

$$
\begin{aligned}
\Delta Re(I_i^A) &= \frac{Re(V_i^A)\Delta P_i^A + Im(V_i^A)\Delta Q_i^A}{(V_i^A)^2} \\
\Delta Im(I_i^A) &= \frac{Im(V_i^A)\Delta P_i^A - Re(V_i^A)\Delta Q_i^A}{(V_i^A)^2}
\end{aligned}
\tag{54}
$$

The Jacobian of the current mismatch equation in three-phase consists of the derivatives with respect to $Re(V_i)$, $Im(V_i)$, $Re(V_j)$ and $Im(V_j)$ in Carthesian coordinates. These derivatives are not shown here. The imaginary parts of the current are ordered first, to make sure that $\mathbf{J}$ is diagonal dominant (because $B_{ij} \gg G_{ij}$). $\mathbf{J}$ is the Jacobian, whose off-diagonal elements are equal to the corresponding elements of the nodal admittance matrix and thus remain constant throughout the iterative process. The diagonal elements are updated every iteration.

Treatment of PV-buses PV-buses should be treated differently. As for PV-buses the V is a known, the derivatives cannot be taken with respect to V but instead are taken with respect to Q. The Jacobian changes from figure 12 to figure 13.

The bold elements in the admittance matrix change to:

$$
Y_{kk}^{abc} = \begin{bmatrix} M & O \\ N & P \end{bmatrix}
\tag{55}
$$

$$
\begin{bmatrix} \Delta I_{m_1}^{abc} \\ \Delta I_{r_1}^{abc} \\ \Delta I_{m_2}^{abc} \\ \Delta I_{r_2}^{abc} \\ \vdots \\ \Delta I_{m_n}^{abc} \\ \Delta I_{r_n}^{abc} \end{bmatrix} = \begin{bmatrix} (\mathbf{Y}_{11}^{\bullet})^{\mathbf{abc}} & \mathbf{Y}_{12}^{\mathbf{abc}} & \cdots & \mathbf{Y}_{1n}^{\mathbf{abc}} \\ \mathbf{Y}_{21}^{\mathbf{abc}} & (\mathbf{Y}_{22}^{\bullet})^{\mathbf{abc}} & \cdots & \mathbf{Y}_{2n}^{\mathbf{abc}} \\ \vdots & \vdots & \vdots & \vdots \\ \mathbf{Y}_{n1}^{\mathbf{abc}} & \mathbf{Y}_{n2}^{\mathbf{abc}} & \cdots & (\mathbf{Y}_{nn}^{\bullet})^{\mathbf{abc}} \end{bmatrix} \cdot \begin{bmatrix} \Delta V_{r_1}^{abc} \\ \Delta V_{m_1}^{abc} \\ \Delta V_{r_2}^{abc} \\ \Delta V_{m_2}^{abc} \\ \vdots \\ \Delta V_{r_n}^{abc} \\ \Delta V_{m_n}^{abc} \end{bmatrix}
$$

Figure 12: Normal Jacobian

$$
\begin{bmatrix} \Delta I_{m_1}^{abc} \\ \Delta I_{r_1}^{abc} \\ \vdots \\ \Delta I_{m_i}^{abc} \\ \Delta I_{r_i}^{abc} \\ \vdots \\ (\mathbf{\Delta I}_{\mathbf{m_k}}^{\bullet})^{\mathbf{abc}} \\ (\mathbf{\Delta I}_{\mathbf{r_k}}^{\bullet})^{\mathbf{abc}} \\ \vdots \\ \Delta I_{m_l}^{abc} \\ \Delta I_{r_l}^{abc} \\ \vdots \end{bmatrix} = \begin{bmatrix} (Y_{11}^{\bullet})^{abc} & \cdots & Y_{1i}^{abc} & \cdots & (\mathbf{Y}_{\mathbf{1k}}^{\bullet\bullet})^{\mathbf{abc}} & \cdots & Y_{1l}^{abc} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ Y_{i1}^{abc} & \cdots & (Y_{ii}^{\bullet})^{abc} & \cdots & (\mathbf{Y}_{\mathbf{ik}}^{\bullet\bullet})^{\mathbf{abc}} & \cdots & Y_{il}^{abc} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ Y_{k1}^{abc} & \cdots & Y_{ki}^{abc} & \cdots & (\mathbf{Y}_{\mathbf{kk}}^{\bullet\bullet})^{\mathbf{abc}} & \cdots & Y_{kl}^{abc} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ Y_{l1}^{abc} & \cdots & Y_{li}^{abc} & \cdots & (\mathbf{Y}_{\mathbf{lk}}^{\bullet\bullet})^{\mathbf{abc}} & \cdots & (Y_{ll}^{\bullet})^{abc} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix} \begin{bmatrix} \Delta V_{r_1}^{abc} \\ \Delta V_{m_1}^{abc} \\ \vdots \\ \Delta V_{r_i}^{abc} \\ \Delta V_{m_i}^{abc} \\ \vdots \\ \mathbf{\Delta V}_{\mathbf{m_k}}^{\mathbf{abc}} \\ \mathbf{\Delta Q}_{\mathbf{k}}^{\mathbf{abc}} \\ \vdots \\ \Delta V_{r_l}^{abc} \\ \Delta V_{m_l}^{abc} \\ \vdots \end{bmatrix}
$$

Figure 13: Voltage dependent Jacobian

and

$$
Y_{lk}^{abc} = \begin{bmatrix} Q & U \\ R & W \end{bmatrix} \tag{56}
$$

The elements of submatrices M and N are the susceptance and conductance of the corresponding element, multiplied with a voltage dependent element. O is a diagonal matrix, 3 by 3 (three-phase), with voltage relations, P is equivalent to minus O. The off-diagonal voltage dependent blocks are similar, but correspond to other susceptance and conductance elements.

The Algorithm Algorithm 3 is the NR-TCIM algorithm including the update of the Jacobian matrix. In case of only PQ nodes, the update of Jacobian is not necessary as it remains constant.

Algorithm 3: Newton-Raphson using TCIM with PV nodes

1. Assemble the nodal admittance matrix $\mathbf{Y}$.
   Initialize the counter $\nu = 0$
   Initialize the voltages and angles V_i^ν and θ_i^ν, $i = 1, n$.
2. Determine the current injections $\mathbf{I} = \mathbf{YV}$
   Determine the active and reactive power mismatches $(\Delta\mathbf{P}, \Delta\mathbf{Q})$
3. If $\max(\Delta\mathbf{P}, \Delta\mathbf{Q}) \leq \epsilon$:
   then Go to step 5
   else Update the Jacobian matrix
   Determine the bus voltage corrections by (53) and update the bus voltages
   Increment iteration counter: $\nu = \nu + 1$
4. Repeat from step 2 until convergence is reached or maximum number of iterations is succeeded

Application of the model The following table specifies on which network configuration the algorithm is tested (and suitable).

Table 3: Specifications of the system network to obtain a convergent network for TCIM ([13])

System network	Distribution
Configuration	Radial
Condition	Unbalanced and balanced
Loading condition	Static, constant power and impedance load model, voltage dependent
R/X ratio	Low
Distributed Generation	No
Ill-conditioned	NS

Convergence characteristics The method is applied on balanced and unbalanced systems and shows that the method is very robust and converges in less iterations than backward/forward method which it is compared to. The structure of the Jacobian matrix is the same as the bus admittance matrix and thus retains it sparsity properties. In case of no cogeneration (no extra PV-buses), the Jacobian will stay constant during the Newton-Raphson iteration.

2.2.3 Optimization by Araujo

Araujo et al [3] introduce two variant methods of TCIM: TCIM-A and TCIM-B. In which for TCIM-A, the ordering is done prior to the solution process (preprocessing step). In TCIM-B, the ordering is done simultaneously with the numerical factorization step at every iteration. In terms of Algorithm 3: TCIM-A adds a line before step 1, where the equations are ordened. TCIM-B changes line 3. to: They state it should also work on weakly meshed networks, although never

4. If $\max(\Delta\mathbf{P}, \Delta\mathbf{Q}) \leq \epsilon$:
   then Go to step 5
   else Determine the bus voltage corrections by (53)
   Update the bus voltages
   Order the equations
   Increment iteration counter: $\nu = \nu + 1$

tested.

2.2.4 Review

Convergence characteristics The work of Sereeter et al [27] puts previous suggested modifications together. It compares the Newton-Raphson method in Carthesian, polar and complex form and with power and current mismatch equations. The result is that six versions of the Newton-Raphson method are described. The NR-TCIM in Polar or Carthesian coordinates are the most robust and fastest methods and more stable to the change of loading conditions and R/X ratios for both balanced and unbalanced networks. The performance of other methods is highly sensitive to these changes.

3 Integrated Networks

3.1 Introduction

The increasing amount of distributed generation, mostly PV (photovoltaic) panels, enables reverse power flow of what was conventionally unidirectional flow. For PV, the voltage is set to 231V instead of the net-voltage of 230 V, to enforce the energy to flow into the grid. Increased production can result in reversed power flow on the distribution grid and, eventually, on the transmission network. Extensive study of an integrated network is therefore recommended.

3.1.1 Methodology

As seen in the first part, transmission and distribution networks differ significantly and are solved with different algorithms where Newton-Raphson and Newton-Raphson-TCIM have the preference. Next, attention should be paid to the integration of the transmission and distribution networks. The objective is to find a suitable model for both networks (phase-frame or sequence-frame, single or three-phase, etc.) and a coupling mechanism between the two networks. Three numerical methods are the object of study. Findings regarding modeling complexity, convergence, and speed will be discussed. The solver is not the topic of investigation, this will be limited to the two versions of Newton-Raphson.
As an integrated network can become very large, it should probably be solved in parallel to gain significant speed-up. Parallelization increases modeling complexity and should be considered when choosing the integration technique.

3.2 Model approaches

As the transmission network is balanced, this network is modeled as a single-phase system. The distribution network is, due to unbalanced loading and unbalanced operation, in general unbalanced and modeled as a three-phase system. Connecting a single-phase to a three-phase system can therefore be complicated. Although the Transmission system is in practical three-phase and can therefore be modeled in this way, this is not a favourable method for computational and memory reasons. Assuming the distribution network to be balanced would allow too many simplifications and results in useless solutions. A unified approach, with respect to different system specifications, seems most optimal.
Some technical challenges that arise when unifying the power flow as a whole [16]:

- Scale of the problem becomes very large

- The grids differ in voltage levels (MW and KW), topology (meshed and radial) and power flow parameters
- The models of both grids are built and maintained in geographically separated systems, which requires algorithms to support geographically distributed computation (This is different in the Netherlands)

Several papers propose different methods to solve the system as an integrated network, we treat some of these methods[2] [16]:

1. A master-slave splitting concept based power flow algorithm
2. An interconnected power flow formulation, see the section on Symmetrical Components
3. A domain decomposition algorithm (not yet covered)

The several methods are tested on certain design and numerical criteria.

- Design Criteria
  - Balanced vs. (fully) unbalanced
  - Boundary injections
  - Parallelity considerations
- Numerical criteria
  - Number of iterations scales independently of number of unknowns
  - Convergence
  - Accuracy

Lastly, it is important to mention that the focus of these models is on steady-state operation and not on any dynamic simulations, such as transients.

3.2.1 The Master-Slave splitting

The master-slave approach solves the two systems iteratively, where convergence is based on the mismatch on the boundary between the two systems [30]. They assume that the total nodal load of the transmission grid is equal to the total three-phase power injected into the root node of the distribution grid and that this load is balanced. The system is solved with an iterative approach between the separate networks by means of a boundary state.

[2] There exist several other techniques to solve integrated networks, such as co-simulation, which are outside the scope of numerical analysis.

Algorithm

step 1: Initialization: determine the boundary state (between T and D) x_B^0 and set iteration counter $\nu = 0$

step 2: Solve the slave sytem: assume x_B^ν is known and solve for state x_S^ν of the slave system. Use X_S^ν to solve for y_B^ν

step 3: Solve the master (T-sytem): assume y_B^ν and solve for the sate $[x_M^\nu \; x_B^\nu]$ of the master

step 4: If $||x_B^{\nu-1} - x_B^\nu|| \leq \epsilon$, convergence is achieved. Otherwise, set $\nu = \nu + 1$.

It is a sequential method, as it is decoupled into transmission and distribution power flow sub-problems.

Networks Thus far, the master-slave approach is only tested on single-phase systems, such that no extra considerations on the boundary node had to be taken into account [30]. If both the transmission and distribution system are modeled as a three-phase system, the master-slave approach can easily be extended to this system, but as explained before, this is not advantageous.
In case of an unbalanced distribution system, the distribution system should be modeled in three-phase and some extra constrains on the boundary should be taken.

An open-source Toolbox The authors of [23] have created a Matlab Toolbox to analyze combined transmission and distribution network models with a master-slave splitting approach. The toolbox is called: TDNetGen. So far, the power flow library of Matlab, called Matpower, is only able to solve separated single-phase systems. Their toolbox connects both networks, but still focus only on single-phase networks.

3.2.2 The interconnected method

An interconnected (sometimes called hybrid) network is a network that consists of a transmission network and one or more distribution networks which is solved as one system. Some extra attention has to be paid on the connection between these two networks, to integrate the system smoothly.

Full three-phase The first, most straight-forward approach is to model both systems completely in three-phase. This should not result in any additional complexity as the systems are physically three-phase. As the data is only given for one phase a, the phases b and c can be easily created by shifting the phase-angle with $2/3\pi$ and $4/3\pi$ respectively.

Single-phase versus three-phase power flow The authors of [31] respect the balanced conditions of the transmission system and keep this system single-phase. To connect it to a three-phase system, they propose to model the transmission system in an equivalent positive sequence frame and inject a zero and a negative sequence on the boundary to couple it to the three-phase system.

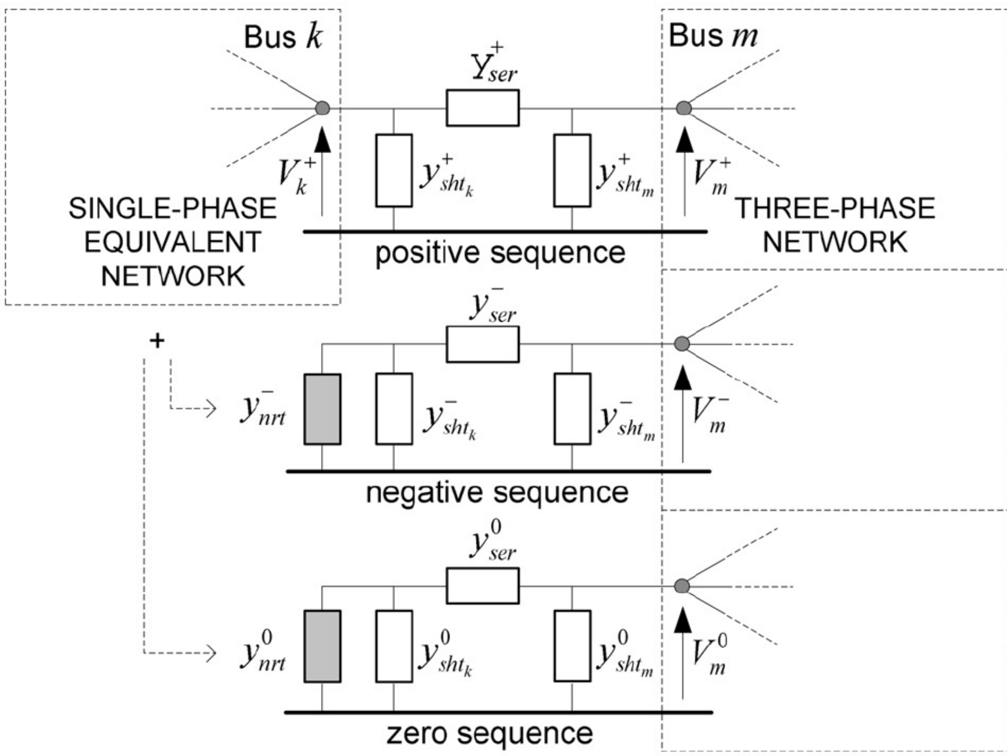

Figure 14: Equivalent π line model

Solution method

- Model T-network in positive sequence frame

$$I^+ = Y^+ V^+$$
$$S^+ = \overline{I^+} V^+$$

- Relation between phase and sequence frame

$$V_k^{abc} = T_1 V_k^+$$
$$I_k^+ = T_2 I_k^{abc}$$

- Transform Y^{abc}-matrix in similar manner by T_1 and T_2:

$$T_1 = [1\ a^2\ a]^T$$
$$T_2 = \frac{1}{3}[1\ a\ a^2]$$

  shift-operator: $a = e^{\frac{2}{3}\pi\iota}$

- Transform J_{abc} by T_3 and T_4:

$$T_3 = \begin{bmatrix} 1 & 0 & a_r & -a_i & a_r & a_i \\ 0 & 1 & a_i & a_r & -a_i & a_r \end{bmatrix}^T$$
$$T_4 = \frac{1}{3}\begin{bmatrix} 1 & 0 & a_r & -a_i & a_r & a_i \\ 0 & 1 & a_i & a_r & -a_i & a_r \end{bmatrix},$$

  where $a_r = -\frac{1}{2}$ and $a_i = \frac{\sqrt{3}}{2}$

After these transformations, the interconnected network can be solved at once.

Three-sequence vs. three-phase systems So far, we considered a full three-phase system and a single-phase connected to a three-phase system. As the assumption of a fully balanced transmission network is not always valid, the authors of [16] suggest to reconsider the model for the transmission network. Especially in dynamic simulations, this assumption is not valid. Although our focus is on steady-state operation, it cannot harm to follow their approach.
They presume that a transmission system contains unbalanced loading and a small portion of unbalanced lines which causes unbalanced operating conditions. They neglect the effect of unbalanced operating conditions but do incorporate unbalanced loading. Balanced lines in combination with unbalanced loading can still be transformed into the sequence frame and is then decoupled into three sequences: the positive, negative and zero sequence.

Advantage of this method is that, the three-sequence model is a decoupled model, where no Jacobian of size $6N$ by $6N$ has to be solved, but a positive sequence model of $2N$ by $2N$ and a zero and a negative sequence of both N by N.
Decoupling of the three-sequence system, allows solving them in parallel. In case of many unbalanced lines, they cannot be neglected and the system cannot be decoupled and have to be solved in three-phase.

Symmetrical systems with unbalanced loading conditions In case of symmetrical systems with unbalanced loading the theory of symmetrical components can be used. The three-phase elements of the system can be described in phasor detail and then transformed according the transformation matrices T_1 and T_2. The same solvers, explained in the next chapter, still perform well on these systems. Sometimes a small amount of unbalanced lines is present, which makes the system not completely symmetrical. The authors of [8] present a way to decouple the system by averaging the mutual impedances along the lines.

Non-symmetrical systems with balanced loading conditions The authors of [1] present a method to solve unbalanced system with balanced loading conditions. Unbalanced lines are handled by eliminating off-diagonal elements in the admittance matrix and replacing them with certain compensation current injections at both ends of the unbalanced line. This idea is first proposed by Ziao-Ping Zhang (Fast three phase load flow methods 2005). They investigated the case of a fully unbalanced system with unbalanced loading, but note that the system will converge but with wrong results.

3.3 Future research

Conventionally, transmission networks are solved by Newton-Raphson power mismatch and distribution networks by Newton-Raphson Current mismatch. When connecting these two methods, it is important to check which solution method is most favourable for both systems.
We recommend to continue with this research and start with a simple test-case to compare the three proposed integration methods. A start can be done with a simple 4-node Distribution network (figure 15) [36] , by assuming branch 1-2 a Transmission line, branch 2-3 the connecting transformer and branch 3-4 a distribution line. Afterwards, the idea can be extended to bigger test-cases. The full three-phase method, the master-slave method and the

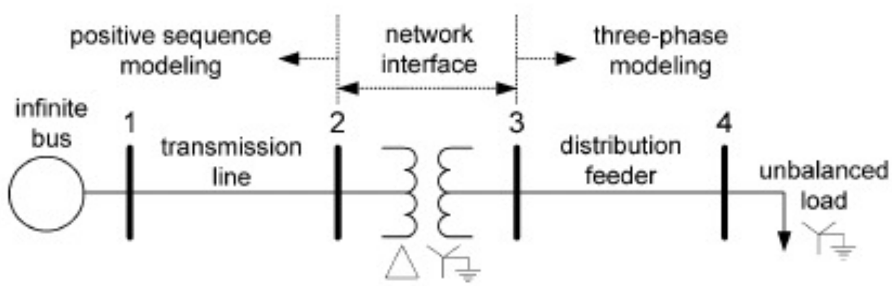

Fig. 6. IEEE Four-Node Test Feeder.

Figure 15: A 4 node test-feeder, provided by [26]

interconnected method should be compared on modeling complexity, convergence, and speed. As Newton-Raphson power mismatch is most favourable to solve transmission networks and Newton-Raphson-TCIM to solve distribution networks, it is highly recommended to test these two different versions as well on the integrated network.

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