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Data-driven output regulation of nonlinear systems via incremental passivity

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Abstract— **This work addresses data-driven output regulation of nonlinear systems with unknown system parameters. Using incremental passivity, output regulation is achieved by interconnecting an incrementally passive internal model and an incrementally passive closed-loop system of the plant. The controller that makes the closed-loop system incrementally passive is characterized by linear matrix inequalities using finite offline data. A numerical example verifies the proposed data-driven regulator design.**

I. Introduction

Direct data-driven control synthesizes controllers directly via experimental data collected from an unknown dynamical system without explicitly identifying an accurate model. Direct data-driven control can be an efficient alternative to indirect data-driven control when a sufficiently accurate model is costly to identify and of little or no interest. Recent developments of direct data-driven control with stability guarantees have been utilizing Willems et al.'s fundamental lemma [1] for characterizing linear and nonlinear controllers via data, such as can be found in [2]–[6]. It is noted that, most of these results focus on the stabilization of a known equilibrium and more complex problems such as tracking of uncertain trajectories have been less addressed. This work considers using finite offline data for output regulation, i.e., reference tracking and disturbance rejection, of a class of nonlinear systems.

Related work. The objectives of output regulation include stabilization of the closed-loop system and asymptotic tracking of a reference, despite external disturbances. Seminal work on linear and nonlinear output regulation includes [7]–[10]. In output regulation problem formulations, the reference and disturbance are exogenous signals generated by an exosystem. The internal model principle is a powerful tool that regulates the tracking error via feedback and is a necessary condition for robustness against model uncertainties for linear output regulation [11]. The regulator design consists of an internal model incorporating the exosystem dynamics and a stabilizer for the augmented system composed of the internal model and the plant. As the construction of the stabilizer depends on the design of the internal model, the stabilizer needs to be redesigned if the exosystem changes, and the overall regulator design can be complicated for nonlinear dynamics. The authors of [12] explored a universal stabilizer design that is independent of the choice of the internal model using incremental passivity. In particular, a stabilizer is developed to make the plant incrementally passive. Then, by interconnection of the stabilizer with an incrementally passive internal model, output regulation is achieved. This approach decouples the designs of the internal model and the stabilizer, and thus simplifies the design of the regulator.

The aforementioned existing work on output regulation relies on a sufficiently accurate model of the plant. When such a model is unavailable, data-driven approaches have also been proposed. For instance, a survey on linear and nonlinear output regulation via adaptive dynamic programming is found in [13]. Using finite input-state data, [14] established necessary and sufficient conditions for the informativity of a given set of data for the algebraic regulator problem. The result of [14] was extended by the authors of [15] to linear robust output regulation, and a dynamic data-driven state feedback regulator was designed. Using data-based contractivity [16], the authors of [17] proposed data-driven regulators for nonlinear systems such that the tracking error of periodic exogenous signals are made arbitrarily small. Assuming unknown linear plant and disturbance model, [18] used a Koopman operator for constructing an extended system and achieved data-driven output regulation.

Contributions. This work proposes a data-driven output regulator design via incremental passivity for a class of nonlinear systems, where all the system parameters are unknown. For a class of known linear exosystems, by designing a feedback stabilizer that makes the closed-loop system incrementally passive and then interconnecting the closed-loop system with an incrementally passive internal model, output regulation is achieved. Thanks to incremental passivity, the designs of the stabilizer and the internal model are independent and substantially simplified. The control gains are obtained by solving a set of linear matrix inequalities (LMIs) dependent on finite offline experimental data, which is computationally efficient. It is noted that this work requires the dynamics of the exosystem but not the initial condition of the exogenous signal, and the exogenous signal is not measured or used for the regulator design.

Outline. The rest of the paper is organized as follows. Section II formulates the data-driven output regulation problem, analyzes the exogenous signals, and reviews incremental passivity. Section III presents the data-driven designs of the stabilizer, the internal model, and the overall regulator. A numerical example is illustrated in Section IV for verification of the proposed approach. Finally, Section V draws the conclusion.

Notation. Throughout the paper, $\mathbb{R}$ denotes the set of

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real numbers and $\mathbb{R}_{\geq 0}$ denotes the set of non-negative real numbers. For a symmetric matrix A, $A \succ (\succeq) 0$ indicates that A is positive (semi-)definite and $A \prec (\preceq) 0$ indicates that A is negative (semi-)definite. $A + (*)^\top$ represents the sum of a square matrix A and its transpose. All elements of vector $\mathbb{1}_n \in \mathbb{R}^n$ are ones.

II. Problem formulation and preliminaries

This section formulates the data-driven output regulation problem and presents preliminaries that are essential for the main result.

A. Data-driven output regulation

Consider the nonlinear system

$$\begin{aligned} \dot{x} &= AZ(x) + Bu + Ew \\ e &= CZ(x) + Fw \end{aligned} \tag{1}$$

with state $x \in \mathbb{R}^n$, input $u \in \mathbb{R}^m$, and error $e \in \mathbb{R}^m$. The known $Z : \mathbb{R}^n \to \mathbb{R}^{nz}$ is a vector in x. Specifically, $Z(x)$ is in the form of $Z(x) = \begin{bmatrix} x^\top & Q(x)^\top \end{bmatrix}^\top$ where $Q(x) \in \mathbb{R}^{(nz-n)}$ denotes the nonlinear part of Z. The uncertain signal $w \in \mathbb{R}^q$ is generated by the exosystem

$$\dot{w} = Sw \tag{2}$$

where matrix $S \in \mathbb{R}^{q\times q}$ is known and the unknown initial condition $w(t_0) \in \mathbb{W} \subset \mathbb{R}^q$ with $\mathbb{W}$ being a compact and forward-invariant set.

This work aims at designing a control input u such that the tracking error e converges to 0 asymptotically despite w. To ensure that the problem has a solution, the following common assumptions are posed.

Assumption 1: All the eigenvalues of S have zero real parts.

Assumption 2: For any $w(t_0) \in \mathbb{W}$, there exist $\mathbf{x}(w)$ and $\mathbf{u}(w)$ that are polynomials in w such that $\mathbf{x}(0) = 0$, $\mathbf{u}(0) = 0$, and

$$\begin{aligned} \frac{\partial \mathbf{x}}{\partial w} Sw &= AZ(\mathbf{x}(w)) + B\mathbf{u}(w) + Ew \\ 0 &= CZ(\mathbf{x}(w)) + Fw. \end{aligned} \tag{3}$$

Remark 1 (On the assumptions): Under Assumption 1, exosystem (2) can generate a combination of a step function with arbitrary amplitude and sinusoidal functions with arbitrary amplitudes and initial phases whose frequencies depend on the eigenvalues of S. Assumption 2 guarantees that there exist steady-state state and input that solve the output regulation problem. Equations (3) are commonly known as regulator equations in the literature on output regulation. It is noted that the approach proposed in this work does not require the solution to the regulator equations. Nonetheless, it is still of our future interest to verify the solvability of (3) via data.

This work assumes that all the system matrices in (1) are unknown, and a data set $\mathcal{DS} := \{\dot{x}(t_k), x(t_k), u(t_k), e(t_k), k = 0, 1, \ldots, T-1\}$ for some $T > 1$ is sampled from one or multiple experiments. The data-driven output regulation problem is formulated as follows.

Problem 1 (Data-driven output regulation): For nonlinear systems (1) with unknown system matrices and known exosystem (2), design a feedback control input u using data set $\mathcal{DS}$, such that all solutions to the closed-loop system are bounded and $\lim_{t\to\infty} e(t) = 0$.

Remark 2 (Unknown nonlinear system): This work assumes that the vector $Z(x)$ consists of known nonlinearities, but the matrices A, B, C, E, and F are unknown. In most existing results on data-driven output regulation, such as [14] and [15], the matrices C, E and F are assumed to be known. For the approach proposed in this work, knowledge of E and F is not needed for controller design, and C can be presented using the error data.

B. Representation of the exogenous signal data

A crucial step of the proposed data-driven control approach is deriving the data-based representation of the closed-loop system. A representation of the exogenous signal data sampled during the experiment(s) is thus needed and has been presented in [16]. For the completeness of this paper, we include the derivation of the representation of the exogenous signal generated by (2) as follows.

Under Assumption 1, one can rearrange the exosystem such that w and S are partitioned as $w = \begin{bmatrix} w_1 & w_2 & \cdots & w_{2q_1} & w_{2q_1+1} & w_{2q_1+2} & \cdots & w_{2q_1+q_2} \end{bmatrix}^\top$ and

$$S = \operatorname{blockdiag}\left(\begin{bmatrix} 0 & \sigma_1 \\ -\sigma_1 & 0 \end{bmatrix}, \ldots, \begin{bmatrix} 0 & \sigma_{q_1} \\ -\sigma_{q_1} & 0 \end{bmatrix}, 0_{q_2\times q_2} \right)$$

where q_1 and q_2 are integers such that $2q_1 + q_2 = q$. As all eigenvalues of S are on the imaginary axis, the signal w has sinusoidal and constant components. To analyze the sinusoidal component of w, let us denote $\overline{w}_i = \begin{bmatrix} w_{2i-1} & w_{2i} \end{bmatrix}^\top \in \mathbb{R}^2$ where $i = 1, \ldots, q_1$. Then, we write

$$\dot{\overline{w}}_i = \overline{S}_i \overline{w}_i \quad \text{with} \quad \overline{S}_i = \begin{bmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{bmatrix}.$$

Solving this linear system using the eigenvalues and eigenvectors of $\overline{S}_i$ gives

$$\overline{w}_i(t) = \underbrace{\begin{bmatrix} \gamma_{i2} & \gamma_{i1} \\ -\gamma_{i1} & \gamma_{i2} \end{bmatrix}}_{:=\gamma_i} \underbrace{\begin{bmatrix} \sin(\sigma_i t) \\ \cos(\sigma_i t) \end{bmatrix}}_{:=\overline{M}_i(t)} = \gamma_i \overline{M}_i(t) \tag{4}$$

with unknown constants γ_{i1} and γ_{i2} depending on the initial condition $\overline{w}_i(t_0)$. Thus, data matrix $W_0 := \begin{bmatrix} w(t_0) & w(t_1) & \cdots & w(t_{T-1}) \end{bmatrix} \in \mathbb{R}^{q\times T}$ can be written as

$$W_0 = \Gamma M_0 \tag{5}$$

where the unknown matrix Γ and known matrix M_0 are

$$\begin{aligned} \Gamma &:= \operatorname{diag}\big(\gamma_1, \ldots, \gamma_{q_1}, w_{2q_1+1}(t_0), \ldots, w_{2q_1+q_2}(t_0)\big) \\ &\quad \in \mathbb{R}^{q\times q}, \\ \overline{M}(t) &:= \begin{bmatrix} \overline{M}_1(t)^\top & \cdots & \overline{M}_{q_1}(t)^\top & \mathbb{1}_{q_2}^\top \end{bmatrix}^\top \in \mathbb{R}^{q\times 1}, \\ M_0 &:= \begin{bmatrix} \overline{M}(t_0) & \cdots & \overline{M}(t_{T-1}) \end{bmatrix} \in \mathbb{R}^{q\times T}. \end{aligned}$$

C. Output regulation via incremental passivity

In what follows, we briefly review the definition of incremental passivity and the condition characterizing incrementally passive systems.

Definition 1 (Incremental passivity [12]): The system

$$\begin{aligned} \dot{x} &= f(x,u,t) \\ y &= h(x,t) \end{aligned} \tag{6}$$

with state $x \in \mathbb{R}^n$, input $u \in \mathbb{R}^m$, and output $y \in \mathbb{R}^p$ is incrementally passive if there exists a C^1 storage function $V(t,x_1,x_2) : \mathbb{R}_{\geq 0} \times \mathbb{R}^{2n} \to \mathbb{R}_{\geq 0}$ such that for any two inputs $u_1(t)$ and $u_2(t)$ and any two solutions $x_1(t)$ and $x_2(t)$ of (6) corresponding to the inputs, the respective outputs $y_1 = h(x_1,t)$ and $y_2 = h(x_2,t)$ satisfy the inequality

$$\begin{aligned} \dot{V}(t,x_1,x_2) &= \frac{\partial V}{\partial t} + \frac{\partial V}{\partial x_1} f(x_1,u_1,t) + \frac{\partial V}{\partial x_2} f(x_2,u_2,t) \\ &\leq (y_1 - y_2)^\top (u_1 - u_2). \end{aligned} \tag{7}$$

Definition 2 (Regular storage function [12]): A storage function $V(t,x_1,x_2)$ is called regular if for any sequence (t_k, x_{1k}, x_{2k}), $k = 1,2,\ldots$, such that x_{2k} is bounded, t_k tends to infinity, and $\|x_{1k}\| \to +\infty$, it holds that $V(t_k,x_{1k},x_{2k}) \to +\infty$ as $k \to +\infty$.

Incremental passivity describes the relationship between any two input-output trajectories of a system. As shown in [12], the feedback interconnection of two incrementally passive systems with regular storage functions is also incrementally passive with a regular storage function. This property lays the foundation for the regulator design. A simple example of a regular storage function is $V(x_1,x_2) = (x_1 - x_2)^\top \mathcal{P}(x_1 - x_2)$ with some $\mathcal{P} \succ 0$.

The following result from [12] gives the condition for characterizing a class of incrementally passive systems.

Lemma 1 ([12]): Consider the system

$$\begin{aligned} \dot{x} &= f(x,w) + Bu \\ e &= Cx + h(w) \end{aligned} \tag{8}$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, and $e \in \mathbb{R}^m$. Suppose that $f(x,w)$ is continuous in w and C^1 in x. If there exists $\mathcal{P} \succ 0$ such that

$$\begin{aligned} &\mathcal{P}\frac{\partial f}{\partial x}(x,w) + \left(\frac{\partial f}{\partial x}(x,w)\right)^\top \mathcal{P} \preceq 0 \quad \forall x \in \mathbb{R}^n,\ w \in \mathbb{W}, \\ &\mathcal{P}B = C^\top \end{aligned}$$

then for any continuous $w \in \mathbb{W}$, system (8) is incrementally passive with input u, output e, and a regular storage function.

In the next section, a result similar to Lemma 1 will be derived for a class of nonlinear systems, which will be subsequently used to construct a data-driven controller that makes the closed-loop system incrementally passive.

III. Data-driven output regulator design

The data-driven output regulation approach proposed in this work consists of designing a controller making the closed-loop system incrementally passive and interconnecting the closed-loop system with an incrementally passive internal model. This section presents the data-driven control design for achieving incremental passivity, and then the overall design for output regulation.

A. Incremental passivation via data

Consider the dynamic controller

$$\begin{aligned} \dot{\xi} &= L\xi + e, \\ u &= KZ(x) + \overline{K}\xi + v \end{aligned} \tag{9}$$

where $\xi \in \mathbb{R}^m$ is the controller state, $v \in \mathbb{R}^m$ is a virtual input, and control gains L, K, and $\overline{K}$ are to be designed. Defining $\zeta = \begin{bmatrix} x^\top & \xi^\top \end{bmatrix}^\top \in \mathbb{R}^{n+m}$ and $\widetilde{Z}(\zeta) = \begin{bmatrix} Z(x)^\top & \xi^\top \end{bmatrix}^\top \in \mathbb{R}^{n_Z+m}$ leads to the closed-loop system

$$\begin{aligned} \dot{\zeta} &= \widetilde{A}\widetilde{Z}(\zeta) + \widetilde{B}v + \widetilde{E}w \\ e &= \widetilde{C}\widetilde{Z}(\zeta) + Fw \end{aligned} \tag{10}$$

where

$$\begin{aligned} \widetilde{A} &:= \begin{bmatrix} A + BK & B\overline{K} \\ C & L \end{bmatrix},\ \widetilde{B} := \begin{bmatrix} B \\ 0_{m\times m} \end{bmatrix}, \\ \widetilde{E} &:= \begin{bmatrix} E \\ F \end{bmatrix},\ \ \widetilde{C} := \begin{bmatrix} C & 0_{m\times m} \end{bmatrix}. \end{aligned}$$

Using Definition 1 and in a similar manner as Lemma 1, a characterization of incremental passivity of (10) is derived.

Lemma 2: If there exists a positive definite matrix $\mathcal{P} \in \mathbb{R}^{(n+m)\times(n+m)}$ such that

$$\mathcal{I}^\top \mathcal{P}\widetilde{A} + \widetilde{A}^\top \mathcal{P}\,\mathcal{I} \preceq 0 \tag{11a}$$

$$\mathcal{I}^\top \mathcal{P}\widetilde{B} = \widetilde{C}^\top \tag{11b}$$

where

$$\mathcal{I} := \begin{bmatrix} I_n & 0_{n\times(n_Z-n)} & 0_{n\times m} \\ 0_{m\times n} & 0_{m\times(n_Z-n)} & I_m \end{bmatrix},$$

then system (10) is incrementally passive with respect to input v, output e, and a regular storage function.

Proof: Define the regular storage function as $V(\zeta_1,\zeta_2) = \frac{1}{2}(\zeta_1-\zeta_2)^\top \mathcal{P}(\zeta_1-\zeta_2)$. The time derivative of the storage function along the solutions ζ_1 and ζ_2 corresponding to the inputs v_1 and v_2 satisfies that

$$\begin{aligned} &\dot{V}(\zeta_1,\zeta_2) \\ &= \frac{1}{2}(\zeta_1-\zeta_2)^\top \mathcal{P}(\dot{\zeta}_1-\dot{\zeta}_2) + \frac{1}{2}(\dot{\zeta}_1-\dot{\zeta}_2)^\top \mathcal{P}(\zeta_1-\zeta_2) \\ &= \frac{1}{2}(\zeta_1-\zeta_2)^\top \mathcal{P}\Big(\widetilde{A}\widetilde{Z}(\zeta_1) + \widetilde{B}v_1 + \widetilde{E}w \\ &\quad -\widetilde{A}\widetilde{Z}(\zeta_2) - \widetilde{B}v_2 - \widetilde{E}w\Big) + (*)^\top \\ &= \frac{1}{2}(\zeta_1-\zeta_2)^\top \mathcal{P}\left(\widetilde{A}\left(\widetilde{Z}(\zeta_1) - \widetilde{Z}(\zeta_2)\right) + \widetilde{B}(v_1-v_2)\right) \\ &\quad + (*)^\top. \end{aligned}$$

Recalling $Z(x) = \begin{bmatrix} x^\top & Q(x)^\top \end{bmatrix}^\top$ and by the definition of $\mathcal{I}$, one has that

$$\begin{aligned} &\zeta_1 - \zeta_2 \\ &= \begin{bmatrix} x_1 - x_2 \\ \xi_1 - \xi_2 \end{bmatrix} = \mathcal{I}\begin{bmatrix} x_1 - x_2 \\ Q(x_1) - Q(x_2) \\ \xi_1 - \xi_2 \end{bmatrix} = \mathcal{I}\left(\widetilde{Z}(\zeta_1) - \widetilde{Z}(\zeta_2)\right). \end{aligned}$$

Therefore, it holds that

$$\begin{aligned}&\dot{V}(\zeta_1,\zeta_2)\\&=\frac{1}{2}\left(\widetilde{Z}(\zeta_1)-\widetilde{Z}(\zeta_2)\right)^\top\left(\mathcal{I}^\top\mathcal{P}\widetilde{A}+\widetilde{A}^\top\mathcal{P}\mathcal{I}\right)\left(\widetilde{Z}(\zeta_1)\right.\\&\quad\left.-\widetilde{Z}(\zeta_2)\right)+\left(\widetilde{Z}(\zeta_1)-\widetilde{Z}(\zeta_2)\right)^\top\mathcal{I}^\top\mathcal{P}\widetilde{B}(v_1-v_2).\end{aligned}$$

The difference between the corresponding e_1 and e_2 can be found as

$$\begin{aligned}&e_1-e_2\\&=\widetilde{C}\widetilde{Z}(\zeta_1)+Fw-\widetilde{C}\widetilde{Z}(\zeta_2)-Fw=\widetilde{C}\left(\widetilde{Z}(\zeta_1)-\widetilde{Z}(\zeta_2)\right).\end{aligned}$$

Under conditions (11a) and (11b), one has that

$$\dot{V}(\zeta_1,\zeta_2)\le(e_1-e_2)^\top(v_1-v_2).$$

This proves that system (10) is incrementally passive with respect to input v, output e, and the regular storage function $V(\zeta_1,\zeta_2)$. ■

Remark 3: Instead of the LMIs derived in Lemma 2, one can also directly use Lemma 1 to derive nonlinear state-dependent conditions characterizing the incremental passivity of (10). Then, if the nonlinearities satisfy the Lipschitz property, one can use an approach similar to [16, Theorem 1] to handle the nonlinearities.

Lemma 2 characterizes sufficient conditions for designing a dynamic controller (9) that renders the closed-loop system (10) incrementally passive. As the conditions depend on system matrices A, B, and C, which are unknown in the setting of this work, data set $\mathcal{DS}$ is used to derive a data-based representation of (10). For this purpose, we first obtain the following data matrices

$$\begin{aligned}Z_0&:=\begin{bmatrix}Z(x(t_0)) & Z(x(t_1)) & \cdots & Z(x(t_{T-1}))\end{bmatrix}\in\mathbb{R}^{n_Z\times T}\\X_1&:=\begin{bmatrix}\dot{x}(t_0) & \dot{x}(t_1) & \cdots & \dot{x}(t_{T-1})\end{bmatrix}\in\mathbb{R}^{n\times T}\\E_0&:=\begin{bmatrix}e(t_0) & e(t_1) & \cdots & e(t_{T-1})\end{bmatrix}\in\mathbb{R}^{m\times T}\\U_0&:=\begin{bmatrix}u(t_0) & u(t_1) & \cdots & u(t_{T-1})\end{bmatrix}\in\mathbb{R}^{m\times T}\end{aligned}$$

from $\mathcal{DS}$ and the known vector $Z(x)$. The following lemma represents (10) via the data matrices.

Lemma 3: Consider nonlinear system (1), controller (9) and data set $\mathcal{DS}$. For any matrices $K\in\mathbb{R}^{m\times n_Z}$, $\overline{K}\in\mathbb{R}^{m\times m}$, and $G=\begin{bmatrix}G_1 & G_2 & G_3\end{bmatrix}\in\mathbb{R}^{T\times(n_Z+2m)}$ with $G_1\in\mathbb{R}^{T\times n_Z}$, $G_2\in\mathbb{R}^{T\times m}$, and $G_3\in\mathbb{R}^{T\times m}$ satisfying

$$\begin{bmatrix}I_{n_Z} & 0_{n_Z\times m} & 0_{n_Z\times m}\\K & \overline{K} & I_m\\0_{q\times n_Z} & 0_{q\times m} & 0_{q\times m}\end{bmatrix}=\begin{bmatrix}Z_0\\U_0\\M_0\end{bmatrix}G, \tag{12}$$

the closed-loop system composed of (1) and (9) can be written as

$$\begin{aligned}\dot{\zeta}&=\widetilde{A}_d\widetilde{Z}(\zeta)+\widetilde{B}_d v+\widetilde{E}w\\e&=\widetilde{C}_d\widetilde{Z}(\zeta)+Fw\end{aligned}\tag{13}$$

where the data-based system matrices are defined as

$$\begin{aligned}&\widetilde{A}_d=\begin{bmatrix}X_1G_1 & X_1G_2\\E_0G_1 & L\end{bmatrix},\ \widetilde{B}_d=\begin{bmatrix}X_1G_3\\0_{m\times m}\end{bmatrix},\\&\widetilde{C}_d=\begin{bmatrix}E_0G_1 & 0_{m\times m}\end{bmatrix}.\end{aligned}\tag{14}$$

Proof: By the dynamics of system (1), the data matrices satisfy that

$$X_1=AZ_0+BU_0+EW_0,\quad E_0=CZ_0+FW_0$$

where $W_0=\Gamma M_0$ based on the analysis in Section II-B. Under (12), it holds that

$$\begin{aligned}A+BK&=\begin{bmatrix}A & B & E\Gamma\end{bmatrix}\begin{bmatrix}I_{n_Z}\\K\\0\end{bmatrix}=\begin{bmatrix}A & B & E\Gamma\end{bmatrix}\begin{bmatrix}Z_0\\U_0\\M_0\end{bmatrix}G_1=X_1G_1,\\B\overline{K}&=\begin{bmatrix}A & B & E\Gamma\end{bmatrix}\begin{bmatrix}0\\\overline{K}\\0\end{bmatrix}=\begin{bmatrix}A & B & E\Gamma\end{bmatrix}\begin{bmatrix}Z_0\\U_0\\M_0\end{bmatrix}G_2=X_1G_2,\\C&=\begin{bmatrix}C & 0 & F\Gamma\end{bmatrix}\begin{bmatrix}I_{n_Z}\\K\\0\end{bmatrix}=\begin{bmatrix}C & 0 & F\Gamma\end{bmatrix}\begin{bmatrix}Z_0\\U_0\\M_0\end{bmatrix}G_1=E_0G_1.\end{aligned}$$

Recall the definitions of matrices $\widetilde{A}$, $\widetilde{B}$, and $\widetilde{C}$ in (10), one can obtain the data-based system matrices given in (14). ■

Having derived the data-based closed-loop representation (13), we present the following result for the data-driven design of controller (9).

Theorem 1: Consider system (1), dynamic controller (9), and data set $\mathcal{DS}$. If there exist matrices $Y_1\in\mathbb{R}^{T\times n_Z}$, $Y_2\in\mathbb{R}^{T\times m}$, $Y_3\in\mathbb{R}^{m\times m}$, $G_3\in\mathbb{R}^{T\times m}$, and positive definite matrix $P_1=\operatorname{diag}(P_{11},P_{12})\in\mathbb{R}^{n_Z\times n_Z}$ with $P_{11}\in\mathbb{R}^{n\times n}$ and $P_{12}\in\mathbb{R}^{(n_Z-n)\times(n_Z-n)}$, such that

$$\begin{bmatrix}P_1 & 0_{n_Z\times m}\\0_{q\times n_Z} & 0_{q\times m}\end{bmatrix}=\begin{bmatrix}Z_0\\M_0\end{bmatrix}\begin{bmatrix}Y_1 & Y_2\end{bmatrix}\tag{15a}$$

$$\begin{bmatrix}0_{n_Z\times m}\\I_m\\0_{q\times m}\end{bmatrix}=\begin{bmatrix}Z_0\\U_0\\M_0\end{bmatrix}G_3\tag{15b}$$

$$\mathcal{I}^\top\begin{bmatrix}X_1Y_1 & X_1Y_2\\E_0Y_1 & Y_3\end{bmatrix}+(*)^\top\preceq 0\tag{15c}$$

$$\begin{bmatrix}(X_1G_3)^\top & 0_{m\times(n_Z-n)}\end{bmatrix}=E_0Y_1\tag{15d}$$

then controller (9) with $L=Y_3P_2^{-1}$, $K=U_0Y_1P_1^{-1}$, and $\overline{K}=U_0Y_2P_2^{-1}$ for any positive definite $P_2\in\mathbb{R}^{m\times m}$ renders the closed-loop system incrementally passive with respect to input v and output e with a regular storage function.

Proof: Let $G_1=Y_1P_1^{-1}$ and $G_2=Y_2P_2^{-1}$, then conditions (15a) and (15b) guarantee the data-based closed-loop representation (14).

Left- and right-multiply $\widetilde{P}^{-1}$, where $\widetilde{P}=\operatorname{diag}(P_1,P_2)=\operatorname{diag}(P_{11},P_{12},P_2)$, to both sides of inequality (15c) gives

$$\begin{aligned}&\widetilde{P}^{-1}\mathcal{I}^\top\begin{bmatrix}X_1Y_1 & X_1Y_2\\E_0Y_1 & Y_3\end{bmatrix}\widetilde{P}^{-1}+(*)^\top\\&=\widetilde{P}^{-1}\mathcal{I}^\top\begin{bmatrix}X_1G_1P_1 & X_1G_2P_2\\E_0G_1P_1 & LP_2\end{bmatrix}\begin{bmatrix}P_1^{-1} & 0\\0 & P_2^{-1}\end{bmatrix}+(*)^\top\\&=\begin{bmatrix}P_{11}^{-1} & 0 & 0\\0 & P_{12}^{-1} & 0\\0 & 0 & P_2^{-1}\end{bmatrix}\mathcal{I}^\top\begin{bmatrix}X_1G_1 & X_1G_2\\E_0G_1 & L\end{bmatrix}+(*)^\top\\&=\begin{bmatrix}P_{11}^{-1} & 0\\0 & 0\\0 & P_2^{-1}\end{bmatrix}\begin{bmatrix}X_1G_1 & X_1G_2\\E_0G_1 & L\end{bmatrix}+(*)^\top\preceq 0.\end{aligned}$$

Define the regular storage function as $V(\zeta_1, \zeta_2) = \frac{1}{2}(\zeta_1 - \zeta_2)^\top P^{-1}(\zeta_1 - \zeta_2)$ where $P = \mathrm{diag}(P_{11}, P_2)$. By the definition of $\mathcal{I}$ and the data-based representation of $\widetilde{A}_d$, one has that

$$\mathcal{I}^\top P^{-1}\widetilde{A}_d = \begin{bmatrix} P_{11}^{-1} & 0 \\ 0 & 0 \\ 0 & P_2^{-1} \end{bmatrix} \begin{bmatrix} X_1G_1 & X_1G_2 \\ E_0G_1 & L \end{bmatrix}.$$

Therefore, condition (15c) ensures that (11a) in Lemma 2 holds with $\mathcal{P} = P^{-1}$.

Condition (15d) is equivalent to

$$\begin{bmatrix}(X_1G_3)^\top & 0_{m\times(n_Z-n)} & 0_{m\times m}\end{bmatrix} = \begin{bmatrix}E_0Y_1 & 0_{m\times m}\end{bmatrix},$$

to the both sides of which we right-multiply $\widetilde{P}^{-1}$ and obtain

$$\begin{bmatrix}(X_1G_3)^\top P_{11}^{-1} & 0_{m\times(n_Z-n)} & 0_{m\times m}\end{bmatrix} = \begin{bmatrix}E_0G_1 & 0_{m\times m}\end{bmatrix}.$$

Again, by the definition of $\mathcal{I}$ and the data-based representations of $\widetilde{B}_d$ and $\widetilde{C}_d$, one has that

$$\mathcal{I}^\top P^{-1}\widetilde{B}_d = \begin{bmatrix}(X_1G_3)^\top P_{11}^{-1} & 0_{m\times(n_Z-n)} & 0_{m\times m}\end{bmatrix}^\top,$$
$$\widetilde{C}_d = \begin{bmatrix}E_0G_1 & 0_{m\times m}\end{bmatrix}.$$

Therefore, condition (15d) ensures that (11b) in Lemma 2 holds. Applying Lemma 2 to (13), the proof is complete. ■

B. Data-driven output regulation

The previous subsection shows the data-driven dynamic controller design that makes the closed-loop system incrementally passive. According to [12, Theorem 1], an interconnection of the incrementally passive closed-loop system and an incremental passive internal model renders the overall system incrementally passive and thus solves the output regulation problem.

The incrementally passive internal model used in this work is

$$\begin{aligned}\dot{\eta} &= S\eta + \alpha\Xi\tilde{e} \\ \tilde{v} &= \Xi^\top\eta\end{aligned} \tag{16}$$

where $\eta \in \mathbb{R}^q$, $\tilde{e} = -e$, and any fixed $\alpha > 0$. The corresponding regular storage function is $V_{IM}(\eta_1, \eta_2) := \frac{1}{2\alpha}\|\eta_1 - \eta_2\|^2$. The interconnection of closed-loop system (10) and internal model (16) is illustrated in Figure 1.

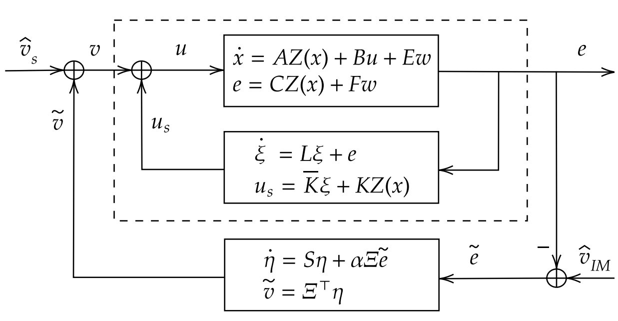

Fig. 1. Interconnection of closed-loop system (10) and internal model (16).

Theorem 2: Consider system (1), exosystem (2), and data set $\mathcal{DS}$. If there exist matrices $Y_1 \in \mathbb{R}^{T\times n_Z}$, $Y_2 \in \mathbb{R}^{T\times m}$, $Y_3 \in \mathbb{R}^{m\times m}$, $G_3 \in \mathbb{R}^{T\times m}$, and positive definite matrix $P_1 = \mathrm{diag}(P_{11}, P_{12}) \in \mathbb{R}^{n_Z\times n_Z}$ with $P_{11} \in \mathbb{R}^{n\times n}$ and $P_{12} \in \mathbb{R}^{(n_Z-n)\times(n_Z-n)}$, such that (15) holds, then the controller

$$\begin{aligned}\dot{\eta} &= S\eta - \alpha\Xi e \\ \dot{\xi} &= L\xi + e \\ u &= KZ(x) + \overline{K}\xi + \Xi^\top\eta - \widehat{K}e\end{aligned} \tag{17}$$

where $L = Y_3P_2^{-1}$, $K = U_0Y_1P_1^{-1}$, $\overline{K} = U_0Y_2P_2^{-1}$, any $\alpha > 0$, and any positive definite matrices $P_2 \in \mathbb{R}^{m\times m}$ and $\widehat{K} \in \mathbb{R}^{m\times m}$, solves the output regulation problem.

Proof: By Theorem 1, the designed controller (9) renders closed-loop system (10) incrementally passive with input v, output e, and regular storage function $V(\zeta_1, \zeta_2)$. As internal model (16) is also incrementally passive with input $\tilde{e}$, output $\tilde{v}$, and regular storage function $V_{IM}(\eta_1, \eta_2)$, the interconnection of (10) and (16) is incrementally passive with input $\hat{v} := \begin{bmatrix}\hat{v}_s^\top & \hat{v}_{IM}^\top\end{bmatrix}^\top$, output $\hat{e} := \begin{bmatrix}e^\top & \tilde{v}^\top\end{bmatrix}^\top$, and a regular storage function. Close the loop by the feedback

$$\hat{v} = \begin{bmatrix}\hat{v}_s \\ \hat{v}_{IM}\end{bmatrix} = \begin{bmatrix}-\widehat{K} & 0_{m\times m} \\ 0_{m\times m} & 0_{m\times m}\end{bmatrix}\begin{bmatrix}e \\ \tilde{v}\end{bmatrix} = \begin{bmatrix}-\widehat{K}e \\ 0_{m\times 1}\end{bmatrix} \tag{18}$$

for any $\widehat{K} \succ 0$. Thus, one obtain that $v = \tilde{v} + \hat{v}_s = \Xi^\top\eta - \widehat{K}e$ and the overall regulator (17). Following [12, Theorem 1], it can be proved that all solutions to the overall closed-loop system under the feedback (18) are bounded, and

$$\lim_{t\to+\infty}\begin{bmatrix}e(t) \\ \tilde{v}(t)\end{bmatrix}^\top \begin{bmatrix}-\widehat{K} & 0_{m\times m} \\ 0_{m\times m} & 0_{m\times m}\end{bmatrix}\begin{bmatrix}e(t) \\ \tilde{v}(t)\end{bmatrix} = 0,$$

which implies that $\lim_{t\to+\infty} e(t)^\top\widehat{K}e(t) = 0$. As $\widehat{K} \succ 0$, it holds that $\lim_{t\to+\infty} e(t) = 0$. Thus, the output regulation problem is solved by (17). ■

Discussion. In the case where the measured data is subject to noise, the relation of the data matrices becomes

$$\begin{aligned}X_1 &= AZ_0 + BU_0 + EW_0 + D_{10} \\ E_0 &= CZ_0 + FW_0 + D_{20}\end{aligned}$$

where D_{10} and D_{20} represent the influence of all measurement noise. This shows that, if the noise can be represented by the exogenous signal w, then it can be handled by the proposed approach. For a more general class of noise, to handle the resulting uncertainty in the data-based closed-loop representation, a robust version of passivation conditions (11) is difficult to derive due to condition (11b). Nevertheless, it is of importance to investigate how to deal with the uncertainty caused by noisy data in our future work.

Recall that one major advantage of output regulation via incremental passivity in the model-based setting is that the designs of the internal model and the stabilizer are independent. In particular, if the exosystem changes, only the internal model needs to be redesigned accordingly, and the stabilizer remains the same. In this work, since the data-based closed-loop representation (13) depends on the exogenous signal data, the design of controller (9) is dependent on the exosystem, but not on the internal model. This means that if the exosystem changes, conditions (15) characterizing the controller remain the same, but data set $\mathcal{DS}$ need to be recollected to update data-based closed-loop presentation. Nevertheless, it is still of interest to investigate how to construct a data-driven controller independent of the exosystem that makes the closed-loop system incrementally passive.

IV. An Example

To verify the proposed data-driven regulator, this section considers the output regulation of the Van der Pol oscillator, which has the dynamics

$$\begin{aligned}\dot{x}_1 &= x_2 + w_2\\ \dot{x}_2 &= -x_1 + (1 - x_1^2)x_2 + u\\ e &= x_2 - w_1,\end{aligned}$$

where exogenous signal w is generated by the exosystem

$$\dot{w} = \begin{bmatrix} 0 & 0.1\\ -0.1 & 0\end{bmatrix} w.$$

It is noted that the dynamics of the Van der Pol oscillator is only used for data collection. One can verify that Assumptions 1 and 2 are satisfied.

Let $Z(x) = \begin{bmatrix} x_1 & x_2 & x_1^2 x_2\end{bmatrix}^\top$ and conduct an experiment with $x(0) = [-0.1 \;\; 0.1]^\top$, $w(0) = [0 \;\; 0.1]^\top$, and $u = \sin(t)$ on the time interval $[0, 10]$. The data set is collected with sampling period 0.5s and the length of the data matrices is $T = 15$. Letting $P_2 = 1$ and solving the condition (15) in MATLAB using YALMIP [19] with MOSEK [20] gives $K = [-0.0196 \;\; -1.5095 \;\; 1.0000]$, $\overline{K} = -1.000$, and $L = -0.4971$. Setting $\alpha = 50$, $\Xi = [5 \;\; 0]^\top$, and $\widehat{K} = 1$, the overall regulator (17) is obtained. Simulation results of the error e from different initial conditions are illustrated in Figure 2, which shows that the errors converge to 0 in all cases.

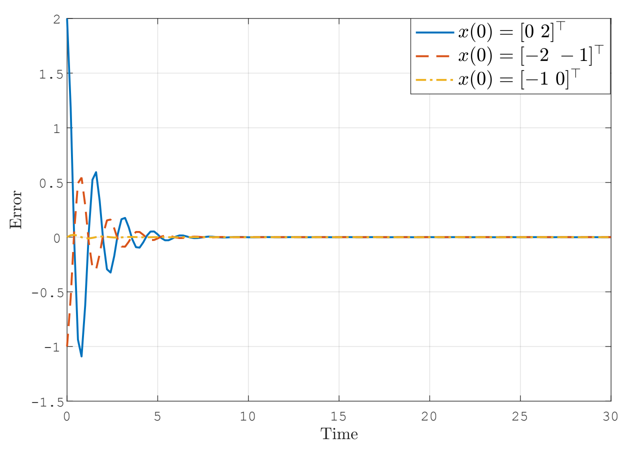

Fig. 2. Error e for different initial conditions.

V. Conclusion and Future Work

This work proposed a data-driven control design that achieves output regulation for a class of unknown nonlinear systems and a known linear exosystem. The design of the regulator contains two parts: a dynamic controller that makes the closed-loop system incrementally passive and an incrementally passive internal model. The dynamic controller is obtained by solving a set of data-based LMIs and is independent of the design of the internal model. Interconnecting the dynamic controller and the internal model leads to an effective data-driven output regulator. Future research will investigate output regulation with noisy data, more general unknown disturbances, and output feedback controllers.

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